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Towards natural and realistic E_7 GUTs in F-theory

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Abstract: We consider phenomenological aspects of a natural class of Standard Model-like supersymmetric F-theory vacua realized through flux breaking of rigid E_7 gauge factors. Three generations of Standard Model matter are realized in many of these vacua. We further find that many other Standard Model-like features are naturally compatible with these constructions. For example, dimension-4 and 5 terms associated with proton decay are ubiquitously suppressed. Many of these features are due to the group theoretical structure of E_7 and associated F-theory geometry. In particular, a set of approximate global symmetries descends from the E_7 group, leading to exponential suppression of undesired couplings.

Keywords: F-Theory, Flux Compactifications, Grand Unification, String and Brane Phenomenology

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Contents

1 Introduction

String theory provides a consistent framework for a unified theory that combines gravity with the other fundamental forces described by quantum field theory. To describe the real world, however, ten-dimensional string theory must be compactified on a real six-dimensional manifold, and various further objects like branes, fluxes, and orientifolds must be incorporated. Such constructions give an enormous number (perhaps on the order of something like 10^{272000} [1]) of string theory vacua, known as the string landscape. As part of the program to realize our Universe in string theory, it has been a long-standing and primary goal to find the structure of the Standard Model (SM) of particle physics within the string landscape. While many (supersymmetric) string vacua have been identified that share many of the principal features of the Standard Model, there is as yet no single vacuum known in the string landscape that reproduces all the observed phenomenological details of our world; for recent reviews of work in this direction, see [2, 3].

Beyond the simple question of the existence of a vacuum matching observed physics, it is perhaps even more important to understand the extent to which the physical features of the Standard Model arise *naturally* in string theory. In other words, we would like to understand the extent to which solutions like the Standard Model are widespread in the string landscape or require extensive fine-tuning. This is a principal focus of this work and the associated research program: we take a top-down perspective on the global set of string vacua and attempt to identify realizations of the Standard Model that are compatible with the most typical structures arising in string theory. We use F-theory [4–6] to study these questions, as this approach gives a global and nonperturbative picture of the largest currently understood set of string vacua. For reviews of F-theory and applications to Standard Model constructions, see [7, 8]. This paper describes some more detailed phenomenological aspects of SM constructions originally presented in [9, 10] that are realized through flux breaking of rigid E_6 and E_7 gauge factors, which are relatively common features in F-theory geometries.

Constructing the detailed Standard Model requires many elements such as the gauge group, the matter content including both chiral matter and the Higgs, the Yukawa couplings, a supersymmetry (SUSY)-breaking mechanism, values of the 19 free parameters, and possibly some room to address beyond-SM problems as well as cosmological aspects such as the density of dark energy. Unfortunately, the current available string theory techniques are far from enough to compute all these features precisely. Among the above SM features, string theory techniques for constructing the gauge group and the chiral spectrum are well-established. While there is some recent progress on the Higgs sector [11–15] and the Yukawa couplings [16] in a large class of F-theory models, so far no fully precise statement on the realization of these features in a way that matches observed physics has been made in this context. On the other hand, incorporating these established features with e.g. SUSY breaking is far beyond our current techniques. Although at this moment no complete realization of the Standard Model has been constructed in any version of string theory, if we can identify a natural class of models that realize a decent portion of the coarsest features of the SM, these structures may naturally correlate with certain other features of the SM or beyond SM physics. We will explore this philosophy in this paper.

One obvious way in which the models studied here (and elsewhere in much of the string theory literature) differ from observed physics is that we focus on solutions with supersymmetry. Supersymmetry has not yet been observed at low (TeV or below) energies in nature, but as a theoretical tool it increases our level of analytic control. By studying solutions with supersymmetry, we can gain some perspective on global aspects of the string landscape. Of course, eventually we need to understand non-supersymmetric solutions to match observed physics. One possibility is that the physics we see is in a broken-symmetry phase of a theory with supersymmetry at energies beyond the TeV scale. Even if supersymmetry is broken at the Planck or string scale, many insights gained by exploring the space of supersymmetric vacua may be relevant to the less controlled non-supersymmetric vacua.

1.1 Natural vs. tuned features

Before describing our results, it is worth clarifying the concept of *naturalness* used in this paper. To obtain vacua with all the SM features considered in this paper, quite a few specific choices must be made in the construction of vacua. A list of such choices in the context of the models studied in this paper is summarized in section 1.3, and the mathematical conditions imposed for such choices are given at the beginning of section 5. The extent to which these different kinds of choices are natural varies, within a hierarchy of naturalness/tuning. Roughly speaking, each of the choices made in constructing a specific class of string vacua can be characterized as belonging to one of the following categories:

1. Physical constraints: these constraints come from string theory itself and must be satisfied in all string compactifications. These constraints ensure physically sensible vacua that have, e.g., Poincaré (or AdS) invariance. Examples include tadpole cancellation and primitivity of fluxes.

2. Ubiquitous/common conditions: let us consider a reasonably large but presumably finite set of string vacua or compactification geometries, such as $\mathcal{N} = 1$ 4D F-theory vacua or the associated set of topologically distinct complex threefold bases that support elliptic Calabi-Yau fourfolds. A condition is common if it holds for an $\mathcal{O}(1)$ fraction of the set of vacua or geometries, considered with a simple counting measure. In particular, the condition is ubiquitous if it holds for a substantial majority. As examples, the existence of rigid E_8 gauge factors in (known) F-theory base geometries is ubiquitous, and that of E_7 gauge factors appears to be common. (See, e.g., [17–19] and discussions below)

3. Fairly likely conditions: sometimes, there are a family of similar conditions, such as possible values of a discrete parameter. Each possible value may only hold for a relatively small fraction of vacua within the above set, so that none of the conditions are ubiquitous. We refer to a condition as being fairly likely if the fraction of vacua or geometries with this property is considerably higher than for most of the other possibilities. As an illustration, we would say that rolling a sum of 6 on a pair of six-sided dice is "fairly likely," although rolling a sum of 7 is slightly more likely. As another example, points near the peaks in a distribution of some discrete parameters correspond to fairly likely conditions. See [20] for more discussions along these lines. As a further example, [9, 10] argued that three generations of chiral matter is fairly likely in this sense in our E_7 model (although, for example, zero generations may be more likely). (See also, e.g., [21].)

4. Natural choices: these are choices for discrete parameters having many possible values that are not (obviously) preferred in any way, but imposing a particular chosen value does not require exponential amounts of tuning. Such choices may hold at the level of, e.g., 0.1% of the given set of vacua. Such choices may be needed for obtaining some *qualitative* features of observed phenomenology in some constructions. For example,

obtaining the SM gauge group and matter representations from flux breaking of E_7 involves some choices of fluxes given by mild linear constraints, which seem to be natural in this sense, although they do not seem to be preferred in any particular way over other choices that would give a variety of other possible groups and representations.

5. Fine-tuning: these are choices involving setting one or more continuous variables to take specific values, or making an exponentially rare choice among discrete possibilities. Vacua based on such choices are increasingly non-generic in the landscape as the number of such tunings increases. In some constructions of string vacua, such choices are needed to obtain certain *qualitative* features of observed phenomenology. For example, a tuned SU(5) or SM gauge group in F-theory involves extensively fine-tuning many moduli to specific values [21] (unless these moduli are somehow automatically tuned by a specific class of flux choices). Notably, it seems that no such fine-tuning is involved in our E_7 models.

6. Technical choices: to facilitate analytic control of the vacua and make some particular calculations manageable, in some cases technical choices are made by restricting attention to some specific relatively simple choices of vacua. These choices are not necessary either for physical or phenomenological consistency, but are made to illustrate specific examples as simply as possible. The features of the models chosen in this way should be representative of some larger class of vacua or geometries. In some situations, technical choices can be made just to simplify calculations that are in principle possible and expected to give qualitatively similar results for all other choices. In other cases, technical choices are made where it is not clear how to do the computation explicitly in general, and/or whether a completely general choice will give qualitatively similar results. If not, some choices or tuning of one of the above types may be implicitly involved. For the specific technical choices made here, we have some confidence that similar results should also hold for a broader class of vacua without those technical choice. Nevertheless, some qualitative simplifications occur based on these choices, thus more explicit further studies are required to understand the extent to which these technical choices are relevant for phenomenologically interesting features. Examples of technical choices include picking some certain topological types for the compactification, which we do in this paper (specifically by choosing models where the gauge divisor is a del Pezzo surface and the matter curve is a $\mathbb{P}^1$) to facilitate and simplify the analysis; these technical choices made here fall in the latter category above that may implicitly involve some more or less natural choice or tuning, as they may affect qualitative aspects of the low-energy physics.

While the term "natural" is used widely in many different ways in the literature, we attempt to use the above classification to be slightly more precise about the types of choices involved in the construction of our models and the realization of phenomenological features. This is a coarse characterization, however, as choices and tunings can occur across a broad spectrum, and we do not attempt to make any precise division between the gradations of "common," "fairly likely," and "natural" conditions. In particular, we do not have a perfect understanding of the class of string geometries or F-theory compactifications, so any attempt

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at classification of this type is necessarily quite imperfect given the current state of knowledge. Moreover, the measure problem on the landscape is not at all understood, so we really do not have any good sense of the proper probability measure to use on the landscape. Nonetheless, in the absence of any known or conjectured dynamical mechanism that would modify these considerations, features that seem to require exponentially large amounts of fine-tuning under a simple counting measure seem likely to occur less frequently in a large string multiverse than features that are ubiquitous, fairly likely, or even natural in the preceding terminology. In principle, even without solving the measure problem, this may give us some insight into the extent to which the Standard Model may be realized naturally in string theory, and what BSM physics may be most naturally associated with those SM structures.

1.2 Review of previous work

In recent years, F-theory has become a particularly promising framework for studying many aspects of string compactifications and phenomenology, as it provides a global description of a large connected class of supersymmetric string vacua. (See [7] for a review.) In particular, F-theory gives 4D $\mathcal{N} = 1$ supergravity models when compactified on elliptically fibered Calabi-Yau (CY) fourfolds Y, corresponding to non-perturbative compactifications of type IIB string theory on general (non-Ricci flat) complex Kähler threefold base manifolds B. The number of such threefold geometries B seems to already be on the order of 10^{3000} for toric bases B [17, 18, 22], without even considering the exponential multiplicity of fluxes possible for each geometry, although the number of flop equivalence classes of bases is somewhat smaller [19]. F-theory is also known to be dual to many other types of string compactifications such as heterotic models. Briefly, F-theory is a strongly coupled version of type IIB string theory with non-perturbative configurations of 7-branes balancing the curvature of the compactification space. The non-perturbative brane physics is encoded geometrically into the elliptically fibered manifold, which can be analyzed using powerful tools from algebraic geometry. The gauge groups and chiral matter content supported on these branes can then be easily determined when combined with flux data.

Applying the above techniques, many SM-like constructions of 4D F-theory models with the gauge group $G_{\rm SM} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)/\mathbb{Z}_6$ have been achieved in the literature. The early literature, starting from [23–26], focused on the breaking of GUT groups of SU(5) and its U(1) extensions [27–31], while there has also been some study of SO(10) [32] and E_6 [33–35] GUTs. (See [7, 8] for more extensive reviews.) These constructions break the GUT group using the so-called hypercharge flux further discussed in [36, 37], which is a kind of "remainder" flux [38, 39] to be reviewed below. Some later constructions tried to construct $G_{\rm SM}$ directly without any symmetry breaking, with the recent culmination of finding 10^{15} explicit solutions of directly tuned $G_{\rm SM}$ with three generations of SM chiral matter (a "quadrillion Standard Models" [40]), based on the "F_{11}" fiber in [41]. These constructions are further generalized in [42, 43]. Although these models nicely capture some of the most important phenomenological features, they face one common issue: in terms of the notions discussed in section 1.1 the gauge groups in these constructions are highly "fine-tuned", namely they are obtained by setting specific values for many complex structure moduli. Furthermore, on most F-theory bases such tuning of $G_{\rm SM}$ is forbidden due to the presence of rigid gauge groups (to be discussed shortly). Even if the tuning is available, it may not be compatible with moduli stabilization by fluxes and/or nonperturbative effects.

A more natural class of SM-like constructions in F-theory comes from rigid gauge groups such as E_7, E_6 [44, 45]. These are gauge groups enforced by strong curvature (to be more precise, very negative normal bundle) on the base, and are present throughout the whole branch of moduli space over that base, hence avoid the issue of tuning moduli. Moreover, statistical studies on (toric) F-theory bases have suggested that these rigid gauge groups are fairly common in the landscape. While the specific base naively associated with the most flux vacua [1] does not contain E_7 or E_6 factors, these gauge factors arise in a substantial fraction of F-theory base geometries enumerated by a simple counting measure (which may or may not distinguish bases related by a flop). The fraction of toric bases for 6D F-theory models that contain rigid E_7 and E_6 factors is more than 50% [46]. The statistics of E_7 and E_6 factors in threefold bases for 4D F-theory models is less well understood; one study found E_7 factors in $\sim 20\%$ of a limited simple of bases [22], and a more detailed analysis of the prevalence of such factors is currently underway [19]. Nonetheless, breaking these gauge groups to $G_{\rm SM}$ should give us a very large set of SM-like constructions. Recently in [9, 10], we have proposed a general class of SM-like models using rigid E_7, E_6 GUT groups in F-theory, with an intermediate SU(5) group. These models enjoy the advantages of being natural and involving little or no fine-tuning. Specifically, a combination of "vertical" and "remainder" fluxes can be used to break the rigid gauge groups in a way that is not transparent in the low-energy field theory, but gives the correct SM gauge group and some chiral matter. Although in many cases the breaking leads to exotic chiral matter, there are large families of models in which the correct SM chiral matter representations are obtained through an intermediate SU(5). The number of generations can easily be small and we have demonstrated that three generations are fairly likely in many of these models. In particular, a fully global explicit construction of such an E_6 model has been given in [10].

1.3 Overview of results

As discussed before, one might hope that there are string realizations of the Standard Model in which most or all of the features observed in nature arise in a relatively natural way. In this paper, we show that apart from the SM gauge group and chiral spectrum, several additional important SM features can be easily obtained in the E_7 (but not E_6) models, with some additional but mild tuning on the geometry and flux background. Specifically, we obtain the following features, each of which depends upon making choices with various extents of naturalness:

- As mentioned above, rigid E_7 factors are quite common in the F-theory landscape, and may be natural or likely depending on the proper vacuum measure. For generic (non-toric) bases, the E_7 gauge group can be broken down to $G_{\rm SM}$ by some natural choices of vertical and remainder fluxes.

- For the models with flux breaking of $E_7 \to G_{\rm SM}$, a set of approximate global U(1) symmetries descend from the E_7, leading to exponential suppression of certain couplings.

- With the $E_7 \to G_{\text{SM}}$ gauge-breaking fluxes, it appears to be fairly likely to have 3 generations of SM chiral matter (although 0, 1, or 2 generations may be more likely), and fairly likely that the exotic $(\mathbf{3},\mathbf{2})_{-5/6}$ representation is removed from the spectrum.

- Due to the use of E_7, there are always candidate Higgs sectors with a string theory origin different from that of chiral matter. Such a structure automatically leads to distinct dynamics between the Higgs and chiral matter, and gives rise to unsuppressed SM Yukawa couplings.

- Under this setup, dimension-4 and 5 proton decay is ubiquitously suppressed to phenomenologically safe levels.

The distinction between the Higgs and chiral matter, the appearance of the approximate global U(1) symmetries, and the ubiquitous suppression of dimension-4 and 5 proton decay are the strongest features of these constructions, in which desirable properties associated with observed physics arise essentially automatically. Most of the remaining features we explore generally require small amounts of discrete tuning. They may involve common, fairly likely or natural choices and do not arise automatically, but do not seem to require extensive fine-tuning.

- There is some automatic splitting between the doublet and triplet masses, although the amount of splitting and the exact masses are unknown.

- Although there are extra charged vector-like exotics in the spectrum, the Yukawa couplings between most of these fields (all besides the triplet Higgs) and the SM matter are exponentially suppressed through the above-mentioned approximate symmetries. We call these fields *inert* vector-like exotics.

- It is plausible that there is some hierarchy in the SM Yukawa couplings, but the exact values are unknown.

- With the setup so far, the model contains three right-handed neutrinos with masses lower than the string/GUT scale. It is plausible but not fully clear that the seesaw mechanism occurs.

To facilitate the discussions and calculations in this paper, we technically choose the gauge divisor to be a del Pezzo surface, and the matter curve to be a $\mathbb{P}^1$. Although we expect similar results for many other choices, these choices do lead to some qualitative simplifications in the analysis, and further work is needed to determine whether low-energy models with similar structure arise for a broader class of gauge divisors and matter curves, and/or to determine how natural or fine-tuned these geometric choices may be.

As an example, we work out an explicit global construction of the E_7 models that realizes all of the above SM features. We emphasize that many of these phenomenological advantages are specific for the E_7 models, and may be (much) harder to realize in other types of SM-like constructions in F-theory. Some of the above features are inherited from the group structure of E_7 itself, regardless of the string theory physics. To the authors' knowledge, however, these group theoretical features have not been noticed in the field theory literature, probably because E_7 itself does not support any chiral matter, if there is no additional input like fluxes from the UV.

