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Type II solutions on $\text{AdS}_3 \times S^3 \times S^3$ with large superconformal symmetry

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ABSTRACT: New local solutions in type II supergravity that are foliations of $\text{AdS}_3 \times S^3 \times S^3$ over an interval and preserve at least large $\mathcal{N} = (4, 0)$ supersymmetry are found. Some cases have compact internal space, some not and one experiences an enhancement to $\mathcal{N} = (4, 4)$. We present two new globally compact solutions with D brane and O plane sources explicitly, one in each of IIA and IIB. The former is part of an infinite family of solutions with D8/O8s back reacted on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$. In the latter the algebra degenerates to small $\mathcal{N} = (4, 0)$ and the internal geometry is bounded between D5s and O5s back reacted on $\text{AdS}_3 \times S^3 \times \mathbb{R}^4$.

KEYWORDS: AdS-CFT Correspondence, D-branes, Extended Supersymmetry

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1 Introduction

The AdS-CFT correspondence has by now shown itself to be a powerful tool to probe the dynamics of theories on both sides of the correspondence. Since its inception it has stimulated progress constructing many CFT_d s and their dual AdS_{d+1} solutions, in many cases embedded into 10 dimensions. One area where progress on the CFT side has somewhat outpaced the other is the AdS_3 - CFT_2 correspondence. This is not to say that progress on the gravity side has not been made (see [1–24] for an incomplete list), merely that there is much more yet to be studied.

Two dimensional CFTs play an important role in physics, in string theory and beyond so there is clear motivation to construct holographic duals. The barrier to this is that when embedded into 10 dimensional supergravity, their internal space is 7-dimensional which is rather large. Progress can be made tractable by assuming extended supersymmetry — in this case the dual geometry will realise an additional R-symmetry reducing the number of undetermined dimensions. An interesting feature of superconformal field theories in 2 dimensions is that a relatively large number of superconformal algebras exist for each number of preserved supercharges, with each preserving a distinct R-symmetry. Those that can be embedded into 10 and 11 dimensions were classified in [25]. Given this, and

the recent G-structure classification of $\mathcal{N} = 1$ AdS₃ solutions in type II supergravity [26], the time seems right to begin to seriously explore the possibilities.

An interesting class of AdS₃ solutions with limited examples exhibiting a compact internal space (as required for a holographic dual of a 2d CFT) are those preserving at least $\mathcal{N} = (4, 0)$ supersymmetry with the so-called “large” superconformal algebra $\mathfrak{d}(2, 1, \alpha)$. This has maximal bosonic sub-algebra $\mathfrak{sl}(2) \oplus \mathfrak{so}(4)$, where the second term is an SO(4) R-symmetry. Large $\mathcal{N} = (4, 0)$ includes a Kac-Moody algebra $\mathfrak{su}(2)_{k_+} \oplus \mathfrak{su}(2)_{k_-}$ (in contrast to small $\mathcal{N} = (4, 0)$ which has just $\mathfrak{su}(2)_k$) and CFTs with this symmetry are characterised by the following relations between their central charge c , the continuous parameter α , and the levels k_{\pm} [27]

$$c = 6 \frac{k_+ k_-}{k_+ + k_-}, \quad \alpha = \frac{k_-}{k_+}. \tag{1.1}$$

Precisely what is assigned to be k_+ and k_- appears ambiguous in c , but this is a manifestation of the fact that $\mathfrak{d}(2, 1, \alpha)$ is isomorphic to $\mathfrak{d}(2, 1, \alpha^{-1})$ — as such the central charge is sufficient to determine the value of α (up to identifying $\alpha \sim \alpha^{-1}$) for a given large $\mathcal{N} = (4, 0)$ CFT₂.

The canonical example of a supergravity solution dual to a CFT₂ with large superconformal symmetry is AdS₃ × S³ × S³ × S¹ (see also [3] for a worldsheet perspective) and its M-theory avatar AdS₃ × S³ × S³ × T² [1, 4] which actually preserve large $\mathcal{N} = (4, 4)$ supersymmetry (a maximal case for AdS₃ [32]) with algebra $\mathfrak{d}(2, 1, \alpha) \oplus \mathfrak{d}(2, 1, \alpha)$, and where α is related to the radii of the 3-spheres. In M-theory such solutions were classified locally then globally in [10–12] where those consistent with a dual CFT₂ were claimed to be locally AdS₃ × S³ × S³ × T² and so all reduce to AdS₃ × S³ × S³ × S¹ in IIA (at least locally). Historically there was some difficulty ascertaining the CFT dual to AdS₃ × S³ × S³ × S¹ [6], though a recent attempt was made in [28]. In large part this difficulty was due to the failure of otherwise likely CFT proposals to reproduce the BPS spectrum of the supergravity solution calculated in [4]. However this computation was recently found to be in error [19] and the corrected spectrum was explicitly shown to match that of a certain symmetric orbifold (\mathcal{S}_k [6]) in [29].¹

