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Agravity

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Abstract: We explore the possibility that the fundamental theory of nature does not contain any scale. This implies a renormalizable quantum gravity theory where the graviton kinetic term has 4 derivatives, and can be reinterpreted as gravity minus an anti-graviton. We compute the super-Planckian RGE of adimensional gravity coupled to a generic matter sector. The Planck scale and a flat space can arise dynamically at quantum level provided that a quartic scalar coupling and its β function vanish at the Planck scale. This is how the Higgs boson behaves for $M_h \approx 125\,\text{GeV}$ and $M_t \approx 171\,\text{GeV}$. Within agravity, inflation is a generic phenomenon: the slow-roll parameters are given by the β-functions of the theory, and are small if couplings are perturbative. The predictions $n_s \approx 0.967$ and $r \approx 0.13$ arise if the inflaton is identified with the Higgs of gravity. Furthermore, quadratically divergent corrections to the Higgs mass vanish: a small weak scale is natural and can be generated by agravity quantum corrections.

Keywords: Cosmology of Theories beyond the SM, Renormalization Group, Models of Quantum Gravity

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Contents

1 Introduction

We propose a general principle that leads to a renormalizable and predictive theory of quantum gravity where all scales are generated dynamically, where a small weak scale coexists with the Planck scale, where inflation is a natural phenomenon. The price to pay is a ghost-like anti-graviton state.

The general principle is: *nature does not possess any scale.* We start presenting how this principle is suggested by two recent experimental results, and next discuss its implementation and consequences.

1) Naturalness. In the past decades theorists assumed that Lagrangian terms with positive mass dimension (the Higgs mass M_h and the vacuum energy) receive big power-divergent quantum corrections, as suggested by Wilsonian computation techniques that attribute physical meaning to momentum shells of loop integrals [1, 2]. According to this point of view, a modification of the SM at the weak scale is needed to make quadratically divergent corrections to M_h^2 naturally small. Supersymmetry seems the most successful possibility, but naturalness got increasingly challenged by the non-observation of any new physics that keeps the weak scale naturally small [3–6].

The naturalness problem can be more generically formulated as a problem of the effective theory ideology, according to which nature is described by a non-renormalizable Lagrangian of the form

$$\mathscr{L} \sim \Lambda^4 + \Lambda^2 |H|^2 + \lambda |H|^4 + \frac{|H|^6}{\Lambda^2} + \cdots \tag{1.1}$$

where, for simplicity, we wrote only the Higgs potential terms. The assumption that $\Lambda \gg M_h$ explains why at low energy $E \sim M_h$ we only observe those terms not suppressed by Λ: the renormalizable interactions. Conservation of baryon number, lepton number, and other successful features of the Standard Model indicate a large $\Lambda \gtrsim 10^{16}\,\text{GeV}$. In this context, gravity can be seen as a non-renormalizable interaction suppressed by $\Lambda \sim M_{\rm Pl} = 1.22\ 10^{19}\,\text{GeV}$.

However, this scenario also leads to the expectation that particles cannot be light unless protected by a symmetry. The Higgs mass should be $M_h^2 \sim \Lambda^2$ and the vacuum energy should be $V \sim \Lambda^4$. In nature, they are many orders of magnitude smaller, and no protection mechanism is observed so far.

We assume that this will remain the final experimental verdict and try to derive the theoretical implications.

Nature is maybe telling us that both super-renormalizable terms and non-renormalizable terms vanish and that only adimensional interactions exists.

2) Inflation. Cosmological observations suggest inflation with a small amount of anisotropies. However, this is a quite unusual outcome of quantum field theory: it requires special models with flat potentials, and often field values above the Planck scale. Let us discuss this issue in the context of Starobinsky-like inflation models [7–19]: a class of inflation models favoured by Planck data [20]. Such models can be described in terms of one scalar S (possibly identified with the Higgs H) with a potential $V(S)$ and a coupling to gravity $-\frac{1}{2}f(S)R$. Going to the Einstein frame (i.e. making field redefinitions such that the graviton kinetic term R gets its canonical coefficient) the potential gets rescaled into $V_E = \bar{M}_{\rm Pl}^4 V/f^2$, where $\bar{M}_{\rm Pl} = M_{\rm Pl}/\sqrt{8\pi} = 2.4\ 10^{18}\,\text{GeV}$ is the reduced Planck mass. Special assumptions such as $V(S) \propto f(S)^2$ make the Einstein-frame potential V_E flat at $S \gg M_{\rm Pl}$, with predictions compatible with present observations [7–19]. However this flattening is the result of a fine-tuning: in presence of generic Planck-suppressed operators V and f and thereby V_E are generic functions of $S/M_{\rm Pl}$.

Nature is maybe telling us that V_E becomes flat at $S \gg M_{\rm Pl}$ because only adimensional terms exists.

The principle. These observations vaguely indicate that nature prefers adimensional terms, so that ideas along these lines are being discussed in the literature [21–46].

We propose a simple concrete principle: *the fundamental theory of nature does not possess any mass or length scale* and thereby only contains 'renormalizable' interactions — i.e. interactions with dimensionless couplings.

This simple assumption solves the two issues above and has strong consequences.

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First, a quasi-flat inflationary potential is obtained because the only adimensional potential is a quartic term $V(S) = \lambda_S|S|^4$ and the only adimensional scalar/gravity coupling is $-\xi_S|S|^2R$, so that $V_E = \bar{M}_{\rm Pl}^4(\lambda_S|S|^4)/(\xi_S|S|^2)^2 = \bar{M}_{\rm Pl}^4\lambda_S/\xi_S^2$ is flat at tree level. At quantum level the parameters λ_S and ξ_S run, such that the slow-roll parameters are the beta-functions of the theory, as discussed in section 4.

Second, power divergences vanish just because of dimensional reasons: they would have mass dimension, but there are no masses. Vanishing of quadratic divergences leads to a modified version of naturalness, where the weak scale can be naturally small even in absence of new physics at the weak scale [47] designed to protect the Higgs mass, such as supersymmetry or technicolor.

In this context scale invariance is just an accidental symmetry, present at tree level because there are no masses. Just like baryon number (a well known accidental symmetry of the Standard Model), scale invariance is broken by quantum corrections.[1] Then, the logarithmic running of adimensional couplings can generate exponentially different scales via dimensional transmutation. This is how the QCD scale arises.

The goal of this paper is exploring if the Planck scale and the electro-weak scale can arise in this context.

The theory. The adimensional principle leads us to consider renormalizable theories of quantum gravity described by actions of the form:

$$S = \int d^4x \sqrt{|\det g|}\left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \mathscr{L}_{\rm SM}^{\rm adim} + \mathscr{L}_{\rm BSM}^{\rm adim}\right]. \tag{1.2}$$

The first two terms, suppressed by the adimensional gravitational couplings f_0 and f_2, are the graviton kinetic terms.[2] The third term, $\mathscr{L}_{\rm SM}^{\rm adim}$, is the adimensional part of the usual Standard Model (SM) Lagrangian:

$$\mathscr{L}_{\rm SM}^{\rm adim} = -\frac{F_{\mu\nu}^2}{4g^2} + \bar{\psi} i \not{D}\psi + |D_\mu H|^2 - (yH\psi\psi + \text{h.c.}) - \lambda_H|H|^4 - \xi_H|H|^2 R \tag{1.3}$$

where H is the Higgs doublet. The last term, $\mathscr{L}_{\rm BSM}^{\rm adim}$, describes possible new physics Beyond the SM (BSM). For example adding a scalar singlet S one would have

$$\mathscr{L}_{\rm BSM}^{\rm adim} = |D_\mu S|^2 - \lambda_S|S|^4 + \lambda_{HS}|S|^2|H|^2 - \xi_S|S|^2 R. \tag{1.4}$$

We ignore topological terms. Non renormalizable terms, the Higgs mass term $\frac{1}{2}M_h^2|H|^2$ and the Einstein-Hilbert term $-M_{\rm Pl}^2 R/16\pi$ are not present in the agravity Lagrangian, because they need dimensionful parameters. The Planck mass can be generated dynamically if, at quantum level, S gets a vacuum expectation value such that $\xi_S\langle S\rangle^2 = M_{\rm Pl}^2/16\pi$. The adimensional parameters of a generic agravity in 3+1 dimensions theory are:

1. the two gravitational couplings f_0 and f_2;

[1]Other attempts along similar lines assume that scale or conformal invariance are exact symmetries at quantum level. However, computable theories do not behave in this way.

