Published for SISSA by Springer

Received: *April 11, 2023*
Accepted: *May 2, 2023*
Published: *May 16, 2023*

A supersymmetric nonlinear sigma model analogue of the ModMax theory

Sergei M. Kuzenko and I.N. McArthur

Department of Physics M013, The University of Western Australia,
35 Stirling Highway, Perth, W.A. 6009, Australia

E-mail: sergei.kuzenko@uwa.edu.au, ian.mcarthur@uwa.edu.au

Abstract: A decade ago, it was shown that associated with any model for $\mathsf{U}(1)$ duality-invariant nonlinear electrodynamics there is a unique $\mathsf{U}(1)$ duality-invariant supersymmetric nonlinear sigma model formulated in terms of chiral and complex linear superfields. Here we study the $\mathcal{N}=1$ superconformal σ-model analogue of the conformal duality-invariant electrodynamics known as the ModMax theory. We derive its dual formulation in terms of chiral superfields and show that the target space is a Kähler cone with $\mathsf{U}(1)\times\mathsf{U}(1)$ being the connected component of the isometry group.

Keywords: Sigma Models, Superspaces, Supersymmetry and Duality

ArXiv ePrint: 2303.15139

https://doi.org/10.1007/JHEP05(2023)127

JHEP05(2023)127

Contents

1 Introduction

Ten years ago, we proposed a family of $\mathsf{U}(1)$ duality-invariant $\mathcal{N}=1$ supersymmetric nonlinear sigma models [1]. Such a σ-model is realised in terms of a chiral scalar Φ, a complex linear scalar Σ, and their conjugates, $\bar{\Phi}$ and $\bar{\Sigma}$,

$$\bar{D}_{\dot{\alpha}}\Phi = 0\,, \qquad \bar{D}^2\Sigma = 0\,. \tag{1.1}$$

The action is of the form

$$S_{\rm CCL} = \int {\rm d}^4x{\rm d}^4\theta\, L(\Phi,\bar{\Phi},\Sigma,\bar{\Sigma})\,. \tag{1.2}$$

The equations of motion for Σ and Φ are

$$\bar{D}_{\dot{\alpha}}\frac{\partial L}{\partial \Sigma} = 0\,, \qquad \bar{D}^2\frac{\partial L}{\partial \Phi} = 0\,. \tag{1.3}$$

We see that the equations of motion have the same functional form as the off-shell constraints but with Φ and Σ interchanged. As a result, "duality" rotations that mix Φ and $\partial L/\partial\Sigma$, and Σ and $\partial L/\partial\Phi$, leave the constraints and the equations of motion invariant.

With the notation $X := (\Phi,\bar{\Phi},\Sigma,\bar{\Sigma})$, the $\mathsf{U}(1)$ duality-invariant supersymmetric nonlinear σ-models are characterised by continuous duality rotations of the form

$$\begin{pmatrix} \Phi' \\ \frac{\partial L(X')}{\partial \Sigma'} \end{pmatrix} = \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} \Phi \\ \frac{\partial L(X)}{\partial \Sigma} \end{pmatrix} \tag{1.4a}$$

and

$$\begin{pmatrix} \Sigma' \\ \frac{\partial L(X')}{\partial \Phi'} \end{pmatrix} = \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} \Sigma \\ \frac{\partial L(X)}{\partial \Phi} \end{pmatrix}\,. \tag{1.4b}$$

As demonstrated in [1], duality invariance of the theory implies that the Lagrangian $L(X) = L(\Phi,\bar{\Phi},\Sigma,\bar{\Sigma})$ must obey the differential equation

$$0 = \Phi\,\Sigma + \bar{\Phi}\,\bar{\Sigma} + \frac{\partial L}{\partial \Phi}\frac{\partial L}{\partial \Sigma} + \frac{\partial L}{\partial \bar{\Phi}}\frac{\partial L}{\partial \bar{\Sigma}}\,. \tag{1.5}$$

