Published for SISSA by Springer

Received: March 11, 2020
Accepted: April 22, 2020
Published: May 11, 2020

On AdS_3 solutions of Type IIB

Achilleas Passias[a] **and Daniël Prins**[b,c]

[a] Département de Physique, École Normale Supérieure, Université PSL, CNRS, 24 Rue Lhomond, 75005 Paris, France

[b] Institut de Physique Théorique, Université Paris Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France

[c] Dipartimento di Fisica, Università di Milano-Bicocca and INFN — Sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy

E-mail: achilleas.passias@phys.ens.fr, daniel.prins@cea.fr

Abstract: We study $\mathcal{N} = 1$ supersymmetric $\mathrm{AdS}_3 \times M_7$ backgrounds of Type IIB supergravity, with non-vanishing axio-dilaton, three-form and five-form fluxes, and a "strict" SU(3)-structure on M_7. We derive the necessary and sufficient conditions for supersymmetry as a set of constraints on the torsion classes of the SU(3)-structure. Given an Ansatz for the three-form fluxes, the problem of also solving the equations of motion involves a "master equation", which generalizes ones that have previously appeared in the literature.

Keywords: AdS-CFT Correspondence, Flux compactifications

ArXiv ePrint: 1910.06326

https://doi.org/10.1007/JHEP05(2020)048

JHEP05(2020)048

Contents

1 Introduction **1**

2 Supersymmetry equations **2**

3 Supersymmetry and G-structures **4**

4 A class of solutions to the supersymmetry equations **6**

4.1 Summary 7

5 A new class of solutions **8**

A Conventions & identities **10**

1 Introduction

Recently, there has been renewed interest in supersymmetric $\mathrm{AdS}_3 \times M_7$ backgrounds of Type IIB supergravity dual to $(0, 2)$ superconformal field theories (SCFTs) in two dimensions [1, 2]. In particular, the authors of [1, 2] studied such backgrounds with only five-form flux [3], and showed the existence of the geometric dual of c-extremization in two-dimensional $(0, 2)$ SCFTs [4, 5].

Motivated by the expectation that a geometric dual of c-extremization should exist for more general backgrounds than the ones considered in [1, 2], we aim to provide, as a first step, a systematic classification of supersymmetric $\mathrm{AdS}_3 \times M_7$ backgrounds of Type IIB supergravity.[1] Such a classification was initiated by the author of [3], with the backgrounds mentioned in the previous paragraph (see also [8]). In [9], this class was extended to also admit a three-form flux satisfying certain conditions, whereas in [10] instead a varying axio-dilaton was included. In this note we extend this classification program further, by allowing for both varying axio-dilaton and three-form fluxes. Although generically we classify supersymmetric backgrounds that are dual to $(0, 1)$ SCFTs[2] for which no principle of c-extremization exists, as we discuss below, our study does lead to solutions dual to $(0, 2)$ SCFTs. We restrict to the case that M_7 is equipped with a "strict" SU(3)-structure, which is equivalent to requiring that the two Majorana supersymmetry parameters on M_7 are orthogonal. Our classification includes as special cases the ones by [1, 3]. The necessary and sufficient conditions for supersymmetry are phrased as restrictions on the torsion classes of the SU(3)-structure, which in seven dimensions is determined by a real one-form v, a real two-form J, and a complex decomposable three-form Ω. The vector dual

[1] A similar expectation for the geometric dual of a-maximization in four dimensions [6] was explored in [7] using generalized geometry.

[2] See [11] for backgrounds with pure NSNS flux.

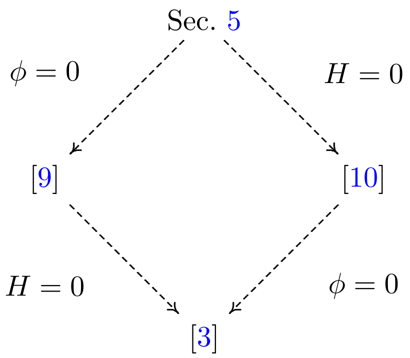

Figure 1. Depiction of the relation between classes of solutions. ϕ is the dilaton, and H the NSNS flux.

to v foliates M_7, and we find that the transverse six-dimensional space M_6 is conformally symplectic.

On $\mathrm{AdS}_3 \times M_7$, a solution to the supersymmetry equations also solves the equations of motion if and only if the Bianchi identities are satisfied by the fluxes (see for example [12]). By making an Ansatz for the three-form fluxes in our solution to the supersymmetry equations, we reduce the problem of finding a solution to the Bianchi identities, and hence the equations of motion, to two conditions: a "master equation" (5.10), which is a partial differential equation for the conformally Kähler metric on M_6, and existence of a primitive (1,2)-form satisfying (5.7). Furthermore, supersymmetry is enhanced to $\mathcal{N} = 2$. Similar master equations (and solutions thereof) associated with Bianchi identities appeared in [3, 9, 10], and the one presented here reduces to the ones of [3, 9, 10] in the appropriate limits.[3] The relation of these classes of solutions, and the corresponding master equations is depicted in figure 1. Solutions to the aforementioned conditions, as well as more general Ansätze will be reported in future work.

The rest of this note is organized as follows. In section 2, we present the supersymmetry equations as a set of equations involving a pair of polyforms on M_7. In section 3, we introduce an SU(3)-structure in seven dimensions, and parameterize the polyforms in terms of it. In section 4, we derive a set of necessary and sufficient conditions for supersymmetry as restrictions on the torsion classes of the SU(3)-structure, and also give expressions for the fluxes in terms of the latter. A summary at the end of this section is included. section 5 presents a class of solutions to the equations of motion following an Ansatz, as described earlier. Our conventions and certain technical details are included in the appendix.

