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Holographic correlation functions for open strings and branes

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Abstract: In this paper, we compute holographically the two-point and three-point functions of giant gravitons with open strings. We consider the maximal giant graviton in S^5 and the string configurations corresponding to the ground states of $Z = 0$ and $Y = 0$ open spin chain, and the spinning string in AdS_5 corresponding to the derivative type impurities in $Y = 0$ or $Z = 0$ open spin chain as well. We treat the D-brane and open string contribution separately and find the corresponding D3-brane and string configurations in bulk which connect the composite operators at the AdS_5 boundary. We apply a new prescription to treat the string state contribution and find agreements for the two-point functions. For the three-point functions of two giant gravitons with open strings and one certain half-BPS chiral primary operator, we find that the D-brane contributions to structure constant are always vanishing and the open string contribution for the $Y = 0$ ground state is in perfect match with the prediction in the free field limit.

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1 Introduction

The holographic computation of the correlation functions has been an important subject in AdS/CFT correspondence. Soon after the proposal of the correspondence [1–3], the two-point and three-point functions of the operators in the protected sectors have been computed on both sides [4–7]. On the field theory side, the correlation functions were discussed in the free field limit, while on the string side, they were computed in the supergravity approximation. The perfect match has been found due to the non-renormalization theorem, strongly supporting the correspondence. However, for the operators beyond the protected sectors, the holographic computation usually becomes difficult.

Very recently, there has been renewed interest in the holographic computation of the correlation functions of semiclassical strings. The semiclassical string states correspond to the composite operators with large quantum numbers in the field theory [8]. And from the dictionary of the AdS/CFT correspondence, at leading order their semiclassical energies, whose quantum corrections are suppressed by the large quantum number, give the quantum dimensions of the corresponding operators, which are the eigenvalues of the dilatation operator and are calculable with the help of integrability. This relation provides not only a nontrivial check of AdS/CFT correspondence beyond the BPS protected sector, but also leads to a better understanding of integrable structures on both sides of the correspondence.

The study of the correlation functions of semi-classical string states is expected to shed light on the string interaction and integrability in super-Yang-Mills theory at non-planar level.

There is a clear physical picture in holographic computation of the correlation functions of semiclassical string states. One needs to insert the string state vertex operators in the string path integral. At strong coupling, the path integral is dominated by a saddle point. Therefore one has to solve the equations of motions with the vertex operators as sources, and find the string configurations whose ends shrink and approach to the boundary of AdS_5 at the insertion points of the composite operators corresponding to the string states. The holographic computation of the two point functions is relatively easy and has been illustrated in [9–11] . The computation for the higher point functions becomes much more difficult, as the string configurations satisfying the appropriate boundary conditions are hard if not impossible to obtain. Nevertheless, using a strategy in the holographic study of operator product expansion(OPE) of Wilson line and Wilson surface operators [12–15], the three-point and four-point functions of two very massive string states and one or two light string states have been investigated [16–19]. For other related studies, see [20–28].

In this article, we would like to study the holographic computation of two-point and three-point functions of the giant gravitons with open strings. The giant gravitons are the D3-banes wrapping trivial cycles without collapsing due to their coupling to the background flux [29–31]. As the open string may end on the D-brane, one can consider the system of the giant gravitons with open strings, which correspond to determinant-like operators mapping to a class of open spin chain [32, 33]. As D-brane is non-perturbative object, it is not clear what kind of string state vertex operators should be inserted in the string path integral. One possibility is to consider the boundary conformal field theory, which may count the presence of D-brane. However, for the case at hand, the string world-sheet picture for the giant graviton wrapping a sphere of finite size seems to make no sense. Nevertheless, inspired by the study of the usual semi-classical closed string state, we expect the similar picture could be carried over, at least for the maximal giant graviton wrapping an S^3 in S^5. As to the leading order of string coupling g_s and α', the D-brane and open string decouple, we may compute their contributions separately. Namely we expect that D-brane contribution is from a "fat" D3-brane configuration whose ends shrink and approach to the insertion points at the AdS boundary. For the open string sector, the contribution is from the open string configuration connecting the insertion points, ending on the brane and satisfying appropriate boundary conditions. For the two-point functions, the computation is relatively simple. For the three-point function, the holographic computation is quite difficult. As in other cases, we consider the cases with two massive states corresponding to the giant gravitons with open strings and one gravity state corresponding to the half-BPS chiral primary operator. Due to the presence of the giant gravitons, we have to consider the contribution of D3-branes to the three-point structure constants, besides the one from open string. It is remarkable that the D-brane contribution is vanishing, possibly due to the BPS property of the giant graviton in our case.

There is another novel feature in our treatment. We propose to use the Routhian to handle the string state contribution. For all the cases studied in the literature, our prescription gives the same answer. But for the cases discussed in this paper, we have to use the Routhian. This seems to suggest that our prescription is more useful.

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In the next section, we give a brief review of open string configurations ending on the maximal giant graviton branes in S^5, which correspond to the open spin chains. In section 3, we discuss the two-point functions of the giant gravitons with open strings, including the cases corresponding to the ground state of $Z = 0$ open spin chain, the ground state of $Y = 0$ open spin chain and the open string in AdS_5 and ending on $Z = 0$ brane. In section 4, we investigate the three-point functions. In the small coupling limit, we compute the three-point functions in the free field limit, and in the strong coupling limit, we holographically compute it in the limit of operator product expansion. We find the perfect match for the structure constant c^Y_{123} involving the ground state operator of the $Y = 0$ open spin chain. Last section is devoted to some conclusions and discussions.

2 Open strings ending on giant graviton branes

In this section we shall be describing giant graviton branes and open strings ending on the maximal giant graviton branes. We shall also comment on the $\mathcal{N} = 4$ SYM operators corresponding to the D-branes plus the open string states. In this note we shall be mainly concerned about giant graviton branes moving in the S^5 part of the 10d spacetime. The S^5 can be described by three complex embedding coordinates (X, Y, Z) satisfying the constraint,

$$|X|^2 + |Y|^2 + |Z|^2 = 1 \tag{2.1}$$

where we set the AdS radius to be unity. Using the parametrization,

$$X = \sin\theta\, \cos\alpha\, e^{i\phi_1}, \qquad Y = \sin\theta\, \sin\alpha\, e^{i\phi_2}, \qquad Z = \cos\theta\, e^{i\phi}\,, \tag{2.2}$$

one finds that the S^5 metric becomes

$$ds^2_{S^5} = d\theta^2 + \cos^2\theta\, d\phi^2 + \sin^2\theta\, d\Omega_3^2\,, \tag{2.3}$$

with

$$d\Omega_3^2 = \cos^2\alpha\, d\phi_1^2 + d\alpha^2 + \sin^2\alpha\, d\phi_2^2\,. \tag{2.4}$$

Defining $r = \sin\theta$, one can alternatively write the S^5 metric as

$$ds^2_{S^5} = \frac{1}{1-r^2}\, dr^2 + (1-r^2)\, d\phi^2 + r^2\, d\Omega_3^2\,, \tag{2.5}$$

where the coordinate r is ranged over the interval [0, 1]. The AdS_5 part can be described by the embedding coordinates Y_I in $R^{2,4}$ satisfying the constraint

$$-Y_0^2 - Y_5^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 = -1\,. \tag{2.6}$$

The global AdS coordinates are related to the embedding coordinates by the relations

$$Y_5 + iY_0 = \cosh\rho\, e^{it_{\rm ads}}\,, \qquad Y_k = \sinh\rho\ \hat{n}_k\,, \tag{2.7}$$

where $k = 1, 2, 3, 4$ and $\hat{n}_k$ with $\hat{n}_k\hat{n}_k = 1$ describes a unit S^3.

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The above three complex coordinates (X, Y, Z) are dual to the following three complex linear combinations

$$X = \Phi^1 + i\Phi^2, \quad Y = \Phi^3 + i\Phi^4, \quad Z = \Phi^5 + i\Phi^6, \tag{2.8}$$

of the six real scalar fields Φ^i in the $\mathcal{N} = 4$ super Yang-Mills side.

