

$B \rightarrow D_1(2420)$ and $B \rightarrow D'_1(2430)$ form factors from QCD light-cone sum rules

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ABSTRACT: We perform the first calculation of form factors in the semileptonic decays $B \rightarrow D_1(2420)\ell\nu_\ell$ and $B \rightarrow D'_1(2430)\ell\nu_\ell$ using QCD light-cone sum rules (LCSRs) with B -meson distribution amplitudes. In this calculation the c -quark mass is finite. Analytical expressions for two-particle contributions up to twist four are obtained. To disentangle the D_1 and D'_1 contributions in the LCSRs, we suggest a novel approach that introduces a combination of two interpolating currents for these charmed mesons. To fix all the parameters in the LCSRs, we use the two-point QCD sum rules for the decay constants of D_1 and D'_1 mesons augmented by a single experimental input, that is the $B \rightarrow D_1(2420)\ell\nu_\ell$ decay width. We provide numerical results for all $B \rightarrow D_1$ and $B \rightarrow D'_1$ form factors. As a byproduct, we also obtain the D_1 - and D'_1 -meson decay constants and predict the lepton-flavour universality ratios $R(D_1)$ and $R(D'_1)$.

KEYWORDS: Bottom Quarks, Semi-Leptonic Decays, Rare Decays

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Contents

1	Introduction	1
2	Two-point QCD sum rules for the 1^+ charmed mesons	3
2.1	Hadronic representation of the two-point correlators	4
2.2	OPE of the two-point correlators	6
2.3	Two-point sum rules and upper bounds	9
3	LCSRs for the $B \rightarrow D_1^{(\prime)}$ form factors	10
3.1	Hadronic representation of the B -to-vacuum correlator	10
3.2	Light-cone OPE of the B -to-vacuum correlator	12
3.3	Light-cone sum rules	14
4	Numerical analysis and predictions	15
4.1	Numerical analysis of the sum rules and f_{D_1} determination	16
4.2	Form factors, decays constants, and LFU ratios	19
5	Conclusion	22
A	B-meson distribution amplitudes	24
B	OPE coefficients and transformation to sum rule	24
C	$B \rightarrow D_1^{(\prime)} \ell \bar{\nu}$ total decay width	27

1 Introduction

The $B \rightarrow D \ell \nu_\ell$ and $B \rightarrow D^* \ell \nu_\ell$ weak semileptonic decays have been broadly studied in the last decades (for a recent review see, e.g., ref. [1]). The predictions of observables in these decays critically depend on our knowledge of the corresponding hadronic form factors. The $B \rightarrow D^{(*)}$ form factors are calculated in lattice QCD with a continuously increasing accuracy [2]. These form factors can also be obtained from QCD sum rules, where the most advanced computations are in refs. [3–5]. In addition, in the heavy-mass limit — that is for $m_c, m_b \rightarrow \infty$ — the $B \rightarrow D^{(*)}$ form factors are constrained and normalized at the zero recoil point corresponding to the maximal momentum transfer to the leptons.

Less explored from both the theoretical and the experimental sides are the B -meson semileptonic transitions to the lowest lying excited charmed mesons $D_0^*, D_1^{(\prime)}, D_2^*$ with the spin-parity $J^P = 0^+, 1^+, 2^+$, respectively. Nonetheless, an accurate knowledge of the

Meson	j	J^P	Mass [MeV]	Width [MeV]
$D_0^*(2300)$	$\frac{1}{2}$	0^+	2343 ± 10	229 ± 16
$D_1(2430) \equiv D_1'$	$\frac{1}{2}$	1^+	2412 ± 9	314 ± 29
$D_1(2420) \equiv D_1$	$\frac{3}{2}$	1^+	2422.1 ± 0.6	31.3 ± 1.9
$D_2^*(2460)$	$\frac{3}{2}$	2^+	2461.1 ± 0.8	47.3 ± 0.8

Table 1. The lowest excited charmed mesons. For definiteness, we quote the masses and total widths of electrically neutral states from [6].

form factors of these transitions is important for several reasons. Firstly, there is a long standing problem of filling the gap between the inclusive $B \rightarrow X_c \ell \nu_\ell$ width and the sum of the exclusive semileptonic widths dominated by the $B \rightarrow D \ell \nu$ and $B \rightarrow D^* \ell \nu$ modes [6]. Secondly, the constraints on the $B \rightarrow D_{0,1,2}^{(*)}$ form factors from the heavy-mass limit are weaker, since we do not have a normalization condition for these cases. Finally, the observed tension in the lepton-flavour universality (LFU) ratios $R(D)$ and $R(D^*)$ demands LFU tests in all channels of the exclusive $b \rightarrow c$ transitions, including $B \rightarrow D_{0,1,2}^{(*)} \ell \nu_\ell$. It is therefore extremely important to calculate the form factors of B transitions to each of these excited charmed mesons.

The masses and widths of the lowest four charm resonances with $J^P = 0^+, 1^+, 2^+$ are collected in table 1. In the heavy mass limit for the charm quark, e.g. within Heavy Quark Effective Theory (HQET), one indeed expects four states with one unit of orbital angular momentum. Since the heavy quark spin decouples, these states fall into two (mass degenerate) spin-symmetry doublets, differing by the total angular momentum j of light degrees of freedom. One doublet with $j = 1/2$ consists of 0^+ and 1^+ states and a second doublet with $j = 3/2$ consists of 1^+ and 2^+ states.

In this work we focus on the two 1^+ mesons D_1 and D_1' . Again, one invokes HQET where the strong transition $|j = 1/2\rangle \rightarrow |j = 1/2\rangle + \pi$ proceeds via S -wave whereas in the transition $|j = 3/2\rangle \rightarrow |j = 1/2\rangle + \pi$ only the D -wave is possible, causing a kinematical suppression. Hence, the $j = 1/2$ state has a significantly larger total width than the $j = 3/2$ state [7, 8]. It is then natural to expect that the observed narrow D_1 (broad D_1') resonance decaying to $D^* \pi$ is predominantly the $j = 3/2$ ($j = 1/2$) state. In reality, a mixing between the two HQET states inevitably takes place if one goes beyond the $m_c \rightarrow \infty$ limit. Note that the mixing pattern of D_1 and D_1' in terms of HQET states is purely non-perturbative. It has been discussed in a model dependent framework (see, e.g., ref. [9] for further details). It is then desirable to calculate the $B \rightarrow D_1$ and $B \rightarrow D_1'$ form factors in a finite c -quark mass framework.

QCD sum rules [10] have been already used to evaluate the $D_1^{(j)}$ decay constant and the $B \rightarrow D_1^{(j)}$ form factors from two-point and three-point correlators, respectively. This was done mostly in HQET (see, e.g., refs. [11–14]), where the separation of the two lowest 1^+ states from each other is straightforward, due to their different orbital angular momentum j . One has to choose an appropriate interpolating current with $j = 1/2$ or $j = 3/2$ and

consider two separate correlators. However, for a finite c -quark mass the two 1^+ states mix in the observed D_1 and D'_1 mesons, making the HQET sum rules inadequate for these mesons. Correlators with a finite c -quark mass were used in, e.g., refs. [15, 16]. Still, these calculations assume that only one 1^+ lowest state is interpolated by the conventional axial current $\bar{c}\gamma_\mu\gamma_5q$. This is however in contradiction with HQET that predicts a second 1^+ state in the same mass region, which has been confirmed by experimental data. Consequently, the assumption that only a single state is present within the duality window of a QCD sum rule cannot be justified.

QCD light-cone sum rules (LCSRs) [17–19] opened up new possibilities to calculate the B -to-charm form factors, especially their version with B -meson distribution amplitudes (DAs) proposed in refs. [20, 21]. These sum rules are derived for a finite c quark mass, which makes them suitable for our task. They can be used in principle for any charmed hadron in the final state, and not only for D and D^* mesons as in, e.g., refs. [3–5]. Nevertheless, we encounter also in this approach the problem of defining a current that interpolates only one of the 1^+ states at a time.

In this paper, we suggest a novel procedure to separate nearby resonances that supersedes the standard LCSR approach. This procedure consists in defining two independent currents with $J^P = 1^+$, which in general interpolate both the D_1 and D'_1 mesons. By finding a suitable linear combinations of these currents that interpolate only one 1^+ state at a time, we can write down the desired LCSRs. However, these linear combinations of currents depend on four unknown decay constants (one for each meson-current combination). Using two-point QCD sum rules, we are able to determine only three out of the four decay constants. The fourth one is determined a posteriori, by using the experimental measurement of the $B \rightarrow D_1\ell\nu_\ell$ decay width. After determining this remaining unknown parameter, we predict the $B \rightarrow D_1$ and $B \rightarrow D'_1$ form factors.

The rest of the paper is organized as follows. In section 2 we derive the two-point sum rules for the decay constants and fix three out of the four decay constants that enter in our calculation. In section 3 we define the dedicated interpolating currents and derive the LCSRs. In section 4 we fix the remaining unknown decay constant by fitting the expression for the $B \rightarrow D_1\ell\nu_\ell$ total decay width to its measured value. Then, we predict the $B \rightarrow D_1^{(\prime)}$ form factors, the $D_1^{(\prime)}$ -mesons decay constants, and the LFU ratios $R(D_1^{(\prime)})$. A series of appendices contain details regarding the B -meson DAs (in A), our analytical results for the LCSRs (in B), and the formulae for the $B \rightarrow D_1^{(\prime)}\ell\nu_\ell$ differential decay widths (in C).

2 Two-point QCD sum rules for the 1^+ charmed mesons

To derive the sum rules for the decay constants of the 1^+ charmed mesons, we construct the two-point correlators

$$\Pi_{\mu\nu}^{(ij)}(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \{ J_\mu^{(i)}(x) J_\nu^{(j)\dagger}(0) \} | 0 \rangle = -g_{\mu\nu} \Pi^{(ij)}(q^2) + q_\mu q_\nu \tilde{\Pi}^{(ij)}(q^2), \quad (2.1)$$

for $i, j = 1, 2$. The currents in eq. (2.1) are defined as

$$J_\mu^{(1)} = (m_c + m_q) \bar{c} \gamma_\mu \gamma_5 q, \tag{2.2}$$

$$J_\mu^{(2)} = i \bar{c} \gamma_5 \overleftrightarrow{D}_\mu q, \tag{2.3}$$

where $\overleftrightarrow{D}_\mu \equiv D_\mu - \overleftarrow{D}_\mu$ and $q = u, d$. Throughout this paper we work in the isospin limit and do not distinguish the flavors of the two light quarks, whose masses are neglected.

The two currents (2.2)–(2.3) are linearly independent since they interpolate different states: while the current $J_\mu^{(1)}$ creates from the vacuum (apart from 0^- states) 1^+ states, for which the light degrees of freedom have exclusively an angular momentum of $j = 1/2$, the current $J_\mu^{(2)}$ interpolates, in addition, 1^+ states with $j = 3/2$. In eq. (2.2) we also added a factor $(m_c + m_q)$ to the dimension-3 operator, such that both currents have the same mass dimension.

The hadronic states with spin-parity $J^P = 1^+$ contribute to both invariant amplitudes in eq. (2.1), since their contributions are proportional to the transverse combination $(q^2 g_{\mu\nu} + q_\mu q_\nu)$. Conversely, the pseudoscalar states — which start from D meson — contribute only to the amplitude multiplying the $q_\mu q_\nu$ structure.¹ Hence, we consider the invariant amplitude $\Pi^{(ij)}(q^2)$ to isolate the axial charmed meson contributions.

We also notice that the currents $J_\mu^{(1)}$ and $J_\mu^{(2)}$ interpolate both the D_1 and the D'_1 mesons. Therefore, to extract the decay constants of these mesons, we need to consider simultaneously the three independent sum rules for the relevant correlators: $\Pi^{(11)}$, $\Pi^{(22)}$, and $\Pi^{(12)}$ since $\Pi^{(12)} = \Pi^{(21)}$. We express these correlators in terms of hadronic dispersion relations in section 2.1, while we compute the same correlators using an operator product expansion (OPE) in section 2.2. These two representations of the correlators are then matched. In addition, semi-global quark-hadron duality is used to remove the contribution of the continuum and further excited states. A Borel transform is then performed to reduce the systematic uncertainty due to quark-hadron duality. The procedure up to this point is a standard one [10] with many successful applications in the literature (see, e.g., the review [22]). In this article, we extend this procedure to deal with the case where there are two mesons with very close masses in the same spin-parity channel, like the D_1 and the D'_1 mesons.