[2]The second term is also known as 'conformal gravity'.

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2. quartic scalar couplings λ;

3. scalar/scalar/graviton couplings ξ;

4. gauge couplings g;

5. Yukawa couplings y.[3]

The graviton $g_{\mu\nu}$ has dimension zero, and eq. (1.2) is the most generic adimensional action compatible with general relativistic invariance. The purely gravitational action just contains two terms: the squared curvature R^2 and the Weyl term $\frac{1}{3}R^2 - R_{\mu\nu}^2$. They are suppressed by two constants, f_0^2 and f_2^2, that are the true adimensional gravitational couplings, in analogy to the gauge couplings g that suppress the kinetic terms for vectors, $-\frac{1}{4}F_{\mu\nu}^2/g^2$. Thereby, the gravitational kinetic terms contain 4 derivatives, and the graviton propagator is proportional to $1/p^4$. Technically, this is how gravity becomes renormalizable. In presence of an induced Planck mass, the graviton propagator becomes

$$\frac{1}{M_2^2 p^2 - p^4} = \frac{1}{M_2^2}\left[\frac{1}{p^2} - \frac{1}{p^2 - M_2^2}\right] \tag{1.5}$$

giving rise to a massless graviton with couplings suppressed by the Planck scale, and to a spin-2 state with mass $M_2^2 = \frac{1}{2}f_2^2 \bar{M}_{\rm Pl}^2$ and negative norm. Effectively, it behaves as an anti-gravity Pauli-Villars regulator for gravity [48]. The Lagrangian can be rewritten in a convoluted form where this is explicit [49–53] (any field with quartic derivatives can be rewritten in terms of two fields with two derivatives). The f_0 coupling gives rise to a spin-0 graviton with positive norm and mass $M_0^2 = \frac{1}{2}f_0^2 \bar{M}_{\rm Pl}^2 + \cdots$. Experimental bounds are satisfied as long as $M_{0,2} \gtrsim$ eV.

At classical level, theories with higher derivative suffer the Ostrogradski instability: the Hamiltonian is not bounded from below [54–57]. At quantum level, creation of negative energy can be reinterpreted as destruction of positive energy: the Hamiltonian becomes positive, but some states have negative norm and are called 'ghosts' [58]. This quantization choice amounts to adopt the same $i\epsilon$ prescription for the graviton and for the anti-graviton, such that the cancellation that leads to renormalizability takes place.

We do not address the potential problem of a negative contribution to the cross-section for producing an odd number of anti-gravitons with mass M_2 above their kinematical threshold. Claims in the literature are controversial [59–73]. Sometimes in physics we have the right equations before having their right interpretation. In such cases the strategy that pays is: proceed with faith, explore where the computations lead, if the direction is right the problems will disappear.

We here compute the one loop quantum corrections of agravity, to explore its quantum behaviour. Can the Planck scale be dynamically generated? Can the weak scale be dynamically generated?

[3]The list would be much shorter for $d \neq 4$. Gauge couplings are adimensional only at $d = 4$. Adimensional scalar self-interactions exist at $d = \{3, 4, 6\}$. Adimensional interactions between fermions and scalars exist at $d = \{3, 4\}$. Adimensional fermion interactions exist at $d = 2$.

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2 Quantum agravity

The quantum corrections to a renormalizable theory are mostly encoded in the renormalization group equations (RGE) for its parameters. Ignoring gravity, a generic adimensional theory of real scalars ϕ_a, Weyl fermions ψ_j and vectors V_A can be written as

$$\mathscr{L} = -\frac{1}{4}(F^A_{\mu\nu})^2 + \frac{(D_\mu \phi_a)^2}{2} + \bar\psi_j i \not{D} \psi_j - (Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d \tag{2.1}$$

and its RGE have been computed up to 2 loops [74–76]. We here compute the one-loop β functions $\beta_p \equiv dp/d\ln\bar\mu$ of all parameters p of a generic agravity theory, obtained adding to (2.1) the generic scalar/graviton coupling

$$-\frac{\xi_{ab}}{2}\phi_a\phi_b R \tag{2.2}$$

as well as the graviton kinetic terms of eq. (1.2) and the minimal gravitational interactions demanded by general relativity.

Previous partial computations found contradictory results and have been performed with ad hoc techniques in a generic background. We instead follow the usual Feynman diagrammatic approach,[4] expanding around a flat background (the background is just an infra-red property which does not affect ultra-violet divergences).

2.1 The graviton propagator

Eq. (1.2) is the most general action containing adimensional powers of the fundamental fields. Concerning the purely gravitational sector, apparently there are extra terms such as D^2R or $R^2_{\mu\nu\alpha\beta}$. However the first one is a pure derivative; and the second one can be eliminated using the topological identity

$$R^2_{\alpha\beta\mu\nu} - 4R^2_{\mu\nu} + R^2 = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta}_{\mu\nu}R^{\gamma\delta}_{\rho\sigma} \simeq 0. \tag{2.3}$$

The combination suppressed by f_2^2 in eq. (1.2) is the square of the Weyl or conformal tensor, defined by subtracting all traces to the Riemann tensor:

$$W_{\mu\nu\alpha\beta} \equiv R_{\mu\nu\alpha\beta} + \frac{1}{2}(g_{\mu\beta}R_{\nu\alpha} - g_{\mu\alpha}R_{\nu\beta} + g_{\nu\alpha}R_{\mu\beta} - g_{\nu\beta}R_{\mu\alpha}) + \frac{1}{6}(g_{\mu\alpha}g_{\nu\beta} - g_{\nu\alpha}g_{\mu\beta})R. \tag{2.4}$$

Indeed

$$\frac{1}{2}W^2_{\alpha\beta\mu\nu} = \frac{1}{2}R^2_{\alpha\beta\mu\nu} - R^2_{\mu\nu} + \frac{1}{6}R^2 \simeq R^2_{\mu\nu} - \frac{1}{3}R^2. \tag{2.5}$$

We expand around the flat-space metric $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ such that

$$R\sqrt{|\det g|} = (\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2)h_{\mu\nu} + \frac{1}{4}(h_{\mu\nu}\partial^2 h_{\mu\nu} - h_{\alpha\alpha}\partial^2 h_{\alpha\alpha} + 2h_{\mu\nu}\partial_\mu\partial_\nu h_{\alpha\alpha} - 2h_{\mu\alpha}\partial_\alpha\partial_\beta h_{\beta\mu}) + \cdots . \tag{2.6}$$

[4] One of the authors (A. Salvio) adapted the public tools [77, 78]; the other author (A. Strumia) employed his own equivalent codes.