The D3 giant graviton brane [29–31] of our main interest below wraps an S^3 part of the S^5 and rotates around the remaining directions of the S^5 while taking a pointlike trajectory given by $\rho = 0$ in the AdS_5 part of the geometry. These D3 branes preserve a half of the supersymmetries. We first consider a maximal sized giant graviton brane. One example of such brane is given by the trajectory $Z = 0$ and $\rho = 0$, which is called as $Z = 0$ brane. This $Z = 0$ brane corresponds to the gauge invariant SYM operator $\mathcal{O}_{D3} = \det Z$, which is a half BPS state. The dimension of this operator is simply given by its engineering dimension N, which is protected against any quantum corrections. Below we shall also consider the $Y = 0$ brane which is related to the $Z = 0$ brane by an appropriate SO(6) rotation and dual to the operator $\mathcal{O}'_{D3} = \det Y$.

The correspondence between open strings ending on the giant graviton brane and the open spin chain operators in the $\mathcal{N} = 4$ SYM theory is first considered in ref. [34] and further developed in refs. [35, 36] (for a recent review, see [37]). There are two classes of open string states of our interest below: one is the open string ending on the $Z = 0$ brane and the other is the open string ending on the $Y = 0$ brane. We shall choose the open string vacuum oriented along Z direction. Then the open string ending on the $Y = 0$ brane takes Neumann boundary condition on the Z plane. The open string can also end on the $Z = 0$ brane with Dirichlet boundary condition on the Z plane; an additional localized boundary degree of freedom is necessary at each boundary of $Z = 0$.

In the $\mathcal{N} = 4$ SYM theory side, the $Y = 0$ brane open spin chain is represented by composite operators [34]

$$\mathcal{O}_Y = \epsilon^{j_1 \dots j_{N-1} A}_{i_1 \dots i_{N-1} B} Y^{i_1}_{j_1} \cdots Y^{i_{N-1}}_{j_{N-1}} (Z \dots Z \chi_1 Z \dots Z \chi_2 Z \dots Z)^B_A, \tag{2.9}$$

where $\chi_1, \chi_2, \dots$ represent other SYM fields of bulk excitations. An elementary excitation of a single impurity is organized by the $SU(1|2)^2$ symmetry. It is clear that the ground state of the $Y = 0$ open spin chain is described by a unique vacuum configuration which is a $\frac{1}{4}$ BPS state preserving the $SU(1|2)^2$ symmetry. The other is the $Z = 0$ brane open spin chain represented by composite SYM operators [34],

$$\mathcal{O}_Z = \epsilon^{j_1 \cdots j_{N-1} A}_{i_1 \cdots i_{N-1} B} Z^{i_1}_{j_1} \cdots Z^{i_{N-1}}_{j_{N-1}} (\chi_L Z \cdots Z \chi_1 Z \cdots Z \chi_2 Z \cdots \chi_R)^B_A \,. \tag{2.10}$$

An important difference of the $Z = 0$ brane case from that of the $Y = 0$ brane is that the $Z = 0$ open spin chain is connected to the giant graviton through boundary impurities χ_L and χ_R. Each boundary state is organized by the representation of $SU(2|2)^2$ symmetry. This elementary magnon involves 16 degenerate states with the energy spectrum [34]

$$E_B = \sqrt{1 + \frac{\lambda}{4\pi^2}} \,. \tag{2.11}$$

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Since the presence of the boundary impurities is essential, the space of corresponding ground states is organized by the 256 degenerate states of $\mathrm{SU}(2|2)_L \otimes \mathrm{SU}(2|2)_R$. Thus there are no remaining supersymmetries for the ground state of the $Z = 0$ open spin chain.

We now turn to the open string description [38, 39] of the open spin chain dynamics. For the AdS_5 part, we use the ansatz

$$\begin{aligned} Y_5 + iY_0 &= \cosh\rho(\sigma)\, e^{i\kappa_s \tau} \\ Y_1 + iY_2 &= \sinh\rho(\sigma)\; e^{i\omega\,\tau} \end{aligned} \tag{2.12}$$

with $Y_3 + iY_4 = 0$. On the other hand, for the S^5 part, we use the ansatz

$$\theta = \theta(\sigma)\,, \qquad \phi = \nu\tau \tag{2.13}$$

with the S^3 coordinates α, ϕ_1 and ϕ_2 being taken as constants. The string equations of motion tell us that

$$\begin{aligned} \rho'' &= (\kappa_s^2 - \omega^2)\sinh\rho\,\cosh\rho\,, \\ \theta'' &= \nu^2 \sin\theta\,\cos\theta\,. \end{aligned} \tag{2.14}$$

The conformal gauge constraints lead to the condition

$$\rho'^2 + \theta'^2 = \kappa_s^2\cosh^2\rho - \omega^2\sinh^2\rho - \nu^2\cos^2\theta\,. \tag{2.15}$$

The first equation of (2.14) may be integrated and leads to

$$\rho'^2 = \kappa_s^2\cosh^2\rho - \omega^2\sinh^2\rho - \nu^2 k^2 \tag{2.16}$$

and the second equation of (2.14) can be integrated to give

$$\theta'^2 = -\nu^2\cos^2\theta + \nu^2 k^2 \tag{2.17}$$

where the integration constant is fixed by the conformal gauge constraint. Now for the $Z = 0$ brane, one has the Dirichlet boundary condition $\rho = 0$ and $\theta = \pi/2$ at $\sigma = 0, \pi$. The solution to the equations (2.16) and (2.17) satisfying the boundary condition at $\sigma = 0$ is given by the Jacobi elliptic function as [39]

$$\cosh\rho = \frac{1}{\mathrm{dn}(\tilde{\omega}\sigma, \tilde{k}^2)}\,, \qquad \sin\theta = \mathrm{dn}(\nu\sigma, k^2) \tag{2.18}$$

where we have introduced

$$\tilde{\omega}^2 \equiv \omega^2 - \nu^2 k^2\,, \qquad \tilde{\kappa}_s^2 \equiv \kappa_s^2 - \nu^2 k^2\,, \qquad \tilde{k}^2 = \frac{\tilde{\kappa}_s^2}{\tilde{\omega}^2}\,. \tag{2.19}$$

Note that the periodicity of the Jacobi elliptic function $\mathrm{dn}(x, k^2)$ is given by $2K(k^2)$ where we have introduced the complete elliptic integral of the first kind $K(k^2)$ defined by

$$K(k^2) = \int_0^1 dx \frac{1}{\sqrt{1-x^2}\sqrt{1-k^2x^2}}\,. \tag{2.20}$$

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Since we are interested in the open string for $0 \leq \sigma \leq \pi$ starting from and ending on the $Z = 0$ brane at $\rho = 0$, we have the relations

$$\tilde{\omega}\pi = 2K(\tilde{k}^2)\,, \qquad \nu\pi = 2K(k^2)\,. \tag{2.21}$$

The corresponding solution describes a string ending on the $Z = 0$ brane spinning around the Z plane of S^5 and $Y_1 + iY_2$ of AdS_5 at the same time. The ground state of the $Z = 0$ open spin chain that does not involve any derivative type letters, e.g. $D_+^S Z$, is described by $\rho(\sigma) = 0$ which leads to the condition $\kappa_s = \nu k$ with $\tilde{\kappa}_s = 0$. Then this ground state configuration carries the energy E and the S^5 R-charge J [38],

$$E = \frac{\sqrt{\lambda}}{2\pi}\kappa_s \int_0^\pi d\sigma \cosh^2\rho = \frac{\sqrt{\lambda}}{\pi}\, k\, K(k^2) \tag{2.22}$$

$$J = \frac{\sqrt{\lambda}}{2\pi}\nu \int_0^\pi d\sigma \cos^2\theta = \frac{\sqrt{\lambda}}{\pi}\left[\, K(k^2) - E(k^2)\,\right]\,, \tag{2.23}$$

where

$$E(k^2) = \int_0^1 dx \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}} \tag{2.24}$$

is the complete elliptic integral of the second kind. It is then straightforward to show that

$$2E_B = E - J = \frac{\sqrt{\lambda}}{\pi}\left(1 - \frac{4}{e^2} e^{-\frac{2\pi J}{\sqrt{\lambda}}} + \cdots\right) \tag{2.25}$$

in the long string limit of large J. This agrees with E_B in (2.11) in the large λ and the large J limit. The J dependent correction in this spectrum stems from the finite size effect of boundary degrees of freedom of the $Z = 0$ brane.

