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Comments about Hamiltonian formulation of non-linear massive gravity with Stückelberg fields

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Abstract: We perform the Hamiltonian analysis of some form of the non-linear massive gravity action that is formulated in the Stückelberg formalism. Following seminal analysis performed in arXiv:1203.5283 [hep-th] we find that this theory possesses one primary constraint which could eliminate one additional mode in this theory. We performed the explicit Hamiltonian analysis of two dimensional non-linear massive gravity and we found that this is theory free from the ghosts.

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Contents

1 Introduction

One of the most challenging problem is to find consistent formulation of massive gravity. The first attempt for construction of this theory is dated to the year 1939 when Fierz and Pauli formulated its version of linear massive gravity [1].[1] However it is very non-trivial task to find a consistent non-linear generalization of given theory and it remains as an intriguing theoretical problem. It is also important to stress that recent discovery of dark energy and associated cosmological constant problem has prompted investigations in the long distance modifications of General Relativity , for review, see [3].

Returning to the theories of massive gravity we should mention that these theories suffer from the problem of the ghost instability, for very nice review, see [4]. Since the General Relativity is completely constrained system there are four constraint equations along the four general coordinate transformations that enable to eliminate four of the six propagating modes of the metric, where the propagating mode corresponds to a pair of conjugate variables. As a result the number of physical degrees of freedom is equal to two which corresponds to the massless graviton degrees of freedom. On the other hand in case of the massive gravity the diffeomorphism invariance is lost and hence the theory contains six propagating degrees of freedom which only five correspond to the physical polarizations of the massive graviton while the additional mode is ghost.

It is natural to ask the question whether it is possible to construct theory of massive gravity where one of the constraint equation and associated secondary constraint eliminates the propagating scalar mode. It is remarkable that linear Fierz-Pauli theory does not suffer from the presence of such a ghost. On the other hand it was shown by Boulware and Deser [5] that ghosts generically reappear at the non-linear level. On the other hand it was shown recently by de Rham and Gobadadze in [6] that it is possible to find such a

[1] For review, see [2].

formulation of the massive gravity which is ghost free in the decoupling limit. Then it was shown in [7] that this action that was written in the perturbative form can be resumed into fully non-linear actions.[2] The general analysis of the constraints of given theory has been performed in [8]. It was argued there that it is possible to perform such a redefinition of the shift function so that the resulting theory still contains the Hamiltonian constraint. Then it was argued that the presence of this constraint allows to eliminate the scalar mode and hence the resulting theory is the ghost free massive gravity. However this analysis was questioned in [31] where it was argued that it is possible that this constraint is the second class constraint so that the phase space of given theory would be odd dimensional. On the other hand in the recent paper [32] very nice analysis of the Hamiltonian formulation of the most general gauge fixed non-linear massive gravity actions was performed with an important conclusion that the Hamiltonian constraints has zero Poisson brackets. Then the requirement of the preservation of this constraint during the time evolution of the system implies an additional constraint. As a result given theory has the right number of constraints for the construction of non-linear massive gravity without additional scalar mode.[3]

All these results suggest that the gauge fixed form of the non-linear massive gravity actions could be ghost free theory. On the other hand the manifest diffeomorphism invariance is lacking and one would like to confirm the same result in the gauge invariant formulation of the massive gravity action using the Stückelberg fields. In fact, it was argued in [30] that for some special cases such a theory possesses an additional primary constraint whose presence implies such a constraint structure of given theory that could eliminate one additional scalar mode. However the limitation of this analysis is that it was not performed for the general metric so that one can still ask the question whether the elimination of the ghost mode occurs in general case as well. In fact, there is a well known example of the theory which seems to be consistent around some background while it is pathological for general background which is the Fierz-Paull theory [1] which is ghost free around the flat background while it contains ghosts in general background [5]. This paper is the first step in the Hamiltonian analysis of Stückelberg formulation of the non-linear massive gravity with general metric. More precisely, we consider one particular class of the non-linear massive action that was presented in [8, 9]. This action is sufficiently simple to be able to find the explicit relation between time derivatives of scalar fields and conjugate momenta while it possesses all interesting properties of the non-linear massive gravity as was shown in [32, 33].[4]

We find the Hamiltonian form of given action and determine primary and secondary constraints of given theory. Then we show that due to the structure of the non-linear massive gravity action this theory possesses one primary constraint that is a consequence of the fact that $\det V^{AB} = \det(\partial_i\phi^A\partial_j\phi^B g^{ij}) = 0)$ as was firstly shown in [29].[5] This result

[2]For related works, see [10–28, 40, 41].

[3]Alternative arguments for the existence of an additional constraints were given in [33] even if the Hamiltonian analysis was not complete and the minimal non-linear massive gravity action was considered only.

[4]We should stress that the bimetric non-linear massive gravity is also manifestly covariant form of the non-linear massive gravity action [20, 21, 32]. In fact, it was argued in these papers that these manifestly diffeomorphism invariant massive gravities are ghost free as well.

[5]We should stress that in the previous versions of given paper the constraint $\det V^{AB} = 0$ was not taken into account and hence wrong conclusions were reached.

JHEP06(2012)170

has a crucial consequence for the structure of the theory. On the one hand this constraint could provide a mechanism for the elimination of an additional scalar mode however on the other hand the condition $\det V^{AB} = 0$ makes the calculation of the algebra of the Hamiltonian constraints very difficult. In fact, we were not able to calculate this algebra in the full generality with exception of the two dimensional case where however two dimensional gravity is trivial. More precisely, the Hamiltonian treatment of two dimensional non-linear massive gravity shows that there are no physical degrees of freedom left and this result coincides with the analysis performed recently in [16, 30].

The structure of this paper is as follows. In the next section 2 we review the main properties of non-linear massive gravity and rewrite it into more tractable form that is suitable for the Hamiltonian analysis which will be performed in section 3. In section 4 we perform the Hamiltonian analysis of two dimensional non-linear massive gravity action. In conclusion 5 we outline our results and suggest possible extension of this work. Finally in the appendix A we perform the calculation of the Poisson brackets of the constraints when we do not impose the condition $\det V^{AB} = 0$. Of course this is not the case of the non-linear massive gravity action but we include this appendix in order to show the complexity of given calculation even if we should again stress that this should be considered as a toy model calculation.