2 Supersymmetry equations

We start with a general bosonic background of Type IIB supergravity invariant under $\mathrm{SO}(2,2)$. The ten-dimensional metric is a warped product of a metric on AdS_3 and a metric on a seven-dimensional Riemannian manifold M_7:

$$g_{10} = e^{2A} g_{\mathrm{AdS}_3} + g_{M_7} \,, \tag{2.1}$$

[3] See [13, 14] for more solutions dual to two-dimensional $(0,2)$ SCFTs.

where A is a function on M_7.[4] Conforming to the SO(2, 2) symmetry, the NSNS field-strength H_{10d} and the RR field-strengths F_{10d}, with F_{10d} denoting their sum in the democratic formulation, are decomposed as

$$H_{10d} = \varkappa e^{3A}\text{vol}_{\text{AdS}_3} + H\,, \qquad F_{10d} = e^{3A}\text{vol}_{\text{AdS}_3} \wedge \star_7\lambda(F) + F\ . \tag{2.2}$$

The magnetic fluxes H and $F = \sum_{p=1,3,5,7} F_p$, are forms on M_7. The operator λ acts on a p-form F_p as $\lambda(F_p) = (-1)^{\lfloor p/2 \rfloor} F_p$. The RR field-strengths are subject to $d_{H_{10d}} F_{10d} = 0$, which decomposes as

$$d_H(e^{3A} \star_7 \lambda(F)) + \varkappa F = 0\,, \qquad d_H F = 0\,, \tag{2.3}$$

where $d_H \equiv d - H\wedge$. We will refer to the first set of equations as equations of motion for F, and to the second one as the Bianchi identities.

In order to study the restrictions imposed by supersymmetry on the above bosonic background, we decompose the supersymmetry parameters of Type IIB supergravity, ϵ_1 and ϵ_2 under Spin$(1,2)\times$ Spin$(7) \subset$ Spin$(1,9)$:[5]

$$\epsilon_1 = \zeta \otimes \chi_1 \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}\,, \qquad \epsilon_2 = \zeta \otimes \chi_2 \otimes \begin{pmatrix} 1 \\ -i \end{pmatrix}\ . \tag{2.4}$$

Here, χ_1 and χ_2 are Majorana Spin(7) spinors; ζ is a Majorana Spin$(1,2)$ spinor satisfying the Killing equation:

$$\nabla_\mu \zeta = \frac{1}{2} m \gamma_\mu \zeta\,, \tag{2.5}$$

where the real constant parameter m is related to the AdS$_3$ radius L_{AdS_3} as $L^2_{\text{AdS}_3} = 1/m^2$. The above decomposition follows the requirement for $\mathcal{N} = 1$ supersymmetry.

The necessary and sufficient conditions for preserving $\mathcal{N} = 1$ supersymmetry can be derived following the derivation for Type IIA supergravity in the appendix of [15], with straightforward modifications. They are expressed in terms of bispinors $\psi_\pm$ defined by

$$\chi_1 \otimes \chi_2^t \equiv \psi_+ + i\psi_-\ . \tag{2.6}$$

Following the Fierz expansion of $\chi_1 \otimes \chi_2^t$, and application of the Clifford map which maps anti-symmetric products of gamma matrices to forms, ψ_+/ψ_- become polyforms on M_7, of even/odd degree.

The supersymmetry restrictions take the form of the following system of equations:

$$2mc_- = -c_+\varkappa\,, \tag{2.7a}$$

$$d_H(e^{A-\phi}\psi_+) = \frac{1}{16} c_- F\,, \tag{2.7b}$$

$$d_H(e^{2A-\phi}\psi_-) + 2me^{A-\phi}\psi_+ = \frac{1}{16} c_+ e^{3A} \star_7 \lambda(F)\,, \tag{2.7c}$$

$$(\psi_+, F)_7 = \frac{m}{2} e^{-\phi}\text{vol}_7\ . \tag{2.7d}$$

[4] We work in string frame.

[5] For the decomposition of the Clifford algebra see the appendix.

JHEP05(2020)048

Here $c_\pm$ are constants defined by the norms of χ_1 and χ_2:

$$c_\pm \equiv e^{\mp A}(||\chi_1||^2 \pm ||\chi_2||^2) \ . \tag{2.8}$$

Furthermore, $(\psi_+, F)_7 \equiv (\psi_+ \wedge \lambda(F))_7$, with $(\cdot)_7$ denoting the restriction to the seven-form component.

In this work we will consider backgrounds with zero electric component for H_{10d} i.e. $\varkappa = 0$, since an electric component can be set to zero by applying an $\mathrm{SL}(2,\mathbb{R})$ duality transformation.[6] Supersymmetry then dictates $c_- = 0$, or equivalently $||\chi_1||^2 = ||\chi_2||^2$. The system of supersymmetry equations thus becomes:

$$d_H(e^{A-\phi}\psi_+) = 0\,, \tag{2.9a}$$

$$d_H(e^{2A-\phi}\psi_-) + 2me^{A-\phi}\psi_+ = \frac{1}{8}e^{3A} \star_7 \lambda(F)\,, \tag{2.9b}$$

$$(\psi_+, F)_7 = \frac{m}{2}e^{-\phi}\mathrm{vol}_7 \ . \tag{2.9c}$$

Without loss of generality we have set $c_+ = 2$ i.e. $||\chi_1||^2 = ||\chi_2||^2 = e^A$.