Any nonlinear σ-model of the form (1.2), which involves chiral and complex linear (CCL) superfields, has a purely chiral formulation which is obtained by performing a superfield Legendre transformation that dualises the complex linear superfield Σ and its conjugate $\bar{\Sigma}$ into a chiral scalar Ψ and its conjugate $\bar{\Psi}$. It is worth recalling its derivation. Starting from the σ-model (1.2), we introduce a first-order action

$$S_{\text{first-order}} = \int \mathrm{d}^4x\mathrm{d}^4\theta \left\{ L(\Phi,\bar{\Phi},\Sigma,\bar{\Sigma}) + \Psi\Sigma + \bar{\Psi}\bar{\Sigma} \right\}, \tag{1.6}$$

where Σ is taken to be an unconstrained complex superfield, while the Lagrange multiplier Ψ is chosen to be chiral,

$$\bar{D}_{\dot{\alpha}}\Psi = 0\,. \tag{1.7}$$

The original CCL σ-model (1.2) is obtained from (1.6) by integrating out the Lagrange multipliers Ψ and $\bar{\Psi}$. Instead, we can integrate out the auxiliary superfield Σ and its conjugate $\bar{\Sigma}$ using the corresponding equation of motion

$$\frac{\partial L}{\partial \Sigma} + \Psi = 0 \tag{1.8}$$

and the conjugate equation. This leads to the chiral formulation

$$S_{\text{chiral}} = \int \mathrm{d}^4x\mathrm{d}^4\theta\, K(\Phi,\bar{\Phi},\Psi,\bar{\Psi})\,, \tag{1.9}$$

where the corresponding Lagrangian K is the Legendre transform of L,

$$K(\Phi,\bar{\Phi},\Psi,\bar{\Psi}) = L(\Phi,\bar{\Phi},\Sigma,\bar{\Sigma}) + \Psi\Sigma + \bar{\Psi}\bar{\Sigma}\,. \tag{1.10}$$

The dual Lagrangian $K(\Phi,\bar{\Phi},\Psi,\bar{\Psi})$ is the Kähler potential for a Kähler target space.

In the chiral formulation, the condition of $\mathsf{U}(1)$ duality invariance (1.5) was shown [1] to become the requirement that $K(\Phi,\bar{\Phi},\Psi,\bar{\Psi})$ is invariant under rigid $\mathsf{U}(1)$ transformations

$$\delta\Phi = -\lambda\Psi\,, \quad \delta\Psi = \lambda\Phi\,, \qquad \lambda \in \mathbb{R}\,. \tag{1.11}$$

Thus the isometry group of the target Kähler space is nontrivial, since it contains the $\mathsf{U}(1)$ subgroup of transformations (1.11).

An important class of duality-invariant nonlinear σ-models (1.2) is described by a Lagrangian of the form

$$L(\Phi,\bar{\Phi},\Sigma,\bar{\Sigma}) = L(\omega,\bar{\omega})\,, \tag{1.12}$$

where the complex variable ω is defined by

$$\omega = \bar{\Phi}\Phi - \bar{\Sigma}\Sigma + \mathrm{i}\,(\Phi\Sigma + \bar{\Phi}\bar{\Sigma}) = (\Phi + \mathrm{i}\bar{\Sigma})\,(\bar{\Phi} + \mathrm{i}\Sigma)\,. \tag{1.13}$$

For such σ-models, the condition for duality invariance, eq. (1.5), was shown [1] to take the form

$$\mathrm{Im}\left\{ \omega - 4\,\omega \left(\frac{\partial L}{\partial \omega}\right)^2 \right\} = 0\,. \tag{1.14}$$