Next we turn to the open string description of the $Y = 0$ open spin chains. At $\sigma = 0,\ \pi$, one has the Dirichlet boundary condition $\rho = 0$ in the AdS_5 side and $\theta' = 0$ for the Z plane which is parallel to the worldvolume directions of the $Y = 0$ brane. Again using the string equations in (2.16) and (2.17), one may construct various solutions but we shall here simply present a rather trivial solution,

$$\theta = 0\,, \qquad \rho = 0 \tag{2.26}$$

with $\kappa_s = \nu$ with $k = 1$. This describes a pointlike string rotating in the equator $r = 1$, which corresponds to the ground state of the $Y = 0$ open string carrying [38]

$$E = J = \frac{1}{2}\sqrt{\lambda}\,\nu\,. \tag{2.27}$$

One may see that there is no finite size correction. Since this vacuum state preserves a quarter of the sixteen supersymmetries, one has the non-renormalization of the ground state energy $E_g = E - J = 0$ using the argument ref. [40]. The operator dual to this ground state $\mathcal{O}_Y^J$ will be given in subsection 4.1.

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3 Two-point functions

3.1 The crucial role played by Routhian

A prescription on the computations of the two-point functions from the semi-classical string solution was given in [10]. One of the points stressed there is that we cannot simply evaluate the on-shell actions of the semiclassical solutions. Instead there are important corrections from convolution with the string-state wave-functions. For point particles and strings only rotating in S^5 part of the background, these corrections just change the action into energy. But for the strings rotating in AdS part as well, the story is more involved. We need to treat certain zero modes carefully. And the proposed corrections in this case reads

$$\Delta S = -\int d\tau d\sigma (\vec{\Pi} - \vec{\Pi}_0) \cdot (\dot{\vec{Y}} - \dot{\vec{Y}}_0) \tag{3.1}$$

where $\vec{Y}_0$ and $\vec{\Pi}_0$ are respectively so called zero-modes of the coordinate $\vec{Y}$ and the momentum $\vec{\Pi}$. It is also stated there that we should use the embedding coordinates Y's which respect the SO(2, 4) symmetry.

Though the above prescription works well for all cases considered there, we found that this turns out to be problematic for more general cases. As an example, for the open string solution in subsection 3.4, we do not obtain any sensible result following the prescription. And the problem is already there for the closed string counterpart of the open string solution. Thus we need some generalization of the prescription.

Through trial and error, we now propose to use the Routhian to handle the string state contribution. Our suggestion is as follows. Whenever there is a conserved quantity Q_a for the solution (except the energy) with the corresponding cyclic coordinate y^a, we perform the Legendre transform

$$L_R = L - \sum_a Q_a \dot{y}^a \,, \tag{3.2}$$

to yield the Routhian. For the further variation of the Routhian, the conserved charge Q_a should be held fixed.

For the strings only spinning inside S^5, the conserved charges are three spins J_1, J_2, J_3 in S^5 and the Routhian coincides with the energy. Our new prescription goes back to the old one in [10] for these strings. It is not hard to see that it is also the case for the string rotating in both AdS_5 and S^5 studied there. However as we have already mentioned, the generalization of the prescription in [10] is needed for more complicated cases.

We also find that, interestingly, the Routhian plays a similar role when we compute the two-point function from a giant graviton (a D3 brane inside S^5) in the next subsection.

3.2 Maximal giant gravitons

In this subsection, we will compute the two-point function of local operators dual to maximal giant graviton which is a D3 moving in S^5. More precisely, the operator we focus on is $\mathcal{O}_{D3} = \det Z$. As we have reviewed, the dual object in the gravity side is the $Z = 0$ giant graviton brane.

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For this computation, we have to use the Poincare coordinate for AdS_5. The corresponding metric is given by

$$ds^2_{AdS_5} = \frac{1}{z^2}\Big[-dt^2 + \sum_{i=1}^{3} dx_i^2 + dz^2 \Big]. \tag{3.3}$$

The two-point function we want to compute in this subsection is

$$\langle \mathcal{O}^\dagger_{D3}(t = t_f, \mathbf{x} = \mathbf{0})\, \mathcal{O}_{D3}(t = 0, \mathbf{x} = \mathbf{0}))\rangle\,, \tag{3.4}$$

and, as in [10], we need to first find the suitable classical D3 brane solution. The worldvolume coordinates of this D3 brane is $(\tau, \theta_1, \theta_2, \theta_3)$, where θ_i are three directions (α, ϕ_1, ϕ_2) of the S^3 inside S^5. The part of nontrivial embedding involves $t(\tau), z(\tau), \phi(\tau)$ directions with r being a constant[1]. The boundary conditions read

$$\big(t(-s/2), z(-s/2)\big) = (0, \epsilon), \qquad \big(t(s/2), z(s/2)\big) = (t_f, \epsilon). \tag{3.5}$$

The resulting induced metric becomes

$$ds^2_{\rm ind} = \left[\frac{-\dot{t}^2 + \dot{z}^2}{z^2} + (1 - r^2)\dot{\phi}^2 \right] d\tau^2 + r^2 d\Omega_3^2 \tag{3.6}$$

where dot denotes a derivative with respect to τ.

The action of D3-brane is given by

$$I_{D3} = I_{\rm DBI} + I_{\rm CS} = -T_{D3} \int \sqrt{-\gamma} + T_{D3} \int P[C_4], \tag{3.7}$$

where γ_{ab} is the induced metric of D3 brane, $P[C_4]$ is the pullback of Ramond-Ramond four-form potential and T_{D3} is the tension of D3-brane,

$$T_{D3} = \frac{1}{(2\pi)^3 g_s \alpha'^2} = \frac{N}{2\pi^2}. \tag{3.8}$$

For the solution at hand, after integration over S^3, the DBI part of the Lagrangian becomes

$$\mathcal{L}_{\rm DBI} = -T_{D3}\Omega_3 r^3 \sqrt{\frac{\dot{t}^2 - \dot{z}^2}{z^2} - (1 - r^2)\dot{\phi}^2}, \tag{3.9}$$

where $\Omega_3 = 2\pi^2$ is the volume of unit S^3. The Chern-Simons part is the same as the one in [29]:

$$\mathcal{L}_{\rm CS} = \dot{\phi} N r^4, \tag{3.10}$$

with

$$\mathcal{L} = \mathcal{L}_{\rm DBI} + \mathcal{L}_{\rm CS}. \tag{3.11}$$

The angular momentum carried by the giant graviton brane is given by

$$J = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{N r^3 (1 - r^2)\dot{\phi}}{\sqrt{\frac{\dot{t}^2 - \dot{z}^2}{z^2} - (1 - r^2)\dot{\phi}^2}} + N r^4. \tag{3.12}$$

[1]We will narrow down to the case with $r = 1$ shortly.

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And one has $J = N$ for the maximal case of $r = 1$.

It can be checked that

$$t = R\tan\kappa\tau + t_0, \qquad z = \frac{R}{\cos\kappa\tau}, \tag{3.13}$$

is the solution of the equations of motion. The boundary condition $z(\pm s/2) = \epsilon$ implies

$$\frac{R}{\cos\frac{\kappa s}{2}} = \epsilon. \tag{3.14}$$

Since ϵ is very small, we can see that s cannot be real. We can write $s = -i\tilde{s}$ with $\tilde{s}$ being real. Then the worldsheet coordinate τ and the spacetime coordinate t have to be purely imaginary. From $t(-s/2) = 0, t(s/2) = t_f$, we get

$$\kappa \sim \frac{2}{\tilde{s}}\log\frac{|t_f|}{\epsilon}. \tag{3.15}$$

As we discussed in the previous subsection in detail, we proposed that it is the Routhian which gives the correlation function. Now the only conserved charge besides the energy is the angular momentum J. It is easy to see that for maximal giant graviton, the Routhian is nothing but the DBI part of the action:

$$\mathcal{L}_{\rm DBI} = -N\kappa = -\frac{2N}{\tilde{s}}\log\frac{|t_f|}{\epsilon} \tag{3.16}$$

and

$$I_{\rm DBI} = \int_{+i\tilde{s}/2}^{-i\tilde{s}/2}\mathcal{L}_{\rm DBI}d\tau = -2iN\log\frac{|t_f|}{\epsilon}. \tag{3.17}$$

We are thus led to

$$\exp(iI_{\rm DBI}) = \left(\frac{|t_f|}{\epsilon}\right)^{-2N} \tag{3.18}$$

which is the expected result of the two-point function $\langle\mathcal{O}^\dagger_{D3}(t_f)\,\mathcal{O}_{D3}(0)\rangle$.

