Published for SISSA by Springer

Received: *May 30, 2011*
Accepted: *June 10, 2011*
Published: *June 27, 2011*

Holographic charge transport in Lifshitz black hole backgrounds

José P. S. Lemos and Da-Wei Pang

Centro Multidisciplinar de Astrofísica-CENTRA, Departamento de Física, Instituto Superior Técnico-IST, Universidade Técnica de Lisboa Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

E-mail: joselemos@ist.utl.pt, dawei.pang@ist.utl.pt

Abstract: We study charge transport properties in a domain-wall geometry, whose near horizon IR geometry is a Lifshitz black hole and whose UV geometry is AdS. The action for the gauge field contains the standard Maxwell term plus the Weyl tensor coupled to Maxwell field strengths. In four dimensions we calculate the conductivity via both the membrane paradigm and Kubo's formula. Precise agreements between both methods are obtained. Moreover, we perform an analysis of the four-dimensional electro-magnetic duality in our domain-wall background and find that the relation between the longitudinal and transverse components of the current-current correlation functions and those of the 'dual' counterparts holds, irrespective of the near horizon IR geometry. Conductivity at extremality is also investigated. Generalizations to higher dimensions are performed.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence

doi:10.1007/JHEP06(2011)122

JHEP06(2011)122

Contents

JHEP06(2011)122

1 Introduction

The AdS/CFT correspondence [1–4], a duality between gravity (AdS) and gauge field theory (CFT), also called the gauge/gravity duality, sets up connections between gravity theory in a certain bulk spacetime and field theory on the boundary of that spacetime. It has been widely recognized that the gauge/gravity duality provides powerful tools for studying dynamics of strongly coupled field theories and physics in the real world. Recently, investigations on applications of the AdS/CFT correspondence to condensed matter physics (AdS/CMT for short) have, due to its great interest, increased enormously [5–9]. For instance, gravity backgrounds which possess non-relativistic symmetries were constructed in [10–12].

One crucial quantity characterizing charge transport properties of condensed matter systems is the conductivity, which can be evaluated via the current-current correlation function of the bulk U(1) gauge field in the dual gravity side. It was found in [13] that the conductivity in the three-dimensional field theory side at zero momentum was a constant with no frequency dependence. The authors of [13] attributed this remarkable result to the electro-magnetic duality of the four-dimensional bulk Einstein-Maxwell theory. Recently in order to acquire a better understanding of this self-duality, Myers, Sachdev and Singh [14] considered a particular form of new higher derivative corrections which involves couplings between the gauge field to the spacetime curvature. The higher order corrections to the conductivity were obtained and they found that although the electro-magnetic self-duality was lost in the presence of higher order corrections, a simple relation between the transverse and longitudinal components of the current-current retarded correlation function and those of the 'dual' counterparts still held.

Since many condensed matter systems possess non-relativistic symmetries, it is desirable to study the conductivity in such non-relativistic backgrounds and to see if the 'duality' relation for the current-current correlation functions still holds. In this paper we consider charge transport properties at Lifshitz fixed points. The background is a domain wall geometry, where the metric becomes a Lifshitz black hole in the IR and an asymptotically AdS spacetime in the UV. The action for the bulk U(1) gauge field contains the ordinary Maxwell term, as well as coupling between the Weyl tensor and the field strengths. First we work in four dimensions and calculate the conductivity via the membrane paradigm, which reduces to the one obtained in [14] when the dynamical exponent $z = 1$. Next we evaluate the conductivity from Kubo's formula, which precisely matches the result obtained via the membrane paradigm. Moreover, we find that the relation between the transverse and longitudinal components of the current-current retarded correlation functions and those of the 'dual' counterparts still holds, irrespective of the IR geometry. We also comment on the conductivity at extremality. Generalizations to higher-dimensional spacetimes are also obtained.

The rest of the paper is organized as follows: we give a brief review on relevant backgrounds in section 2. Then we focus on charge transport properties in four dimensions in section 3. Firstly we calculate the conductivity using the membrane paradigm in section 3.1, where we find that although the Weyl corrections do not contribute in the $z = 2$ case, it is indeed a coincidence which can be seen by considering more general actions. Next in section 3.2 we reconsider the conductivity by evaluating the retarded current-current correlation functions and find precise agreement with the result obtained in section 3.1. A simple relation between the transverse and longitudinal components of the current-current retarded correlation functions and those of the 'dual' counterparts is derived in section 3.3, which agrees with that obtained in [14]. In section 3.4 conductivity at extremality is investigated. Higher-dimensional generalizations are evaluated in section 4 and discussions on other related issues are given in section 5.

2 Preliminaries

In this section we review some relevant backgrounds before proceeding. First of all, the starting point in [14] was the four-dimensional planar Schwarzschild-AdS$_4$ black hole [15–17],

$$ds^2 = \frac{r^2}{L^2}(-f(r)dt^2 + dx^2 + dy^2) + \frac{L^2 dr^2}{r^2 f(r)}\,, \tag{2.1}$$

where $f(r) = 1 - r_0^3/r^3$. On the other hand, after integrating by parts and imposing the identities $\nabla_{[a}F_{bc]} = R_{[abc]d} = 0$, the most general four-derivative action contains the following terms,

$$\begin{aligned} I_4 = \int d^4x\sqrt{-g}[&\alpha_1 R^2 + \alpha_2 R_{ab}R^{ab} + \alpha_3 (F^2)^2 + \alpha_4 F^4 + \alpha_5 \nabla^a F_{ab}\nabla^c F_c{}^b \\ &+ \alpha_6 R_{abcd}F^{ab}F^{cd} + \alpha_7 R^{ab}F_{ac}F_b{}^c + \alpha_8 R F^2], \end{aligned} \tag{2.2}$$

where $F^2 = F_{ab}F^{ab}, F^4 = F^a{}_b F^b{}_c F^c{}_d F^d{}_a$. If we focus on the conductivity, which means that only the current-current two-point functions are relevant, we can just consider the

JHEP06(2011)122

effects of the α_6, α_7 and α_8 terms. Furthermore, after taking a particular linear combination of these three terms, the effective action for bulk Maxwell field turns out to be

$$I_{\rm vec} = \frac{1}{g_4^2}\int d^4x\sqrt{-g}[-\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}], \tag{2.3}$$

where C_{abcd} denotes the Weyl tensor. One advantage of taking this particular combination is that the asymptotic geometry will not be modified, as the Weyl tensor vanishes in pure AdS space. Then the DC conductivity in the presence of higher order corrections is given by

$$\sigma_{\rm DC} = \frac{1}{g_4^2}(1+4\gamma). \tag{2.4}$$

We shall consider the following domain-wall geometry

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{r^2}{R_0^2}(dx^2+dy^2), \tag{2.5}$$

where R_0 denotes certain length scale. The IR region is described by a Lifshitz black hole (see, e.g., [18–28]),

$$ds^2_{\rm IR} = -\frac{r^{2z}}{L^2}f(r)dt^2 + \frac{L^2dr^2}{r^2f(r)} + \frac{r^2}{L^2}(dx^2+dy^2), \quad f(r) = 1-\frac{r_0^{z+2}}{r^{z+2}}, \tag{2.6}$$

where z is the dynamical exponent. The above background possesses the following Lifshitz scaling symmetry at extremality when $f(r)=1$,

$$t \to \lambda^z t, \quad r \to \frac{r}{\lambda}, \quad \vec{x} \to \lambda\vec{x}. \tag{2.7}$$