While the E_7 models considered in this paper have quite a few phenomenological advantages over some other stringy realizations of the Standard Model, we note that these models potentially still suffer from the following issues, in light of which extra care must be taken when interpreting the results presented here. First, these models contain many vector-like exotics that cannot be removed by fluxes (except the most dangerous $(\mathbf{3},\mathbf{2})_{-5/6}$, which is fairly likely to be absent). In particular, these exotics include other copies of the Higgs field. From the effective field theory perspective, we generically expect these exotics to get heavy masses near the GUT/string scale such that they do not affect the low-energy phenomenology. On the other hand, this expectation in general may not be true in string theory, and it is important to develop further techniques to ensure the right masses. Although it may be possible, we do not see any reason in these models why one of the Higgs doublet pairs should get much lower masses than the other copies. In other words, there is no totally clear solution to the μ-problem in our setup. Next, these E_7 models have codimension-3 $(4, 6)$ singularities on the base, which correspond to an extra family of flux and may be associated to an extra sector of strongly coupled superconformal and chiral matter [47–51]. We can easily control the flux such that this sector is non-chiral, but since we understand very little about this sector, further studies are needed to ensure that this sector does not affect phenomenology.

1.4 Outline of paper

This paper is organized as follows. In section 2, we review fluxes in F-theory, which are the central tools in our construction of E_7 models. We first discuss the notion of vertical and remainder fluxes, and the constraints satisfied by these fluxes. We then describe how these fluxes break a nonabelian gauge group (known as "flux breaking"), determine the chiral spectrum, and tell us something about the vector-like spectrum. Despite the difficulty of computing the vector-like spectrum in general, the vector-like spectrum can be completely determined in some special cases. We discuss how our E_7 models easily fit into these cases, so that we can fully compute the matter spectrum in our models.

To initiate our discussions on semi-realistic E_7 GUTs in F-theory, we first review our previous work on the E_7 models [9, 10] in section 3. We describe the geometry and fluxes needed to get the SM gauge group and three generations of SM chiral matter from a rigid E_7.

In section 4, we discuss various phenomenological aspects of the E_7 models, namely the vector-like matter, Yukawa couplings, proton decay, the Higgs sector, the neutrino sector, and gauge coupling unification. One of our main tools is the Stückelberg mechanism [23, 52, 53] used in our flux breaking of E_7 to G_{SM}, which leaves several approximate U(1) global symmetries. We discuss these symmetries in detail and study how they constrain the couplings and mass terms[1] in the low-energy theory. These constraints, plus some additional tuning on the fluxes, lead to many of the above phenomenological advantages, especially the ubiquitous suppression of proton decay. We also discuss the vector-like matter that appears in the spectrum. We demonstrate how to easily remove the most dangerous $(\mathbf{3},\mathbf{2})_{-5/6}$ vector-like

[1]Throughout the paper, "mass terms" refer to the μ-term and other similar terms in the superpotential, which involve two different fields.

exotic, and discuss why most other vector-like exotics are *inert*. Although we cannot make any precise statements, we discuss various possible origins of the vector-like (including the Higgs) masses. Based on such discussions, we make some brief comments about the Higgs sector, the neutrino sector, and gauge coupling unification.

After describing the recipe of getting semi-realistic E_7 GUTs F-theory, in section 5 we write down an explicit global construction of a E_7 model that achieves all the above phenomenological features. This example demonstrates the fact that these features can indeed be obtained through some mild tuning on the geometry and the fluxes, but without the necessity of fine-tuning any moduli. Therefore, it is reasonable to regard these features as being natural in the string landscape. To emphasize various advantages and disadvantages of the E_7 models, in section 6 we briefly compare our models with other SM-like F-theory constructions in the literature. We finally conclude in section 7. In appendices A and B, we discuss several technical tools that are useful in the construction in section 5.

2 Fluxes in 4D F-theory models

In this section, we review vertical and remainder fluxes in 4D F-theory models, and how these fluxes determine the gauge group and matter spectrum. Except in section 2.4 on vector-like matter, all the content in this section has been discussed in depth in [10]. Here we only recap the essential facts for our construction of E_7 models and set up the notation. Interested readers can refer to [10] for more background information.

2.1 Vertical and remainder fluxes

To describe the flux backgrounds, we first need some basic geometric facts about the compactifications. As mentioned in section 1, we consider F-theory compactified on a CY fourfold Y, which is an elliptic fibration on a threefold base B. Nonabelian gauge groups arise when sufficiently high degrees of singularities are developed in the elliptic fibers over divisors on B (denoted by D_α), called gauge divisors Σ. When this happens, Y itself is also singular and we need to consider its resolution $\hat{Y}$ such that we can study cohomology and intersection theory. Let the total gauge group be G, where G has no U(1) factors before flux breaking. In the main part of this paper, we always study the simple case where $G = E_7$, but for generality we assume any simple Lie group G in this section. The nonabelian group G is supported on a gauge divisor Σ, and the resolution results in exceptional divisors $D_{1\leq i\leq \mathrm{rank}(G)}$ in $\hat{Y}$. For ADE (or simply-laced) groups, the intersection structure of these divisors matches the Dynkin diagram of G, where each exceptional divisor corresponds to a Dynkin node [54, 55]. By the Shioda-Tate-Wazir theorem [56, 57], the divisors D_I on $\hat{Y}$ are spanned by the zero section D_0 of the elliptic fibration, pullbacks of base divisors $\pi^* D_\alpha$ (which we also call D_α depending on context), and the exceptional divisors D_i.[2] Although the choice of resolution is not unique, our analysis and results are clearly resolution-independent [50].

Now we are ready to understand fluxes. These are most easily understood by considering the dual M-theory picture of the F-theory models, that is, M-theory compactified on the

[2]If G has U(1) factors, there are also divisors associated with these factors coming from the Mordell-Weil group of rational sections with nonzero rank.

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resolved fourfold $\hat{Y}$ (reviewed in [7]). In the M-theory perspective, fluxes are characterized by the three-form potential C_3 and its field strength $G_4 = dC_3$. The data of G_4 flux, which can be studied with well-established tools, is sufficient for constructing our E_7 models.

In general, G_4 is a discrete flux that takes values in the fourth cohomology $H^4(\hat{Y}, \mathbb{R})$. The quantization condition on G_4 is slightly subtle and is given by [58]

$$G_4 + \frac{1}{2} c_2(\hat{Y}) \in H^4(\hat{Y}, \mathbb{Z}) \,, \tag{2.1}$$

where $c_2(\hat{Y})$ is the second Chern class of $\hat{Y}$. In general (and particularly for E_7 models), $c_2(\hat{Y})$ can be odd (i.e., non-even), in which case the discrete quantization of G_4 contains a half-integer shift. When we construct an E_7 model explicitly in section 5, we will make use of an odd $c_2(\hat{Y})$ and half-integer fluxes. More details will be discussed in that section.

Next, to preserve the minimal amount of SUSY in 4D, G_4 must live in the middle cohomology i.e. $G_4 \in H^{2,2}(\hat{Y}, \mathbb{R}) \cap H^4(\hat{Y}, \mathbb{Z}/2)$. Supersymmetry also imposes the condition of primitivity [59, 60]:

$$J \wedge G_4 = 0 \,, \tag{2.2}$$

where J is the Kähler form of $\hat{Y}$. This is automatically satisfied when the geometric gauge group is not broken, but when the gauge group is broken by *vertical* flux (to be discussed below), this condition stabilizes some (but not all) Kähler moduli; stabilizing these moduli within the Kähler cone imposes additional flux constraints.

We also have the D3-tadpole condition [61] that must be satisfied for a consistent vacuum solution:

$$\frac{\chi(\hat{Y})}{24} - \frac{1}{2} \int_{\hat{Y}} G_4 \wedge G_4 = N_{D3} \in \mathbb{Z}_{\geq 0} \,, \tag{2.3}$$

where $\chi(\hat{Y})$ is the Euler characteristic of $\hat{Y}$, and N_{D3} is the number of D3-branes, or M2-branes in the dual M-theory. To preserve SUSY and stability, we forbid the presence of anti-D3-branes i.e. $N_{D3} \geq 0$. The integrality of N_{D3} is guaranteed by eq. (2.1). This condition has an immediate consequence on the sizes of fluxes. Since in general $h^{2,2} > 2\chi/3 \gg \chi/24$, if we randomly turn on flux in the whole middle cohomology such that the tadpole constraint is satisfied, a generic flux configuration vanishes or has small magnitude in most of the $h^{2,2}$ independent directions. In this sense, the tadpole contributed by fluxes along some particular directions can be treated as a rough estimate on the amount of fine-tuning on fluxes. We leave a more precise and detailed analysis of these considerations to future work.

We now consider the orthogonal decomposition of the middle cohomology [39]:

$$H^4(\hat{Y}, \mathbb{C}) = H^4_{\text{hor}}(\hat{Y}, \mathbb{C}) \oplus H^{2,2}_{\text{vert}}(\hat{Y}, \mathbb{C}) \oplus H^{2,2}_{\text{rem}}(\hat{Y}, \mathbb{C}) \,. \tag{2.4}$$

The horizontal subspace comes from the complex structure variation of the holomorphic 4-form Ω, while the vertical subspace is spanned by products of harmonic $(1,1)$-forms (which are Poincaré dual to divisors, denoted by $[D_I]$)

$$H^{2,2}_{\text{vert}}(\hat{Y}, \mathbb{C}) = \text{span}\left(H^{1,1}(\hat{Y}, \mathbb{C}) \wedge H^{1,1}(\hat{Y}, \mathbb{C})\right) \,. \tag{2.5}$$

Components that do not belong to the horizontal or vertical subspaces are referred to as remainder flux. In the construction here we use a specific type of remainder flux. Consider a curve $C_{\rm rem} \in H_{1,1}(\Sigma, \mathbb{Z})$ in Σ, such that its pushforward $\iota_* C_{\rm rem} \in H_{1,1}(B, \mathbb{Z})$ is trivial, where $\iota : \Sigma \to B$ is the inclusion map. While such a curve cannot be realized on toric divisors on toric bases, it has been suggested that such curves do exist on "typical" bases [39], so that toric geometry may be insufficiently generic for this class of constructions; understanding this question of typicality is an important problem for further study. In any case, we now restrict each D_i (considered as a fibration over Σ) onto $C_{\rm rem}$, giving a surface in $\hat{Y}$. Its Poincaré dual (in $\hat{Y}$) $[D_i|_{C_{\rm rem}}]$ is a $(2,2)$-form, but since $C_{\rm rem}$ cannot be obtained by intersections of base divisors, we must have

$$[D_i|_{C_{\rm rem}}] \in H^{2,2}_{\rm rem}(\hat{Y}, \mathbb{C})\,. \tag{2.6}$$

Here we explain more about vertical flux (denoted by $G_4^{\rm vert}$), as there are more constraints specifically on vertical flux such that G_4 dualizes to a consistent F-theory background that preserves Poincaré invariance. Combining eqs. (2.5) and (2.1) gives the integral vertical subspace $H^{2,2}_{\rm vert}(\hat{Y}, \mathbb{R}) \cap H^4(\hat{Y}, \mathbb{Z})$.[3] We focus primarily here on the vertical subspace spanned by integer multiples of forms $[D_I] \wedge [D_J]$

$$H^{2,2}_{\rm vert}(\hat{Y}, \mathbb{Z}) := {\rm span}_{\mathbb{Z}} \left(H^{1,1}(\hat{Y}, \mathbb{Z}) \wedge H^{1,1}(\hat{Y}, \mathbb{Z}) \right)\,. \tag{2.7}$$

While this subspace does not necessarily include all lattice points in the full vertical cohomology $H^{2,2}_{\rm vert}(\hat{Y}, \mathbb{C}) \cap H^4(\hat{Y}, \mathbb{Z})$ of the same dimension, this subspace is sufficient for us to construct the E_7 models, and we leave the analysis of the full vertical subspace to future work.

Now we set up some notations for vertical flux. We expand

$$G_4^{\rm vert} = \phi_{IJ} [D_I] \wedge [D_J]\,, \tag{2.8}$$

and work with integer (or half-integer if c_2 is odd) flux parameters ϕ_{IJ}. We denote the integrated flux as [62]

$$\Theta_{\Lambda\Gamma} = \int_{\hat{Y}} G_4 \wedge [\Lambda] \wedge [\Gamma]\,, \tag{2.9}$$

where Λ, Γ are arbitrary linear combinations of D_I; subscripts $0, i, \alpha, \ldots$ refer to the basis divisors $D_0, D_i, D_\alpha, \ldots$. In this paper, we use the following resolution-independent formula to relate $\Theta_{i\alpha}$ to $\phi_{i\alpha}$ [50, 63]:[4]

$$\Theta_{i\alpha} = -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_\beta \phi_{j\beta}\,, \tag{2.10}$$

where κ^{ij} is the inverse Killing metric of G, and "dots" denote the intersection product. In the E_7 models, the only vertical flux parameters we turn on have indices of type $\phi_{i\alpha}$; although it is also possible to turn on nontrivial ϕ_{ij}, we always turn them off for reasons to be discussed below.

[3] We remind readers that the quantity $G_4^{\rm vert} + c_2(\hat{Y})/2$ instead of $G_4^{\rm vert}$ lives in this subspace. Note that $c_2(\hat{Y})$ is always vertical.

[4] Indices appearing twice are summed over, while other summations are indicated explicitly.

Now we write down the extra constraints for vertical flux. To preserve Poincaré symmetry after dualizing, we require [64]

$$\Theta_{0\alpha} = \Theta_{\alpha\beta} = 0\,. \tag{2.11}$$

If the whole geometric gauge symmetry is preserved, a necessary condition is that

$$\Theta_{i\alpha} = 0\,, \tag{2.12}$$

for all i, α, otherwise flux breaking occurs. This condition is not sufficient when there is nontrivial remainder flux, which will be discussed more in section 2.2. When flux breaking occurs, i.e. eq. (2.12) is violated, the condition (2.11) for Poincaré symmetry is unchanged, but there are extra constraints from primitivity, which are demonstrated in later sections.

Much of the above discussion on vertical flux extends naturally to the type of remainder flux G_4^{rem} we need. Similarly, we expand

$$G_4^{\text{rem}} = \phi_{ir} \left[D_i|_{C_{\text{rem}}} \right] , \tag{2.13}$$

and work with integer ϕ_{ir}. In this paper, we always turn on remainder flux with a single C_{rem} only, so we do not specify the choice of C_{rem} in the flux parameters; instead we just label them by "r". Eq. (2.10) straightforwardly generalizes to remainder flux by replacing the triple intersection on the base with the intersection of two C_{rem}'s on Σ.

2.2 Flux breaking

With the knowledge of vertical and remainder fluxes, we now describe the breaking of geometric gauge groups with these fluxes, a.k.a. flux breaking. This kind of breaking has been used as early as [24] (see also [7]), and is recently developed in depth in [10]. In this paper, we only list the results essential for our analysis on the E_7 models, and we refer readers to [10] for full technical details.

Let us first study vertical flux. Recall that we need $\Theta_{i\alpha} = 0$ for all i, α to preserve the whole G. Now we break G into a smaller group G' by turning on some nonzero $\phi_{i\alpha}$. Such flux breaks some of the roots in G. It also induces masses for some Cartan gauge bosons by the Stückelberg mechanism [52, 53], hence breaks some combinations of Cartan U(1)'s in G. Let α_i be the simple roots of G, and T_i be the Cartan generators associated with α_i i.e. in the co-root basis. The root $b_i\alpha_i$ is preserved under the breaking if

$$\sum_i b_i \langle \alpha_i, \alpha_i \rangle \, \Theta_{i\alpha} = 0\,, \tag{2.14}$$

for all α. Here $\langle ., . \rangle$ denotes the inner product of root vectors. Moreover, the corresponding linear combination of Cartan generators

$$\sum_i b_i \langle \alpha_i, \alpha_i \rangle \, T_i\,, \tag{2.15}$$

is preserved. These generators form a nonabelian gauge group $G' \subset G$ after breaking. Note that for ADE groups like E_7, $\langle \alpha_i, \alpha_i \rangle$ is the same for all i, and we will drop this factor in the above conditions. Below we will use a simple version of the breaking: we turn on $\Theta_{i'\alpha} \neq 0$ for some set of Dynkin indices $i' \in I'$ and some α, in a generic way such that eq. (2.14) is satisfied only when $b_{i'} = 0$ for all $i' \in I'$. Then G' is given by removing the corresponding nodes in the Dynkin diagram of G. The simple roots of G' are directly descended from G and are given by $\alpha_{i \notin I'}$. In the E_7 models, one can show that all flux breaking routes to G_{SM} (or SU(5) before including remainder flux) are related to this simple version of breaking by automorphisms.

There are additional constraints on vertical flux breaking coming from primitivity, since eq. (2.2) is not automatically satisfied when there is vertical flux breaking and $\Theta_{i\alpha} \neq 0$ for some i, α. To understand these constraints, we consider the F-theory limit where the fibers shrink to zero volume. Hence we can expand J of $\hat{Y}$ as $J \to \pi^* J_B = t^\alpha [D_\alpha]$, where the Kähler moduli t^α are restricted to the positive Kähler cone. Now primitivity requires that

$$\int_{\hat{Y}} [D_i] \wedge J \wedge G_4 = t^\alpha \Theta_{i\alpha} = 0 \,, \tag{2.16}$$

which is true only for specific choices of J when there is vertical flux breaking. The condition of primitivity then stabilizes some but not all Kähler moduli in J; consistently stabilizing these moduli within the Kähler cone imposes additional flux constraints.

