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Holographic three-point functions of giant gravitons

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Abstract: Working within the AdS/CFT correspondence we calculate the three-point function of two giant gravitons and one pointlike graviton using methods of semiclassical string theory and considering both the case where the giant gravitons wrap an $S^3 \subset S^5$ and the case where the giant gravitons wrap an $S^3 \subset \mathrm{AdS}_5$. We likewise calculate the correlation function in $\mathcal{N} = 4$ SYM using two Schur polynomials and a single trace chiral primary. We find that the gauge and string theory results have structural similarities but do not match perfectly, and interpret this in terms of the Schur polynomials' inability to interpolate between dual giant and pointlike gravitons.

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Contents

1 Introduction

The integrable structures underlying the $\mathcal{N} = 4$ Super-Yang-Mills (SYM) theory are a continuing source of fascination, in large part due to the promise they hold of leading to the complete solution of this non-trivial four-dimensional gauge theory. Since their appearance in [1, 2], the main focus of investigation has been the *spectral problem*, i.e. the question of obtaining the exact spectrum of anomalous dimensions of gauge-invariant operators of the theory. For a review of the state of the art and current challenges in this field, we refer to the collection of articles in [3]. The spectrum, however, is only part of the information required to (at least in principle) solve the theory, the other essential ingredient being the set of all three-point functions between the fundamental operators. It thus natural to look beyond the spectrum and ask whether integrability plays any role in specifying the correlation functions of the theory. Answering this question is also likely to be crucial in attempts to probe integrability beyond the planar limit, which involves considering interacting strings (and three-point vertices thereof) in the dual description.[1]

Understanding the role of integrability in the calculation of $\mathcal{N} = 4$ SYM correlation functions has recently been receiving a growing amount of attention. On the gauge side

[1]See the review [4] for a recent discussion of these issues.

of the AdS/CFT correspondence, and following earlier work in [5–7], it was shown in [8] that the one-loop Bethe equations can be used to simplify the calculation of tree-level three-point functions of certain non-protected operators.

On the gravity side, the computation of two-point functions of spinning strings was considered in [9], and recently taken-up again in [10], as well as in [11] using the formalism of semiclassical vertex operators [12, 13]. However, the extension to general semiclassical three-point functions appears to rely on finding the precise geometric solution interpolating between the insertion points of the corresponding operators on the boundary of AdS, which seems to lie beyond current capabilities. As argued in [14, 15], this problem can be avoided provided one restricts to correlation functions involving only two "heavy" operators (for which the classical string trajectory is known), while treating the remaining "light"operators as a perturbation. With this simplifying assumption, several cases of three- and higher-order correlation functions, involving different types of semiclassical strings and various choices for the light operators, have been considered in the literature [16–23].

The semiclassical string solutions discussed above correspond to single-trace gauge theory operators with large quantum numbers, which can also be described as spin chains with certain amounts of excitations. Although the dimension of these states is much larger than 1 in order to justify the semiclassical approximation (in particular, the dimension is $\sim \sqrt{\lambda}$ with λ the 't Hooft coupling), it is also necessarily much smaller than the rank of the gauge group N. However, the gauge theory also contains operators whose dimension scales as N in the large-N limit. A class of such operators was identified in [24] with the *giant gravitons* of [25], which are D3-branes wrapping an S^3 in the S^5 (their radius stabilised by the presence of five-form flux in the geometry), spinning along a circle in the S^5 and located at the centre of AdS_5. The mapping of these giant gravitons, as well as the dual giant gravitons wrapping an S^3 in AdS_5 constructed in [26], to the gauge theory was put in a more general context in the work of [27]: they correspond to *Schur polynomial* operators, which are specific combinations of traces of one of the $\mathcal{N} = 4$ SYM scalars forming an orthogonal basis for any N.[2]

Each Schur polynomial is labeled by a Young tableau corresponding to a specific representation of U(N). As argued in [27], S^5 giant gravitons of dimension $k \leq N$ are mapped to the *antisymmetric* representation with k boxes, while the dual AdS_5 ones map to the *symmetric* representation with k boxes. The fact that the dimension of the antisymmetric representation is bounded from above by N corresponds to the fact that the angular momentum of the S^5 graviton has an upper bound: its radius increases with angular momentum but cannot become larger than that of the S^5. Giant gravitons which saturate this bound are called *maximal*.

Given that giant gravitons are heavy, semiclassical objects, one can ask whether the approach of [14] can be applied to correlation functions of giant gravitons. A calculation involving giant gravitons was recently performed in [28], which considered correlation func-

[2]See section 2 for the precise definition of these operators.

tions of two S^5 giant gravitons with open strings attached (as the heavy states), with the light state being dual to a chiral primary operator. This work computed two-point functions for both maximal and non-maximal giant gravitons, but three-point functions only in the maximal case, where the giant gravitons were essentially inert (their role being that of providing the open string endpoints).

In this work we consider a different type of correlation function of two giant graviton operators with a chiral primary. On the string side, we will take as our heavy states either AdS_5 or S^5 giant gravitons (not necessarily maximal). The light state will be an AdS scalar field of dimension $\Delta = J \ll \sqrt{N}$ dual to a chiral primary. On the gauge theory side, the operators corresponding to the giant gravitons are Schur polynomials with dimension k of order N, in the large-N limit. We compute the three-point function of these operators with a chiral primary operator, using established gauge theory techniques. This quantity is expected to be protected from quantum corrections, owing to the shared 1/2 BPS supersymmetry of the operators, however we do not find perfect agreement with the string theoretic results. In the concluding section we discuss in detail this discrepancy which we believe hinges on the inability of the Schur polynomials to interpolate between giant and pointlike gravitons.

The plan of the paper is as follows: In the next section we introduce the operators that we will consider on the gauge theory side and compute the three-point functions in question using Schur polynomial techniques. Then, in section 3 we describe the string theory computation of the same quantities using the approach of [14]. In the concluding section we compare the two results and discuss open problems and directions for future work. We have also included an appendix containing a simple computation of the holographic S^5 giant graviton two-point function.

2 Three-point functions from gauge theory

In this section we will compute three-point functions involving two types of half-BPS operators: single trace chiral primaries (which in the following we will simply call "chiral primaries") and Schur polynomial operators. Three-point functions of such operators (all built up of the same $\mathcal{N} = 4$ SYM chiral field Z) are expected to be protected to all orders [29] and can thus be exactly calculated in the gauge theory.