2.1 Hadronic representation of the two-point correlators

The hadronic spectrum of the correlators defined in eq. (2.1) — considering only the $J^P = 1^+$ channel — consists of the two low-lying resonances D_1 and D'_1 and the continuum with further excited states. This spectrum is markedly different from the one in a “typical” QCD sum rule with a single ground-state resonance. The currently available experimental data suggest that the well established $D_1(2420)$ resonance is narrow, whereas the $D'_1(2430)$ resonance is very broad and has a mass only about 10 MeV smaller (see table 1). Therefore, the hadronic dispersion relation for the three correlators (2.1) after performing the Borel

¹The situation similar to the QCD sum rule for the light axial meson first obtained in ref. [10]. The diagonal sum rules for (conventional) heavy-light axial currents were considered in ref. [16].

transform reads

$$\Pi_{\text{had}}^{(11)}(M^2) = f_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'}^2 m_{D_1'}^4 e^{-m_{D_1'}^2/M^2} + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(11)}(s), \quad (2.4)$$

$$\Pi_{\text{had}}^{(12)}(M^2) = f_{D_1} g_{D_1} m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'} g_{D_1'} m_{D_1'}^4 e^{-m_{D_1'}^2/M^2} + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(12)}(s), \quad (2.5)$$

$$\Pi_{\text{had}}^{(22)}(M^2) = g_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + g_{D_1'}^2 m_{D_1'}^4 e^{-m_{D_1'}^2/M^2} + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(22)}(s), \quad (2.6)$$

where $s_{\text{th}} = (m_D + 2m_\pi)^2 \simeq (m_{D^*} + m_\pi)^2$ is the lowest threshold of hadronic continuum states in this channel and M^2 is the Borel parameter. The decay constants introduced in the above equations are defined as

$$\begin{aligned} \langle 0 | J_\mu^{(1)} | D_1(q, \lambda) \rangle &= m_{D_1}^2 \varepsilon_\mu^{(D_1)} f_{D_1}, & \langle 0 | J_\mu^{(1)} | D_1'(q, \lambda) \rangle &= m_{D_1'}^2 \varepsilon_\mu^{(D_1')} f_{D_1'}, \\ \langle 0 | J_\mu^{(2)} | D_1(q, \lambda) \rangle &= m_{D_1}^2 \varepsilon_\mu^{(D_1)} g_{D_1}, & \langle 0 | J_\mu^{(2)} | D_1'(q, \lambda) \rangle &= m_{D_1'}^2 \varepsilon_\mu^{(D_1')} g_{D_1'}, \end{aligned} \quad (2.7)$$

where $\varepsilon_\mu^{(D_1^{(i)})} \equiv \varepsilon_\mu^{(D_1^{(i)})}(p, \lambda)$ is the polarization vector of the $D_1^{(i)}$ mesons. In eq. (2.4)–(2.6), we have isolated the ground-state resonances and attributed a generic hadronic spectral density $\rho_{\text{cont}}^{(ij)}(s)$ to the rest of the hadronic spectrum including continuum and excited states.

To make this hadronic representations more accurate, we take into account the large total width of the D_1' meson, replacing the zero-width resonance by a Breit-Wigner form with the energy-dependent width. After performing the Borel transform, this corresponds to the replacement of a simple exponential in eq. (2.4)–(2.6) by the following expression:

$$e^{-m_{D_1'}^2/M^2} \rightarrow \mathcal{E}(\Gamma_{D_1'}, M^2) = \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \left[\frac{1}{\pi} \frac{\sqrt{s} \Gamma_{D_1'}(s)}{(s - m_{D_1'}^2)^2 + s \Gamma_{D_1'}^2(s)} \right]. \quad (2.8)$$

The formula for the energy dependence of $\Gamma_{D_1'}(s)$ is approximated by the S -wave phase-space factor for the dominant decay channel $D_1' \rightarrow D^* \pi$ with the largest phase space:

$$\Gamma_{D_1'}(s) = \Gamma_{D_1'}^{\text{tot}} \left[\frac{\lambda^{1/2}(s, m_{D^*}^2, m_\pi^2) m_{D_1'}}{\lambda^{1/2}(m_{D_1'}^2, m_{D^*}^2, m_\pi^2) \sqrt{s}} \right], \quad (2.9)$$

where λ is the Källén function. In the narrow-width limit, that is for $\Gamma_{D_1'}^{\text{tot}} \rightarrow 0$, it is easy to show that $\mathcal{E}(\Gamma_{D_1'}, M^2) = e^{-m_{D_1'}^2/M^2}$. The ansatz (2.8) can be interpreted as a result of the resummation of the $D^* \pi$ intermediate states strongly coupled to the D_1' resonance. In other words, we effectively take into account the most important $D^* \pi$ continuum state with the lowest threshold in the hadronic spectrum of the correlator.² Since the spectral

²This interpretation of the energy-dependent width for a broad ρ resonance is explained, e.g., in ref. [23], while for a similar resonance ansatz in the LCSRs for $B \rightarrow 2\pi$ form factors see ref. [24].

density of this state spans up to $s \rightarrow \infty$, in the final sum rules the upper limit of integration over s in eq. (2.8) is replaced by the effective threshold of the quark-hadron duality. This means that the part of the $D^*\pi$ -state contribution above that threshold is subtracted as a part of the duality approximation. We do not apply the replacement (2.8) for the D_1 meson, since its width is relatively small and hence the narrow-width limit is a reasonable approximation in this case.

Finally, eq. (2.4)–(2.6) become

$$\Pi_{\text{had}}^{(11)}(M^2) = f_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'}^2 m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2) + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(11)}(s), \quad (2.10)$$

$$\Pi_{\text{had}}^{(12)}(M^2) = f_{D_1} g_{D_1} m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'} g_{D_1'} m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2) + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(12)}(s), \quad (2.11)$$

$$\Pi_{\text{had}}^{(22)}(M^2) = g_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + g_{D_1'}^2 m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2) + \int_{s_{\text{th}}}^{\infty} ds e^{-s/M^2} \rho_{\text{cont}}^{(22)}(s). \quad (2.12)$$

2.2 OPE of the two-point correlators

We compute the correlators (2.1) by expanding the time-ordered product for $x \sim 0$, that is using a short-distance OPE. The leading power contribution to this OPE is reduced to the perturbative calculation of the correlator. The higher-power terms — which are typically suppressed by inverse powers of M^2 — are given by a series of QCD vacuum condensates multiplied by the corresponding Wilson coefficient. As a result, the correlators can be decomposed as

$$\Pi_{\text{OPE}}^{(ij)}(q^2) = \Pi_{\text{pert}}^{(ij)}(q^2) + \Pi_{\text{cond}}^{(ij)}(q^2). \quad (2.13)$$

We discuss the calculation of $\Pi_{\text{pert}}^{(ij)}$ and $\Pi_{\text{cond}}^{(ij)}$ in the remainder of this section.

The spectral densities of the correlators $\Pi_{\text{pert}}^{(ij)}$, defined as $\rho^{(ij)}(s) \equiv (1/\pi)\text{Im}\Pi^{(ij)}(s)$, at leading order (LO) read

$$\rho_{\text{pert}}^{(11)}(s) = \frac{m_c^2 (s - m_c^2)^2 (m_c^2 + 2s)}{8\pi^2 s^2} \theta(s - m_c^2), \quad (2.14)$$

$$\rho_{\text{pert}}^{(12)}(s) = -\frac{m_c^2 (s - m_c^2)^3}{8\pi^2 s^2} \theta(s - m_c^2), \quad (2.15)$$

$$\rho_{\text{pert}}^{(22)}(s) = \frac{(s - m_c^2)^4}{8\pi^2 s^2} \theta(s - m_c^2). \quad (2.16)$$

The next-to-leading order (NLO) gluon radiative correction to $\Pi_{\text{pert}}^{(11)}$ is known (see, e.g., ref. [16]). This correction is numerically not large, in contrast to the case of b -flavored currents [25]. To obtain the same correction for the correlators containing the current $J_\mu^{(2)}$ one would have to perform a dedicated calculation of two-loop diagrams, which is out of

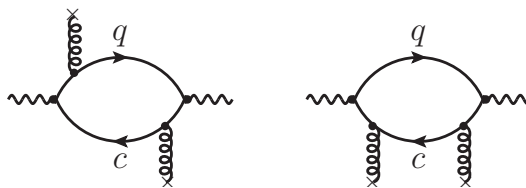


Figure 1. The diagrams with nonvanishing contributions to the gluon condensate term in the OPE of the two-point correlators. The crossed lines denote vacuum gluons.

the scope of this work. Hence, for consistency, we do not include the NLO corrections in the OPE.

The QCD vacuum condensates are organized in series with increasing mass dimension of the respective operators. We consider contributions up to and including $d = 5$ condensates:

$$\Pi_{\text{cond}}^{(ij)} = \Pi_{\bar{q}q}^{(ij)} + \Pi_{GG}^{(ij)} + \Pi_{\bar{q}Gq}^{(ij)}. \tag{2.17}$$

Here $\Pi_{\bar{q}q}^{(ij)}$, $\Pi_{GG}^{(ij)}$, and $\Pi_{\bar{q}Gq}^{(ij)}$ are the quark, gluon, and quark-gluon condensate contributions, respectively.

To calculate the $d = 3$ quark condensate terms we use the vacuum average

$$\langle 0 | \bar{q}_{\alpha i} q_{\beta}^k | 0 \rangle = \frac{1}{4N_c} \langle \bar{q}q \rangle \delta_i^k \delta_{\alpha\beta}.$$

We find

$$\Pi_{\bar{q}q}^{(11)}(q^2) = \langle \bar{q}q \rangle \frac{m_c^3}{m_c^2 - q^2}, \quad \Pi_{\bar{q}q}^{(12)}(q^2) = \Pi_{\bar{q}q}^{(22)}(q^2) = 0. \tag{2.18}$$

Note that including the next-to-leading term in the expansion of quark field at $x = 0$, namely

$$\bar{q}(x) = \bar{q}(0) + x^\mu \bar{q}(x) \overleftarrow{D}_\mu \Big|_{x=0} + \frac{1}{2} x^\mu x^\nu \bar{q}(x) \overleftarrow{D}_\mu \overleftarrow{D}_\nu \Big|_{x=0} + \dots, \tag{2.19}$$

generates contributions in eq. (2.18) proportional to the light-quark mass and hence they can be neglected.

The $d = 4$ contributions to eq. (2.18) proportional to the gluon condensate density

$$\langle GG \rangle \equiv \frac{\alpha_s}{\pi} \langle 0 | G_{\mu\nu}^a G^{\mu\nu a} | 0 \rangle$$

are conveniently calculated adopting the Fock-Schwinger gauge, defined as

$$(x - x_0)^\mu A_\mu^a(x) = 0$$

for $x_0 = 0$ (see, e.g., the reviews [26, 27]). In figure 1 we display the diagrams which provide non vanishing contributions to the gluon condensate term in the OPE. The remaining diagram where the light quark emits two gluons vanishes in the adopted limit $m_q = 0$.

Also the additional diagrams for $\Pi^{(12),(22)}$ with a gluon emitted from the vertex $J_\mu^{(2)}$ with covariant derivative are vanishing. The diagrams in figure 1 yield

$$\Pi_{GG}^{(11)}(q^2) = -\frac{1}{12}\langle GG \rangle \frac{m_c^2}{m_c^2 - q^2}, \quad (2.20)$$

$$\Pi_{GG}^{(12)}(q^2) = \frac{1}{24(q^2)^2}\langle GG \rangle m_c^2 \left((m_c^2 - 3q^2) \log \left(\frac{m_c^2}{m_c^2 - q^2} \right) - q^2 \right), \quad (2.21)$$

$$\Pi_{GG}^{(22)}(q^2) = \frac{1}{24(q^2)^2}\langle GG \rangle \left(-m_c^2 q^2 + (-4m_c^2 q^2 + m_c^4 + 3(q^2)^2) \log \left(\frac{m_c^2}{m_c^2 - q^2} \right) \right). \quad (2.22)$$

In the last equation we omit the constant terms since they vanish after performing the Borel transform. The functions in eq. (2.21)–(2.22) that multiply the gluon condensate densities develop imaginary parts at $q^2 = s > m_c^2$:

$$\rho_{GG}^{(12)}(s) = \frac{m_c^2(m_c^2 - 3s)}{24s^2} \theta(s - m_c^2), \quad (2.23)$$

$$\rho_{GG}^{(22)}(s) = \frac{(m_c^4 - 4m_c^2 s + 3s^2)}{24s^2} \theta(s - m_c^2). \quad (2.24)$$

Therefore, the gluon condensate contributions (2.21) and (2.22) can be represented in the form of an unsubtracted dispersion relation:

$$\Pi_{GG}^{(12),(22)}(q^2) = \langle GG \rangle \int_{m_c^2}^{\infty} \frac{ds}{s - q^2} \rho_{GG}^{(12),(22)}(s). \quad (2.25)$$

We treat these contributions in the same way as the perturbative contributions of eq. (2.14)–(2.16), i.e. we include them into the OPE spectral density.