As discussed in [10], the above flux constraints lead to an important necessary condition for consistent vertical flux breaking: let r be the number of linearly independent D_α's appearing in the set of all homologically independent surfaces in the form of $S_{i\alpha} = D_i \cdot D_\alpha$ (for any i of the given G). Now consider the $(r \times \mathrm{rank}(G))$ matrix $\Theta_{(\alpha_a)(i)}$ (where a and i are the indices for rows and columns respectively). The condition (2.16) asserts that $t^\alpha \Theta_{\alpha i} = 0$. Since the solution to primitivity thus requires a nontrivial left null space of the matrix, the rank of the matrix is at most $r - 1$. Moreover from eq. (2.14), the rank of the matrix is also the change in rank of G during flux breaking. Therefore, we require

$$r \geq \mathrm{rank}(G) - \mathrm{rank}(G') + 1 \,. \tag{2.17}$$

Note that when remainder flux breaking is not available, and all divisors in Σ descend from intersections in B, we have $r = h^{1,1}(\Sigma)$. In the E_7 models, however, we require the presence of remainder flux and r is smaller than $h^{1,1}(\Sigma)$. This condition means that we must have a sufficiently large r in order to get a desired G', hence imposing constraints on the possible geometries that support a given vertical flux breaking.

So far we have focused on the nonabelian part of the broken gauge group, but there may also be U(1) factors in the broken gauge group like G_{SM}. In the formalism of flux breaking, there are two ways to get U(1) factors: for U(1) factors not along any roots, we can use vertical flux to get these factors using the Stückelberg mechanism; for a recent application see [65]. On the other hand, U(1) factors like the hypercharge in G_{SM} are along some roots of a higher gauge group (SU(5) in the case of hypercharge), and remainder flux is necessary for obtaining these factors. Therefore, we proceed as follows: if we turn on

$$G_4^{\mathrm{rem}} = \phi_{ir} [D_i|_{C_{\mathrm{rem}}}] \,, \tag{2.18}$$

for some C_{rem} satisfying the property mentioned in section 2.1, G is broken into the commutant of $T = \phi_{ir} T_i$ within G. The difference is that the remainder flux does not turn on any $\Theta_{i\alpha}$, so

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there is no Stückelberg mechanism and all the U(1) factors in the commutant are preserved. In other words, breaking using remainder flux never decreases the rank of the gauge group, while breaking using vertical flux always decreases the rank. Note that when G is a rigid gauge group, Σ is a rigid divisor and supports remainder flux breaking only when embedded into a non-toric base. This follows because for a toric base B, toric divisors span the cone of effective divisors, so any rigid effective divisor Σ is toric, and toric curves in a toric Σ span $h^{1,1}(\Sigma)$.

2.3 Chiral matter

It is well known that vertical flux induces some chiral matter, and the same is true for vertical flux that breaks the gauge group. Perhaps more strikingly, even if the unbroken gauge group G does not have any complex representations and does not support chiral matter, there may still be some chiral matter after *vertical* flux breaking [10]. The famous index formula states that for a weight β in representation R, its chiral index χ_β is [30, 62, 66, 67]

$$\chi_\beta = \int_{S(\beta)} G_4^{\text{vert}} \,, \tag{2.19}$$

where $S(\beta)$ is called the matter surface of β. When R is localized on a matter curve C_R, $S(\beta)$ is the fibration of the blowup $\mathbb{P}^1$ corresponding to β over C_R. Here we recall that a matter curve is a curve on Σ where the fiber singularity is enhanced, resulting in additional fibral curves in the resolution, which corresponds to matter multiplets in the 4D theory.

Since weights differ by roots, given a weight β in R of G, it is useful to expand $\beta = -b_i\alpha_i$. Hence we can decompose its matter surface $S(\beta)$ as [7]

$$S(\beta) = S_0(R) + b_i \; D_i|_{C_R} \,, \tag{2.20}$$

where S_0 only depends on R but not β. $S_0(R)$ corresponds to the flux that gives chiral matter without breaking G, when G supports chiral matter. In contrast, when G itself does not support chiral matter as in the E_7 models, $S_0(R)$ is trivial in homology. From now on we will ignore $S_0(R)$ and focus on the second term of eq. (2.20). Matter curves in general can be written as $C_R = \Sigma \cdot D_R$ for some divisor D_R. Then,

$$\int_{S(\beta)} G_4^{\text{vert}} = b_i \Theta_{iD_R} \,. \tag{2.21}$$

We can replace the i summation with $i' \in I'$ since the other terms vanish. When G is broken to G', R decomposes into different irreducible representations R' in G', which can be labelled by $b_{i'}$. In general, different $b_{i'}$ and different R can give rise to the same irreducible representation R'. The total chiral index $\chi_{R'}$ for R' is then

$$\chi_{R'} = \sum_R \sum_{b_{i'}} b_{i'} \Theta_{i' D_R} \,. \tag{2.22}$$

Note that this expression is nontrivial even when R is not complex.

There are also adjoint chiral multiplets (apart from the vector multiplet of the gauge fields) living on the bulk of Σ, and matter curves or surfaces for this representation are not well-defined. Nevertheless, it has been shown that adjoint matter can also become chiral after flux breaking, and the chiral indices are given by setting $S_0(\text{Adj}) = 0$ and replacing C_R by K_Σ [68]. Here K denotes the canonical class. By the adjunction formula, $K_\Sigma = \Sigma \cdot (K_B + \Sigma)$ and we should set $D_R = K_B + \Sigma$.

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2.4 Vector-like matter

To fully understand the phenomenology of these models, it is important to study the vector-like spectrum in addition to the chiral spectrum. One of the main reasons for this is that in a realistic model we need to realize the Higgs sector, while avoiding dangerous vector-like exotics. While the techniques for computing the chiral spectrum are already at hand as above, computing the vector-like spectrum in general requires not only the G_4 flux, but the full information of C_3 in terms of line bundles on Σ and C.[5] In many cases, these things are hard to compute and some of the relevant technology has only been developed fairly recently [11–14]. Fortunately, our models admit several important simplifications such that the G_4 flux itself already determines the vector-like spectrum.

Let us first focus on vector-like matter that lives on the bulk of Σ. We follow the formalism in [68, 69]. At least in most cases, the full C_3 can be captured by an algebraic complex 2-cycle $\mathcal{A}$ in the Chow group $\mathrm{CH}^2(\hat{Y})$ (algebraic cycles modulo rational equivalence instead of homological equivalence), with homology class is $[G_4]$ [66]. We consider the restriction of $\mathcal{A}$ onto a Cartan divisor D_i, given by the intersection product $\mathcal{A} \cdot D_i \in \mathrm{CH}^2(D_i)$. Its projection onto Σ, given by $\pi_*(\mathcal{A} \cdot D_i) \in \mathrm{CH}^1(\Sigma)$, is a curve on Σ associated with the line bundle

$$L_i = \mathcal{O}_\Sigma\left(\pi_*\left(\mathcal{A} \cdot D_i\right)\right) . \tag{2.23}$$

Now for each weight $\beta = -b_i\alpha_i$ of the adjoint, we define the line bundle

$$L_\beta = \otimes_i L_i^{b_i} . \tag{2.24}$$

Then the chiral and anti-chiral multiplicities for β are counted by the following sheaf cohomologies [23, 24]

$$\text{chiral}: \quad H^0\left(\Sigma, L_\beta \otimes K_\Sigma\right) \oplus H^1\left(\Sigma, L_\beta\right) \oplus H^2\left(\Sigma, L_\beta \otimes K_\Sigma\right) , \tag{2.25}$$

$$\text{anti}-\text{chiral}: \quad H^0\left(\Sigma, L_\beta\right) \oplus H^1\left(\Sigma, L_\beta \otimes K_\Sigma\right) \oplus H^2\left(\Sigma, L_\beta\right) . \tag{2.26}$$

Notice that the sheaf cohomologies for chiral and anti-chiral matter are related by Serre duality. To calculate their dimensions, we apply the following two simplifications [24]. First, for fluxes with *nontrivial* L_β and satisfying primitivity (i.e. preserving SUSY), we have $H^0(\Sigma, L_\beta) = H^2(\Sigma, L_\beta \otimes K_\Sigma) = 0$. Next, we assume that Σ is a rational surface with effective $-K_\Sigma$,[6] in this case, we have $H^2(\Sigma, L_\beta) = H^0(\Sigma, L_\beta \otimes K_\Sigma) = 0$. Therefore, the exact multiplicities n_β and $n_{-\beta}$ are fully determined by $h^1(\Sigma, L_\beta)$ and $h^1(\Sigma, L_\beta \otimes K_\Sigma)$ respectively. Since only H^1 is nontrivial, the multiplicities are also captured by the topological Euler characteristics $\chi(\Sigma, L_\beta)$ and $\chi(\Sigma, L_\beta \otimes K_\Sigma)$. These are fully determined by $c_1(L_\beta)$,

[5] In general, these are described by sheaves when there are more severe singularities on Σ and/or C. In this paper, we only consider completely smooth geometry on the bases, so the description by line bundles is sufficient.

[6] As shown below, the condition that $-K_\Sigma$ is effective is a reasonable simplifying assumption in the context of rigid gauge groups. All toric surfaces are rational and have effective $-K_\Sigma$. We assume these conditions on Σ in the rest of the paper; in much of the paper we restrict attention to the special case where Σ is a (generally non-toric) del Pezzo surface. Further work would be needed to understand the detailed structure of the resulting models when these technical conditions are relaxed.

given by the Hirzebruch-Riemann-Roch theorem: [23, 24, 70]

$$\begin{aligned} n_\beta &= -\chi\left(\Sigma, L_\beta\right) \\ &= \frac{1}{2}\left[c_1\left(L_\beta\right)\right]\cdot K_\Sigma - \frac{1}{2}\left[c_1\left(L_\beta\right)\right]^2 - 1 \\ &= \frac{1}{2}\chi_\beta - \frac{1}{2}\left[c_1\left(L_\beta\right)\right]^2 - 1\,, \end{aligned} \tag{2.27}$$

where χ_β is the chiral index for β given in section 2.2, and the Poincaré dual is taken with respect to Σ. The expression for $n_{-\beta}$ is the same except that the sign of the first term is flipped; indeed we get back $n_\beta - n_{-\beta} = \chi_\beta$. In our models where only gauge-breaking fluxes are turned on, we can read off $c_1\left(L_\beta\right)$ from eq. (2.10):

$$\left[c_1\left(L_\beta\right)\right] = -b_i \kappa^{ij}\left(\phi_{j\alpha}\Sigma\cdot D_\alpha + \phi_{jr}C_{\mathrm{rem}}\right)\,. \tag{2.28}$$

Combining eqs. (2.27) and (2.28), this gives us a formula to compute the exact matter multiplicities from the bulk of Σ in terms of the vertical and remainder flux parameters.

Now we turn to vector-like matter localized on matter curves. Similarly for a weight $\beta \in R$ supported on C_R, we consider the pullback of $\mathcal{A}$ onto a matter surface $S(\beta)$ given by $\mathcal{A}\cdot S(\beta) \in \mathrm{CH}^2\left(S(\beta)\right)$. Its projection onto C_R, given by $\pi_*(A\cdot S(\beta)) \in \mathrm{CH}^1\left(C_R\right)$ defines a line bundle for β:

$$L_\beta = \mathcal{O}_{C_R}\left(\pi_*\left(\mathcal{A}\cdot S(\beta)\right)\right)\,. \tag{2.29}$$

Then the chiral and anti-chiral multiplicities for β are counted by the following sheaf cohomologies

$$\text{chiral}: \quad H^0\left(C_R, L_\beta \otimes \sqrt{K_{C_R}}\right)\,, \tag{2.30}$$

$$\text{anti}-\text{chiral}: \quad H^1\left(C_R, L_\beta \otimes \sqrt{K_{C_R}}\right)\,, \tag{2.31}$$

where $\sqrt{K_{C_R}}$ is the spin bundle on C_R. These sheaf cohomologies are more subtle than those for the bulk of Σ. While they are well understood when the matter curve has genus 0 or 1, for irreducible curves with higher genus, these cohomologies have complicated dependence on moduli, and their dimensions can jump at special points in the moduli space. For reducible curves, there can also be vector-like pairs between different irreducible components of the curves, if the total chiral index is split into different components accordingly. Instead of running into all these subtleties, below we just focus on a special case where the matter curve is simply a $\mathbb{P}^1$. In this case, there can never be any vector-like pairs from the matter curve, since for $\mathbb{P}^1$ only one of the H^0, H^1 is nontrivial, depending on the sign of the line bundle. This means in particular that the vector-like spectrum is independent of the choice of spin bundle, significantly simplifying the analysis. As shown below, this geometry is not hard to achieve in our E_7 models, and we leave the generalizations to more complicated matter curves in future work.

So far, we have discussed the vector-like multiplicities for each weight separately. On the other hand, we recall that weights from different $b_{i'}$ and R can contribute to the same chiral R' in eq. (2.22) during flux breaking. Similarly, these different weights can form

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vector-like pairs after flux breaking, even if each weight is purely chiral. This effect can occur on both matter curves and the bulk of Σ. Such vector-like matter has qualitatively different behavior from that obtained from sheaf cohomologies, and has interesting phenomenological implications. More details will be discussed in later sections.

3 Review of E_7 GUTs in F-theory

With the above background knowledge, now we are ready to describe the E_7 models. For completeness, first we briefly review our previous work [9, 10] on E_7 models, namely describing how the SM gauge group and chiral spectrum can be realized in a natural way through flux breaking of a rigid E_7 factor. We refer readers to those two papers for more details.

3.1 Flux breaking of rigid E_7 factors

Recall that gauge groups in F-theory arise from sufficiently high degrees of fiber singularities on a gauge divisor Σ. For an E_7 gauge group, the (singular) elliptic CY fourfold Y is described by a certain form of Weierstrass model [54, 55, 71]. Treating the elliptic curve as the CY hypersurface in $\mathbb{P}^{2,3,1}$ with homogeneous coordinates $[x : y : z]$, Y is given by the locus of

$$y^2 = x^3 + s^3 f_3 x z^4 + s^5 g_5 z^6 \,, \tag{3.1}$$

where s, f_3, g_5 are sections of line bundles $\mathcal{O}(\Sigma), \mathcal{O}(-4K_B - 3\Sigma), \mathcal{O}(-6K_B - 5\Sigma)$ on the base B, and the gauge divisor Σ supporting the E_7 factor is given by $s = 0$. There is adjoint matter **133** arising from excitations localized around the bulk of Σ. There is also fundamental matter **56** localized on the curve $s = f_3 = 0$, or $C_{\mathbf{56}} = -\Sigma \cdot (4K_B + 3\Sigma)$ in terms of the intersection product, when the curve is nontrivial in homology. When Σ has a sufficiently negative normal bundle N_Σ, singularities of the elliptic fibration are enforced on Σ, and the Weierstrass model for Y is automatically restricted to the form (3.1). A rigid E_7 is then realized on Σ. To be precise, we can consider the following divisors on Σ (not on B) [45]:

$$\begin{aligned} F_k &= -4K_\Sigma + (4-k)N_\Sigma \,, \\ G_l &= -6K_\Sigma + (6-l)N_\Sigma \,, \end{aligned} \tag{3.2}$$

where k, l are integers. Then there is a rigid E_7 on Σ if F_k, G_l are effective for $k \geq 3, l \geq 5$ only. A simple way to satisfy this condition is to consider effective $-K_\Sigma$ and $-N_\Sigma$, such that $-3K_\Sigma + N_\Sigma$ is not effective but $-4K_\Sigma + N_\Sigma$ is effective. As discussed previously, the natural choice of effective $-K_\Sigma$ matches nicely with the simplification we made in section 2.4 for computing vector-like spectrum, and we assume that this condition holds.

After setting up the geometry, we now turn on the flux background. We break E_7 to G_{SM} in two steps. Since remainder flux preserves U(1)'s along the roots but vertical flux does not, we first break E_7 to SU(5) with vertical flux, then break SU(5) to G_{SM} with remainder flux. The latter flux is very similar to the hypercharge flux in traditional SU(5) GUTs in F-theory. To perform the first step of breaking, we turn on nonzero $\Theta_{i'\alpha}$ for some α and $i' = 4, 5, 6$ subject to the flux constraints listed in section 2.1, see figure 1. In terms of flux parameters $\phi_{i\alpha}$, we turn on

$$\phi_{1\alpha} = 2n_\alpha \,, \quad \phi_{2\alpha} = 4n_\alpha \,, \quad \phi_{3\alpha} = 6n_\alpha \,, \quad \phi_{4\alpha} = 5n_\alpha \,, \quad \phi_{7\alpha} = 3n_\alpha \,. \tag{3.3}$$

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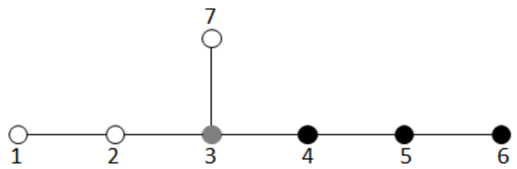

Figure 1. The Dynkin diagram of E_7. The Dynkin node labelled i corresponds to the exceptional divisor D_i. The solid nodes are the ones we break to get the Standard Model gauge group and chiral matter. Node 3 (in gray) is broken by remainder flux while the others are broken by vertical flux.