2.1 Single trace chiral primaries

Let us consider single-trace operators built from a single complex scalar field Z, i.e.

$$\mathcal{O}^J = \mathrm{Tr} Z^J, \tag{2.1}$$

These operators are dual to point-like strings moving along an equator of S^5 with angular momentum J. Their two- and three-point functions are protected and can be calculated exactly, see for instance [30], and read

$$\langle \mathrm{Tr} Z^J\, \mathrm{Tr}\bar{Z}^J\rangle = \frac{1}{J+1}\left\{\frac{\Gamma(N+J+1)}{\Gamma(N)} - \frac{\Gamma(N+1)}{\Gamma(N-J)}\right\} \tag{2.2}$$

$$= J\,N^J\left\{1+\binom{J+1}{4}\frac{1}{N^2}+\ldots\right\}. \tag{2.3}$$

$$\langle \mathrm{Tr} Z^J\, \mathrm{Tr} Z^K \mathrm{Tr}\bar{Z}^{J+K}\rangle = \frac{1}{J+K+1}\left\{\frac{\Gamma(N+J+K+1)}{\Gamma(N)} - \frac{\Gamma(N+J+1)}{\Gamma(N-K)} + \frac{\Gamma(N+1)}{\Gamma(N-J-K)} - \frac{\Gamma(N+K+1)}{\Gamma(N-J)}\right\} \tag{2.4}$$

$$= N^{J+K-1} J\,K\,(J+K) \times\left\{1+\frac{1}{3!N^2}\binom{K+J-1}{2}\left[\binom{K}{2}+\binom{J}{2}-1\right]+\ldots\right\}.$$

Here we have left out the trivial dependence on space-time coordinates and the 't Hooft coupling constant. Hence we get for the CFT structure constant

$$\mathcal{C}_{J,K,K+J} \equiv \frac{\langle \mathcal{O}^J\mathcal{O}^K\bar{\mathcal{O}}^{J+K}\rangle}{\sqrt{\langle \mathcal{O}^J\bar{\mathcal{O}}^J\rangle\langle \mathcal{O}^K\bar{\mathcal{O}}^K\rangle\langle \mathcal{O}^{J+K}\bar{\mathcal{O}}^{J+K}\rangle}} \tag{2.5}$$

$$= \frac{1}{N}\sqrt{J\,K\,(J+K)}\left[1+\mathcal{O}\left(\frac{1}{N^2}\right)\right]. \tag{2.6}$$

This is the well known expression for the three-point function of three chiral primaries of the type given in eq. (2.1). This object can also be viewed as a two point function of a single trace operator and a double trace operator (since the contractions needed are the same in both cases) and we notice the well-known fact that single and multi-trace operators are orthogonal to the leading order in $\frac{1}{N}$ provided J is not too big. For large values of J single trace operators mix with multi trace operators and a more convenient basis is the basis of Schur polynomials $\chi_R(Z)$ described below [27].

2.2 Schur polynomials

The Schur polynomial $\chi_R(Z)$ of a complex matrix Z is defined as

$$\chi_{R_n}(Z) = \frac{1}{n!}\sum_{\sigma\in S_n}\chi_{R_n}(\sigma)\, Z_{i_1}^{i_{\sigma(1)}}\ldots Z_{i_n}^{i_{\sigma(n)}}. \tag{2.7}$$

Here R_n denotes an irreducible representation of $\mathrm{U}(N)$ described in terms of a Young tableau with n boxes. The sum is over all elements of the symmetric group S_n and $\chi_{R_n}(\sigma)$ is the character of the element σ in the representation R_n. Notice that there is no limit in which the Schur polynomial reduces to a chiral primary operator.[3] Schur polynomials are

[3]The Schur polynomials $\chi_{R_n}(Z)$ have the general structure

$$\chi_{R_n}(Z) = c_{0,n}\mathrm{Tr}Z^n + c_{1,n}\mathrm{Tr}Z\mathrm{Tr}Z^{n-1}+\ldots+c_{n,n}\,(\mathrm{Tr}Z)^n, \tag{2.8}$$

where the c's are constants independent of N and the sum is over all partitions.

again 1/2-BPS operators with protected two- and three-point functions. These correlation functions have been calculated exactly and read [27]

$$\langle \chi_R(Z)\chi_S(\bar{Z})\rangle = \delta_{R,S} \prod_{i.j\in R} (N-i+j), \tag{2.9}$$

$$\langle \chi_R(Z)\chi_S(Z)\chi_T(\bar{Z})\rangle = g(R,S;T) \prod_{i,j\in T} (N-i+j), \tag{2.10}$$

where $g(R,S,T)$ is the Littlewood-Richardson coefficient which counts the multiplicity with which the representation T appears in the tensor product of the representations R and S. Furthermore, the product $\prod_{i,j\in R}$ goes over all boxes of the Young tableau of the representation R with i denoting the row number and j the column number. Hence the Schur polynomials provide an orthogonal basis of operators. The string theory duals of Schur polynomials are collections of giant gravitons, i.e. D3-branes which wrap an S^3 of either S^5 or AdS_5 [24, 27]. The cleanest examples are the Schur polynomials of the symmetric and the antisymmetric representations. When the number of boxes, k, in the Young tableau of the representation is large (i.e. $k \sim \mathcal{O}(N)$, with $N \to \infty$), the Schur polynomial of the symmetric representation is dual to a single giant graviton moving on S^5 with angular momentum k and wrapping an $S^3 \subset \mathrm{AdS}_5$. For the antisymmetric case the giant graviton instead wraps an $S^3 \subset S^5$ [27].

Let us denote the Schur polynomial for the symmetric representation with k boxes as $\chi_k^S(Z)$ and the Schur polynomial for the antisymmetric representation with k boxes as $\chi_k^A(Z)$. Then we find for the corresponding two and three-point functions

$$\langle \chi_k^S(\bar{Z})\chi_k^S(Z)\rangle = \prod_{j=1}^{k}(N-1+j), \tag{2.11}$$

$$\langle \chi_k^A(\bar{Z})\chi_k^A(Z)\rangle = \prod_{i=1}^{k}(N-i+1), \tag{2.12}$$

$$\langle \chi_k^S(\bar{Z})\chi_{k-J}^S(Z)\chi_J^S(Z)\rangle = \prod_{j=1}^{k}(N-1+j), \tag{2.13}$$

$$\langle \chi_k^A(\bar{Z})\chi_{k-J}^A(Z)\chi_J^A(Z)\rangle = \prod_{i=1}^{k}(N-i+1), \tag{2.14}$$

since for these cases $g(R,S;T) = 1$. Notice that for the antisymmetric case we have that $k \leq N$ while in the symmetric case k is unbounded.