Truncating the short-distance OPE at $d = 5$, we also take into account the quark-gluon condensate:

$$\langle \bar{q}Gq \rangle \equiv \langle 0 | \bar{q} g_s G_{\mu\nu}^a t^a \sigma^{\mu\nu} q | 0 \rangle = m_0^2 \langle \bar{q}q \rangle.$$

The calculation of this contribution — although technically more complicated than for the quark condensate — is well documented in the reviews such as refs. [26, 27] and hence we do not dwell on details. We only mention that here again the diagrams with a gluon emitted from the vertex containing a covariant derivative vanish. The results for the quark-gluon condensate contributions are

$$\begin{aligned} \Pi_{\bar{q}Gq}^{(11)}(q^2) &= -\frac{1}{2} m_0^2 \langle \bar{q}q \rangle \frac{m_c^5}{(m_c^2 - q^2)^3}, & \Pi_{\bar{q}Gq}^{(22)}(q^2) &= \frac{3}{8} m_0^2 \langle \bar{q}q \rangle \frac{m_c}{(m_c^2 - q^2)}, \\ \Pi_{\bar{q}Gq}^{(12)}(q^2) &= \frac{1}{12} m_0^2 \langle \bar{q}q \rangle \frac{m_c q^2}{(m_c^2 - q^2)^2}. \end{aligned} \quad (2.26)$$

Finally, we note that neglecting the $d = 6$ four-quark condensate contributions is justified because in the correlators of heavy-light currents these corrections are in general numerically negligible (for example in the sum rule for the D^* decay constant of ref. [25]).

Following the standard procedure to derive a sum rule, we perform a Borel transform of the results listed in this section. We obtain the following expressions for the OPE of the three correlators to the adopted accuracy:

$$\Pi_{\text{OPE}}^{(11)}(M^2) = \int_{m_c^2}^{\infty} ds e^{-s/M^2} \rho_{\text{pert}}^{(11)}(s) + \left[\langle \bar{q}q \rangle m_c^3 - \frac{1}{12} \langle GG \rangle m_c^2 - \frac{1}{4} m_0^2 \langle \bar{q}q \rangle \frac{m_c^5}{M^4} \right] e^{-m_c^2/M^2}, \quad (2.27)$$

$$\Pi_{\text{OPE}}^{(12)}(M^2) = \int_{m_c^2}^{\infty} ds e^{-s/M^2} \left[\rho_{\text{pert}}^{(12)}(s) + \langle GG \rangle \rho_{GG}^{(12)}(s) \right] - \frac{1}{12} m_0^2 \langle \bar{q}q \rangle m_c \left(1 - \frac{m_c^2}{M^2} \right) e^{-m_c^2/M^2}, \quad (2.28)$$

$$\Pi_{\text{OPE}}^{(22)}(M^2) = \int_{m_c^2}^{\infty} ds e^{-s/M^2} \left[\rho_{\text{pert}}^{(22)}(s) + \langle GG \rangle \rho_{GG}^{(22)}(s) \right] + \frac{3}{8} m_0^2 \langle \bar{q}q \rangle m_c e^{-m_c^2/M^2}. \quad (2.29)$$

2.3 Two-point sum rules and upper bounds

To obtain the two-point sum rules for the decay constants, we match the hadronic representation of the correlators given in section 2.1 to their respective OPE calculations given in section 2.2. We also use semi-global quark-hadron duality to remove the contributions of the continuum and excited states encoded in the functions $\rho_{\text{cont}}^{(ij)}$. This implies that the upper limit of the integrals in the OPE results (2.27)–(2.29) and, simultaneously, the upper limit in the expression for $\mathcal{E}(\Gamma_{D_1'}, M^2)$ are replaced by the respective effective thresholds $s_0^{(ij)}$, whose choice is discussed in section 4. The resulting sum rules for the three two-point correlators read

$$f_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'}^2 m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2, s_0^{(11)}) = \Pi_{\text{OPE}}^{(11)}(M^2, s_0^{(11)}), \quad (2.30)$$

$$f_{D_1} g_{D_1} m_{D_1}^4 e^{-m_{D_1}^2/M^2} + f_{D_1'} g_{D_1'} m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2, s_0^{(12)}) = \Pi_{\text{OPE}}^{(12)}(M^2, s_0^{(12)}), \quad (2.31)$$

$$g_{D_1}^2 m_{D_1}^4 e^{-m_{D_1}^2/M^2} + g_{D_1'}^2 m_{D_1'}^4 \mathcal{E}(\Gamma_{D_1'}, M^2, s_0^{(22)}) = \Pi_{\text{OPE}}^{(22)}(M^2, s_0^{(22)}). \quad (2.32)$$

Since there are only three independent sum rules for four unknown decay constants, it is not possible to determine all of them. We then consider eq. (2.30)–(2.32) as a system of equations to express $f_{D_1'}$, g_{D_1} , and $g_{D_1'}$ as a function of f_{D_1} . This system has four distinct solutions, which are discussed in detail in section 4. These decay constants serve as input parameters for the LCSRs, which are derived in the next section.

Even though we cannot calculate the individual decay constants with the information inferred from the sum rules, we can still set bounds on their value. In fact, the two diagonal correlators $\Pi_{\mu\nu}^{(11)}$ and $\Pi_{\mu\nu}^{(22)}$ have positive definite spectral densities. Hence, independent of the quark-hadron duality assumption, the following model-independent upper bounds are valid for the squared decay constants (see, e.g., ref. [28] where the upper bounds for the

$D_{(s)}$ decay constants were obtained):

$$\begin{aligned}
 f_{D_1}^2 &< \frac{\Pi_{\text{OPE}}^{(11)}(M^2)}{m_{D_1}^4 e^{-m_{D_1}^2/M^2}}, & f_{D'_1}^2 &< \frac{\Pi_{\text{OPE}}^{(11)}(M^2)}{m_{D'_1}^4 \mathcal{E}(\Gamma_{D'_1}, M^2)}, \\
 g_{D_1}^2 &< \frac{\Pi_{\text{OPE}}^{(22)}(M^2)}{m_{D_1}^4 e^{-m_{D_1}^2/M^2}}, & g_{D'_1}^2 &< \frac{\Pi_{\text{OPE}}^{(22)}(M^2)}{m_{D'_1}^4 \mathcal{E}(\Gamma_{D'_1}, M^2)}.
 \end{aligned}
 \tag{2.33}$$

The numerical value of these bounds are presented in section 4.

3 LCSRs for the $B \rightarrow D_1^{(\prime)}$ form factors

Here we derive the LCSRs with B -meson DAs for the $B \rightarrow D_1$ and $B \rightarrow D'_1$ form factors. This method is well established and has already been applied to several B -meson transitions. For the derivation of our sum rules we follow refs. [3, 5].

In analogy with the two-point sum rules considered in the previous section, we start by defining suitable B -to-vacuum correlators:

$$\mathcal{F}_{\mu\nu}^{(R)}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | \mathcal{T} \{ J_\nu^{(R)\dagger}(x), J_\mu^w(0) \} | \bar{B}(p+q) \rangle, \quad (R = D_1, D'_1). \tag{3.1}$$

Here, $J_\mu^w = \bar{c}\gamma_\mu(1 - \gamma_5)b$ is the weak current with the momentum q while $p+q$ is the momentum of the B meson state, so that $(p+q)^2 = m_B^2$. The interpolating currents $J_\mu^{(D_1)}$ or $J_\mu^{(D'_1)}$ with momentum p are chosen such that they only interpolate the D_1 or D'_1 meson, respectively. This can be easily achieved by combining the decay constants defined in eq. (2.7):

$$J_\mu^{(D_1)} = J_\mu^{(1)} - \frac{f_{D'_1}}{g_{D'_1}} J_\mu^{(2)}, \quad J_\mu^{(D'_1)} = J_\mu^{(1)} - \frac{f_{D_1}}{g_{D_1}} J_\mu^{(2)}. \tag{3.2}$$

Again, the correlators (3.1) can be both expressed in terms of hadronic quantities and computed using — in this case — a light-cone OPE. The details of these calculations are given in section 3.1 and section 3.2, respectively. The LCSRs are then obtained by matching the results of the hadronic representation and OPE calculation and using semi-global quark-hadron duality.

3.1 Hadronic representation of the B -to-vacuum correlator

To obtain the hadronic dispersion relation for the correlators (3.1), we calculate the imaginary part with respect to the variable p^2 , by inserting a complete set of hadronic states in eq. (3.1):

$$\text{Im}_{p^2} \mathcal{F}_{\text{had}, \mu\nu}^{(R)}(p, q) = \frac{1}{2} \sum_h d\tau_h (2\pi)^4 \delta^{(4)}(p_h - p) \langle 0 | J_\nu^{(R)\dagger} | \overline{h(p)} \rangle \langle h(p) | J_\mu^w | \bar{B}(p+q) \rangle, \tag{3.3}$$

where the overline denotes the sum over different polarizations of a given intermediate state h . The contributions of the D_1 and D'_1 mesons to eq. (3.3) read

$$\text{Im}_{p^2} \mathcal{F}_{\text{had}, \mu\nu}^{(R)}(p, q) = \pi \delta(s - m_R^2) \langle 0 | J_\nu^{(R)\dagger} | \overline{R(p)} \rangle \langle R(p) | J_\mu^w | \bar{B}(p+q) \rangle + \dots, \tag{3.4}$$

where the ellipsis stands for contributions from other hadronic states with the same quantum numbers. Using the definitions in eq. (2.7), one can easily find that

$$\begin{aligned}\langle 0|J_\nu^{(D_1)}|D_1(p)\rangle &= m_{D_1}^2 \varepsilon_\nu^{(D_1)} \left(f_{D_1} - \frac{f_{D_1'}}{g_{D_1}} g_{D_1} \right), \\ \langle 0|J_\nu^{(D_1')}|D_1'(p)\rangle &= m_{D_1'}^2 \varepsilon_\nu^{(D_1')} \left(f_{D_1'} - \frac{f_{D_1}}{g_{D_1}} g_{D_1'} \right),\end{aligned}\tag{3.5}$$

and, by construction,

$$\langle 0|J_\nu^{(D_1')}|D_1(p)\rangle = \langle 0|J_\nu^{(D_1)}|D_1'(p)\rangle = 0.$$

Furthermore, we express the hadronic matrix elements of the weak current in terms of the $B \rightarrow D_1^{(\prime)}$ form factors:

$$\begin{aligned}\langle R(p, \varepsilon)|J_\mu^w|\bar{B}(p+q)\rangle &= -i\varepsilon_\mu^{(R)*} (m_B + m_R) V_1^{BR}(q^2) + i(2p+q)_\mu (\varepsilon^{(R)*} \cdot q) \frac{V_2^{BR}(q^2)}{m_B + m_R} \\ &+ iq_\mu (\varepsilon^{(R)*} \cdot q) \frac{2m_R}{q^2} \left(V_3^{BR}(q^2) - V_0^{BR}(q^2) \right) - \epsilon_{\mu\nu\alpha\beta} \varepsilon^{(R)*\nu} p^\alpha q^\beta \frac{2A^{BR}(q^2)}{m_B + m_R},\end{aligned}\tag{3.6}$$

where $2m_R V_3^{BR}(q^2) = (m_B + m_R) V_1^{BR}(q^2) - (m_B - m_R) V_2^{BR}(q^2)$ and $V_0^{BR}(0) = V_3^{BR}(0)$, and we adopt the convention $\epsilon^{0123} = 1$. Note that our form factor definitions are analogous to the conventional definitions of the $B \rightarrow D^*$ form factors of, e.g., ref. [3]. There are now one axial-vector and three independent vector form factors instead of one vector and three axial-vector form factors in the $B \rightarrow D^*$ case, since the parity of the final state is opposite.