3.3 Nonmaximal giant gravitons in S^5

We begin with the D3 brane action

$$\mathcal{L} = -\frac{T_{D3}}{2}\sqrt{-\gamma}\left(\gamma^{ab}\partial_a X^m\partial_b X^n G_{mn}(X) - 2\right) + \mathcal{L}_{\rm CS} \tag{3.19}$$

where γ_{ab} is the world volume metric which is dynamical now. Since $\mathcal{L}_{\rm CS}$ is metric independent, the equations for the worldvolume metric γ_{ab} become

$$\gamma_{ab}\left(-1 + \frac{1}{2}\gamma^{cd}\partial_c X^m\partial_d X^n G_{mn}(X)\right) = \partial_a X^m\partial_b X^n G_{mn}(X)\,. \tag{3.20}$$

Taking trace of this equation, one finds

$$\gamma^{cd}\partial_c X^m\partial_d X^n G_{mn}(X) = 4\,. \tag{3.21}$$

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We partially solve the problem by the choice $\gamma_{ij}d\sigma^i d\sigma^j = d\Omega_3^2$ and integrate the Lagrangian density over the S^3. The resulting Lagrangian then becomes

$$L = -\frac{N}{2}r^3\, b\, \left(-\left(\frac{-\dot{t}^2 + \dot{z}^2}{z^2} + (1-r^2)\dot{\phi}^2 \right) \frac{1}{b^2} + 1 \right) + Nr^4\dot{\phi} \tag{3.22}$$

where $b = \sqrt{-\gamma_{00}}$. We have set $\dot{r} = 0$ consistently since we are interested in the solution with fixed radius. The angular momentum

$$J = Nr^3(1-r^2)\frac{\dot{\phi}}{b} + Nr^4, \tag{3.23}$$

is a conserved quantity as a result of the equation of motion. By the Legendre transform, one now introduce the Routhian

$$L_R = L - \dot{\phi}J = \frac{N}{2}r^3 \left(\frac{-\dot{t}^2 + \dot{z}^2}{z^2\, b} - b\left(\frac{(J/N - r^4)^2}{(1-r^2)r^6} + 1 \right) \right) \tag{3.24}$$

One may check that all the resulting equations of motion are equivalent to the ones got from the Lagrangian before the transformation. The equation for b reads

$$-\frac{-\dot{t}^2 + \dot{z}^2}{z^2\, b^2} - \left(\frac{(J/N - r^4)^2}{(1-r^2)r^6} + 1 \right) = 0\,. \tag{3.25}$$

Using this, the equation for r becomes

$$6r^2 \left(\frac{(J/N - r^4)^2}{(1-r^2)r^6} + 1 \right) + r^3 \frac{d}{dr}\left(\frac{(J/N - r^4)^2}{(1-r^2)r^6} \right) = 0\,, \tag{3.26}$$

whose stable solution is $r^2 = J/N$. Inserting this solution to L_R, one gets the effective Lagrangian for the AdS part:

$$L_R = \frac{N}{2}(J/N)^{\frac{3}{2}} \left(\frac{-\dot{t}^2 + \dot{z}^2}{z^2\, b} - bN/J \right)\,. \tag{3.27}$$

The desired solution of this system is given by (3.13) with $b = \kappa\sqrt{J/N}$, which leads to

$$L_R = -J\kappa\,. \tag{3.28}$$

The treatment afterward is the same as the maximal giant case.

3.4 The two-point functions from giant gravitons with open strings

Let us consider the two-point function $\langle \mathcal{O}_Z^{\dagger J}(x)\, \mathcal{O}_Z^J(0) \rangle$ with the operator $\mathcal{O}_Z$ corresponding to the ground state of $Z = 0$ open spin chain without involving any derivative type operators D^M. The open string solution dual to the single operator is give in section 2. We can use the idea in [10] to find the open string solution for two-point function:

$$\begin{aligned} x(\tau) &= R\tanh\kappa\tau + x_0,\ z(\tau) = \frac{R}{\cosh\kappa\tau}, \\ \sin\theta(\sigma) &= \mathrm{dn}(\nu\sigma, k^2)\,, \quad \phi = \nu\tau\,, \end{aligned} \tag{3.29}$$

with $i\kappa \equiv \kappa_s = \nu k$. The boundary conditions $x(-s/2) = 0,\ x(s/2) = x,\ z(\pm s/2) = \epsilon$ give the following constraints:

$$x_0 = x/2,\ x = 2R\tanh\frac{\kappa s}{2},\ \frac{R}{\epsilon} = \cosh\frac{\kappa s}{2}. \tag{3.30}$$

There are two parts of contributions to this two-point function. The first part is from the Routhian of the D3-brane which is discussed in subsection 3.2. The second part is from the Routhian of the open string which is just the same as the energy of the open string. The computations are the same as the ones for the closed string case in [10]. And finally we obtain

$$\langle \mathcal{O}_Z^{\dagger J}(x)\mathcal{O}_Z^J(0)\rangle = |x|^{-2N-2E_{\rm open}}, \tag{3.31}$$

with $E_{\rm open}$ given in (2.22). The computation of the two-point function $\langle \mathcal{O}_Y^{\dagger J}(x)\,\mathcal{O}_Y^J(0)\rangle$ from the point-like open string solution in section 2 can be treated in a similar manner. The string solution is

$$\begin{aligned} x(\tau) &= R\tanh\kappa\tau + x_0,\ z(\tau) = \frac{R}{\cosh\kappa\tau}, \\ \theta &= 0\,, \qquad \phi = \nu\tau\,, \qquad i\kappa = \nu, \end{aligned} \tag{3.32}$$

together with similar constraints from the boundary conditions.

Now we turn to the open string inside AdS_5 [39]. We try the same ansatz as in (2.12)

$$\begin{aligned} Y_5 + iY_0 &= \cosh\rho(\sigma)\, e^{i\kappa_s\tau} \\ Y_1 + iY_2 &= \sinh\rho(\sigma)\ e^{i\omega\,\tau}, \end{aligned} \tag{3.33}$$

with $Y_3 = Y_4 = 0$ to describe the spinning open-string solution ending on the giant graviton branes. We choose the range of string coordinate σ to be $0 \le \sigma \le \pi$. For the S^5 part, we assume that the string stays at a point in S^5 that satisfies the required open-string boundary conditions. The contribution from this part vanishes in this case.

The equation of motion for the string becomes

$$\rho'' = (\kappa_s^2 - \omega^2)\sinh\rho\cosh\rho\,. \tag{3.34}$$

Integration of this equation leads to

$$\rho'^2 = \kappa_s^2\cosh^2\rho - \omega^2\sinh^2\rho + a\,, \tag{3.35}$$

where a is the integration constant. We also have the conformal gauge constraint

$$\dot{\vec{Y}}\cdot\dot{\vec{Y}} + \vec{Y}'\cdot\vec{Y}' = 0 \tag{3.36}$$

$$\dot{\vec{Y}}\cdot\vec{Y}' = 0. \tag{3.37}$$

The latter condition is satisfied automatically for any ansatz of the form in (3.33) and the former one is solved by setting $a = 0$. Below for the later purpose, we shall relax this condition $a = 0$ and solve the equation of motion for general a.