2 Non-linear massive gravity

Our goal is to study non-linear massive gravity action [7] in the version that appears in [8, 9]

$$S = M_p^2 \int d^4x \sqrt{-\hat{g}}^{(4)}R(\hat{g}) - \frac{1}{4}M_p^2 m^2 \int d^4x \sqrt{-\hat{g}}\mathcal{U}(\hat{g}^{-1}f)\,. \tag{2.1}$$

Note that by definition $\hat{g}^{\mu\nu}$ and $f_{\mu\nu}$ transform under general diffeomorphism transformations $x'^{\mu} = x'^{\mu}(x)$ as

$$\hat{g}'^{\mu\nu}(x') = \hat{g}^{\rho\sigma}(x)\frac{\partial x'^{\mu}}{\partial x^{\rho}}\frac{\partial x'^{\nu}}{\partial x^{\sigma}}\,, \quad f'_{\mu\nu}(x') = f_{\rho\sigma}(x)\frac{\partial x^{\rho}}{\partial x'^{\mu}}\frac{\partial x^{\sigma}}{\partial x'^{\nu}}\,. \tag{2.2}$$

Now the requirement that the combination $\hat{g}^{-1}f$ has to be diffeomorphism invariant implies that the potential $\mathcal{U}$ has to contain the trace over space-time indices. Further, it is convenient to parameterize the tensor $f_{\mu\nu}$ using four scalar fields ϕ^A and some fixed auxiliary metric $\bar{f}_{\mu\nu}(\phi)$ so that

$$f_{\mu\nu} = \partial_\mu\phi^A\partial_\nu\phi^B\bar{f}_{AB}(\phi)\,, \tag{2.3}$$

where the metric f_{AB} is invariant under diffeomorphism transformation $x'^{\mu} = x^{\mu}(x')$ which however transforms as a tensor under reparametrizations of ϕ^A. In what follows we consider $\bar{f}_{AB} = \eta_{AB}$, where $\eta_{AB} = \mathrm{diag}(-1, 1, 1, 1)$.

The fundamental ingredient of the non-linear massive gravity is the potential term. The most general forms of this potential were derived in [7, 9]. Let us consider the minimal

JHEP06(2012)170

form of the potential introduced in [7]

$$\begin{aligned}\mathcal{U}(g,H) &= -4\left(\langle\mathcal{K}\rangle^2-\langle\mathcal{K}^2\rangle\right) = \\ &= -4\left(\sum_{n\geq 1} d_n\langle H^n\rangle\right)^2 - 8\sum_{n\geq 2} d_n\langle H^n\rangle\,,\end{aligned} \tag{2.4}$$

where we now have

$$\begin{aligned}&H_{\mu\nu}=\hat{g}_{\mu\nu}-\partial_\mu\phi^A\partial_\nu\phi^B\eta_{AB}\,, && H^\mu_\nu=\hat{g}^{\mu\alpha}H_{\alpha\nu}\,,\\ &\mathcal{K}^\mu_\nu=\delta^\mu_\nu-\sqrt{\delta^\mu_\nu-H^\mu_\nu}=-\sum_{n=1}^{\infty}d_n(H^n)^\mu_\nu\,, && d_n=\frac{(2n)!}{(1-2n)(n!)^2 4^n}\,.\end{aligned} \tag{2.5}$$

and where $(H^n)^\mu_\nu = H^\mu_{\alpha_1}H^{\alpha_1}_{\alpha_2}\dots H^{\alpha_{n-1}}_\nu$. Using this explicit form of the potential we present an important observation related to the fact that the potential has to be defined using the trace over curved space-time indices which is a consequence of the requirement of the diffeomorphism invariance of the non-linear massive gravity action written in the Stückelberg field. Then we observe

$$\begin{aligned}H^\mu_\mu &= \delta^\mu_\mu-\hat{g}^{\mu\rho}\partial_\rho\phi^A\partial_\mu\phi_A\equiv\delta^A_A-\mathbf{A}^A{}_A=\mathcal{H}^A{}_A\,,\quad \mathcal{H}^A{}_B=\delta^A_B-\hat{g}^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi_B\,,\\ H^\mu_\nu H^\nu_\mu &= \delta^\mu_\mu-\eta^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi_A-\eta^{\mu\nu}\partial_\nu\phi^A\partial_\mu\phi_A+(\eta^{\mu\nu}\partial_\nu\phi^A\partial_\mu\phi_B)(\eta^{\sigma\nu}\partial_\sigma\phi^B\partial_\nu\phi_A)=\\ &= (\delta^A_B-\mathbf{A}^A{}_B)(\delta^B_A-\mathbf{A}^B{}_A)=\mathcal{H}^A{}_B\mathcal{H}^B{}_A\,,\\ \langle H^n\rangle^\mu_\mu &= H^\mu_{\nu_1}H^{\nu_1}_{\mu_2}\dots H^{\mu_{n-1}}_\nu=\mathcal{H}^A_{B_1}\mathcal{H}^{B_1}_{B_2}\dots\mathcal{H}^{B_{n-1}}_A\,.\end{aligned} \tag{2.6}$$

These observation implies that the potential can be written in the form

$$\begin{aligned}\mathcal{U} &= -4\left(\sum_{n\geq 1}d_n\langle\mathcal{H}^n\rangle\right)^2-8\sum_{n\geq 2}d_n\langle\mathcal{H}^n\rangle=\\ &= -4(\langle\hat{\mathcal{K}}\rangle^2-\langle\hat{\mathcal{K}}^2\rangle)=-4(\hat{\mathcal{K}}^A{}_A)^2+4\hat{\mathcal{K}}^A{}_B\hat{\mathcal{K}}^B{}_A\,,\end{aligned} \tag{2.7}$$

where we defined

$$\hat{\mathcal{K}}^A{}_B=\delta^A_B-\sqrt{\delta^A_B-\mathbf{A}^A{}_B}\,. \tag{2.8}$$

We mean that the trace over Lorentz indices is more convenient for the Hamiltonian treatment since we can easily implement the ADM decomposition of the space time metric. Explicitly, we use 3+1 notation [34][6] and write the four dimensional metric components as

$$\begin{aligned}&\hat{g}_{00}=-N^2+N_ig^{ij}N_j\,, && \hat{g}_{0i}=N_i\,, && \hat{g}_{ij}=g_{ij}\,,\\ &\hat{g}^{00}=-\frac{1}{N^2}\,, && \hat{g}^{0i}=\frac{N^i}{N^2}\,, && \hat{g}^{ij}=g^{ij}-\frac{N^iN^j}{N^2}\,.\end{aligned} \tag{2.9}$$