The values of $\phi_{5\alpha}, \phi_{6\alpha}$, if sufficiently generic, do not affect the gauge group, but they will be fixed by other flux and phenomenological constraints. Here we define a new set of flux parameters n_α, which can be integers or half-integers depending on the parity of $c_2(\hat{Y})$. At this point the gauge group has been broken to SU(5). To perform the second step of breaking, we similarly turn on the remainder flux

$$\phi_{1r} = 2n_r\,, \quad \phi_{2r} = 4n_r\,, \quad \phi_{3r} = 6n_r\,, \quad \phi_{7r} = 3n_r\,, \tag{3.4}$$

and $\phi_{4r}, \phi_{5r}, \phi_{6r}$ plays the same role as $\phi_{5\alpha}, \phi_{6\alpha}$. Here n_r is always integer. Under the construction of rigid E_7, we require a non-toric base B to ensure the existence of C_{rem}, hence this remainder flux. After the remainder flux breaking, the remaining unbroken gauge group is

$$G_{\text{SM}} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)/\mathbb{Z}_6\,. \tag{3.5}$$

3.2 Chiral spectrum in flux-broken E_7 models

It is straightforward to calculate the chiral spectrum given the above fluxes. Since only the vertical flux induces chiral matter, we can analyze the matter content by breaking $E_7 \to \text{SU}(5)$, where the **56** breaks into a combination of **5**, **10**, uncharged singlets and conjugate representations, and **133** includes these as well as the adjoint **24**. Since the adjoint of SU(5) is non-chiral, the only chiral representations we expect for G_{SM} after the whole breaking are the Standard Model representations, which descend from the **5**, **10** of SU(5),

$$Q = (\mathbf{3},\mathbf{2})_{1/6}\,, \quad \bar{U} = (\bar{\mathbf{3}},\mathbf{1})_{-2/3}\,, \quad \bar{D} = (\bar{\mathbf{3}},\mathbf{1})_{1/3}\,, \quad L = (\mathbf{1},\mathbf{2})_{-1/2}\,, \quad \bar{E} = (\mathbf{1},\mathbf{1})_1\,. \tag{3.6}$$

Using eq. (2.22), we indeed get the anomaly-free combination of SM chiral matter from vertical flux. It will be useful to separate the contributions from **56** and **133** to the total chiral index, i.e. $\chi_{(\mathbf{3},\mathbf{2})_{1/6}} = \chi^{\mathbf{56}}_{(\mathbf{3},\mathbf{2})_{1/6}} + \chi^{\mathbf{133}}_{(\mathbf{3},\mathbf{2})_{1/6}}$, where each contribution is anomaly-free by itself. The fundamental **56** gives

$$\chi^{\mathbf{56}}_{(\mathbf{3},\mathbf{2})_{1/6}} = \Sigma \cdot (4K_B + 3\Sigma) \cdot D_\alpha n_\alpha\,, \tag{3.7}$$

and the adjoint **133** gives

$$\chi^{\mathbf{133}}_{(\mathbf{3},\mathbf{2})_{1/6}} = 2\Sigma \cdot (K_B + \Sigma) \cdot D_\alpha n_\alpha\,. \tag{3.8}$$

Note that the total chiral indices only depend on n_α but not $\phi_{5\alpha}, \phi_{6\alpha}$. This is no longer true when we look at the chiral indices for each weight in the phenomenological analysis below. An important feature of these chiral indices is that they have a linear Diophantine structure in the quantized flux parameters, with the coefficients not being very large. If we randomly pick some small values of n_α (bounded by the tadpole constraint (2.3)), generically different terms in the chiral indices will cancel each other, resulting in small chiral indices. Therefore, small chiral indices are preferred in these models, and it is not hard to achieve three generations of SM chiral matter.

In most cases, the above Weierstrass model also has codimension-3 singularities at the locus $s = f_3 = g_5 = 0$. Traditionally, codimension-3 singularities are interpreted as Yukawa couplings in the low-energy theory. In E_7 models, however, these are so-called non-minimal singularities (with degrees $(4, 6)$ or higher) where the fiber becomes non-flat, i.e. its dimension jumps. Such singularities can no longer be interpreted as Yukawa couplings;[7] this is also manifest by noticing that $\mathbf{56}^3$ does not contain any singlets, hence cannot form any gauge-invariant couplings. Instead, there is an extra family of vertical flux associating to the non-flat fiber with nontrivial ϕ_{ij} components [50]. Analogous to codimension-2 $(4, 6)$ singularities in 6D F-theory models [72, 73], there has been evidence that this flux switches on an extra sector of strongly coupled superconformal *chiral* matter, given by M2-branes wrapping curves on the non-flat fiber [51]. For our phenomenological purpose, we can always set this flux to zero i.e. $\phi_{ij} = 0$ for all i, j, such that the extra sector becomes non-chiral and probably does not affect the Standard Model sector. We should warn readers, however, that without further studies on these extra sectors, we cannot precisely rule out the possibility that these sectors ruin the desired phenomenology.

4 Phenomenology of E_7 GUTs

So far, we have studied the gauge group and the chiral spectrum in the above class of E_7 models. In this section, we start to analyze the phenomenological aspects of these models in more detail. The presence of approximate global symmetries descending from the underlying E_7 group suppresses certain couplings, with significant implications for phenomenology of these models; in particular, we show that proton decay is automatically suppressed. We focus further on the Higgs sector and the interactions in these models. We also discuss vector-like exotics in these models and the circumstances under which they can be phenomenologically safe. We consider the extent to which the various features of the E_7 flux-broken models possibly, or even naturally, match with observed phenomenology. The analysis in this section, together with an explicit construction of an example model in section 5, are the main results of this paper. These results provide evidence that natural and realistic E_7 GUTs can be realized in F-theory. On the other hand, due to limits on existing technologies for computing detailed aspects of F-theory models (such as the specific values of couplings), most of the analysis in this section is purely qualitative.

[7]It was pointed out in [49] that these singularities may give rise to quartic or higher order couplings. Nevertheless, the singularities in [49] have an unusual local geometry where a curve intersects another curve three times. We do not see any such intersections or any evidence of such higher order couplings in our models.

4.1 Approximate global symmetries

Our starting point is based on considering approximate U(1) global symmetries that arise in the E_7 flux-broken models. These approximate symmetries directly originate from vertical flux breaking, and control the structure of interactions and mass terms in the low-energy theory, as well as leading to many of the phenomenologically attractive features of these models such as suppression of proton decay. Recall that during vertical flux breaking from E_7 to $G_{\rm SM}$, the Cartan gauge bosons along the generators T_4, T_5, T_6 get masses from the Stückelberg mechanism [23, 52, 53]. These masses explicitly break the corresponding Cartan U(1)'s of the nonabelian gauge symmetry, but the matter interactions descending from the unbroken gauge group still respect the global parts of the broken U(1) gauge symmetries (at least for the symmetries without mixed anomalies with the remaining gauge group). These U(1) global symmetries are broken only by D3/M5-instanton effects, which are exponentially suppressed in Kähler moduli [74–77]. In the low-energy theory, these effects turn on exponentially suppressed mass terms and interactions that violate the global symmetries. Since these effects are small, the symmetries still remain as approximate global symmetries in the theory. This scenario is consistent with the No Global Symmetries Conjecture [78, 79]. Note that without more details of the model, we cannot quantitatively specify the extent to which a certain quantity is suppressed by these symmetries. Throughout this paper, we only adopt the qualitative picture of exponential suppression, and leave efforts towards explicit calculations of these quantities for future work.

To study the implications of these U(1) symmetries, it is important to understand the branching rules from E_7 to $G_{\rm SM}$ in the presence of these additional U(1) charges. Below we use the basis (Y, b_4, b_5, b_6) for the U(1) charges, where Y is the SM hypercharge.[8] The branching rules are

$$\begin{aligned}\mathbf{56}\to\,&(\mathbf{1},\mathbf{1})_{0,5/2,2,3/2}+(\mathbf{1},\mathbf{1})_{0,5/2,2,1/2}+(\mathbf{1},\mathbf{1})_{0,5/2,1,1/2}+(\mathbf{1},\mathbf{1})_{1,3/2,1,1/2}\\&+(\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2}\\&+(\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2}+(\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2}+(\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2}+(\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2}\\&+\text{conjugates}\,,\end{aligned}\tag{4.1}$$

$$\begin{aligned}\mathbf{133}\to\,&(\mathbf{8},\mathbf{1})_{0,0,0,0}+(\mathbf{1},\mathbf{3})_{0,0,0,0}+4\times(\mathbf{1},\mathbf{1})_{0,0,0,0}\\&+\Big[(\mathbf{1},\mathbf{1})_{0,0,0,1}+(\mathbf{1},\mathbf{1})_{0,0,1,0}+(\mathbf{1},\mathbf{1})_{0,0,1,1}+(\mathbf{1},\mathbf{1})_{1,-1,0,0}+(\mathbf{1},\mathbf{1})_{1,-1,-1,0}\\&\quad+(\mathbf{1},\mathbf{1})_{1,-1,-1,-1}+(\mathbf{3},\mathbf{2})_{-5/6,0,0,0}+(\mathbf{3},\mathbf{2})_{1/6,-1,0,0}+(\mathbf{3},\mathbf{2})_{1/6,-1,-1,0}+(\mathbf{3},\mathbf{2})_{1/6,-1,-1,-1}\\&\quad+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,0,0}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,0}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,-1}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,0}\\&\quad+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,-1}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-2,-1}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,3,2,1}+(\mathbf{1},\mathbf{2})_{-1/2,-2,-1,0}\\&\quad+(\mathbf{1},\mathbf{2})_{-1/2,-2,-1,-1}+(\mathbf{1},\mathbf{2})_{-1/2,-2,-2,-1}+(\mathbf{1},\mathbf{2})_{-1/2,3,2,1}+\text{conjugates}\Big]\,.\end{aligned}\tag{4.2}$$

[8]One direction of the additional U(1) symmetries actually has mixed anomalies with the SM gauge group (see also, e.g. [76], for a similar situation). In the example below, the anomaly-free directions are spanned by $-4b_4/5+b_5$ and $2b_4/5+b_6$. It turns out that excluding the anomalous symmetry does not affect the selection rules below at all, so for completeness we still use all the U(1) charges in labeling the charges.

It is then straightforward to apply the rule that only terms with all net U(1) charges vanishing are unsuppressed in the superpotential of the low-energy theory. Notice that there are three copies of $\bar{D} = (\bar{\mathbf{3}}, \mathbf{1})_{1/3}$ and $L = (\mathbf{1}, \mathbf{2})_{-1/2}$ (or the SU(5) fundamental before remainder flux breaking) in each of the decompositions $\mathbf{56}, \mathbf{133}$, which are distinguished by having different U(1) charges. Without further information or inputs, three families of SM chiral matter arising in a given model in general may be distributed within the three copies. As shown below, such a distribution may lead to phenomenological inconsistencies, and some extra tuning must be done to avoid those issues.

4.2 Vector-like exotics

As in all GUT models, the E_7 models face the issue of having (many) vector-like exotics that have not been observed in experiments. Although we have chosen a $\mathbb{P}^1$ matter curve to ensure that there are no vector-like pairs on the matter curve, generically there are many vector-like matter fields on the bulk of Σ. Therefore, all representations in eq. (4.2), except the SM adjoint representations, generically have nontrivial vector-like matter multiplicities. Interestingly, these bulk vector-like fields involve the usual MSSM Higgs fields H_u, H_d, which indeed play the role of a SM Higgs sector in the discussion below. This feature does not seem to happen in models with smaller GUT groups. On the other hand, there are also inert Higgs fields H'_u, H'_d, which have the same representation $(\mathbf{1}, \mathbf{2})_{\pm 1/2}$ under the SM gauge group, but do not have the right additional U(1) charges to form unsuppressed Yukawa couplings with SM matter (see section 4.3). There are also similar sets of fields for the triplet Higgs T_u, T_d, T'_u, T'_d, as well as vector-like fields in other exotic representations, namely $(\mathbf{3}, \mathbf{2})_{-5/6}, (\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}, (\mathbf{1}, \mathbf{1})_1$. In particular, the exotic $(\mathbf{3}, \mathbf{2})_{-5/6}$ ruins phenomenology by causing proton decay and spoiling gauge coupling unification (see also section 4.7), and must be removed from the spectrum. We will also discuss the phenomenological safety of other vector-like exotics as we proceed in later sections.

As seen in section 2.4, the multiplicities of these vector-like fields are controlled by the fluxes. Unfortunately, it has been shown in [25] that for GUT groups higher or equal to SO(10), it is impossible to remove all the vector-like exotics by tuning the fluxes. Nevertheless, it was pointed out in [10] that it is easy to remove the most dangerous $(\mathbf{3}, \mathbf{2})_{-5/6}$.

Now we show that in the E_7 models, this representation is reasonably likely to be removed from the spectrum, at least for certain kinds of gauge divisor Σ. First, we notice that the representation and its conjugate have $(b_3, b_4, b_5, b_6) = (\pm 1, 0, 0, 0)$. Recall that the vertical flux we turn on breaks directions 4, 5, 6. We then see that $\chi_{(\mathbf{3},\mathbf{2})_{-5/6}} = 0$, since $\Theta_{i\alpha} \neq 0$ only for $i = 4, 5, 6$ in (2.14), and the vertical flux does not contribute in eq. (2.28). In other words, the multiplicity is purely controlled by the remainder flux in eq. (3.4), given by[9]

$$n_{(\mathbf{3},\mathbf{2})_{-5/6}} = -\frac{1}{2}(5n_r - \phi_{4r})^2 C_{\text{rem}}^2 - 1\,. \tag{4.3}$$

Therefore, $n_{(\mathbf{3},\mathbf{2})_{-5/6}} = 0$ if $5n_r - \phi_{4r} = \pm 1$ and $C_{\text{rem}}^2 = -2$. Interestingly, some choices of the remainder flux with the smallest tadpole satisfy these conditions. Consider the tadpole

$$\frac{1}{2}[G_4^{\text{rem}}]^2 = -\frac{1}{2} C_{\text{rem}}^2 \kappa^{ij} \phi_{ir} \phi_{jr}\,. \tag{4.4}$$

[9]When $\phi_{4r} = 5n_r$, L_β becomes trivial and eq. (2.27) no longer applies.

As demonstrated in section 5, in many cases Σ is a del Pezzo surface and the available C_{rem} with the least negative self-intersection has $C_{\text{rem}}^2 = -2$. Going forward we assume these technical conditions, which also imply the condition discussed earlier that $-K_\Sigma$ is effective. Further work would be needed to generalize the analysis to the situation when these technical conditions are relaxed. In this situation, $\kappa^{ij}\phi_{ir}\phi_{jr}$ is minimized by e.g.

$$\phi_{ir} = (2, 4, 6, 4, 2, 1, 3)\,, \tag{4.5}$$

which indeed leads to $n_{(\mathbf{3},\mathbf{2})_{-5/6}} = 0$. From now on, we always assume this choice of remainder flux, with tadpole

$$\frac{1}{2}[G_4^{\text{rem}}]^2 = 4\,. \tag{4.6}$$

How about other vector-like exotics? Unlike the above, vertical flux also contributes to the multiplicities of other vector-like exotics. Comparing to remainder flux, vertical flux satisfies more constraints like primitivity. We also need vertical flux to get the right chiral spectrum and, to be discussed below, the right interactions. After fulfilling these more important requirements, we find no more room to remove the remaining vector-like exotics; i.e., generically there is a nontrivial or even large contribution to n_β from vertical flux. Since remainder flux is orthogonal to vertical flux, there is no remainder flux that can cancel the contribution from vertical flux. Therefore, we expect that all the other vector-like exotics are present in our models. Fortunately, below we will show that these vector-like exotics, including the triplet Higgs $(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$, which potentially mediates dangerous proton decay, can still be phenomenologically acceptable if some additional assumptions and tunings are made.

4.3 List of Yukawa couplings

With the selection rules section 4.1, we can now list the Yukawa couplings that are not suppressed by the approximate global symmetries. Recall that in a general 4D F-theory model, there are three types of Yukawa couplings on a gauge divisor Σ [24]. First, there are Yukawa couplings between three fields on the bulk of Σ (denoted by $\Sigma\Sigma\Sigma$), but it has been shown in [24] that these couplings are all absent when $-K_\Sigma$ is effective, which is assumed in our E_7 models as discussed in section 2.4. The second type of Yukawa couplings are between one field on the bulk of Σ and two fields on matter curves (denoted by ΣCC). These couplings are generically present, and we will simply assume that all couplings of this type that satisfy the symmetry constraints are present. Finally, there are Yukawa couplings between three fields on matter curves (denoted by CCC). These couplings are characterized by codimension-3 singularities of the fibration. Nevertheless as discussed in last section, the codimension-3 singularities in the E_7 models cannot be interpreted as Yukawa couplings. In conclusion, there are only ΣCC-type couplings in our theory.[10] Note that this UV structure of Yukawa couplings is quite different from that of other previous SM-like constructions in F-theory, and we expect new phenomenological features in the IR to arise from this structure.

[10]This coupling structure, from the low-energy perspective, can also be understood as a kind of R-symmetry, where the fields on the bulk of Σ have R-charge 1, and those on matter curves have R-charge 1/2. We thank Jesse Thaler for this comment.

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We now investigate how the SM Yukawa couplings can arise from ΣCC-type couplings. In principle, we can localize the vector-like Higgs on matter curves more general than $\mathbb{P}^1$. If we make such a choice, however, the general (including both diagonal and off-diagonal) Yukawa couplings will require the non-existent CCC- and/or $\Sigma\Sigma C$-type couplings apart from ΣCC-type couplings. Therefore, to reproduce the SM Yukawa couplings with mixing between all three generations, it is necessary to localize the Higgs on the bulk of Σ, and all SM chiral matter on the matter curve $C_{\mathbf{56}}$. This choice of localization also matches with the fact that, from the discussion of section 2.4, generically there are many vector-like fields on the bulk of Σ, but no such pairs on $\mathbb{P}^1$ matter curves. We also notice that this choice of Higgs is only available when the GUT group is as large as E_7, such that the adjoint includes the Higgs after breaking. On the other hand, the chiral matter induced by vertical flux breaking descends from both **133** and **56**, and can be localized on both Σ and $C_{\mathbf{56}}$. Therefore to reproduce the SM Yukawa couplings, we need to impose the flux constraint

$$\chi^{\mathbf{133}}_{(\mathbf{3},\mathbf{2})_{1/6}} = 2\Sigma \cdot (K_B + \Sigma) \cdot D_\alpha n_\alpha = 0\,. \tag{4.7}$$

Below we will see that this constraint can be easily satisfied. It is worth emphasizing that this choice of localization automatically implies very different low-energy physics between the Higgs and chiral matter, due to their distinct geometric origins.