2.3 Two Schur polynomials and one single trace operator

Recently, it has been understood from the string theory side how to calculate by semiclassical methods three-point functions which involve two massive string states and one light one dual to a chiral primary operator of the type $\mathrm{Tr}Z^J$ [10, 14]. In the present paper we will study the case where the two heavy operators are giant gravitons. In the field theory language these three-point functions can be calculated exactly using the results of the sections above. The properly normalized three-point functions are

$$C^S_{k,k-J,J} \equiv \frac{\langle \chi^S_k(\bar{Z})\chi^S_{k-J}(Z)\mathrm{Tr}Z^J\rangle}{\sqrt{\langle \chi^S_k(\bar{Z})\chi^S_k(Z)\rangle\langle \chi^S_{k-J}(\bar{Z})\chi^S_{k-J}(Z)\rangle\langle \mathrm{Tr}\bar{Z}^J\mathrm{Tr}Z^J\rangle}}, \tag{2.15}$$

and similarly for $C^A_{k,k-J,J}$. We have already calculated the relevant norms above, cf. eqs. (2.2) and (2.9). To calculate the expectation value in the numerator we expand $\mathrm{Tr}\, Z^J$ in the basis of Schur polynomials. Noting that by definition

$$\mathrm{Tr}Z^J = \mathrm{Tr}(\sigma_0 Z), \tag{2.16}$$

where σ_0 is the cyclic permutation we have

$$\mathrm{Tr}\, Z^J = \sum_{R_J} \chi_{R_J}(\sigma_0)\chi_{R_J}(Z), \tag{2.17}$$

where the sum goes over all possible irreducible representations R_J corresponding to Young tableaux with J boxes, see e.g. [31]. Inserting the sum instead of $\mathrm{Tr}Z^J$ in the expectation values $\langle \chi^A_k(\bar{Z})\chi^A_{k-J}(Z)\,\mathrm{Tr}\, Z^J\rangle$ and $\langle \chi^S_k(\bar{Z})\chi^S_{k-J}(Z)\,\mathrm{Tr}\, Z^J\rangle$ it is clear from (2.10) that only the completely antisymmetric representation contributes in the former case and only the completely symmetric representation in the latter. The character $\chi_{R_J}(\sigma_0)$ can be written down in closed form for hook diagrams, i.e. Young diagrams for which only the first row can have more than one box. Denoting the number of boxes in the first row of the hook diagram as $J-m$ it holds that $\chi^{\mathrm{hook}}_{R_J}(\sigma_0) = (-1)^m$. Hence for the cases of interest to us we have

$$\chi^S_J(\sigma_0) = 1, \qquad \chi^A_J(\sigma_0) = (-1)^{J-1}. \tag{2.18}$$

This implies[4]

$$\langle \chi^S_k(\bar{Z})\chi^S_{k-J}(Z)\,\mathrm{Tr}\, Z^J\rangle = \prod_{j=1}^{k}(N-1+j), \tag{2.19}$$

$$\langle \chi^A_k(\bar{Z})\chi^A_{k-J}(Z)\,\mathrm{Tr}\, Z^J\rangle = (-1)^{J-1}\prod_{i=1}^{k}(N-i+1). \tag{2.20}$$

Dividing with the relevant norms we hence find the structure constants

$$C^S_{k,k-J,J} = \frac{\sqrt{\prod_{p=k-J+1}^{k}(N+p-1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}, \tag{2.21}$$

$$C^A_{k,k-J,J} = (-1)^{(J-1)}\frac{\sqrt{\prod_{p=k-J+1}^{k}(N-p+1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}, \tag{2.22}$$

[4]The $(-1)^J$ part of the prefactor in the antisymmetric case could be removed since one can equally well define the gauge theory dual of the antisymmetric giant graviton with angular momentum k to be $(-1)^k\chi^A_k(Z)$. However, in the following we will follow the usual definition in the Schur operator literature and keep the alternating sign.

JHEP06(2011)085

where the quantities in the denominators are nothing but $\sqrt{\langle \text{Tr}\bar{Z}^J \text{Tr}Z^J\rangle}$ which is given exactly in equation (2.2). In other words we have exact expressions for $C^S_{k,k-J,J}$ and $C^A_{k,k-J,J}$. Now, we are interested in the situation where the Schur polynomials correspond to large Young tableaux and where the chiral primary is a small operator, i.e. the limit

$$N \to \infty, \qquad k \to \infty, \qquad \frac{k}{N} \text{ finite}, \qquad J \ll k, \tag{2.23}$$

and in particular $J \ll \sqrt{N}$. In this limit we find for the structure constants

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2}, \tag{2.24}$$

$$C^A_{k,k-J,J} = (-1)^{(J-1)}\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}. \tag{2.25}$$

Notice that this result does not reduce to the chiral primary result in any limit (in accordance with the fact that a chiral primary operator can not be obtained as a limit of a single Schur polynomial). Furthermore, we note that for the antisymmetric representation we have the constraint $k \leq N$ while for the symmetric case k is unbounded.