Beyond the cases with maximal supersymmetry (for AdS₃), solutions with $\mathcal{N} = (4, 0)$ were constructed from AdS₃ × S³ × S³ × S¹ using T-duality (and its non-abelian counterpart) in [15, 16] and a class of AdS₃ × S² × S² × CY₂ solutions in M-theory was found in [18]. Another interesting example is a flow from AdS₅ × T^{1,1} to a twice T-dualised version of AdS₃ × S³ × S³ × S¹ preserving $\mathcal{N} = (4, 2)$ [14] (other flows across dimensions were found in [10–12], but these exhibit large $\mathcal{N} = (4, 4)$). Finally, somewhat related to this story, there is also a family of $\mathcal{N} = (2, 0)$ solution in IIB that are AdS₃ × S³ × S³ × S¹ only topologically [9].

In this work new AdS₃ preserving large $\mathcal{N} = (4, 0)$ supersymmetry will be constructed that are neither locally AdS₃ × S³ × S³ × S¹ nor related to it by duality. To do this one needs to arrange for the internal space to realise an SO(4) R-symmetry. There are several ways to arrange for this to happen with products of 2 and 3-spheres. Here it will

¹Actually the match between \mathcal{S}_k and AdS₃ × S³ × S³ × S¹ additionally requires that some flux charges are tuned.

be assumed that the R-symmetry is realised with a foliation of $S^3 \times S^3$ over an interval.² Generically, such solutions will have a flavour $SO(4)$ in addition to the R-symmetry.³ The reasons to make this choice are two fold: i) In short, it is the easiest example to look at. However this simplicity will allow for a complete local description of all such solution in type II supergravity.⁴ Additionally this should aid the process of finding a CFT dual. ii) With $S^3 \times S^3$ there is the possibility of an enhancement to large $\mathcal{N} = (4, 4)$ supersymmetry, thereby generalising the classification of [10–12] to type II supergravity. One should appreciate though that the assumption of $S^3 \times S^3$ limits the scope of this work to a small portion of the space of possible solutions with large $\mathcal{N} = (4, 0)$. It will turn out that this portion is far from empty, but one should view this as a first step in a much broader classification endeavour with most solutions lying outside this ansatz.

The method used here to find new solutions with large $\mathcal{N} = (4, 0)$ supersymmetry shall be to construct spinors manifestly realising the bosonic sub-algebra of $\mathfrak{d}(2, 1, \alpha)$

$$\mathfrak{sl}(2) \oplus \mathfrak{so}(4).$$

The first factor will be realised by Killing spinors on AdS_3 — it requires a little more work to construct general spinors on the internal space that manifestly transform under the action of $SO(4)$. Having such spinors we shall then find every solution with an $S^3 \times S^3$ factor consistent with them. This follows the line of reasoning of the earlier works [33–37], where many of the technical details exploited here were originally worked out. Here it will be possible to give the explicit local form of every type II solution consistent with the $SO(4)$ spinor. Strictly speaking, as one is not imposing the entire superconformal algebra $\mathfrak{d}(2, 1, \alpha)$, the solutions that follow could in fact preserve some other algebra with bosonic sector containing $\mathfrak{sl}(2) \oplus \mathfrak{so}(4)$. One possibility is a larger R-symmetry but given the ansatz for the internal space, the only possibilities for enhanced R-symmetries (other than $SO(4) \times SO(4)$) are $SO(8)$ and $Spin(7)$ [25] which require the internal space to become S^7 [26]. However S^7 is never realised by the solutions constructed here. The other possibility is that the $SO(4)$ R-symmetry of the geometry is realising an $SU(2)$ R-symmetry of the dual CFT and an additional $SU(2)$ outer automorphism symmetry, as is the case with $AdS_2 \times S^3 \times T^4$. Such solutions are degenerate cases of $\mathfrak{d}(2, 1, \alpha)$ with $\alpha \rightarrow 0$ where small $\mathcal{N} = (4, 0)$ is recover — we will find one such example in our analysis. That leaves the question of how one calculates α — one could proceed as in [39] and carefully map bi-linears of the spinors on $AdS_3 \times S^3 \times S^3$ to the algebra and compute α directly. However, for the examples with compact internal space (the only ones dual to well defined 2d CFTs), there is an easier way. One simply computes the holographic central charge, and then read α off from (1.1) — this will be the route followed here.