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To quantise the theory we follow the Fadeev-Popov procedure adding the gauge fixing term

$$S_{\rm gf} = -\frac{1}{2\xi_g}\int d^4x\ f_\mu \partial^2 f_\mu, \qquad f_\mu = \partial_\nu h_{\mu\nu}. \tag{2.7}$$

We choose a non-covariant term quadratic in $h_{\mu\nu}$, such that gauge fixing does not affect the graviton couplings. At quadratic level the purely gravitational action is

$$S = \frac{1}{2}\int d^4k\ k^4\ h_{\mu\nu}\Big[-\frac{1}{2f_2^2}P^{(2)}_{\mu\nu\rho\sigma} + \frac{1}{f_0^2}P^{(0)}_{\mu\nu\rho\sigma} + \frac{1}{2\xi_g}(P^{(1)}_{\mu\nu\rho\sigma} + 2P^{(0w)}_{\mu\nu\rho\sigma})\Big]h_{\rho\sigma} \tag{2.8}$$

where

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}T_{\mu\rho}T_{\nu\sigma} + \frac{1}{2}T_{\mu\sigma}T_{\nu\rho} - \frac{T_{\mu\nu}T_{\rho\sigma}}{d-1} \tag{2.9a}$$

$$P^{(1)}_{\mu\nu\rho\sigma} = \frac{1}{2}(T_{\mu\rho}L_{\nu\sigma} + T_{\mu\sigma}L_{\nu\rho} + T_{\nu\rho}L_{\mu\sigma} + T_{\nu\sigma}L_{\mu\rho}) \tag{2.9b}$$

$$P^{(0)}_{\mu\nu\rho\sigma} = \frac{T_{\mu\nu}T_{\rho\sigma}}{d-1} \tag{2.9c}$$

$$P^{(0w)}_{\mu\nu\rho\sigma} = L_{\mu\nu}L_{\rho\sigma} \tag{2.9d}$$

are projectors over spin-2, spin-1 and spin-0 components of $h_{\mu\nu}$, written in terms of $d = 4-2\epsilon$, $T_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu/k^2$ and $L_{\mu\nu} = k_\mu k_\nu/k^2$. Their sum equals unity: $(P^{(2)} + P^{(1)} + P^{(0)} + P^{(0w)})_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\sigma}\eta_{\rho\nu})$. Inverting the kinetic term of eq. (2.8) we find the graviton propagator

$$D_{\mu\nu\,\rho\sigma} = \frac{i}{k^4}\Big[-2f_2^2P^{(2)}_{\mu\nu\rho\sigma} + f_0^2P^{(0)}_{\mu\nu\rho\sigma} + 2\xi_g(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2}P^{(0w)}_{\mu\nu\rho\sigma})\Big]. \tag{2.10}$$

Gravitational ghost couplings. One needs to path-integrate over Fadeev-Popov ghosts η_α and $\bar\eta_\mu$ with action

$$S_{\rm ghost} = \int d^4x\, d^4y\, \bar\eta_\mu(x)\frac{\delta f_\mu(x)}{\delta\xi_\alpha(y)}\eta_\alpha(y). \tag{2.11}$$

By performing an infinitesimal transformation $x_\mu \to x'_\mu = x_\mu + \xi_\mu(x)$ one finds the transformation of $h_{\mu\nu}$ at first order in ξ_μ and at a fixed point x_μ:

$$\delta h_{\mu\nu} = -(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu) - (h_{\alpha\mu}\partial_\nu + h_{\alpha\nu}\partial_\mu + (\partial_\alpha h_{\mu\nu}))\xi_\alpha. \tag{2.12}$$

The ghost action then is

$$S_{\rm ghost} = \int d^4x\,\{\partial_\alpha\bar\eta_\mu(\partial_\alpha\eta_\mu + \partial_\mu\eta_\alpha) + \partial_\nu\bar\eta_\mu[h_{\alpha\mu}\partial_\nu\eta_\alpha + h_{\alpha\nu}\partial_\mu\eta_\alpha + (\partial_\alpha h_{\mu\nu})\eta_\alpha]\}. \tag{2.13}$$

In order to verify gravitational gauge-independence we will perform all computations using a more general gauge fixing, given by eq. (2.7) with $f_\mu = \partial_\nu(h_{\mu\nu} - c_g\frac{1}{2}\eta_{\mu\nu}h_{\alpha\alpha})$.[5]

We fix gauge invariance of vectors V_μ adding to the Lagrangian the standard ξ-gauge term $-f_V^2/2(1-\xi_V)$ with $f_V = \partial_\mu V_\mu$, such that the vector propagator is $i(-g_{\mu\nu} + \xi_V k_\mu k_\nu/k^2)/k^2$. Such term does not depend on gravitons, so that the gauge-invariances of spin-2 and spin-1 particles are fixed independently.

[5]The graviton propagator becomes

$$D_{\mu\nu\,\rho\sigma} = \frac{i}{k^4}\Big[-2f_2^2P^{(2)} + f_0^2\Big(P^{(0)} + \frac{\sqrt{3}c_gT^{(0)}}{2-c_g} + \frac{3c_g^2P^{(0w)}}{(2-c_g)^2}\Big) + 2\xi_g\Big(P^{(1)} + \frac{2P^{(0w)}}{(2-c_g)^2}\Big)\Big]_{\mu\nu\rho\sigma}$$

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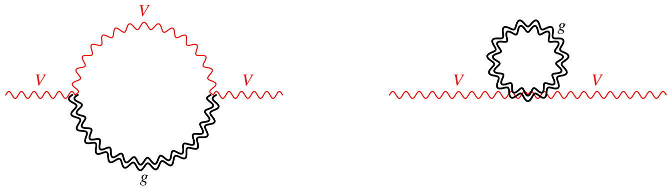

Figure 1. Gravitational corrections to the running of the gauge couplings.

2.2 Wave-function renormalizations

We write the gauge-covariant derivatives as $D_\mu \phi_a = \partial_\mu \phi_a + i\theta^A_{ab} V^A_\mu \phi_b$ when acting on scalars and as $D_\mu \psi_j = \partial_\mu \psi_j + i t^A_{jk} V^A_\mu \psi_k$ when acting on fermions (the gauge couplings are contained in the matrices θ^A and t^A). In this paper we adopt dimensional regularization and the modified minimal subtraction renormalization scheme $\overline{\rm MS}$, with energy scale $\bar\mu$. The anomalous dimensions γ of the fields are defined as $\gamma = \frac{1}{2} d \ln Z / d \ln \bar\mu$ in terms of the wave-function renormalization constants $Z = 1 + \delta Z$. Gravitational couplings of fermions are derived following the formalism of [79, 80]. By computing the one-loop corrections to the matter kinetic terms we find the one-loop anomalous dimension of scalars

$$\begin{aligned}(4\pi)^2 \gamma^S_{ab} = &\ {\rm Tr}\, Y^a Y^{\dagger b} - (2+\xi_V)\theta^A_{ac}\theta^A_{cb} + \\ &+ f_0^2 \left(\frac{3\,(c_g-1)^2}{4\,(c_g-2)^2}\delta_{ab} + \frac{3 c_g \xi_{ab}}{c_g-2} \right) + \frac{3c_g^2 - 12 c_g + 13}{4(c_g-2)^2}\xi_g \delta_{ab}\end{aligned} \tag{2.14}$$

and of fermions

$$\begin{aligned}(4\pi)^2 \gamma^F = &\ \frac{1}{2} Y^a Y^{\dagger a} + (1-\xi_V) t^A t^A + \\ &- \frac{25}{16} f_2^2 + f_0^2 \frac{22 - 16 c_g + 7 c_g^2}{16(c_g-2)^2} + 3\xi_g \frac{22 - 20 c_g + 5 c_g^2}{16(c_g-2)^2}\end{aligned} \tag{2.15}$$

where the first lines show the well known matter terms [74–76], and the second lines show the new terms due to agravity, as computed for generic gauge-fixing parameters ξ_g and c_g. Vectors are discussed in the next section.

2.3 RGE for the gauge couplings

The one-loop correction to the kinetic term of vectors describes the RGE for the gauge couplings. The two new gravitational Feynman diagrams are shown in figure 1. They have

where $T^{(0)}_{\mu\nu\rho\sigma} = (T_{\mu\nu} L_{\rho\sigma} + L_{\mu\nu} T_{\rho\sigma})/\sqrt{d-1}$. The gauge of eq. (2.7) corresponds to $c_g = 0$; the gauge used in [48] corresponds to $c_g = 1$, which is a convenient choice in Einstein gravity. In the generic gauge $c_g \neq 0$ the ghost Lagrangian is

$$\bar\eta_\mu \left[\partial^2 \eta_{\alpha\mu} + (1-c_g)\partial_\alpha \partial_\mu + \partial_\nu h_{\alpha\mu}\partial_\nu + \partial_\nu h_{\alpha\nu}\partial_\mu + \partial_\nu(\partial_\alpha h_{\mu\nu}) - c_g \partial_\mu h_{\alpha\nu}\partial_\nu - \frac{c_g}{2}\partial_\mu(\partial_\alpha h_{\nu\nu}) \right] \eta_\alpha$$

so that the ghost propagator is

$$-\frac{i}{k^2}\left[\eta_{\mu\nu} + \frac{c_g - 1}{2 - c_g}\frac{k_\mu k_\nu}{k^2} \right].$$

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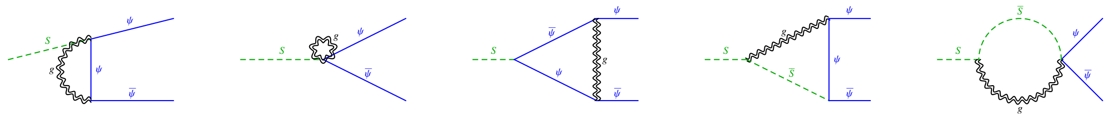

Figure 2. Gravitational corrections to the running of the Yukawa couplings.

opposite logarithmic divergences, so that their sum is finite. This cancellation was noticed in [81] in more general contexts, and seems due to the fact that gravitons have no gauge charge.[6]

In conclusion, the one-loop RGE for the gauge couplings do not receive any gravitational correction and the usual one-loop RGE for the gauge couplings remain valid also above the Planck scale. Within the SM, the hypercharge gauge couplings hits a Landau pole at $\bar{\mu} \sim 10^{41}$ GeV.