Substituting the definitions of hadronic matrix elements (3.5)–(3.6) in eq. (3.4) we obtain

$$\begin{aligned}\frac{1}{\pi} \text{Im}_{p^2} \mathcal{F}_{\text{had}, \mu\nu}^{(D_1)}(p, q) &= m_{D_1}^2 h_{D_1} \left[i \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{D_1}^2} \right) (m_B + m_{D_1}) V_1^{BD_1}(q^2) \right. \\ &- i(2p+q)_\mu \left(q_\nu - p_\nu \frac{(q \cdot p)}{m_{D_1}^2} \right) \frac{V_2^{BD_1}(q^2)}{m_B + m_{D_1}} \\ &- i \left(q_\mu q_\nu - q_\mu p_\nu \frac{(q \cdot p)}{m_{D_1}^2} \right) \frac{2m_{D_1}}{q^2} \left(V_3^{BD_1}(q^2) - V_0^{BD_1}(q^2) \right) \\ &\left. + \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \frac{2A^{BD_1}(q^2)}{m_B + m_{D_1}} \right] \delta(p^2 - m_{D_1}^2) + \dots,\end{aligned}\tag{3.7}$$

$$\begin{aligned}\frac{1}{\pi} \text{Im}_{p^2} \mathcal{F}_{\text{had}, \mu\nu}^{(D_1')}(p, q) &= m_{D_1'}^2 h_{D_1'} \left[i \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{D_1'}^2} \right) (m_B + m_{D_1'}) V_1^{BD_1'}(q^2) \right. \\ &- i(2p+q)_\mu \left(q_\nu - p_\nu \frac{(q \cdot p)}{m_{D_1'}^2} \right) \frac{V_2^{BD_1'}(q^2)}{m_B + m_{D_1'}}\end{aligned}$$

$$\begin{aligned}
 & -i \left(q_\mu q_\nu - q_\mu p_\nu \frac{(q \cdot p)}{m_{D'_1}^2} \right) \frac{2m_{D'_1}}{q^2} \left(V_3^{BD'_1}(q^2) - V_0^{BD'_1}(q^2) \right) \\
 & + \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta \frac{2A^{BD'_1}(q^2)}{m_B + m_{D'_1}} \left[\frac{1}{\pi} \frac{\sqrt{p^2} \Gamma_{D'_1}(p^2)}{(p^2 - m_{D'_1}^2)^2 + p^2 \Gamma_{D'_1}^2(p^2)} + \dots \right] \quad (3.8)
 \end{aligned}$$

Here, as in section 2.1, we take into account the width of the D'_1 state and, for the sake of brevity, introduce a notation for the two combinations of decay constants:

$$h_{D_1} \equiv \left(f_{D_1} - \frac{f_{D'_1}}{g_{D'_1}} g_{D_1} \right), \quad h_{D'_1} \equiv \left(f_{D'_1} - \frac{f_{D_1}}{g_{D_1}} g_{D'_1} \right).$$

For future convenience, we decompose the correlator $\mathcal{F}_{\mu\nu}^{(R)}(p, q)$ in a set of independent Lorentz structures $\mathcal{L}_{\mu\nu}(p, q)$ multiplied by the corresponding invariant amplitudes $\mathcal{F}_{\mathcal{L}}^{(R)}(p^2, q^2)$:

$$\mathcal{F}_{\mu\nu}^{(R)}(p, q) = \sum_{\mathcal{L}} \mathcal{L}_{\mu\nu}(p, q) \mathcal{F}_{\mathcal{L}}^{(R)}(p^2, q^2). \quad (3.9)$$

From the above decomposition we specifically choose the Lorentz structures

$$\mathcal{L}_{\mu\nu} = g_{\mu\nu}, \quad p_\mu q_\nu, \quad q_\mu q_\nu, \quad \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta, \quad (3.10)$$

since they are free from the contributions of pseudoscalar charmed mesons. Indeed, the hadronic matrix elements $\langle 0 | J_\nu^{(R)\dagger} | D(p) \rangle$ are proportional to the momentum p_ν , which does not appear in eq. (3.10). Finally, we can write down a hadronic dispersion relation for each function $\mathcal{F}_{\mathcal{L}}^{(R)}(p^2, q^2)$:

$$\mathcal{F}_{\text{had}, \mathcal{L}}^{(R)}(p^2, q^2) = \frac{1}{\pi} \int_{s_{\text{th}}}^{\infty} ds \frac{\text{Im}_{p^2} \mathcal{F}_{\text{had}, \mathcal{L}}^{(R)}(s, q^2)}{s - p^2}, \quad (3.11)$$

where s_{th} is the same threshold as in the hadronic dispersion relations for the two-point correlator considered in section 2.1.

3.2 Light-cone OPE of the B -to-vacuum correlator

The calculation of the light-cone OPE for the correlators (3.1) is analogous to the one performed in refs. [3, 5, 21] for the $B \rightarrow P$ or $B \rightarrow V$ form factors, where $P = \pi, K, D$ and $V = \rho, K^*, D^*$. The main difference with these calculations is the choice of the interpolating current, which has already been discussed at the beginning of this section.

It is more convenient to consider separately the correlators

$$\mathcal{F}_{\mu\nu}^{(i)}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | \mathcal{T} \left\{ J_\nu^{(i)\dagger}(x), J_\mu^{\text{w}}(0) \right\} | \bar{B}(p+q) \rangle, \quad (i = 1, 2) \quad (3.12)$$

which are related to the ones in eq. (3.1) through the following equations:

$$\mathcal{F}_{\mu\nu}^{(D_1)} = \mathcal{F}_{\mu\nu}^{(1)} - \frac{f_{D'_1}}{g_{D'_1}} \mathcal{F}_{\mu\nu}^{(2)}, \quad \mathcal{F}_{\mu\nu}^{(D'_1)} = \mathcal{F}_{\mu\nu}^{(1)} - \frac{f_{D_1}}{g_{D_1}} \mathcal{F}_{\mu\nu}^{(2)}. \quad (3.13)$$

We expand the correlators $\mathcal{F}_{\mu\nu}^{(i)}$ at near light-cone separations $x^2 \simeq 0$, which in momentum space implies that our calculation is valid for $p^2 \ll m_c^2$ and $q^2 \ll (m_b + m_c)^2$. Using the Wick's theorem to compute the time-ordered product in eq. (3.12) and expanding the c -quark propagator near the light-cone, we obtain

$$\mathcal{F}_{\text{OPE},\mu\nu}^{(1)}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | m_c \bar{q}(x) \gamma_\nu \gamma_5 i S_c(x, 0) \Gamma_\mu^{\text{w}} b(0) | \bar{B}(p+q) \rangle, \quad (3.14)$$

$$\mathcal{F}_{\text{OPE},\mu\nu}^{(2)}(p, q) = \int d^4x e^{ip \cdot x} \langle 0 | \bar{q}(x) \gamma_5 (D_\nu - \overleftarrow{D}_\nu) i S_c(x, 0) \Gamma_\mu^{\text{w}} b(0) | \bar{B}(p+q) \rangle, \quad (3.15)$$

where $\Gamma_\mu^{\text{w}} = \gamma_\mu(1 - \gamma_5)$. We neglect the gluon emission effects which generate quark-antiquark-gluon (three-particle) components of the B meson DAs. Indeed, their contributions turned out to be numerically irrelevant in LCSRs for the $B \rightarrow D^{(*)}$ form factors [5, 29]. We also do not take into account the $O(\alpha_s)$ corrections which can be relevant [4] but would demand involved loop diagram calculations which are out of our scope. Thus, the covariant derivatives in eq. (3.15) can be replaced with partial derivatives and in both correlation functions the free c -quark propagator can be used:

$$S_c(x, 0) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \frac{\not{k} + m_c}{k^2 - m_c^2}. \quad (3.16)$$

Using these approximations, eq. (3.14)–(3.15) can be written as

$$\mathcal{F}_{\text{OPE},\mu\nu}^{(1)}(p, q) = -m_c \int d^4x e^{ip \cdot x} \langle 0 | \bar{q}^\alpha(x) b^\beta(0) | \bar{B}(p+q) \rangle [\gamma_\nu \gamma_5 S_c(x, 0) \Gamma_\mu^{\text{w}}]_{\alpha\beta}, \quad (3.17)$$

$$\begin{aligned} \mathcal{F}_{\text{OPE},\mu\nu}^{(2)}(p, q) = & i \int d^4x e^{ip \cdot x} \left\{ \langle 0 | \bar{q}^\alpha(x) b^\beta(0) | \bar{B}(p+q) \rangle [\gamma_5 \partial_\nu S_c(x, 0) \Gamma_\mu^{\text{w}}]_{\alpha\beta} \right. \\ & \left. - \langle 0 | \bar{q}^\alpha(x) \overleftarrow{\partial}_\nu b^\beta(0) | \bar{B}(p+q) \rangle [\gamma_5 S_c(x, 0) \Gamma_\mu^{\text{w}}]_{\alpha\beta} \right\}, \end{aligned} \quad (3.18)$$

where α, β are Dirac indices. Eq. (3.18) is further simplified integrating by parts its second line:

$$\begin{aligned} \mathcal{F}_{\text{OPE},\mu\nu}^{(2)}(p, q) = & i \int d^4x e^{ip \cdot x} \langle 0 | \bar{q}^\alpha(x) b^\beta(0) | \bar{B}(p+q) \rangle \\ & \times \left\{ 2 [\gamma_5 \partial_\nu S_c(x, 0) \Gamma_\mu^{\text{w}}]_{\alpha\beta} + ip_\nu [\gamma_5 S_c(x, 0) \Gamma_\mu^{\text{w}}]_{\alpha\beta} \right\}. \end{aligned} \quad (3.19)$$

This allows us to write the correlators $\mathcal{F}_{\text{OPE},\mu\nu}^{(1)}$ and $\mathcal{F}_{\text{OPE},\mu\nu}^{(2)}$ in the same compact form

$$\mathcal{F}_{\text{OPE},\mu\nu}^{(i)}(p, q) = \int d^4x e^{ip \cdot x} \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} [\Gamma_\nu^{(i)} \gamma_5 \frac{\not{k} + m_c}{m_c^2 - k^2} \Gamma_\mu^{\text{w}}]_{\alpha\beta} \langle 0 | \bar{q}^\alpha(x) b^\beta(0) | \bar{B}(p+q) \rangle, \quad (3.20)$$

where

$$\Gamma_\nu^{(1)} = m_c \gamma_\nu, \quad \Gamma_\nu^{(2)} = p_\nu - 2k_\nu.$$

To proceed, we approximate the B -to-vacuum matrix element in the above expression by its HQET limit, replacing the b -quark field by the heavy-quark effective field with

velocity $v = (p + q)/m_B$, i.e. $b(0) \rightarrow h_v(0)$. This non-local B -to-vacuum matrix element can now be expanded in B -meson light-cone DAs of increasing twist. We emphasize that even though we apply the HQET approximation for the b quark and light degrees of freedom within B meson, we still treat the virtual c quark in the correlation functions as a full QCD object with a finite mass.

In our calculation of correlation functions we specifically use the DAs given in ref. [30] up to twist four. In addition, we include the twist-five DA g_- in the Wandzura-Wilczek limit as in ref. [5]. For the reader's convenience we collect the relevant formulae in appendix A. Using the expressions given there, taking the traces, and isolating the Lorentz structures $\mathcal{L}_{\mu\nu}$ listed in (3.10), eq. (3.20) can be written as

$$\mathcal{F}_{\text{OPE},\mathcal{L}}^{(i)}(p^2, q^2) = f_B m_B \sum_k \int_0^\infty d\sigma \frac{I_{\mathcal{L}}^{(i,k)}(\sigma, q^2)}{(p^2 - s)^k}, \quad (i = 1, 2) \quad (3.21)$$

where

$$\sigma \equiv \frac{\omega}{m_B}, \quad s(\sigma, q^2) \equiv \sigma m_B^2 + \frac{m_c^2 - \sigma q^2}{\bar{\sigma}}, \quad \bar{\sigma} \equiv 1 - \sigma. \quad (3.22)$$

The functions $I_{\mathcal{L}}^{(i,k)}$ are linear combinations of the four B -meson DAs:

$$I_{\mathcal{L}}^{(i,k)}(\sigma, q^2) = \sum_{\psi} \mathcal{C}_{\mathcal{L},\psi}^{(i,k)}(\sigma, q^2) \psi(m_B \sigma), \quad (3.23)$$

for $\psi = \phi_+, g_+, \bar{\Phi}_\pm, \bar{G}_\pm$. Our results for the coefficients $\mathcal{C}_{\mathcal{L},\psi}^{(i,k)}$ are collected in appendix B. This completes our OPE calculation for the correlators $\mathcal{F}_{\mu\nu}^{(1)}$ and $\mathcal{F}_{\mu\nu}^{(2)}$.