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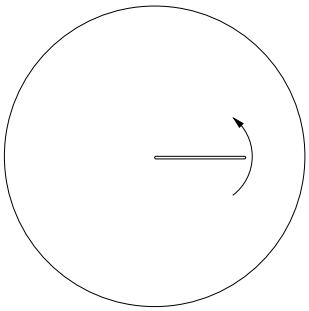

Figure 1. The open string starting from and ending on the giant graviton at $\rho = 0$ is rotating in the (Y_1, Y_2) plane.

The equation (3.35) is solved by

$$\cosh\rho(\sigma) = \frac{1}{\mathrm{dn}(\bar{\omega}\sigma, k^2)}\,, \tag{3.38}$$

where we have defined

$$\bar{k}^2 \equiv \frac{\bar{\kappa}_s^2}{\bar{\omega}^2}\,, \quad \bar{\omega}^2 \equiv \omega^2 + a\,, \quad \bar{\kappa}_s^2 \equiv \kappa_s^2 + a\,. \tag{3.39}$$

Since we are considering the open string starting from and ending on the giant graviton at $\rho = 0$, we find the relation

$$\bar{\omega}\pi = 2K(\bar{k}^2)\,. \tag{3.40}$$

The energy and the angular momentum are given by

$$\begin{aligned} E &= \frac{\sqrt{\lambda}}{2\pi}\kappa_s \int_0^\pi d\sigma \cosh^2\rho, \\ S &= \frac{\sqrt{\lambda}}{2\pi}\omega \int_0^\pi d\sigma \sinh^2\rho \end{aligned} \tag{3.41}$$

which are related to each other by

$$E = \frac{\kappa_s}{2}\sqrt{\lambda} + \frac{\kappa_s}{\omega}S\,. \tag{3.42}$$

Using the solution in (3.38), one finds

$$\begin{aligned} E &= \frac{\sqrt{\lambda}}{\pi}\frac{\kappa_s}{\bar{\omega}}\frac{E(\bar{k}^2)}{1-\bar{k}^2}\,, \\ S &= \frac{\sqrt{\lambda}}{\pi}\frac{\omega}{\bar{\omega}}\left(\frac{E(\bar{k}^2)}{1-\bar{k}^2} - K(\bar{k}^2)\right). \end{aligned} \tag{3.43}$$

As in [10], we do the transformation $Y_0 \to iY_4, Y_4 \to iY_0, \kappa_s \to -i\kappa$ and get

$$\begin{aligned} Y_4 &= \cosh\rho(\sigma)\sinh\kappa\tau,\ Y_5 = \cosh\rho(\sigma)\cosh\kappa\tau, \\ Y_1 &= \sinh\rho(\sigma)\cos\omega\tau,\ Y_2 = \sinh\rho(\sigma)\sin\omega\tau,\ Y_0 = Y_3 = 0. \end{aligned} \tag{3.44}$$

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In Poincare coordinate, this solution is

$$\begin{aligned} x_1 &= e^{\kappa\tau}\tanh\rho(\sigma)\cos\omega\tau, \\ x_2 &= e^{\kappa\tau}\tanh\rho(\sigma)\sin\omega\tau, \\ z &= \frac{e^{\kappa\tau}}{\cosh\rho(\sigma)}\,, \end{aligned} \tag{3.45}$$

with $t = x_3 = 0$.

Now we use the same conformal transformation as the one used in [10] and get

$$\begin{aligned} x_1 &= \frac{\tanh\rho(\sigma)\cos\omega\tau e^{\kappa\tau} + \frac{1}{R}e^{2\kappa\tau}}{1 + \frac{2}{R}\tanh\rho(\sigma)\cos\omega\tau e^{\kappa\tau} + \frac{1}{R^2}e^{2\kappa\tau}}, \\ x_2 &= \frac{\tanh\rho(\sigma)\sin\omega\tau e^{\kappa\tau}}{1 + \frac{2}{R}\tanh\rho(\sigma)\cos\omega\tau e^{\kappa\tau} + \frac{1}{R^2}e^{2\kappa\tau}}, \\ z &= \frac{\frac{1}{\cosh\rho(\sigma)}e^{\kappa\tau}}{1 + \frac{2}{R}\tanh\rho(\sigma)\cos\omega\tau e^{\kappa\tau} + \frac{1}{R^2}e^{2\kappa\tau}}, \end{aligned} \tag{3.46}$$

again with $t = x_3 = 0$. Notice that for fixed τ, the value of z depends on σ for the current case. So we need a slight modification of the boundary conditions used in [10]. We choose the following boundary conditions:

$$x_1(\tau_i,\sigma) = 0, \quad x_1(\tau_f,\sigma) = x, \quad x_2(\tau_i,\sigma) = x_2(\tau_f,\sigma) = 0, \tag{3.47}$$

$$\min{}_{0\le\sigma\le\pi}\, z(\tau_i,\sigma) = \min{}_{0\le\sigma\le\pi} z(\tau_f,\sigma) = \epsilon. \tag{3.48}$$

From these, we get

$$x = R, \quad \kappa = \frac{1}{s}(2\log R - 2\log\epsilon + \log(1-k^2)), \tag{3.49}$$

where $s = \tau_f - \tau_i$ and we have used the fact that the minimum value of $\mathrm{dn}(\bar{\omega}\sigma, \mathrm{k}^2)$ is given by $\sqrt{1-k^2}$ for our spinning string. Now we rescale ϵ, such that

$$x = R, \quad \kappa = \frac{1}{s}(2\log R - 2\log\epsilon). \tag{3.50}$$

We remark here that one can also use other boundary condition by changing the value of z at $t = t_i$, t_f into the maximal value of z or the mean value of z or the value of z at any σ. These boundary conditions give the same results if we always use the rescaled ϵ like above.

As we discussed in subsection 3.1, we need to use the Routhian to compute the two-point functions. For the present problem, the conserved quantity is the angular momentum S and the corresponding coordinate ϕ is defined by $Y_1 + iY_2 = |Y_1 + iY_2|e^{i\phi}$ with $\dot{\phi} = \omega$. Evaluation of the action can be expressed as

$$I = \frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma \left(\kappa^2\cosh^2\rho + \omega^2\sinh^2\rho - \rho'^2\right). \tag{3.51}$$

This leads to the corrected action of the form,

$$I_R = I - \int d\tau S\omega = \frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma \left(\kappa^2\cosh^2\rho - \omega^2\sinh^2\rho - \rho'^2\right) \tag{3.52}$$

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where we use the expression for the angular momentum in (3.41). Using the equation of motion in (3.35), the above expression can be reduced to

$$I_R = \frac{\sqrt{\lambda}}{2\pi}\int d\tau d\sigma(\kappa^2\cosh^2\rho - a) = 2\left(\frac{\kappa}{2}\sqrt{\lambda} + \frac{\kappa}{\omega}S - \frac{a}{4\kappa}\right)\log R/\epsilon. \tag{3.53}$$

We would then like to minimize the above expression with respect to the modular parameter s that is proportional to $\kappa = i\kappa_s$. One further note that $\bar{\omega}$ will be completely determined as a function of $\bar{k}$ by (3.40). Using the definition of $\bar{k}$, $\bar{\kappa}_s$ is also function of only $\bar{k}$. Using the expression in (3.43), we find also that ω is completely fixed as a function of $\bar{k}$ when we fix S as a constant. Noting $a = \bar{\omega}^2 - \omega^2$, one finds that a is also completely determined as a function of $\bar{k}$. Thus the extremization condition of I_R with respect to modular parameter s is equivalent to that of extremization of I_R with respect to k. As summarized in the appendix, $dI_R/d\bar{k}^2 = 0$ gives $a = 0$, which is the desired solution of the Virasoro constraint. One then has

$$e^{i(I_R^{D3}+I_R)} \propto \frac{1}{(x/\epsilon)^{2\Delta}} \tag{3.54}$$

with $\Delta = N + \frac{\kappa_s}{2}\sqrt{\lambda} + \frac{\kappa_s}{\omega}S$ including the contributions from the D3-brane.

4 Three-point functions

4.1 Field theory side

In this subsection, we discuss the computation of three-point function in $\mathcal{N} = 4$ SYM theory, which will be used for the later comparison with the semiclassical computation.