Now, assuming that the SM chiral spectrum is supported on $C_{\mathbf{56}}$, we can easily list all couplings that do not violate the approximate global symmetries. For simplicity, here we first ignore the couplings involving uncharged singlets under the SM gauge group; these singlets may play the role of right-handed neutrinos and will be studied in section 4.6. We then have the SM Yukawa couplings:[11]

$$\begin{aligned}
H_uQ\bar{U}: &\quad (\mathbf{1},\mathbf{2})_{1/2,-3,-2,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2}\,, \\
H_dQ\bar{D}: &\quad \begin{cases} (\mathbf{1},\mathbf{2})_{-1/2,-2,-2,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2}\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2}\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,0} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2}\,, \end{cases} \\
H_dL\bar{E}: &\quad \begin{cases} (\mathbf{1},\mathbf{2})_{-1/2,-2,-2,-1} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \times (\mathbf{1},\mathbf{1})_{1,3/2,1,1/2}\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,-1} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \times (\mathbf{1},\mathbf{1})_{1,3/2,1,1/2}\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,0} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \times (\mathbf{1},\mathbf{1})_{1,3/2,1,1/2}\,, \end{cases}
\end{aligned} \tag{4.8}$$

where the first representation in each product is the up and down Higgs H_u, H_d. We also have

[11] Similar to mass terms, the couplings described here are terms in the superpotential W; Yukawa couplings between one boson and two fermions come as usual from the contributions to the potential V of the form $(\partial^2 W/\partial\phi_i\partial\phi_j)\psi_i\psi_j$ and its conjugate.

a number of other exotic couplings. There are couplings involving the triplet Higgs $(\mathbf{3},\mathbf{1})_{-1/3}$:

$$
\begin{aligned}
T_uQQ: \quad & (\mathbf{3},\mathbf{1})_{-1/3,-3,-2,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \,, \\
T_u\bar{U}\bar{E}: \quad & (\mathbf{3},\mathbf{1})_{-1/3,-3,-2,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2} \times (\mathbf{1},\mathbf{1})_{1,3/2,1,1/2} \,, \\
T_dQL: \quad & \begin{cases} (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-2,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \,, \\ (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,-1} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \,, \\ (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,0} \times (\mathbf{3},\mathbf{2})_{1/6,3/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \,, \end{cases} \\
T_d\bar{U}\bar{D}: \quad & \begin{cases} (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-2,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \,, \\ (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2} \,, \\ (\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,0} \times (\bar{\mathbf{3}},\mathbf{1})_{-2/3,3/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2} \,. \end{cases}
\end{aligned} \tag{4.9}
$$

These couplings are always present together with the SM Yukawa couplings, but the ones with triplet Higgs mediate dimension-5 proton decay and need extra attention. Note that there are unique sets of additional U(1) charges for H_u, T_u. This uniquely identifies these fields in the decomposition (4.2) as

$$
H_u = (\mathbf{1},\mathbf{2})_{1/2,-3,-2,-1} \,, \quad T_u = (\bar{\mathbf{3}},\mathbf{1})_{-1/3,-3,-2,-1} \,. \tag{4.10}
$$

On the other hand, there are three possible fields with distinct approximate U(1) charges for each of H_d, T_d, each of which couples to $\bar{D}, L$ in one of the three possible copies of the SU(5) fundamentals. The choices of these charges will be discussed below.

There are also couplings involving other types of vector-like exotics, namely $(\mathbf{3},\mathbf{2})_{1/6}, (\mathbf{3},\mathbf{1})_{2/3}, (\mathbf{1},\mathbf{1})_1$:

$$
\begin{aligned}
& (\mathbf{3},\mathbf{2})_{1/6,-1,-1,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \,, \\
& (\mathbf{3},\mathbf{2})_{1/6,-1,-1,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \,, \\
& (\mathbf{3},\mathbf{2})_{1/6,-1,-1,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \,, \\
& (\mathbf{3},\mathbf{2})_{1/6,-1,-1,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \,, \\
& (\mathbf{3},\mathbf{2})_{1/6,-1,0,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \,, \\
& (\mathbf{3},\mathbf{2})_{1/6,-1,0,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \,, \\
& (\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2} \,, \\
& (\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2} \,, \\
& (\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,0,0} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2} \,, \\
& (\mathbf{1},\mathbf{1})_{1,-1,-1,-1} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \,, \\
& (\mathbf{1},\mathbf{1})_{1,-1,-1,0} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \,, \\
& (\mathbf{1},\mathbf{1})_{1,-1,0,0} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,1/2} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,0,-1/2} \,.
\end{aligned} \tag{4.11}
$$

These couplings induce, e.g., additional proton decay and may not be compatible with phenomenology. On the other hand, all these couplings mix distinct copies of $(\bar{\mathbf{3}},\mathbf{1})_{1/3}$

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and/or $(\mathbf{1},\mathbf{2})_{-1/2}$ with different U(1) charges, while the couplings in eqs. (4.8) and (4.9) relate H_d, T_d in a given copy with matter fields within the same corresponding copy. As shown in section 4.5.2, extra tuning of the model is available such that the couplings in eq. (4.11) are absent, and we assume such absence throughout the paper. All the other couplings without uncharged singlets and not listed above, if present, are exponentially suppressed by the approximate global symmetries.

4.4 Proton decay

Before building semi-realistic Higgs sector and Yukawa couplings, there is an important issue to be resolved: as in every GUT model, there is a possibility of couplings that give a proton decay rate that exceeds experimental limits. In particular, since the triplet Higgs cannot be removed from the spectrum, the couplings in last subsection naively seem to suggest that the E_7 models will suffer from excess proton decay mediated by dimension-5 operators. We now show that, fortunately, both dimension-4 and 5 proton decay are ubiquitously suppressed in the E_7 models. This feature in some sense "comes for free" with the construction of these models.

First, dimension-4 proton decay in the MSSM is driven by the R-parity violating terms in the superpotential:

$$W \supset \alpha_1 Q L \bar{D} + \alpha_2 L L \bar{E} + \alpha_3 \bar{D}\bar{D}\bar{U}\,, \tag{4.12}$$

where we have used the notation in eq. (3.6). These are couplings between three chiral fields, which all descend from **56** under the assumption of eq. (4.7). Hence from the geometric perspective, dimension-4 proton decay requires CCC-type couplings, which are absent in the E_7 models. The absence of these couplings is also natural from the form of the fields in the symmetry-broken theory: since the approximate global charges b_4, b_6 of all fields (including all copies of $\bar{D}, L$) appearing in eq. (4.12) are half-integers, none of the interactions of this type have vanishing net b_4, b_6 charges, so all such interactions violate one of the approximate global symmetries. Thus, dimension-4 proton decay is automatically absent in the E_7 models (with the caveat that we do not completely understand the $(4,6)$ singularities; it is not fully clear whether the interplay between these singularities and flux breaking would modify this conclusion). In fact, it was already pointed out in [80] that dimension-4 proton decay can be eliminated in this way only when the GUT group is E_7 or E_8; this suppression arises in heterotic and M-theory models as well as in F-theory.

Now we turn to dimension-5 proton decay. Such decay in conventional supersymmetric GUTs comes from the following terms

$$W \supset \lambda_1 T_u Q Q + \lambda_2 T_d Q L + M T_u T_d\,, \tag{4.13}$$

where T_u, T_d are the triplet Higgs, and M is a large mass close to the GUT scale M_{GUT}. Integrating out T_u, T_d, we then get the dimension-5 operator $QQQL/M$, which is only suppressed by $1/M$ and leads to an unacceptable rate of proton decay. Nevertheless, the E_7 models are different from conventional GUTs in the following sense: although the first two terms in eq. (4.13) are not suppressed, we see from eq. (4.9) that T_u, T_d never have opposite additional U(1) charges, hence the mass term in eq. (4.13) is exponentially suppressed. Instead, T_u, T_d have their own vector-like partners, denoted by T'_d, T'_u, with opposite additional U(1) charges. These "primed" fields are *inert*, i.e. their Yukawa interactions with SM chiral matter are exponentially suppressed, but they give T_u, T_d large masses by the conventional mass terms. Now the superpotential schematically has the form of

$$W \supset \lambda_1 T_u Q Q + \lambda_2 T_d Q L + M T_u T_d' + M T_u' T_d + m T_u T_d + m T_u' T_d' \,, \tag{4.14}$$

where m is exponentially suppressed compared to M. The operator we get by integrating out the triplet Higgs and the vector-like partners is roughly $(m/M)\, QQQL/M$, which is indeed further exponentially suppressed by the factor m/M. Therefore, as long as M is sufficiently large (probably close to M_{GUT}) and m/M is sufficiently small, the E_7 models are safe from overly dangerous proton decay.

Regarding dimension-6 proton decay mediated by gauge bosons along the broken directions of the gauge group, we do not see an obvious mechanism of suppression. The gauge bosons typically have masses around the KK/GUT scale, which may be sufficiently high to evade the current experimental bounds [81]. Moreover, fluxes on Σ can make the ground state wavefunction of the gauge bosons more localized, and suppress its wavefunction overlap (hence the coupling) to the chiral matter on $C_{\mathbf{56}}$ [82]. Note that such suppression may not be exponential [83], but may already be sufficient for our purposes due to the high GUT scale. Despite all these heuristic arguments, more techniques and explicit calculations are still required to determine the exact rate of proton decay, which is essential for realistic model building.

4.5 Higgs and Yukawa sectors

Now we turn to the Higgs and Yukawa sectors in the E_7 models. Even with the Yukawa couplings in eq. (4.8) that have the right representations, it is not guaranteed that those couplings resemble the structure of Higgs and Yukawa sectors in the Standard Model, due to various differences of the E_7 models from the conventional Standard Model. Below we explain each of these differences, and write down the necessary conditions for realizing the SM Higgs and Yukawa sectors.

4.5.1 Doublet-triplet splitting and the Higgs masses

Apart from proton decay, one of the important questions in general GUT models is the doublet-triplet splitting problem, or why the masses of doublet and triplet Higgs are separated by many orders of magnitude. In F-theory GUTs, this splitting in principle can be explained by the presence of hypercharge flux [25]. For the tuned SU(5) GUTs, however, the explicit realization of such splitting can be difficult; see section 6 for further discussion. In contrast, the doublet and triplet Higgs in the E_7 model live on the bulk of Σ and always receive mass splitting from hypercharge flux, which is also localized on a (remainder) surface on Σ. Therefore, the doublet and triplet masses are automatically split once we break E_7 to G_{SM}, although the amount of splitting is still unknown and new techniques must be developed for finding out the Higgs mass spectrum.

Still, what controls the mass terms before the splitting by hypercharge flux? Similar to the triplet Higgs in section 4.4, the conventional μ-term i.e. $\mu H_u H_d$ is exponentially suppressed. This suppression, however, does not mean that the μ-problem is solved, since the Higgs

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can still get large masses from terms $H_uH'_d, H_dH'_u$, when H_u, H_d have their own vector-like partners H'_d, H'_u. On the other hand, it means that there is some vector-like matter with light masses when such vector-like partners do not exist. Indeed, such a scenario generically arises for H_d, and the essence of this effect lies in vertical flux breaking: although we have imposed that the total chiral index of fields arising from **133** vanishes, there can still be nontrivial chiral surpluses for each of the three copies of doublet and triplet Higgs in this representation (see section 4.1). Suppose we have the following spectrum for the three copies of Higgs fields:

$$\begin{aligned} (\mathbf{1},\mathbf{2})_{-1/2,-2,-2,-1} &: n_1\,, \quad (\mathbf{1},\mathbf{2})_{1/2,2,2,1} : n'_1\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,-1} &: n_2\,, \quad (\mathbf{1},\mathbf{2})_{1/2,2,1,1} : n'_2\,, \\ (\mathbf{1},\mathbf{2})_{-1/2,-2,-1,0} &: n_3\,, \quad (\mathbf{1},\mathbf{2})_{1/2,2,1,0} : n'_3\,, \end{aligned} \tag{4.15}$$

where n_i, n'_i denote the multiplicities. Eq. (4.7) implies that $n_1+n_2+n_3 = n'_1+n'_2+n'_3$, hence the spectrum is non-chiral under the SM gauge group. On the other hand, generically we have $n_i \neq n'_i$ and the spectrum would be chiral if the additional U(1)'s were gauge symmetries, i.e. the Stückelberg mechanism was absent. If $n_i \neq n'_i$ for some i, there must be a field direction in the i-th copy that cannot acquire any mass terms within the same copy. It can only get mass terms from fields in other copies. Since they do not have opposite additional U(1) charges, the resulting mass terms are exponentially suppressed, leading to a light doublet Higgs H_d.

It is tempting to use the above mechanism to solve the μ-problem. Unfortunately, the μ-problem cannot be solved in this way for two reasons. First, only H_d, but not H_u, has three copies in the branching rule. In other words, this mechanism for producing light H_d cannot produce a light H_u. Second, the above mechanism relies on vertical flux breaking, which only breaks E_7 to SU(5) instead of G_{SM}. This means that whenever a light H_d is produced in this way, there must also be a light T_d. Although there may still be doublet-triplet splitting from hypercharge flux, the mass of the T_d is still exponentially suppressed. Such T_d directly interacts with SM chiral matter and ruins the argument in section 4.4, i.e. there is still too much dimension-5 proton decay even with the exponential suppression in section 4.4. In this sense, we should even avoid any light H_d or T_d produced in this way. As discussed in section 4.5.2, we will arrange the fluxes such that only one copy of H_d interacts with SM chiral matter. Without loss of generality, let us pick the copy with $(b_4, b_5, b_6) = (2, 2, 1)$. Then avoiding light H_d and T_d coming from the above mechanism is achieved by the flux constraint

$$\chi^{\mathbf{133}}_{(\overline{\mathbf{3}},\mathbf{1})_{1/3,-2,-2,-1}} = 0\,. \tag{4.16}$$

This is another linear constraint on the flux parameters, similar to the ones for breaking the gauge group or inducing three generations of chiral matter. This new constraint, however, is the first constraint that involves the previously unused flux parameters $\phi_{5\alpha}, \phi_{6\alpha}$. Therefore, given the gauge group and total chiral spectrum, there is always still some room in the E_7 models for satisfying this new constraint.

The above still does not explain the origin of light masses in the SM Higgs sector. Sadly, in the current construction of our E_7 models, there is still no obvious solution to the μ-problem. This is understandable, however, since the Higgs masses in F-theory are very complicated quantities to calculate. Traditionally, the Higgs masses come from the vevs of some fields localized on divisors other than Σ but intersecting with Σ. These fields behave as singlets and couple to the vector-like matter on Σ. Nevertheless, the vevs or potential of these fields depends on many factors, including but not limited to the detailed couplings between these fields, the D-term potential, the nonperturbative superpotential, and most importantly, soft SUSY breaking [84]. Therefore without understanding more basic issues like moduli stabilization and SUSY breaking in F-theory, no precise statements on these vector-like masses can be made. On the other hand, given such a complicated origin of the Higgs masses, it is reasonable to expect that some hierarchy is generated and brings some of the Higgs to light scales. At the same time, we should not allow more than one pair of Higgs to be at the electroweak scale, although generically there are many vector-like fields with the same representation. This is because when more than one Higgs field couples to SM chiral matter in the same way, the flavor basis generically does not align with the Higgs mass basis. Such misalignment produces tree-level flavor-changing neutral currents (FCNCs), which are not observed in experiments.[12] In conclusion, to reproduce the SM Higgs sector, it is far from clear how to realize exactly one pair of light Higgs doublets among all the Higgs fields. This is a major shortcoming of the E_7 models, and we hope to give a better explanation for this Higgs hierarchy in the future.

As a remark, there are still many *inert* Higgs fields in the other two copies. In particular, there can be multiple light inert Higgs fields, coming from pairing chiral surpluses between the copies or other ways. Fortunately since they are inert, there is no tight constraint on these fields. We note that the current experimental lower bound on H'_u, H'_d masses is around 100 GeV [81].

4.5.2 Structure of Yukawa couplings

One of the most dangerous features in the E_7 model is that there are three copies of $(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$ and $(\mathbf{1}, \mathbf{2})_{-1/2}$ in the branching rules in section 4.1, with different additional U(1) charges. The generic case where the three generations of chiral matter are distributed in all the copies is not phenomenologically acceptable for various reasons. First, generically there are chiral differences for each of the copies, as in (4.15). While these add up to the three generations in the total chiral spectrum, as demonstrated in section 4.5.1 they can also form light vector-like exotics between different copies. Next, having different copies in the chiral spectrum turns on an unsuppressed set of exotic couplings in eq. (4.11), including additional proton decay. More seriously, multiple light Higgs fields are required to generate unsuppressed SM Yukawa couplings for all the copies, see eq. (4.8).[13] As discussed in section 4.5.1, such a Higgs sector again leads to unacceptable FCNCs. Therefore to avoid all the above issues, we must arrange all three generations of chiral matter to be within the same copy. For the choice of light Higgs with $(b_4, b_5, b_6) = (2, 2, 1)$ in section 4.5.1, for example, we should choose the corresponding

[12] We thank Jesse Thaler for pointing out this issue.

[13] While there is indeed some hierarchy between Yukawa couplings in the observed Standard Model, we do not expect the hierarchy to be as large as the exponential suppression from the approximate global symmetries. Therefore, if we only use one Higgs for multiple copies with exponentially suppressed couplings, very probably it will not give the right flavor structure.

copy of chiral matter with $(b_4, b_5, b_6) = (1/2, 1, 1/2)$ and impose the following flux constraints:

$$\chi^{\mathbf{56}}_{(\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,1/2}} = \chi^{\mathbf{56}}_{(\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,0,-1/2}} = 0\,. \tag{4.17}$$

Again, these constraints are mild tuning on the remaining flux parameters $\phi_{5\alpha}, \phi_{6\alpha}$, which is generically achievable. Nevertheless, it will be interesting to see whether there is a more fundamental reason that leads to such choice of fluxes.