Note also that 4−dimensional scalar curvature has following decomposition

$${}^{(4)}R=K_{ij}\mathcal{G}^{ijkl}K_{kl}+{}^{(3)}R\,, \tag{2.10}$$

[6] For review, see [35].

where ${}^{(3)}R$ is three-dimensional spatial curvature, K_{ij} is extrinsic curvature defined as

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i)\,, \tag{2.11}$$

where ∇_i is covariant derivative built from the metric components g_{ij}. Note also that $\mathcal{G}^{ijkl}$ is de Witt metric defined as

$$\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - g^{ij}g^{kl}\,, \quad \mathcal{G}_{ijkl} = \frac{1}{2}(g_{ik}g_{jl} + g_{il}g_{jk}) - \frac{1}{2}g_{ij}g_{kl}\,. \tag{2.12}$$

Finally note that in (2.10) we omitted terms proportional to the covariant derivatives which induce the boundary terms that vanish for suitable chosen boundary conditions. Using this notation we have

$$\mathbf{A}^A{}_B = -\nabla_n\phi^A\nabla_n\phi_B + g^{ij}\partial_i\phi^A\partial_j\phi_B\,, \quad \nabla_n\phi^A = \frac{1}{N}(\partial_t\phi^A - N^i\partial_i\phi^A)\,. \tag{2.13}$$

We would like to stress that there is an another issue with the construction of the Hamiltonian formalism. To see this note that the general potential term contains the matrix $\sqrt{\mathbf{A}}$ that can be defined as power series $\sqrt{\mathbf{A}^A{}_B} = \sum_n c_n(\mathbf{A}^n)^A{}_B$ with appropriate coefficients c_n. Then the variation of this expression is equal to

$$\delta\sqrt{\mathbf{A}^A{}_B} = c_1\delta\mathbf{A}^A{}_B + c_1(\delta\mathbf{A}^A{}_C\mathbf{A}^C{}_B + \mathbf{A}^A{}_C\delta\mathbf{A}^C{}_B) + \dots \tag{2.14}$$

that clearly cannot be written as $\delta\sqrt{\mathbf{A}^A{}_B} = \frac{1}{2}\delta\mathbf{A}^A{}_C\left((\sqrt{\mathbf{A}})^{-1}\right)^C{}_B$. However the situation improves when we consider the potential term that depends on $\sqrt{\mathbf{A}^A{}_A}$ since then we have[7]

$$\delta\sqrt{\mathbf{A}^A{}_A} = c_1\delta\mathbf{A}^A_A + 2c_1\delta\mathbf{A}^A{}_B\mathbf{A}^B{}_A + 3c_3\delta\mathbf{A}^A{}_B\mathbf{A}^B{}_C\mathbf{A}^C{}_A + \dots = \frac{1}{2}\delta\mathbf{A}^A{}_B\left((\sqrt{\mathbf{A}})^{-1}\right)^B{}_A\,. \tag{2.20}$$

For that reason we restrict ourselves to the Hamiltonian analysis of the non-linear massive gravity action with the potential term

$$\mathcal{U} = 4\mathrm{Tr}\sqrt{\mathbf{A}}\,. \tag{2.21}$$

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[7] Note that this result is consistent with the definition of the square root of the matrix given in [7, 9]

$$(\sqrt{\mathbf{A}})^A{}_B(\sqrt{\mathbf{A}})^B{}_C = \mathbf{A}^A{}_C \tag{2.15}$$

since

$$\delta(\sqrt{\mathbf{A}})^A{}_B + (\sqrt{\mathbf{A}})^A{}_B\delta(\sqrt{\mathbf{A}})^B{}_C((\sqrt{\mathbf{A}})^{-1})^C{}_D = \delta(\mathbf{A})^A{}_B((\sqrt{\mathbf{A}})^{-1})^B{}_D\,. \tag{2.16}$$

Taking the trace of given equation we immediately obtain (2.20). Note also that due to the matrix nature of objects $\mathbf{A}$ and $\mathbf{B}$ the following relation is not valid

$$\sqrt{\mathbf{AB}} = \sqrt{\mathbf{A}}\sqrt{\mathbf{B}} \tag{2.17}$$

unless $\mathbf{A}$ and $\mathbf{B}$ commute. On the other hand since obviously $\mathbf{A}$ and $\mathbf{A}^{-1}$ commute the equation (2.17) gives

$$\sqrt{\mathbf{A}}\sqrt{\mathbf{A}^{-1}} = \mathbf{I} \tag{2.18}$$

which implies following important relation

$$\left(\sqrt{\mathbf{A}}\right)^{-1} = \sqrt{\mathbf{A}^{-1}}\,. \tag{2.19}$$

JHEP06(2012)170

In fact, the Hamiltonian analysis of the gauge fixed form of this non-linear massive gravity was firstly performed in [31] and then in [32, 33]. It was shown in these two papers that this theory possesses interesting properties that allow to eliminate one additional scalar mode. Then our goal is to analyze given theory written in the manifest diffeomorphism invariant form and try to identify possible additional constraints that could eliminate one scalar mode. Explicitly, we would like to perform the Hamiltonian analysis of the following action

$$S = M_p^2 \int d^3\mathbf{x} dt N\sqrt{g}\left[K_{ij}\mathcal{G}^{ijkl}K_{kl} + {}^{(3)}R - m^2 \mathrm{Tr}\sqrt{\mathbf{A}}\right] . \tag{2.22}$$