Generically, such solutions are always accompanied by various matter fields and the form of $f(r)$ is determined by the matter fields. However, here we just write down the metric as above so that it becomes Schwarzschild-AdS$_4$ when $z=1$. Combining (2.5) and (2.6), we can find that

$$e^{-\chi(r)} = r^{2z-2}, \quad g(r) = \frac{r^2 f(r)}{L^2}, \quad R_0 = L. \tag{2.8}$$

The UV geometry is chosen to be AdS so that it will not be modified by the higher order corrections (2.3) and we can still perform calculations in the context of AdS/CFT. Such a domain-wall geometry holographically describes a RG flow towards a nontrivial IR Lifshitz fixed point.

In this paper the action for the Maxwell field is still given by (2.3) and the equation of motion reads

$$\nabla_a[F^{ab} - 4\gamma L^2 C^{abcd}F_{cd}] = 0. \tag{2.9}$$

We also list the non-vanishing components of the Weyl tensor for later convenience

$$\begin{aligned} C_{trtr} &= \frac{e^{-\chi(r)}}{12r^2}F(r), & C_{titj} &= -\frac{e^{-\chi(r)}}{24R_0^2}g(r)F(r)\delta_{ij}, \\ C_{rirj} &= \frac{1}{24R_0^2}\frac{F(r)}{g(r)}\delta_{ij}, & C_{ijkl} &= -\frac{r^2}{12R_0^4}F(r)\delta_{ik}\delta_{jl}, \end{aligned} \tag{2.10}$$

where $i,j,k,l = x,y$ and

$$\begin{aligned} F(r) =\ & r[-g'(r)(4+3\chi'(r)) + 2rg''(r)] \\ & + g(r)(4 + 2r\chi'(r) + r^2\chi'^2(r) - 2r^2\chi''(r)). \end{aligned} \tag{2.11}$$

JHEP06(2011)122

3 Charge transport in four dimensions

We study charge transport properties in a four-dimensional domain-wall background, which is the most interesting case. In section 3.1 we calculate the conductivity using the membrane paradigm and verify the result via Kubo's formula in section 3.2. A simple relation between the longitudinal and transverse parts of the current-current correlation functions and those of the 'dual' counterparts is derived in section 3.3. In addition, we briefly discuss the conductivity at zero temperature in section 3.4.

3.1 DC conductivity from the membrane paradigm

In this subsection, we calculate the DC conductivity via the membrane paradigm, following [29, 30]. Such a prescription can be seen as a generalization of the analysis in [31, 32] to incorporate the following general action

$$I = \int d^4x\sqrt{-g}(-\frac{1}{8g_4^2}F_{ab}X^{abcd}F_{cd}), \tag{3.1}$$

where the tensor X^{abcd} possesses the following symmetries $X^{abcd} = X^{[ab][cd]} = X^{cdab}$. For our particular example,

$$X_{ab}{}^{cd} = I_{ab}{}^{cd} - 8\gamma L^2 C_{ab}{}^{cd}, \tag{3.2}$$

where

$$I_{ab}{}^{cd} = \delta_a{}^c\delta_b{}^d - \delta_a{}^d\delta_b{}^c, \tag{3.3}$$

so that the above action reduces to the conventional Maxwell action when $\gamma = 0$.

Extensions to the general action (3.1) are straightforward. We still define the stretched horizon at $r = r_H$, where $r_H > r_0$ and $r_H - r_0 \ll r_0$. The corresponding conserved current is given by

$$j^a = \frac{1}{4}n_b X^{abcd}F_{cd}|_{r=r_0}, \tag{3.4}$$

where n_a is an ourward-pointing radial unit vector. According to Ohm's law at the stretched horizon, the DC conductivity reads

$$\sigma = \frac{1}{g_4^2}\sqrt{-g}\sqrt{-X^{txtx}X^{rxrx}}|_{r=r_0}. \tag{3.5}$$

Plugging (2.10) and (2.11) into the above expression, we can arrive at

$$\sigma = \frac{1}{g_4^2}[1 - \frac{4}{3}\gamma(z^2 - 4)]. \tag{3.6}$$

When $z = 1$, the conductivity turns out to be

$$\sigma = \frac{1}{g_4^2}[1 + 4\gamma]\,, \tag{3.7}$$

which agrees with that obtained [14].

It can be easily seen that when $z = 2$, $\sigma = 1/g_4^2$, which means that the conductivity is not corrected by the higher order terms. One may wonder if this fact implies some

JHEP06(2011)122

underlying physics or just a coincidence. To answer this question, we can consider a more general form of corrections

$$\tilde{I}_{\rm vec}=\frac{1}{g_4^2}\int d^4x\sqrt{-g}[-\frac{1}{4}F_{ab}F^{ab}+\gamma L^2(c_1R_{abcd}F^{ab}F^{cd}+c_2R_{ab}F^{ac}F^b{}_c+c_3RF^{ab}F_{ab})], \quad (3.8)$$

where $c_i, i=1,2,3$ are constants. Now the tensor X^{abcd} becomes

$$\tilde{X}^{abcd}=(g^{ac}g^{bd}-g^{ad}g^{bc})-8\gamma L^2\Big[c_1R^{abcd}+\frac{c_2}{4}(R^{ac}g^{bd}-R^{ad}g^{bc}+R^{bd}g^{ac}-R^{bc}g^{ad}) +\frac{c_3}{2}R(g^{ac}g^{bd}-g^{ad}g^{bc})\Big], \quad (3.9)$$

and the conductivity is given by

$$\tilde{\sigma}=\frac{1}{g_4^2}\sqrt{-g}\sqrt{-\tilde{X}^{txtx}\tilde{X}^{rxrx}}|_{r=r_0}=\frac{1}{g_4^2}[1+2\gamma(z+2)(2c_1+(c_2+4c_3)(z+1))]. \quad (3.10)$$

It can be seen that $z=2$ also leads to nontrivial higher order corrections for general c_i's. In particular, when $c_1=1, c_2=-2, c_3=1/3$, the tensor $\tilde{X}^{abcd}=(g^{ac}g^{bd}-g^{ad}g^{bc})-8\gamma L^2C^{abcd}$, and the conductivity is given by

$$\tilde{\sigma}=\frac{1}{g_4^2}\left[1-\frac{4}{3}\gamma(z^2-4)\right], \quad (3.11)$$

which agrees with (3.6). Hence the 'non-renormalization' of the conductivity is just due to our particular choice of the higher order corrections.

