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Magnetic fixed points and emergent supersymmetry

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Abstract: We establish in perturbation theory the existence of fixed points along the renormalization group flow for QCD with an adjoint Weyl fermion and scalar matter reminiscent of magnetic duals of QCD [1–3]. We classify the fixed points by analyzing their basin of attraction. We discover that among these there are stable supersymmetric ones emerging from a generic nonsupersymmetric renormalization group flow once the mass operators have been properly subtracted away. We therefore conclude that four dimensional supersymmetry can emerge as a fixed point theory from a nonsupersymmetric Lagrangian. Our results suggest that supersymmetry can be viewed as an emergent phenomenon in four dimensional field theory complementing recent discoveries in lower number of dimensions [4, 5].

Keywords: Renormalization Group, Supersymmetry and Duality, Duality in Gauge Field Theories

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Contents

1 Introduction

Gauge theories constitute the building blocks of our present understanding of natural phenomena. The Standard Model (SM) of high energy particle interactions is, in fact, entirely based on a semi-simple gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The SM accounts for roughly four percent of the known universe. However, several puzzles remain still unexplained. For example, why do we observe, at least, three generations of elementary particles. Besides, the remaining 94% of the universe, of which 22% dark matter, and 72% dark energy, remains largely unknown. It is therefore natural to expect that new gauge theories, or extensions of the present ones play a fundamental role in explaining the unknown side of the universe. The space of four dimensional gauge theories at our disposal, without theoretical prejudice, is very large and moreover a large fraction of it is still *terra incognita* due to our limited methods to tackle nonperturbative dynamics.

In the late nineties, in a series of ground breaking papers [1, 2], Seiberg provided strong support for the existence of duality at long distances between two fundamental supersymmetric theories both exhibiting long distance conformality. Seiberg's proposal is an electric-magnetic type duality between two different asymptotically free gauge theories, and the region in their external parameter space, such as the number of colors and flavors, where they are both asymptotically free and duality should hold is called the conformal window. One of the most relevant results put forward by Seiberg has been the identification of the conformal window for supersymmetric QCD. Supersymmetry is, however, quite special and the existence of such a duality does not automatically imply the existence of nonsupersymmetric duals.

The exploration of the possible existence of nonsupersymmetric gauge duals providing a consistent picture of the phase diagram as a function of number of colors and flavors has started [3, 6, 6–9] and recently several analytic predictions have been provided for the conformal window of nonsupersymmetric gauge theories using different approaches [10–41]. The first goal of this work is to establish in perturbation theory the existence of fixed points along the renormalization group flow for QCD with an adjoint Weyl fermion and scalar matter reminiscent of magnetic duals of QCD [1–3]. We discover, for the first time, a large number of calculable fixed points and classify them by analyzing their basin of attraction. Interestingly we discover supersymmetric stable fixed points to emerge in the infrared along nonsupersymmetric flows once the mass operators have been properly subtracted away. The phenomenological interest of our studies relies also on the fact that theories similar to the ones investigated here and featuring infrared fixed points have been used to construct sensible extensions of the standard model of particle interactions of technicolor type passing precision data and known as Minimal Walking Technicolor models [10, 15]. The theory studied here has also been used in [42] as a toy model-like dual of a Pati-Salam extension of the SM.

Our results suggest that supersymmetry can emerge in four dimensional field theory complementing similar discoveries in lower number of dimensions [4, 5].

2 Magnetic setup of QCD with one adjoint fermion.

To introduce the models analyzed in this work, we start with a brief review related to duality arguments that lead us to the analysis in this work. Our results are, however, independent of the existence of gauge duality.

In [3] the possible existence of a magnetic dual for a nonsupersymmetric electric theory was put forward. This electric theory is a scalarless SU(N) gauge theory with N_f Dirac fermions and N larger than two, as in QCD, but with an extra Weyl fermion transforming according to the adjoint representation of the gauge group. The quantum global symmetry of the electric theory is therefore:

$$\mathrm{SU_L(N_f)} \times \mathrm{SU_R(N_f)} \times \mathrm{U_V(1)} \times \mathrm{U_{AF}(1)} \ . \tag{2.1}$$

At the classical level there is one more $\mathrm{U_A}(1)$ symmetry destroyed by quantum corrections due to the Adler-Bell-Jackiw anomaly. Of the three independent U(1) symmetries only two survive, a vector like $\mathrm{U_V}(1)$ and an axial-like anomaly free (AF) $\mathrm{U_{AF}}(1)$. The spectrum of the theory and the global transformations are summarized in table 1.

A magnetic dual was constructed based on the minimal spectrum of composite states and gauge group structure from the constraints:

- Matching all of the 't Hooft anomaly conditions, for *any* number of colors and flavors, which was accomplished for the first time.

- Allow for consistent flavor decoupling both in the electric and in the magnetic theory.

Fields	[SU(N)]	$\mathrm{SU_L(N_f)}$	$\mathrm{SU_R(N_f)}$	$\mathrm{U_V(1)}$	$\mathrm{U_{AF}(1)}$
λ	Adj	1	1	0	1
Q	$\square$	$\square$	1	1	$-\frac{N}{N_f}$
$\widetilde{Q}$	$\overline{\square}$	1	$\overline{\square}$	-1	$-\frac{N}{N_f}$
G_μ	Adj	1	1	0	0

Table 1. Field content of the electric theory and field transformation properties. The squared brackets around SU(N) indicate that this is the gauge group.

Fields	[SU(X)]	$\mathrm{SU_L(N_f)}$	$\mathrm{SU_R(N_f)}$	$\mathrm{U_V(1)}$	$\mathrm{U_{AF}(1)}$
λ_m	Adj	1	1	0	1
q	$\square$	$\overline{\square}$	1	$\frac{N_f-X}{X}$	$-\frac{X}{N_f}$
$\widetilde{q}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-X}{X}$	$-\frac{X}{N_f}$
M	1	$\square$	$\overline{\square}$	0	$-\frac{N_f-2X}{N_f}$
$\widetilde{\phi}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-X}{X}$	$\frac{N_f-X}{N_f}$
ϕ	$\square$	$\overline{\square}$	1	$\frac{N_f-X}{X}$	$\frac{N_f-X}{N_f}$
G_μ	Adj	1	1	0	0

Table 2. Field content of the magnetic theory and field transformation properties. The four upper fields are Weyl spinors in the $(1/2, 0)$ representation of the Lorentz group. The two ϕ-fields are complex scalars and G_μ are the gauge bosons.

- Ensure *duality involution*; by dualizing the magnetic theory one should recover the gauge structure of the electric theory.

The proposed nonsupersymmetric SU(X) magnetic gauge theory [3] is summarized in table 2. It was shown that one has to have $X = N_f - N$. The spectrum in table 2 is nonsupersymmetric. This should be clear, since there is no complex scalar partner of M. Furthermore it was shown that one can build the gauge singlet states using the electric variables. Subsequently in [42] it was argued that one could also add the complex scalar H (appearing now in table 3). This is possible since it does not affect the anomaly conditions and can be built naturally out of the electric fermionic variable as follows [42]:

$$H \sim Q\lambda\lambda\widetilde{Q} \ . \tag{2.2}$$

It is crucial to be able to construct all these states directly from the electric fermionic variables. This demonstrates that supersymmetry is not a fundamental ingredient in order to construct these states. H in [42] plays the phenomenologically relevant role of the SM-like Higgs, elementary in terms of the magnetic variables. Although when adding the new complex scalar field the spectrum of the dual theory looks supersymmetric, the full theory is not since the couplings, in the ultraviolet, are not taken to respect supersymmetric relations.

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Fields	[SU(X)]	$\mathrm{SU_L(N_f)}$	$\mathrm{SU_R(N_f)}$	$\mathrm{U_V(1)}$	$\mathrm{U_{AF}(1)}$
λ_m	Adj	1	1	0	1
q	$\square$	$\overline{\square}$	1	$\frac{N_f-X}{X}$	$-\frac{X}{N_f}$
$\widetilde{q}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-X}{X}$	$-\frac{X}{N_f}$
M	1	$\square$	$\overline{\square}$	0	$-\frac{N_f-2X}{N_f}$
H	1	$\square$	$\overline{\square}$	0	$\frac{2X}{N_f}$
$\widetilde{\phi}$	$\overline{\square}$	1	$\square$	$-\frac{N_f-X}{X}$	$\frac{N_f-X}{N_f}$
ϕ	$\square$	$\overline{\square}$	1	$\frac{N_f-X}{X}$	$\frac{N_f-X}{N_f}$
G_μ	Adj	1	1	0	0

Table 3. Field content of the magnetic theory with the addition of the Higgs-field, H and the field transformation properties. The four upper fields are Weyl spinors in the $(1/2, 0)$ representation of the Lorentz group.

Motivated by the duality arguments above we move to investigate the actual existence of the fixed points in the presumed magnetic theory.

3 Magnetic potential of the theory and beta functions

We start by considering the Yukawa terms of the magnetic theory in table 3:

$$\mathcal{L}_Y = y_\lambda \phi^* \lambda_m q + y_{\widetilde{\lambda}} \widetilde{\phi}^* \lambda_m \widetilde{q} + y_{\widetilde{M}} \widetilde{\phi} M q + y_M \phi M \widetilde{q} + y_H \widetilde{q} H q + \text{h.c.} \tag{3.1}$$

Quartic ϕ^4-interactions do not affect our results to the perturbative order we will consider. These interactions will be discussed in the end. We will be working in a mass-independent scheme and can therefore, in the spirit of Coleman and E. Weinberg [43] (see also the discussion by Gildener and S. Weinberg in [44]), work with the massless theory.