3 Three point function from string theory

In this section we will calculate the three-point function structure constants considered in the previous sections using the AdS/CFT dictionary put forth in [10, 14]. We work under the assumption that the holographic two-point function of the giant gravitons are given by the D-brane solutions [25, 32] continued to the Euclidean Poincaré patch, in the same way that semiclassical spinning strings were argued to represent the two-point functions of the associated operators in [10, 14]. The calculation proceeds by varying the Euclidean D-brane actions in accordance with the supergravity fluctuations corresponding to the small operator in the desired three-point function, and then evaluating those fluctuations on the Wick-rotated giant graviton solutions, described in the Poincaré patch.[5]

3.1 Giant graviton on S^5

We begin by reviewing the giant graviton [25] with worldvolume $\mathbb{R}(\subset \text{AdS}_5) \times S^3(\subset S^5)$. We begin in Lorentzian signature $(-,+,\ldots,+)$. The metric of $\text{AdS}_5 \times S^5$ can be taken as

$$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\widetilde{\Omega}_3^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\Omega_3^2. \tag{3.1}$$

The action for the D3-brane is (in units where the AdS radius is set to 1)

$$S_{D3} = -\frac{N}{2\pi^2}\int d^4\sigma \left(\sqrt{-g} - P[C_4]\right), \tag{3.2}$$

where $g_{ab} = \partial_a X^M \partial_b X_M$, where $a,b = 0,\ldots,3$ label the worldvolume coordinates and where X^M are the embedding coordinates. Note that there is no B-field in our background,

[5] Such fluctuation calculations for D-branes were first performed in [33] in the context of Wilson loops in higher representations.

and we also will not be turning on worldvolume gauge fields. The four-form potential C_4 which will be important for the giant graviton has its legs entirely in the S^5, and may be taken as [32]

$$C_{\phi\chi_1\chi_2\chi_3} = \cos^4\theta\,\mathrm{Vol}(\Omega_3), \tag{3.3}$$

where the χ_i are angles covering the $S^3 \subset S^5$ and where $\mathrm{Vol}(\Omega_3)$ indicates its volume element.

One takes the ansatz

$$\rho = 0, \quad \sigma^0 = t, \quad \phi = \phi(t), \quad \sigma^i = \chi_i, \tag{3.4}$$

and obtains

$$S = \int dt\, L = -N \int dt \left[\cos^3\theta\sqrt{1-\dot\phi^2\sin^2\theta} - \dot\phi\,\cos^4\theta\right]. \tag{3.5}$$

Independence of ϕ leads to a conserved angular momentum

$$k \equiv \frac{\delta L}{\delta\dot\phi} = \frac{N\dot\phi\sin^2\theta\cos^3\theta}{\sqrt{1-\dot\phi^2\sin^2\theta}} + N\cos^4\theta. \tag{3.6}$$

The action may be rewritten in terms of k, to give

$$S = N\int dt\,\frac{\cos^4\theta}{\sin\theta}\frac{l-\cos^2\theta}{\sqrt{(l-\cos^4\theta)^2+\sin^2\theta\cos^6\theta}}, \tag{3.7}$$

where $l \equiv k/N$. One may also introduce an energy defined by

$$E \equiv \dot\phi k - L = \frac{N}{\sin\theta}\sqrt{(l-\cos^4\theta)^2+\sin^2\theta\cos^6\theta}, \tag{3.8}$$

and which notably removes the WZ part of the action. The energy is minimized by

$$\cos^2\theta = l, \qquad E_{\text{min.}} = k, \qquad S_{\text{min.}} = 0, \tag{3.9}$$

and by plugging this value in to (3.6), one finds that

$$\dot\phi = 1. \tag{3.10}$$

3.2 Giant graviton on AdS_5

We turn next to the giant graviton [32] with worldvolume $\mathbb{R}\times S^3(\subset \mathrm{AdS}_5)$. We begin in Lorentzian signature $(-,+,\ldots,+)$. The metric of $\mathrm{AdS}_5\times S^5$ can be taken as in (3.1). The action for the anti-D3-brane[6] is (in units where the AdS radius is set to 1)

$$S_{D3} = -\frac{N}{2\pi^2}\int d^4\sigma\left(\sqrt{-g}+P[C_4]\right), \tag{3.11}$$

where $g_{ab} = \partial_a X^M \partial_b X_M$, where $a,b = 0,\ldots,3$ label the worldvolume coordinates and where X^M are the embedding coordinates. The four-form potential C_4 which will be important for this giant graviton has its legs entirely in the AdS_5, and may be taken as [32]

$$C_{t\tilde\chi_1\tilde\chi_2\tilde\chi_3} = -\sinh^4\rho\,\mathrm{Vol}(\tilde\Omega_3), \tag{3.12}$$

[6]It is the anti-D3-brane which is dual to the large symmetric representation gauge theory operator [32].

where the $\widetilde{\chi}_i$ are angles covering the $S^3 \subset \mathrm{AdS}_5$ and where $\mathrm{Vol}(\widetilde{\Omega}_3)$ indicates its volume element.

One takes the ansatz

$$\rho = \text{const.}, \quad \sigma^0 = t, \quad \sigma^i = \widetilde{\chi}_i, \quad \phi = \phi(t), \quad \theta = \frac{\pi}{2}, \tag{3.13}$$

and obtains

$$S = \int dt\, L = -N \int dt \left[\sinh^3\rho \sqrt{\cosh^2\rho - \dot{\phi}^2} - \sinh^4\rho\right]. \tag{3.14}$$

Independence of ϕ leads to a conserved angular momentum

$$\widetilde{k} \equiv \frac{\delta L}{\delta \dot{\phi}} = \frac{N\dot{\phi}\sinh^3\rho}{\sqrt{\cosh^2\rho - \dot{\phi}^2}}. \tag{3.15}$$

The action may be rewritten in terms of $\widetilde{k}$, to give

$$S = -N \int dt \cosh\rho \sinh^4\rho \left[\sinh^2\rho \sqrt{\frac{1}{\sinh^6\rho + \widetilde{l}^2}} - 1\right], \tag{3.16}$$

where $\widetilde{l} \equiv \widetilde{k}/N$. One may also introduce an energy defined by

$$E \equiv \dot{\phi}\widetilde{k} - L = N\left[\cosh\rho\sqrt{\sinh^6\rho + \widetilde{l}^2} - \sinh^4\rho\right]. \tag{3.17}$$

The energy is minimized by

$$\sinh^2\rho = \widetilde{l}, \qquad E_{\text{min.}} = \widetilde{k}, \qquad S_{\text{min.}} = 0, \tag{3.18}$$

and by plugging this value in to (3.15), one finds that

$$\dot{\phi} = 1. \tag{3.19}$$

3.3 Coordinates

We can map the global coordinates (3.1) of section 3.1 into the Poincaré patch as follows. Take as a simplification AdS_3, for which the factor $d\widetilde{\Omega}_3^2 = d\psi^2$, then we have that

$$\begin{aligned} z &= \frac{R}{\cosh\rho\cos t - \sinh\rho\cos\psi}, \\ x^0 &= \frac{R\cosh\rho\sin t}{\cosh\rho\cos t - \sinh\rho\cos\psi}, \qquad x^1 = \frac{R\sinh\rho\sin\psi}{\cosh\rho\cos t - \sinh\rho\cos\psi}, \end{aligned} \tag{3.20}$$

where the metric of the Poincaré patch is

$$ds^2 = \frac{-(dx^0)^2 + (dx^1)^2 + dz^2}{z^2}. \tag{3.21}$$

On the path of the S^5 giant graviton we have $\rho = 0$. Continuing to Euclidean AdS, so that $t \to t_E = -it$ and $x^0 \to x_E^0 = -ix^0$ we have that

$$z = \frac{R}{\cosh t_E}, \quad x_E^0 = R\tanh t_E, \quad x^1 = 0 \tag{3.22}$$

which gives the trajectory of [10], if we identify the Euclidean time direction in the Poincaré patch with the spatial direction in which the operators are separated on the boundary. Note that the operator separation is given by $L = 2R$ [10].