3.3 Light-cone sum rules

To obtain the LCSRs for the $B \rightarrow D_1^{(\prime)}$ form factors we match the hadronic representations of the correlators from section 3.1 onto their respective OPE expressions presented in section 3.2. In addition, we use semi-global quark-hadron duality to eliminate the contribution of the continuum and excited states in the hadronic dispersion relation. After adopting this approximation and performing the Borel transform, the OPE result (3.21) can be written as [5, 31]

$$\begin{aligned} \mathcal{F}_{\text{OPE},\mathcal{L}}^{(i)}(\hat{M}^2, \hat{s}_0, q^2) &= f_B m_B \sum_{k=1}^4 \frac{(-1)^k}{(k-1)!} \left\{ \int_0^{\sigma_0} d\sigma e^{-s(\sigma, q^2)/\hat{M}^2} \frac{1}{(\hat{M}^2)^{k-1}} I_{\mathcal{L}}^{(i,k)}(\sigma, q^2) \right. \\ &\quad \left. + \left[e^{-s(\sigma, q^2)/\hat{M}^2} \sum_{j=1}^{k-1} \frac{1}{(\hat{M}^2)^{k-j-1}} \frac{1}{s'} \left(\frac{d}{d\sigma} \frac{1}{s'} \right)^{j-1} I_{\mathcal{L}}^{(i,k)}(\sigma, q^2) \right]_{\sigma=\sigma_0} \right\}, \quad (i = 1, 2), \quad (3.24) \end{aligned}$$

where \hat{s}_0 is the LCSRs effective threshold. We assume a universal duality interval for all LCSRs. In the equation above, we have introduced the following notation:

$$\begin{aligned} \left(\frac{d}{d\sigma} \frac{1}{s'} \right)^n f(\sigma) &\equiv \left(\frac{d}{d\sigma} \frac{1}{s'} \left(\frac{d}{d\sigma} \frac{1}{s'} \cdots f(\sigma) \right) \right), & s' &\equiv \frac{ds}{d\sigma}, \\ \sigma(q^2, s) &= \frac{m_B^2 - q^2 + s - \sqrt{4(m_c^2 - s)m_B^2 + (m_B^2 - q^2 + s)^2}}{2m_B^2}, & \sigma_0 &\equiv \sigma(q^2, \hat{s}_0). \end{aligned}$$

Finally, we can write down the LCSRs $B \rightarrow D_1$ and $B \rightarrow D'_1$ form factors:

$$m_{D_1}^2 h_{D_1} \frac{2A^{BD_1}(q^2)}{m_B + m_{D_1}} e^{-m_{D_1}^2/\hat{M}^2} = \mathcal{F}_{\text{OPE}, \epsilon_{\mu\nu\rho\sigma}}^{(D_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.25)$$

$$i m_{D_1}^2 h_{D_1} (m_B + m_{D_1}) V_1^{BD_1}(q^2) e^{-m_{D_1}^2/\hat{M}^2} = \mathcal{F}_{\text{OPE}, g_{\mu\nu}}^{(D_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.26)$$

$$-i 2m_{D_1}^2 h_{D_1} \frac{V_2^{BD_1}(q^2)}{m_B + m_{D_1}} e^{-m_{D_1}^2/\hat{M}^2} = \mathcal{F}_{\text{OPE}, p_\mu q_\nu}^{(D_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.27)$$

$$-2i m_{D_1}^3 h_{D_1} \frac{V_3^{BD_1}(q^2) - V_0^{BD_1}(q^2)}{q^2} e^{-m_{D_1}^2/\hat{M}^2} = \mathcal{F}_{\text{OPE}, r_\mu q_\nu}^{(D_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.28)$$

$$m_{D'_1}^2 h_{D'_1} \frac{2A^{BD'_1}(q^2)}{m_B + m_{D'_1}} \mathcal{E}(\Gamma_{D'_1}, \hat{M}^2, \hat{s}_0) = \mathcal{F}_{\text{OPE}, \epsilon_{\mu\nu\rho\sigma}}^{(D'_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.29)$$

$$i m_{D'_1}^2 h_{D'_1} (m_B + m_{D'_1}) V_1^{BD'_1}(q^2) \mathcal{E}(\Gamma_{D'_1}, \hat{M}^2, \hat{s}_0) = \mathcal{F}_{\text{OPE}, g_{\mu\nu}}^{(D'_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.30)$$

$$-i 2m_{D'_1}^2 h_{D'_1} \frac{V_2^{BD'_1}(q^2)}{m_B + m_{D'_1}} \mathcal{E}(\Gamma_{D'_1}, \hat{M}^2, \hat{s}_0) = \mathcal{F}_{\text{OPE}, p_\mu q_\nu}^{(D'_1)}(\hat{M}^2, \hat{s}_0, q^2), \quad (3.31)$$

$$-2i m_{D'_1}^3 h_{D'_1} \frac{V_3^{BD'_1}(q^2) - V_0^{BD'_1}(q^2)}{q^2} \mathcal{E}(\Gamma_{D'_1}, \hat{M}^2, \hat{s}_0) = \mathcal{F}_{\text{OPE}, r_\mu q_\nu}^{(D'_1)}(\hat{M}^2, \hat{s}_0, q^2). \quad (3.32)$$

Here, the equations (2.8), (3.7)–(3.10), and (3.13) have been used. We have also introduced the shorthand notation

$$\mathcal{F}_{\text{OPE}, \mathcal{L}}^{(D_1)} = \mathcal{F}_{\text{OPE}, \mathcal{L}}^{(1)} - \frac{f_{D'_1}}{g_{D'_1}} \mathcal{F}_{\text{OPE}, \mathcal{L}}^{(2)}, \quad \mathcal{F}_{\text{OPE}, \mathcal{L}}^{(D'_1)} = \mathcal{F}_{\text{OPE}, \mathcal{L}}^{(1)} - \frac{f_{D_1}}{g_{D_1}} \mathcal{F}_{\text{OPE}, \mathcal{L}}^{(2)}, \quad (3.33)$$

and

$$\mathcal{F}_{\text{OPE}, r_\mu q_\nu}^{(R)} \equiv \mathcal{F}_{\text{OPE}, q_\mu q_\nu}^{(R)} - \frac{1}{2} \mathcal{F}_{\text{OPE}, p_\mu q_\nu}^{(R)}, \quad (R = D_1, D'_1). \quad (3.34)$$

Note that to extract the form factors A^{BR} , V_1^{BR} , and V_2^{BR} we have selected the Lorentz structures $\epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta$, $g_{\mu\nu}$, and $p_\mu q_\nu$, respectively. The form factor difference $V_3^{BR} - V_0^{BR}$ is extracted by taking the linear combination of the Lorentz structures $q_\mu q_\nu$ and $p_\mu q_\nu$.

4 Numerical analysis and predictions

Turning to the numerical analysis, we note that each of the LCSRs for a $B \rightarrow D_1^{(\prime)}$ form factor presented in eq. (3.25)–(3.32) depends on all four decay constants of $D_1^{(\prime)}$ -mesons. In a standard LCSR with B -meson DAs, a form factor is multiplied by a single decay constant of the final-state hadron. This decay constant can be replaced by its analytical expression inferred from the two-point sum rule. However, here we only have at our disposal three two-point sum rules (2.30)–(2.32). Hence, one of the decay constants, which we choose to be f_{D_1} , remains a free parameter to be fixed from an additional external input specified below. The two-point sum rules provide three relations that allow us to express the three decay constants $f_{D'_1}$, g_{D_1} and $g_{D'_1}$ as functions of f_{D_1} . In more details, we adopt the following procedure:

- We set the input parameters and evaluate the OPE part of the two-point sum rules in eqs. (2.30)–(2.32).
- Employing these sum rules, we express the decay constants $f_{D_1'}$, g_{D_1} , and $g_{D_1'}$ as functions of f_{D_1} , i.e. $f_{D_1'} = f_{D_1'}(f_{D_1})$, $g_{D_1} = g_{D_1}(f_{D_1})$ and $g_{D_1'} = g_{D_1'}(f_{D_1})$.
- Specifying the input parameters (including the parameters of B meson DAs, the interval of Borel mass \hat{M} and the threshold \hat{s}_0), we evaluate the OPE part of the LCSRs $\mathcal{F}_{\text{OPE},\mathcal{L}}^{(i)}$ in eq. (3.24). This is done for negative values of the momentum transfer squared, where the OPE in the adopted approximation is valid. Using conformal mapping and a z expansion, we extrapolate these results to positive q^2 values.
- The extrapolated OPE results, together with the hadron masses and expressions for the decay constants, are substituted in the LCSRs (3.25)–(3.32) yielding the $B \rightarrow D_1^{(\prime)}$ form factors as functions of f_{D_1} .
- Employing these expressions for $B \rightarrow D_1$ form factors, we compute the $B \rightarrow D_1 \ell \nu$ decay width as function of f_{D_1} . Matching this computation with the corresponding experimental value, we determine the numerical value of f_{D_1} .
- Finally, using f_{D_1} , extracted with the procedure outlined above we evaluate the numerical values of all $B \rightarrow D_1^{(\prime)}$ form factors and predict the lepton-flavour universality ratios $R(D_1^{(\prime)})$. As a byproduct, the values of other decay constants are calculated as well.

4.1 Numerical analysis of the sum rules and f_{D_1} determination

The input parameters that we use for the numerical analysis outlined above are listed in table 2. For the parameters determining the OPE such as the c -quark mass, the vacuum condensate values and the characteristics of B meson DAs, we quote their sources. Our choice of the sum-rule specific parameters deserves separate comments. For instance, for the normalization scale we use a typical interval established in the analyses of other correlation functions with a virtual c -quark. In these cases (see, e.g., refs. [25, 32]) not only the LO, as here, but also the NLO gluon radiative corrections were taken into account. Importantly, their numerical effect turned out to be mild, indicating a good convergence of perturbative series at these particular scales.

In addition, to evaluate the OPE part of the two-point sum rules, that is the r.h.s. of the eq. (2.30)–(2.32), we need to choose a suitable interval for the Borel parameter M^2 and to determine the effective thresholds $s_0^{(ij)}$. The Borel parameter has to be chosen such that both the contributions of states above $D_1^{(\prime)}$ and higher-power terms in the OPE are sufficiently suppressed. An indicator of the goodness of this interval is a mild variation of the sum rule result. For the sum rules (2.30)–(2.32) the requirements listed above are fulfilled for the interval quoted in table 2. Moreover, we find that the same interval represents a reasonable choice for the Borel parameter \hat{M}^2 in LCSRs as well.

Concerning the choice of the duality threshold, the commonly used procedure consists in taking the ratio between the derivative of a sum rule over $-1/M^2$ and the initial sum rule.

Parameter	Value/Interval	Ref.
normalization scale	$\mu = 1.5 \text{ GeV} \text{ (1.3–2.5) GeV}$	[25, 32]
c -quark mass	$m_c(\mu = 1.5 \text{ GeV}) = 1.205 \pm 0.035 \text{ GeV}$	[6]
quark condensate	$\langle \bar{q}q \rangle(\mu = 1.5 \text{ GeV}) = -(0.278 \pm 0.022 \text{ GeV})^3$	[2]
Ratio $\langle \bar{q}Gq \rangle / \langle \bar{q}q \rangle$ Gluon condensate	$m_0^2 = 0.8 \pm 0.2 \text{ GeV}^2$ $\langle GG \rangle = 0.012^{+0.006}_{-0.012} \text{ GeV}^4$	[33]
B -meson decay constant	$f_B = 189.4 \pm 1.4 \text{ MeV}$	[34]
Parameters of the B -meson DAs ³	$\lambda_B = 0.460 \pm 0.110 \text{ GeV}$ $\lambda_E^2 = 0.03 \pm 0.02 \text{ GeV}^2$ $\lambda_H^2 = 0.06 \pm 0.03 \text{ GeV}^2$	[35] [37]
Borel parameters	$M^2 = (2.5\text{--}3.5) \text{ GeV}^2, \quad \hat{M}^2 = M^2$	
Duality thresholds	$s_0^{(22)} = (7.20 \pm 0.65) \text{ GeV}^2$ $\hat{s}_0 = s_0 = s_0^{(11)} = s_0^{(12)} = s_0^{(22)}$.	

Table 2. Input values used in the numerical analysis of the two-point sum rules and the LCSRs.

For the charmed meson channel this procedure was used, for example, in the LCSRs for the $B \rightarrow D^{(*)}$ form factors. Note that here, apart from the conventional axial interpolating current, we deal with a nonstandard current with a derivative. It is therefore important to clarify if a correlator of these currents reveals a peculiar duality threshold. To find that out, we have considered the sum rule for the correlator $\Pi^{(22)}$ and applied the differentiation procedure to establish the value of $s_0^{(22)}$. The resulting interval for which the masses of the D_1 and D_1' mesons (neglecting their small difference) are reproduced, is displayed in table 2. It also turns out to be in the same ballpark as the thresholds for various two-point sum rules and LCSRs with D^* mesons (see, e.g., [3, 5, 25, 32]). Guided by this affinity and by the fact that the correlators $\Pi^{(11)}$ and $\Pi^{(12)}$ yield similar values for the threshold, we simplify our numerical analysis adopting one and the same threshold for all remaining sum rules.