The Yukawa-sector contributes to the two-loop beta function of the gauge coupling in the following manner:

$$\beta(g) = \frac{dg}{d\ln\mu} = -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4} - \beta_Y \frac{g^3}{(4\pi)^4} + \mathcal{O}(g^7) \tag{3.2}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{2}{3} \sum_r T(r) N_f(r) - \frac{1}{6} \sum_s T(s) N_f(s) \tag{3.3}$$

$$\beta_1 = \frac{34}{3} C_2(G)^2 - \sum_r \left[\frac{10}{3} C_2(G) + 2C_2(r)\right] T(r) N_f(r) - \sum_s \left[\frac{1}{3} C_2(G) + 2C_2(r)\right] T(s) N_f(s) \tag{3.4}$$

$$\beta_Y = \frac{1}{d(G)} \sum_r \mathrm{Tr}\left[C_2(r) Y^j Y_j^\dagger\right] \tag{3.5}$$

where r denotes the representation of fermions and s denotes the representation of the *real* scalars. $T(\cdot)$ is the trace normalization of the group generators, $C_2(\cdot)$ is the quadratic Casimir of these and $d(G)$ is the dimension of the gauge group. We refer to appendix A for a careful derivation of the Yukawa contribution β_Y to the running of the gauge coupling. The result is:

$$\beta(\alpha_g) = -2\alpha_g^2 \left[\beta_0 + \alpha_g \beta_1 + (\alpha_\lambda + \alpha_{\widetilde{\lambda}}) \frac{3X^2-1}{4X} N_f + \left(\frac{\alpha_M + \alpha_{\widetilde{M}}}{2} + \alpha_H\right) N_f^2\right], \tag{3.6}$$

with

$$\beta_0 = 3X - N_f, \quad \beta_1 = 6X^2 - 7N_f X + \frac{3N_f}{X}, \tag{3.7}$$

and where we used the notation

$$\alpha_i \equiv \frac{|\, y_i \,|^2}{(4\pi)^2}.$$

We now consider the running of the Yukawa couplings. The one loop beta function for the Yukawa couplings is given by [46, 47]:

$$\begin{aligned}(4\pi)^2 \beta(Y^j) = &\frac{1}{2}\left[Y_2^\dagger(r) Y^j + Y^j Y_2(r)\right] \\ &+ 2Y^k Y_j^\dagger Y^k + \frac{1}{2} Y^k \mathrm{Tr}\left[Y_k^\dagger Y^j + Y_j^\dagger Y^k\right] - 3g^2 \{C_2(r), Y^j\}.\end{aligned} \tag{3.8}$$

Here Y^j is the Yukawa coupling matrix defined by the particular interaction:

$$\mathcal{L}_Y \sim Y^j_{\alpha\beta} \phi_j \psi^\alpha \chi^\beta,$$

where roman indices contract over the scalar gauge-flavor overall index and the greek indices α, β are again gauge-flavor indices but reserved for the Weyl fermions ψ and χ. $Y_2(r)$ is the group invariant:

$$Y_2(r) \equiv Y_j^\dagger Y^j.$$

For each scalar contraction we multiply, row by column, the Yukawa matrices over the fermion indices. We again report the derivation of the beta function for each Yukawa coupling in appendix A. To the second order in the couplings the set of beta function equations reads:

$$\beta(\alpha_g) = -2\alpha_g^2 \left[\beta_0 + \alpha_g \beta_1 + (\alpha_\lambda + \alpha_{\widetilde{\lambda}}) \frac{3X^2-1}{4X} N_f + \left(\frac{\alpha_M + \alpha_{\widetilde{M}}}{2} + \alpha_H\right) N_f^2\right] \tag{3.9}$$

$$\begin{aligned}\beta(\alpha_\lambda) = &2\alpha_\lambda \left[\frac{3(X^2-1)}{4X}\alpha_\lambda + (\alpha_\lambda + \alpha_{\widetilde{\lambda}})\frac{N_f}{4} + N_f\left(\frac{\alpha_{\widetilde{M}} + \alpha_H}{2} + \alpha_M\right) - 3\alpha_g \frac{3X^2-1}{2X}\right] \\ &- 4N_f \frac{y_M y_{\widetilde{M}} y_\lambda y_{\widetilde{\lambda}}}{(4\pi)^2}\end{aligned} \tag{3.10}$$

$$\begin{aligned}\beta(\alpha_M) = &2\alpha_M \left[\frac{3N_f + X}{2}\alpha_M + \frac{X}{2}\alpha_{\widetilde{M}} + \frac{N_f}{2}\alpha_H + \frac{X^2-1}{2X}\left(\frac{\alpha_{\widetilde{\lambda}}}{2} + \alpha_\lambda\right) - 3\alpha_g \frac{X^2-1}{2X}\right] \\ &- 4\frac{X^2-1}{2X}\frac{y_M y_{\widetilde{M}} y_\lambda y_{\widetilde{\lambda}}}{(4\pi)^2}\end{aligned} \tag{3.11}$$

$$\beta(\alpha_H) = 2\alpha_H \left[\frac{\alpha_M + \alpha_{\widetilde{M}} + 2\alpha_H}{2} N_f + \left(\frac{\alpha_\lambda + \alpha_{\widetilde{\lambda}}}{2} - 6\alpha_g\right) C_2(\square) + X\alpha_H\right] \tag{3.12}$$

$$\beta(\alpha_{\widetilde{\lambda}}) = \beta_{\alpha_\lambda}\left(y_\lambda \leftrightarrow y_{\widetilde{\lambda}}, y_{\widetilde{M}} \leftrightarrow y_M\right), \tag{3.13}$$

$$\beta(\alpha_{\widetilde{M}}) = \beta_{\alpha_M}\left(y_M \leftrightarrow y_{\widetilde{M}}, y_{\widetilde{\lambda}} \leftrightarrow y_\lambda\right), \tag{3.14}$$

where:

$$\beta(\alpha_i) \equiv \frac{2y_i}{(4\pi)^2}\beta(y_i) \ . \tag{3.15}$$

We impose CP invariance and therefore set all the Yukawa phases to zero. We discuss the full set of equations, including the phases, in the appendix A.

In the supersymmetric limit $\alpha_\lambda = \alpha_{\widetilde{\lambda}} = 2\alpha_g$, $\alpha_M = \alpha_{\widetilde{M}} = \alpha_H$ the beta function system simplifies to:

$$\beta(\alpha_g) = -2\alpha_g^2\left[\beta_0 + \beta_1'\alpha_g + 2N_f^2\alpha_M\right] \tag{3.16}$$
$$\beta(\alpha_\lambda) = -2\alpha_\lambda\left[\beta_0\alpha_g\right] \tag{3.17}$$
$$\beta(\alpha_M) = 2\alpha_M\left[(2N_f + X)\alpha_M - 4\alpha_g C_2(\Box)\right] \tag{3.18}$$
$$\beta(\alpha_H) = 2\alpha_H\left[(2N_f + X)\alpha_H - 4\alpha_g C_2(\Box)\right], \tag{3.19}$$

where $\beta_1' \equiv \beta_1 + (3X^2 - 1)N_f/X = 6X^2 - 4N_f X + 2N_f/X$, which is the well-known two-loop term of pure super QCD.

This result shows, as expected, that supersymmetry stays unbroken along the renormalization flow. Also note how the running of the gaugino Yukawa coupling has collapsed to the one-loop result for the running of the gauge coupling. These results are in agreement with the known result from supersymmetry: the all-orders supersymmetric beta functions for Seiberg's magnetic dual are:

$$\begin{aligned} \beta^s(\alpha_g) &= -2\alpha_g^2\frac{[\beta_0 + N_f\gamma_0]}{1 - 2X\alpha_g} \\ \beta^s(\alpha_M) &= \alpha_M\left[\gamma_M + 2\gamma_0\right], \end{aligned} \tag{3.20}$$

where the one-loop expressions for the anomalous dimensions of the chiral superfields read:

$$\begin{aligned} \gamma_0 &= -4C_2(\Box)\alpha_g + 2N_f\alpha_M + \dots \\ \gamma_M &= 2X\alpha_M + \dots \end{aligned} \tag{3.21}$$

by which it is readily seen that the one-loop expansion of $\beta^s(\alpha_g)$ and $\beta^s(\alpha_M)$ are in agreement with the expressions in equations (3.16)–(3.19).

4 Magnetic fixed point analysis

Just below the critical number of flavors where the theory looses asymptotic freedom ($\beta_0 = 0$) a Banks-Zaks perturbatively stable infrared fixed point (IRFP) emerges once the Yukawa interactions are set to zero. The well known expression for the value of the gauge coupling at this fixed point is:

$$\alpha_g^{*BZ} = -\frac{\beta_0}{\beta_1} \ . \tag{4.1}$$

In this phase the theory shows large distance conformality. We now investigate whether the perturbatively stable IRFP persists when we turn on the Yukawa interactions. From eq. (3.9), the perturbative fixed point value for the gauge coupling reads:

$$\alpha_g^* = -\frac{4X\beta_0 + (\alpha_\lambda^* + \alpha_{\widetilde{\lambda}}^*)(3X^2-1)N_f + 2X(\alpha_M^* + \alpha_{\widetilde{M}}^* + 2\alpha_H^*)N_f^2}{4X\beta_1} \ . \tag{4.2}$$

Perturbative consistency is ensured by the smallness of the Yukawa couplings at the fixed point. There is, clearly, a symmetry between the tilded and untilded Yukawa coupling constants at the beta functions level. This symmetry persists at the fixed point effectively reducing the space of solutions.

The fixed points associated to zeros of $\beta(\alpha_H)$ are:

$$\alpha_H^* = 0 \quad \wedge \quad \alpha_H^* = \frac{12C_2(\square)\alpha_g^* - C_2(\square)(\alpha_\lambda^* + \alpha_{\widetilde{\lambda}}^*) - N_f(\alpha_M^* + \alpha_{\widetilde{M}}^*)}{2(N_f + X)}. \tag{4.3}$$

A perturbative nontrivial infrared fixed point for all the couplings is achieved when the number of flavors are such that asymptotic freedom is almost lost. To express this point mathematically we introduce the small parameter ϵ as follows:

$$\beta_0 \geq 0 \Rightarrow N_f \leq 3X, \qquad N_f \equiv 3X(1-\epsilon) \ .$$

We report in table 4 and table 5 the explicit solutions for the fixed points as a function of ϵ. There are 18 physical solutions for the system (3.9)–(3.14), where 10 of these correspond to the class $\alpha_H^* = 0$ and the remaining 8 to $\alpha_H^* \neq 0$. The solutions obtained exchanging tilded couplings with untilted ones

$$\alpha_\lambda \leftrightarrow \alpha_{\widetilde{\lambda}} \quad \alpha_M \leftrightarrow \alpha_{\widetilde{M}} \, ,$$

are indicated in the tables 4 and 5 by an asterisk.

Solution (5) in table 4 corresponds to the perturbative infrared fixed point for super QCD while the last solution (6) in table 5 corresponds to the perturbative fixed point for Seiberg's magnetic dual and it is the only one with all nonvanishing couplings. Interestingly, when setting to zero $\alpha_H^* = 0$ (solution 7 in table 4) we find yet another fixed point corresponding to the infrared stable fixed point of the non-supersymmetric magnetic dual theory proposed in [3] and shown in table 2. This lends further support to the proposed non-supersymmetric duality.