3 Supersymmetry and G-structures

A nowhere-vanishing Majorana spinor χ on M_7 defines a G_2-structure for TM_7. A pair of nowhere-vanishing Majorana spinors χ_1, χ_2 define a $G_2 \times G_2$-structure on the generalized tangent bundle $TM_7 \oplus T^*M_7$. If χ_1, χ_2 are parallel, the $G_2 \times G_2$-structure reduces to a G_2-structure, whereas if χ_1, χ_2 are orthogonal it reduces to a "strict" SU(3)-structure. This can be illustrated by the decomposition of χ_2 in terms of χ_1 (taking χ_1, χ_2 to be of equal norm):

$$\chi_2 = \sin\theta\chi_1 - i\cos\theta v_m\gamma^m\chi_1\,, \tag{3.1}$$

where v is a real one-form with $||v|| = 1$, and $\theta \in [0, \pi/2]$. As θ varies from 0 to $\pi/2$, the $G_2 \times G_2$-structure varies from a "strict" SU(3)-structure, to an "intermediate" SU(3)-structure, to a G_2-structure. In this work we will consider the first case, i.e. $\theta = 0$.

An SU(3)-structure on M_7 is defined by a real one-form v, a real two-form J, and a complex decomposable three-form Ω, all nowhere-vanishing, satisfying[7]

$$v \lrcorner J = v \lrcorner \Omega = 0\,, \quad \Omega \wedge J = 0\,, \quad \frac{i}{8}\Omega \wedge \overline{\Omega} = \frac{1}{3!}J \wedge J \wedge J \ . \tag{3.2}$$

These forms can be expressed as bilinears in terms of the spinors (χ_1, χ_2); see appendix A for our conventions. The one-form v gives a foliation of M_7 with leaves M_6; accordingly, we define the volume form as $\mathrm{vol}_7 \equiv \frac{1}{3!}v \wedge J \wedge J \wedge J$ and locally decompose the metric on M_7 as

$$g_{M_7} = v \otimes v + g_{M_6} \ . \tag{3.3}$$

[6] We thank N. Macpherson for pointing this out.
[7] $X \lrcorner \omega_{(k)} \equiv \frac{1}{k-1!}X^n\omega_{nm_1\ldots m_{k-1}}dx^{m_1} \wedge \ldots \wedge dx^{m_{k-1}}$.

Existence of an SU(3)-structure ensures that all forms on M_7 decompose into irreducible representations of SU(3). In particular, the local k-forms with no component along v can be decomposed into primitive (p, q)-forms.[8]

We may also apply this decomposition to the exterior derivatives of the SU(3)-structure $\{v,\ J,\ \Omega\}$ itself. Doing so, we find a parameterization in terms of torsion classes. These constitute the components of the intrinsic torsion of the SU(3)-structure expressed in irreducible representations of SU(3). Specifically, we have (see for example [16])

$$\begin{aligned} dv &= RJ + T_1 + \mathrm{Re}(\overline{V_1} \lrcorner \Omega) + v \wedge W_0\,, \\ dJ &= \frac{3}{2}\mathrm{Im}(\overline{W}_1 \Omega) + W_3 + W_4 \wedge J + v \wedge \left(\frac{2}{3}\mathrm{Re}EJ + T_2 + \mathrm{Re}(\overline{V_2} \lrcorner \Omega) \right)\,, \\ d\Omega &= W_1 J \wedge J + W_2 \wedge J + \overline{W_5} \wedge \Omega + v \wedge (E\Omega - 2V_2 \wedge J + S)\ . \end{aligned} \tag{3.4}$$

The real scalar R and the complex scalars E and W_1 transform in the **1** representation of SU(3). The complex $(1, 0)$-forms V_1, V_2 and W_5 transform in the **3**, and the real one-forms W_0 and W_4 in the $\mathbf{3} + \overline{\mathbf{3}}$. The real primitive (1,1)-forms T_1 and T_2, and the complex primitive (1,1)-form W_2 transform in the **8**. Finally, the real primitive $(2, 1) + (1, 2)$-form W_3 transforms in the $\mathbf{6} + \overline{\mathbf{6}}$, and the complex primitive $(2, 1)$-form S in the **6**.

In order to solve the supersymmetry equations, we parameterize the polyforms $\psi_\pm$ as defined in (2.6) in terms of the SU(3)-structure data. Making use of (3.1), (A.7), (A.8) we find that in the general case,

$$\begin{aligned} \psi_+^{G_2 \times G_2} &= \frac{1}{8} e^A \left[\mathrm{Im}(e^{i\theta} e^{iJ}) + v \wedge \mathrm{Re}(e^{i\theta}\Omega) \right]\,, \\ \psi_-^{G_2 \times G_2} &= \frac{1}{8} e^A \left[v \wedge \mathrm{Re}(e^{i\theta} e^{iJ}) + \mathrm{Im}(e^{i\theta}\Omega) \right]\,, \end{aligned} \tag{3.5}$$

for $||\chi_1||^2 = ||\chi_2||^2 = e^A$. As stated earlier, we will study the case of a strict SU(3)-structure for which $\theta = 0$ and hence

$$\begin{aligned} \psi_+ &= \frac{1}{8} e^A \left[\mathrm{Im}(e^{iJ}) + v \wedge \mathrm{Re}(\Omega) \right]\,, \\ \psi_- &= \frac{1}{8} e^A \left[v \wedge \mathrm{Re}(e^{iJ}) + \mathrm{Im}(\Omega) \right]\ . \end{aligned} \tag{3.6}$$

Substituting the above expressions in the supersymmetry equations (2.9), we will derive the restrictions on the intrinsic torsion of the SU(3)-structure imposed by supersymmetry.