The membrane paradigm also determines the charge diffusion constant

$$D=-\sqrt{-g}\sqrt{-X^{txtx}X^{rxrx}}|_{r=r_0}\int_{r_0}^{\infty}\frac{dr}{\sqrt{-g}X^{trtr}}. \quad (3.12)$$

However, here we cannot evaluate the charge diffusion constant in a similar way, as we are studying the domain-wall geometry and we only explicitly know the IR and the UV geometries. It can be seen that the $r\to\infty$ limit of (2.6) leads to Lifshitz metric rather than AdS metric, which means that we cannot calculate the charge diffusion constant by naively applying this formula.

3.2 DC Conductivity from Kubo's formula

In this subsection, we reconsider the DC conductivity by making use of Kubo's formula, which can be seen as a check of consistency for the result obtained via the membrane paradigm. According to Kubo's formula, in the hydrodynamic limit the conductivity can be determined in terms of the retarded current-current correlation function

$$\sigma_{\rm DC}=-\lim_{\omega\to0}\frac{1}{\omega}{\rm Im}G^R_{xx}(\omega,\vec{k}=0), \quad (3.13)$$

where

$$G^R_{xx}(\omega,\vec{k}=0)=-i\int dtd\vec{x}e^{i\omega t}\theta(t)\langle[J_x(x),J_x(0)]\rangle. \quad (3.14)$$

JHEP06(2011)122

Here J_x denotes the CFT current dual to the bulk gauge field A_x. In this subsection we also introduce a new radial coordinate u, in which the domain wall metric can be written as

$$ds^2 = -g(u)e^{-\chi(u)}dt^2 + \frac{du^2}{g(u)} + \frac{R_0^2}{u^2}(dx^2+dy^2). \tag{3.15}$$

The IR geometry can be expressed as

$$ds^2_{\rm IR} = -\frac{r_0^{2z}}{L^2u^{2z}}f(u)dt^2 + \frac{L^2du^2}{u^2f(u)} + \frac{r_0^2}{L^2u^2}(dx^2+dy^2), \quad f(u) = 1-u^{z+2}, \tag{3.16}$$

where the horizon locates at $u=1$. Notice that $u=0$ does not correspond to the asymptotic boundary. Comparing the above two metrics we can find

$$e^{-\chi(u)} = \frac{r_0^{2z}}{u^{2z+2}}, \quad g(u) = \frac{u^2f(u)}{L^2}, \quad R_0 = \frac{r_0}{L}. \tag{3.17}$$

The non-vanishing components of the Weyl tensor are given as follows

$$\begin{aligned} C_{tutu} &= \frac{e^{-\chi(u)}}{12u^2}F(u), & C_{titj} &= -\frac{R_0^2e^{-\chi(u)}}{24u^4}g(u)F(u)\delta_{ij}, \\ C_{uiuj} &= \frac{R_0^2}{24u^4}\frac{F(u)}{g(u)}\delta_{ij}, & C_{ijkl} &= -\frac{R_0^4}{12u^6}F(u)\delta_{ik}\delta_{jl}, \end{aligned} \tag{3.18}$$

where $i,j,k,l = x,y$ and

$$\begin{aligned} F(u) = {} & u[g'(u)(4-3u\chi'(u)) + 2ug''(u)] \\ & -g(u)(4+2u\chi'(u) - u^2\chi'^2(u) + 2u^2\chi''(u)), \end{aligned} \tag{3.19}$$

Consider gauge field fluctuations of the following form

$$A_a(t,u,x) = \int \frac{d^3q}{(2\pi)^3}e^{-i\omega t+iqx}A_a(u,q), \tag{3.20}$$

where we have chosen the three-momentum vector $\mathbf{q}^\mu = (\omega, q, 0)$ and the gauge $A_u(u,q) = 0$. From (3.13), it can be seen that to calculate the conductivity, it is sufficient to set $q=0$ in subsequent calculations. The y-component of the generalized Maxwell equation reads

$$A_y'' + \frac{M'(u)}{M(u)}A_y' + \frac{e^{\chi(u)}}{g(u)^2}\omega^2A_y = 0, \tag{3.21}$$

where

$$M(u) = (1 - \frac{\gamma L^2}{3u^2}F(u))e^{-\chi(u)/2}g(u). \tag{3.22}$$

On the other hand, the boundary action is given by

$$I_y = -\frac{1}{2g_4^2}\int d^3x\sqrt{-g}g^{uu}g^{yy}(1-8\gamma L^2C_{uy}{}^{uy})A_y\partial_uA_y|_{u\to u_b}, \tag{3.23}$$

where u_b denotes the boundary, as the $u\to 0$ limit of (3.16) also leads to Lifshitz geometry rather than AdS. Therefore the corresponding retarded Green's function is given by [33]

$$G^R_{yy} = -\frac{1}{g_4^2}\sqrt{-g}g^{uu}g^{yy}(1-8\gamma L^2C_{uy}{}^{uy})\frac{A_y(u,-q)\partial_uA_y(u,q)}{A_y(u,-q)A_y(u,q)}\Big|_{u\to u_b}. \tag{3.24}$$

JHEP06(2011)122

It can be easily seen that

$$\sqrt{-g}g^{uu}g^{yy}(1-8\gamma L^2 C_{uy}{}^{uy}) = M(u), \tag{3.25}$$

As argued previously, since we do not know the explicit domain-wall metric, we cannot obtain concrete forms of the correlation functions. However, as noted in [34, 35], there exists a shortcut to calculate the conductivity. First of all, for a general second order differential equation

$$Y''(u) + A(u)Y'(u) + B(u)Y(u) = 0, \tag{3.26}$$

there exists a conserved quantity

$$Q(u) = e^{\int A}(\bar{Y}\partial_u Y - Y\partial_u \bar{Y}). \tag{3.27}$$

For our case, the conserved quantity $Q(u)$ is given by

$$Q(u) = M(u)(\bar{A}_y \partial_u A_y - A_y \partial_u \bar{A}_y), \tag{3.28}$$

thus the imaginary part of the correlation function turns out to be

$$\mathrm{Im}G^R_{yy} = -\frac{1}{2ig_4^2}\frac{Q(u)}{|A_y(u)|^2}|_{u\to u_b}, \tag{3.29}$$

where again u_b denotes the boundary. The solution for A_y can be written as

$$A_y(u) = (1-u)^{-\frac{i\omega}{4\pi T}} y(u), \tag{3.30}$$

where the exponent $-i\omega/(4\pi T)$ is determined by solving (3.21) in the near horizon region and imposing the incoming boundary condition. Since $Q(u)$ is a conserved quantity, we can evaluate it at the horizon $u = 1$. Therefore

$$\mathrm{Im}G^R_{yy} = -\frac{\omega}{g_4^2}\left(1 - \frac{4}{3}\gamma(z^2-4)\right)\frac{|y(1)|^2}{|y(u_b)|^2}. \tag{3.31}$$

Moreover, in the low frequency limit, the solution to (3.21) is simply $y(u) = \text{const}$. Finally we arrive at

$$\sigma = -\frac{1}{\omega}\mathrm{Im}G^R_{yy} = \frac{1}{g_4^2}\left[1 - \frac{4}{3}\gamma(z^2-4)\right], \tag{3.32}$$

which agrees with that obtained before.