3 Hamiltonian analysis

In this section we perform the Hamiltonian analysis of the action (2.22). For the General Relativity part of the action the procedure is standard. Explicitly, the momenta conjugate to N, N_i are the primary constraints of the theory

$$\pi_N(\mathbf{x}) \approx 0 , \quad \pi^i(\mathbf{x}) \approx 0 \tag{3.1}$$

while the Hamiltonian takes the form

$$\begin{aligned} H^{GR} &= \int d^3\mathbf{x}(N\mathcal{H}_T^{GR} + N^i\mathcal{H}_i^{GR}) , \\ \mathcal{H}_T &= \frac{1}{\sqrt{g}M_p^2}\pi^{ij}\mathcal{G}_{ijkl}\pi^{kl} - M_p^2\sqrt{g}{}^{(3)}R , \\ \mathcal{H}_i &= -2g_{ik}\nabla_l\pi^{kl} , \end{aligned} \tag{3.2}$$

where π^{ij} are momenta conjugate to g_{ij} with following non-zero Poisson brackets

$$\left\{g_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{y})\right\} = \frac{1}{2}\left(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k\right)\delta(\mathbf{x}-\mathbf{y}) . \tag{3.3}$$

Finally note that π_N, π^i have following Poisson brackets with N, N_i

$$\{N(\mathbf{x}), \pi_N(\mathbf{y})\} = \delta(\mathbf{x}-\mathbf{y}) , \quad \{N_i(\mathbf{x}), \pi^j(\mathbf{y})\} = \delta_i^j\delta(\mathbf{x}-\mathbf{y}) . \tag{3.4}$$

Now we proceed to the Hamiltonian analysis of the scalar field part of the action. Note that in $3+1$ formalism the matrix $\mathbf{A}^A_{\ B}$ takes the form

$$\mathbf{A}^A_{\ B} = -\nabla_n\phi^A\nabla_n\phi_B + g^{ij}\partial_i\phi^A\partial_j\phi_B \equiv K^A_{\ B} + V^A_{\ B} , \tag{3.5}$$

where

$$\begin{aligned} K^A_{\ B} &= -\nabla_n\phi^A\nabla_n\phi_B , \qquad & K_{AB} &= \eta_{AC}K^C_{\ B} = K_{BA} , \\ V^A_{\ B} &= g^{ij}\partial_i\phi^A\partial_j\phi_B , \qquad & V^{AB} &= V^A_{\ C}\eta^{CB} = V^{BA} . \end{aligned} \tag{3.6}$$

Then the conjugate momenta p_A are equal to

$$\begin{aligned} p_A &= -\frac{M_p^2m^2}{2}\sqrt{g}\frac{\delta\mathbf{A}^C_{\ D}}{\delta\partial_t\phi^A}(\mathbf{A}^{-1/2})^D_{\ C} = \\ &= \frac{M_p^2m^2}{2}\sqrt{g}(\nabla_n\phi_C(\mathbf{A}^{-1/2})^C_{\ A} + \eta_{AC}(\mathbf{A}^{-1/2})^C_{\ B}\nabla_n\phi^B) , \quad \mathbf{A}^{-1/2} = (\sqrt{\mathbf{A}})^{-1} . \end{aligned} \tag{3.7}$$

JHEP06(2012)170

Note that using the symmetry of $\mathbf{A}_{AB} = \mathbf{A}_{BA}$ we can write (3.7) in simpler form

$$p_A = M_p^2 m^2 \sqrt{g} (\mathbf{A}^{-1/2})_{AB} \nabla_n \phi^B \, . \tag{3.8}$$

Using this expression we derive following relation

$$\begin{aligned} \frac{1}{g M_p^4 m^4} p_A p_B &= (\mathbf{A}^{-1/2})_{AC} (\nabla_n \phi^C \nabla_n \phi^D) (\mathbf{A}^{-1/2})_{DB} = \\ &= (\mathbf{A}^{-1/2})_{AC} (V^{CD} - \mathbf{A}^{CD}) (\mathbf{A}^{-1/2})_{DB} \end{aligned} \tag{3.9}$$

which implies

$$\Pi_{AB} = (\mathbf{A}^{-1/2})_{AC} V^{CD} (\mathbf{A}^{-1/2})_{DB} \, , \tag{3.10}$$

where we introduced the matrix Π_{AB} defined as

$$\Pi_{AB} = \frac{1}{g m^4 M_p^4} p_A p_B + \eta_{AB} \, . \tag{3.11}$$

Note that when we multiply (3.10) by V from the right we obtain (we use matrix notation)

$$\Pi V = (\mathbf{A}^{-1/2} V)(\mathbf{A}^{-1/2} V) \tag{3.12}$$

which implies

$$\mathbf{A}^{-1/2} V = \sqrt{\Pi V} \, . \tag{3.13}$$

This relation will be important below. The important point of our analysis is that V^{AB} has the rank 3 as was firstly explicitly stressed in [29]. In fact, if we introduce the 4×3 matrix $W_i^A = \partial_i \phi^A$ and its transpose matrix $(W^T)^i{}_A = \partial_i \phi^A$ which is 3×4 matrix we can write

$$V^{AB} = W_i^A g^{ij} (W^T)_j{}^B \, . \tag{3.14}$$

Then since W_i^A, g^{ij} have the rank 3 we obtain that V^{AB} has the rank 3 as well. As a result $\det V = 0$. In other words V is not invertible matrix. This fact has an fundamental consequence for the Hamiltonian structure of given theory. On the other hand we can multiply (3.10) by V from the left so that

$$V \Pi = (V \mathbf{A}^{-1/2})(V \mathbf{A}^{-1/2}) \tag{3.15}$$

that now implies

$$\sqrt{V \Pi} = V \mathbf{A}^{-1/2} \, . \tag{3.16}$$

With the help of these results it is easy to determine corresponding Hamiltonian

$$\begin{aligned} \mathcal{H}^{sc} &= \partial_t \phi^A p_A - \mathcal{L}_{sc} = M_p^2 m^2 \sqrt{g} N V^{AB} (\mathbf{A}^{-1/2})_{BA} + N^i p_A \partial_i \phi^A = \\ &= N M_p^2 m^2 \sqrt{g} \sqrt{\Pi_{AB} V^{BA}} + N^i p_A \partial_i \phi^A \equiv N \mathcal{H}_T^{sc} + N^i \mathcal{H}_i^{sc} \end{aligned} \tag{3.17}$$

using (3.13) and using an obvious relation $\mathrm{Tr}\sqrt{V\Pi} = \mathrm{Tr}\sqrt{\Pi V}$. With the help of these results we find the final form of the Hamiltonian formulation of the action (2.22)

$$H = \int d^3\mathbf{x} (N \mathcal{H}_T + N^i \mathcal{H}_i) \, , \tag{3.18}$$

JHEP06(2012)170

where

$$\mathcal{H}_T = \mathcal{H}_T^{GR} + \mathcal{H}_T^{sc}\,, \quad \mathcal{H}_i = \mathcal{H}_i^{GR} + \mathcal{H}_i^{sc}\,, \tag{3.19}$$

where the explicit forms of these terms is given in (3.2) and in (3.17). Then note that the requirement of the preservation of the primary constraints $\pi_N \approx 0\,, \pi^i \approx 0$ implies an existence of the secondary constraints

$$\mathcal{H}_T(\mathbf{x}) \approx 0\,, \quad \mathcal{H}_i(\mathbf{x}) \approx 0\,. \tag{3.20}$$