2.4 RGE for the Yukawa couplings

Summing the diagrams of figure 2 gives a divergent correction that depends on the gauge fixing parameters and on the scalar/graviton couplings ξ. Adding the fermion and scalar wave function renormalizations of section 2.2 such dependencies cancel. We find the one-loop RGE:

$$(4\pi)^2 \frac{dY^a}{d\ln\bar{\mu}} = \frac{1}{2}(Y^{\dagger b}Y^bY^a + Y^aY^{\dagger b}Y^b) + 2Y^bY^{\dagger a}Y^b + \\ +Y^b\,\mathrm{Tr}(Y^{\dagger b}Y^a) - 3\{C_{2F}, Y^a\} + \frac{15}{8}f_2^2Y^a. \tag{2.16}$$

where $C_{2F} = t^At^A$, and the latter term is the contribution due to agravity and has the opposite sign with respect to the analogous multiplicative term due to gauge interactions. Specializing eq. (2.16) to the SM, we find the one-loop RGE for the top quark Yukawa coupling:

$$(4\pi)^2 \frac{dy_t}{d\ln\bar{\mu}} = \frac{9}{2}y_t^3 + y_t\left(\frac{15}{8}f_2^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2\right). \tag{2.17}$$

We know of no previous computation in the literature.

2.5 RGE for the quartic couplings

Tens of Feynman diagrams contribute to the scalar 4-point function at one loop. After summing them and taking into account the scalar wave-function renormalization of eq. (2.14)

[6] Other authors try to interpret ambiguous power-divergent corrections to gauge couplings from Einstein gravity as gravitational power-running RGE, with possible physical consequences such as an asymptotically free hypercharge [82]. We instead compute the usual unambiguous logarithmic running, in the context of theories where power divergences vanish.

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the gauge dependence disappears and we find the one-loop RGE:

$$(4\pi)^2 \frac{d\lambda_{abcd}}{d\ln\bar{\mu}} = \sum_{\text{perms}} \left[\frac{1}{8}\lambda_{abef}\lambda_{efcd} + \frac{3}{8}\{\theta^A,\theta^B\}_{ab}\{\theta^A,\theta^B\}_{cd} - \operatorname{Tr} Y^a Y^{\dagger b} Y^c Y^{\dagger d} + \right.$$

$$+\frac{5}{8}f_2^4\xi_{ab}\xi_{cd} + \frac{f_0^4}{8}\xi_{ae}\xi_{cf}(\delta_{eb}+6\xi_{eb})(\delta_{fd}+6\xi_{fd}) + \tag{2.18}$$

$$\left. + \frac{f_0^2}{4!}(\delta_{ae}+6\xi_{ae})(\delta_{bf}+6\xi_{bf})\lambda_{efcd} \right] + \lambda_{abcd}\left[\sum_k (Y_2^k - 3C_2^k) + 5f_2^2 \right],$$

where the first sum runs over the 4! permutations of $abcd$ and the second sum over $k = \{a,b,c,d\}$, with Y_2^k and C_2^k defined by

$$\operatorname{Tr}(Y^{\dagger a}Y^b) = Y_2^a \delta^{ab}, \qquad \theta^A_{ac}\theta^A_{cb} = C^a_{2S}\delta_{ab}. \tag{2.19}$$

RGE for quartics have been computed in the literature in some models [83]; we find a simpler result where the f_2^2 term does not depend on ξ and where the f_0^2 term vanishes if $\xi_{ab} = -\delta_{ab}/6$.

Specializing our general eq. (2.18) to the case of the SM Higgs doublet plus a complex scalar singlet S with action given by eq. (1.2), the RGE become:

$$(4\pi)^2 \frac{d\lambda_S}{d\ln\bar{\mu}} = 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2}\left(5f_2^4 + f_0^4(1+6\xi_S)^2\right) + \lambda_S\left(5f_2^2 + f_0^2(1+6\xi_S)^2\right), \tag{2.20}$$

$$(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\bar{\mu}} = -\xi_H\xi_S\left(5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)\right) - 4\lambda_{HS}^2 + \lambda_{HS}\Bigg(8\lambda_S + 12\lambda_H + 6y_t^2 +$$

$$+5f_2^2 + \frac{f_0^2}{6}\left[(6\xi_S+1)^2 + (6\xi_H+1)^2 + 4(6\xi_S+1)(6\xi_H+1)\right]\Bigg), \tag{2.21}$$

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\bar{\mu}} = \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2}\left(5f_2^4 + f_0^4(1+6\xi_H)^2\right) +$$

$$+\lambda_H\left(5f_2^2 + f_0^2(1+6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right). \tag{2.22}$$

2.6 RGE for the scalar/graviton couplings

We extract the one-loop RGE for the ξ parameters from the one-loop correction to the graviton$_{\mu\nu}$/scalar/scalar vertex. At tree level, two different Lagrangian terms contribute to such vertex:

a) one contribution comes from the scalar kinetic term (when the graviton momentum is vanishing and the two scalars have momenta $\pm p$);

b) one contribution comes from ξ terms (when the graviton has a non-vanishing momentum k and one scalar has zero momentum).

We compute both contributions. We find that the correction a) reproduces the scalar wave-function renormalization already computed in section 2.2, including the correct tensorial structure $\frac{1}{2}p^2\eta_{\mu\nu} - p_\mu p_\nu$, provided that the graviton field is renormalized as follows:

$$h_{\mu\nu} \to \frac{1}{\sqrt{Z_{TL}}}\left(h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h_{\alpha\alpha}\right) + \frac{1}{\sqrt{Z_T}}\frac{1}{4}\eta_{\mu\nu}h_{\alpha\alpha} \tag{2.23}$$

The wave-function renormalization Z_T and Z_{TL} differ because we used a simple gravitational gauge-fixing term that breaks general relativity but respects special relativity: thereby distinct representations of the Lorentz group (the trace and the traceless part of $h_{\mu\nu}$), get different renormalizations. We find the one-loop results

$$Z_T = 1 + \frac{1}{(4\pi)^2\epsilon}\frac{12c_g - 13 - 3c_g^2}{(c_g-2)^2}\xi_g, \tag{2.24}$$

$$Z_{TL} = 1 + \frac{1}{(4\pi)^2\epsilon}\left[\frac{10}{9}\frac{c_g-4}{c_g-2}f_2^2 - \frac{2}{9}\frac{4-3c_g+2c_g^2}{(c_g-2)^2)}f_0^2 - \frac{2}{3}\frac{9-8c_g+2c_g^2}{(c_g-2)^2}\xi_g\right]. \tag{2.25}$$

We verified that we find the same Z_{TL} by computing the graviton renormalization constant from the one-loop graviton/vector/vector vertex.