[8] A primitive k-form $\omega^{(k)}$ satisfies $J \lrcorner \omega^{(k)} = 0$ for $k = 2, 3$, whereas k-forms with $k = 0, 1$ are primitive by definition. The (p, q) decomposition of k-form ω is defined by

$$\begin{aligned} \omega^{(p,q)}_{m_1 \ldots m_k} &= \frac{k!}{p!q!} (\Pi^+)_{[m_1}{}^{n_1} \ldots (\Pi^+)_{m_p]}{}^{n_p} (\Pi^-)_{[m_{p+1}}{}^{n_{p+1}} \ldots (\Pi^-)_{m_k]}{}^{n_k} \omega_{n_1 \ldots n_k}\,, \\ (\Pi^\pm)_m{}^n &= \frac{1}{2} (\delta^n_m \mp i J_m{}^n - v_m v^n)\ . \end{aligned}$$

4 A class of solutions to the supersymmetry equations

In this section, we derive a class of solutions to the supersymmetry equations (2.9) by inserting the strict SU(3)-structure polyforms (3.6).

The first constraint (2.9a) yields

$$d\left(e^{2A-\phi}J\right)=0\,, \tag{4.1a}$$

$$d\left(e^{2A-\phi}v\wedge\mathrm{Re}\Omega\right)-e^{2A-\phi}H\wedge J=0\,, \tag{4.1b}$$

$$d\left(e^{2A-\phi}J\wedge J\wedge J\right)+3!\,e^{2A-\phi}H\wedge v\wedge\mathrm{Re}\Omega=0\;. \tag{4.1c}$$

These in turn determine

$$\begin{aligned}0&=W_1=W_3=V_2=T_2\,,\\ 2dA-d\phi&=-W_4-\frac{2}{3}\mathrm{Re}E\,v\;.\end{aligned} \tag{4.2}$$

Upon decomposing the NSNS field-strength H with respect to the SU(3)-structure as

$$\begin{aligned}H=&\,H^R\mathrm{Re}\Omega+H^I\mathrm{Im}\Omega+\left(H^{(1,0)}+H^{(0,1)}\right)\wedge J+H^{(2,1)}+H^{(1,2)}\\&+v\wedge\left(H_v^{(1,1)}+H_v^0J+H_v^{(0,1)}\lrcorner\Omega+H_v^{(1,0)}\lrcorner\overline{\Omega}\right)\,,\end{aligned} \tag{4.3}$$

where $H^{(2,1)}$ and $H_v^{(1,1)}$ are primitive, we also find expressions for several of the components in terms of torsion classes from (4.1). Using (A.12), we find:

$$\begin{aligned}&H^I=-\frac{1}{3}\mathrm{Re}E\,,\qquad H^{(1,0)}=V_1\,,\\ &H_v^{(1,1)}=-\mathrm{Re}W_2\,,\qquad H_v^0=0\;,\qquad H_v^{(1,0)}=\frac{1}{2i}\left(W_4^{(1,0)}+W_0^{(1,0)}-W_5\right)\;.\end{aligned} \tag{4.4}$$

The exterior derivatives of the the SU(3)-structure tensors now read

$$dv=RJ+\mathrm{Re}(\overline{V_1}\lrcorner\Omega)+T_1+v\wedge W_0\,, \tag{4.5a}$$

$$dJ=-d(2A-\phi)\wedge J\,, \tag{4.5b}$$

$$d\Omega=W_2\wedge J+(\overline{W_5}+E\,v)\wedge\Omega+v\wedge S\;. \tag{4.5c}$$

We define a rescaled metric $g_{M_7}=e^{-2A+\phi}\check{g}_{M_7}$ and rescale the SU(3)-structure tensors accordingly as $\{v,\,J,\,\Omega\}=\{e^{-A+\phi/2}\check{v},\,e^{-2A+\phi}\check{J},\,e^{-3A+3\phi/2}\check{\Omega}\}$ to obtain

$$d\check{v}=\check{R}\check{J}+\mathrm{Re}(\overline{\check{V}}_1\lrcorner\check{\Omega})+\check{T}_1+\check{v}\wedge\check{W}_0\,, \tag{4.6a}$$

$$d\check{J}=0\,, \tag{4.6b}$$

$$d\check{\Omega}=\check{W}_2\wedge\check{J}+(\overline{\check{W}}_5+i\mathrm{Im}\check{E}\,\check{v})\wedge\check{\Omega}+\check{v}\wedge\check{S}\,, \tag{4.6c}$$

where

$$\check{W}_0=W_0+\frac{1}{2}W_4\,,\qquad \check{W}_5=W_5-\frac{3}{2}W_4\,, \tag{4.7}$$

and

$$
\begin{aligned}
R &= e^{A-\frac{\phi}{2}}\check{R}\,, \qquad & \mathrm{Im}E &= e^{A-\frac{\phi}{2}}\mathrm{Im}\check{E}\,, \qquad & V_1 &= \check{V}_1\,, \\
W_2 &= e^{-A+\frac{\phi}{2}}\check{W}_2\,, \qquad & T_1 &= e^{-A+\frac{\phi}{2}}\check{T}_1\,, \qquad & S &= e^{-2A+\phi}\check{S}\,.
\end{aligned} \tag{4.8}
$$

We note that the condition $d\check{J}=0$ means that the six-dimensional leaves M_6 transverse to $\check{v}$ admit a symplectic structure.