5 Stability analysis

The stability of the fixed points reported in tables 4 and 5 is determined by first evaluating the following matrix

$$\omega_{ij} = \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g*} \tag{5.1}$$

at the fixed point of interest denoted in brief by g^*. If the matrix ω_{ij} has only real and positive eigenvalues the fixed point is said to be stable. We note as before that quantum

#	(*) $\alpha_\lambda \leftrightarrow \alpha_{\widetilde{\lambda}}, \quad \alpha_M \leftrightarrow \alpha_{\widetilde{M}}$	$\mathcal{O}(\epsilon)$-expansion
1	$\alpha_g^* = \frac{(3X-N_f)X}{N_f(7X^2-3)-6X^3}$	$= \frac{X\epsilon}{5X^2-3}$
	$\alpha^*_{\lambda,\widetilde{\lambda},M,\widetilde{M}} = 0$	
$2^{(*)}$	$\alpha_g^* = \frac{2(3X-N_f)(X(N_f+3X)-3)X}{2N_f^2X(7X^2-3)+3N_f(5-14X^2+X^4)-36X^3(X^2-1)}$	$= \frac{2X(2X^2-1)\epsilon}{5-16X^2+11X^4}$
	$\alpha_\lambda^* = \frac{12(3X-N_f)(3X^2-1)X}{2N_f^2X(7X^2-3)+3N_f(5-14X^2+X^4)-36X^3(X^2-1)}$	$= \frac{4X(3X^2-1)\epsilon}{5-16X^2+11X^4}$
	$\alpha^*_{\widetilde{\lambda},M,\widetilde{M}} = 0$	
$3^{(*)}$	$\alpha_g^* = \frac{2(3X-N_f)(3N_f+X)X}{3N_f^2(13X^2-5)-12X^4-2N_fX(3+11X^2)}$	$= \frac{20X\epsilon}{91X^2-51}$
	$\alpha_M^* = \frac{6(3X-N_f)(X^2-1)X}{3N_f^2(13X^2-5)-12X^4-2N_fX(3+11X^2)}$	$= \frac{6(X^2-1)\epsilon}{X(91X^2-51)}$
	$\alpha^*_{\lambda,\widetilde{\lambda},\widetilde{M}} = 0$	
4	$\alpha_g^* = \frac{(3X-N_f)(3N_f+2X)X}{2(N_f^2(9X^2-3)-6X^4-N_fX(3+2X^2))}$	$= \frac{11X\epsilon}{2(23X^2-12)}$
	$\alpha_M^* = \alpha^*_{\widetilde{M}} = \frac{3(3X-N_f)(X^2-1)}{2(N_f^2(9X^2-3)-6X^4-N_fX(3+2X^2))}$	$= \frac{3(X^2-1)\epsilon}{2X(23X^2-12)}$
	$\alpha^*_{\lambda,\widetilde{\lambda}} = 0$	
5	$\alpha_g^* = \frac{(3X-N_f)(3X^2+2N_fX-3)X}{2(9X^3-9X^5+N_f^2X(7X^2-3)+N_f(3-6X^2-9X^4))}$	$= \frac{X\epsilon}{2(X^2-1)}$
	$\alpha_\lambda^* = \alpha^*_{\widetilde{\lambda}} = \frac{3(3X-N_f)(3X^2-1)X}{2(9X^3-9X^5+N_f^2X(7X^2-3)+N_f(3-6X^2-9X^4))}$	$= \frac{X\epsilon}{X^2-1} = 2\alpha_g^* + \mathcal{O}(\epsilon^2)$
	$\alpha^*_{M,\widetilde{M}} = 0$	
$6^{(*)}$	$\alpha_g^* = \frac{2(3X-N_f)(3(N_f^2-1)X-8N_f+9N_fX^2+3X^3)X}{3N_f^3X(13X^2-5)-36X^4(X^2-1)+3N_fX(5+18X^2-31X^4)+2N_f^2(18-59X^2+9X^4))}$	$= \frac{2X(19X^2-9)\epsilon}{41-141X^2+100X^4}$
	$\alpha_\lambda^* = \frac{12(3X-N_f)(3X^3-2N_f-X+8N_fX^2)}{3N_f^3X(13X^2-5)-36X^4(X^2-1)+3N_fX(5+18X^2-31X^4)+2N_f^2(18-59X^2+9X^4))}$	$= \frac{4X(27X^2-7))\epsilon}{41-141X^2+100X^4}$
	$\alpha^*_{\widetilde{M}} = \frac{6(3X-N_f)(N_fX-2)(X^2-1)}{3N_f^3X(13X^2-5)-36X^4(X^2-1)+3N_fX(5+18X^2-31X^4)+2N_f^2(18-59X^2+9X^4))}$	$= \frac{(6X^2-4)\epsilon}{100X^3-41X}$
	$\alpha^*_{\widetilde{\lambda},M} = 0$	
7	$\alpha_g^* = \frac{(3X-N_f)(3(N_f^2-1)X-4N_f+6N_fX^2+3X^3)}{2(9X^3-9X^5+N_f^3(9X^2-3)+N_f^2(X-15X^3)+N_f(3+6X^2-21X^4))}$	$= \frac{X(16X^2-5)\epsilon}{2(4X^4-5X^2+1)}$
	$\alpha_\lambda^* = \alpha^*_{\widetilde{\lambda}} = \frac{3(3X-N_f)(5N_fX^2-2N_f-X+3X^3)}{9X^3-9X^5+N_f^3(9X^2-3)+N_f^2(X-15X^3)+N_f(3+6X^2-21X^4)}$	$= \frac{X(18X^2-7))\epsilon}{4X^4-5X^2+1}$
	$\alpha_M^* = \alpha^*_{\widetilde{M}} = \frac{3(3X-N_f)(N_fX+3X^2-2)(X^2-1)}{2X(9X^3-9X^5+N_f^3(9X^2-3)+N_f^2(X-15X^3)+N_f(3+6X^2-21X^4))}$	$= \frac{(3X^2-1)\epsilon}{X(4X^2-1)}$

Table 4. Fixed point solutions on the critical surface $\alpha_H^* = 0$. Solution 2,3 and 6 are doubled by symmetry property (*). Note that to linear order in ϵ solution 5 is the super QCD fixed point.

effects will induce scalar masses which in general will drive the RG flow away from any fixed point. This is cured by tuning the renormalized mass to zero, as usually done, and is the only tuning we are assuming.

In the tables 6 and 7 we present the eigenvalues corresponding to the fixed points of tables 4 and 5 respectively for $X = 4$ and $\epsilon = 0.05$. For convenience, we have marked the cells with negative eigenvalues in gray.

There is a rich structure of fixed points classified according to the relevant directions clearly visible from the list of associated eigenvectors reported in the appendix B. The

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#	(*) $\alpha_\lambda \leftrightarrow \alpha_{\widetilde{\lambda}}$, $\alpha_M \leftrightarrow \alpha_{\widetilde{M}}$	$\mathcal{O}(\epsilon)$expansion
1	$\alpha_g^* = \frac{(3X-N_f)(N_f+X)}{4N_f^2X-6X^3+N_f(X^2-3)}$ $\alpha_H^* = \frac{(3X-N_f)(X^2-1)}{X(4N_f^2X-6X^3+N_f(X^2-3))}$ $\alpha^*_{\lambda,\widetilde{\lambda},M,\widetilde{M}} = 0$	$= \frac{4X\epsilon}{11X^2-3}$ $= \frac{3(X^2-1)\epsilon}{X(11X^2-3)}$
$2^{(*)}$	$\alpha_g^* = \frac{(3X-N_f)(6X(X^2-1)+N_f(7X^2+2XN_f-5))}{2XN_f^2(5X^2+4XN_f-13)-36X^3(X^2-1)-3N_f(4X^2+9X^4-5)}$ $\alpha_H^* = \frac{3(3X-N_f)(X^2-1)(3X^2+2XN_f-5)}{X(2XN_f^2(5X^2+4XN_f-13)-36X^3(X^2-1)-3N_f(4X^2+9X^4-5))}$ $\alpha_\lambda^* = \frac{12X(3X-N_f)(3X^2+2XN_f-1)}{2XN_f^2(5X^2+4XN_f-13)-36X^3(X^2-1)-3N_f(4X^2+9X^4-5)}$ $\alpha^*_{\widetilde{\lambda},M,\widetilde{M}} = 0$	$= \frac{X(15X^2-7)\epsilon}{5-26X^2+21X^4}$ $= \frac{(9X^2-5)\epsilon}{X(21X^2-5)}$ $= \frac{4X(9X^2-1)\epsilon}{5-26X^2+21X^4}$
$3^{(*)}$	$\alpha_g^* = \frac{(3X-N_f)(2X^2+8XN_f+5N_f^2)}{20N_f^3X-12X^4+N_f^2(17X^2-15)-2N_fX(17X^2+3)}$ $\alpha_H^* = \frac{3(3X-N_f)(2X+5N_f)(X^2-1)}{X(20N_f^3X-12X^4+N_f^2(17X^2-15)-2N_fX(17X^2+3))}$ $\alpha_M^* = \frac{6(3X-N_f)(X^2-1)}{20N_f^3X-12X^4+N_f^2(17X^2-15)-2N_fX(17X^2+3)}$ $\alpha^*_{\lambda,\widetilde{\lambda},\widetilde{M}} = 0$	$= \frac{71X\epsilon}{193X^2-51}$ $= \frac{51(X^2-1)\epsilon}{X(193X^2-51)}$ $= \frac{6(X^2-1)\epsilon}{X(193X^2-51)}$
4	$\alpha_g^* = \frac{(3X-N_f)(2N_f+X)(N_f+2X)}{2(4N_f^3X-6X^4+N_f^2(7X^2-3)-N_fX(3+8X^2))}$ $\alpha_H^* = \frac{3(3X-N_f)(N_f+X)(X^2-1)}{X(4N_f^3X-6X^4+N_f^2(7X^2-3)-N_fX(3+8X^2))}$ $\alpha_M^* = \alpha^*_{\widetilde{M}} = \frac{3(3X-N_f)(X^2-1)}{2(4N_f^3X-6X^4+N_f^2(7X^2-3)-N_fX(3+8X^2))}$ $\alpha^*_{\lambda,\widetilde{\lambda}} = 0$	$= \frac{35X\epsilon}{2(47X^2-12))}$ $= \frac{12(X^2-1)\epsilon}{X(47X^2-12)}$ $= \frac{3(X^2-1)\epsilon}{2X(47X^2-12)}$
5	$\alpha_g^* = \frac{(3X-N_f)(3X(X^2-1)+2N_f(2X^2+XN_f-1))}{2(4N_fX+3X^2-3)(N_f^2X-3X^3-N_f(X^2+1))}$ $\alpha_H^* = \frac{3(3X-N_f)(N_fX-1)(X^2-1)}{X(4N_fX+3X^2-3)(N_f^2X-3X^3-N_f(X^2+1)}$ $\alpha_\lambda^* = \alpha^*_{\widetilde{\lambda}} = \frac{3X(3X-N_f)(3X^2+2XN_f-1)}{(4N_fX+3X^2-3)(N_f^2X-3X^3-N_f(X^2+1))}$ $\alpha^*_{M,\widetilde{M}} = 0$	$= \frac{X(11X^2-3)\epsilon}{2(5X^2-1)(X^2-1)}$ $= \frac{(3X^2-1)\epsilon}{X(5X^2-1)}$ $= \frac{X(9X^2-1)\epsilon}{(5X^2-1)(X^2-1)}$
6	$\alpha_g^* = \frac{(3X-N_f)(2N_f+X)(N_f^2+2N_fX+3X^2-3)}{2(4N_f^4X+9X^3-9X^5+N_f^3(X^2-3)-N_f^2X(3+23X^2)+3N_f(1+4X^2-9X^4))}$ $\alpha_H^* = \frac{3(3X-N_f)(N_fX-N_f^2+1)(X^2-1)}{X(4N_f^4X+9X^3-9X^5+N_f^3(X^2-3)-N_f^2X(3+23X^2)+3N_f(1+4X^2-9X^4))}$ $\alpha_\lambda^* = \alpha^*_{\widetilde{\lambda}} \frac{3(3X-N_f)(X+2N_f)(3X^2+XN_f-1)}{4N_f^4X+9X^3-9X^5+N_f^3(X^2-3)-N_f^2X(3+23X^2)+3N_f(1+4X^2-9X^4))}$ $\alpha_M^* = \alpha^*_{\widetilde{M}} = \frac{3(3X-N_f)(3N_fX+3X^2-2)(X^2-1)}{2X(4N_f^4X+9X^3-9X^5+N_f^3(X^2-3)-N_f^2X(3+23X^2)+3N_f(1+4X^2-9X^4))}$	$= \frac{7X\epsilon}{2(X^2-1)}$ $= \frac{\epsilon}{X}$ $= \frac{7X\epsilon}{X^2-1} = 2\alpha_g^* + \mathcal{O}(\epsilon^2)$ $= \frac{\epsilon}{X} = \alpha_H^* + \mathcal{O}(\epsilon^2)$