The crucial question is the existence of additional constraints in the theory. The existence of these constrains would be important for the elimination of an additional scalar mode. We should expect that this constraint is the primary constraint between momenta and coordinates of the scalar fields and the question is whether we can find such a relation. For example, we can try to calculate $\det \Pi$. Using

$$\begin{aligned}
&\det\left(\eta_{AB} + \frac{1}{gM_p^4m^4}p_Ap_B\right) = \det \eta_{AC} \det\left(\delta^C_{\ B} + \frac{1}{gM_p^4m^4}p^Cp_B\right) = \\
&= \det \eta \exp \mathrm{Tr} \ln\left(\delta^A_{\ B} + \frac{1}{gM_p^4m^4}p^Ap_B\right) = \det \eta \exp \ln\left(1 + \frac{1}{gM_p^4m^4}p_Ap^A\right) = \\
&= -\left(1 + \frac{1}{gM_p^4m^4}p_Ap^A\right)
\end{aligned} \tag{3.21}$$

and hence from (3.10) we obtain

$$1 + \frac{1}{gM_p^4m^4}p_Ap^A = -\frac{\det V}{\det \mathbf{A}}\,. \tag{3.22}$$

We see that the upper equation implies the primary constraint on condition when $\det V = 0$. Then (3.22) implies the primary constraint

$$\mathcal{C} : \frac{1}{gM_p^4m^4}p_Ap^A + 1 = 0\,. \tag{3.23}$$

The fact that $\det V^{AB} = 0$ is however crucial for the calculation of the algebra of the constraints. Unfortunately we are not able to perform this calculation in case of the four dimensional case due to the absence of the inverse matrix V^{-1}. We demonstrate the complexity of such a calculation in the appendix when we abandon the condition $\det V^{AB} = 0$. However we are able to perform this calculation in case of two dimensional massive gravity which we perform in the next section.

4 Two dimensional massive gravity

Two dimensional massive gravity is exceptional also from the fact that the gravity is trivial so that the dynamical content is hidden in the scalar degrees of freedom only. In order to determine the physical number of degrees of freedom we have to find an appropriate

JHEP06(2012)170

structure of constraints. To proceed we have to determine the form of the Hamiltonian constraint (3.17). An explicit calculation gives

$$\mathcal{H}_T = M_p^2 m^2 \sqrt{g} \mathrm{Tr}\sqrt{\Pi V} = \sqrt{\frac{1}{\omega}(p_A \partial\phi^A)^2 + M_p^4 m^4 \partial\phi_A \partial\phi^A}\,, \tag{4.1}$$

where we introduced following notation

$$g_{11} = \omega\,, \quad \det g = \omega\,, \quad g^{11} = \frac{1}{\omega}\,, \quad \partial_1 = \partial\,. \tag{4.2}$$

As a result the extended Hamiltonian takes the form

$$\begin{aligned} H &= \int dx (N\mathcal{H}_T + N^1 \mathcal{H}_S + u^\omega \pi^\omega + u^N \pi_N + u_1 \pi^1 + \Gamma\mathcal{C})\,, \\ \mathcal{H}_T &= \sqrt{\frac{1}{\omega}(p_A \partial\phi^A)^2 + M_p^4 m^4 \partial\phi_A \partial\phi^A} \equiv \sqrt{\mathbf{A}}\,, \quad \mathcal{H}_S = p_A \partial\phi^A - 2\omega\nabla\pi^\omega\,, \end{aligned} \tag{4.3}$$

where we included the primary constraint $\pi^\omega \approx 0$ into the definition of the spatial diffeomorphism constraint. Note the this theory possesses following four primary constraints

$$\pi_N \approx 0\,, \quad \pi^1 \approx 0\,, \quad \pi^\omega \approx 0\,, \quad \mathcal{C} = \frac{1}{\omega M_p^4 m^4} p_A p^A + 1 = 0\,. \tag{4.4}$$

First of all the requirement of the preservation of the primary constraints π_N, π^1 implies following secondary ones

$$\mathcal{H}_T \approx 0\,, \quad \mathcal{H}_S \approx 0\,. \tag{4.5}$$

We see that $\mathcal{H}_T \approx 0, \mathcal{H}_S \approx 0$ are the secondary constraints. Then we have to check the consistency of all primary and secondary constraints during the time evolution of the system. The time evolution of the constraints π_N, π^1 does not generate new conditions while the requirement of the preservation of the constraint π^ω implies

$$\begin{aligned} \partial_t \pi^\omega &= \{\pi^\omega, H\} = \frac{N(p_A \partial\phi^A)^2}{\omega^2 \sqrt{\mathbf{A}}} - \Gamma\triangle = \\ &= \frac{N(\mathcal{H}_S + 2\omega\nabla\pi^\omega)^2}{\omega^2\sqrt{\mathbf{A}}} - \Gamma\triangle \approx -\Gamma\triangle = 0\,, \end{aligned} \tag{4.6}$$

where we used following Poisson bracket

$$\{\mathcal{C}(\mathbf{x}), \pi^\omega(\mathbf{y})\} = \frac{1}{\omega^2} p_A p^A(\mathbf{x}) \delta(\mathbf{x}-\mathbf{y}) \equiv \triangle(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) \tag{4.7}$$

and we used the fact that we have to determine the evolution on the constraint surface. On the other hand the time evolution of the constraint $\mathcal{C}$ is equal to

$$\partial_t \mathcal{C} = \{\mathcal{C}, H\} \approx \{\mathcal{C}, \mathbf{T}_T(N)\} + \int dx u^\omega \{\mathcal{C}, \pi^\omega(x)\} = \{\mathcal{C}, \mathbf{T}_T(N)\} + u^\omega \triangle = 0\,, \tag{4.8}$$

where we used the fact that

$$\{\mathbf{T}_S(N^1), \mathcal{C}\} = -N^1 \partial\mathcal{C} \tag{4.9}$$

which vanishes on the constraint surface and where $\{\mathcal{C},\mathbf{T}_T(N)\}$ is equal to

$$\{\mathbf{T}_T(N),\mathcal{C}\}=2\partial\left(\frac{N(p_A\partial\phi^A)p_B}{\omega\sqrt{\mathbf{A}}}\right)\frac{p^B}{\omega}+2\partial\left(N\frac{\partial\phi^A}{\sqrt{\mathbf{A}}}\right)\frac{p_A}{\omega}\,. \tag{4.10}$$