Turning to the second constraint (2.9b) we obtain:

$$e^{3A}\star_7 F_7 = 0\,, \tag{4.9a}$$

$$e^{3A}\star_7 F_5 = d\left(e^{3A-\phi}v\right) + 2me^{2A-\phi}J\,, \tag{4.9b}$$

$$-e^{3A}\star_7 F_3 = d\left(e^{3A-\phi}\mathrm{Im}\Omega\right) - e^{3A-\phi}H\wedge v + 2me^{2A-\phi}v\wedge\mathrm{Re}\Omega\,, \tag{4.9c}$$

$$e^{3A}\star_7 F_1 = -\frac{1}{2}d\left(e^{3A-\phi}v\wedge J\wedge J\right) - e^{3A-\phi}H\wedge\mathrm{Im}\Omega - \frac{1}{3}me^{2A-\phi}J\wedge J\wedge J\,. \tag{4.9d}$$

From these equations, employing (4.4) and (4.5) and the set of identities (A.10), (A.11), we can obtain expressions for the magnetic RR fluxes F_p, $p=1,3,5,7$. We give these in (4.14) in the summary below.

Finally, the third constraint (2.9c) reads

$$F_5\wedge J - F_3\wedge v\wedge\mathrm{Re}\Omega - \frac{1}{3!}F_1\wedge J\wedge J\wedge J = 4me^{-A-\phi}\mathrm{vol}_7\,, \tag{4.10}$$

and plugging in the expressions for the RR fields we conclude that

$$3R + 6me^{-A} + 4H^R + 2\mathrm{Im}E = 0\,. \tag{4.11}$$

4.1 Summary

Let us summarize our results. The differential constraints imposed on the SU(3)-structure by supersymmetry are:

$$dv = RJ + \mathrm{Re}(\overline{V_1}\lrcorner\Omega) + T_1 + v\wedge W_0\,, \tag{4.12a}$$

$$dJ = -d(2A-\phi)\wedge J\,, \tag{4.12b}$$

$$d\Omega = W_2\wedge J + (\overline{W_5} + E\,v)\wedge\Omega + v\wedge S\,. \tag{4.12c}$$

The expression for the NSNS field is:

$$
\begin{aligned}
H = &-\frac{1}{4}\left(3R + 6me^{-A} + 2\mathrm{Im}E\right)\mathrm{Re}\Omega - \frac{1}{3}\mathrm{Re}E\;\mathrm{Im}\Omega + 2\mathrm{Re}V_1\wedge J + 2\mathrm{Re}(H^{(2,1)}) \\
&+ v\wedge\left(-\mathrm{Re}W_2 + \mathrm{Im}\left((W_4^{(1,0)} + W_0^{(1,0)} - W_5)\lrcorner\overline{\Omega}\right)\right)\,.
\end{aligned} \tag{4.13}
$$

The expressions for the RR fields are:

$$e^{\phi}F_1 = \big(2\mathrm{Im}E + 4me^{-A}\big)\, v + 2\mathrm{Im}(X_1^{(1,0)})\,, \tag{4.14a}$$

$$\begin{aligned} e^{\phi}F_3 = &\frac{1}{4}\big(-2me^{-A} + 3R - 2\mathrm{Im}E\big)\,\mathrm{Im}\Omega - 2\mathrm{Im}V_1\wedge J + v\wedge \mathrm{Im}W_2 \\ &+ 2\mathrm{Im}(H^{(2,1)}) - \mathrm{Re}S + X_3\lrcorner(v\wedge \mathrm{Re}\Omega)\,, \end{aligned} \tag{4.14b}$$

$$\begin{aligned} e^{\phi}F_5 = &\frac{1}{2}\big(R + 2me^{-A}\big)\, v\wedge J\wedge J - \mathrm{Im}(X_5^{(1,0)})\wedge J\wedge J \\ &- v\wedge J\wedge T_1 + 2v\wedge \mathrm{Re}V_1\wedge \mathrm{Im}\Omega\,, \end{aligned} \tag{4.14c}$$

$$e^{\phi}F_7 = 0\,, \tag{4.14d}$$

with

$$\begin{aligned} X_1 &\equiv dA + W_0 + 3W_4 - 2(W_5 + \overline{W_5})\,, \\ X_3 &\equiv dA - W_4 + W_5 + \overline{W_5}\,, \\ X_5 &\equiv dA - W_0 - W_4\,. \end{aligned} \tag{4.15}$$

The above solution to the supersymmetry equations also solves the equations of motion if and only if the Bianchi identities for the NSNS and RR fields are imposed in addition.