Table 5. Fixed point solutions on the critical surface $\alpha_H^* \neq 0$. Solutions 2 and 3 are doubled by symmetry property (*). Note that to linear order in ϵ solution 6 is the fixed point of Seiberg's magnetic dual.

dimensions of the critical surfaces are dictated by the number of positive eigenvalues. Once the unstable directions, in the coupling space, are removed the remaining subset of couplings generate a critical d-dimensional surface with d given by the number of positive

#	ν_1	ν_2	ν_3	ν_4	ν_5	ν_6
1	-0.049	-0.049	-0.031	-0.016	-0.016	0.0034
2	0.20	-0.048	-0.045	-0.015	0.0063	0.0011
3	-0.049	-0.044	0.035	-0.029	-0.015	0.0036
4	-0.064	0.038	0.030	-0.027	-0.023	0.0039
5	0.30	0.092	0.063	-0.060	-0.058	0.0089
6	0.20	-0.062	-0.043	0.031	0.021	0.0065
7	3.0	1.8	1.4	1.1	-0.14	0.033

Table 6. Eigenvalues corresponding to the fixed points in table 4 for $X = 4$ and $\epsilon = 0.05$ case.

#	ν_1	ν_2	ν_3	ν_4	ν_5	ν_6
1	0.12	-0.067	-0.067	-0.0073	-0.0073	0.0058
2	0.37	0.17	-0.078	0.026	0.012	0.00038
3	0.12	-0.067	-0.064	0.015	-0.0075	0.0058
4	0.12	-0.074	-0.053	0.015	0.014	0.058
5	0.73	0.26	0.18	0.16	-0.081	0.018
6	21	5.4	4.0	4.0	0.77	0.052

Table 7. Eigenvalues corresponding to the fixed points in table 5 for $X = 4$ and $\epsilon = 0.05$ case.

eigenvalues containing the nontrivial fixed points. As an example consider the first fixed point of table 6. Here we have a critical line with an infrared stable fixed point and five unstable directions. The critical *surface* in this case is just a line.

Another interesting example is constituted by the fixed point solution 7 of table 4. In this case we find a five-dimensional critical surface and one unstable sixth direction parallel to the y_H axis. We plot in figure 1 the projections of the renormalization group flow around this solution for a subset of two couplings at the time. The other filled small circles in the figure correspond to the projections on the chosen planes of fixed points such as the Seiberg's magnetic dual, indicated by the SUSY label on the plot, or the super QCD Banks-Zaks fixed point labelled by susyBZ.

The fixed point featuring all positive eigenvalues is the one corresponding to Seiberg's magnetic dual.

In the analysis so far the quartic interactions among the scalar fields were not considered, since they do not affect the running of the gauge and Yukawa couplings to the leading order in perturbation theory. Nevertheless, one should consider the evolution of the quartic couplings to test the emergence of supersymmetry. We test the stability of the

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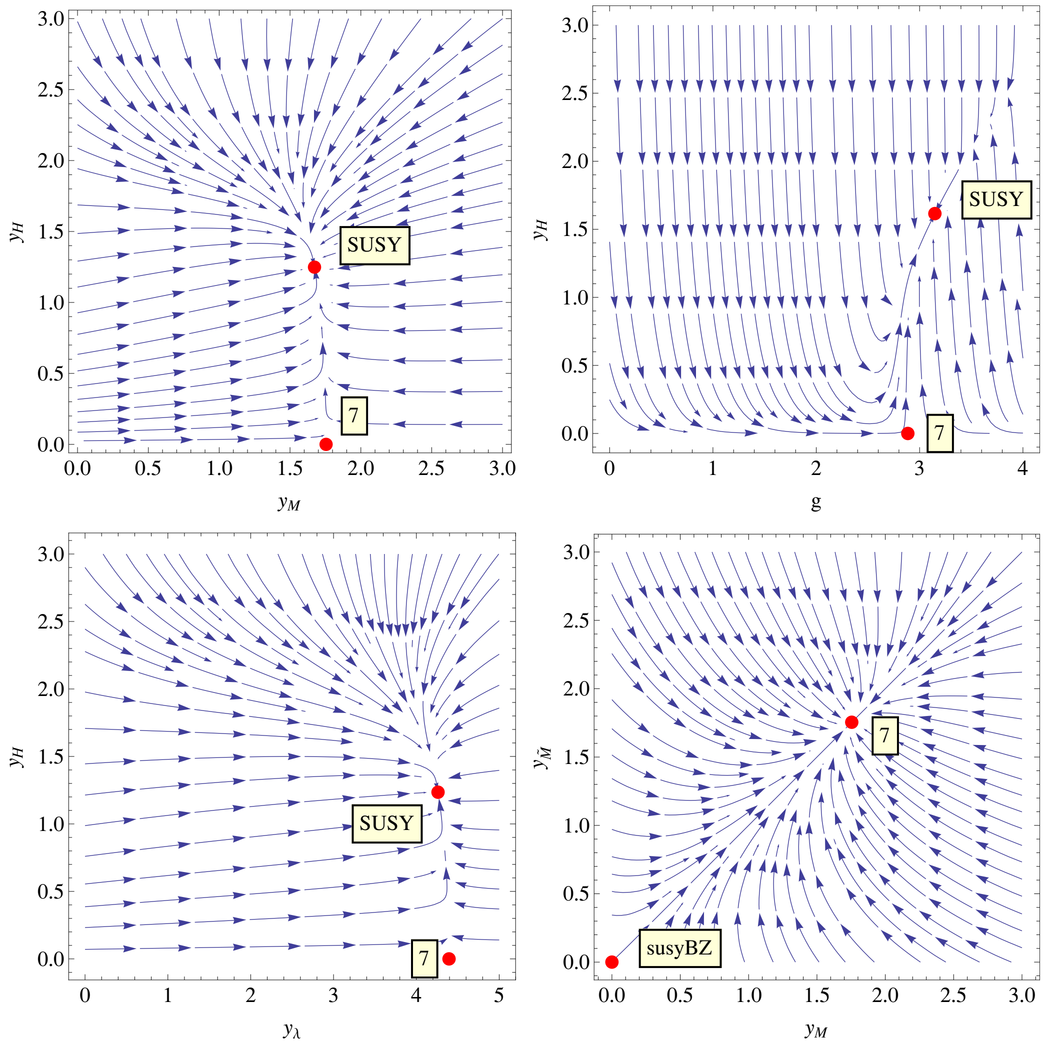

Figure 1. Projections of RG flows for $X = 4$ and $\epsilon = 0.05$ in the planes (from left to right and top to bottom): (y_M, y_H), $(g,\ y_H)$, (y_λ, y_H) and $(y_M,\ y_{\widetilde{M}})$. The other couplings are kept fixed at the fixed point value of solution 7 given in table 4. Besides solution 7 clearly indicated in the plot we also plot the projections of the Seiberg's magnetic dual indicated by the SUSY label on the plot, and the super QCD perturbative fixed point indicated by susyBZ.

quartic sector restricted to the squark-like fields ϕ and $\widetilde{\phi}$ of the magnetic theory given in table 2. The potential is:

$$\begin{aligned}\mathcal{L}_{\text{quartic}} = &-\frac{4\pi}{2} T^a_{ij} T^a_{kl} \Big[u(\phi_i^{A,*}\phi_j^A)(\phi_k^{B,*}\phi_l^B) + \widetilde{u}(\widetilde{\phi}_i^A \widetilde{\phi}_j^{A,*})(\widetilde{\phi}_k^B \widetilde{\phi}_l^{B,*}) - 2w(\phi_k^{A,*}\phi_l^A)(\widetilde{\phi}_k^{B,*}\widetilde{\phi}_l^B) \Big] \\ &- \frac{1}{2} \Big[\eta(\phi_i^{A,*}\phi_i^A)(\phi_k^{B,*}\phi_k^B) + \widetilde{\eta}(\widetilde{\phi}_i^A \widetilde{\phi}_i^{A,*})(\widetilde{\phi}_k^B \widetilde{\phi}_k^{B,*}) - 2\rho(\phi_i^{A,*}\phi_i^A)(\widetilde{\phi}_k^{B,*}\widetilde{\phi}_k^B) \Big], \end{aligned} \quad (5.2)$$

where A and B are flavor indices and i, j, k, l are gauge indices. Supersymmetric QCD is recovered for $u = \widetilde{u} = w = \alpha_g$ and $\eta = \rho = 0$.