In the case of the AdS_5 giant graviton we must use the generalization of the coordinate transformation to AdS_5

$$\begin{aligned} z &= \frac{R}{\cosh\rho\cos t - n_0 \sinh\rho}, \\ x^0 &= \frac{R\cosh\rho\sin t}{\cosh\rho\cos t - n_0\sinh\rho}, \qquad \vec{x} = \frac{R\,\vec{n}\sinh\rho}{\cosh\rho\cos t - n_0\sinh\rho}, \end{aligned} \tag{3.23}$$

where the $S^3 \subset AdS_5$ is given by the embedding coordinates $n_I = (n_0, \vec{n})$, $n_I n_I = 1$.

We remind the reader of the Euclidean form of the D-brane action[7]

$$S^E_{D3} = \frac{N}{2\pi^2}\int d^4\sigma\,(\sqrt{g} - iP[C_4])\,, \tag{3.24}$$

and note that the four-form potential with legs in the AdS_5 part of the geometry (3.12) gains a $-i$ under the Wick rotation, $C_4^{\rm AdS} \to -iC_4^{\rm AdS}$, due to having a leg in the temporal direction; the potential on S^5 is unaffected. Plugging the Wick-rotated solutions into the Euclidean action always yields a real result, since the angle $\phi = -it$ compensates for the factor of i in the Wess-Zumino term for the giant graviton on S^5, whose four-form potential has a leg in the ϕ direction.

3.4 Supergravity fluctuations

The supergravity modes that we are interested in are fluctuations of the 4-form potentials, as well as the spacetime metric, and are dual to chiral primary operators with R-charge Δ in $\mathcal{N} = 4$ SYM [34–36]. The fluctuations are[8]

$$\begin{aligned} \delta g_{\mu\nu} &= \left[-\frac{6\,\Delta}{5}\,g_{\mu\nu} + \frac{4}{\Delta+1}\,\nabla_{(\mu}\nabla_{\nu)}\right]\,s^\Delta(X)\,Y_\Delta(\Omega), \\ \delta g_{\alpha\beta} &= 2\,\Delta\,g_{\alpha\beta}\,s^\Delta(X)\,Y_\Delta(\Omega), \\ \delta C_{\mu_1\mu_2\mu_3\mu_4} &= -4\,\epsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5}\,\nabla^{\mu_5}\,s^\Delta(X)\,Y_\Delta(\Omega), \\ \delta C_{\alpha_1\alpha_2\alpha_3\alpha_4} &= 4\epsilon_{\alpha\alpha_1\alpha_2\alpha_3\alpha_4} s^\Delta(X)\nabla^\alpha Y_\Delta(\Omega)\,, \end{aligned} \tag{3.25}$$

where μ, ν are AdS_5 and α, β are S^5 indices. The symbol X indicates coordinates on AdS_5 and Ω coordinates on the S^5. The $Y_\Delta(\Omega)$ are the spherical harmonics on the five-sphere, while $s^\Delta(X)$ have arbitrary profile and represent a scalar field propagating on AdS_5 space with mass squared $= \Delta(\Delta - 4)$, where Δ labels the representation of SO(6) and must be an integer greater than or equal to 2.

The bulk-to-boundary propagator for s^Δ is given in [34], with normalization from [35]. It is

$$\sqrt{\frac{\alpha_0}{B_\Delta}}\frac{z^\Delta}{((x - x_B)^2 + z^2)^\Delta} \simeq \sqrt{\frac{\alpha_0}{B_\Delta}}\frac{z^\Delta}{x_B^{2\Delta}}, \tag{3.26}$$

[7] The anti-D-brane action has a flipped sign on the WZ part.

[8] The traceless symmetric double covariant derivative is defined as $\nabla_{(\mu}\nabla_{\nu)} \equiv \frac{1}{2}(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu) - \frac{1}{5}g_{\mu\nu}\,g^{\rho\sigma}\nabla_\rho\nabla_\sigma$.

JHEP06(2011)085

where we have indicated the limit where the boundary insertion x_B^μ is taken infinitely far away from the giant graviton; this is the limit we will be interested in. The normalization is given by

$$\alpha_0 = \frac{\Delta - 1}{2\pi^2}, \qquad B_\Delta = \frac{2^{3-\Delta} N^2 \Delta(\Delta - 1)}{\pi^2(\Delta + 1)^2}. \tag{3.27}$$

3.5 Antisymmetric giant graviton

We consider the coupling of the supergravity fluctuations to the Euclidean action. The DBI part, given by

$$S_{\rm DBI} = \frac{N}{2\pi^2} \int d^4\sigma \sqrt{g}, \tag{3.28}$$

gives the following variation

$$\delta S_{\rm DBI} = \frac{N}{2} \cos^2\theta \int dt \, Y_\Delta(\Omega) \left(\frac{4}{\Delta+1} \partial_t^2 - \frac{2\Delta(\Delta-1)}{\Delta+1} - 8\Delta \sin^2\theta + 6\Delta \right) s^\Delta. \tag{3.29}$$