We begin our discussion with the correlation functions involving the vacuum-state operator of the $Y = 0$ open spin chain,

$$\mathcal{O}_Y^J = d_N^J \frac{1}{(N-1)!}\epsilon^{j_1\cdots j_{N-1}A}_{i_1\cdots i_{N-1}B} Y^{i_1}_{j_1}\cdots Y^{i_{N-1}}_{j_{N-1}}(Z^J)^B_A\,. \tag{4.1}$$

In our convention, the normalization factor d_N^J is defined by the two-point function

$$\langle \mathcal{O}_Y^{\dagger J}(x)\,\mathcal{O}_Y^J(0)\rangle = \frac{1}{(x^2)^{N+J-1}} \tag{4.2}$$

with a unit normalization. Note that the position space propagator of the fields X, Y and Z is given by

$$I(x) = \frac{g_{\rm YM}^2}{4\pi^2}\frac{1}{x^2} \equiv \frac{s_2}{x^2}\,, \tag{4.3}$$

in the standard convention of the $\mathcal{N} = 4$ SYM theory.

The relevant Feynman diagram of the free field contractions is depicted in figure 2: The part involving separated parallel lines represents the contractions of elementary fields inside the subdeterminant parts of the operators whereas the remaining square represents the contractions in the $(Z^J)^A_B$ parts. With the operator of the normalization $\mathcal{O}_Y^J/d_N^J$, the subdeterminant part has $(N-1)!$ independent ways of contractions for given indices A

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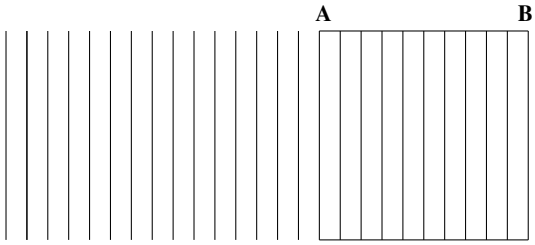

Figure 2. The relevant Feynman diagram of free field evaluation for the two-point function of the $Y = 0$ ground state is depicted. We illustrate here the case of $N = 16$ and $J = 11$.

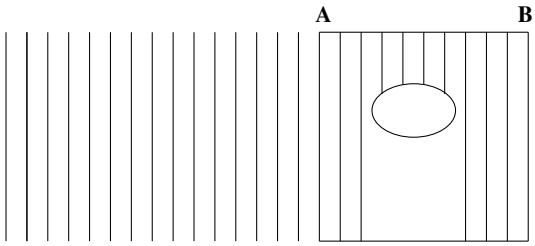

Figure 3. The relevant Feynman diagram of free field evaluation for c_{123}^{Y} is depicted here. We illustrate the case of $N = 16$, $J = 7$ and $\ell = 4$.

and B and the $(Z^J)_B^A$ part involves N^{J-1} factor from the closed loops of indices. Further including the N^2 possibilities of the indices (A, B), one has

$$\langle \mathcal{O}_Y^{\dagger J}(x)\, \mathcal{O}_Y^J(0)\rangle \; (d_N^J)^{-2} = \big[\, I(x)\,\big]^{N+J-1} N^2\, N^{J-1}\, (N-1)!\,. \tag{4.4}$$

Therefore, the normalization factor is determined as

$$(d_N^J)^{-2} = (s_2)^{N+J-1}\, N^J\, N!\,. \tag{4.5}$$

This computation is valid for all range of λ, which can be justified using the non-renormalization argument of $1/2, 1/4$ and $1/8$ BPS operators for the two-point functions in ref. [40].

Next we turn to the evaluation of the 1/4 BPS three-point correlation function

$$\langle \mathcal{O}_Y^{\dagger J+\ell}(x_1)\, \mathcal{O}_Y^J(x_2) \mathcal{O}_{Z^\ell}(x_3)\rangle = c_{123}^Y\, \frac{1}{(x_{12}^2)^{N+J-1}\,(x_{13}^2)^\ell} \tag{4.6}$$

where $\mathcal{O}_{Z^\ell}$ denotes the half BPS chiral primary operator oriented in the Z direction given by $\mathcal{O}_{Z^\ell} = c_\ell \,\mathrm{tr}\, Z^\ell$ with $c_\ell = (2\pi)^\ell/(\sqrt{\ell}\lambda^{\ell/2})$.

The corresponding Feynman diagram for the free field evaluation is illustrated in figure 3. We note that there are $J-1$ independent ways to put the operator $\mathcal{O}_{Z^\ell}$ inside the box and ℓ independent ways to pick a particular Z in $\mathrm{tr}\, Z^\ell$. The remaining counting of the combinatoric factors is similar to that of the two-point function. Therefore, one is led to

$$c_{123}^Y = d_N^{J+\ell} d_N^J c_\ell\, N!\, N^{J+\ell-1}\, (s_2)^{N+J+\ell-1}\, (J-1)\, \ell = \frac{1}{N}\sqrt{\ell}\,(J-1)\,, \tag{4.7}$$

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which is again valid for all range of the 't Hooft coupling λ due to the nonrenormalization of the 1/4-BPS three-point correlation function [40].

This kind of non-renormalization argument does not apply for the three-point function involving the operator $\mathcal{O}_Z^J$ which represents a particular ground state of the $Z = 0$ open spin chain carrying the (Z plane) R-charge $J + N - 1$. This is because the three-point function of our interest below

$$\langle \mathcal{O}_Z^{\dagger J+\ell}(x_1)\, \mathcal{O}_Z^J(x_2)\mathcal{O}_{Z^\ell}(x_3)\rangle = c_{123}^Z\, \frac{1}{(x_{12}^2)^{N+J-1+2E_B}\,(x_{13}^2)^\ell}\,, \tag{4.8}$$

does not preserve any of the sixteen supersymmetries. We expect generic perturbative renormalization of the three-point function and do not have any field theoretic means to compute the corresponding structure constant in the strongly coupled regime.

4.2 D brane contribution to the structure constants

In the following two subsections, we shall concern about the holographic computations of the structure constants c_{123}^Y and c_{123}^Z. This subsection will be devoted to the D-brane contributions c_{D3}. And we will focus on the open string contributions c_{string} in the next subsection. We shall compute the three-point functions in the limit of the operator product expansion (OPE)

$$R_{12} = |x_1 - x_2| \;\ll\; L_{12} \equiv \frac{1}{2}(x_1 + x_2)\,, \tag{4.9}$$

with the choice of $x_3 = 0$. Our prescription of holographic computations of these three-point functions is along the way in [16, 17]. The light chiral primary operator is dual to specific fluctuations of background fields in string theory side. This fluctuation will lead to the fluctuation of the classical D3-brane and open string solutions used in the calculation of the two-point functions. The variant of the on-shell action of the coupled system of D-brane and open string will give the three-point functions in the large N and large λ limit. This can be understood as that we treat the brane and string as source for the supergravity fields and read off the OPE coefficients from the coupling to the brane and string of the bulk supergravity modes dual to the light operators [12, 14].

To the leading order of g_s and α', the classical onshell action of the coupled system of the brane and the string is the sum of the brane action I_{D3} and the string action I_{string}. So the variation of the on-shell action of the coupled system is equal to $\delta I_{D3} + \delta I_{\text{string}}$. This leads to the fact that the OPE coefficient c_{123} is the sum of c_{D3} and c_{string}.