Building on the influential 1981 work by Gaillard and Zumino [2], the general theory of $\mathsf{U}(1)$ duality-invariant models for nonlinear electrodynamics in four dimensions was developed in the mid 1990s [3–6] and the early 2000s [7–9] (see also [10]), including the case of duality-invariant theories with higher derivatives [11]. In general, nonlinear electrodynamics is described by a Lagrangian of the form

$$L_{\rm NLED}(F_{ab}) = L(\boldsymbol{\omega}, \bar{\boldsymbol{\omega}})\,, \tag{1.15}$$

where we have introduced

$$-\,\boldsymbol{\omega} := F_{\alpha\beta}F^{\alpha\beta} = \frac{1}{4}F^{ab}F_{ab} + \frac{\rm i}{4}\,F^{ab}\tilde{F}_{ab}\,, \tag{1.16}$$

with F_{ab} being the electromagnetic field strength. The Gaillard-Zumino-Gibbons-Rasheed condition for $\mathsf{U}(1)$ duality invariance of nonlinear electrodynamics (1.15) is

$$\widetilde{G}^{ab}G_{ab} + \widetilde{F}^{ab}F_{ab} = 0\,, \qquad \widetilde{G}^{ab}(F) := \frac{1}{2}\,\varepsilon^{abcd}\,G_{cd}(F) = 2\,\frac{\partial L(F)}{\partial F_{ab}}\,, \tag{1.17}$$

originally given in [3, 5, 6].[1] As shown in [11], this condition for $\mathsf{U}(1)$ duality invariance of nonlinear electrodynamics can be expressed in the form of (1.14) with a ω replaced by $\boldsymbol{\omega}$. This means that any $\mathsf{U}(1)$ duality-invariant Lagrangian for nonlinear electrodynamics will also generate a $\mathsf{U}(1)$ duality-invariant supersymmetric σ-model by replacing $\boldsymbol{\omega}$ defined in (1.16) by ω defined in (1.13).

Three years ago, a unique conformal $\mathsf{U}(1)$ duality-invariant nonlinear electrodynamics was constructed [13] (see also [14]) and called the ModMax theory. It is described by the Lagrangian

$$L_{\rm conf}(\boldsymbol{\omega}, \bar{\boldsymbol{\omega}}) = \frac{1}{2}(\boldsymbol{\omega} + \bar{\boldsymbol{\omega}})\cosh\gamma + \sqrt{\boldsymbol{\omega}\,\bar{\boldsymbol{\omega}}}\,\sinh\gamma\,, \tag{1.18}$$

with γ a coupling constant. This model does not possess a weak-field expansion, which is why such theories had not been considered earlier.[2] The $\mathcal{N}=1$ supersymmetric extension of the ModMax theory (1.18) was constructed in [16], and alternative derivations of the resulting theory were given in [15] using the approaches advocated in [11, 17, 18]. It should be mentioned that every $\mathsf{U}(1)$ duality-invariant nonlinear electrodynamics described by the relations (1.15) and (1.17) is contained in a $\mathsf{U}(1)$ duality-invariant model for the $\mathcal{N}=1$ vector multiplet proposed in [11, 17].

2 Chiral formulation

Our aim in this paper is to study the supersymmetric σ-model analogue[3] of the ModMax theory (1.18),

$$L(\omega, \bar{\omega}) = \frac{1}{2}(\omega + \bar{\omega})\cosh\gamma + \sqrt{\omega\,\bar{\omega}}\,\sinh\gamma\,, \tag{2.1}$$

where ω is defined by (1.13). In terms of the original dynamical variables $X = (\Phi, \bar{\Phi}, \Sigma, \bar{\Sigma})$, the Lagrangian is given by

$$L(X) = (\Phi\bar{\Phi} - \Sigma\bar{\Sigma})\cosh\gamma + \sqrt{(\Phi^2 + \bar{\Sigma}^2)(\bar{\Phi}^2 + \Sigma^2)}\,\sinh\gamma\,. \tag{2.2}$$

[1] Actually, the self-duality equation was derived for the first time by Bialynicki-Birula [12], but unfortunately his work was largely unnoticed.

[2] Using the Ivanov-Zupnik formalism [7–10], this theory was re-derived in appendix A of [15].

[3] Inspired by the Australian cinematographic tradition, it seems suitable to call (2.1) the MadMax σ-model, although we will not pursue this terminology.

JHEP05(2023)127

It is a superconformal field theory. In the framework of [19], the superconformal transformation laws of Φ and Σ are given by eq. (6.1).