3.3 EM duality in four dimensions

In this subsection we discuss electro-magnetic duality in our domain-wall background, which can be seen as extensions of [14] to more general cases. Generally speaking, current conservation and spatial rotational invariance fix the following general structure of the retarded Green's functions

$$G^R_{\mu\nu}(\mathbf{q}) = \sqrt{\mathbf{q^2}}(P^T_{\mu\nu}K^T(\omega,q) + P^L_{\mu\nu}K^L(\omega,q)), \tag{3.33}$$

JHEP06(2011)122

where $\mathbf{q}^\mu = (\omega, q^x, q^y), q^2 = [(q^x)^2+(q^y)^2]^{1/2}, \mathbf{q}^2 = q^2-\omega^2$. Here $P^T_{\mu\nu}$ and $P^L_{\mu\nu}$ are orthogonal projection operators given by

$$P^T_{tt} = P^T_{ti} = P^T_{it} = 0, \quad P^T_{ij} = \delta_{ij} - \frac{q_i q_j}{q^2}, \quad P^L_{\mu\nu} = \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{|\mathbf{q}|^2}\right) - P^T_{\mu\nu}, \tag{3.34}$$

where i, j are spatial indices and μ, ν denote the whole spacetime indices. Let us choose $\mathbf{q}^\mu = (\omega, q, 0)$ for simplicity, then we have

$$G^R_{yy}(\omega, q) = \sqrt{q^2-\omega^2}K^T(\omega, q), \quad G^R_{tt}(\omega, q) = -\frac{q^2}{\sqrt{q^2-\omega^2}}K^L(\omega, q) \tag{3.35}$$

It was observed in [13] that at the leading order level, i.e. in the standard four-dimensional Maxwell theory, K^T and K^L satisfied the following simple relation

$$K^T(\omega, q)K^L(\omega, q) = \text{const},$$

which signifies self-duality of the theory. As a result, the conductivity was a fixed constant.

Following [14], we introduce a Lagrangian multiplier B_a

$$I = \int d^4x\sqrt{-g}\left(-\frac{1}{8g_4^2}F_{ab}X^{abcd}F_{cd} + \frac{1}{2}\varepsilon^{abcd}B_a\partial_b F_{cd}\right), \tag{3.36}$$

where ε_{abcd} is the totally antisymmetric tensor with $\epsilon_{0123} = \sqrt{-g}$. After integrating by parts in the second term and some other manipulations, the action can be written as

$$I = \int d^4x\sqrt{-g}\left(-\frac{1}{8\hat{g}_4^2}\hat{X}^{abcd}G_{ab}G_{cd}\right), \tag{3.37}$$

where $G_{ab} \equiv \partial_a B_b - \partial_b B_a$ denotes the new field strength, $\hat{g}_4^2 \equiv 1/g_4^2$ and

$$\hat{X}^{cd}_{ab} = -\frac{1}{4}\varepsilon_{ab}{}^{ef}(X^{-1})_{ef}{}^{gh}\varepsilon_{gh}{}^{cd}. \tag{3.38}$$

Here and in the following the hatted quantities denote those in the 'dual' theory. The field strengths F_{ab} and G_{ab} are related by

$$F_{ab} = \frac{g^2}{4}(X^{-1})_{ab}{}^{cd}\varepsilon_{cd}{}^{ef}G_{ef}, \tag{3.39}$$

In standard Maxwell theory, the two actions and the corresponding equations of motion for A_a and B_a are identical, which means that the Maxwell theory is self-dual. Moreover, the duality relation between F_{ab} and G_{ab} is the usual Hodge dual.

In general $\hat{X} \neq X$, which means that self-duality is lost. The corresponding equations of motion are given by

$$\nabla_a(X^{abcd}F_{cd}) = 0, \quad \nabla_a(\hat{X}^{abcd}G_{cd}) = 0. \tag{3.40}$$

It can be seen that in the small γ limit

$$(X^{-1})_{ab}{}^{cd} = I_{ab}{}^{cd} + 8\gamma L^2 C_{ab}{}^{cd} + O(\gamma^2). \tag{3.41}$$

JHEP06(2011)122

Furthermore, using traceless properties of the Weyl tensor, we obtain

$$\hat{X}^{cd}_{ab} = (X^{-1})_{ab}{}^{cd} + O(\gamma^2), \tag{3.42}$$

Introducing index pairs $A, B \in \{tx, ty, tu, xy, xu, yu\}$ we write

$$X_A{}^B = \mathrm{diag}(X_1, X_2, X_3, X_4, X_5, X_6), \tag{3.43}$$

and

$$\hat{X}^B_A = \mathrm{diag}\left(\frac{1}{X_6}, \frac{1}{X_5}, \frac{1}{X_4}, \frac{1}{X_3}, \frac{1}{X_2}, \frac{1}{X_1}\right). \tag{3.44}$$

The relation between F_{ab} and G_{ab} (3.39) becomes

$$F_A = g_4^2 (X^{-1})_A{}^B \varepsilon_B{}^C G_C \,. \tag{3.45}$$

Here our background is shown in (3.15) and the non-vanishing components of the Weyl tensor are given in (3.18). Furthermore, fluctuations of gauge field are still presented in (3.20). Therefore the Maxwell equation $\nabla_a(X^{abcd}F_{cd}) = 0$ reads

$$\partial_u\left(\frac{e^{\chi(u)/2}}{u^2}X_3 A_t'\right) - \frac{e^{\chi(u)/2}X_1}{R_0^2 g(u)}(\omega q A_x + q^2 A_t) = 0, \tag{3.46}$$

$$A_t' + \frac{e^{-\chi(u)}g(u)u^2}{R_0^2}\frac{qX_5}{\omega X_3}A_x' = 0, \tag{3.47}$$

$$\partial_u(e^{-\chi(u)/2}g(u)X_5 A_x') + \frac{e^{\chi(u)/2}}{g(u)}X_1(\omega^2 A_x + \omega q A_t) = 0, \tag{3.48}$$

$$\partial_u(e^{-\chi(u)/2}g(u)X_6 A_y') + \frac{e^{\chi(u)/2}}{g(u)}X_2\omega^2 A_y - \frac{e^{-\chi(u)/2}u^2}{R_0^2}X_4 q^2 A_y = 0. \tag{3.49}$$