The fluctuations of the background $AdS_5 \times S^5$ metric $g_{\mu\nu}, g_{\alpha\beta}$ corresponds to the half-BPS chiral primary operators are [6]

$$h_{\mu\nu} = \Big(-\frac{6}{5}\,\ell\, g_{\mu\nu} s_I + \frac{4}{\ell+1}\nabla_{(\mu}\nabla_{\nu)} s_I \Big)\mathcal{Y}^I, \tag{4.10}$$

$$h_{\alpha\beta} = 2\,\ell\, g_{\alpha\beta}\, s_I\, \mathcal{Y}^I, \tag{4.11}$$

where we use $\mu, \nu, \ldots$ for the coordinate indices of the AdS_5 part and $\alpha, \beta, \ldots$ for the coordinate indices of the S^5 part, and the round parentheses of $(\mu\nu)$ denotes the symmetric

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traceless part. The fluctuation of the background four-form potential is [6]

$$a_{\mu_1\cdots\mu_4} = -4\epsilon_{\mu_1\cdots\mu_5}\nabla^{\mu_5}s_I\mathcal{Y}^I, \tag{4.12}$$

$$a_{\alpha_1\cdots\alpha_4} = 4\epsilon_{\alpha_1\cdots\alpha_5}s_I\nabla^{\alpha_5}\mathcal{Y}^I. \tag{4.13}$$

The spherical harmonic function $\mathcal{Y}^I$ in the S^5 coordinate space is entirely fixed by the choice of the chiral primary operators: for $\mathcal{O}_{Z^\ell} = c_\ell \mathrm{tr}(Z^\ell)$ and $\mathcal{O}_{Y^\ell} = c_\ell \mathrm{tr}(Y^\ell)$, the corresponding harmonic functions are given by $2^{-\ell/2}\,Z^\ell$ and $2^{-\ell/2}\,Y^\ell$ respectively.

The linear operation s^I is related to a source $s_0(\vec{x}')$ at the boundary of AdS_5 through the following relation: $s^I(\vec{x},y) = \int d^4\vec{x}'\, G_\ell(\vec{x}';\vec{x},y)s_0^I(\vec{x}')$ where the bulk-to-boundary propagator G_ℓ is given by

$$G_\ell(\vec{x}';\vec{x},y) = g_\ell\left(\frac{y}{y^2+|\vec{x}-\vec{x}'|^2}\right)^\ell, \qquad g_\ell = \frac{\ell+1}{2^{2-\ell/2}N\sqrt{\ell}}\,. \tag{4.14}$$

In the OPE limit $R_{12} \ll L_{12}$, one has

$$G_\ell \simeq g_\ell\,\frac{z^\ell}{L_{12}^{2\ell}}, \tag{4.15}$$

and also

$$h_{\mu\nu} \simeq \left(-2\ell g_{\mu\nu}s_I + \frac{4\ell}{z^2}\delta^z_\mu\delta^z_\nu s_I\right)\mathcal{Y}^I. \tag{4.16}$$

The three-point function we want to compute is proportional to $\frac{\delta I_{D3}}{\delta s_0(\vec{x}'=0)}$.

We begin with the DBI action of the D3-brane

$$I_{\mathrm{DBI}} = -T_{D3}\int\sqrt{\gamma}\,. \tag{4.17}$$

Its variation due to the above fluctuation of the background fields is given by

$$\begin{aligned}\delta I_{\mathrm{DBI}} &= -\frac{1}{2}T_{D3}\int d^4\sigma\sqrt{\gamma}\,\gamma^{ab}(h_{\mu\nu}\partial_a X^\mu\partial_b X^\nu + h_{\alpha\beta}\partial_a X^\alpha\partial_b X^\beta)\\ &= -\frac{1}{2}T_{D3}\int d^4\sigma\sqrt{\gamma}\left(-2\ell s_I\mathcal{Y}^I + \frac{4\ell s_I\mathcal{Y}^I}{\kappa^2 z^2}\left(\frac{\partial z}{\partial\tilde{\tau}}\right)^2 + 6\ell s_I\mathcal{Y}^I\right)\\ &= -T_{D3}\int d^4\sigma\sqrt{\gamma}\,2\,\ell\,s_I\,\mathcal{Y}^I\left(2-\frac{1}{\cosh^2\kappa\tilde{\tau}}\right),\end{aligned} \tag{4.18}$$

where $\tilde{\tau} = i\tau$, and we know that $\tilde{\tau}$ is real from subsection 3.2.

Following the prescription just reviewed, replacing s_I by the bulk-to-boundary propagator,

$$G_\ell \simeq g_\ell\frac{z^\ell}{L_{12}^{2\ell}} = \frac{g_\ell R_{12}^\ell}{2^\ell L_{12}^{2\ell}\cosh^\ell\kappa\tilde{\tau}}, \tag{4.19}$$

one finds that the contribution of the DBI part of the action to the structure constant takes the form

$$c_{\mathrm{DBI}} = \frac{g_\ell\,\ell N}{2^{\ell-1}}\int d\tilde{\tau}\,\kappa\,\frac{\mathcal{Y}^I}{\cosh^\ell\kappa\tilde{\tau}}\left(-2+\frac{1}{\cosh^2\kappa\tilde{\tau}}\right). \tag{4.20}$$

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The variation of the Chern-Simons term is given by

$$\delta I_{\rm WZ} = \frac{N}{2\pi^2}\int 4\sin^3\theta\cos\theta\sin\alpha\cos\alpha s^I(-\partial_\theta \mathcal{Y}^I)\dot{\phi}\, d\tau\wedge d\phi_1\wedge d\alpha\wedge d\phi_2\,, \tag{4.21}$$

and the contribution from this part to the structure constant is proportional to

$$c_{\rm WZ} = \frac{g_\ell\, N}{2^{\ell+1}\pi^2}\int \frac{4\sin^3\theta\cos\theta\sin\alpha\cos\alpha(-\partial_\theta\mathcal{Y}^I)\dot{\phi}}{\cosh^\ell\kappa\tilde{\tau}}\, d\tilde{\tau}\wedge d\phi_1\wedge d\alpha\wedge d\phi_2. \tag{4.22}$$

Now we use the above formula to show that the contribution from the $Z=0$ ($Y=0$) brane part to the structure constants c^Z_{D3} (c^Y_{D3}) is vanishing. We first consider the case of c^Z_{D3} where the corresponding spherical harmonic function is $2^{-\ell/2}\,Z^\ell = 2^{-\ell/2}\cos^\ell\theta e^{i\ell\phi}$. This is identically zero on the $Z=0$ brane which is at $\theta=\pi/2$. So the contribution from the DBI part is vanishing. As for the Chern-Simons part, we notice that $\cos\theta\,\partial_\theta(Z^\ell)$ vanishes on the brane for $\ell\geq 1$, and this leads to the vanishing of the contributions since $\ell\geq 2$ for the chiral primary operators.

The $Y=0$ brane contribution c^Y_{D3} can be related to the the $Z=0$ brane contribution with the 1/2 BPS operator $\mathcal{O}_{Y^\ell} = c_\ell {\rm tr}(Y^\ell)$ by an appropriate SO(6) rotation. The corresponding spherical harmonic function is given by $2^{-\ell/2}Y^\ell = 2^{-\ell/2}\sin^\ell\theta\sin^\ell\alpha e^{i\ell\phi_2}$. The contribution from the DBI part is proportional to the integral

$$c_{\rm DBI}\ \propto\ \int_0^{2\pi} d\phi_2\ e^{i\ell\phi_2} \tag{4.23}$$

which is zero for $\ell\neq 0$. As for the contribution from the Chern-Simons part, we have $\partial_\theta(Y^\ell)=0$ on the $Z=0$ brane and the corresponding contribution again vanishes. This shows that $c^Y_{D3}=c^Z_{D3}=0$.

4.3 Open string contributions

In this subsection we compute the open string contribution to the structure constants c^Y_{123} and c^Z_{123}. Since we are dealing with the three-point functions of one light chiral primary operator and two heavy operators represented by semiclassical string trajectory, the method developed in ref. [16, 17] is appropriate. The derivation of the corresponding formula is originally for the closed string state. The derivation for our open string states is then essentially the same as the closed string case in ref. [16] but simply replacing the closed string trajectory by the open string one. One finds that

$$c_{\rm string} = \frac{2^\ell(\ell+1)\sqrt{\ell\lambda}}{8\pi N R_{12}^\ell}\int d^2\sigma \mathcal{Y}^I z^\ell\left[\frac{(\partial\vec{x})^2-(\partial z)^2}{z^2} - h_{\alpha\beta}\partial X^\alpha\partial X^\beta\right] \tag{4.24}$$

where one evaluates the above integral with the on-shell open string trajectory in $AdS_5\times S^5$ and ℓ is the dimension of the light chiral primary operator as before. The rest is basically straightforward: we use the semiclassical solution of the open string state constructed in the previous section to evaluate the contribution to the structure constant.