We will be interested in the spherical harmonic

$$Y_\Delta(\Omega) = \frac{\sin^\Delta\theta \, e^{i\Delta\phi}}{2^{\Delta/2}} = \frac{\sin^\Delta\theta \, e^{\Delta t}}{2^{\Delta/2}}, \tag{3.30}$$

which corresponds to the gauge theory operator $\mathrm{Tr}Z^\Delta$. Replacing the field s^Δ with the bulk to boundary propagator (3.26), namely

$$s^\Delta \to \frac{\Delta+1}{N\Delta^{\frac{1}{2}} 2^{2-\frac{\Delta}{2}}} \frac{z^\Delta}{x_B^{2\Delta}}, \tag{3.31}$$

we obtain

$$\delta S_{\rm DBI} = \frac{\cos^2\theta \sin^\Delta\theta \, (\Delta+1)\sqrt{\Delta}}{2} \int dt \, \frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^\Delta t} \left(2\cos^2\theta - \frac{1}{\cosh^2 t} \right), \tag{3.32}$$

where $\mathcal{R} \equiv R/x_B^2$, see section 3.3. We now turn our attention to the Wess-Zumino coupling. Because the relevant legs of C_4 are in S_5 we require only the fluctuation $\delta C_{\phi\chi_1\chi_2\chi_3}$

$$\delta C_{\phi\chi_1\chi_2\chi_3} = 4\epsilon_{\theta\phi\chi_1\chi_2\chi_3} s^\Delta \nabla^\theta Y_\Delta(\Omega) = 2^{-\frac{\Delta}{2}+2} \epsilon_{\theta\phi\chi_1\chi_2\chi_3} \Delta s^\Delta (\sin\theta)^{\Delta-1} \cos\theta e^{\Delta t}. \tag{3.33}$$

Therefore the variation of the Wess-Zumino part is

$$\begin{aligned} \delta S_{WZ} &= -2^{-\frac{\Delta}{2}+2} N\Delta \int dt \, e^{\Delta t} \sin^\Delta\theta \cos^4\theta s^\Delta \\ &= -\cos^4\theta \sin^\Delta\theta \, (\Delta+1)\sqrt{\Delta} \int dt \, \frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^\Delta t}. \end{aligned} \tag{3.34}$$

Adding the variations of the DBI and Wess-Zumino terms we find

$$\delta S = -\frac{\cos^2\theta \, (\sin\theta)^\Delta (\Delta+1)\sqrt{\Delta}}{2} \int_{-\infty}^{\infty} dt \, \frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^{\Delta+2} t} = -(2\mathcal{R})^\Delta \sqrt{\Delta} \cos^2\theta \sin^\Delta\theta, \tag{3.35}$$

or (recalling that $\cos^2\theta = k/N$, $\Delta = J$) in terms of gauge theory quantities, the three-point function structure constant is given by

$$C^A_{k,k-J,J} = \sqrt{J} \frac{k}{N} \left(1 - \frac{k}{N} \right)^{J/2}. \tag{3.36}$$

3.6 Symmetric giant graviton

We write the metric on $S^3 \subset \mathrm{AdS}_5$ as

$$d\widetilde{\Omega}_3^2 = d\vartheta^2 + \cos^2\vartheta d\phi_1^2 + \sin^2\vartheta d\phi_2^2, \tag{3.37}$$

so that embedding coordinates are given by

$$n_I = (\cos\vartheta\sin\phi_1, \cos\vartheta\cos\phi_1, \sin\vartheta\sin\phi_2, \sin\vartheta\cos\phi_2). \tag{3.38}$$

The variation of the Lagrangian density is

$$\begin{aligned}\delta\mathcal{L} =& \frac{N}{4\pi^2}\sinh^2\rho\cos\vartheta\sin\vartheta\left[-2\Delta s + h_{tt} + h_{\vartheta\vartheta} + \frac{h_{\phi_1\phi_1}}{\cos^2\vartheta} + \frac{h_{\phi_2\phi_2}}{\sin^2\vartheta}\right] \\ &- \frac{2N}{\pi^2}\cosh\rho\sinh^3\rho\cos\vartheta\sin\vartheta\,\partial_\rho s.\end{aligned} \tag{3.39}$$

where the second line is the WZ part of the variation, and

$$h_{\mu\nu} = \frac{2}{\Delta+1}\Big[2\nabla_\mu\nabla_\nu - \Delta(\Delta-1)g_{\mu\nu}\Big]s, \tag{3.40}$$

where $s = s^\Delta Y_\Delta$, while

$$\begin{aligned}\nabla_t\nabla_t s &= \left(\partial_t^2 + \cosh\rho\sinh\rho\,\partial_\rho\right)s, \\ \nabla_\vartheta\nabla_\vartheta s &= \left(\partial_\vartheta^2 + \cosh\rho\sinh\rho\,\partial_\rho\right)s, \\ \nabla_{\phi_1}\nabla_{\phi_1} s &= \left(\partial_{\phi_1}^2 + \cos^2\vartheta\cosh\rho\sinh\rho\,\partial_\rho - \cos\vartheta\sin\vartheta\,\partial_\vartheta\right)s, \\ \nabla_{\phi_2}\nabla_{\phi_2} s &= \left(\partial_{\phi_2}^2 + \sin^2\vartheta\cosh\rho\sinh\rho\,\partial_\rho + \cos\vartheta\sin\vartheta\,\partial_\vartheta\right)s.\end{aligned} \tag{3.41}$$

Now we may replace the field s with the bulk-to-boundary propagator (3.26)

$$s \to \frac{\Delta+1}{2^2\sqrt{\Delta}N}\frac{\mathcal{R}^\Delta e^{\Delta t}}{(\cosh\rho\cosh t - \cos\vartheta\sin\phi_1\sinh\rho)^\Delta}, \tag{3.42}$$

where $\mathcal{R} \equiv R/x_B^2$, see section 3.3. There is a great simplification which occurs between the DBI and WZ pieces of the variation of the action, which leads to

$$\begin{aligned}\delta S =& -\int_{-\infty}^{\infty} dt \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\pi/2} d\vartheta \\ &\times \frac{\sqrt{\Delta}(\Delta+1)}{4\pi^2}\cos\vartheta\sin\vartheta\sinh^2\rho\frac{\mathcal{R}^\Delta e^{\Delta t}}{(\cosh\rho\cosh t - \cos\vartheta\sin\phi_1\sinh\rho)^{\Delta+2}},\end{aligned} \tag{3.43}$$