The equations of motion for B_a can be simply obtained by replacing $A_a \to B_a$ and $X_i \to \hat{X}_i$. In addition, the components of the ε tensor are listed below

$$\begin{aligned}
\varepsilon_{tx}{}^{yu} &= e^{-\chi(u)/2}g(u), & \varepsilon_{tu}{}^{xy} &= -e^{-\chi(u)/2}g(u),\\
\varepsilon_{tu}{}^{xy} &= e^{-\chi(u)/2}\frac{u^2}{R_0^2}, & \varepsilon_{xy}{}^{tu} &= -e^{\chi(u)/2}\frac{R_0^2}{u^2},\\
\varepsilon_{xu}{}^{ty} &= \frac{e^{\chi(u)/2}}{g(u)}, & \varepsilon_{yu}{}^{tx} &= -\frac{e^{\chi(u)/2}}{g(u)}.
\end{aligned} \tag{3.50}$$

Then we can explicitly work out the relation between F_{ab} and G_{ab},

$$\begin{aligned}
F_{tx} &= g_4^2\frac{e^{-\chi(u)/2}}{X_1}g(u)G_{yu}, & F_{ty} &= -g_4^2\frac{e^{-\chi(u)/2}}{X_2}g(u)G_{xu},\\
F_{tu} &= g_4^2\frac{e^{-\chi(u)/2}u^2}{R_0^2 X_3}G_{xy}, & F_{xy} &= -g_4^2\frac{R_0^2 e^{-\chi(u)/2}}{u^2 X_4}G_{tu},\\
F_{xu} &= g_4^2\frac{e^{\chi(u)/2}}{g(u)X_5}G_{ty}, & F_{yu} &= -g_4^2\frac{e^{\chi(u)/2}}{g(u)X_6}G_{tx}.
\end{aligned} \tag{3.51}$$

JHEP06(2011)122

The boundary action can be written as follows

$$I_b = \frac{1}{2g_4^2}\int d^4x \left(e^{\chi(u)/2}\frac{R_0^2}{u^2}X_3 A_t A_t' - e^{-\chi(u)/2}g(u)X_5 A_x A_x' - e^{-\chi(u)/2}g(u)X_6 A_y A_y' \right)|_{u\to u_b}. \tag{3.52}$$

Thus the retarded Green's functions are given by [33]

$$G^R_{tt} = \frac{R_0^2}{g_4^2}\frac{e^{\chi(u)/2}X_3}{u^2}\frac{\delta A_t'}{\delta A_t^b}|_{u\to u_b}, \tag{3.53}$$

$$G^R_{xx} = -\frac{1}{g_4^2}e^{-\chi(u)/2}g(u)X_5\frac{\delta A_x'}{\delta A_x^b}|_{u\to u_b}, \tag{3.54}$$

$$G^R_{tx} = \frac{1}{2g_4^2}\left[\frac{e^{\chi(u)}/2}{u^2}X_3\frac{\delta A_t'}{\delta A_x^b} - e^{-\chi(u)/2}g(u)X_5\frac{\delta A_x'}{\delta A_x^b}\right]|_{u\to u_b}, \tag{3.55}$$

$$G^R_{yy} = -\frac{1}{g_4^2}e^{-\chi(u)/2}g(u)X_6\frac{\delta A_y'}{\delta A_y^b}|_{u\to u_b}. \tag{3.56}$$

Let us focus on the yy-component of the retarded Green's function. The solution for $A_y(u)$ can be written in an abstract form $A_y(u) = \psi(u)A_y^b$, where A_y^b denotes its boundary value. Therefore it can be easily seen that $\psi(u_b) = 1$ and

$$G^R_{yy} = -\frac{1}{g_4^2}e^{-\chi(u_b)/2}g(u_b)X_6(u_b)\psi'(u_b). \tag{3.57}$$

Recall that

$$F_{xy} = -\frac{g_4^2}{X_4}e^{\chi(u)/2}\frac{R_0^2}{u^2}G_{tu}\,, \tag{3.58}$$

therefore

$$B_t' = C_1 u^2 e^{-\chi(u)/2}X_4\psi(u), \tag{3.59}$$

where C_1 is some undetermined constant. Moreover, the equation for B_t can be deduced from (3.46),

$$\partial_u\left(\frac{e^{\chi(u)/2}}{u^2}\hat{X}_3 B_t'\right) - \frac{e^{\chi(u)/2}\hat{X}_1}{R_0^2 g(u)}(\omega q B_x + q^2 B_t) = 0, \tag{3.60}$$

which leads to

$$C_1 = \frac{e^{\chi(u_b)/2}(\omega q B_x^b + q^2 B_t^b)}{R_0^2 g(u_b)X_6(u_b)\psi'(u_b)}, \tag{3.61}$$

where we have used the fact that $\hat{X}_3 = 1/X_4$ and $\hat{X}_1 = 1/X_6$. Then the retarded Green's function for B_t is given by

$$\hat{G}^R_{tt} = \frac{R_0^2}{\hat{g}_4^2}\frac{e^{\chi(u)/2}\hat{X}_3}{u^2}\frac{\delta B_t'}{\delta B_t^b} = \frac{g_4^2 e^{\chi(u_b)/2}q^2}{g(u_b)X_6(u_b)\psi'(u_b)}. \tag{3.62}$$

Finally we arrive at

$$\hat{G}^R_{tt}G^R_{yy} = -q^2, \quad \Rightarrow \quad K^T(\omega,q)\hat{K}^L(\omega,q) = 1, \tag{3.63}$$

while we can also obtain $K^L(\omega,q)\hat{K}^T(\omega,q) = 1$ in a parallel way. Our results indicate thus that such a simple duality relation still holds in our domain-wall geometry, irrespective of the IR near horizon geometry.

JHEP06(2011)122

3.4 DC conductivity at zero temperature

Up to now we have discussed the conductivity at finite temperature, while the conductivity at extremality can be studied in a somewhat different way. In this case the asymptotic geometry is still AdS, but the near horizon geometry is Lifshitz metric. Notice that the Weyl tensor vanishes in AdS spacetime, so the asymptotic solution of the gauge field is still given by

$$A_y = A_y^{(0)} + \frac{A_y^{(1)}}{r^{d-1}}\,. \tag{3.64}$$

It was observed in [38] that the equation of motion for A_y can be recast into a Schrödinger equation

$$-A_{y,ss} + V(s)A_y = \omega^2 A_y\,, \tag{3.65}$$

where s denotes some redefinition of the radial coordinate. The conductivity can be expressed in terms of the reflection coefficient $\mathcal{R}$

$$\sigma = \frac{1-\mathcal{R}}{1+\mathcal{R}}. \tag{3.66}$$

The general strategy can be summarized as follows: we solve the Schrödinger equation in the near horizon region and the asymptotic region respectively and then match the two solutions in certain intermediate region. Thus the reflection coefficient can be determined and the conductivity is obtained. In our specific background, let us consider the four-dimensional case as an example. Recall that the equation of motion for A_y is given by

$$\partial_r[e^{-\chi(r)/2}g(r)G(r)A_y'] + \frac{e^{\chi(r)/2}}{g(r)}G(r)\omega^2 A_y = 0, \quad G(r) = 1 - \frac{\gamma L^2}{3r^2}F(r).$$