Next, we compute the correction b). After adding to it the scalar and the graviton wave-function renormalization, we find that the total correction has the correct tensorial structure $k^2\eta_{\mu\nu} - k_\mu k_\nu$ and corresponds to the following one-loop RGE for the ξ parameters:

$$\begin{aligned}(4\pi)^2\frac{d\xi_{ab}}{d\ln\bar\mu} &= \frac{1}{6}\lambda_{abcd}\,(6\xi_{cd}+\delta_{cd}) + (6\xi_{ab}+\delta_{ab})\sum_k\left[\frac{Y_2^k}{3} - \frac{C_{2S}^k}{2}\right] + \\ &\quad - \frac{5f_2^4}{3f_0^2}\xi_{ab} + f_0^2\xi_{ac}\left(\xi_{cd}+\frac{2}{3}\delta_{cd}\right)(6\xi_{db}+\delta_{db}).\end{aligned} \tag{2.26}$$

RGE for the ξ term have been computed in the literature in some models [83–86]; we find different and simpler gravitational terms.

Specialising our general eq. (2.6) to the case of the SM Higgs doublet plus a complex scalar singlet S with action given by eq. (1.2), the RGE become:

$$(4\pi)^2\frac{d\xi_S}{d\ln\bar\mu} = (1+6\xi_S)\frac{4}{3}\lambda_S - \frac{2\lambda_{HS}}{3}(1+6\xi_H) + \frac{f_0^2}{3}\xi_S(1+6\xi_S)(2+3\xi_S) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_S, \tag{2.27}$$

$$\begin{aligned}(4\pi)^2\frac{d\xi_H}{d\ln\bar\mu} &= (1+6\xi_H)\left(2y_t^2 - \frac{3}{4}g_2^2 - \frac{3}{20}g_1^2 + 2\lambda_H\right) - \frac{\lambda_{HS}}{3}(1+6\xi_S) + \\ &\quad + \frac{f_0^2}{3}\xi_H(1+6\xi_H)(2+3\xi_H) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_H.\end{aligned} \tag{2.28}$$

2.7 RGE for the agravitational couplings

The RGE for the couplings f_0, f_2 are computed summing the one loop corrections to the graviton kinetic term at 4th order in the external momentum k from: a) the graviton rainbow and seagull diagrams; b) the gravitational ghost; c) the graviton wave function renormalization of eq. (2.23). After combining all these ingredients we find a tensorial structure equal to the structure of the graviton kinetic term of eq. (2.8) that thereby can be interpreted as a renormalization of f_0 and f_2. Adding also the matter contributions (that separately have the correct tensorial structure) we obtain:

$$(4\pi)^2\frac{df_2^2}{d\ln\bar\mu} = -f_2^4\left(\frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60}\right), \tag{2.29a}$$

$$(4\pi)^2\frac{df_0^2}{d\ln\bar\mu} = \frac{5}{3}f_2^4 + 5f_2^2f_0^2 + \frac{5}{6}f_0^4 + \frac{f_0^4}{12}(\delta_{ab}+6\xi_{ab})(\delta_{ab}+6\xi_{ab}). \tag{2.29b}$$

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Here N_V, N_f, N_s are the number of vectors, Weyl fermions and real scalars. In the SM $N_V = 12$, $N_f = 45$, $N_s = 4$. Unlike in the gauge case, all the contributions to the RGE of f_2 have the same sign, such that f_2 is always asymptotically free. This result agrees with [87] (see [81, 84, 85, 88] for results with different signs). The matter contributions had been computed in [89–91] leading to the concept of 'induced gravity' [92], which in our language corresponds to the RGE running of f_2. Concerning the pure gravitational effect, agravity differs from gravity. The coupling f_0 is asymptotically free only for $f_0^2 < 0$ which leads to a tachionic instability $M_0^2 < 0$.

3 Dynamical generation of the Planck scale

Having determined the quantum behaviour of agravity, we can now study if the Planck scale can be generated dynamically. The possible ways are:

a) **Non-perturbative**: the couplings f_2 or ξ become non-perturbative when running down to low energy: the Planck scale can be generated in a way similar to how the QCD scale is generated.

b) **Perturbative**: a quartic λ_S runs in such a way that S gets a vacuum expectation value $\xi\langle S\rangle^2 = \bar{M}_{\rm Pl}^2/2$.

We focus on the second, perturbative, mechanism. In the usual Coleman-Weinberg case, S acquires a vev if its quartic λ_S becomes negative when running down to low energy, and $\langle S\rangle$ is roughly given by the RGE scale at which $\lambda(\bar{\mu})$ becomes negative. This can be understood by noticing that the quantum effective potential is roughly given by $V_{\rm eff} = \lambda_S(\bar{\mu} \approx S)|S|^4$. This means that the vacuum energy is always negative, in contrast to the observed near-vanishing vacuum energy.

In the gravitational case the situation is different, precisely because the effective Lagrangian contains the $-\frac{1}{2}f(S)R$ term, that should generate the Planck scale. The field equation for the scalar S in the homogeneous limit is

$$V' + \frac{f'(S)}{2}R = 0 \tag{3.1}$$

and the (trace of the) gravitational equation is

$$fR + 4V = \mathcal{O}(R^2/f_{0,2}^2) \tag{3.2}$$

where, around the phenomenologically desired flat-space solution, we can neglect the R^2 term with respect to the induced Einstein term. By eliminating R we obtain the minimum equation for S:

$$V' - \frac{2f'}{f}V = 0 \tag{3.3}$$

or, equivalently,

$$V_E' = 0 \qquad \text{where} \qquad V_E = \bar{M}_{\rm Pl}^4 \frac{V}{f^2} \tag{3.4}$$

is called Einstein-frame potential because eq. (3.4) can be obtained by performing a field redefinition $g^E_{\mu\nu} = g_{\mu\nu} \times f/\bar{M}^2_{\rm Pl}$ such that the coefficient of R_E in the Lagrangian has the canonical Einstein value. Under this transformation[7] the Lagrangian for the modulus of the scalar $|S| = s/\sqrt{2}$ becomes

$$\sqrt{\det g}\left[\frac{(\partial_\mu s)^2}{2} - \frac{f}{2}R - V\right] = \sqrt{\det g_E}\left[K\frac{(\partial_\mu s)^2}{2} - \frac{\bar{M}^2_{\rm Pl}}{2}R_E - V_E\right]. \tag{3.5}$$

where

$$K = \bar{M}^2_{\rm Pl}\left(\frac{1}{f} + \frac{3f'^2}{2f^2}\right). \tag{3.6}$$

The non-canonical factor K in the kinetic term for s can be reabsorbed by defining a canonically normalised Einstein-frame scalar $s_E(s)$ as $ds_E/ds = \sqrt{K}$ such that the Lagrangian becomes

$$\sqrt{\det g_E}\left[\frac{(\partial_\mu s_E)^2}{2} - \frac{\bar{M}^2_{\rm Pl}}{2}R_E - V_E(s_E)\right]. \tag{3.7}$$

In the agravity scenario $f(S)$ is approximatively given by $f(S) = \xi_S(\bar{\mu} \approx s)s^2$ and V_S by $\lambda_S(\bar{\mu} \approx s)s^4/4$. Thereby the Einstein-frame potential is given by

$$V_E(S) = \frac{\bar{M}^4_{\rm Pl}}{4}\frac{\lambda_S(s)}{\xi^2_S(s)} \tag{3.8}$$

and the vacuum equation is

$$\frac{\beta_{\lambda_S}(s)}{\lambda_S(s)} - 2\frac{\beta_{\xi_S}(s)}{\xi_S(s)} = 0 \tag{3.9}$$

where $\beta_p = dp/d\ln_\mu$ are the β functions of the couplings p. This equation is significantly different from the analogous equation of the usual non-gravitational Coleman-Weinberg mechanism, which is $\lambda_S(s) = 0$.

Furthermore, we want a nearly-vanishing cosmological constant.

Unlike in the non-gravitational Coleman-Weinberg case, where V is always negative at the minimum, in the agravity context $V(s) = \lambda_S(s)s^4/4$ can be vanishing at the minimum, provided that $\lambda_S(s) = 0$ at the minimum. This equation has the same form as the Coleman-Weinberg minimum condition, but its origin is different: it corresponds to demanding a negligible cosmological constant.