The gauge and Yukawa couplings run to the IR supersymmetric fixed point corresponding to $\alpha_\lambda = \alpha_{\widetilde{\lambda}} = 2\alpha_g$. We assume these relations to hold to constrain the flow of the

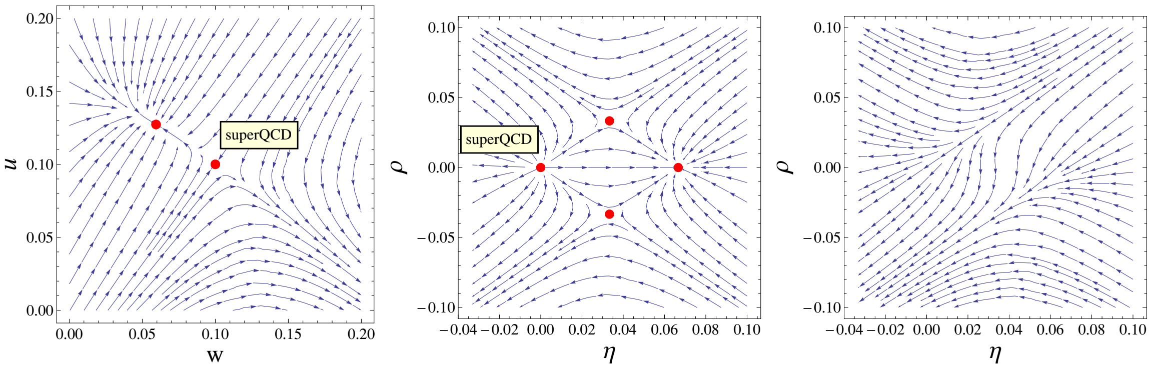

Figure 2. Renormalization group flow of quartic couplings. The dots show the various fixed points. In the u, w system (or equivalently $\widetilde{u}, w$) there are two fixed points, one is the supersymmetric fixed point $w^* = u^* = a_g$, which is stable in the direction $u = w$ and unstable in the orthogonal direction $u = -w$. The other fixed point is stable in the u, w system, which is decoupled from the other quartic couplings. For the flows in the η, ρ system (or equivalently $\widetilde{\eta}, \rho$) we have set the values of the $u, \widetilde{u}, w$ couplings to either two fixed points; the middle figure corresponds to the supersymmetric fixed point in $u, \widetilde{u}, w$, the right figure corresponds to the stable fixed point in $u, \widetilde{u}, w$. Note that the stable fixed point in $u, \widetilde{u}, w$ is unstable in the other quartic couplings.

Yukawa couplings. We then investigate the theory near the asymptotically free boundary, i.e. $N_f/X = 3$ and in the large X and N_f limit. The rescaled couplings in this limit reads

$$\alpha_g \to \alpha_g/X\,, \quad u \to u/X\,, \quad w \to w/X\,, \quad \eta \to \eta/X^2\,, \quad \rho \to \rho/X^2\,. \tag{5.3}$$

The beta functions of the rescaled quartic couplings read

$$\beta_u \equiv \frac{8\pi}{X}\frac{\mathrm{d}u}{\mathrm{d}\ln\mu^2} = u^2 + 3(u^2+w^2) - 5\alpha_g^2 - 2u\alpha_g\,, \tag{5.4}$$

$$\beta_w \equiv \frac{8\pi}{X}\frac{\mathrm{d}w}{\mathrm{d}\ln\mu^2} = -w^2 + 6wu - 3\alpha_g^2 - 2w\alpha_g\,, \tag{5.5}$$

$$\beta_\eta \equiv 4\pi\frac{\mathrm{d}\eta}{\mathrm{d}\ln\mu^2} = \frac{1}{2}(u^2-\alpha_g^2) + 3(\eta^2+\rho^2) - 2\eta\alpha_g\,, \tag{5.6}$$

$$\beta_\rho \equiv 4\pi\frac{\mathrm{d}\rho}{\mathrm{d}\ln\mu^2} = \frac{1}{2}(\alpha_g^2-w^2) + 6\rho\eta - 2\rho\alpha_g\,, \tag{5.7}$$

$$\beta_{\widetilde{u}} = \beta_u(u\to\widetilde{u})\,, \quad \beta_{\widetilde{\eta}} = \beta_\eta(u\to\widetilde{u}, \eta\to\widetilde{\eta})\,. \tag{5.8}$$

Note that the running of the couplings, $u, \widetilde{u}$ and w are independent of the other quartic couplings.

There is a fixed point with $u = \widetilde{u} = w = \alpha_g$ and $\eta = \rho = 0$ corresponding to the supersymmetric limit. The renormalization group flow reaches this supersymmetric fixed point along the direction $u = \widetilde{u} = w$, provided that the couplings η, $\widetilde{\eta}$ and ρ are kept zero. Any perturbation away from this tuning will not lead in the IR to the emergence of supersymmetry. This is summarized with more details in figure 2.

6 Physical results and conclusions

We uncovered the full spectrum of perturbative fixed points associated to a SU(X) nonsupersymmetric gauge theory featuring scalars and fermions, of the type summarized in table 3. Although the noninteracting field theory has a supersymmetric looking spectrum we did not assume the bare couplings to respect supersymmetry, but instead assumed tuning to the massless theory. This is similar to what is routinely done in condensed matter physics [4, 5]. We then analyzed the fixed points of the beta functions to leading order, which are renormalization-scheme independent. We discovered supersymmetric and nonsupersymmetric fixed points. We further analyzed their stability and discovered that in the gauge-Yukawa sector:

- When the bare dimensionless couplings are nonzero the theory flows to Seiberg's magnetic dual fixed point. This occurs on supersymmetric and nonsupersymmetric renormalization group flow.
- When the Yukawa couplings y_M, $y_{\widetilde{M}}$ and y_H are all set to zero the theory flows to the super QCD fixed point, as above, independently of whether the renormalization group flow is supersymmetric.
- When all the Yukawa couplings are set to zero we achieve the perturbative nonsupersymmetric fixed point in the gauge coupling.
- When y_M and $y_{\widetilde{M}}$ are nonzero and $y_H = 0$ we discover a new nonsupersymmetric fixed point. This can be identified with the magnetic dual fixed point for nonsupersymmetric gauge theories proposed in [3]. Another fixed point emerges when $y_M = y_{\widetilde{M}} = 0$ and $y_H \neq 0$.
- All the other fixed points emerge similarly by setting to zero different Yukawas.

We finally considered the scalar quartic sector and found the necessary conditions to have supersymmetry as an emergent phenomenon, which read:

- The supersymmetric quartic couplings must be aligned according to the supersymmetric relations, but are not necessarily in supersymmetric relation with the gauge and Yukawa couplings.
- Supersymmetry breaking quartic operators must in general be tuned to zero.

The results above indicates that supersymmetry can emerge as a fixed point theory from a nonsupersymmetric Lagrangian, once the mass operators and supersymmetry breaking operators have been properly subtracted. This relevant result demonstrates that supersymmetry can be viewed as an emergent phenomenon in field theory.[1] Our results,

[1]The phenomenon of emergent supersymmetry in four dimensions was also suggested earlier, e.g. in [48–50]. Our results are obtained using trustable perturbative computations while the earlier works make assumptions regarding nonperturbative nonsupersymmetric strong dynamics which remain to be verified.

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among other things can be used to argue that one does not need to fine-tune the bare dimensionless couplings when performing Lattice simulations aimed to study supersymmetry on the Lattice [51, 52]. As mentioned earlier similar examples of nonsupersymmetric gauge theories flowing to supersymmetric fixed points were discovered recently in lower number of space-time dimensions [4, 5] further reinforcing the possibility that supersymmetry could be an emergent phenomenon.

It is also possible to speculate that duality is not a prerogative of supersymmetry given that the theory features many nonsupersymmetric fixed points, depending on the details of the interactions. Therefore these new fixed points could describe the nonperturbative physics of electric dual gauge theories like the one envisioned in [3].

Acknowledgments

We thank Simon Catterall, Stefano Di Chiara, Marco Nardecchia and Erich Poppitz for enlightening discussions and comments.

A Derivation of the beta functions

We follow the notation of Machacek and Vaughn [45, 46], who derived the general expressions to two-loop order for the running of the gauge coupling and Yukawa couplings for a general gauge theory with real scalars and Majorana fermions. Quartic scalar couplings do not contribute to the running of the gauge and Yukawa couplings in any general field theory to the order in perturbation theory we will be working. The running of these couplings to the perturbative order we are working here are therefore irrelevant.