By introducing

$$\frac{\partial}{\partial s} = e^{-\chi/2}g\frac{\partial}{\partial r}, \quad \Psi = \sqrt{G(r)}A_y, \tag{3.67}$$

the above equation turns out to be of Schrödinger form

$$-\partial_s^2\Psi + V(s)\Psi = \omega^2\Psi, \quad V(s) = \frac{1}{\sqrt{G(r)}}\partial_s^2\sqrt{G(r)}. \tag{3.68}$$

However, it can be seen that

$$G(r)_{\rm IR} = 1 - \frac{1}{3}\gamma L^2, \quad G(r)_{\rm UV} = 1, \tag{3.69}$$

which leads to a trivial potential $V(s) = 0$. Therefore we can easily obtain $\mathcal{R} = 0$ and $\sigma = 1$.

4 Charge transport in higher dimensions

In this section we calculate the conductivity in a general $(d+2)$-dimensional spacetime, where we apply the same techniques adopted in section 3. It was observed in [36] that in general $(d+2)$-dimensional background, the electrical conductivity and charge susceptibility are fixed by the central charge in a universal manner. However, due to our lack of

JHEP06(2011)122

understanding on the conformal field theory side, the relations between the conductivity and the central charge are still unclear. Furthermore, conductivity in asymptotically Lifshitz spacetimes was also studied in [37], where the focus was on the leading order effective action. In addition, since higher-dimensional electro-magnetic duality is not so powerful as its four-dimensional counterparts, we will not consider it.

4.1 DC Conductivity from the membrane paradigm

Considering the following $(d+2)$-dimensional domain-wall geometry

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{r^2}{R_0^2}\sum_{i=1}^{d} dx_i^2, \tag{4.1}$$

whose IR near horizon metric is given by

$$ds^2_{\rm IR} = -\frac{r^{2z}}{L^2}f(r)dt^2 + \frac{L^2dr^2}{r^2 f(r)} + \frac{r^2}{L^2}\sum_{i=1}^{d} dx_i^2, \quad f(r) = 1 - \frac{r_0^{z+d}}{r^{z+d}}\,. \tag{4.2}$$

It becomes Schwarzschild-AdS$_{d+2}$ when $z = 1$. It can be seen that here we still have

$$e^{-\chi(r)} = r^{2z-2}, \quad g(r) = \frac{r^2 f(r)}{L^2}, \quad R_0 = L, \tag{4.3}$$

The UV geometry is still fixed to be AdS. In the background (4.1), the non-vanishing components of the Weyl tensor are given as follows

$$\begin{aligned} C_{trtr} &= \frac{(d-1)e^{-\chi(r)}}{4(d+1)r^2}F(r), & C_{titj} &= -\frac{(d-1)e^{-\chi(r)}}{4d(d+1)R_0^2}g(r)F(r)\delta_{ij}, \\ C_{rirj} &= \frac{d-1}{4d(d+1)R_0^2}\frac{F(r)}{g(r)}\delta_{ij}, & C_{ijkl} &= -\frac{r^2}{2d(d+1)R_0^4}F(r)\delta_{ik}\delta_{jl}, \end{aligned} \tag{4.4}$$

where $i, j, k, l = x_1, \cdots, x_d$ and $F(r)$ is still given by (2.11). Following the procedures exhibited in section 3, we obtain the conductivity

$$\begin{aligned} \sigma &= \frac{1}{g_{d+2}^2}\sqrt{-g}\sqrt{-X^{txtx}X^{rxrx}}|_{r=r_0}, \\ &= \frac{1}{g_{d+2}^2}\left(\frac{r_0}{L}\right)^{d-2}\left[1 - \frac{4(d-1)\gamma}{d(d+1)}(2z(z-1) + d(z-d-2))\right]. \end{aligned} \tag{4.5}$$

This reduces to (3.6) when $d = 2$.

4.2 DC Conductivity from Kubo's formula

To evaluate the conductivity from Kubo's formula, we introduce a new radial coordinate u,

$$ds^2 = -g(u)e^{-\chi(u)}dt^2 + \frac{du^2}{g(u)} + \frac{R_0^2}{u^2}\sum_{i=1}^{d} dx_i^2. \tag{4.6}$$

JHEP06(2011)122

The IR metric can be written as follows in the u-coordinate

$$ds^2_{\rm IR} = -\frac{r_0^{2z}}{L^2 u^{2z}} f(u) dt^2 + \frac{L^2 du^2}{u^2 f(u)} + \frac{r_0^2}{L^2 u^2} \sum_{i=1}^{d} dx_i^2, \quad f(u) = 1 - u^{z+d}, \tag{4.7}$$

where the horizon locates at $u = 1$. Comparing the two metrics we can obtain

$$e^{-\chi(u)} = \frac{r_0^{2z}}{u^{2z+2}}, \quad g(u) = \frac{u^2 f(u)}{L^2}, \quad R_0 = \frac{r_0}{L}, \tag{4.8}$$

The corresponding non-vanishing components of the Weyl tensor are given by

$$\begin{aligned} C_{tutu} &= \frac{(d-1)e^{-\chi(u)}}{4(d+1)u^2} F(u), & C_{titj} &= -\frac{(d-1)R_0^2 e^{-\chi(u)}}{4d(d+1)u^4} g(u) F(u) \delta_{ij}, \\ C_{uiuj} &= \frac{(d-1)R_0^2}{4d(d+1)u^4} \frac{F(u)}{g(u)} \delta_{ij}, & C_{ijkl} &= -\frac{R_0^4}{2d(d+1)u^6} F(u) \delta_{ik}\delta_{jl}, \end{aligned} \tag{4.9}$$

where $i, j, k, l = x, y$ and $F(u)$ is still given by (3.19). Hence the generalized Maxwell equation in $(d+2)$-dimensions reads

$$A_y'' + \frac{M_{d+2}'(u)}{M_{d+2}(u)} A_y' + \frac{e^{\chi(u)}}{g(u)^2} \omega^2 A_y = 0, \tag{4.10}$$

where

$$M_{d+2}(u) = \left(1 - \frac{2\gamma(d-1)L^2}{d(d+1)u^2} F(u)\right) \frac{e^{-\chi(u)/2}}{u^{d-2}} g(u). \tag{4.11}$$

On the other hand, the retarded Green's function turns out to be

$$G^R_{yy} = -\frac{1}{g_4^2} \sqrt{-g} g^{uu} g^{yy} (1 - 8\gamma L^2 C_{uy}{}^{uy}) \frac{A_y(u,-q)\partial_u A_y(u,q)}{A_y(u,-q)A_y(u,q)} \Big|_{u \to u_b}. \tag{4.12}$$