After this choice of chiral matter, the remaining couplings in the low-energy theory are just the SM Yukawa couplings and their counterparts with the triplet Higgs. It would be even more informative if we can get the values of the Yukawa couplings. Although calculating those values is beyond our current F-theory technologies, we can gather some of their qualitative features. Unlike the conventional F-theory models with CCC-type couplings, the use of ΣCC-type couplings means that the Yukawa couplings are supported on the whole $C_{\mathbf{56}}$ instead of points on it. If the Higgs wavefunction is nearly uniform on $C_{\mathbf{56}}$, the Higgs will interact with all three generations of chiral matter in the same way, thus the Yukawa couplings will be undesiredly close to an identity matrix.[14] Nevertheless, especially with the presence of bulk fluxes, we expect the Higgs wavefunction to be non-uniform and peak in some smaller region. A simple scenario would be that the region intersects with $C_{\mathbf{56}}$ in a connected small but finite range. This scenario is then similar to the case of a single Yukawa point studied in e.g. [85, 86], where the small nonperturbative correction is now due to the finite size of the interaction region. In this way, the Yukawa hierarchy is generated as in the SU(5) F-theory GUTs. On the other hand, to really compute the Yukawa couplings, we first need to understand the Higgs wavefunction profile and its possible correlations with the exponentially low Higgs mass. Once we understand these issues, we may be able to use the ultra-local approach developed in [87–90] to computing Yukawa couplings within the intersecting region, but understanding those issues remains very challenging. In more complicated scenarios where there are multiple disconnected interaction regions, they are similar to the case of multiple Yukawa points. The arguments in [16] then suggest that there is also some Yukawa hierarchy, although the methods in [16] do not straightforwardly generalize to our models due to the use of flux breaking. In summary, it is possible that there is some hierarchy between the Yukawa couplings. This hierarchy may match with the observed Yukawa hierarchy, but explicitly computing the Yukawa matrix in our models will be an important future step for realistic model building.

4.6 Neutrino sector

Here we turn to the neutrino sector and make some brief comments. From eq. (4.1), we see that **56** gives three copies of singlets that can be right-handed neutrinos. Since in the above we have restricted the leptons into one copy, only one copy of the singlets $(\mathbf{1},\mathbf{1})_{0,5/2,1,1/2}$ have unsuppressed Yukawa couplings with SM chiral matter:

$$\begin{aligned}&(\mathbf{1},\mathbf{2})_{1/2,-3,-2,-1} \times (\mathbf{1},\mathbf{2})_{-1/2,1/2,1,1/2} \times (\mathbf{1},\mathbf{1})_{0,5/2,1,1/2}\,,\\ &(\mathbf{3},\mathbf{1})_{-1/3,-3,-2,-1} \times (\bar{\mathbf{3}},\mathbf{1})_{1/3,1/2,1,1/2} \times (\mathbf{1},\mathbf{1})_{0,5/2,1,1/2}\,.\end{aligned} \tag{4.18}$$

The other two copies of singlets have nontrivial multiplicities but belong to inert matter.

[14] We thank Jonathan Heckman for pointing this out.

We can obtain the multiplicity of right-handed neutrinos from its "chiral index". It sounds strange to calculate a "chiral index" for non-chiral matter; the correct interpretation is that the right-handed neutrinos carry additional U(1) charges, hence "would be" chiral matter if we ignore the Stückelberg mechanism. Since there are no vector-like exotics on the matter curve, the chiral index is the same as the exact multiplicity, which remains unchanged under the symmetry breaking. Now from the flux constraints we imposed in previous sections, we see that the chiral index is fixed to be

$$\chi_{(\mathbf{1},\mathbf{1})_{0,5/2,1,1/2}} = 3\,. \tag{4.19}$$

Therefore, we have three right-handed neutrinos, which favorably combine with the left-handed ones to give three Dirac neutrinos and a square PMNS matrix. The above Yukawa couplings then give the usual Dirac mass terms after electroweak symmetry breaking. This scenario is more or less the same as conventional GUTs with SO(10) gauge group or above.

There are also Majorana mass terms from the ΣCC-type couplings involving two right-handed neutrinos and a bulk singlet. Since all right-handed neutrinos have the same additional U(1) charges, the Majorana masses are always exponentially suppressed (compared to string/GUT scale) by the additional U(1) symmetries. In fact, similar suppression was already used in some early type II [75, 76] and F-theory [25] SM-like model. It was estimated in those references that the exponential suppression factor might be around 10^{-6} to 10^{-4}, which is much more mild than the electroweak hierarchy. This is not incompatible with the observational constraints on the seesaw mechanism. We emphasize that, however, these numerical estimates are very crude, and without more explicit computations of the masses and couplings, we cannot make fully precise statements on how the left-handed neutrinos get very small masses.

4.7 Gauge coupling unification

Here we briefly comment on the possibility of gauge coupling unification in our models. Despite the use of E_7 in the construction of models, whether gauge coupling unification is present in any useful sense is far from obvious. From the point of view of the world-volume theory on the IIB 7-branes supporting the E_7 gauge theory (as in, e.g., [24]), it should be possible to find a classical description of flux breaking through turning on flux (T-dual to turning on an adjoint scalar as in, e.g., [91]). From this perspective, at sufficiently high energies the world-volume E_7 gauge symmetry would be effectively restored, and the expected extra gauge bosons would become relatively light, so there is some sense in which gauge coupling unification might be expected. Note, however, that the quantization of flux means that the background flux will give a mass scale $m_{\mathrm{KK}} = 1/l_{\mathrm{KK}}$, where l_{KK} is the compactification scale, so that this unification only occurs much above the KK scale. Furthermore, in the nonperturbative F-theory regime, where there is no weakly coupled description, it is not clear that the 7-brane world-volume theory can be meaningfully separated from string theory in the bulk space. Thus, we do not necessarily expect unification even at the compactification scale. To understand some of the issues, we first clarify the meaning of gauge coupling unification in our string theory context.

There are two separate aspects. First at the GUT scale M_{GUT},[15] the gauge couplings in our models are clearly unified if flux breaking is absent. The coupling is given by the volume of the gauge divisor:

$$\frac{1}{g^2} \simeq \mathrm{vol}\,(\Sigma)\ . \tag{4.20}$$

It is estimated from observations that $1/\alpha \simeq 24$ at M_{GUT} [92]; we simply assume that the divisor volume is stabilized to this particular value by certain mechanisms. On the other hand, the remainder flux breaks SU(5) to G_{SM} and induces some splitting of gauge couplings at M_{KK}. It is then important to understand such splitting and determine its size. Such splitting has been understood in type IIB models [36, 93]: the splitting between the SU(3) and SU(2) gauge couplings is

$$\frac{1}{\alpha_2\,(M_{\mathrm{GUT}})} - \frac{1}{\alpha_3\,(M_{\mathrm{GUT}})} \simeq -\frac{1}{10 g_s}\left[c_1\,(L_3)\right]^2 = \frac{1}{5 g_s}(n_{Q'}+1)\,, \tag{4.21}$$

where g_s is the string coupling, and $n_{Q'}$ is the number of vector-like pairs in the exotic representation $(\mathbf{3},\mathbf{2})_{-5/6}$; recall that we have set $n_{Q'}=0$ in previous sections. There is also a unification-like relation

$$\frac{1}{\alpha_Y\,(M_{\mathrm{GUT}})} = \frac{5}{3}\frac{1}{\alpha_1\,(M_{\mathrm{GUT}})} = \frac{1}{\alpha_2\,(M_{\mathrm{GUT}})} + \frac{2}{3}\frac{1}{\alpha_3\,(M_{\mathrm{GUT}})}\,. \tag{4.22}$$

All the above, however, cannot be directly applied to F-theory models, especially when the models, like our E_7 models, are intrinsically strongly coupled and have no type IIB limit. This is because the axio-dilaton varies over the internal space and the meaning of the $1/g_s$ correction is no longer clear. The worldvolume theory, which was used to derive the type IIB result, also needs to be reconsidered in F-theory setups. In addition, there may be large stringy threshold corrections to the gauge kinetic functions due to the strong coupling nature of these models. All these subtleties imply that the splitting at M_{GUT} may not be small even if we set $n_{Q'}=0$.

Next, at scales lower than M_{GUT}, the RG flow of the SM gauge couplings are affected by the vector-like exotics. The RG flow depends on both the representations and the masses of the vector-like exotics. Since the remainder flux already breaks the GUT group at M_{GUT}, it is possible that some vector-like exotics are light and do not form GUT multiplets. These exotics seriously alter the RG flow and may ruin gauge coupling unification. The existence of such exotics, however, depends crucially on uncontrolled aspects of the models such as SUSY breaking. It is also possible that the presence of these exotics compensates the above splitting at M_{GUT} and makes the couplings apparently unified from the bottom-up perspective. Therefore, so far we cannot make any definite statement on how the vector-like spectrum may affect the RG flow.

In conclusion, the gauge couplings in our models are affected by a number of uncontrolled aspects, thus gauge coupling unification is not guaranteed in our models. From this perspective, the unification of the observed gauge couplings in ordinary MSSM looks like an accident if our models really describe our Universe. Nevertheless, this conclusion mainly comes from our inability to compute non-topological details of our models. A more careful string theory analysis in the future may reveal that the observed unification is in fact not an accident at all.

[15] In the string theory context, M_{GUT} may be around the KK scale or string scale depending on model details.

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5 Explicit global constructions of E_7 GUTs

In all the above sections, we have written down many necessary constraints on the geometry and fluxes for constructing semi-realistic E_7 GUTs in F-theory. It remains important to see whether all these constraints can be satisfied simultaneously within a 4D F-theory model. In this section, we provide an explicit global construction of such a model, using the tools of toric hypersurfaces. The construction here is a generalization of that in [37]. It is also the first explicit example of a rigid E_7 GUT (rigid E_6 GUTs were presented in [10]). Although we only present a single example here, the same construction can be generalized to large class of F-theory compactifications. Before writing down such an explicit model, it is useful to first review the geometric and flux constraints we want to achieve:

- Σ as a del Pezzo surface supporting both rigid E_7 (with effective $-K_\Sigma$) and hypercharge flux, and $C_{\mathbf{56}} = -\Sigma \cdot (4K_B + 3\Sigma)$ as a $\mathbb{P}^1$, to enable explicit computations and interesting phenomenology. The first requirement demands that Σ is a rigid divisor on a non-toric base.

- The general flux constraints in section 2.1: flux quantization, primitivity for vertical flux, and tadpole cancellation. In particular, we should look for flux configurations with minimal tadpole.

- A vertical flux breaking $E_7 \to \mathrm{SU}(5)$ and a remainder (hypercharge) flux breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$. In particular we need $r \geq 4$, see section 2.2.

- $\chi^{\mathbf{133}}_{(\mathbf{3},\mathbf{2})_{1/6}} = 0$ and $\chi^{\mathbf{56}}_{(\mathbf{3},\mathbf{2})_{1/6}} = 3$ for the total chiral spectrum.

- All three families of chiral $(\bar{\mathbf{3}},\mathbf{1})_{1/3}$ and $(\mathbf{1},\mathbf{2})_{-1/2}$ coming from the same copy, i.e. eq. (4.17), to avoid exotic vector-like spectrum and couplings.

- The copy of bulk $(\bar{\mathbf{3}},\mathbf{1})_{1/3}$ and $(\mathbf{1},\mathbf{2})_{-1/2}$ that interacts with the chiral matter being itself non-chiral, i.e. eq. (4.16), to avoid light vector-like exotics.

- $[c_1(L_3)]^2 = -2$ for hypercharge flux, to remove the exotic $(\mathbf{3},\mathbf{2})_{-5/6}$.

As always, we set the flux associated to the non-flat fiber to zero.

Now we write down an explicit F-theory model that satisfies all the above constraints. As in [10], we choose the base B through the following procedure. We start with an auxilliary toric threefold A with $h^{1,1}(A) = 4$. Then the ambient fourfold X is a $\mathbb{P}^1$-bundle over A with a certain normal bundle, and B is a certain hypersurface in X. The geometry of B can be analyzed using the techniques in appendix A. With appropriate choices in the above procedure, we can construct B containing a rigid Σ with $r = 4$ and nontrivial remainder flux.

Let us first construct the ambient space X. We choose A to be a $\mathbb{P}^1$-bundle over the del Pezzo surface dP_2, which has a toric description. Let us first introduce the notations. Within dP_2 i.e. blowup of $\mathbb{P}^2$ at two generic points, let e_1, e_2 be the exceptional curves from the blowup, and $h = f + e_1 + e_2$ be the hyperplane. The intersection numbers are $f^2 = e_1^2 = e_2^2 = -1, f \cdot e_1 = f \cdot e_2 = 1, e_1 \cdot e_2 = 0$. Now on A, we denote σ as the dP_2 section and E_1, E_2, F as the $\mathbb{P}^1$-fibers along e_1, e_2, f respectively. We choose the normal bundle of

Toric ray	Divisor
$(1,0,0,0)$	$F_{E_1}+F_F$
$(0,1,0,0)$	$F_{E_2}+F_F$
$(-1,-1,-1,0)$	F_F
$(-1,0,-1,-3)$	F_{E_1}
$(0,-1,-1,0)$	F_{E_2}
$(0,0,-1,-4)$	F_σ
$(0,0,1,0)$	$F_\sigma+F_{E_1}+F_{E_2}+F_F$
$(0,0,0,-1)$	σ_A
$(0,0,0,1)$	$\sigma_A+4F_\sigma+3F_{E_1}$

Table 1. The toric rays and the corresponding divisors in the toric construction of the ambient fourfold X.

the $\mathbb{P}^1$-bundle to be $N_\sigma=-h$, and the anticanonical class is $-K_A=2\sigma+3E_1+3E_2+4F$. The intersection numbers on A follow straightforwardly from those on dP_2 and the relation $\sigma\cdot(\sigma+F+E_1+E_2)=0$. Finally we let the fourfold X be a $\mathbb{P}^1$-bundle over A with normal bundle $N_A=-4\sigma-3E_1$. We denote σ_A as the section and F_I be the fiber along $I\in\{\sigma,E_1,E_2,F\}$. The anticanonical class is $-K_X=2\sigma_A+6F_\sigma+6F_{E_1}+3F_{E_2}+4F_F$. Again, the intersection numbers follow from those on A and the relation $\sigma_A\cdot(\sigma_A+4F_\sigma+3F_{E_1})=0$. Note that with these choices of normal bundles, there is a unique triangulation such that X is a smooth and projective toric variety. The toric rays of X are listed in table 1.

We now choose the threefold base B as a hypersurface in X with irreducible class $B=\sigma_A+5F_\sigma+5F_{E_1}+2F_{E_2}+3F_F$. By abuse of notation, we use B to denote both the base and its divisor class in X. By adjunction $-K_B=B\cdot(\sigma_A+F_\sigma+F_{E_1}+F_{E_2}+F_F)$. Using the techniques in appendix A, one can check that $h^{1,1}(B)=h^{1,1}(X)=5$. In particular, in this situation the divisors of B are spanned by intersections in X. The intersection numbers of these divisors relevant to our purpose are

$$B\cdot\sigma_A\cdot F_I\cdot F_J=\begin{pmatrix}-2&1&1&0\\1&-1&0&1\\1&0&-1&1\\0&1&1&-1\end{pmatrix}. \tag{5.1}$$

Now we consider the divisor $\Sigma=B\cdot\sigma_A$. It is also the hypersurface in A with class $\sigma+2E_1+2E_2+3F$. This class is irreducible and does not have any base locus (considered as a hypersurface in A), so it is a well-defined irreducible gauge divisor. We compute

$$\begin{aligned}-K_\Sigma&=B\cdot\sigma_A\cdot(F_\sigma+F_{E_1}+F_{E_2}+F_F)\\&=\sigma_A\cdot(2F_\sigma\cdot F_{E_1}+2F_\sigma\cdot F_{E_2}+3F_\sigma\cdot F_F+3F_{E_1}\cdot F_F)\,,\end{aligned} \tag{5.2}$$

$$\begin{aligned}N_\Sigma&=B\cdot\sigma_A^2\\&=-\sigma_A\cdot(7F_\sigma\cdot F_{E_1}+4F_\sigma\cdot F_{E_2}+8F_\sigma\cdot F_F+3F_{E_1}\cdot F_F)\,.\end{aligned} \tag{5.3}$$

Therefore by eq. (3.2), we see that Σ is indeed a rigid divisor supporting E_7. The matter curve is

$$C_{\mathbf{56}} = \Sigma \cdot (-4K_B - 3\Sigma) \tag{5.4}$$

$$= B \cdot \sigma_A \cdot (\sigma_A + 4F_\sigma + 4F_{E_1} + 4F_{E_2} + 4F_F)$$

$$= B \cdot \sigma_A \cdot (F_{E_1} + 4F_{E_2} + 4F_F)\ . \tag{5.5}$$

Notice that the divisor $E_1 + 4E_2 + 4F$ in A is also irreducible and does not have any base locus. Therefore, $C_{\mathbf{56}}$ is also irreducible with genus

$$g = 1 + \frac{1}{2} C_{\mathbf{56}} \cdot (C_{\mathbf{56}} + K_\Sigma) = 0\,, \tag{5.6}$$

which means that the matter curve is simply a $\mathbb{P}^1$.