Note that this Poisson bracket vanishes on the constraint surface $\mathcal{H}_S\approx 0\,,\pi^\omega\approx 0$ as well. As a result we find that (4.6) and (4.8) determine that Lagrange multipliers u^ω,Γ are equal to zero.

Finally we have to determine the algebra of constraints $\mathcal{H}_T,\mathcal{H}_S$. As usual we calculate the Poisson brackets of their smeared forms and we find

$$\{\mathbf{T}_T(M),\mathbf{T}_T(N)\}=\frac{1}{4}\int dxdy\frac{N(x)}{\sqrt{\mathbf{A}(x)}}\left\{\mathbf{A}(x),\mathbf{A}(y)\right\}\frac{N(y)}{\sqrt{\mathbf{A}(y)}}\,. \tag{4.11}$$

To proceed we calculate following Poisson bracket

$$\begin{aligned}
&\{\mathbf{A}(x),\mathbf{A}(y)\}=\\
&=-4\frac{1}{\omega(x)}(p_A\partial\phi^A)(x)\left[\partial\phi^A(x)p_A(y)\partial_y\delta(x-y)-\right.\\
&\quad\left.-p_A(x)\partial\phi^A(y)\partial_x\delta(x-y)\right]\frac{1}{\omega(y)}(p_A\partial\phi^A)(y)+\\
&\quad+4\frac{M_p^4m^4}{\omega(x)}(p_A\partial\phi^A)(x)\left[(\partial\phi^A(x)\partial\phi_A(y)\partial_y\delta(x-y)-\right.\\
&\quad\left.-(\partial\phi^A(y)\partial\phi_A(x)\partial_x\delta(x-y)\right]\frac{1}{\omega(y)}(p_A\partial\phi^A)(y)\,.
\end{aligned} \tag{4.12}$$

In the calculation of the Poisson bracket of the smeared Hamiltonian constraints we use the fact the Poisson bracket $\{\mathbf{A}(x),\mathbf{A}(y)\}$ contains derivative of the delta functions that give non-zero contribution when they act on N and M respectively. As a result we obtain

$$\begin{aligned}
\{\mathbf{T}_T(M),\mathbf{T}_T(N)\}&=\int dx(M\partial N-N\partial M)\frac{1}{\mathbf{A}\omega}\times\\
&\quad\times[(p_A\partial\phi^A)^3+M_p^4m^4(p_A\partial\phi^A)((\partial\phi^A\partial\phi_A)]=\\
&=\mathbf{T}_S\left(\frac{1}{\omega}(N\partial M-M\partial N)\right)+2\int dx(N\partial M-M\partial N)\omega\nabla\pi^\omega\,.
\end{aligned} \tag{4.13}$$

The algebra of constraints $\mathbf{T}_S$ takes the standard form

$$\left\{\mathbf{T}_S(M^1),\mathbf{T}_S(N^1)\right\}=\mathbf{T}_S(M^1\partial N^1-N^1\partial M^1)\,. \tag{4.14}$$

Finally we determine the Poisson bracket

$$\begin{aligned}
\left\{\mathbf{T}_S(M^1),\mathbf{T}_T(N)\right\}&=\int dx\left\{\mathbf{T}_S(M^1),\mathcal{H}_T(x)\right\}N=\\
&=-\int dx(M^1\partial\mathcal{H}_T+\mathcal{H}_T\partial X^1)=\mathbf{T}_T(M^1\partial N)
\end{aligned} \tag{4.15}$$

using

$$\left\{\mathbf{T}_S(M^1),\mathcal{H}_T(x)\right\}=-M^1\partial\mathcal{H}_T-\mathcal{H}_T\partial M^1\,. \tag{4.16}$$

Finally we have to analyze the stability of the constraints $\mathcal{H}_T$ and $\mathcal{H}_S$. In fact, the constraint $\mathbf{T}_S(M^1)$ is preserved due to the fact that the Poisson brackets between all constraints vanish on the constraint surface. On the other hand the time evolution of the constraint is given by the equation

$$\partial_t \mathbf{T}_T(M) = \{\mathbf{T}_T(M), H\} \approx \int dx (u^\omega \{\mathbf{T}_T(M), \pi^\omega\} + \Gamma \{\mathbf{T}_T(M), \mathcal{C}\}) = 0 \tag{4.17}$$

using the fact that u^ω and Γ vanish.

At this stage we have finished the analysis of the time development of the constraints with following result. We have four first class constraints

$$\pi_N \approx 0\,, \quad \pi^1 \approx 0, \quad \mathcal{H}_T \approx 0\,, \quad \mathcal{H}_1 \approx 0 \tag{4.18}$$

and the second class constraints

$$\pi^\omega \approx 0\,, \quad \mathcal{C} \approx 0\,. \tag{4.19}$$

Using the second class constraints we can eliminate π^ω and ω as functions of p_A. Then the gauge fixing of the four first class constrains completely eliminate the physical degrees of freedom N, π_N, N^1, π_1 and p_A, ϕ^A.

5 Conclusion

In this section we outline our results and suggest the possible extension of this work. We performed the Hamiltonian analysis of some particular model of non-linear massive gravity action written in the Stückelberg picture [30]. We found corresponding Hamiltonian. Then following [29] where it was explicitly shown an existence of the primary and corresponding secondary constraint in the version of the non-linear massive gravity action presented in [33] we find corresponding primary constraint of the theory. Unfortunately due to the fact that this constraint is a consequence of the singularity of the matrix V^{AB} we were not able to determine algebra of all constraints and identify secondary constraints for the case of four dimensional non-linear massive gravity action. On the other hand we were able to complete the Hamiltonian analysis of two dimensional non-linear massive gravity where we showed that the algebra of the Hamiltonian and spatial diffeomorphism constraints is in agreement with the basic principles of geometrodynamics [36–38]. We also identified an additional constraints and we show that there are no physical degrees of freedom with agreement with the analysis presented in [16, 30].