As discussed in the Introduction, we can perform a Legendre transformation to dualise the complex linear superfield Σ and its conjugate $\bar{\Sigma}$ into a chiral scalar Ψ and its conjugate $\bar{\Psi}$. In terms of the superfields $X_D := (\Phi, \bar{\Phi}, \Psi, \bar{\Psi})$, the dual formulation is determined by the action

$$L_D(X_D) = L(X) + \Psi\Sigma + \bar{\Psi}\bar{\Sigma}\,. \tag{2.3}$$

The equation of motion $\frac{\partial L_D(X_D)}{\partial \Sigma} = 0$ yields

$$\Psi = \bar{\Sigma}\cosh\gamma - \frac{\mathrm{i}}{2}(\Phi + \mathrm{i}\bar{\Sigma})\left(\frac{\bar{\omega}}{\omega}\right)^{\frac{1}{2}}\sinh\gamma + \frac{\mathrm{i}}{2}(\Phi - \mathrm{i}\bar{\Sigma})\left(\frac{\omega}{\bar{\omega}}\right)^{\frac{1}{2}}\sinh\gamma \tag{2.4}$$

$$= \bar{\Sigma}\cosh\gamma - \Sigma\left(\frac{\Phi^2 + \bar{\Sigma}^2}{\bar{\Phi}^2 + \Sigma^2}\right)^{\frac{1}{2}}\sinh\gamma\,. \tag{2.5}$$

Defining

$$X = \left(\frac{\Phi^2 + \bar{\Sigma}^2}{\bar{\Phi}^2 + \Sigma^2}\right)^{\frac{1}{2}} \tag{2.6}$$

and noting that $\bar{X} = \frac{1}{X}$,

$$\begin{pmatrix}\Psi\\ \bar{\Psi}\end{pmatrix} = \begin{pmatrix}\cosh\gamma & -X\sinh\gamma\\ -\frac{1}{X}\sinh\gamma & \cosh\gamma\end{pmatrix}\begin{pmatrix}\bar{\Sigma}\\ \Sigma\end{pmatrix}. \tag{2.7}$$

Inverting this relation,

$$\begin{pmatrix}\bar{\Sigma}\\ \Sigma\end{pmatrix} = \begin{pmatrix}\cosh\gamma & X\sinh\gamma\\ \frac{1}{X}\sinh\gamma & \cosh\gamma\end{pmatrix}\begin{pmatrix}\Psi\\ \bar{\Psi}\end{pmatrix}. \tag{2.8}$$

Substituting the expression above for Σ and $\bar{\Sigma}$ into the definition (2.6) of X, one obtains the quadratic equation

$$X^2 = \frac{\Phi^2 + \Psi^2}{\bar{\Phi}^2 + \bar{\Psi}^2} \quad\Rightarrow\quad X = \pm\left(\frac{\Phi^2 + \Psi^2}{\bar{\Phi}^2 + \bar{\Psi}^2}\right)^{\frac{1}{2}}. \tag{2.9}$$

This therefore allows us, using equation (2.8), to express the complex linear superfields Σ and $\bar{\Sigma}$ in terms of the chiral superfields Φ, $\bar{\Phi}$, Ψ and $\bar{\Psi}$, as required.

In order to compute the dual Lagrangian $L_D(X_D)$, we note that using the original definition (2.6) of X, we can write

$$\sqrt{\omega\,\bar{\omega}} = X\,(\bar{\Phi}^2 + \Sigma^2), \tag{2.10}$$

and equivalently,

$$\sqrt{\omega\,\bar{\omega}} = \frac{1}{X}\,(\Phi^2 + \bar{\Sigma}^2). \tag{2.11}$$

Thus we can express $\sqrt{\omega\,\bar{\omega}}$ in the symmetric form

$$\sqrt{\omega\,\bar{\omega}} = \frac{X}{2}\,(\bar{\Phi}^2 + \Sigma^2) + \frac{1}{2X}\,(\Phi^2 + \bar{\Sigma}^2). \tag{2.12}$$