We can now study the constraints on vertical flux. First, we study primitivity by expanding the Kähler form of B using a basis of base divisors:

$$\begin{aligned}[J_B] = B \cdot (&t_1 (F_{E_1} + F_F) + t_2 (F_{E_2} + F_F) + t_3 (F_{E_1} + F_{E_2} + F_F) \\ &+ t_4 (F_\sigma + F_{E_1} + F_{E_2} + F_F) + t_5 (\sigma_A + 4F_\sigma + 4F_{E_1} + 4F_{E_2} + 4F_F)\ ,\end{aligned} \tag{5.7}$$

where t_1, t_2, t_3, t_4, t_5 are linear combinations of Kähler moduli, and may be negative inside the Kähler cone of B in general. While determining the exact Kähler cone of a hypersurface in a toric variety can be subtle, the Kähler cone of B must contain that of X [94]. For simplicity, we look for a solution of the primitivity constraints in the Kähler cone of X only. By a direct toric computation, one can check that the Kähler cone of X is given by $t_1, t_2, t_3, t_4, t_5 > 0$. The independent $S_{i\alpha}$ are $S_{i\sigma}, S_{iE_1}, S_{iE_2}, S_{iF}$, where we have simplified the notation and denoted $S_{i(B\cdot F_I)}$ as S_{iI}. The primitivity condition is then

$$t_1 (\phi_{iE_2} + \phi_{i\sigma}) + (t_2 + 3t_5)(\phi_{iE_1} + \phi_{i\sigma}) + (t_3 + t_5)(\phi_{iF} + 2\phi_{i\sigma}) + t_4 (\phi_{iF} + \phi_{iE_1} + \phi_{iE_2}) = 0\,, \tag{5.8}$$

for all i. A necessary but not sufficient condition for satisfying primitivity is that there must be some coefficients in eq. (5.8) with opposite signs for each i. Below we will find explicit solutions to primitivity and check that the solutions are within the Kähler cone.

Next, we require the total chiral indices to be

$$\chi^{\mathbf{133}}_{(\mathbf{3},\mathbf{2})_{1/6}} = -2\,(n_F + n_{E_1} + n_{E_2}) = 0\,, \tag{5.9}$$

$$\chi^{\mathbf{56}}_{(\mathbf{3},\mathbf{2})_{1/6}} = -n_F - 3n_{E_1} - 5n_\sigma = 3\,. \tag{5.10}$$

Here the n_I parameterize the fluxes through eq. (3.3). To understand what values of n_I we should turn on to get the right total chiral spectrum, we should first look at flux quantization, since for E_7 models $c_2(\hat{Y})$ is not necessarily even. We can calculate $c_2(\hat{Y})$ using the techniques in [50], which involve picking a particular resolution of the E_7 models, but the parity of $c_2(\hat{Y})$ is expected to be resolution-independent. We outline the procedure in appendix B, while here we only apply the result, which tells us that we can turn on half-integers n_{E_1}, n_F and integers n_{E_2}, n_σ (similarly for $\phi_{6\alpha}$) to guarantee flux quantization. Note that these may not be the only choices of n_I, since the structure of $H^4(\hat{Y}, \mathbb{Z})$ is subtle and may include elements with fractional coefficients. Also note that these choices of n_I do not necessarily mean that the vertical flux does not belong to $H^4(\hat{Y}, \mathbb{Z})$, since as we will see its tadpole is still integer. Now with these choices of n_I, we see that an almost minimal flux configuration $(n_\sigma, n_{E_1}, n_{E_2}, n_F) = (0, -3/2, 0, 3/2)$ already gives the above two chiral indices and is consistent with primitivity.

There are more flux conditions that constraint the values of $\phi_{5\alpha}, \phi_{6\alpha}$. First, by flux quantization we should turn on half-integers ϕ_{6E_1}, ϕ_{6F} and integers for the remaining parameters. Primitivity still constraints their values nontrivially. Moreover, to put the chiral matter into the copy $(b_4, b_5, b_6) = (1/2, 1, 1/2)$, we need to impose eq. (4.17), or in terms of flux parameters

$$-\phi_{5F} - 3\phi_{5E_1} - 5\phi_{5\sigma} = 12\,, \quad -\phi_{6F} - 3\phi_{6E_1} - 5\phi_{6\sigma} = 6\,. \tag{5.11}$$

We also need to avoid light vector-like exotics in the bulk copy $(b_4, b_5, b_6) = (2, 2, 1)$. Eq. (4.16) then leads to

$$\phi_{5F} + \phi_{5E_1} + \phi_{5E_2} = 0\,. \tag{5.12}$$

These are mild linear constraints on the flux parameters $\phi_{5\alpha}, \phi_{6\alpha}$. Although there are multiple solutions to these linear constraints, we should seek for solutions with minimal tadpole. By a brute force search, we find that one of the optimal solutions is

$$(\phi_{5\sigma}, \phi_{5E_1}, \phi_{5E_2}, \phi_{5F}, \phi_{6\sigma}, \phi_{6E_1}, \phi_{6E_2}, \phi_{6F}) = \left(0, -5, 2, 3, 1, -\frac{7}{2}, 1, -\frac{1}{2}\right)\,, \tag{5.13}$$

which consistently stabilizes the Kähler moduli at $t_1 = t_2 + 3t_5 = t_3 + t_5 = 3t_4$. Together with n_I, this vertical flux gives a tadpole

$$\frac{1}{2}\left[G_4^{\text{vert}}\right] \cdot \left[G_4^{\text{vert}}\right] = 32\,. \tag{5.14}$$

As a comparison, if we do not impose eqs. (4.16) and (4.17) i.e. primitivity is the only constraint on $\phi_{5\alpha}, \phi_{6\alpha}$, the minimal tadpole is

$$\frac{1}{2}\left[G_4^{\text{vert}}\right] \cdot \left[G_4^{\text{vert}}\right] = 20\,, \tag{5.15}$$

given by e.g.

$$(\phi_{5\sigma}, \phi_{5E_1}, \phi_{5E_2}, \phi_{5F}, \phi_{6\sigma}, \phi_{6E_1}, \phi_{6E_2}, \phi_{6F}) = \left(0, -4, -1, 5, 1, -\frac{5}{2}, -1, \frac{3}{2}\right)\,. \tag{5.16}$$

Therefore, we see that the vertical flux we need to turn on is slightly non-generic.

Let us now turn to remainder flux. It can be shown that Σ is a del Pezzo surface dP_6 and supports remainder flux. First recall that Σ is a hypersurface in A with class $\sigma + 2E_1 + 2E_2 + 3F$. In other words, Σ is the vanishing locus

$$xP + yP' = 0\,, \tag{5.17}$$

in A, where P, P' are sections of $\mathcal{O}_A(2E_1 + 2E_2 + 3F), \mathcal{O}_A(E_1 + E_2 + 2F)$ respectively, and x, y are the homogeneous coordinates of the $\mathbb{P}^1$ in A. For generic points in the dP_2, eq. (5.17) has a unique solution, representing a single point in $\mathbb{P}^1$. On the other hand, there are $(2e_1 + 2e_2 + 3f) \cdot (e_1 + e_2 + 2f) = 4$ points in dP_2 such that $P = P' = 0$, and eq. (5.17) represents the whole $\mathbb{P}^1$. Therefore, the geometry of Σ is dP_2 blown up in 4 generic points i.e. a dP_6.

$(\mathbf{1},\mathbf{2})_{-1/2,3,2,1}$	18	$(\mathbf{1},\mathbf{2})_{1/2,-3,-2,-1}$	**18**
$(\mathbf{1},\mathbf{2})_{-1/2,-2,-2,-1}$	**4**	$(\mathbf{1},\mathbf{2})_{1/2,2,2,1}$	4
$(\mathbf{1},\mathbf{2})_{-1/2,-2,-1,-1}$	19	$(\mathbf{1},\mathbf{2})_{1/2,2,1,1}$	16
$(\mathbf{1},\mathbf{2})_{-1/2,-2,-1,0}$	24	$(\mathbf{1},\mathbf{2})_{1/2,2,1,0}$	27
$(\bar{\mathbf{3}},\mathbf{1})_{1/3,3,2,1}$	21	$(\mathbf{3},\mathbf{1})_{-1/3,-3,-2,-1}$	**21**
$(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-2,-1}$	**3**	$(\mathbf{3},\mathbf{1})_{-1/3,2,2,1}$	3
$(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,-1}$	16	$(\mathbf{3},\mathbf{1})_{-1/3,2,1,1}$	13
$(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1,0}$	21	$(\mathbf{3},\mathbf{1})_{-1/3,2,1,0}$	24
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,-1}$	6	$(\mathbf{3},\mathbf{1})_{2/3,1,1,1}$	3
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1,0}$	1	$(\mathbf{3},\mathbf{1})_{2/3,1,1,0}$	4
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,0,0}$	19	$(\mathbf{3},\mathbf{1})_{2/3,1,0,0}$	19
$(\mathbf{3},\mathbf{2})_{1/6,-1,-1,-1}$	5	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6,1,1,1}$	2
$(\mathbf{3},\mathbf{2})_{1/6,-1,-1,0}$	0	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6,1,1,0}$	3
$(\mathbf{3},\mathbf{2})_{1/6,-1,0,0}$	16	$(\bar{\mathbf{3}},\mathbf{2})_{-1/6,1,0,0}$	16
$(\mathbf{3},\mathbf{2})_{-5/6,0,0,0}$	0	$(\bar{\mathbf{3}},\mathbf{2})_{5/6,0,0,0}$	0
$(\mathbf{1},\mathbf{1})_{1,-1,-1,-1}$	6	$(\mathbf{1},\mathbf{1})_{-1,1,1,1}$	3
$(\mathbf{1},\mathbf{1})_{1,-1,-1,0}$	1	$(\mathbf{1},\mathbf{1})_{-1,1,1,0}$	4
$(\mathbf{1},\mathbf{1})_{1,-1,0,0}$	15	$(\mathbf{1},\mathbf{1})_{-1,1,0,0}$	15

Table 2. The representations and multiplicities of vector-like matter originated from the adjoint **133** on the bulk of gauge divisor. Only the bold multiplicities correspond to fields interacting with the SM chiral matter without exponential suppression. All the other fields are inert vector-like exotics. Note that there are nontrivial linear relations between these numbers implied by the formulas in section 2.4.

To construct the remainder flux, notice that the four exceptional curves on Σ from blowing up dP_2 (denoted by e_3 to e_6) are all $\mathbb{P}^1$ fibers in A, hence all have the same class in B. Therefore, we can choose e.g. $C_{\text{rem}} = e_3 - e_4$ (or any difference $e_i - e_j$ for distinct $i, j = 3, 4, 5, 6$) with $C_{\text{rem}}^2 = -2$, and turn on the remainder flux specified in section 4.2. Therefore, we need a total tadpole of 36 to satisfy all the flux constraints. Using the techniques in [50], we find that $\chi(\hat{Y}) = 1176$ and $\chi(\hat{Y})/24 = 49 > 36$, so tadpole cancellation is satisfied. Unfortunately, there seems to be not much room to achieve full moduli stabilization, but the situation should improve if we consider more complicated geometries.

Having the full flux configuration, it is now straightforward to also compute the vector-like spectrum. For simplicity again we ignore the uncharged singlets. Since $C_{\mathbf{56}}$ is a $\mathbb{P}^1$, all vector-like pairs comes from the bulk of Σ. Using the formula for n_β in section 2.4, we get the vector-like spectrum as in table 2. It is clear that there are too many doublet and triplet Higgs that are not inert, and it is important to understand how the mass hierarchy is produced, such that we only see one doublet Higgs (pair) at the electroweak scale. There are also a number of light and inert vector-like exotics from the table.

In conclusion, we have obtained an explicit F-theory model with the SM gauge group from rigid E_7, three families of SM chiral matter with qualitatively standard Yukawa couplings and suppressed proton decay, and excess numbers of heavy Higgs with some doublet-triplet splitting. The flux configuration requires some but not too much fine-tuning. We emphasize again that most analysis in this section depends on the local geometry only. We expect that many of the F-theory threefold bases contain local geometries that are the same or similar to the above, so this explicit construction can be easily generalized to large class of 4D F-theory compactifications.

6 Comparing with other F-theory constructions in the literature

As we have pointed out a number of times in the previous sections, the E_7 models have many features that are distinct from previous SM-like constructions in F-theory. This distinction makes the E_7 models a new interesting class of models to be studied in depth in the future. In this section, we explain in more detail some of the specific differences between the models presented here and the F-theory models with tuned G_{SM} or SU(5) reviewed briefly in section 1, as well as the rigid E_6 GUTs.

6.1 Tuned models

There have been many SM-like F-theory constructions using tuned gauge groups such as G_{SM} or SU(5) (again, for reviews see [2, 3, 7, 8]). The most obvious difference between those constructions and the E_7 models presented here has been discussed in section 1: namely, fine-tuning of many complex structure moduli is required to obtain G_{SM} or SU(5) geometrically, while the presence of rigid E_7 only depends on the normal bundle of Σ instead of any moduli. It seems that rigid E_7 factors are relatively abundant in the landscape. Although the measure on the landscape has never been clear, a naive counting measure on the (singular) geometries suggests a large exponential dominance of geometries supporting rigid E_7 factors over those supporting tuned gauge factors. Note, however, that it is possible that certain flux choices may in some situations force complex structure moduli to a tuned locus with an enhanced gauge group; further investigation of this possibility is needed to clarify the level of tuning really involved in geometrically tuned constructions beyond the level suggested by the analysis of e.g., [21]. Due to the moduli-independence of the relevant gauge group, it may also be easier to incorporate a full analysis of moduli stabilization in the rigid models than the tuned ones.

Another significant difference regards the Yukawa couplings. In many SM-like F-theory constructions, some selection rules are required to get rid of exotic couplings. The Yukawa couplings in tuned G_{SM} or SU(5) models come from CCC-type couplings, and all the matter fields are localized on matter curves. Therefore, the selection rules are usually obtained by engineering a set of multiple matter curves where different types of matter localize separately, or some additional U(1)'s by tuning the global geometry. In the E_7 models, selection rules that remove exotic couplings automatically follow from the use of ΣCC-type couplings, and easily separate the chiral matter from vector-like matter including the Higgs. There are also approximate U(1)'s from the Stückelberg mechanism that arrive without additional tuning. Therefore, the selection rules needed to match expectations from observed physics are more easily realized in the E_7 models than in other constructions. An example is the proton decay suppression described in section 4.4.

The means of realizing the Higgs in the two classes of models is also qualitatively different. In tuned G_{SM} or SU(5), the Higgs comes from some vector-like matter on matter curves. Such a construction requires explicit specification of the sheaf cohomology groups in eq. (2.30), which are in general very hard to compute since they are moduli-dependent quantities. More exotic tools like root bundles [11–15] may also be needed in the construction. In many cases, there is no Higgs in the low-energy theory unless some further tuning is done. In SU(5), we also need the Higgs matter surfaces to have remainder components, such that the hypercharge flux is present on the Higgs curves and doublet-triplet splitting can be achieved. Generic matter surfaces, however, are purely vertical unless further tuning on moduli is done, and global examples of such scenarios are rare in the literature (see e.g. [37]). In contrast, in the E_7 models we can instead realize the Higgs as bulk vector-like matter, which generically has nonzero multiplicities that are easily calculated from the fluxes. In this situation, there are already Higgs fields with some doublet-triplet splitting without any further tuning, but the issue becomes having too many instead of too few Higgs fields. It is less clear how to make one pair of the Higgs exponentially lighter in the E_7 models, while in the tuned models there can be exactly one pair of Higgs, and thus the way to obtain the Higgs hierarchy may be clearer.

Because of the use of flux breaking and E_7, there are many further differences between these two classes of models in terms of computational abilities. First, in tuned G_{SM} or SU(5) the total chiral spectrum is controlled by one flux parameter only. In many cases the chiral indices contain large prefactors, and three generations of chiral matter cannot easily be obtained using integer fluxes, unless more nontrivial (and less completely understood) quantization conditions are used, as in [40, 43]. In the E_7 models, however, many flux parameters from vertical flux breaking contribute to the chiral indices, giving a linear Diophantine structure. As a result, it is natural to get three generations of chiral matter just by generic integer fluxes.

Specifically for SU(5), the removal of exotic $(\mathbf{3},\mathbf{2})_{-5/6}$ appears to be harder. This is because the form of hypercharge flux is completely fixed to be $\phi_{ir} \propto (2,4,6,3)$ and there is no free flux parameter like ϕ_{4r} as in the E_7 models. Therefore, there must be a factor of 5 in $c_1(L_3)$, and we need to use fractional line bundles to satisfy the condition $[c_1(L_3)]^2 = -2$ for removing the exotic $(\mathbf{3},\mathbf{2})_{-5/6}$. In contrast, as in section 4.2, this condition in E_7 is already satisfied by a fairly likely choice of integer remainder flux. On the other hand, there can be some controlled scenario in SU(5) where all the vector-like exotics are removed, while in E_7 we cannot remove most of the vector-like exotics; we can at best arrange them into inert fields.

In conclusion, we see that the E_7 models are not only more natural in the landscape, but also possess a number of phenomenological advantages over the tuned models. These models demonstrate how using naturalness as the guiding philosophy can help us discover more semi-realistic features in the landscape. On the other hand, these models still have their own shortcomings especially regarding the heavy mass spectrum, due to the lack of computational technologies.

6.2 Rigid E_6 GUTs

In [9, 10], it was proposed that the rigid construction of SM-like models works equally well for both E_7 and E_6, since these two gauge groups are similarly abundant in the landscape. While the two GUT groups share some features such as the naturalness of the gauge group and three generations of chiral matter, E_6 behaves differently when coming to Yukawa couplings.