5 A new class of solutions

We make the following Ansatz:

$$H + ie^{\phi}F_3 = 2H^{(2,1)}\,, \tag{5.1}$$

and recall that $H^{(2,1)}$ is primitive. This leads to $v\lrcorner dA = 0 = v\lrcorner d\phi$ and the following restrictions on the torsion classes:

$$\begin{gathered} 0 = \mathrm{Re}E = V_1 = W_2 = S = W_5 - W_0^{(1,0)} - W_4^{(1,0)}\,, \\ \mathrm{Im}E = -2me^{-A}\,, \quad R = -\frac{2}{3}me^{-A}\,, \quad W_0 = -dA\,, \quad W_4 = -2dA + d\phi\,. \end{gathered} \tag{5.2}$$

We thus have

$$dv = -\frac{2}{3}me^{-A}J + T_1 - v\wedge dA\,, \tag{5.3a}$$

$$dJ = -d(2A - \phi)\wedge J\,, \tag{5.3b}$$

$$d\Omega = \big(-3dA + d\phi - 2ime^{-A}v\big)\wedge\Omega\,, \tag{5.3c}$$

or in terms of the rescaled SU(3)-structure

$$d\check{v} = -\frac{2}{3}me^{-2A+\phi/2}\check{J} + \check{T}_1 - \check{v}\wedge\left(2dA - \frac{1}{2}d\phi\right)\,, \tag{5.4a}$$

$$d\check{J} = 0\,, \tag{5.4b}$$

$$d\check{\Omega} = \left(-\frac{1}{2}d\phi - 2ime^{-2A+\phi/2}\check{v}\right)\wedge\check{\Omega}\,. \tag{5.4c}$$

From the differential equations for $\{\check{J}\,,\check{\Omega}\}$ we conclude that $\check{M}_6$ is Kähler. In what follows we will introduce the exterior derivative d_6 on $\check{M}_6$, Dolbeault operators ∂, $\bar{\partial}$ so that $d_6 = \partial + \bar{\partial}$, and $d_6^c = i(\bar{\partial} - \partial)$. The remaining RR fields read

$$\begin{aligned} F_1 &= -d_6^c e^{-\phi}\,, \\ F_5 &= \frac{2}{3} m e^{-6A+3\phi/2} \check{v} \wedge \check{J}^2 + \frac{1}{2} d_6^c (e^{-4A+\phi}) \wedge \check{J} \wedge \check{J} - e^{-4A+\phi} \check{v} \wedge \check{J} \wedge \check{T}_1 \,. \end{aligned} \tag{5.5}$$

Let us now examine the Bianchi identities. The first Bianchi identity, $dF_1 = 0$, enforces

$$\partial\bar{\partial} e^{-\phi} = 0\,, \tag{5.6}$$

which is solved by setting $\phi = -\log(\varphi + \overline{\varphi})$, with φ holomorphic. Next, the three-form Bianchi identities $dH = 0$ and $dF_3 - H \wedge F_1 = 0$ yield the constraints

$$\partial H^{(1,2)} + \bar{\partial} H^{(2,1)} = \bar{\partial} H^{(1,2)} = \partial H^{(1,2)} - \partial\phi \wedge H^{(1,2)} + \bar{\partial}\phi \wedge H^{(2,1)} = 0\,. \tag{5.7}$$

In analyzing the Bianchi identity for F_5, we will invoke the results of [10]. The authors of [10] study supersymmetric solutions which descend from the solutions analyzed here, upon setting $H^{(2,1)} = 0$. However, even when $H^{(2,1)} \neq 0$, the SU(3)-structure of [10] and the expressions for F_1 and F_5 can be identified with the ones presented in this section. The map identifying the tensors there (left-hand side), with the tensors here (right-hand side) is

$$\begin{aligned} &P = \partial\phi\,, \qquad && Q = -\frac{1}{2} d_6^c \phi\,, \qquad && F^{(2)} = -e^{3A} \star_7 F_5\,, \\ &\Delta = A - \frac{1}{4}\phi\,, \qquad && e^{2\Delta} K = \check{v}\,, \qquad && \widetilde{g}_6 = m^2 \check{g}_6\,, \end{aligned} \tag{5.8}$$

and in particular, (4.9b) is identified with (2.58) of [10].[9] The authors of [10] showed that the Bianchi identity for F_5, $dF_5 = 0$, amounts to

$$\nabla^2 (R - 2|\partial\phi|^2) - \frac{1}{2} R^2 + R_{ij} R^{ij} + 2|\partial\phi|^2 R - 4 R_{ij} \partial^i \phi \bar{\partial}^j \phi = 0\,, \tag{5.9}$$

which they refer to as the "master equation". In the above, R and R_{ij} are respectively the Ricci scalar and the Ricci tensor of $\check{g}_6$, and contractions are also made using $\check{g}_6$. This master equation generalizes the one derived in [3] by including a varying axio-dilaton. For the case at hand the Bianchi identity of F_5 is $dF_5 = H \wedge F_3$, and the term on the right-hand side (a "transgression" term) modifies the master equation, which now becomes:

$$\nabla^2 (R - 2|\partial\phi|^2) - \frac{1}{2} R^2 + R_{ij} R^{ij} + 2|\partial\phi|^2 R - 4 R_{ij} \partial^i \phi \bar{\partial}^j \phi - \frac{8}{3} e^{-\phi} H^{(2,1)}_{ijk} (H^{(1,2)})^{ijk} = 0\,. \tag{5.10}$$

As noted above, in the limit $H^{(2,1)} = 0$ the present class of solutions and master equation (5.10) reduce to the ones of [10]. Further setting the axio-dilaton to zero, they

[9] One has to take into account that we work in the string frame whereas the Einstein frame is used in [10]. In addition, we use a different orientation on AdS_3.

reduce to the ones studied in [3]. Starting with the latter, the authors of [9] "turned on" a three-form flux $G = G^{(1,2)}$, and taking the limit of vanishing axio-dilaton we recover their results. See figure 1.

Finally, the supersymmetry preserved by the class of solutions in this section enhances to $\mathcal{N} = 2$, and the dual field theories are $(0,2)$ SCFTs [17]. The vector field dual to v generates a U(1) symmetry of the solutions, corresponding to the R-symmetry of the $(0,2)$ SCFTs. Thus we expect that a geometric dual of c-extremization exists for this class of solutions and would be very interesting to identify it.