Therefore one can find that

$$\sqrt{-g} g^{uu} g^{yy} (1 - 8\gamma L^2 C_{uy}{}^{uy}) = R_0^{d-2} M_{d+2}(u), \tag{4.13}$$

For our general $(d+2)$-dimensional case, the conserved quantity in (3.27) is given by

$$Q(u) = M_{d+2}(u)(\bar{A}_y \partial_u A_y - A_y \partial_u \bar{A}_y), \tag{4.14}$$

which leads to the following expression for the retarded Green's function

$$\mathrm{Im} G^R_{yy} = -\frac{R_0^{d-2}}{2i g_{d+2}^2} \frac{Q(u)}{|A_y(u_b)|^2}. \tag{4.15}$$

Furthermore, the general solution to A_y can still be written as

$$A_y(u) = (1-u)^{-\frac{i\omega}{4\pi T}} y(u). \tag{4.16}$$

Thus we can obtain

$$\mathrm{Im} G^R_{yy} = -\frac{\omega R_0^{d-2}}{g_{d+2}^2} \left[1 - \frac{4(d-1)\gamma}{d(d+1)} (2z(z-1) + d(z-d-2))\right] \frac{|y(1)|^2}{|y(u_b)|^2}. \tag{4.17}$$

JHEP06(2011)122

Finally, in the low frequency limit the solution to (4.10) is simply $y(u) = \text{const}$, which results in

$$\sigma = -\frac{1}{\omega}\text{Im}G^R_{yy} = \frac{r_0^{d-2}}{g^2_{d+2}L^{d-2}}\left[1 - \frac{4(d-1)\gamma}{d(d+1)}(2z(z-1) + d(z-d-2))\right]. \tag{4.18}$$

It can be seen that once again this result agrees with the one obtained via the membrane paradigm.

5 Summary and discussion

The full background geometry is required when calculating the retarded Green's functions via AdS/CFT. However, we can still acquire some knowledge about the transport coefficients from a domain-wall geometry. In this paper we computed conductivity in the presence of Weyl corrections in a domain-wall background, whose near horizon IR geometry is Lifshitz black hole and asymptotic geometry is AdS. We obtained the conductivity via both the membrane paradigm and Kubo's formula. By making use of a shortcut, the conductivity derived from Kubo's formula can be solely expressed in terms of quantities at the horizon. The results obtained via both approaches precisely match in four as well as in higher dimensions. Moreover, it was shown in [14] that in four dimensions, although self-duality was lost in higher derivative theories, a simple relation for the longitudinal and transverse components of the current-current correlation functions and those of the dual counterparts, $K^L(\omega, q)\hat{K}^T(\omega, q) = 1$, still held. Here we show that this simple relation also holds in our domain-wall background, irrespective of the IR near horizon geometry.

Similar backgrounds were also investigated in [39] and [40], where the authors considered charged dilaton black branes in Einstein-Maxwell-Dilaton theory, whose near horizon geometry was Lifshitz metric and asymptotic geometry was AdS. One crucial difference was that due to the nontrivial background U(1) gauge field, the potential in the Schrödinger equation was also nontrivial, which lead to a universal conductivity $\text{Re}\sigma \sim \omega^2$ in four dimensions. If we want to consider Weyl corrections to the conductivity in such a background, it would be necessary to work out the perturbed metric, as the nontrivial background gauge field would back-react on the leading order solution. Holographic properties of charged black holes in higher derivative theories were studied in [41–43] and transport properties in extremal charged black hole backgrounds were considered in [44–48].

One can also consider the following type of higher order corrections instead

$$I'_{\text{vec}} = \frac{1}{\tilde{g}_4^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{ab}F^{ab} + \alpha L^2(R_{abcd}F^{ab}F^{cd} - 4R_{ab}F^{ac}F^b{}_c + RF^{ab}F_{ab})\right], \tag{5.1}$$

which arises from the Kaluza-Klein reduction of five-dimensional Gauss-Bonnet gravity. It was observed in [14] that by combining the Einstein equation in the neutral black hole background $R_{ab} = -3/L^2 g_{ab}$ and the definition of the Weyl tensor, the action (5.1) becomes

$$I'_{\text{vec}} = \frac{1+8\alpha}{\tilde{g}_4^2}\int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{ab}F^{ab} + \frac{\alpha}{1+8\alpha}L^2 C_{abcd}F^{ab}F^{cd}\right]. \tag{5.2}$$

JHEP06(2011)122

It can be easily seen that the resulting action is equivalent to (2.3) with the following identifications

$$g_4^2 = \frac{\tilde{g}_4^2}{1+8\alpha}, \quad \gamma = \frac{\alpha}{1+8\alpha}. \tag{5.3}$$

Therefore the charge transport properties are identical. However, here the Einstein equation in the IR Lifshitz black hole background cannot have such a simple expression and thus the two actions are generically not equivalent. It would be interesting to study charge transport coefficients in a different theory e.g. (5.1) and to see the effects of higher order corrections on the conductivity.

Acknowledgments

DWP acknowledges an FCT (Portuguese Science Foundation) grant. This work was also funded by FCT through projects CERN/FP/109276/2009 and PTDC/FIS/098962/2008.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [*Adv. Theor. Math. Phys.* **2** (1998) 231] [hep-th/9711200] [SPIRES].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [SPIRES].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [SPIRES].

[4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [SPIRES].

[5] S.A. Hartnoll, *Lectures on holographic methods for condensed matter physics*, *Class. Quant. Grav.* **26** (2009) 224002 [arXiv:0903.3246] [SPIRES].

[6] C.P. Herzog, *Lectures on holographic superfluidity and superconductivity*, *J. Phys.* **A 42** (2009) 343001 [arXiv:0904.1975] [SPIRES].

[7] J. McGreevy, *Holographic duality with a view toward many-body physics*, *Adv. High Energy Phys.* **2010** (2010) 723105 [arXiv:0909.0518] [SPIRES].

[8] G.T. Horowitz, *Introduction to holographic superconductors*, arXiv:1002.1722 [SPIRES].

[9] S. Sachdev, *Condensed matter and AdS/CFT*, arXiv:1002.2947 [SPIRES].

[10] D.T. Son, *Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [SPIRES].

[11] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053] [SPIRES].

[12] S. Kachru, X. Liu and M. Mulligan, *Gravity duals of Lifshitz-like fixed points*, *Phys. Rev.* **D 78** (2008) 106005 [arXiv:0808.1725] [SPIRES].

[13] C.P. Herzog, P. Kovtun, S. Sachdev and D.T. Son, *Quantum critical transport, duality and M-theory*, *Phys. Rev.* **D 75** (2007) 085020 [hep-th/0701036] [SPIRES].

JHEP06(2011)122

[14] R.C. Myers, S. Sachdev and A. Singh, *Holographic quantum critical transport without self-duality*, *Phys. Rev.* **D 83** (2011) 066017 [arXiv:1010.0443] [SPIRES].