Using the inputs in table 2, we evaluate the OPE part of the two-point sum rules and LCSRs in eq. (2.30)–(2.32) and (3.24), respectively. Inspecting the light-cone OPE in the LCSRs, we observe that for $q^2 < 0$ there is a strong suppression of the higher-twist contributions (of the DAs g_{\pm}). However, at $q^2 \geq 0$ these contributions almost reach the level of the lower-twist ones (of the DAs ϕ_{\pm}). To ensure a maximal predictivity of the OPE results, we follow the same strategy as in ref. [5] and calculate the OPE at negative values, choosing the points $q^2 = \{-20.0, -15.0, -10.0, -5.0\} \text{ GeV}^2$.

We then extrapolate these OPE results to the semileptonic region $0 < q^2 < (m_B - m_{D_1^{(\prime)}})^2$ using a parametrization similar to the one proposed in refs. [40]. Each function

³For the parameter λ_B we use the value obtained from a QCD sum rule in ref. [35]. This value has recently been confirmed in ref. [36]. For the parameters λ_E^2 and λ_H^2 we use the results of ref. [37], which are in agreement — due to the large uncertainties — with an independent recent calculation of ref. [38].

J^P	Form factors	$B_c(J^P)$ resonance masses [GeV]
0^+	V_0^{BR}	6.70
1^-	V_1^{BR}, V_2^{BR}	6.33
1^+	A^{BR}	6.74

Table 3. The lowest-lying $B_c(J^P)$ resonances in the form factor parametrizations. The masses are taken from the refs. [6, 39].

$\mathcal{F}_{\text{OPE},\mathcal{L}}^{(i)}$ is written as

$$\mathcal{F}_{\text{OPE},\mathcal{L}}^{(i)}(q^2) = \frac{1}{1 - \frac{q^2}{m_{J^P}^2}} \sum_{k=0}^K \alpha_{\mathcal{L}}^{(k)} [z(q^2) - z(0)]^k, \quad (4.1)$$

where

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (4.2)$$

with the parameters

$$t_+ = (m_B + m_D)^2, \quad t_0 = (m_B + m_D)(\sqrt{m_B} - \sqrt{m_D})^2. \quad (4.3)$$

The parametrization (4.1) isolates the $B_c(J^P)$ resonances located below the thresholds of the continuum $b\bar{c}$ states. The masses m_{J^P} of these resonances, listed in table 3, have been calculated in lattice QCD. One can see from eq. (3.25)–(3.32) that each Lorentz structure is related to a certain form factor, which in turn has certain spin-parity quantum numbers of the $b\bar{c}$ states in the timelike region. For the function $\mathcal{F}_{\text{OPE},r_{\mu}q\nu}^{(i)}$, which corresponds to the form factors with both the 0^+ and the 1^- states, we take for simplicity the mass m_{0^+} . Since $q_{\text{max}}^2 = (m_B - m_{D_1^{(i)}})^2 \ll m_{0^+}^2$ this assumption has no significant numerical effect. Note that we truncate the series in eq. (4.1) already at $K = 1$. Given the uncertainties of the LCSRs, a third coefficient in this expansion should not affect significantly the results of the extrapolation.

Having the OPE results in the semileptonic region, we obtain the $B \rightarrow D_1^{(\prime)}$ form factors in this region as functions of f_{D_1} using eq. (3.25)–(3.32). Then, we calculate the total semileptonic width $\Gamma(B \rightarrow D_1 \ell \nu) \equiv \Gamma_{D_1}^{\text{th}}$ as a function of f_{D_1} . The formulas for the decay width expressed via form factors are given in appendix C. We determine f_{D_1} by fitting $\Gamma_{D_1}^{\text{th}}(f_{D_1})$ to its experimental value obtained using the corrected experimental measurement quoted in ref. [41]

$$\mathcal{B}(B^+ \rightarrow \bar{D}_1^0 \ell^+ \bar{\nu}_\ell) = (0.67 \pm 0.05) \cdot 10^{-2}, \quad (\ell = e, \mu) \quad (4.4)$$

and $\tau_{B^\pm} = 1.638$ ps [6].

Note that, as we have already mentioned in section 2.3, there are four solutions expressing the decay constants $f_{D_1'}$, g_{D_1} , and $g_{D_1'}$ in terms of f_{D_1} . This is due to the fact

that eq. (2.30) and eq. (2.32) are quadratic in the decay constants. However, only two of these solutions lead to phenomenologically different results. The other two solutions can be related to the former two via a redefinition of the form-factor phase which is not observable. As a result, the two independent solutions that the fit to eq. (4.4) yields are

$$\text{sol. 1: } f_{D_1} = (60 \pm 20) \text{ MeV}, \tag{4.5}$$

$$\text{sol. 2: } f_{D_1} = (95 \pm 25) \text{ MeV}. \tag{4.6}$$

We stress that the difference between the two solutions is not only in the resulting value of f_{D_1} , but also in the functional dependence of the other decay constants and the form factors on f_{D_1} . For each of these solutions we obtain numerical results for the full set of $B \rightarrow D_1^{(\prime)}$ form factors and $D_1^{(\prime)}$ decay constants. These results are presented in the next subsection.

The twofold ambiguity that emerges in our approach could be resolved by using additional experimental data. However, the form factors obtained for both the solutions, (4.5) and (4.6), predict similar intervals of the $B \rightarrow D_1' \ell \nu_\ell$ width which is not used in the fit (see section 4.2). These intervals, within large errors, are in agreement with the measured value. Hence, the currently achieved accuracy of both LCSRs and experimental widths is not sufficient to distinguish the two solutions. Certain angular observables might be able to resolve this twofold ambiguity, due to their different dependence on the form factors in contrast to the total decay width. Alternatively, further theoretical inputs could also simplify the extraction of the $B \rightarrow D_1^{(\prime)}$ form factors within our approach. For instance, if one of the $D_1^{(\prime)}$ decay constants was known from lattice QCD, we would not need to perform any fit to extract f_{D_1} and thus we would be able to compute the form factor with no ambiguity.

Note that a systematic comparison of our results with the ones obtained in refs. [41, 42] in the framework of HQET is not straightforward and demands additional studies which are beyond our scope. The evident reason for that is that in HQET the D_1 and D_1' resonances are assumed to be pure $j = 3/2$ and $j = 1/2$ states, respectively, whereas our approach does not use this assignment. Still, comparing the slopes of differential distributions obtained in [41] with our predictions, we observe a better agreement when our form factors are obtained using “sol. 1”.

4.2 Form factors, decays constants, and LFU ratios

We predict the $B \rightarrow D_1$ and $B \rightarrow D_1'$ form factors using the OPE results obtained in the previous subsection for the two alternative solutions emerging from our analysis. The plots of these form factors are given in figure 2, where it is also possible to observe the difference between the two solutions in eq. (4.5)–(4.6) of the fit to f_{D_1} . For a practical use, we fit our form factor results to the following parametrization:

$$F^{BR}(q^2) = \frac{F^{BR}(0)}{1 - \frac{q^2}{m_{jP}^2}} \left\{ 1 + \beta_F \left[z(q^2) - z(0) \right] \right\}, \tag{4.7}$$

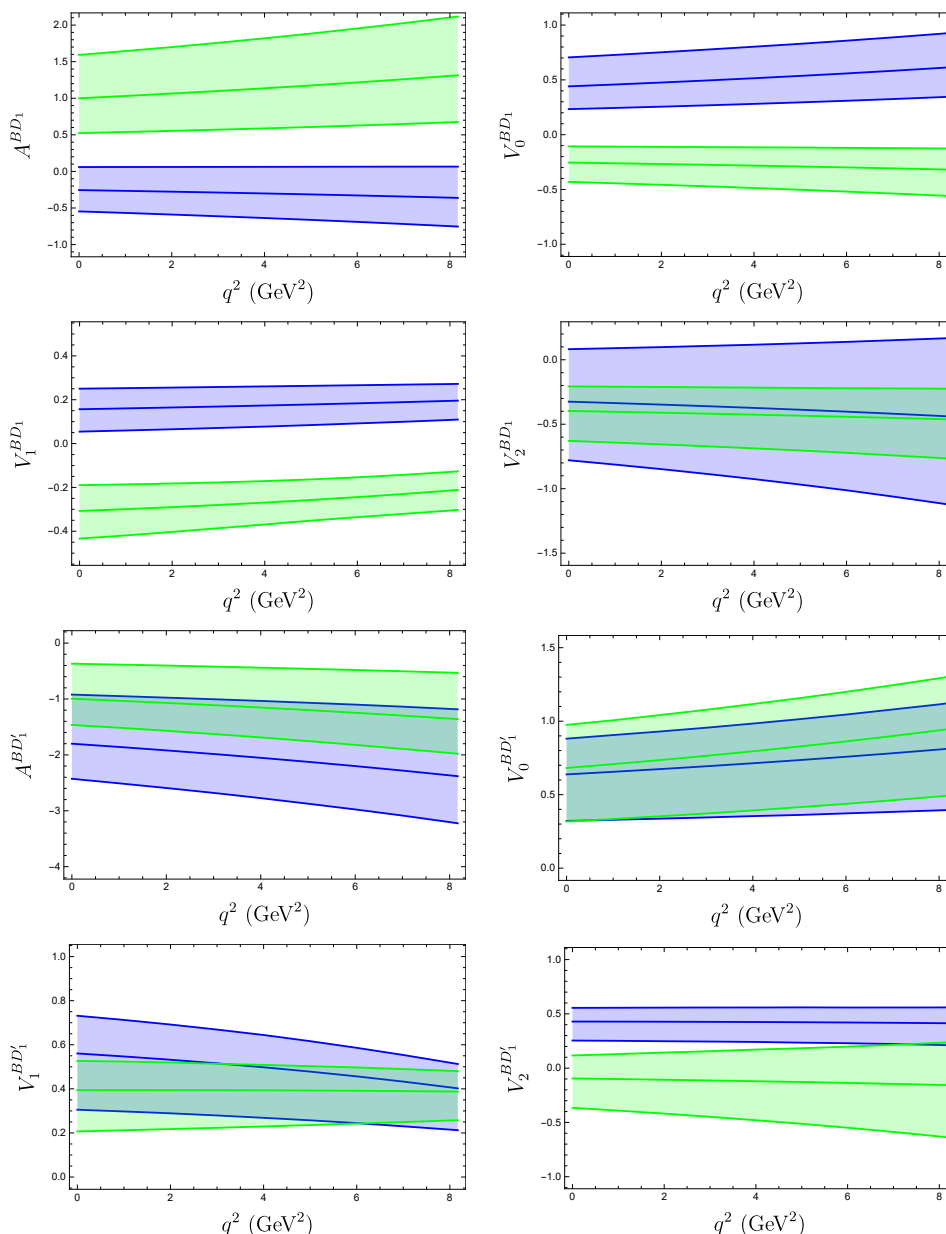


Figure 2. The q^2 -dependence of the $B \rightarrow D_1$ and $B \rightarrow D_1'$ form factors. The central values and 68% probability envelopes in blue (green) are the results obtained using sol. 1 (sol. 2).

for $F^{BR} = A^{BR}, V_0^{BR}, V_1^{BR}, V_2^{BR}$, ($R = D_1, D_1'$). Here, we use the same definition of the variable z as in eq. (4.2). The central values, the uncertainties, and the correlation of the coefficients in eq. (4.7) are given in table 4.