Acknowledgments

We would like to thank N. Macpherson and A. Tomasiello for useful discussions. The work of A.P. is supported by is supported by Agence Nationale de la Recherche LabEx grant ENS-ICFP ANR-10-LABX-0010/ANR-10-IDEX-0001-02 PSL. D.P. has been financially supported in part by INFN and by the ERC Starting Grant 637844-HBQFTNCER

A Conventions & identities

Clifford algebra decomposition. The ten-dimensional gamma matrices are decomposed as follows

$$\Gamma_\mu = e^A \gamma_\mu^{(3)} \otimes \mathbb{I} \otimes \sigma_3 \,, \qquad \Gamma_m = \mathbb{I} \otimes \gamma_m \otimes \sigma_1 \,, \tag{A.1}$$

where $\gamma_\mu^{(3)}$ span Cliff(1,2), γ_m span Cliff(7) and the indices are spacetime indices.

We take $\gamma_\mu^{(3)}$ to be real, and γ_m imaginary and antisymmetric. In particular we have

$$\gamma_m = \gamma_m^\dagger \,, \qquad \gamma_m = -\gamma_m^t \,, \qquad \gamma_m = -\gamma_m^* \tag{A.2}$$

$$\gamma_0^{-1}\gamma_\mu^{(3)}\gamma_0 = -(\gamma_\mu^{(3)})^\dagger \,, \qquad \gamma_0^{-1}\gamma_\mu^{(3)}\gamma_0 = -(\gamma_\mu^{(3)})^t \,, \qquad \gamma_\mu^{(3)} = (\gamma_\mu^{(3)})^* \,. \tag{A.3}$$

It follows that the ten-dimensional intertwiners A, C, and D defined by

$$A^{-1}\Gamma_M A = \Gamma_M^\dagger \,, \qquad C^{-1}\Gamma_M C = -\Gamma_M^t \,, \qquad D^{-1}\Gamma_M D = \Gamma_M^* \,, \tag{A.4}$$

are

$$A = \gamma_0 \otimes \mathbb{I} \otimes \sigma_1 \,, \qquad C = \gamma_0 \otimes \mathbb{I} \otimes \mathbb{I} \,, \qquad D = \mathbb{I} \otimes \mathbb{I} \otimes \sigma_3 \,. \tag{A.5}$$

Spin(7). As noted above, we work with the Majorana representation of Cliff(7), for which the gamma matrices are imaginary and antisymmetric. The charge-conjugate of a Spin(7) spinor is the complex conjugate, and a Majorana spinor is real. The basis elements of Cliff(7) are related via the identity

$$\gamma_{m_1\ldots m_k} = \frac{i}{(7-k)!}(-1)^{k(k-1)/2}\,\epsilon_{m_1\ldots m_k m_{k+1}\ldots m_7}\gamma^{m_{k+1}\ldots m_7} \,. \tag{A.6}$$

As discussed in section 3, a pair of nowhere-vanishing Spin(7) Majorana spinors χ_1 and χ_2 define an SU(3)-structure $\{v,\, J,\, \Omega\}$ in seven dimensions. For the strict SU(3)-structure ($\theta = 0$, or equivalently, $\chi_1^t\chi_2 = 0$), we introduce a Dirac spinor η as

$$\chi_1 = \frac{1}{\sqrt{2}}(\eta + \eta^*) \,, \qquad \chi_2 = \frac{i}{\sqrt{2}}(\eta^* - \eta) \,. \tag{A.7}$$

The bilinears that can be constructed using η are $\eta^\dagger\gamma_{m_1\dots m_k}\eta$ and $\eta^t\gamma_{m_1\dots m_k}\eta$. In terms of the SU(3)-structure, η satisfies

$$\begin{aligned} \eta^\dagger\eta &= e^A\,, & \eta^t\eta &= 0\,,\\ \eta^\dagger\gamma_m\eta &= e^A v_m\,, & \eta^t\gamma_m\eta &= 0\,,\\ \eta^\dagger\gamma_{mn}\eta &= ie^A J_{mn}\,, & \eta^t\gamma_{mn}\eta &= 0\,,\\ \eta^\dagger\gamma_{mnp}\eta &= 3ie^A v_{[m}J_{np]}\,, & \eta^t\gamma_{mnp}\eta &= -ie^A\Omega_{mnp}\,. \end{aligned} \tag{A.8}$$

These can then be used to deduce the expressions (3.6) for the polyforms. In the case of the G_2-structure ($\theta = \pi/2$, or $\chi_1 = \chi_2$), we may instead consider $\chi_1 = \chi_2 = \frac{1}{\sqrt{2}}(\eta + \eta^*)$, where η satisfies the above equations.