[15] J.P.S. Lemos, *Two-dimensional black holes and planar general relativity*, *Class. Quant. Grav.* **12** (1995) 1081 [gr-qc/9407024] [SPIRES].

[16] J.P.S. Lemos, *Cylindrical black hole in general relativity*, *Phys. Lett.* **B 353** (1995) 46 [gr-qc/9404041] [SPIRES].

[17] J.P.S. Lemos and V.T. Zanchin, *Rotating charged black string and three dimensional black holes*, *Phys. Rev.* **D 54** (1996) 3840 [hep-th/9511188] [SPIRES].

[18] M. Taylor, *Non-relativistic holography*, arXiv:0812.0530 [SPIRES].

[19] U.H. Danielsson and L. Thorlacius, *Black holes in asymptotically Lifshitz spacetime*, *JHEP* **03** (2009) 070 [arXiv:0812.5088] [SPIRES].

[20] R.B. Mann, *Lifshitz topological black holes*, *JHEP* **06** (2009) 075 [arXiv:0905.1136] [SPIRES].

[21] D.-W. Pang, *A note on black holes in asymptotically Lifshitz spacetime*, arXiv:0905.2678 [SPIRES].

[22] G. Bertoldi, B.A. Burrington and A. Peet, *Black holes in asymptotically Lifshitz spacetimes with arbitrary critical exponent*, *Phys. Rev.* **D 80** (2009) 126003 [arXiv:0905.3183] [SPIRES].

[23] G. Bertoldi, B.A. Burrington and A.W. Peet, *Thermodynamics of black branes in asymptotically Lifshitz spacetimes*, *Phys. Rev.* **D 80** (2009) 126004 [arXiv:0907.4755] [SPIRES].

[24] K. Balasubramanian and J. McGreevy, *An analytic Lifshitz black hole*, *Phys. Rev.* **D 80** (2009) 104039 [arXiv:0909.0263] [SPIRES].

[25] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, *Lifshitz black hole in three dimensions*, *Phys. Rev.* **D 80** (2009) 104029 [arXiv:0909.1347] [SPIRES].

[26] R.-G. Cai, Y. Liu and Y.-W. Sun, *A Lifshitz black hole in four dimensional R^2 gravity*, *JHEP* **10** (2009) 080 [arXiv:0909.2807] [SPIRES].

[27] E. Ayon-Beato, A. Garbarz, G. Giribet and M. Hassaine, *Analytic Lifshitz black holes in higher dimensions*, *JHEP* **04** (2010) 030 [arXiv:1001.2361] [SPIRES].

[28] G. Bertoldi, B.A. Burrington and A.W. Peet, *Thermal behavior of charged dilatonic black branes in AdS and UV completions of Lifshitz-like geometries*, *Phys. Rev.* **D 82** (2010) 106013 [arXiv:1007.1464] [SPIRES].

[29] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *Viscosity bound violation in higher derivative gravity*, *Phys. Rev.* **D 77** (2008) 126006 [arXiv:0712.0805] [SPIRES].

[30] A. Ritz and J. Ward, *Weyl corrections to holographic conductivity*, *Phys. Rev.* **D 79** (2009) 066003 [arXiv:0811.4195] [SPIRES].

[31] P. Kovtun, D.T. Son and A.O. Starinets, *Holography and hydrodynamics: Diffusion on stretched horizons*, *JHEP* **10** (2003) 064 [hep-th/0309213] [SPIRES].

[32] N. Iqbal and H. Liu, *Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm*, *Phys. Rev.* **D 79** (2009) 025023 [arXiv:0809.3808] [SPIRES].

JHEP06(2011)122

[33] D.T. Son and A.O. Starinets, *Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications*, *JHEP* **09** (2002) 042 [hep-th/0205051] [SPIRES].

[34] S. Caron-Huot, P. Kovtun, G.D. Moore, A. Starinets and L.G. Yaffe, *Photon and dilepton production in supersymmetric Yang-Mills plasma*, *JHEP* **12** (2006) 015 [hep-th/0607237] [SPIRES].

[35] A. Nata Atmaja and K. Schalm, *Photon and dilepton production in soft wall AdS/QCD*, *JHEP* **08** (2010) 124 [arXiv:0802.1460] [SPIRES].

[36] P. Kovtun and A. Ritz, *Universal conductivity and central charges*, *Phys. Rev.* **D 78** (2008) 066009 [arXiv:0806.0110] [SPIRES].

[37] D.-W. Pang, *Conductivity and diffusion constant in Lifshitz backgrounds*, *JHEP* **01** (2010) 120 [arXiv:0912.2403] [SPIRES].

[38] G.T. Horowitz and M.M. Roberts, *Zero temperature limit of holographic superconductors*, *JHEP* **11** (2009) 015 [arXiv:0908.3677] [SPIRES].

[39] K. Goldstein, S. Kachru, S. Prakash and S.P. Trivedi, *Holography of charged dilaton black holes*, *JHEP* **08** (2010) 078 [arXiv:0911.3586] [SPIRES].

[40] C.-M. Chen and D.-W. Pang, *Holography of charged dilaton black holes in general dimensions*, *JHEP* **06** (2010) 093 [arXiv:1003.5064] [SPIRES].

[41] R.C. Myers, M.F. Paulos and A. Sinha, *Holographic hydrodynamics with a chemical potential*, *JHEP* **06** (2009) 006 [arXiv:0903.2834] [SPIRES].

[42] S. Cremonini, K. Hanaki, J.T. Liu and P. Szepietowski, *Black holes in five-dimensional gauged supergravity with higher derivatives*, *JHEP* **12** (2009) 045 [arXiv:0812.3572] [SPIRES].

[43] R.-G. Cai and D.-W. Pang, *Holography of charged black holes with RF^2 corrections*, arXiv:1104.4453 [SPIRES].

[44] M.F. Paulos, *Transport coefficients, membrane couplings and universality at extremality*, *JHEP* **02** (2010) 067 [arXiv:0910.4602] [SPIRES].

[45] M. Edalati, J.I. Jottar and R.G. Leigh, *Transport coefficients at zero temperature from extremal black holes*, *JHEP* **01** (2010) 018 [arXiv:0910.0645] [SPIRES].

[46] R.-G. Cai, Y. Liu and Y.-W. Sun, *Transport coefficients from extremal Gauss-Bonnet black holes*, *JHEP* **04** (2010) 090 [arXiv:0910.4705] [SPIRES].

[47] S.K. Chakrabarti, S. Jain and S. Mukherji, *Viscosity to entropy ratio at extremality*, *JHEP* **01** (2010) 068 [arXiv:0910.5132] [SPIRES].

[48] M. Edalati, J.I. Jottar and R.G. Leigh, *Shear modes, criticality and extremal black holes*, *JHEP* **04** (2010) 075 [arXiv:1001.0779] [SPIRES].

JHEP06(2011)122