In summary, agravity can generate the Planck scale while keeping the vacuum energy vanishing provided that

$$\begin{cases} \lambda_S(s) = 0 & \text{(vanishing cosmological constant)}, \\ \beta_{\lambda_S}(s) = 0 & \text{(minimum condition)}, \\ \xi_S(s)s^2 = \bar{M}^2_{\rm Pl} & \text{(observed Planck mass)}. \end{cases} \tag{3.10}$$

The minimum equation of eq. (3.9) has been simplified taking into account that λ_S nearly vanishes at the minimum.

[7] Quantum corrections change when changing frame. In a generic context this leads to ambiguities. In the agravity context the fundamental action is given by eq. (1.2), thereby quantum corrections must be computed in the 'Jordan frame'.

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RGE running of the $\overline{\rm MS}$ quartic Higgs coupling in the SM

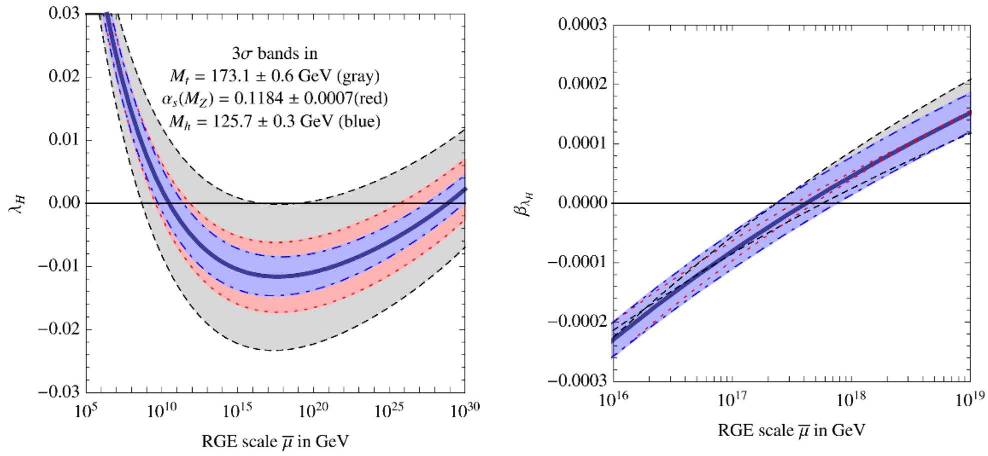

Figure 3. Running of the quartic Higgs coupling in the SM [93]. Agravity corrections can increase $\beta_{\lambda_H} = d\lambda_H/d\ln\bar{\mu}$ and thereby λ_H at scales above $M_{0,2}$. Parameterizing the effective potential as $V_{\rm eff}(h) \equiv \lambda_{\rm eff}(h)h^4/4$, the β function of $\lambda_{\rm eff}$ vanishes at a scale a factor of few higher than the β function of λ_H.

In the present scenario the cosmological constant can be naturally suppressed down to about M_0^2: even making it as light as possible, $M_0 \sim$ eV, the cosmological constant is at least 60 orders of magnitude larger than the observed value. Thereby we just invoke a huge fine-tuning without trying to explain the smallness of the cosmological constant.

Models. In words, the quartic λ_S must run in such a way that it vanishes together with its β function around the Planck mass. Is such a behaviour possible? The answer is yes; for example this is how the Higgs quartic λ_H can run in the Standard Model (see figure 3a, upper curve). Its β function vanishes at the scale where the gauge coupling contribution to β_{λ_H} in eq. (2.22) compensates the top Yukawa contribution. Figure 3b shows that this scale happens to be close to the Planck mass. Although we cannot identify the Higgs field with the S field — the Higgs vev is at the weak scale so that the possible second minimum of the Higgs potential at the Planck mass is not realised in nature — the fact that the conditions of eq. (3.10) are realised in the SM is encouraging in showing that they can be realised and maybe points to a deeper connection.

By considering the generic RGE of agravity, one can see that in the pure gravitational limit the conditions of eq. (3.10) cannot be satisfied, so the scalar S must have extra gauge and Yukawa interactions, just like the Higgs. Clearly, many models are possible.

A predictive model with no extra parameters is obtained by introducing a second copy of the SM and by imposing a Z_2 symmetry, spontaneously broken by the fact that the mirror Higgs field, identified with S, lies in the Planck-scale minimum while the Higgs field lies in the weak scale minimum. The mirror SM photon would be massless. Depending on the thermal history of the universe, a heavy mirror SM particle, such as a mirror neutrino or electron, could be a Dark Matter candidate. The interactions between such dark matter candidate with the visible sector are suppressed by λ_{HS}; as we will discuss in section 5, the smallness of λ_{HS} is implied by a mechanism to understand the hierarchy between the Planck and the electroweak scales.

4 Inflation

Inflation with a small amplitude of perturbations is not a typical outcome of quantum field theory: it needs potentials with special flatness properties and often super-Planckian vacuum expectation values. Agravity allows to compute the effective action at super-Planckian vacuum expectation values, and potentials are flat at tree level, when expressed in the Einstein frame. At loop level quantum corrections lead to deviations from flatness, proportional to the β functions of the theory computed in section 2: thereby perturbative couplings lead to quasi-flat potentials, as suggested by inflation.

All scalar fields in agravity are inflaton candidates: the scalar component of the graviton χ, the Higgs boson h, the Higgs of gravity s. Computing their quantum-generated effective action is non trivial: for example, disentangling the kinetic mixing between s and χ around the minimum of the effective potential, we find mass eigenstates

$$M_{\pm}^2 = \frac{m_s^2 + m_\chi^2}{2} \pm \frac{1}{2}\sqrt{(m_s^2+m_\chi^2)^2 - 4\frac{m_s^2 m_\chi^2}{1+6\xi_S}} \tag{4.1}$$

where $m_s^2 \equiv \langle s\rangle^2 b/4$, $m_\chi^2 \equiv \langle s\rangle^2 f_0^2(1+6\xi_S)\xi_S/2$ and $b=\beta(\beta_{\lambda_S})$ is the β function of the β function of λ_S evaluated at $\langle s\rangle$. More generically, one can eliminate the $R^2/6f_0^2$ term in the Lagrangian by adding an auxiliary field χ with action $-\sqrt{|\det g|}(R+3f_0^2\chi/2)^2/6f_0^2$. Next, by performing a Weyl rescaling $g_{\mu\nu}^E = g_{\mu\nu}\times(\xi_S s^2+\chi)/\bar M_{\rm Pl}^2$ one obtains a canonical Einstein-Hilbert term, mixed kinetic terms for s and χ, as well as their effective potential. The square of the Weyl tensor remains unaffected, and does not contribute to classical cosmological evolution equations.

Various regimes for inflation are possible. In the limit where h or χ feel the vacuum expectation value of s as a constant mass term, one obtains Starobinsky inflation and Higgs ξ-inflation [7–19]: agravity dictates how they can hold above the Planck scale. Leaving a full analysis to a future work, we here want to explore the possibility that the inflaton is the field s that dynamically generates the Planck scale, as discussed in section 3.

In the limit where the spin 0 graviton χ is heavy enough that we can ignore its kinetic mixing with s, we can easily convert s into a scalar s_E with canonical kinetic term, as discussed in eq. (3.7). Then, the usual formalism of slow-roll parameters allows to obtain the inflationary predictions. The slow-roll parameters ϵ and η are given by the β functions of the theory. At leading order we find:

$$\epsilon \equiv \frac{\bar M_{\rm Pl}^2}{2}\left(\frac{1}{V_E}\frac{\partial V_E}{\partial s_E}\right)^2 = \frac{1}{2}\frac{\xi_S}{1+6\xi_S}\left[\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right]^2, \tag{4.2a}$$

$$\eta \equiv \bar M_{\rm Pl}^2\frac{1}{V_E}\frac{\partial^2 V_E}{\partial s_E^2} = \frac{\xi_S}{1+6\xi_S}\left[\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2\frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5+36\xi_S}{1+6\xi_S}\frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7+48\xi_S}{1+6\xi_S}\frac{\beta_{\lambda_S}\beta_{\xi_S}}{2\lambda_S\xi_S}\right]. \tag{4.2b}$$

The scalar amplitude A_s, its spectral index n_s and the tensor-to-scalar ratio $r = A_t/A_s$ are predicted as

$$n_s = 1 - 6\epsilon + 2\eta, \qquad A_s = \frac{V_E/\epsilon}{24\pi^2 \bar{M}_{\rm Pl}^4}, \qquad r = 16\epsilon \tag{4.3}$$

where all quantities are evaluated at about $N \approx 60$ e-folds before the end of inflation, when the inflation field $s_E(N)$ was given by

$$N = \frac{1}{\bar{M}_{\rm Pl}^2} \int_0^{s_E(N)} \frac{V_E(s_E)}{V_E'(s_E)} ds_E. \tag{4.4}$$

In any given agravity model the running of λ_S and of ξ_S and consequently the inflationary predictions can be computed numerically.