Identities. The SU(3)-structure is normalized as follows:

$$\begin{aligned} \Omega_{mpq}\overline{\Omega}^{npq} &= 2^3\left(\delta_m{}^n - iJ_m{}^n - v_m v^n\right)\,, \quad \Omega_{mnp}\overline{\Omega}^{mnp} = 3!2^3\,,\\ \epsilon_{m_1\dots m_7} &= \frac{7!}{3!2^3} v_{[m_1}J_{m_2m_3}J_{m_4m_5}J_{m_6m_7]}\,. \end{aligned} \tag{A.9}$$

Given the above normalization, we derive a number of identities necessary to obtain the RR fields $F_{1,3,5}$ from their Hodge duals $\star_7 F_{1,3,5}$. Duals of the SU(3)-structure are given by

$$\star_7 J = \frac{1}{2} v\wedge J\wedge J\,, \quad \star_7(v\wedge J) = \frac{1}{2}J\wedge J\,, \quad \star_7\Omega = iv\wedge\Omega\,. \tag{A.10}$$

Duals for arbitrary primitive (p,q)-forms $\omega^{(p,q)}$ are given by

$$\begin{aligned} \star_7(\omega^{(1,0)}\wedge J) &= iv\wedge\omega^{(1,0)}\wedge J\,, & \star_7(\omega^{(1,0)}\wedge J\wedge J) &= 2iv\wedge\omega^{(1,0)}\,,\\ \star_7(v\wedge(\omega^{(0,1)}\lrcorner\Omega)) &= -i\omega^{(0,1)}\wedge\Omega\,, & \star_7(\omega^{(0,1)}\lrcorner\Omega) &= -iv\wedge\omega^{(0,1)}\wedge\Omega\,,\\ \star_7\omega^{(1,1)} &= -v\wedge J\wedge\omega^{(1,1)}\,, & \star_7(\omega^{(1,1)}\wedge J) &= -v\wedge\omega^{(1,1)}\,,\\ \star_7\omega^{(2,1)} &= -iv\wedge\omega^{(2,1)}\,. \end{aligned} \tag{A.11}$$

We also make use of the identities

$$(\omega^{(0,1)}\lrcorner\Omega)\wedge J = -i\omega^{(0,1)}\wedge\Omega\,, \quad (\omega^{(0,1)}\lrcorner\Omega)\wedge\overline{\Omega} = 4\omega^{(0,1)}\wedge J\wedge J\,, \tag{A.12}$$

in order to obtain the components of H.

References

[1] C. Couzens, J.P. Gauntlett, D. Martelli and J. Sparks, *A geometric dual of c-extremization*, *JHEP* **01** (2019) 212 [arXiv:1810.11026] [INSPIRE].

[2] J.P. Gauntlett, D. Martelli and J. Sparks, *Toric geometry and the dual of c-extremization*, *JHEP* **01** (2019) 204 [arXiv:1812.05597] [INSPIRE].

JHEP05(2020)048

[3] N. Kim, *AdS_3 solutions of IIB supergravity from D3-branes*, *JHEP* **01** (2006) 094 [hep-th/0511029] [INSPIRE].

[4] F. Benini and N. Bobev, *Exact two-dimensional superconformal R-symmetry and c-extremization*, *Phys. Rev. Lett.* **110** (2013) 061601 [arXiv:1211.4030] [INSPIRE].

[5] F. Benini and N. Bobev, *Two-dimensional SCFTs from wrapped branes and c-extremization*, *JHEP* **06** (2013) 005 [arXiv:1302.4451] [INSPIRE].

[6] K.A. Intriligator and B. Wecht, *The exact superconformal R symmetry maximizes a*, *Nucl. Phys.* **B 667** (2003) 183 [hep-th/0304128] [INSPIRE].

[7] M. Gabella and J. Sparks, *Generalized geometry in AdS/CFT and volume minimization*, *Nucl. Phys.* **B 861** (2012) 53 [arXiv:1011.4296] [INSPIRE].

[8] J.P. Gauntlett and N. Kim, *Geometries with Killing spinors and supersymmetric AdS solutions*, *Commun. Math. Phys.* **284** (2008) 897 [arXiv:0710.2590] [INSPIRE].

[9] A. Donos, J.P. Gauntlett and N. Kim, *AdS solutions through transgression*, *JHEP* **09** (2008) 021 [arXiv:0807.4375] [INSPIRE].

[10] C. Couzens, D. Martelli and S. Schäfer-Nameki, *F-theory and AdS_3/CFT_2 $(2,0)$*, *JHEP* **06** (2018) 008 [arXiv:1712.07631] [INSPIRE].

[11] L. Eberhardt, *Supersymmetric AdS_3 supergravity backgrounds and holography*, *JHEP* **02** (2018) 087 [arXiv:1710.09826] [INSPIRE].

[12] D. Prins and D. Tsimpis, *Type IIA supergravity and M-theory on manifolds with* SU(4) *structure*, *Phys. Rev.* **D 89** (2014) 064030 [arXiv:1312.1692] [INSPIRE].

[13] J.P. Gauntlett, N. Kim and D. Waldram, *Supersymmetric AdS_3, AdS_2 and bubble solutions*, *JHEP* **04** (2007) 005 [hep-th/0612253] [INSPIRE].

[14] F. Benini, N. Bobev and P.M. Crichigno, *Two-dimensional SCFTs from D3-branes*, *JHEP* **07** (2016) 020 [arXiv:1511.09462] [INSPIRE].

[15] G. Dibitetto et al., *AdS_3 solutions with exceptional supersymmetry*, *Fortsch. Phys.* **66** (2018) 1800060 [arXiv:1807.06602] [INSPIRE].

[16] D. Cassani, P. Koerber and O. Varela, *All homogeneous $N = 2$ M-theory truncations with supersymmetric AdS_4 vacua*, *JHEP* **11** (2012) 173 [arXiv:1208.1262] [INSPIRE].

[17] C. Couzens, *$\mathcal{N} = (0,2)$ AdS_3 solutions of Type IIB and F-theory with generic fluxes*, arXiv:1911.04439 [INSPIRE].

JHEP05(2020)048