In ref. [47] is showed how to use the expressions of Machacek and Vaughn for complex scalars and Weyl fermions. We need only to do a slight rewriting of $\mathcal{L}_Y$ given in eq. (3.1) to follow their notation:

$$\begin{aligned}\mathcal{L}_Y &= y_\lambda \phi^* T^a q \lambda^a_m - y_{\widetilde{\lambda}} \widetilde{q} T^a \widetilde{\phi}^* \lambda^a_m + y_M \phi M \widetilde{q} + y_{\widetilde{M}} \widetilde{\phi} M q + y_H H q \widetilde{q} + h.c. \\ &= Y^{\alpha,a}_j \phi^{*j} q_\alpha \lambda^a_m + \widetilde{Y}^j_{\alpha,a} \widetilde{\phi}^*_j \widetilde{q}^\alpha \lambda^a_m + U^j_\alpha \phi_j M \widetilde{q}^\alpha + \widetilde{U}^\alpha_j \widetilde{\phi}^j M q_\alpha + V^\alpha_\beta H q_\alpha \widetilde{q}^\beta + \text{h.c.},\end{aligned} \tag{A.1}$$

where $a = 1, \ldots, d(G)$ is the gauge index reserved for the adjoint Majorana fermion with $d(G)$ the dimension of its representation, greek gauge indices $\alpha, \beta, \ldots$ are reserved for the Weyl fermions and roman gauge indices $i, j, \ldots$ are reserved for the complex scalars. Flavor indices have been suppressed. The Yukawa matrices are defines as follows, once we take flavor indices into account with l, l' denoting the indices of $\mathrm{SU(N_f)_L}$ and r, r' indices of the $\mathrm{SU(N_f)_R}$ global symmetries:

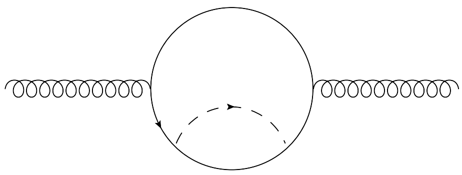
$$Y^{\alpha l,a}_{jl'} = y_\lambda (T^a)_j{}^\alpha \delta^{l'}_l, \tag{A.2a}$$
$$\widetilde{Y}^{jr'}_{\alpha r,a} = -y_{\widetilde{\lambda}} (T^a)_\alpha{}^j \delta^{r'}_r, \tag{A.2b}$$
$$U^{jl',r'}_{\alpha r,l} = y_M \delta^j_\alpha \delta^{l'}_l \delta^{r'}_r, \tag{A.2c}$$
$$\widetilde{U}^{\alpha l,r}_{jr',l'} = y_{\widetilde{M}} \delta^\alpha_j \delta^l_{l'} \delta^r_{r'}, \tag{A.2d}$$
$$V^{\alpha l',r'}_{\beta r,l} = y_H \delta^\alpha_\beta \delta^{l'}_l \delta^{r'}_r, \tag{A.2e}$$

where T^a are the generators of the gauge group. The minus sign of $y_{\widetilde{\lambda}}$-term has been chosen to facilitate the connection with supersymmetry, i.e. for SUSY $y_\lambda = y_{\widetilde{\lambda}} = i\sqrt{2}g$.

We will first derive the Yukawa contribution to the 2-loop beta function, given in eq. (3.2)–(3.5), in particular:

$$\beta_Y = \frac{1}{d(G)} \sum_r \mathrm{Tr}\left[C_2(r) Y^j {Y_j}^\dagger\right] , \tag{A.3}$$

where $C_2(r)$ is the quadratic Casimir of the spinor representations r and $d(G)$ is the dimension of the gauge group. It is instructive to consider the contribution from the Yukawa sector to the two-loop beta function of the gauge coupling diagrammatically. There is only one type of diagram that has non-vanishing contribution, which is:



where the solid line represents spinor fields, and the dashed line represents the scalar(s) coupled via the Yukawa interactions. Note that the scalar-gauge interactions do not contribute.

The fermion M is not coupled to the gauge fields. Thus we get:

$$\begin{aligned}
\frac{\beta_Y}{(4\pi)^2} &= \frac{1}{(4\pi)^2 d(G)} \mathrm{Tr}\left[C_2(\lambda)\left\{Y^j {Y_j}^\dagger + \widetilde{Y}^j \widetilde{Y}_j^\dagger\right\} + C_2(q)\left\{Y^j {Y_j}^\dagger + \widetilde{U}^j \widetilde{U}_j^\dagger + VV^\dagger\right\}\right.\\
&\qquad\qquad \left. + \, C_2(\widetilde{q})\left\{\widetilde{Y}^j \widetilde{Y}_j^\dagger + U^j {U_j}^\dagger + VV^\dagger\right\}\right]\\
&= (\alpha_\lambda + \alpha_{\widetilde{\lambda}}) T(\Box) N_f \left[C_2(\Box) + C_2(G)\right] + (\alpha_M + \alpha_{\widetilde{M}} + 2\alpha_H) N_f^2 \frac{C_2(\Box) X}{d(G)}\\
&= (\alpha_\lambda + \alpha_{\widetilde{\lambda}}) \frac{N_f}{2} \frac{3X^2 - 1}{2X} + \left(\frac{\alpha_M + \alpha_{\widetilde{M}}}{2} + \alpha_H\right) N_f^2,
\end{aligned} \tag{A.4}$$

where we used the notation

$$\alpha_i \equiv \frac{|\, y_i \,|^2}{(4\pi)^2}.$$

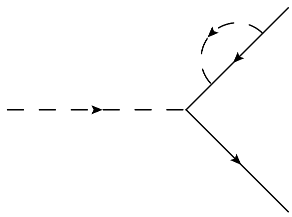
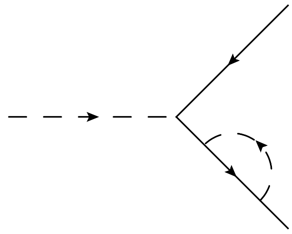
We now consider the running of the Yukawa couplings. The one loop beta function for the Yukawa couplings is given in [46, 47]:

$$\begin{aligned}
(4\pi)^2 \beta(Y^j) = \frac{1}{2}\left[Y_2^\dagger(r) Y^j + Y^j Y_2(r)\right] + 2Y^k Y_j^\dagger Y^k + \frac{1}{2} Y^k \mathrm{Tr}\left[Y_k^\dagger Y^j + Y_j^\dagger Y^k\right]&\\
- \, 3g^2 \{C_2(r), Y^j\}.&
\end{aligned} \tag{A.5}$$

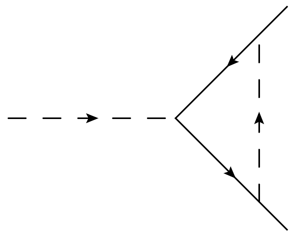
where $Y_2(r)$ is the group invariant:

$$Y_2(r) \equiv Y_k^\dagger Y^k \tag{A.6}$$

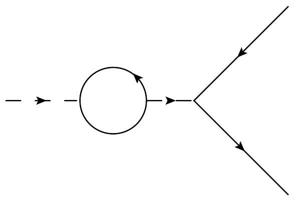
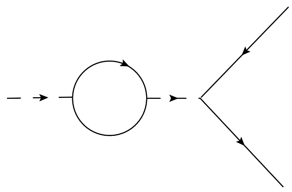
It is again instructive to consider each term in the beta function in terms of the diagrams they originate from. The first two terms in the bracket corresponds to the fermion leg self-energy contribution from the Yukawa sector, and the expression tells us that they each contribute with a factor of $\frac{1}{2}$ to the beta function:

$\frac{1}{2}$  $+\frac{1}{2}$ 

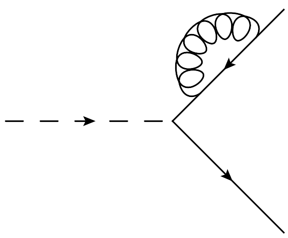
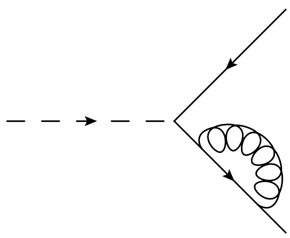
The next term in the beta function is the vertex correction from the Yukawa sector:



The fourth term is the scalar leg self energy contribution from the Yukawa sector:

 $+$ 

Note the different directions of the arrow in the fermion loops. For Dirac fermions the above contribution comes naturally with a factor of two. Finally the gauge sector has a non-vanishing contribution only to the fermion leg self energy, and each term contributes with a factor of -3:

-3  -3 

We thus find the one-loop coefficient of the beta function for Y^j to be:

$$(4\pi)^2\beta(Y^j) = \frac{1}{2}\left[\left\{Y_2^\dagger(q) + \widetilde{U}_2^\dagger(q) + V_2^\dagger(q)\right\} Y^j + Y^j\left\{Y_2(\lambda) + \widetilde{Y}_2(\widetilde{\lambda})\right\}\right] + 2\widetilde{U}^i U_j^\dagger \widetilde{Y}^i$$
$$+ Y^k \mathrm{Tr}\left[Y_k^\dagger Y^j\right] + Y^k \mathrm{Tr}\left[U^k U_j^\dagger\right] - 3g^2\left\{C_2(q)Y^j + Y^j C_2(\lambda)\right\}. \quad \text{(A.7)}$$

Inserting the definitions of the Yukawa-matrices, we find:

$$(4\pi)^2\beta(Y^j) = \left[\frac{1}{2}\left\{C_2(\square) \mid y_\lambda \mid^2 + N_f \mid \tilde{y}_M \mid^2 + N_f \mid y_H \mid^2 + T(\square) N_f (\mid y_\lambda \mid^2 + \mid y_{\widetilde{\lambda}} \mid^2)\right\}\right.$$
$$\left. + C_2(\square) \, |y_\lambda|^2 + N_f \, |y_M|^2 - 3g^2 \left\{C_2(\square) + C_2(G)\right\}\right] Y^j - 2N_f y_M^* y_{\widetilde{M}} y_{\widetilde{\lambda}} (T^a)_\alpha{}^j \delta_l^{l'}$$

Note that $(T^a)_\alpha{}^j \delta_l^{l'}$ in the last term is the matrix part of Y^j, which cancels on both sides of the equation once we do the rewriting $\beta(Y^j) = (T^a)_\alpha{}^j \delta_l^{l'} \beta(y_\lambda)$.

The beta function for $\widetilde{Y}^j$ is obtained via:

$$y_\lambda \leftrightarrow \widetilde{y}_\lambda \quad y_M \leftrightarrow \widetilde{y}_M$$

$\beta(U^j)$ and $\beta(\widetilde{U}^j)$ are related by a similar transformation. We compute the former:

$$(4\pi)^2\beta(U^j) = \frac{1}{2}\left[\left\{U_2^\dagger(M) + \widetilde{U}_2^\dagger(M)\right\} U^j + U^j \left\{U_2(\widetilde{q}) + \widetilde{Y}_2(\widetilde{q}) + V_2(\widetilde{q})\right\}\right] + 2\widetilde{Y}^i Y_j^\dagger \widetilde{U}^i$$
$$+ U^k \mathrm{Tr}\left[U_k^\dagger U^j\right] + U^k \mathrm{Tr}\left[Y_k^\dagger Y^j\right] - 3g^2 U^j C_2(\widetilde{q}). \tag{A.8}$$

Using the expression for the matrices we deduce:

$$\beta(U^j) = \left[\frac{1}{2}\left\{X\alpha_M + X\alpha_{\widetilde{M}} + N_f\alpha_M + C_2(\square)\alpha_{\widetilde{\lambda}} + N_f\alpha_H\right\}\right.$$
$$\left. + N_f\alpha_M + C_2(\square)\alpha_\lambda - 3\alpha_g C_2(\square)\right] U^j - \frac{2C_2(\square)}{(4\pi)^2} y_{\widetilde{\lambda}} y_\lambda^* y_{\widetilde{M}} \delta_\alpha^j \delta_l^{l'} \delta_r^{r'}, \tag{A.9}$$

where we used $\alpha_i = \mid y_i \mid^2 / (4\pi)^2$. As before $\delta_\alpha^j \delta_l^{l'} \delta_r^{r'}$ is the matrix part of $\mathrm{U^j}$, which cancels on both sides of the equation.