It is very unhappy that we were not able to finish the Hamiltonian analysis of four dimensional non-linear massive gravity theory due to the singular nature of the matrix V^{AB} especially in the light of the very nice proof of the existence of the primary and the secondary constraints that was performed in [29] in the case of the version of the non-linear massive gravity action presented in [33]. To finish such an analysis is very desirable and it is the main goal of our future work. It would be also very nice to perform the Hamiltonian analysis of the general form of the non-linear massive gravity action with the Stückelberg fields and we hope to return to this problem in future.

JHEP06(2012)170

A Toy model: algebra of constraints in case of det $V^{AB} \neq 0$

In this appendix we perform the calculation of the algebra of constraints in case when we do not impose the condition det $V^{AB} = 0$ even if these calculations are not directly related to the case of the non-linear massive gravity action studied in the main body of this paper. The goal of this appendix is to demonstrate the complexity of given analysis. To begin with note that the fact that det $V^{AB} \neq 0$ implies the existence inverse matrix V^{-1}. In this case we find an important relation

$$V^{-1}\sqrt{V\Pi} = \mathbf{A}^{-1/2} = \sqrt{\Pi V}V^{-1} \tag{A.1}$$

which will be useful when we calculate the algebra of constraints.

Let us consider the smeared form of the constraints (3.20)

$$\mathbf{T}_T(N) = \int d^3\mathbf{x} N\mathcal{H}_T \,, \quad \mathbf{T}_S(N^i) = \int d^3\mathbf{x} N^i\mathcal{H}_i \,. \tag{A.2}$$

The goal of this section is to determine Poisson brackets among these constraints. Note that in case of the General Relativity part of the constraints we have following Poisson brackets

$$\begin{aligned}
\left\{\mathcal{H}_T^{GR}(\mathbf{x}), \mathcal{H}_T^{GR}(\mathbf{y})\right\} &= -\left[\mathcal{H}^i_{GR}(\mathbf{x})\frac{\partial}{\partial x^i}\delta(\mathbf{x}-\mathbf{y}) - \mathcal{H}^i_{GR}(\mathbf{y})\frac{\partial}{\partial y^i}\delta(\mathbf{x}-\mathbf{y})\right] , \\
\left\{\mathcal{H}_T^{GR}(\mathbf{x}), \mathcal{H}_i^{GR}(\mathbf{y})\right\} &= \mathcal{H}_T^{GR}(\mathbf{y})\frac{\partial}{\partial x^i}\delta(\mathbf{x}-\mathbf{y}) , \\
\left\{\mathcal{H}_i^{GR}(\mathbf{x}), \mathcal{H}_j^{GR}(\mathbf{y})\right\} &= \left[\mathcal{H}_j^{GR}(\mathbf{x})\frac{\partial}{\partial x^i}\delta(\mathbf{x}-\mathbf{y}) - \mathcal{H}_i(\mathbf{y})\frac{\partial}{\partial y^j}\delta(\mathbf{x}-\mathbf{y})\right] .
\end{aligned} \tag{A.3}$$

The calculation of the Poisson brackets that contains scalar phase space degrees of freedom is more involved. However it is easy to find the Poisson bracket between generators of spatial diffeomorphisms

$$\left\{\mathcal{H}_i^{sc}(\mathbf{x}), \mathcal{H}_j^{sc}(\mathbf{y})\right\} = \left[\mathcal{H}_j^{sc}(\mathbf{x})\frac{\partial}{\partial x^i}\delta(\mathbf{x}-\mathbf{y}) - \mathcal{H}_i^{sc}(\mathbf{y})\frac{\partial}{\partial y^j}\delta(\mathbf{x}-\mathbf{y})\right] \tag{A.4}$$

that together with the Poisson bracket on the third line in (A.3) implies following form of Poisson bracket between smeared form of the diffeomorphism constraints

$$\left\{\mathbf{T}_S(N^i), \mathbf{T}_S(M^j)\right\} = \mathbf{T}_S(N^j\partial_j M^i - M^j\partial_j N^i) \,. \tag{A.5}$$

In case of Hamiltonian constraint the situation is not so easy. Explicitly, from the definition of the Poisson bracket we find

$$\begin{aligned}
&\{\mathbf{T}_T^{sc}(N), \mathbf{T}_T^{sc}(M)\} = \int d^3\mathbf{x} d^3\mathbf{y} N(\mathbf{x}) \left\{\mathcal{H}_T^{sc}(\mathbf{x}), \mathcal{H}_T^{sc}(\mathbf{y})\right\} M(\mathbf{y}) = \\
&= -M_p^4 m^4 \int d^3\mathbf{x} d^3\mathbf{y} d^3\mathbf{z} N(\mathbf{x}) M(\mathbf{y}) \left(\sqrt{g}(\mathbf{x})\frac{\delta(\sqrt{\Pi V})_A^{\ A}(\mathbf{x})}{\delta p_X(\mathbf{z})}\frac{\delta(\sqrt{\Pi V})_C^{\ C}(\mathbf{y})}{\delta\phi^X(\mathbf{z})}\sqrt{g}(\mathbf{x}) - \right. \\
&\quad \left. -\sqrt{g}(\mathbf{y})\frac{\delta(\sqrt{\Pi V})_C^{\ C}(\mathbf{y})}{\delta p_X(\mathbf{z})}\frac{\delta(\sqrt{\Pi V})_A^{\ A}(\mathbf{x})}{\delta\phi^X(\mathbf{z})}\sqrt{g}(\mathbf{x})\right) .
\end{aligned} \tag{A.6}$$