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Let us now start with the open string contribution to the structure constant c^Y_{123}. Using the open string trajectory in (3.32), one has

$$\begin{aligned}\frac{1}{z^2}\Big[(\partial\vec{x})^2-(\partial z)^2\Big] &= \frac{2\kappa^2}{\cosh^2\kappa\tau}-\kappa^2\\ h_{\alpha\beta}\,\partial X^\alpha\partial X^\beta &= -\kappa^2\end{aligned} \tag{4.25}$$

with $\mathcal{Y}^I = 2^{-\ell/2}\cos^\ell\theta e^{i\ell\phi} = 2^{-\ell/2}e^{-\ell\kappa\tau}$. A straightforward computation then leads to the expression,

$$c^Y_{\rm string} = \frac{J\sqrt{\ell}\,(\ell+1)}{N\,2^{\ell+1}}\kappa\int_{-\infty}^{\infty}d\tau\frac{e^{-\ell\kappa\tau}}{\cosh^{\ell+2}\kappa\tau} = \frac{1}{N}J\sqrt{\ell}\,. \tag{4.26}$$

Since the $Y=0$ brane contribution is vanishing as computed in the previous section, our final semiclassical result is given by

$$c^Y_{123} = c^Y_{D3} + c^Y_{\rm string} = \frac{1}{N}J\sqrt{\ell}\,, \tag{4.27}$$

which agrees with the field theory result in the large J limit.

Let us now turn to the case of the $Z=0$ open string contribution to c^Z_{123}: By a straightforward evaluation using the solution (3.29), one has

$$\begin{aligned}\frac{1}{z^2}\Big[(\partial\vec{x})^2-(\partial z)^2\Big] &= \frac{2\kappa^2}{\cosh^2\kappa\tau}-\kappa^2\\ h_{\alpha\beta}\,\partial X^\alpha\partial X^\beta &= \kappa^2-2\kappa^2\,\mathrm{sn}^2(\nu\sigma,k^2)\end{aligned} \tag{4.28}$$

with $\mathcal{Y}^I = 2^{-\ell/2}\,k^\ell\,\mathrm{sn}^\ell(\nu\sigma,k^2)e^{-\ell\kappa\tau/k}$. This leads to the expression

$$\begin{aligned}c^Z_{\rm string} = \frac{J(\ell+1)\sqrt{\ell}}{N\,2^{\ell+2}}\frac{k^\ell}{K(k^2)-E(k^2)}\int_{-\infty}^{\infty}dx\frac{e^{-\ell x/k}}{\cosh^\ell x}\\ \times\int_0^{2K(k^2)}ds\;\mathrm{sn}^\ell(s,k^2)\Big(\mathrm{sn}^2(s,k^2)-\tanh^2 x\Big)\,,\end{aligned} \tag{4.29}$$

for the $Z=0$ open string trajectory. Since $k<1$, the x integral diverges which implies a divergent $c^Z_{\rm string}=\infty$. Therefore we have a trouble in this case since our semiclassical value of c^Z_{123} diverges whereas the field theoretic one should be finite for a given λ. We shall comment on this issue in the last section.

5 Discussions

In this paper, we studied the holographic computation of two- and three-point functions of giant graviton with open strings. We discussed the string configurations corresponding to the ground states of $Z=0$ and $Y=0$ open spin chain, and the spinning string in AdS_5 corresponding to the derivative type impurities in $Z=0$ open spin chain as well. For the two-point functions, we found both the D3-brane configuration and the open string configurations connecting two insertions of the composite operators at the boundary. There is only one subtle point: We had to use the Routhian rather than energy directly

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in the computation. For the three-point functions, we consider two giant gravitons with open strings coupled to a half-BPS chiral primary operator. In the field theory side, we computed them in the free field limit. In the limit of OPE and with the configurations for the computation of two-point functions, we computed the three-point functions holographically. In the holographic computation, we carefully took the D-brane contribution into account, which turned out to be vanishing. For the point-like string dual to the ground state of $Y = 0$ open spin chain, we found perfect agreement. The reason of this agreement may be the nonrenormalization of this three-point correlation function [40] which we mentioned in the field theory computations. But for the string dual to the $Z = 0$ open spin chain, we obtained a divergent structure constant from holographic computation. This divergence may be canceled by the $1/N$ corrections to the contributions of D-brane because of the attached open strings. These $1/N$ contributions also include quantum corrections of D branes and there is currently no available method to compute these quantum corrections for generic, non-BPS states of D branes. We also wish that an improved understanding of the prescription for three-point functions will shed light on this subtle issue.

The computation in this paper is based on the picture that both the D-brane and open string contributes to the computation. This is reasonable as the dual operators include both the part corresponding to the giant graviton and the part corresponding to the open spin chain. However, this picture should be changed for other kinds of open spin chain [41–43]. For example, in the case studied in [41, 42], there are two kinds of integrable spin chain, closed one and open one, which decouple to the leading order. In other words, the presence of D7-brane just provides boundary conditions for the open semiclassical string. To compute the correlation functions of the open string state, the contribution from D-brane could be neglected safely.

In our work, we discussed the giant gravitons in S^5. It would be nice to consider the case with the giant graviton in AdS_5. It is also interesting to compute holographically the correlation functions of these giant gravitons and non-local operators, such as Wilson loops.

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A Extremal condition for I_R

In this appendix, we shall show that the extremal condition for I_R with respect to s is given by $a = 0$. Using the relation (3.40) and the expression in (3.43), ω and $\bar{\kappa}_s^2$ are expressed as

$$\sqrt{\lambda}\,\omega = \frac{2S\,K(x)}{\frac{E(x)}{1-x} - K(x)}\,, \qquad \bar{\kappa}_s^2 = \frac{4x}{\pi^2}\,K^2(x)\,, \tag{A.1}$$

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Where $x = \bar{k}^2$ and S is taken as a constant in k. Then a can be expressed as a function of x by

$$\bar{\omega}^2 - \omega^2 = a = \frac{4K^2(x)}{\pi^2}\left[1 - \frac{\pi^2 S^2}{\lambda}\Big(\frac{E(x)}{1-x} - K(x)\Big)^{-2}\right]. \tag{A.2}$$

Note also

$$\frac{da}{dx}\Big|_{a=0} = \frac{8}{\pi^2}\,K^2(x)\,F(x) \tag{A.3}$$

where

$$F(x) = \frac{d}{dx}\log\left(\frac{E(x)}{1-x} - K(x)\right) = \frac{1}{2(1-x)}\,\frac{\frac{2E(x)}{1-x} - K(x)}{\frac{E(x)}{1-x} - K(x)}\,. \tag{A.4}$$

We then like to show that

$$A(x) \equiv \frac{d}{dx}\left(\frac{\kappa}{2}\sqrt{\lambda} + \frac{\kappa}{\omega}S - \frac{a}{4\kappa}\right)\Big|_{a=0,\kappa=i\kappa_s} = 0\,. \tag{A.5}$$

This can be rearranged to

$$i\kappa_s\,A(x) = -\frac{\sqrt{\lambda}}{4}\frac{d}{dx}\bar{\kappa}_s^2 + \left[-\frac{\pi}{4K(x)}\frac{d}{dx}\big(\bar{\kappa}_s^2 - a\big) + x\frac{d\omega}{dx}\right]S\,. \tag{A.6}$$

We also note that

$$\frac{d}{dx}\bar{\kappa}_s^2 = \frac{4}{\pi^2}\frac{K(x)E(x)}{1-x} \tag{A.7}$$

$$\sqrt{\lambda}\,\frac{d\omega}{dx} = \frac{d}{dx}\left(\frac{2K(x)S}{\frac{E(x)}{1-x} - K(x)}\right) - \frac{2K(x)S}{\frac{E(x)}{1-x} - K(x)}F(x)\,. \tag{A.8}$$

Inserting these two together with (A.3) into $i\kappa_s A(x)$ in (A.6), one finds that $i\kappa_s A = 0$, which proves that $a = 0$ is the extremal condition for I_R.

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