JHEP05(2023)127

Substituting this into

$$L_D(X_D) = \frac{1}{2}(\omega + \bar{\omega})\, \cosh\gamma + \sqrt{\omega\,\bar{\omega}}\, \sinh\gamma + \Psi\Sigma + \bar{\Psi}\bar{\Sigma} \tag{2.13}$$

and using the expressions (2.8) for Σ and $\bar{\Sigma}$, we can obtain the Kähler potential $L_D(X_D) = K(\Phi, \bar{\Phi}, \Psi, \bar{\Psi})$. The result is

$$K(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) = (\Phi\,\bar{\Phi} + \Psi\,\bar{\Psi})\cosh\gamma + \sqrt{(\Phi^2 + \Psi^2)\,(\bar{\Phi}^2 + \bar{\Psi}^2)}\,\sinh\gamma\,. \tag{2.14}$$

The target space of the σ-model is a Kähler cone, following the terminology of [20], in particular the Kähler potential obeys the homogeneity condition

$$\left(\Phi\frac{\partial}{\partial\Phi} + \Psi\frac{\partial}{\partial\Psi}\right)K(\Phi, \bar{\Phi}, \Psi, \bar{\Psi}) = K(\Phi, \bar{\Phi}, \Psi, \bar{\Psi})\,. \tag{2.15}$$

With the notation $\phi^i = (\Phi, \Psi)$ and $\bar{\phi}^{\bar{i}} = (\bar{\Phi}, \bar{\Psi})$, the Kähler metric $g_{i\bar{j}}(\phi, \bar{\phi})$ is

$$g = (g_{i\bar{j}}) = \mathbb{1}_2 \cosh\gamma + \frac{\phi\,\phi^\dagger}{\sqrt{\phi^{\mathrm{T}}\phi\phi^\dagger\bar{\phi}}}\sinh\gamma\,, \tag{2.16}$$

where ϕ is viewed as a column-vector. A short calculation gives

$$\det(g_{i\bar{j}}) = \cosh^2\gamma + \frac{\Phi\bar{\Phi} + \Psi\bar{\Psi}}{\sqrt{(\Phi^2 + \Psi^2)\,(\bar{\Phi}^2 + \bar{\Psi}^2)}}\cosh\gamma\sinh\gamma\,. \tag{2.17}$$

For $\gamma \neq 0$, the matrix elements of (2.16) are nonsingular in the domain $\Phi^2 + \Psi^2 \neq 0$, which we identify with the target space $\mathcal{M}^4$ of the σ-model. It follows from (2.16) and (2.17) that the metric is positive definite on $\mathcal{M}^4$ provided $\gamma > 0$. It is interesting that the condition $\gamma \geq 0$ also naturally occurs for the ModMax theory [13], since for $\gamma < 0$ superluminal propagation becomes possible.

The connected component of the isometry group G of the Kähler cone $\mathcal{M}^4$ is $\mathsf{U}(1) \times \mathsf{U}(1)$. It consists of holomorphic transformations of the form

$$\phi \to g\phi\,, \qquad g = \mathrm{e}^{\mathrm{i}\alpha}\mathrm{e}^{-\mathrm{i}\lambda\sigma_2}\,, \quad \alpha, \lambda \in \mathbb{R}\,, \tag{2.18}$$

where σ_2 is one of the Pauli matrices $\sigma_I = (\sigma_1, \sigma_2, \sigma_3)$. The λ-transformation is a finite version of (1.11). The α-transformation is generated by the homothetic conformal Killing vector

$$\chi = \chi^i\partial_i + \bar{\chi}^{\bar{i}}\partial_{\bar{i}}\,, \qquad \chi^i = \phi^i\,, \qquad \partial_i = \frac{\partial}{\partial\phi^i}\,, \quad \partial_{\bar{i}} = \frac{\partial}{\partial\bar{\phi}^{\bar{i}}}\,. \tag{2.19}$$

In general, a Kähler cone possesses a homothetic conformal Killing vector with the properties [20]

$$\nabla_j\chi^i = \delta_j{}^i\,, \qquad \nabla_{\bar{j}}\chi^i = 0\,, \tag{2.20}$$

with ∇ the torsion-free covariant derivative on $\mathcal{M}^4$. In particular, χ is holomorphic. The properties of χ include the following:

$$\chi_i := g_{i\bar{j}}\bar{\chi}^{\bar{j}} = \partial_i K, \quad \chi^i\partial_i K = K \quad \Longrightarrow \quad K = g_{i\bar{j}}\chi^i\bar{\chi}^{\bar{j}}\,. \tag{2.21}$$

The Kähler potential (2.14) is positive in the domain $\Phi^2 + \Psi^2 \neq 0$.