4.1 Agravity inflation: analytic approximation

We consider a simple analytic approximation that encodes the main features of this scenario. As discussed in section 3 and summarised in eq. (3.10), dynamical generation of the Planck scale with vanishing cosmological constant demands that the quartic λ_S as well as its β function vanish at a scale $\langle s \rangle = \bar{M}_{\rm Pl}/\sqrt{\xi_S}$. Thereby, around such minimum, we can approximate the running parameters as

$$\lambda_S(\bar{\mu} \approx s) \approx \frac{b}{2} \ln^2 \frac{s}{\langle s \rangle}, \qquad \xi_S(\bar{\mu}) \approx \xi_S. \tag{4.5}$$

We neglect the running of ξ_S, given that it does not need to exhibit special features. The coefficient $b = \beta(\beta_{\lambda_S})$ can be rewritten as $b \equiv g^4/(4\pi)^4$, where g^4 is the sum of quartic powers of the adimensional couplings of the theory. It can be computed in any given model. With this approximated running the slow-roll parameters of eq.s (4.2) simplify to

$$\epsilon \approx \eta \approx \frac{2\xi_S}{1 + 6\xi_S} \frac{1}{\ln^2 s/\langle s \rangle} = \frac{2\bar{M}_{\rm Pl}^2}{s_E^2}. \tag{4.6}$$

The latter equality holds because, within the assumed approximation, the explicit expression for the Einstein-frame scalar s_E is

$$s_E = \bar{M}_{\rm Pl} \sqrt{\frac{1 + 6\xi_S}{\xi_S}} \ln \frac{s}{\langle s \rangle}. \tag{4.7}$$

The Einstein-frame potential gets approximated, around its minimum, as a quadratic potential:

$$V_E = \frac{\bar{M}_{\rm Pl}^4}{4} \frac{\lambda_S}{\xi_S^2} \approx \frac{M_s^2}{2} s_E^2 \qquad \text{with} \qquad M_s = \frac{g^2 \bar{M}_{\rm Pl}}{2(4\pi)^2} \frac{1}{\sqrt{\xi_S(1 + 6\xi_S)}}. \tag{4.8}$$

Notice that the eigenvalue M_- of eq. (4.1) indeed reduces to M_s, in the limit where it is much lighter than the other eigenvalue $M_0^2 \simeq \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2 (1 + 6\xi_S)$.

Inserting the value of s_E at $N \approx 60$ e-folds before the end of inflation, $s_E(N) \approx 2\sqrt{N}\bar{M}_{\rm Pl}$, we obtain the predictions:

$$n_s \approx 1 - \frac{2}{N} \approx 0.967, \qquad r \approx \frac{8}{N} \approx 0.13, \qquad A_s \approx \frac{g^4 N^2}{24\pi^2 \xi_S (1 + 6\xi_S)}. \tag{4.9}$$

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Such predictions are typical of quadratic potentials, and this is a non-trivial fact.

Indeed, vacuum expectation values above the Planck scale, $s_E \approx 2\sqrt{N}\bar{M}_{\rm Pl}$, are needed for inflation from a quadratic potential and, more generically, if the tensor/scalar ratio is above the Lyth bound [94]. This means that, in a generic context, higher order potential terms suppressed by the Planck scale become important, so that the quadratic approximation does not hold.

Agravity predicts physics above the Planck scale, and a quadratic potential is a good approximation, even at super-Planckian vev, because coefficients of higher order terms are dynamically suppressed by extra powers of the loop expansion parameters, roughly given by $g^2/(4\pi)^2$. Higher order terms are expected to give corrections of relative order $g^2\sqrt{N}/(4\pi)^2$, which are small if the theory is weakly coupled.

4.2 Numerical model-dependent computation

We here consider the specific model presented in section 3, where the scalar S is identified with the Higgs doublet of a mirror sector which is an exact copy of the SM, with the only difference that S sits in the Planck-scale minimum of the SM effective potential.

This model predicts that the β function coefficient in eq. (4.5) equals $g^4 \approx 1.0$ provided that we can neglect the gravitational couplings f_0, f_2 with respect to the known order-one SM couplings y_t, g_3, g_2, g_1.

Thereby the observed scalar amplitude $A_s = 2.2\ 10^{-9}$ [20] is reproduced for $\xi_S \approx 210$. A large ξ_S is perturbative as long as it is smaller than $1/f_{0,2}$.

We notice that ξ_S is not a free parameter, within the context of the SM mirror model: the vev of the Higgs mirror s is given by the RGE scale at which β_{λ_S} vanishes (see figure 3b), and in order to reproduce the correct Planck scale with $\xi_S \approx 210$ one needs $\langle s\rangle = \bar{M}_{\rm Pl}/\sqrt{\xi_S} = 1.6\ 10^{17}\,\text{GeV}$. The fact that this condition can be satisfied (within the uncertainties) is a test of the model.

The inflaton mass $M_s \approx 1.4\ 10^{13}\,\text{GeV}$ is below the Planck scale because suppressed by the β-functions of the theory, see eq. (4.8).

The model allows to compute the full inflationary potential from the full running of λ_S (shown in figure 3) and of ξ_S. The computation is conveniently performed in the Landau gauge $\xi_V = 0$, given that the gauge-dependence of the effective potential gets canceled by the gauge-dependence of the scalar kinetic term [95]. By performing a numerical computation we find a more precise prediction $r \approx 0.128$ for $N \approx 60$. This is compatible with the expected accuracy of the quadratic approximation, estimated as $g^2\sqrt{N}/(4\pi)^2 \approx 5\%$ in section 4.1.

In conclusion, we identified the inflaton with the field that dynamically generates the Planck scale. In the agravity context, such field must have a dimensionless logarithmic potential: this is why our predictions for $r \approx 8/N \approx 0.13$ differ from the tentative prediction $r \approx 12/N^2 \approx 0.003$ of a generic ξ-inflation model with mass parameters in the potential [7–19].

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5 Dynamical generation of the Weak scale

In section 3 we discussed how the Planck scale can be dynamically generated. We now discuss how it is possible to generate also the electro-weak scale, such that it *naturally* is much below the Planck scale. 'Naturally' here refers to the modified version of naturalness adopted in [47], where quadratically divergent corrections are assumed to vanish, such that no new physics is needed at the weak scale to keep it stable. The present work proposed a theoretical motivation for the vanishing of power divergences: they have mass dimension, and thereby must vanish if the fundamental theory contains no dimensionful parameters. This is the principle that motivated our study of adimensional gravity.

In this scenario, the weak scale can be naturally small, and the next step is exploring what can be the physical dynamical origin of the small ratio $M_h^2/M_{\rm Pl}^2 \sim 10^{-34}$. The dynamics that generates the weak scale can be:

a) **around the weak scale**, with physics at much high energy only giving negligible finite corrections to the Higgs mass. Models of this type have been proposed in the literature [21–46], although the issue of gravitational corrections has not been addressed. Such models lead to observable signals in weak-scale experiments.

b) **much above the weak scale**. For example, Einstein gravity naively suggests that any particle with mass M gives a finite gravitational correction to the Higgs mass at three [96] and two loops:

$$\delta M_h^2 \sim \frac{y_t^2 M^6}{(4\pi)^6 M_{\rm Pl}^4} + \frac{\xi_H M^6}{(4\pi)^4 M_{\rm Pl}^4} \tag{5.1}$$

which is of the right order of magnitude for $M \sim 10^{14}\,$GeV.