We predict also the decay constants $f_{D_1'}$, g_{D_1} , and $g_{D_1'}$ using the two-point sum rules derived in section 2. For the two solutions we obtain:

$$\text{sol. 1: } f_{D_1'} = -110 \pm 28 \text{ MeV}, \quad g_{D_1} = -110 \pm 38 \text{ MeV}, \quad g_{D_1'} = -22 \pm 31 \text{ MeV}. \quad (4.8)$$

$$\text{sol. 2: } f_{D_1'} = 5 \pm 34 \text{ MeV}, \quad g_{D_1} = 0 \pm 34 \text{ MeV}, \quad g_{D_1'} = -136 \pm 44 \text{ MeV}. \quad (4.9)$$

The predicted values for the decay constants together with the fitted values (4.5), (4.6) for

		$F^{BR}(0)$	β_F	Correlation
sol. 1	A^{BD_1}	-0.27 ± 0.29	-3.15 ± 1.76	0.03
	$V_0^{BD_1}$	0.44 ± 0.20	-3.41 ± 1.26	0.04
	$V_1^{BD_1}$	0.16 ± 0.10	1.69 ± 1.38	0.01
	$V_2^{BD_1}$	-0.32 ± 0.38	-4.19 ± 6.29	0.01
	$A^{BD'_1}$	-1.69 ± 0.77	-1.82 ± 0.74	0.04
	$V_0^{BD'_1}$	0.60 ± 0.32	-0.75 ± 1.21	-0.04
	$V_1^{BD'_1}$	0.53 ± 0.22	9.98 ± 1.05	-0.02
	$V_2^{BD'_1}$	0.40 ± 0.15	5.86 ± 3.42	-0.02
sol. 2	A^{BD_1}	1.00 ± 0.45	-1.82 ± 0.50	-0.06
	$V_0^{BD_1}$	-0.26 ± 0.15	-0.26 ± 1.63	0.02
	$V_1^{BD_1}$	-0.31 ± 0.12	9.32 ± 1.73	-0.02
	$V_2^{BD_1}$	-0.39 ± 0.19	2.26 ± 2.36	0.04
	$A^{BD'_1}$	-0.92 ± 0.61	-3.28 ± 1.26	-0.03
	$V_0^{BD'_1}$	0.66 ± 0.33	-3.71 ± 2.37	0.03
	$V_1^{BD'_1}$	0.37 ± 0.19	3.74 ± 3.25	0.01
	$V_2^{BD'_1}$	-0.12 ± 0.25	-5.84 ± 4.42	0.02

Table 4. The central values, 1σ uncertainties, and correlations of the coefficients of the form factor parametrization (4.7).

f_{D_1} , lead to the following observation. If the solution 1 is adopted, then the interpolating current $J_\mu^{(1)}$ has a larger overlap with the broad resonance D'_1 than with the narrow resonance D_1 . Simultaneously, the current $J_\mu^{(2)}$ with derivative of quark field has a larger decay constant with D_1 than with D'_1 . The solution 2 clearly manifests an opposite situation. The observed correlation between the structure of the interpolating currents and the decay constants deserves further investigation.

Furthermore, the values (4.9) together with the ones for f_{D_1} in eq. (4.5)–(4.6) satisfy the upper bounds given in section 2.3. Evaluating the r.h.s. of the inequalities (2.33) we obtain

$$\begin{aligned}
 |f_{D_1}| &< 181 \text{ MeV}, & |f_{D'_1}| &< 220 \text{ MeV}, \\
 |g_{D_1}| &< 266 \text{ MeV}, & |g_{D'_1}| &< 323 \text{ MeV},
 \end{aligned}
 \tag{4.10}$$

where, to stay on the conservative side, we have increased the central values of these bounds by one standard deviation.

By construction, our calculated value of the $B \rightarrow D_1 \ell \nu$ total decay width reproduces the measured one in eq. (4.4). On the other hand, our predictions of the $B \rightarrow D'_1 \ell \nu$

($\ell = e, \mu$) width converted into branching fraction are

$$\text{sol. 1: } \mathcal{B}(B^+ \rightarrow \bar{D}_1^0 \ell^+ \bar{\nu}_\ell) = (2.0_{-1.4}^{+2.8}) \cdot 10^{-2}, \quad (4.11)$$

$$\text{sol. 2: } \mathcal{B}(B^+ \rightarrow \bar{D}_1^0 \ell^+ \bar{\nu}_\ell) = (1.7_{-1.1}^{+2.1}) \cdot 10^{-2}. \quad (4.12)$$

They are still compatible with the measured value $\mathcal{B}(B^+ \rightarrow \bar{D}_1^0 \ell^+ \bar{\nu}_\ell) = (0.2 \pm 0.05) \cdot 10^{-2}$ [6, 41] but only within large uncertainties.

Currently, a very important observable for a semileptonic exclusive B decay is the corresponding lepton-flavour-universality (LFU) ratio. In our case, these ratios are defined as

$$R(D_1^{(\prime)}) = \frac{\Gamma(B \rightarrow D_1^{(\prime)} \tau \nu)}{\Gamma(B \rightarrow D_1^{(\prime)} \ell \nu)} \quad (4.13)$$

with $\ell = e, \mu$. Our predictions are the same for both solutions and read

$$R(D_1) = 0.10 \pm 0.02, \quad (4.14)$$

$$R(D_1') = 0.10 \pm 0.03. \quad (4.15)$$

The relatively small uncertainties of these results indicate a partial cancellation of parametric uncertainties in the ratios of widths. These predictions are in agreement with the results of refs. [41, 43].

5 Conclusion

In this paper, we have performed the first calculation of the $B \rightarrow D_1$ and the $B \rightarrow D_1'$ form factors using QCD light-cone sum rules (LCSRs) with B -meson distribution amplitudes (DAs).

In order to disentangle the D_1 and D_1' mesons, which are close in mass and are both 1^+ states, we have set up a novel approach that combines QCD two-point sum rules and LCSRs. This approach consists in defining two currents that interpolate both D_1 and D_1' states and finding a linear combinations of these currents, which interpolate each of these states individually. This implies that four decay constants are needed as inputs to evaluate the $B \rightarrow D_1^{(\prime)}$ form factors from the LCSRs with the properly defined currents. Using three independent two-point sum rules, we have related three decay constants to the fourth one, which we have chosen to be f_{D_1} and which is then determined a posteriori using the experimental measurement of the $B \rightarrow D_1 \ell \nu$ ($\ell = e, \mu$) branching ratio.

Our new results include analytical expressions for the diagonal and non-diagonal two-point correlators of the two interpolating currents obtained with local OPE and used to derive the two-point sum rules. In addition, we have calculated the light-cone OPE in terms of B -meson DAs for the two vacuum-to- B correlators formed by the same interpolating currents with the weak $b \rightarrow c$ current and used them to derive LCSRs.

A drawback of our approach is that it yields to a twofold ambiguity, which could be resolved in future with more precise experimental data and/or further independent inputs from theory. For example, if a lattice QCD result for one of the decay constants would

be available, we would neither have a twofold ambiguity nor need to use experimental information. Hence, it is an important, albeit challenging task for lattice QCD to isolate the narrow D_1 resonance and calculate its decay constant of at least one of the interpolating currents.

Our main numerical results include the full set of the $B \rightarrow D_1^{(\prime)}$ form factors in the whole semileptonic region. Even though these results suffer from large uncertainties caused also by the twofold ambiguity, we have found that important observables — such as the lepton flavour universality ratios $R(D_1)$ and $R(D_1')$ — can be predicted with a moderate error. It will be interesting to use our results for comparison with the future measurements of semileptonic $B \rightarrow D^* \pi \ell \nu_\ell$ decays which are accessible, e.g., with the Belle II detector. An angular analysis disentangling S and D waves of the $D^* \pi$ final state in these decays will allow to obtain more accurate information on the positions and widths of both 1^+ states. Especially important is to confirm the properties of the broad resonance D_1' which are currently less certain. Another type of processes where our predictions could be used are various nonleptonic B decays with D_1 and D_1' in the final state observed as $D^* \pi$ resonances. Here both LHCb and Belle-II measurements are possible. To give just one example: the $B_{(s)} \rightarrow D^* \pi \pi (K)$ modes where the predicted decay constants $f_{D_1^{(\prime)}}$ of the axial current can be used in the factorizable approximation to the decay amplitudes.

Future improvements of the approach suggested in this paper are possible and realistic. They include a more precise calculation of the OPE for both two-point sum rules and LCSRs, for instance by taking into account perturbative radiative corrections and, for the LCSRs, also the subdominant three-particle contributions. Moreover, an improvement of the input parameters of B -meson DAs, especially of its first inverse moment, will also help to pin down the overall parametric uncertainty of our results. Achieving this precision of OPE in future would also demand a more refined analysis of the duality thresholds. One should use a threshold fixing procedure with the inverse Borel-mass differentiation for each sum rule separately.

Furthermore, a relation to HQET in the context of QCD sum rules remains a very important issue. Starting from the correlators with a finite c -quark mass considered in this paper and expanding them in the powers of inverse mass, could provide a link to the heavy-quark limit of the resulting sum rules and to get insight into the mixing pattern of $j = 1/2$ and $j = 3/2$ states. Studying this mixing pattern could also help to resolve the twofold ambiguity of our results. We plan to return to this problem in future.

In conclusion, let us mention that the LCSR method with B meson DAs and finite c quark mass is universal enough to be used also for the B -meson transitions to the other excited charmed mesons listed in table 1, which will be another natural continuation of this work.

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A B -meson distribution amplitudes

Following the most complete analysis available for the B -meson DAs in ref. [30], we define the two-particle B -meson DAs as

$$\begin{aligned} \langle 0 | \bar{q}^\alpha(x) h_v^\beta(0) | \bar{B}(v) \rangle = & -\frac{if_B m_B}{4} \int_0^\infty d\omega \left\{ (1 + \psi) \left[\phi_+(\omega) - g_+(\omega) \partial_\lambda \partial^\lambda \right. \right. \\ & \left. \left. + \frac{1}{2} \left(\bar{\Phi}_\pm(\omega) - \bar{G}_\pm(\omega) \partial_\lambda \partial^\lambda \right) \gamma^\rho \partial_\rho \right] \gamma_5 \right\}^{\beta\alpha} e^{-il \cdot x} \Big|_{l=\omega v}. \end{aligned} \quad (\text{A.1})$$

The derivatives $\partial_\mu \equiv \partial/\partial l^\mu$ are understood to act on the hard-scattering kernel. We have also introduced the notation

$$\bar{\Phi}_\pm(\omega) \equiv \int_0^\omega d\tau (\phi_+(\tau) - \phi_-(\tau)), \quad \bar{G}_\pm(\omega) \equiv \int_0^\omega d\tau (g_+(\tau) - g_-(\tau)). \quad (\text{A.2})$$

The DAs ϕ_+ , ϕ_- , g_+ , and g_- are of twist two, three, four, and five, respectively. For the twist two, three, and four DAs we use the exponential model given in section 5.1 of ref. [30]. For the twist five DA, which is not specified in ref. [30], we use its Wandzura-Wilczek approximation given in eqs. (A.7)–(A.8) of ref. [5].

B OPE coefficients and transformation to sum rule

In this appendix we list the coefficients $\mathcal{C}_{\mathcal{L},\psi}^{(i,k)}$ defined in eq. (3.23), which enter the master formula (3.24) to compute the OPE of the correlators $\mathcal{F}_{\mathcal{L}}^{(i)}$:

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q}, \phi_+}^{(1,1)} = -\frac{m_c}{\bar{\sigma}}, \quad (\text{B.1})$$

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q}, \bar{\Phi}_\pm}^{(1,2)} = \frac{m_c^2}{\bar{\sigma}^2}, \quad (\text{B.2})$$

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q}, g_+}^{(1,2)} = -\frac{4m_c}{\bar{\sigma}^2}, \quad (\text{B.3})$$

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q}, g_+}^{(1,3)} = \frac{8m_c^3}{\bar{\sigma}^3}, \quad (\text{B.4})$$

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q}, \bar{G}_\pm}^{(1,4)} = -\frac{24m_c^4}{\bar{\sigma}^4}, \quad (\text{B.5})$$

$$\mathcal{C}_{g_{\mu\nu}, \phi_+}^{(1,1)} = \frac{im_c (\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{2\bar{\sigma}^2}, \quad (\text{B.6})$$

$$\mathcal{C}_{g_{\mu\nu},\bar{\Phi}_{\pm}}^{(1,1)} = -\frac{im_c^2}{2\bar{\sigma}^2}, \quad (\text{B.7})$$

$$\mathcal{C}_{g_{\mu\nu},\bar{\Phi}_{\pm}}^{(1,2)} = -\frac{im_c^2(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{2\bar{\sigma}^3}, \quad (\text{B.8})$$

$$\mathcal{C}_{g_{\mu\nu},g_+}^{(1,1)} = \frac{2im_c}{\bar{\sigma}^2}, \quad (\text{B.9})$$

$$\mathcal{C}_{g_{\mu\nu},g_+}^{(1,2)} = -\frac{2im_c(-\bar{\sigma}^2 m_B^2 + m_c^2 + q^2)}{\bar{\sigma}^3}, \quad (\text{B.10})$$

$$\mathcal{C}_{g_{\mu\nu},g_+}^{(1,3)} = -\frac{4im_c^3(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{\bar{\sigma}^4}, \quad (\text{B.11})$$

$$\mathcal{C}_{g_{\mu\nu},\bar{G}_{\pm}}^{(1,2)} = \frac{4im_B m_c}{\bar{\sigma}^2}, \quad (\text{B.12})$$

$$\mathcal{C}_{g_{\mu\nu},\bar{G}_{\pm}}^{(1,3)} = -\frac{4i(2\bar{\sigma} m_B - 3m_c)m_c^3}{\bar{\sigma}^4}, \quad (\text{B.13})$$