JHEP05(2023)127

The isometry group G of $\mathcal{M}^4$ also includes a discrete holomorphic transformation that may be chosen as $g = \sigma_1$.[4] By multiplying σ_1 with certain elements of the subgroup $\mathsf{U}(1) \times \mathsf{U}(1) \subset G$, one observes that G also includes the following group elements: $\pm\sigma_I$ and $\pm \mathrm{i}\sigma_I$. In particular, the isometry group includes the non-abelian quaternion group $Q_8 = \{\pm \mathrm{i}\sigma_I, \pm \mathbb{1}_2\}$.

For completeness, we reproduce the component version of the σ-model action (1.9), see e.g. [21] for the technical details:[5]

$$S = -\int \mathrm{d}^4x \left\{ \partial^a \varphi^i g_{i\bar{j}} \partial_a \bar{\varphi}^{\bar{j}} + \mathrm{i}\lambda^{\alpha i} g_{i\bar{j}} \nabla_{\alpha\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}\bar{j}} - \hat{F}^i g_{i\bar{j}} \bar{\hat{F}}^{\bar{j}} - \frac{1}{4}(\lambda^i\lambda^j)(\bar{\lambda}^{\bar{k}}\bar{\lambda}^{\bar{l}}) R_{i\bar{k}j\bar{l}} \right\} . \tag{2.22}$$

Here we have defined the component fields of ϕ^i in the conventional way

$$\varphi^i := \phi^i| , \qquad \lambda^i_\alpha := \frac{1}{\sqrt{2}} D_\alpha \phi^i| , \qquad F^i := -\frac{1}{4} D^2 \phi^i| \tag{2.23}$$

and have made use of the complex field

$$\hat{F}^i := F^i - \frac{1}{2} \Gamma^i{}_{jk} \lambda^j \lambda^k \tag{2.24}$$

which transforms covariantly under holomorphic reparametrisations. The Kähler metric in (2.22) depends on the physical scalar fields, $g_{i\bar{j}}(\varphi, \bar{\varphi})$.

The formalism of [1] admits a natural extension to $\mathcal{N} = 1$ supergravity. In particular, the superconformal sigma model (2.2) and its dual (2.14) can be coupled to conformal supergravity.

Acknowledgments

We are grateful to Dmitri Sorokin for useful comments and suggestions. The work of SMK is supported in part by the Australian Research Council, projects DP200101944 and DP230101629.

References

[1] S.M. Kuzenko and I.N. McArthur, *Self-dual supersymmetric nonlinear sigma models*, *JHEP* **09** (2013) 042 [arXiv:1306.3407] [INSPIRE].

[2] M.K. Gaillard and B. Zumino, *Duality rotations for interacting fields*, *Nucl. Phys. B* **193** (1981) 221 [INSPIRE].

[4] The isometry group G also includes the anti-holomorphic discrete transformation $\phi \to \bar{\phi}$.

[5] We recall that the Christoffel symbols Γ^i_{jk} and the curvature tensor $R_{i\bar{j}k\bar{l}}$ are given by the expressions $\Gamma^i_{jk} = g^{i\bar{l}} \partial_j \partial_k \partial_{\bar{l}} K$ and $R_{i\bar{j}k\bar{l}} = \partial_i \partial_k \partial_{\bar{j}} \partial_{\bar{l}} K - g^{m\bar{n}} \partial_i \partial_k \partial_{\bar{n}} K \partial_{\bar{j}} \partial_{\bar{l}} \partial_m K$.