JHEP06(2011)085

where we have included the spherical harmonic $Y = e^{\Delta t}/2^{\Delta/2}$. We may re-cast the integral as follows

$$\begin{aligned}
\delta S &= -\frac{\sqrt{\Delta}(\Delta+1)}{2\pi}\frac{\sinh^2\rho}{\cosh^{\Delta+2}\rho}\int_{-\infty}^{\infty}dt\int_0^{2\pi}d\phi_1\int_0^1 d\lambda\frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^{\Delta+2}t}\frac{\lambda}{\left[1-\frac{\lambda\sin\phi_1\tanh\rho}{\cosh t}\right]^{\Delta+2}}\\
&= -\frac{\sqrt{\Delta}(\Delta+1)}{2\pi}\frac{\sinh^2\rho}{\cosh^{\Delta+2}\rho}\int_{-\infty}^{\infty}dt\int_0^{2\pi}d\phi_1\int_0^1 d\lambda\frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^{\Delta+2}t}\\
&\qquad\times\lambda\sum_{k=0}^{\infty}\left(\frac{\lambda\sin\phi_1\tanh\rho}{\cosh t}\right)^k\frac{\Gamma(\Delta+k+2)}{\Gamma(k+1)\Gamma(\Delta+2)}\\
&= -\frac{\sqrt{\Delta}}{2\pi\Gamma(\Delta+1)}\frac{\sinh^2\rho}{\cosh^{\Delta+2}\rho}\int_{-\infty}^{\infty}dt\int_0^{2\pi}d\phi_1\frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^{\Delta+2}t}\\
&\qquad\times\sum_{k=0}^{\infty}\frac{1}{k+2}\left(\frac{\sin\phi_1\tanh\rho}{\cosh t}\right)^k\frac{\Gamma(\Delta+k+2)}{\Gamma(k+1)}\\
&= -\frac{\sqrt{\Delta}}{2\Gamma(\Delta+1)}\frac{1}{\cosh^\Delta\rho}\sum_{k=0}^{\infty}\int_{-\infty}^{\infty}dt\frac{\mathcal{R}^\Delta e^{\Delta t}}{\cosh^{\Delta+2+2k}t}\frac{1}{2^{2k}}\frac{\Gamma(\Delta+2k+2)}{\Gamma(k+2)\Gamma(k+1)}\tanh^{2k+2}\rho\\
&= -\frac{(2\mathcal{R})^\Delta}{\sqrt{\Delta}\Gamma(\Delta)}\frac{1}{\cosh^\Delta\rho}\sum_{k=0}^{\infty}\tanh^{2k+2}\rho\frac{\Gamma(\Delta+k+1)}{\Gamma(k+2)}\\
&= -\frac{(2\mathcal{R})^\Delta}{\sqrt{\Delta}}\Big(\cosh^\Delta\rho-\cosh^{-\Delta}\rho\Big).
\end{aligned} \tag{3.44}$$

In terms of gauge theory quantities, recalling that $\sinh^2\rho = k/N$, $\Delta = J$, the three-point function structure constant is given by

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right). \tag{3.45}$$

4 Discussion and conclusion

We have obtained the three-point function involving two giant gravitons and one pointlike graviton from the gauge theory as well as from the string theory side. On the string theory side the calculation was carried out in a semiclassical approximation where the two giant gravitons were heavy and the pointlike one light. Both the giant gravitons and the pointlike graviton were moving with given angular velocities on S^5. In the case where the two giant gravitons were wrapping an $S^3 \subset \mathrm{AdS}_5$ we found for the three-point function

$$C^{S,\text{string}}_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right). \tag{4.1}$$

Here k is the S^5 angular momenta of the giant gravitons and J the S^5 angular momentum of the pointlike graviton. First, we notice that in the limit where the size of the giant gravitons shrinks to zero, i.e. $\frac{k}{N} \to 0$, we recover, as we should, the result for the three point function of three pointlike gravitons, cf. eq. (2.6)

$$C^{S,\text{string}}_{k,k-J,J} \to \frac{\sqrt{J}\,k}{N}, \qquad \text{for} \qquad \frac{k}{N} \to 0. \tag{4.2}$$

Secondly, we observe that as the (unrestricted) angular momentum of the giant gravitons becomes large, i.e. $\frac{k}{N} \to \infty$ the three-point function turns into the gauge theory three-point function involving two Schur polynomials and one chiral primary, cf. eqn (2.24)

$$C^{S,\text{string}}_{k,k-J,J} \to \frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2} = C^{S,gauge}_{k,k-J,J} \qquad \text{for} \qquad \frac{k}{N} \to \infty. \tag{4.3}$$

Although a more natural parameter region to consider would be $\frac{k}{N} \sim 1$, this matching at large k is an interesting coincidence. It may point to an easing, in the symmetric case, of the problems of the Schur polynomial description of giant gravitons discussed below.

In the case where the giant graviton was wrapping an $S^3 \subset S^5$ we found from the string theory calculations

$$C^{A,\text{string}}_{k,k-J,J} = \sqrt{J}\,\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2}. \tag{4.4}$$

Again, we notice that we correctly recover the three-point function of three point like gravitons in the limit where the size of the giant gravitons shrink to zero, i.e.

$$C^{A,\text{string}}_{k,k-J,J} \to \frac{\sqrt{J}\,k}{N}, \qquad \text{for} \qquad \frac{k}{N} \to 0. \tag{4.5}$$

The gauge theory analysis gave the result

$$C^{A,gauge}_{k,k-J,J} = (-1)^{(J-1)}\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}. \tag{4.6}$$

In this case the giant graviton angular momentum k has to satisfy the bound of $\frac{k}{N} \leq 1$. For $\frac{k}{N} = 1$ we find that both the string theory and gauge theory result are exactly equal to zero but considering the limit $\frac{k}{N} \to 1$ the two results differ by a factor proportional to J.[9]

Our interpretation of the mismatch between the gauge and string theory calculations centers on the validity of the Schur polynomials as duals of the giant gravitons. It is well known that the Schur polynomials should only describe the giant gravitons when the size of the operators k is of order N, in the large-N limit [24]. Furthermore, the Schur polynomials do not reduce to chiral primaries in the small-$\frac{k}{N}$ limit, and are therefore disconnected from the pointlike limit of the giant gravitons. When calculating a three-point function with a small operator such as the pointlike graviton dual, i.e. the chiral primary $\text{Tr}Z^J$, we are probing the chiral-primary content of the OPE of two Schur polynomials $\chi_k(\bar{Z})$ and $\chi_{k-J}(Z)$, schematically

$$\chi_k(\bar{Z}(0))\,\chi_{k-J}(Z(x)) = \ldots + C^{\text{gauge}}_{k,k-J,J}\text{Tr}\bar{Z}^J(0)\,x^{-2J} + \ldots. \tag{4.7}$$

[9] As mentioned previously, the $(-1)^J$ part of the prefactor is convention dependent and could be removed by a different normalisation.