The renormalization of the y_H coupling reads:

$$(4\pi)^2\beta(V) = \left[\frac{1}{2}\left\{Y_2^\dagger(q) + \widetilde{U}_2^\dagger(q) + V_2^\dagger(q)\right\} V + V\left\{U_2(\widetilde{q}) + \widetilde{Y}_2(\widetilde{q}) + V_2(\widetilde{q})\right\}\right]$$
$$+ V\mathrm{Tr}\left[V^\dagger V\right] - 3g^2\left\{C_2(q)V + VC_2(\widetilde{q})\right\}$$
$$\beta(V) = \left[\frac{\alpha_M + \alpha_{\widetilde{M}} + 2\alpha_H}{2} N_f + \left(\frac{\alpha_\lambda + \alpha_{\widetilde{\lambda}}}{2} - 6\alpha_g\right) C_2(\square) + X\alpha_H\right] V \tag{A.10}$$

6.1	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	0	1.0	0	0
ν_2	0	0	1.0	0	0	0
ν_3	0	0	0	0	0	1.0
ν_4	0	1.0	0	0	0	0
ν_5	1.0	0	0	0	0	0
ν_6	0	0	0	0	1.0	0

6.2	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	0	−53.	1.0	0
ν_2	0	0	1.0	0	0	0
ν_3	0	0	0	0	0	1.0
ν_4	1.0	0	0	0	0	0
ν_5	0	0	0	1.8	1.0	0
ν_6	0	1.0	0	0	0	0

6.3	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	1.0	0	0	0
ν_2	0	0	0	1.0	0	0
ν_3	0	−46.	0	0	1.0	0
ν_4	0	0	0	0	0	1.0
ν_5	1.0	0	0	0	0	0
ν_6	0	0.61	0	0	1.0	0

Table B.1. Eigenvectors associated to ν_i given in table 6 and corresponding to the fixed points 1,2 and 3 of table 4.

We summarize now the results for beta functions, after having cancelled the matrices:

$$\beta(\alpha_g) = -2\alpha_g \left[\beta_0 + \alpha_g\beta_1 + N_f T(\square)\left[C_2(\square) + C_2(G)\right](\alpha_\lambda + \alpha_{\widetilde{\lambda}}) + \frac{C_2(\square) X N_f^2}{d(G)}(\alpha_M + \alpha_{\widetilde{M}} + 2\alpha_H)\right] \tag{A.11a}$$

$$\begin{aligned}\beta(y_\lambda) = &|y_\lambda| e^{i\theta_\lambda}\left[\frac{3}{2}C_2(\square)\alpha_\lambda + \frac{T(\square)N_f}{2}(\alpha_\lambda + \alpha_{\widetilde{\lambda}}) + N_f\left(\frac{\alpha_{\widetilde{M}} + \alpha_H}{2} + \alpha_M\right)\right.\\ &\left. -3\alpha_g\{C_2(\square) + C_2(\mathrm{G})\}\right] - 2N_f\frac{|y_M||y_{\widetilde{M}}||y_{\widetilde{\lambda}}|}{(4\pi)^2}e^{i(\theta_{\widetilde{\lambda}} + \Delta\theta_M)}\end{aligned} \tag{A.11b}$$

$$\begin{aligned}\beta(y_M) = &| y_M | e^{i\theta_M}\left[\frac{3}{2}N_f\left(\alpha_M + \frac{\alpha_H}{3}\right) + \frac{X}{2}(\alpha_M + \alpha_{\widetilde{M}}) + C_2(\square)\left(\frac{\alpha_{\widetilde{\lambda}}}{2} + \alpha_\lambda\right) - 3\alpha_g C_2(\square)\right]\\ &- 2C_2(\square)\frac{|y_{\widetilde{M}}||y_\lambda||y_{\widetilde{\lambda}}|}{(4\pi)^2}e^{i(\theta_{\widetilde{M}} + \Delta\theta_\lambda)}\end{aligned} \tag{A.11c}$$

$$\beta(\alpha_H) = 2\alpha_H\left[\frac{N_f}{2}(\alpha_M + \alpha_{\widetilde{M}} + 2\alpha_H) + C_2(\square)\left(\frac{\alpha_\lambda + \alpha_{\widetilde{\lambda}}}{2} - 6\alpha_g\right) + X\alpha_H\right] \tag{A.11d}$$

$$\beta(\alpha_{\widetilde{\lambda}}) = \beta_{\alpha_\lambda}\left(y_\lambda \leftrightarrow y_{\widetilde{\lambda}}, y_{\widetilde{M}} \leftrightarrow y_M\right), \quad \beta(\alpha_{\widetilde{M}}) = \beta_{\alpha_M}\left(y_M \leftrightarrow y_{\widetilde{M}}, y_{\widetilde{\lambda}} \leftrightarrow y_\lambda\right), \tag{A.11e}$$

where we have made explicit the phases of the Yukawa's couplings:

$$\begin{gathered} y_i = | y_i | e^{i\theta_i} \\ \Delta\theta_M = \theta_{\widetilde{M}} - \theta_M, \qquad \Delta\theta_\lambda = \theta_{\widetilde{\lambda}} - \theta_\lambda. \end{gathered}$$

Note also that we define

$$\beta(\alpha_i) \equiv \frac{2\bar{y}_i}{(4\pi)^2}\beta(y_i),$$

where $\bar{y}_i$ is the complex conjugate of y_i.

We have considered the case of real couplings in the main text.

B Tables

We provide here the eigenvectors associated to the eigenvalues given in tables 6 and 7. Shaded cells indicate unstable directions, i.e. corresponding to negative eigenvalues.

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6.4	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	1.0	1.0	0	0
ν_2	−23.	−23.	0	0	1.0	0
ν_3	−1.0	1.0	0	0	0	0
ν_4	0	0	0	0	0	1.0
ν_5	0	0	−1.0	1.0	0	0
ν_6	0.58	0.58	0	0	1.0	0

6.5	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	−22.	−22.	1.0	0
ν_2	0	0	−1.0	1.0	0	0
ν_3	−1.0	1.0	0	0	0	0
ν_4	1.0	1.0	0	0	0	0
ν_5	0	0	0	0	0	1.0
ν_6	0	0	1.5	1.5	1.0	0

Table B.2. Eigenvectors associated to ν_i given in table 6 and corresponding to the fixed points 4 and 5 of table 4.

6.6	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	−2.2	0	0	−49.	1.0	0
ν_2	0	0.22	1.0	0	0	0
ν_3	0	0	0	0	0	1.0
ν_4	−40.	0	0	13.	1.0	0
ν_5	0	−0.73	1.0	0	0	0
ν_6	0.47	0	0	1.8	1.0	0

6.7	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	−0.035	−0.035	−3.1	−3.1	1.0	0
ν_2	−0.41	0.41	−1.0	1.0	0	0
ν_3	0.40	−0.40	−1.0	1.0	0	0
ν_4	−2.0	−2.0	1.7	1.7	1.0	0
ν_5	0	0	0	0	0	1.0
ν_6	0.64	0.64	1.6	1.6	1.0	0

Table B.3. Eigenvectors associated to ν_i given in table 6 and corresponding to the fixed points 6 and 7 of table 4.

7.1	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	0	0	−0.052	1.0
ν_2	0	0	0	1.0	0	0
ν_3	0	0	1.0	0	0	0
ν_4	0	1.0	0	0	0	0
ν_5	1.0	0	0	0	0	0
ν_6	0	0	0	0	1.1	1.0

7.2	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	0	5.2	−0.28	1.0
ν_2	0	0	0	−1.3	−0.057	1.0
ν_3	0	0	1.0	0	0	0
ν_4	0	1.0	0	0	0	0
ν_5	0	0	0	2.0	1.2	1.0
ν_6	1.0	0	0	0	0	0

Table B.4. Eigenvectors associated to ν_i given in table 7 and corresponding to the fixed points 1 and 2 of table 5.

7.3	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0.15	0	0	−0.054	1.0
ν_2	0	0	1.0	0	0	0
ν_3	0	0	0	1.0	0	0
ν_4	0	−6.0	0	0	0.094	1.0
ν_5	1.0	0	0	0	0	0
ν_6	0	0.47	0	0	1.2	1.0

7.4	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0.16	0.16	0	0	−0.057	1.0
ν_2	0	0	1.0	1.0	0	0
ν_3	0	0	−1.0	1.0	0	0
ν_4	−1.0	1.0	0	0	0	0
ν_5	−2.9	−2.9	0	0	0.093	1.0
ν_6	0.49	0.49	0	0	1.2	1.0

Table B.5. Eigenvectors associated to ν_i given in table 7 and corresponding to the fixed points 3 and 4 of table 5.

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7.5	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0	0	3.9	3.9	−0.54	1.0
ν_2	0	0	−0.93	−0.93	−0.076	1.0
ν_3	0	0	−1.0	1.0	0	0
ν_4	−1.0	1.0	0	0	0	0
ν_5	1.0	1.0	0	0	0	0
ν_6	0	0	1.8	1.8	1.3	1.0

7.6	y_M	$y_{\widetilde{M}}$	y_λ	$y_{\widetilde{\lambda}}$	g	y_H
ν_1	0.41	0.41	5.1	5.1	−7.4	1.0
ν_2	−0.44	0.44	−1.0	1.0	0	0
ν_3	0.37	−0.37	−1.0	1.0	0	0
ν_4	1.7	1.7	−3.8	−3.8	−1.2	1.0
ν_5	−0.38	−0.38	−0.26	−0.26	0.072	1.0
ν_6	1.1	1.1	2.8	2.8	1.9	1.0

Table B.6. Eigenvectors associated to ν_i given in table 7 and corresponding to the fixed points 5 and 6 of table 5.