JHEP06(2012)170

To proceed further we use the relations

$$
\begin{aligned}
\frac{\delta(\sqrt{\Pi V})_A^{\ A}(\mathbf{x})}{\delta p_X(\mathbf{z})} &= \frac{1}{2}\frac{\delta(\Pi V)_A^{\ B}(\mathbf{x})}{\delta p_X(\mathbf{x})}((\Pi V)^{-1/2})_B^{\ A} = \\
&= \frac{1}{2gM_p^4 m^4}(\delta_A^X p_C + p_A\delta_C^X)V^{CB}\left((\Pi V)^{-1/2}\right)_B^{\ A}(\mathbf{x})\delta(\mathbf{x}-\mathbf{z}) \,, \\
\frac{\delta(\sqrt{V\Pi})^C_{\ C}(\mathbf{y})}{\delta\phi^X(\mathbf{z})} &= \frac{1}{2}\frac{\delta(V\Pi)^C_{\ D}(\mathbf{y})}{\delta\phi^X(\mathbf{z})}((V\Pi)^{-1/2})^D_{\ C}(\mathbf{y}) = \\
&= \frac{1}{2}[g^{ij}\partial_{y^i}\delta(\mathbf{z}-\mathbf{y})\delta_X^C\partial_{y^j}\phi^E(\mathbf{y}) + g^{ij}\partial_{y^i}\phi^C(\mathbf{y})\partial_{y^j}\delta_X^E\delta(\mathbf{z}-\mathbf{y})] \\
&\quad \times\Pi_{ED}\left((V\Pi)^{-1/2}\right)^D_{\ C}(\mathbf{y}) \,.
\end{aligned} \tag{A.7}
$$

Then with the help of (A.7) we can determine the Poisson bracket (A.6). First of all these Poisson brackets contain the derivative of the delta functions. We perform integration by parts. Then the non-zero contribution arises from the derivative of the smeared functions N and M. As a result we obtain

$$
\begin{aligned}
&\{\mathbf{T}_T^{sc}(N), \mathbf{T}_T^{sc}(M)\} = \frac{1}{4}\int d^3\mathbf{x}(N\partial_i M - M\partial_i N)g^{ij}\times \\
&\quad \times\left(p_A(\sqrt{(V\Pi)^{-1}}V)^{AB} + p_A(V\sqrt{(\Pi V)^{-1}})^{AB}\right)\times \\
&\quad \times((\Pi\sqrt{(V\Pi)^{-1}})_{BC}\partial_j\phi^C + (\sqrt{(\Pi V)^{-1}}\Pi)_{BC}\partial_j\phi^C) = \\
&= \int d^3\mathbf{x}(N\partial_i M - M\partial_i N)g^{ij}p_A(V\sqrt{(\Pi V)^{-1}})^{AB}(\sqrt{(\Pi V)^{-1}}\Pi)_{BC}\partial_j\phi^C) = \\
&= \int d^3\mathbf{x}(N\partial_i M - M\partial_i N)g^{ij}p_A(V(\Pi V)^{-1}\Pi)^A_{\ B}\partial_j\phi^B = \\
&= \mathbf{T}_S^{sc}((N\partial_j M - M\partial_j N)g^{ji}) \,,
\end{aligned} \tag{A.8}
$$

where we used the symmetry of Π and V and the relations

$$
\sqrt{(V\Pi)^{-1}}V = V\sqrt{(\Pi V)^{-1}}, \quad \Pi\sqrt{(V\Pi)^{-1}} = \sqrt{(\Pi V)^{-1}}\Pi \tag{A.9}
$$

that follow from (A.1). If we combine this result with the Poisson brackets between smeared form of the General Relativity Hamiltonian constraints we find the final result

$$
\{\mathbf{T}_T(N), \mathbf{T}_T(M)\} = \mathbf{T}_S((N\partial_j M - M\partial_j N)g^{ji}) \,. \tag{A.10}
$$

Finally we calculate the Poisson bracket between $\mathbf{T}_S(N^i)$ and $\mathbf{T}_T^{sc}(N)$. Using

$$
\begin{aligned}
\left\{\mathbf{T}_S(N^i), p_A(\mathbf{x})\right\} &= -N^i\partial_i p_A(\mathbf{x}) - \partial_i N^i p_A(\mathbf{x}) \,, \\
\left\{\mathbf{T}_S(N^i), \phi^A(\mathbf{x})\right\} &= -N^i\partial_i\phi^A(\mathbf{x}) \,, \\
\left\{\mathbf{T}_S(N^i), \sqrt{g}(\mathbf{x})\right\} &= -N^i\partial_i\sqrt{g}(\mathbf{x}) - \partial_i N^i\sqrt{g}(\mathbf{x}) \,, \\
\left\{\mathbf{T}_S(N^i), g^{ij}(\mathbf{x})\right\} &= -N^k\partial_k g^{ij}(\mathbf{x}) + \partial_k N^i g^{kj}(\mathbf{x}) + g^{ik}\partial_k N^j(\mathbf{x}) \,, \\
\left\{\mathbf{T}_S(N^i), \Pi_{AB}(\mathbf{x})\right\} &= -N^i\partial_i\Pi_{AB}(\mathbf{x}) \,, \\
\left\{\mathbf{T}_S(N^i), V^{AB}(\mathbf{x})\right\} &= -N^k\partial_k V^{AB}(\mathbf{x}) \,.
\end{aligned} \tag{A.11}
$$

With the help of these results it is easy to find

$$\left\{\mathbf{T}_S(N^i), \mathcal{H}_T^{sc}(\mathbf{x})\right\} = -\partial_k N^k \mathcal{H}_T^{sc}(\mathbf{x}) - N^k \partial_k \mathcal{H}_T^{sc}(\mathbf{x}) \,. \tag{A.12}$$

Collecting this result with the Poisson bracket between diffeomorphism constraint and Hamiltonian constraint of General Relativity we obtain

$$\left\{\mathbf{T}_S(N^i), \mathbf{T}_T(N)\right\} = \mathbf{T}_T(\partial_k N N^k) \,. \tag{A.13}$$

To conclude, we found that the theory with $\det V \neq 0$ possesses four constraints $\mathcal{H}_T(\mathbf{x}) \approx 0\,, \mathcal{H}_i(\mathbf{x}) \approx 0$ which are the first class constraints as follows from the Poisson brackets (A.5), (A.10) and (A.13). We also showed that the crucial presumption for these calculations was the regularity of the matrix V^{AB} which of course is not the case of four dimensional non-linear massive gravity.

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JHEP06(2012)170