While E_7 models do not have any CCC-type couplings, in E_6 models the gauge group only gets enhanced to E_8 at codimension-3 singularities, which are well-defined Yukawa points giving CCC-type couplings. Moreover, the branching rules from E_6 to G_{SM} including the additional U(1) charges (b_4, b_5) are

$$\begin{aligned}\mathbf{27} \to\ & (\mathbf{1},\mathbf{1})_{0,5/3,4/3}+(\mathbf{1},\mathbf{1})_{0,5/3,1/3}+(\mathbf{1},\mathbf{1})_{1,2/3,1/3}\\ &+(\mathbf{3},\mathbf{2})_{1/6,2/3,1/3}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,2/3,1/3}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-1/3,1/3}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-1/3,-2/3}\\ &+(\bar{\mathbf{3}},\mathbf{1})_{1/3,4/3,2/3}+(\mathbf{1},\mathbf{2})_{-1/2,-1/3,1/3}+(\mathbf{1},\mathbf{2})_{-1/2,-1/3,-2/3}+(\mathbf{1},\mathbf{2})_{-1/2,4/3,2/3}\,,\end{aligned} \tag{6.1}$$

$$\begin{aligned}\mathbf{78} \to\ & (\mathbf{8},\mathbf{1})_{0,0,0}+(\mathbf{1},\mathbf{3})_{0,0,0}+3\times(\mathbf{1},\mathbf{1})_{0,0,0}\\ &+[(\mathbf{1},\mathbf{1})_{0,0,1}+(\mathbf{1},\mathbf{1})_{1,-1,0}+(\mathbf{1},\mathbf{1})_{1,-1,-1}+(\mathbf{3},\mathbf{2})_{-5/6,0,0}+(\mathbf{3},\mathbf{2})_{1/6,-1,0}+(\mathbf{3},\mathbf{2})_{1/6,-1,-1}\\ &\quad+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,0}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3,-1,-1}+(\bar{\mathbf{3}},\mathbf{1})_{1/3,-2,-1}+(\mathbf{1},\mathbf{2})_{-1/2,-2,-1}+\text{conjugates}]\,.\end{aligned} \tag{6.2}$$

Unlike E_7, one can check that there is no suitable field on the bulk of Σ that can play the role of Higgs, so the Higgs must be localized on the matter curve. The Yukawa couplings in E_6 models are more similar to the tuned models and many results in section 4 do not apply to E_6 models, while E_6 models also suffer from many vector-like exotics. In this sense, E_7 models are fundamentally different from any other F-theory GUT models.

7 Conclusion

In this paper, we have studied various phenomenological aspects of E_7 GUTs in 4D F-theory compactifications. These models were proposed in [9, 10] as a large class of natural SM-like constructions, since rigid E_7 gauge factors are moduli independent and common in the F-theory landscape. Vertical and remainder fluxes are used to break E_7 to the SM gauge group, and appear fairly likely to induce three generations of SM chiral matter. Here we have shown that the use of E_7 and flux breaking also naturally implies several more phenomenologically favorable features, including suppression of proton decay, doublet-triplet splitting, and Higgs candidates with the right structure of SM Yukawa couplings due to approximate global symmetries descending from the E_7 Cartan generators. For the first time, we have written down an explicit global construction of such E_7 models that achieve all the above features. The construction of these features is qualitatively distinct from other SM-like constructions in the previous F-theory literature. In particular, only mild tuning on the discrete data of the geometry and the flux background is involved in the construction. The results in this paper give us strong hints towards SM constructions in F-theory that are both realistic and natural. In other words, these results appear to be compatible with the hypothesis that our Universe can be described as a *natural* solution in the string landscape.

These results, on the other hand, are still far from complete in realizing the full details of the Standard Model in string theory. Since the E_7 models are inherently strongly coupled and there is extensive use of fluxes, the machinery for computing continuous parameters, or at least the moduli dependence of continuous parameters, is very limited. While we can more or less fully specify the discrete data in the E_7 models, our arguments are at best qualitative when it comes to questions on Yukawa couplings, mass scales, etc. Such limitations lead to a number of shortcomings of the E_7 models, especially the unavoidable presence of (mostly inert) vector-like exotics with masses not determined. The Higgs hierarchy problem, i.e. the μ-problem, also remains unsolved in our models.

There are many challenges to answering these important questions. First, we need to develop new tools beyond the ultra-local approach [87–90] to compute the moduli dependence of various quantities. After that, we still need to understand more fundamental questions like the realization of moduli stabilization and SUSY breaking in F-theory, which by themselves are essential components of realistic model building. Solutions to these questions also involve tackling some open problems such as computing the Kähler potential in F-theory. All these tasks are particularly challenging when there is no weakly coupled type IIB limit for the E_7 models. Nonetheless, some insight into aspects like gauge coupling unification may be possible by considering the 8D world-volume theory on the E_7 7-branes, where flux breaking should have a classical (if nonperturbative) description.

There are also several extensions of the E_7 models presented in this paper that should be investigated further. Throughout the paper, we have made several strong assumptions on the geometry, such as restricting the matter curve to be $\mathbb{P}^1$, to enable more interesting computations on the discrete data. It will be interesting, although technically challenging, to relax these assumptions and explore more behavior of the E_7 models. It is also important to study the statistics of these E_7 models in the F-theory landscape. By scanning through a large set of F-theory bases and flux configurations, we can quantitatively analyze the genericity of different features of the E_7 models, which further sheds light on where our Universe sits in the landscape.

We hope to address some of these questions in future studies.

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A Toric hypersurfaces

In this appendix, we explain how to count $h^{1,1}$, or the number of divisors, of a threefold hypersurface in an ambient toric fourfold, following the general approach of Danilov and Khovanskii [95].[16] This technique is useful in section 5. To simplify the discussion, we focus on simple cases where there is a triangulation such that both the ambient space and the hypersurface are smooth. We also assume that the hypersurface does not have any base locus.

The geometry of the hypersurface can be understood from its stratification. First we look at the stratification of the ambient space. A d-dimensional toric variety is given by a disjoint union of algebraic tori $(\mathbb{C}^*)^k$, where $0 \leq k \leq d$. These algebraic tori, called strata,

[16] We thank Manki Kim for teaching us these techniques.

are associated with the cones of a toric (polyhedral) fan. To be more precise, for a toric fan Σ with n-dimensional cones $\sigma^{(n)} \in \Sigma(n)$ (where $0 \leq n \leq d$), the toric variety $\mathbb{P}_\Sigma$ is given by

$$\mathbb{P}_\Sigma = \coprod_n \coprod_{\sigma^{(n)} \in \Sigma(n)} T_{\sigma^{(n)}} \,, \quad T_{\sigma^{(n)}} \cong (\mathbb{C}^*)^{d-n} \,. \tag{A.1}$$

Notice that the unique $\sigma^{(0)}$ corresponds to the prime stratum $(C^*)^d$, which is the defining feature of toric varieties. The one-dimensional cones $\sigma^{(1)}$ are also given by the toric rays $\vec{v}$, associated with prime toric divisors $D_{\vec{v}}$.

Now consider a hypersurface Z as a divisor in $\mathbb{P}_\Sigma$. We abuse notation and use Z to also denote its divisor class:

$$Z = \sum_{\vec{v}} a_{\vec{v}} D_{\vec{v}} \,. \tag{A.2}$$

Note that the prime toric divisors are not all independent and there are multiple choices of the coefficients $a_{\vec{v}}$ for the same Z; our final results are independent of such a choice. We assume that all the strata of $\mathbb{P}_\Sigma$ intersect Z transversely. Then Z admits the following stratification

$$Z = \coprod_n \coprod_{\sigma^{(n)} \in \Sigma(n)} Z_{\sigma^{(n)}} \,, \quad Z_{\sigma^{(n)}} = Z \cap T_{\sigma^{(n)}} \,. \tag{A.3}$$

Note that the dimension of the strata $Z_{\sigma^{(n)}}$ is $d - n - 1$. To understand the geometry of $Z_{\sigma^{(n)}}$, it is useful to construct the Newton polytope Δ of Z

$$\Delta = \{\vec{m} \mid \vec{m} \cdot \vec{v} \geq -a_{\vec{v}}, \forall \vec{v} \in \Sigma(1)\} \,. \tag{A.4}$$

The Newton polytope encodes information about the holomorphic sections of the line bundle $\mathcal{O}_{\mathbb{P}_\Sigma}(Z)$. Below we restrict to the case where this line bundle is big, i.e. Δ is also d-dimensional.

The faces of Δ encode the geometry of Z in the following way. From Δ we can construct the so-called normal fan $\Sigma(\Delta)$, where each k-dimensional face $\Theta^{(k)}$ is associated with a $(d-k)$-dimensional cone in $\Sigma(\Delta)$.[17] The resulting toric variety $\mathbb{P}_{\Sigma(\Delta)}$ is a blowdown of $\mathbb{P}_\Sigma$, which is singular in general. The corresponding blowdown of Z is denoted by $Z(\Delta)$. An important fact is that $Z(\Delta)$ is an ample divisor in $\mathbb{P}_{\Sigma(\Delta)}$. Now given the one-to-one correspondence between faces of Δ and cones of $\Sigma(\Delta)$, we can write the stratification of $Z(\Delta)$ as (again, assuming all strata of $\mathbb{P}_{\Sigma(\Delta)}$ intersect $Z(\Delta)$ transversely)

$$Z(\Delta) = \coprod_k \coprod_{\Theta^{(k)}} Z_{\Theta^{(k)}} \,. \tag{A.5}$$

Note that the dimension of the strata $Z_{\Theta^{(k)}}$ is $k-1$. Now including the blowups from $Z(\Delta)$ back to Z, the stratification of Z can be written as

$$Z = Z_{\Theta^{(d)}} \coprod_{\Theta^{(d-1)}} Z_{\Theta^{(d-1)}} \coprod_{k \geq 2} \coprod_{\Theta^{(d-k)}} E_{\Theta^{(d-k)}} \times Z_{\Theta^{(d-k)}} \,, \tag{A.6}$$

where

$$E_{\Theta^{(d-k)}} = \coprod_{i=0}^{k-1} \left(\coprod (C^*)^i \right) \,, \tag{A.7}$$

[17] The explicit construction of $\Sigma(\Delta)$ is more complicated but is not important for our purpose.

is the exceptional set associated with $\Theta^{(d-k)}$ resulting from the blowups. The geometry of $Z_{\sigma^{(n)}}$ can then be seen by comparing eqs. (A.3) and (A.6).

A great advantage of studying the stratification of Z is that the Hodge numbers of Z can be computed using the Hodge-Deligne numbers together with the stratification [39, 95] (see also [96] for more recent review and applications). In our case where Z is smooth, the Hodge-Deligne numbers are just certain signed combinations of the Hodge numbers, but they behave nicely under disjoint unions and products. One can then compute the Hodge-Deligne numbers of Z by combining those of its strata, which are easy to get. Although the formulas for general Hodge numbers are more complicated, it can be shown that for $d \geq 4$, $h^{1,1}(Z)$ is simply given by counting the irreducible components of $Z_{\sigma^{(1)}}$. In terms of Δ, we should look at the faces

$$\Theta_i = \{\vec{m} \mid \vec{m} \cdot \vec{v} \geq -a_{\vec{v}}, \forall \vec{v} \neq \vec{v}_i; \vec{m} \cdot \vec{v}_i = -a_{\vec{v}_i}\} \, . \tag{A.8}$$

All Θ_i's are nontrivial when Z does not have any base locus, but they can have different dimensions and contribute differently to $h^{1,1}(Z)$:

- $\dim(\Theta_i) = 0$: $Z_{\sigma^{(1)}}$ is given by the components in $E_{\Sigma^{(0)}} \times Z_{\Sigma^{(0)}}$. For generic moduli, however, $Z_{\Sigma^{(0)}}$ is an empty set and such Θ_i does not contribute to $h^{1,1}(Z)$.

- $\dim(\Theta_i) = 1$: $Z_{\sigma^{(1)}}$ is given by the components in $E_{\Sigma^{(1)}} \times Z_{\Sigma^{(1)}}$. Notice that Z_{Θ_i} is a degree $n = l^*(\Theta_i) + 1$ hypersurface in $\mathbb{C}^*$, where l^* denotes the number of interior points. For generic moduli, this hypersurface is a collection of n points, so there are n copies of an irreducible component in $Z_{\sigma^{(1)}}$, contributing n to $h^{1,1}(Z)$.

- $\dim(\Theta_i) = k \geq 2$: $Z_{\sigma^{(1)}}$ is given by the components in $E_{\Sigma^{(k)}} \times Z_{\Sigma^{(k)}}$. We see that $Z_{\sigma^{(1)}}$ is irreducible, contributing 1 to $h^{1,1}(Z)$.

Finally, the above procedure overcounts $h^{1,1}(Z)$ by d, since there are d linear relations between prime toric divisors in $\mathbb{P}_\Sigma$. One can check that the above procedure reproduces the famous Batyrev formula for toric hypersurface Calabi-Yau manifolds [97].

For applications in section 5, it is now clear that to obtain $h^{1,1}(Z) = h^{1,1}(\mathbb{P}_\Sigma)$, we can pick Z such that $\dim(\Theta_i) \geq 2$ for all rays $\vec{v}_i$. It is straightforward to check this condition for the example in section 5.

B Flux quantization

In this appendix, we compute $c_2(\hat{Y})$ and determine the impact of flux quantization on the vertical flux parameters in section 5. The computation of $c_2(\hat{Y})$ involves an explicit choice of resolution. We consider the singular Weierstrass model in eq. (3.1), and resolve it by performing blowups. We denote

$$Y_1 \overset{(x,y,s|e_1)}{\longrightarrow} Y \, , \tag{B.1}$$

as the blowup from Y to Y_1 by the redefinition

$$x \to x e_1 \, , \quad y \to y e_1 \, , \quad s \to s e_1 \, . \tag{B.2}$$

The resulting locus $e_1 = 0$ is a divisor in the ambient space, denoted by E_1. Using the same notation, we can then write down the resolution as the following steps [50, 98, 99]:

$$\hat{Y} \xrightarrow{(e_4,e_5|e_7)} Y_6 \xrightarrow{(e_2,e_4|e_6)} Y_5 \xrightarrow{(e_2,e_3|e_5)} Y_4 \xrightarrow{(y,e_3|e_4)} Y_3 \xrightarrow{(x,e_2|e_3)} Y_2 \xrightarrow{(y,e_1|e_2)} Y_1 \xrightarrow{(x,y,s|e_1)} Y\,. \tag{B.3}$$

This resolution smooths out all singularities on Y up to codimension 3. The exceptional divisors on $\hat{Y}$ are given by

$$\begin{aligned}
D_1 &= (E_1 - E_2) \cap \hat{Y}\,,\\
D_2 &= (-E_1 + 2E_2 - E_3 - E_5 - E_6) \cap \hat{Y}\,,\\
D_3 &= (E_1 - 2E_2 + E_3 + 2E_5 + E_6 - E_7) \cap \hat{Y}\,,\\
D_4 &= E_7 \cap \hat{Y}\,,\\
D_5 &= (E_3 - E_4 - E_5) \cap \hat{Y}\,,\\
D_6 &= (-E_3 + 2E_4 + E_5 - E_6 - E_7) \cap \hat{Y}\,,\\
D_7 &= (-E_1 + 2E_2 - E_3 - 2E_5 + E_7) \cap \hat{Y}\,.
\end{aligned} \tag{B.4}$$

Using the above information, we can then compute $c_2(\hat{Y})$ using the techniques in [50, 98]. The computation involves a pushforward formula from $\hat{Y}$ to Y for the total Chern class, and homology relations to relate all $D_i \cdot D_j$ to $D_i \cdot D_\alpha$. The result is

$$\begin{aligned}
[c_2(\hat{Y})] = [c_2(B)] &+ 11\pi^* K_B^2\\
&+ (-12D_0 + 14D_1 + 30D_2 + 48D_3 + 41D_4 + 28D_5 + 17D_6 + 27D_7) \cdot \pi^* K_B\\
&+ (2D_1 + 6D_2 + 12D_3 + 12D_4 + 8D_5 + 6D_6 + 8D_7) \cdot \pi^*\Sigma\,.
\end{aligned} \tag{B.5}$$

It is known that the first row of the above is even [100]. Therefore, the potentially odd terms are $D_i \cdot \pi^* K_B$ for $i = 4, 6, 7$. To determine the parity of these terms, it is more convenient to work with their pushforward $\pi_*(D_i \cdot \pi^* K_B) = \Sigma \cdot K_B$. For the model in section 5, we calculate

$$\Sigma \cdot K_B = K_\Sigma + N_\Sigma = -\sigma_A \cdot (9F_\sigma \cdot F_{E_1} + 6F_\sigma \cdot F_{E_2} + 11F_\sigma \cdot F_F + 6F_{E_1} \cdot F_F)\,, \tag{B.6}$$

which has odd coefficients. Notice that

$$\Sigma \cdot (F_{E_1} + F_F)|_B = B \cdot \sigma_A \cdot (F_{E_1} + F_F) = \sigma_A \cdot (F_\sigma \cdot F_{E_1} + F_\sigma \cdot F_F + 2F_{E_1} \cdot F_F)\,, \tag{B.7}$$

has the same parity as $\Sigma \cdot K_B$, so the pullback

$$(D_4 + D_6 + D_7) \cdot \pi^* \left(F_{E_1} + F_F\right)|_B\,, \tag{B.8}$$

has the same parity as $[c_2(\hat{Y})]$. We see that flux quantization as in eq. (2.1) can be satisfied by turning on half-integer ϕ_{iE_1} and ϕ_{iF} for $i = 4, 6, 7$. From eq. (3.3), this is the same as half-integer flux parameters $n_{E_1}, n_F, \phi_{6E_1}, \phi_{6F}$.

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