[3] G.W. Gibbons and D.A. Rasheed, *Electric-magnetic duality rotations in nonlinear electrodynamics*, Nucl. Phys. B **454** (1995) 185 [hep-th/9506035] [INSPIRE].

[4] G.W. Gibbons and D.A. Rasheed, SL$(2, R)$ *invariance of non-linear electrodynamics coupled to an axion and a dilaton*, Phys. Lett. B **365** (1996) 46 [hep-th/9509141].

[5] M.K. Gaillard and B. Zumino, *Selfduality in nonlinear electromagnetism*, Lect. Notes Phys. **509** (1998) 121 [hep-th/9705226] [INSPIRE].

[6] G.W. Gibbons and D.A. Rasheed, SL$(2, R)$ *invariance of nonlinear electrodynamics coupled to an axion and a dilaton*, Phys. Lett. B **365** (1996) 46 [hep-th/9509141] [INSPIRE].

[7] E.A. Ivanov and B.M. Zupnik, $N = 3$ *supersymmetric Born-Infeld theory*, Nucl. Phys. B **618** (2001) 3 [hep-th/0110074] [INSPIRE].

[8] E.A. Ivanov and B.M. Zupnik, *New representation for Lagrangians of selfdual nonlinear electrodynamics*, in the proceedings of the 4th *international workshop on supersymmetry and quantum symmetries: 16*th *Max Born symposium*, (2002), p. 235 [hep-th/0202203] [INSPIRE].

[9] E.A. Ivanov and B.M. Zupnik, *New approach to nonlinear electrodynamics: dualities as symmetries of interaction*, Phys. Atom. Nucl. **67** (2004) 2188 [hep-th/0303192] [INSPIRE].

[10] E.A. Ivanov and B.M. Zupnik, *Bispinor auxiliary fields in duality-invariant electrodynamics revisited*, Phys. Rev. D **87** (2013) 065023 [arXiv:1212.6637] [INSPIRE].

[11] S.M. Kuzenko and S. Theisen, *Nonlinear selfduality and supersymmetry*, Fortsch. Phys. **49** (2001) 273 [hep-th/0007231] [INSPIRE].

[12] I. Bialynicki-Birula, *Nonlinear electrodynamics: variations on a theme by Born and Infeld*, in *Quantum theory of particles and fields*, B. Jancewicz and J. Lukierski eds., World Scientific (1983), p. 31 [INSPIRE].

[13] I. Bandos, K. Lechner, D. Sorokin and P.K. Townsend, *A non-linear duality-invariant conformal extension of Maxwell's equations*, Phys. Rev. D **102** (2020) 121703 [arXiv:2007.09092] [INSPIRE].

[14] B.P. Kosyakov, *Nonlinear electrodynamics with the maximum allowable symmetries*, Phys. Lett. B **810** (2020) 135840 [arXiv:2007.13878] [INSPIRE].

[15] S.M. Kuzenko, *Superconformal duality-invariant models and $N = 4$ SYM effective action*, JHEP **09** (2021) 180 [arXiv:2106.07173] [INSPIRE].

[16] I. Bandos, K. Lechner, D. Sorokin and P.K. Townsend, *ModMax meets SUSY*, JHEP **10** (2021) 031 [arXiv:2106.07547] [INSPIRE].

[17] S.M. Kuzenko and S. Theisen, *Supersymmetric duality rotations*, JHEP **03** (2000) 034 [hep-th/0001068] [INSPIRE].

[18] S.M. Kuzenko, *Duality rotations in supersymmetric nonlinear electrodynamics revisited*, JHEP **03** (2013) 153 [arXiv:1301.5194] [INSPIRE].

[19] S.M. Kuzenko, *On superconformal projective hypermultiplets*, JHEP **12** (2007) 010 [arXiv:0710.1479] [INSPIRE].

[20] G.W. Gibbons and P. Rychenkova, *Cones, triSasakian structures and superconformal invariance*, Phys. Lett. B **443** (1998) 138 [hep-th/9809158] [INSPIRE].

[21] J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton University Press, Princeton, NJ, U.S.A. (1992) [INSPIRE].