We can certainly trust the terms in the OPE involving operators of dimension $\mathcal{O}(N)$, but it is not clear that the small-operator content of the operators $\chi_k(Z)$, i.e. the structure constants $C^{\text{gauge}}_{k,k-J,J}$, are in fact themselves dual to the three-point functions of two giant gravitons and one pointlike graviton, defined holographically in string theory. Indeed we find that the string theory results interpolate smoothly to the limit where all three gravitons are pointlike, whereas, without surprise, the gauge theory results fail to do so.

The fact that for the symmetric case we get a string three-point function which nicely interpolates between the gauge theory three-point function of single trace chiral primaries and that of Schur polynomials raises the question of whether in the gauge theory language one can construct an (orthogonal) basis of operators which interpolates between single trace operators and the Schur polynomials in such a way that the gauge theory and string theory three-point functions can be exactly matched for all values of $\frac{k}{N}$. The question of the existence of such an interpolating basis has been brought up before, see e.g. [24, 37], but still lacks resolution. The string theoretic results given here will hopefully serve as a benchmark for any such future construction.

In view of the above discrepancy, it might also be useful to take a moment to recapitulate why the gravity calculation we performed does indeed provide the leading (in the $1/N$ expansion) contribution to the three-point function. For instance, let us consider a process in which the light state coming from the boundary first decays into two light states in the bulk, which then reach the giant graviton worldvolume. Compared to the case we examined above, such a process will receive additional factors of N^2 (from the bulk vertex), $(1/N^2)^2$ (from the two bulk-to-bulk propagators) and N (from the additional integration over the worldvolume), and will thus be subleading as $1/N$. Similarly one can check that all other contributions to the three-point function are also subleading, so our gravity computation is indeed expected to capture the leading part of the three-point function of a chiral primary dual with two giant gravitons at large N.

Although we have not emphasized the point, it is possible that there exist subtleties in the generalization of the methods developed for the holographic two-point functions involving semi-classical strings in [10] to the case of D-branes, ultimately affecting the procedure used here to calculate holographic three-point functions. For this reason, a first principles derivation using D-branes, as was presented in [10] for strings, would be welcome. We make some progress toward this goal in appendix A where the holographic two-point function for giant gravitons on S^5 is derived.

Finally, let us mention that this study, while being concerned only with 1/2 BPS objects, opens up the avenue for studying holographic three-point functions of a variety of extended objects in the form of branes, an interesting example being the systems of non-BPS giant gravitons moving with two angular momenta on S^5 and being dual to so-called restricted Schur polynomials, see [38] and references therein.

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A A shortcut to the S^5 giant graviton two-point function

In this appendix we show that the calculation of the holographic S^5 graviton two-point function can easily be performed by making use of the fact that, as noted in [39], all the dependence on the S^5 directions can be integrated out, leaving a point particle on AdS$_5$ with effective mass equal to the dimension of the giant graviton.

Let us start with the DBI action (3.11) for the giant graviton and take the embedding to be $\sigma^0 = t, \sigma^i = \chi_i, \phi = \phi(t)$ but leave the AdS part general. Integrating over the three-sphere coordinates we find

$$S_{D3} = -N\cos^3\theta \int \mathrm{d}t\sqrt{-G_{MN}\dot{X}^M\dot{X}^N} + N\cos^4\theta \int \dot{\phi}\mathrm{d}t\,. \tag{A.1}$$

Now, following [39], we switch to a Polyakov-type formulation, by introducing an einbein e:

$$S_{D3} = \frac{1}{2}\int \mathrm{d}t\left(\frac{1}{e}G_{MN}\dot{X}^M\dot{X}^N - m^2 e\right) + N\cos^4\theta\int\dot{\phi}\mathrm{d}\sigma_0, \tag{A.2}$$

where we have defined an effective mass,

$$m = N\cos^3\theta\,. \tag{A.3}$$

To go back to the original action one simply solves for $e = \frac{1}{m}\sqrt{-G_{MN}\dot{X}^M\dot{X}^N}$ and substitutes back. Now let us separate the AdS$_5$ from the S^5 part as

$$G_{MN}\dot{X}^M\dot{X}^N = g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + \sin^2\theta\,\dot{\phi}^2, \tag{A.4}$$

to obtain

$$S = \frac{1}{2}\int \mathrm{d}t\left(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - m^2 e + \frac{1}{e}\sin^2\theta\dot{\phi}^2 + 2N\cos^4\theta\dot{\phi}\right)\,. \tag{A.5}$$

The conjugate momentum to ϕ is

$$k = \frac{1}{e}\dot{\phi}\sin^2\theta + N\cos^4\theta, \tag{A.6}$$

which of course agrees with (3.6) if we substitute the on-shell value of e (together with the form of the metric (3.1) and $\rho = 0$). Now fix θ to the value θ_0 minimising the energy (3.9)

$$\cos^2\theta_0 = l = \frac{k}{N}\,. \tag{A.7}$$

Substituting k from (A.6) and solving for $\dot{\phi}$ we get

$$\dot{\phi} = eN\cos^2\theta_0\,. \tag{A.8}$$

It is easy to see that this would lead to (3.10) were we to use the on-shell value of the einbein. Substituting $\dot{\phi}$ into (A.5) we finally find

$$-m^2 e + \frac{1}{e}\sin^2\theta_0\dot{\phi}^2 + 2N\cos^4\theta_0\dot{\phi} = e\ N^2\cos^4\theta_0 = e\ M_{D3}^2\,, \tag{A.9}$$

JHEP06(2011)085

to conclude [39] that the giant graviton can be described by a particle moving in AdS_5 with mass:

$$M_{D3} = N \cos^2 \theta_0 = k \; . \tag{A.10}$$

Standard point-particle techniques (reviewed in [10]) now straightforwardly give for the two-point function

$$G(0, \epsilon; x, \epsilon) = \left(\frac{|x|}{\epsilon} \right)^{-2M_{D3}} = \left(\frac{|x|}{\epsilon} \right)^{-2k} , \tag{A.11}$$

which is the expected answer.[10]

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JHEP06(2011)085

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