$$\mathcal{C}_{g_{\mu\nu},\bar{G}_{\pm}}^{(1,4)} = \frac{12im_c^4(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{\bar{\sigma}^5}, \quad (\text{B.14})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},\phi_+}^{(1,1)} = -\frac{2i(\bar{\sigma} - 1)m_c}{\bar{\sigma}}, \quad (\text{B.15})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},\bar{\Phi}_{\pm}}^{(1,2)} = -\frac{2i(\bar{\sigma} - 1)^2 m_B m_c}{\bar{\sigma}^2}, \quad (\text{B.16})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},g_+}^{(1,2)} = -\frac{8i(\bar{\sigma} - 1)m_c}{\bar{\sigma}^2}, \quad (\text{B.17})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},g_+}^{(1,3)} = \frac{16i(\bar{\sigma} - 1)m_c^3}{\bar{\sigma}^3}, \quad (\text{B.18})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},\bar{G}_{\pm}}^{(1,3)} = -\frac{16i(\bar{\sigma} - 1)^2 m_B m_c}{\bar{\sigma}^3}, \quad (\text{B.19})$$

$$\mathcal{C}_{q_{\mu}q_{\nu},\bar{G}_{\pm}}^{(1,4)} = \frac{48i(\bar{\sigma} - 1)^2 m_B m_c^3}{\bar{\sigma}^4}, \quad (\text{B.20})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},\phi_+}^{(1,1)} = \frac{i(1 - 2\bar{\sigma})m_c}{\bar{\sigma}}, \quad (\text{B.21})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},\bar{\Phi}_{\pm}}^{(1,2)} = -\frac{i(2m_B \bar{\sigma}^2 - 2m_B \bar{\sigma} - m_c)m_c}{\bar{\sigma}^2}, \quad (\text{B.22})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},g_+}^{(1,2)} = -\frac{4i(2\bar{\sigma} - 1)m_c}{\bar{\sigma}^2}, \quad (\text{B.23})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},g_+}^{(1,3)} = \frac{8i(2\bar{\sigma} - 1)m_c^3}{\bar{\sigma}^3}, \quad (\text{B.24})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},\bar{G}_{\pm}}^{(1,3)} = -\frac{16i(\bar{\sigma} - 1)m_B m_c}{\bar{\sigma}^2}, \quad (\text{B.25})$$

$$\mathcal{C}_{p_{\mu}q_{\nu},\bar{G}_{\pm}}^{(1,4)} = \frac{24i(2m_B \bar{\sigma}^2 - 2m_B \bar{\sigma} - m_c)m_c^3}{\bar{\sigma}^4}, \quad (\text{B.26})$$

$$\mathcal{C}_{\epsilon_{\mu\nu\rho q},\bar{\Phi}_{\pm}}^{(2,1)} = \frac{1}{\bar{\sigma}}, \quad (\text{B.27})$$

$$\mathcal{C}_{\epsilon\mu\nu\rho q, \bar{G}_\pm}^{(2,2)} = \frac{4}{\bar{\sigma}^2}, \quad (\text{B.28})$$

$$\mathcal{C}_{\epsilon\mu\nu\rho q, \bar{G}_\pm}^{(2,3)} = -\frac{8m_c^2}{\bar{\sigma}^3}, \quad (\text{B.29})$$

$$\mathcal{C}_{g\mu\nu, \bar{\Phi}_\pm}^{(2,1)} = -\frac{i(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{2\bar{\sigma}^2}, \quad (\text{B.30})$$

$$\mathcal{C}_{g\mu\nu, g_+}^{(2,1)} = \frac{4im_B}{\bar{\sigma}}, \quad (\text{B.31})$$

$$\mathcal{C}_{g\mu\nu, \bar{G}_\pm}^{(2,1)} = -\frac{2i}{\bar{\sigma}^2}, \quad (\text{B.32})$$

$$\mathcal{C}_{g\mu\nu, \bar{G}_\pm}^{(2,2)} = -\frac{2i(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B - m_c^2 - q^2)}{\bar{\sigma}^3}, \quad (\text{B.33})$$

$$\mathcal{C}_{g\mu\nu, \bar{G}_\pm}^{(2,3)} = \frac{4im_c^2(\bar{\sigma}^2 m_B^2 - 2\bar{\sigma} m_c m_B + m_c^2 - q^2)}{\bar{\sigma}^4}, \quad (\text{B.34})$$

$$\mathcal{C}_{q_\mu q_\nu, \phi_+}^{(2,1)} = -\frac{2i(\bar{\sigma} - 1)((\bar{\sigma} - 1)m_B + m_c)}{\bar{\sigma}}, \quad (\text{B.35})$$

$$\mathcal{C}_{q_\mu q_\nu, \bar{\Phi}_\pm}^{(2,1)} = \frac{4i(\bar{\sigma} - 1)}{\bar{\sigma}}, \quad (\text{B.36})$$

$$\mathcal{C}_{q_\mu q_\nu, \bar{\Phi}_\pm}^{(2,2)} = -\frac{2i(\bar{\sigma} - 1)m_c((\bar{\sigma} - 1)m_B + m_c)}{\bar{\sigma}^2}, \quad (\text{B.37})$$

$$\mathcal{C}_{q_\mu q_\nu, g_+}^{(2,2)} = -\frac{8i(\bar{\sigma} - 1)(2(\bar{\sigma} - 1)m_B + m_c)}{\bar{\sigma}^2}, \quad (\text{B.38})$$

$$\mathcal{C}_{q_\mu q_\nu, g_+}^{(2,3)} = \frac{16i(\bar{\sigma} - 1)m_c^2((\bar{\sigma} - 1)m_B + m_c)}{\bar{\sigma}^3}, \quad (\text{B.39})$$

$$\mathcal{C}_{q_\mu q_\nu, \bar{G}_\pm}^{(2,2)} = \frac{16i(\bar{\sigma} - 1)}{\bar{\sigma}^2}, \quad (\text{B.40})$$

$$\mathcal{C}_{q_\mu q_\nu, \bar{G}_\pm}^{(2,3)} = -\frac{16i(\bar{\sigma} - 1)m_c((\bar{\sigma} - 1)m_B + 2m_c)}{\bar{\sigma}^3}, \quad (\text{B.41})$$

$$\mathcal{C}_{q_\mu q_\nu, \bar{G}_\pm}^{(2,4)} = \frac{48i(\bar{\sigma} - 1)m_c^3((\bar{\sigma} - 1)m_B + m_c)}{\bar{\sigma}^4}, \quad (\text{B.42})$$

$$\mathcal{C}_{p_\mu q_\nu, \phi_+}^{(2,1)} = -\frac{2i(\bar{\sigma} - 1)(\bar{\sigma} m_B + m_c)}{\bar{\sigma}}, \quad (\text{B.43})$$

$$\mathcal{C}_{p_\mu q_\nu, \bar{\Phi}_\pm}^{(2,1)} = 4i - \frac{3i}{\bar{\sigma}}, \quad (\text{B.44})$$

$$\mathcal{C}_{p_\mu q_\nu, \bar{\Phi}_\pm}^{(2,2)} = -\frac{2i(\bar{\sigma} - 1)m_c(\bar{\sigma} m_B + m_c)}{\bar{\sigma}^2}, \quad (\text{B.45})$$

$$\mathcal{C}_{p_\mu q_\nu, g_+}^{(2,2)} = -\frac{8i(\bar{\sigma} - 1)(2\bar{\sigma} m_B + m_c)}{\bar{\sigma}^2}, \quad (\text{B.46})$$

$$\mathcal{C}_{p_\mu q_\nu, g_+}^{(2,3)} = \frac{16i(\bar{\sigma} - 1)m_c^2(\bar{\sigma} m_B + m_c)}{\bar{\sigma}^3}, \quad (\text{B.47})$$

$$\mathcal{C}_{p_\mu q_\nu, \bar{G}_\pm}^{(2,2)} = \frac{4i(4\bar{\sigma} - 3)}{\bar{\sigma}^2}, \quad (\text{B.48})$$

$$\mathcal{C}_{p_\mu q_\nu, \bar{G}_\pm}^{(2,3)} = -\frac{8im_c(2m_B\bar{\sigma}^2 - 2m_B\bar{\sigma} + 4m_c\bar{\sigma} - 3m_c)}{\bar{\sigma}^3}, \quad (\text{B.49})$$

$$\mathcal{C}_{p_\mu q_\nu, \bar{G}_\pm}^{(2,4)} = \frac{48i(\bar{\sigma} - 1)m_c^3(\bar{\sigma}m_B + m_c)}{\bar{\sigma}^4}. \quad (\text{B.50})$$

C $B \rightarrow D_1^{(\prime)} \ell \bar{\nu}$ total decay width

The differential distribution for the semileptonic decay $B \rightarrow D_1^{(\prime)} \ell \bar{\nu}$ with respect to the lepton-neutrino invariant mass square q^2 can be written as

$$\begin{aligned} \frac{d\Gamma(B \rightarrow D_1^{(\prime)} \ell \nu_\ell)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192m_B^3 \pi^3} q^2 \lambda^{1/2}(m_B^2, m_{D_1^{(\prime)}}^2, q^2) \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\ &\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)^2 \{|H_+^{(\prime)}|^2 + |H_-^{(\prime)}|^2 + |H_0^{(\prime)}|^2\} + \frac{3m_\ell^2}{2q^2} |H_t^{(\prime)}|^2 \right]. \end{aligned} \quad (\text{C.1})$$

Here, the helicity amplitudes are

$$H_{+,-}^{(\prime)} = i(m_B + m_{D_1^{(\prime)}}) V_1^{BD_1^{(\prime)}}(q^2) \mp \frac{i\lambda^{1/2}(m_B^2, m_{D_1^{(\prime)}}^2, q^2)}{m_B + m_{D_1^{(\prime)}}} A^{BD_1^{(\prime)}}(q^2), \quad (\text{C.2})$$

$$H_0^{(\prime)} = i \frac{m_B + m_{D_1^{(\prime)}}}{2m_{D_1^{(\prime)}} \sqrt{q^2}} \left((m_{D_1^{(\prime)}}^2 + q^2 - m_B^2) V_1^{BD_1^{(\prime)}}(q^2) + \frac{\lambda(m_B^2, m_{D_1^{(\prime)}}^2, q^2)}{(m_B + m_{D_1^{(\prime)}})^2} V_2^{BD_1^{(\prime)}}(q^2) \right), \quad (\text{C.3})$$

$$H_t^{(\prime)} = -i \frac{\lambda^{1/2}(m_B^2, m_{D_1^{(\prime)}}^2, q^2)}{\sqrt{q^2}} V_0^{BD_1^{(\prime)}}. \quad (\text{C.4})$$

Equation (C.1) coincides with the formulas given in [41]. The relations between their HQET basis of form factors and our basis are

$$A^{BD_1} = \frac{i(m_B + m_{D_1})}{2\sqrt{m_B m_{D_1}}} f_A, \quad (\text{C.5})$$

$$V_1^{BD_1} = \frac{i\sqrt{m_B m_{D_1}}}{m_B + m_{D_1}} f_{V_1}, \quad (\text{C.6})$$

$$V_2^{BD_1} = -\frac{i(m_B + m_{D_1})}{2\sqrt{m_B m_{D_1}}} \left[f_{V_3} + \frac{m_{D_1}}{m_B} f_{V_2} \right], \quad (\text{C.7})$$

$$V_0^{BD_1} = \frac{i}{2\sqrt{m_B m_{D_1}}} \left[m_B f_{V_1} + \frac{m_B^2 - m_{D_1}^2 + q^2}{2m_B} f_{V_2} + \frac{m_B^2 - m_{D_1}^2 - q^2}{2m_{D_1}} f_{V_3} \right], \quad (\text{C.8})$$

$$A^{BD_1'} = \frac{i(m_B + m_{D_1'})}{2\sqrt{m_B m_{D_1'}}} g_A, \quad (\text{C.9})$$

$$V_1^{BD_1'} = \frac{i\sqrt{m_B m_{D_1'}}}{m_B + m_{D_1'}} g_{V_1}, \quad (\text{C.10})$$

$$V_2^{BD'_1} = -\frac{i(m_B + m_{D'_1})}{2\sqrt{m_B m_{D'_1}}} \left[g_{V_3} + \frac{m_{D'_1}}{m_B} g_{V_2} \right], \quad (\text{C.11})$$

$$V_0^{BD'_1} = \frac{i}{2\sqrt{m_B m_{D'_1}}} \left[m_B g_{V_1} + \frac{m_B^2 - m_{D'_1}^2 + q^2}{2m_B} g_{V_2} + \frac{m_B^2 - m_{D'_1}^2 - q^2}{2m_{D'_1}} g_{V_3} \right]. \quad (\text{C.12})$$

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