In the context of agravity we can address the issue of gravitational corrections, and propose a scenario where the weak scale is generated from the Planck scale. It is convenient to divide the computation into 3 energy ranges

1) *Low energies*: at RGE scales below the mass $M_{0,2}$ of the heavy gravitons, agravity can be neglected and the usual RGE of the SM apply. The Higgs mass parameter receives a multiplicative renormalization:

$$(4\pi)^2 \frac{dM_h^2}{d\ln\bar\mu} = M_h^2 \beta_{M_h}^{\rm SM}, \qquad \beta_{M_h}^{\rm SM} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}. \tag{5.2}$$

2) *Intermediate energies* between $M_{0,2}$ and $M_{\rm Pl}$: agravity interactions cannot be neglected but M_h and $M_{\rm Pl}$ appear in the effective Lagrangian as apparent dimensionful parameters. We find that their RGE are gauge-dependent because the unit of mass is gauge dependent. The RGE for adimensional mass-ratios are gauge-independent

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and we find[8]

$$(4\pi)^2 \frac{d}{d\ln\bar\mu}\frac{M_h^2}{\bar M_{\rm Pl}^2} = -\xi_H[5f_2^4 + f_0^4(1+6\xi_H)] - \frac{1}{3}\left(\frac{M_h^2}{\bar M_{\rm Pl}^2}\right)^2(1+6\xi_H) + \\ + \frac{M_h^2}{\bar M_{\rm Pl}^2}\left[\beta_{M_h}^{\rm SM} + 5f_2^2 + \frac{5}{3}\frac{f_2^4}{f_0^2} + f_0^2(\frac{1}{3}+6\xi_H+6\xi_H^2)\right]. \quad (5.3)$$

The first term is crucial: it describes corrections to M_h proportional to $M_{\rm Pl}$. A naturally small [47] weak scale arises provided that the agravity couplings are small:

$$f_0, f_2 \approx \sqrt{\frac{4\pi M_h}{M_{\rm Pl}}} \sim 10^{-8}. \quad (5.4)$$

The mass of the spin-2 graviton ghost is $M_2 = f_2\bar M_{\rm Pl}/\sqrt{2} \approx 3\ 10^{10}\,$GeV. The spin-0 massive component of the graviton mixes with the other scalars giving rise to the mass eigenvalues of eq. (4.1). Experimental bounds are safely satisfied.

3) *Large energies* above the Planck mass: the theory is adimensional and the RGE of section 2 apply. According to the Lagrangian of eq. (1.4), the quartic coupling $\lambda_{HS}|H|^2|S|^2$ leads to a Higgs mass term $\frac12 M_h^2|H|^2$ given by $M_h^2 = \lambda_{HS}\langle s\rangle^2$. Ignoring gravity, λ_{HS} can be naturally arbitrarily small, because it is the only interaction that couples the SM sector with the S sector. Within agravity, a non vanishing λ_{HS} is unavoidably generated by RGE running at one-loop order, as shown by its RGE in eq. (2.21), which contains the non-multiplicative contribution:

$$(4\pi)^2\frac{d\lambda_{HS}}{d\ln\bar\mu} = -\xi_H\xi_S[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)] + \cdots. \quad (5.5)$$

For $\xi_S = 0$ this equation is equivalent to (5.3). We need to assume that the mixed quartic acquires its minimal natural value, $\lambda_{HS} \sim f_{0,2}^4$ (for simplicity we do not consider the possibility of values of $\xi_{H,S} = \{0, -1/6\}$ that lead to special cancellations).

In conclusion, agravity unavoidably generates a contribution to the Higgs mass given by

$$M_h^2 \approx \frac{\bar M_{\rm Pl}^2\xi_H}{(4\pi)^2}[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)]\ell \quad (5.6)$$

where ℓ is a positive logarithmic factor.

This alternative understanding of the Higgs mass hierarchy problem relies on the smallness of some parameters. All parameters assumed to be small are naturally small, just like the Yukawa coupling of the electron, $y_e \sim 10^{-6}$, is naturally small. These small parameters do not receive unnaturally large quantum corrections. No fine-tuned cancellations are necessary.

At perturbative level, this is clear from the explicit form of the one-loop RGE equations derived in section 2: quantum corrections to f_0, f_2 are proportional to cubic powers of f_0, f_2, and higher order loop corrections are even more suppressed.

[8]We also verified that the RGE for the ratio of scalar to fermion masses is gauge invariant. We cannot comparare our eq. (5.3) with gauge-depend RGE for $M_{\rm Pl}$ computed in the literature [81, 84, 85, 87, 97] with discrepant results, given that we use a different gauge.

At non perturbative level, a black hole of mass M might give a quantum correction of order $\delta M_h^2 \sim M_{\rm BH}^2 e^{-S_{\rm BH}}$ where $S_{\rm BH} = M_{\rm BH}^2/2\bar{M}_{\rm Pl}^2$ is the black hole entropy. Black holes with Planck-scale mass might give an unnaturally large correction, $\delta M_h^2 \gg M_h^2$, ruining naturalness. Planck-scale black holes do not exist in agravity, where the minimal mass of a black hole is $M_{\rm BH} \gtrsim \bar{M}_{\rm Pl}/f_{0,2}$, as clear from the fact that the massive anti-gravitons damp the $1/r$ Newton behaviour of the gravitational potential at $r \lesssim 1/M_{0,2}$:

$$V = -\frac{Gm}{r}\left[1 - \frac{4}{3}e^{-M_2 r} + \frac{1}{3}e^{-M_0 r}\right]. \tag{5.7}$$

Thereby, non-perturbative quantum corrections are expected to be negligible in agravity, because exponentially suppressed as $e^{-1/f_{0,2}^2}$.

The Higgs of gravity s has a mass M_s which can be anywhere between the weak scale and the Planck scale, depending on how large are the gauge and Yukawa couplings within its sector. Its couplings to SM particles are always negligibly small. In the model where s is the Higgs of a mirror copy of the SM, its mass is a few orders of magnitude below the Planck scale.

As a final comment, we notice that accidental global symmetries (a key ingredient of axion models) are a natural consequence of the dimensionless principle. In the usual scenario, ad hoc model building is needed in order to suppress explicit breaking due to mass terms or non-renormalizable operators [98]. An axion can be added to agravity compatibly with finite naturalness along the lines of [47].

6 Conclusions

In conclusion, we proposed that the fundamental theory contains no dimensionful parameter. Adimensional gravity (agravity for short) is renormalizable because gravitons have a kinetic term with 4 derivatives and two adimensional coupling constants f_0 and f_2.

The theory predicts physics above the Planck scale. We computed the RGE of a generic agravity theory, see eqs. (2.16), (2.18), (2.6) and (2.29). We found that quantum corrections can dynamically generate the Planck scale as the vacuum expectation value of a scalar s, that acts as the Higgs of gravity. The cosmological constant can be tuned to zero. This happens when a running quartic coupling and its β function both vanish around the Planck scale, as summarised in eq. (3.10). The quartic coupling of the Higgs in the SM can run in such a way, see figure 3.

The graviton splits into the usual massless graviton, into a massive spin 2 anti-graviton, and into a scalar. The spin 2 state is a ghost, to be quantised as a state with positive kinetic energy but negative norm.

The lack of dimensional parameters implies successful quasi-flat inflationary potentials at super-Planckian vacuum expectation values: the slow-roll parameters are the β functions of the theory. Identifying the inflaton with the Higgs of gravity leads to predictions $n_s \approx 0.967$ for the spectral index and $r \approx 0.13$ for the tensor/scalar amplitude ratio.

The Higgs of gravity can also be identified with the Higgs of the Higgs: if $f_0, f_2 \sim 10^{-8}$ are small enough, gravitational loops generate the observed weak scale. In this context,

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a weak scale much smaller than the Planck scale is natural: all small parameters receive small quantum corrections. In particular, quadratic divergences must vanish in view of the lack of any fundamental dimensionful parameter, circumventing the usual hierarchy problem.

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