References

[1] N. Seiberg, *Exact results on the space of vacua of four-dimensional SUSY gauge theories*, *Phys. Rev.* **D 49** (1994) 6857 [hep-th/9402044] [INSPIRE].

[2] N. Seiberg, *Electric-magnetic duality in supersymmetric nonAbelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149] [INSPIRE].

[3] M. Mojaza, M. Nardecchia, C. Pica and F. Sannino, *Dual of QCD with One Adjoint Fermion*, *Phys. Rev.* **D 83** (2011) 065022 [arXiv:1101.1522] [INSPIRE].

[4] S.-S. Lee, *Emergence of supersymmetry at a critical point of a lattice model*, *Phys. Rev.* **B 76** (2007) 075103 [cond-mat/0611658] [INSPIRE].

[5] S.-S. Lee, *TASI Lectures on Emergence of Supersymmetry, Gauge Theory and String in Condensed Matter Systems*, arXiv:1009.5127 [INSPIRE].

[6] F. Sannino, *QCD Dual*, *Phys. Rev.* **D 80** (2009) 065011 [arXiv:0907.1364] [INSPIRE].

[7] G. 't Hooft, *Recent Developments in Gauge Theories*, Plenum Press, (1980), pg. 135; reprinted in *Unity of Forces in the Universe Vol. II*, A. Zee ed., World Scientific, (1982), pg. 1004.

[8] J. Terning, *'t Hooft anomaly matching for QCD*, *Phys. Rev. Lett.* **80** (1998) 2517 [hep-th/9706074] [INSPIRE].

[9] F. Sannino, *Higher Representations Duals*, *Nucl. Phys.* **B 830** (2010) 179 [arXiv:0909.4584] [INSPIRE].

[10] F. Sannino and K. Tuominen, *Orientifold theory dynamics and symmetry breaking*, *Phys. Rev.* **D 71** (2005) 051901 [hep-ph/0405209] [INSPIRE].

[11] D.K. Hong, S.D. Hsu and F. Sannino, *Composite Higgs from higher representations*, *Phys. Lett.* **B 597** (2004) 89 [hep-ph/0406200] [INSPIRE].

[12] H. Gies and J. Jaeckel, *Chiral phase structure of QCD with many flavors*, *Eur. Phys. J.* **C 46** (2006) 433 [hep-ph/0507171] [INSPIRE].

[13] J. Braun and H. Gies, *Running coupling at finite temperature and chiral symmetry restoration in QCD*, *Phys. Lett.* **B 645** (2007) 53 [hep-ph/0512085] [INSPIRE].

[14] J. Braun and H. Gies, *Chiral phase boundary of QCD at finite temperature*, *JHEP* **06** (2006) 024 [hep-ph/0602226] [INSPIRE].

[15] D.D. Dietrich and F. Sannino, *Conformal window of* $\mathrm{SU}(N)$ *gauge theories with fermions in higher dimensional representations*, Phys. Rev. **D 75** (2007) 085018 [hep-ph/0611341] [INSPIRE].

[16] T.A. Ryttov and F. Sannino, *Conformal Windows of* $\mathrm{SU}(N)$ *Gauge Theories, Higher Dimensional Representations and The Size of The Unparticle World*, Phys. Rev. **D 76** (2007) 105004 [arXiv:0707.3166] [INSPIRE].

[17] T.A. Ryttov and F. Sannino, *Supersymmetry inspired QCD* β*-function*, Phys. Rev. **D 78** (2008) 065001 [arXiv:0711.3745] [INSPIRE].

[18] F. Sannino, *Dynamical Stabilization of the Fermi Scale: Phase Diagram of Strongly Coupled Theories for (Minimal) Walking Technicolor and Unparticles*, arXiv:0804.0182 [INSPIRE].

[19] E. Poppitz and M. Ünsal, *Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD*, JHEP **12** (2009) 011 [arXiv:0910.1245] [INSPIRE].

[20] F. Sannino, *Conformal Dynamics for TeV Physics and Cosmology*, Acta Phys. Polon. **B 40** (2009) 3533 [arXiv:0911.0931] [INSPIRE].

[21] J. Braun and H. Gies, *Scaling laws near the conformal window of many-flavor QCD*, JHEP **05** (2010) 060 [arXiv:0912.4168] [INSPIRE].

[22] O. Antipin and K. Tuominen, *Resizing the Conformal Window: A* β*-function Ansatz*, Phys. Rev. **D 81** (2010) 076011 [arXiv:0909.4879] [INSPIRE].

[23] O. Antipin and K. Tuominen, *Constraints on Conformal Windows from Holographic Duals*, Mod. Phys. Lett. **A 26** (2011) 2227 [arXiv:0912.0674] [INSPIRE].

[24] M. Jarvinen and F. Sannino, *Holographic Conformal Window — A Bottom Up Approach*, JHEP **05** (2010) 041 [arXiv:0911.2462] [INSPIRE].

[25] M. Mojaza, C. Pica and F. Sannino, *Hot Conformal Gauge Theories*, Phys. Rev. **D 82** (2010) 116009 [arXiv:1010.4798] [INSPIRE].

[26] J. Alanen, K. Kajantie and K. Tuominen, *Thermodynamics of quasiconformal theories from gauge/gravity duality*, Phys. Rev. **D 82** (2010), no. 5 055024 [arXiv:1003.5499].

[27] H.S. Fukano and F. Sannino, *Conformal window of gauge theories with four-fermion interactions and ideal walking technicolor*, Phys. Rev. **D 82** (2010), no. 3 035021 [arXiv:1005.3340].

[28] C. Pica and F. Sannino, β*-function and Anomalous Dimensions*, Phys. Rev. **D 83** (2011) 116001 [arXiv:1011.3832] [INSPIRE].

[29] C. Pica and F. Sannino, *Ultraviolet and infrared zeros of gauge theories at the four-loop order and beyond*, Phys. Rev. **D 83** (2011), no. 3 035013 [arXiv:1011.5917].

[30] M.T. Frandsen, T. Pickup and M. Teper, *Delineating the conformal window*, Phys. Lett. **B 695** (2011) 231 [arXiv:1007.1614] [INSPIRE].

[31] T.A. Ryttov and R. Shrock, *Higher-Loop Corrections to the Infrared Evolution of a Gauge Theory with Fermions*, Phys. Rev. **D 83** (2011) 056011 [arXiv:1011.4542] [INSPIRE].

[32] N. Chen, T.A. Ryttov and R. Shrock, *Patterns of Dynamical Gauge Symmetry Breaking*, Phys. Rev. **D 82** (2010) 116006 [arXiv:1010.3736] [INSPIRE].

[33] T.A. Ryttov and R. Shrock, *Infrared Evolution and Phase Structure of a Gauge Theory Containing Different Fermion Representations*, Phys. Rev. **D 81** (2010) 116003 [*Erratum ibid.* **D 82** (2010) 059903] [arXiv:1006.0421] [INSPIRE].

JHEP06(2013)037

[34] T.A. Ryttov and R. Shrock, *Higher extended technicolor representations and fermion generations*, *European Physical Journal C* **71** (2011) 1523 [arXiv:1005.3844].

[35] J. Braun, C.S. Fischer and H. Gies, *Beyond Miransky Scaling*, *Phys. Rev.* **D 84** (2011) 034045 [arXiv:1012.4279] [INSPIRE].

[36] M. Jarvinen and F. Sannino, *Extreme Technicolor and The Walking Critical Temperature*, *JHEP* **02** (2011) 081 [arXiv:1009.5380] [INSPIRE].

[37] F. Sannino, *Conformal windows of Sp(2N) and* SO(N) *gauge theories*, *Phys. Rev.* **D 79** (2009), no. 9 096007 [arXiv:0902.3494].

[38] T.A. Ryttov and F. Sannino, *Conformal House*, *Int. J. Mod. Phys.* **A 25** (2010) 4603 [arXiv:0906.0307] [INSPIRE].

[39] F. Sannino, *Mass deformed S parameter in conformal theories*, *Phys. Rev.* **D 82** (2010), no. 8 081701 [arXiv:1006.0207].

[40] F. Sannino, *Magnetic S Parameter*, *Physical Review Letters* **105** (2010), no. 23 232002 [arXiv:1007.0254].

[41] S. Di Chiara, C. Pica and F. Sannino, *Flavor Dependence of the S-parameter*, *Phys. Lett.* **B 700** (2011) 229 [arXiv:1008.1267] [INSPIRE].

[42] F. Sannino, *The Standard Model is Natural as Magnetic Gauge Theory*, *Mod. Phys. Lett.* **A 26** (2011) 1763 [arXiv:1102.5100] [INSPIRE].

[43] S.R. Coleman and E.J. Weinberg, *Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*, *Phys. Rev.* **D 7** (1973) 1888 [INSPIRE].

[44] E. Gildener and S. Weinberg, *Symmetry Breaking and Scalar Bosons*, *Phys. Rev.* **D 13** (1976) 3333 [INSPIRE].

[45] M.E. Machacek and M.T. Vaughn, *Two Loop Renormalization Group Equations in a General Quantum Field Theory. 1. Wave Function Renormalization*, *Nucl. Phys.* **B 222** (1983) 83 [INSPIRE].

[46] M.E. Machacek and M.T. Vaughn, *Two Loop Renormalization Group Equations in a General Quantum Field Theory. 2. Yukawa Couplings*, *Nucl. Phys.* **B 236** (1984) 221 [INSPIRE].

[47] M. Luo, H. Wang and Y. Xiao, *Two-loop renormalization group equations in general gauge field theories*, *Phys. Rev.* **D 67** (2003), no. 6 065019 [arXiv:hep-ph/0211440].

[48] M.J. Strassler, *Nonsupersymmetric theories with light scalar fields and large hierarchies*, hep-th/0309122 [INSPIRE].

[49] H.-S. Goh, M.A. Luty and S.-P. Ng, *Supersymmetry without supersymmetry*, *JHEP* **01** (2005) 040 [hep-th/0309103] [INSPIRE].

[50] R. Sundrum, *SUSY Splits, But Then Returns*, *JHEP* **01** (2011) 062 [arXiv:0909.5430] [INSPIRE].

[51] S. Catterall, E. Dzienkowski, J. Giedt, A. Joseph and R. Wells, *Perturbative renormalization of lattice $N = 4$ super Yang-Mills theory*, *JHEP* **04** (2011) 074 [arXiv:1102.1725] [INSPIRE].

[52] S. Catterall, *Supersymmetric lattices*, arXiv:1005.5346 [INSPIRE].

JHEP06(2013)037