The outline of the paper is as follows: in section 2 we explicitly construct general spinors that transform in the fundamental representation of one of the two available independent $SO(4)$ isometries on $S^3 \times S^3$, that are also singlets under the action of the other — this ensures we are consistent with $\mathcal{N} = (4, 0)$ supersymmetry and an $SO(4)$ R-symmetry. In

²Solutions with a similar local foliation were recently constructed in [30, 31] by utilising Romans $F_{(4)}$ gauged supergravity.

³We shall impose that this entire $SO(4) \times SO(4)$ is preserved by the remaining physical fields also.

⁴The same methods could also be used to probe the space of M theory solutions.

section 3 we use G-structure techniques to extract geometric conditions from the $SO(4)$ spinors that all solutions should obey, and in sections 4 and 5 we find all local solutions that follow. The most interesting of these are clearly those that can be used to construct global solutions with compact internal space. We explicitly construct two such examples (though in IIA infinitely many are possible): a new $\mathcal{N} = (4, 0)$ massive IIA solution with large superconformal symmetry in section 4.1 and a new $\mathcal{N} = (4, 0)$ IIB solution in section 5.1 with small superconformal symmetry and $SU(2)$ outer automorphism symmetry. The IIA solution is constructed by gluing two locally non compact solutions together with a D8 brane defect. Further new local solutions preserving $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (4, 0)$ can be found in sections 4.2 and 5.2 respectively, but these are neither compact nor related to flows across dimensions (at not least obviously). In section 4.2 we speculate that these might also be used to construct globally compact solutions by using (this time) smeared Dp or NS5 brane defects for $p < 8$ to glue them together, however a detailed study is beyond the scope here. Finally appendix A details the conventions used throughout and B proves a claim made in 2.

Given the results here and [26], where AdS_3 solutions with exceptional R-symmetries were studied, it has become clear that such R-symmetry based spinor constructions are a powerful tool to study AdS_3 solutions with extended supersymmetry.

2 Realising an $SO(4)$ R-symmetry on $S^3 \times S^3$

In this section we will construct Killing spinors that realise the bosonic sub-algebra of $\mathfrak{d}(2, 1, \alpha)$, the conventions used and explicit representations of the of what follows can be found in appendix A.

We are interested in large $\mathcal{N} = (4, 0)$ AdS_3 solutions preserving an $SO(4)$ R-symmetry. Any AdS_3 solution can be expressed in the form

$$\begin{aligned}
 ds^2 &= e^{2A} ds^2(AdS_3) + ds^2(M_7), \\
 F &= f + e^{3A} \text{Vol}(AdS_3) \wedge \star_7 \lambda(f), \quad H = h_0 \text{Vol}(AdS_3) + H_3,
 \end{aligned}
 \tag{2.1}$$

where F is the RR polyform⁵ and H the NS 3-form fluxes. Both of these decompose in terms of their magnetic components f, H_3 which are defined on M_7 only and electric counterparts with legs on all AdS_3 directions ensuring that fluxes respect the isometry of AdS_3 . The Bianchi identity $dH = 0$ fixes h_0 to be constant, while the electric component of F is fixed such the lower and higher fluxes are correctly related under 10d hodge duality — for this reason the operator λ is defined such that

$$\lambda(X_n) = (-)^{\frac{n}{2}(n-1)} X_n
 \tag{2.2}$$

when acting on an n -form. Finally the AdS warp factor e^{2A} , and likewise the dilaton Φ have support on M_7 only and the RR polyform obeys

$$dF - H \wedge F = 0,
 \tag{2.3}$$

away from localised sources.

⁵In IIA it is $F = F_0 + F_2 + F_4 + F_6 + F_8 + F_{10}$, in IIB it is $F = F_1 + F_3 + F_5 + F_7 + F_9$.

As we seek solutions preserving $\mathcal{N} = (4, 0)$ the 10 dimensional Majorana spinors may be written as

$$\epsilon_1 = \sum_{I=1}^4 \zeta^I \otimes v_+ \otimes \chi_1^I, \quad \epsilon_2 = \sum_{I=1}^4 \zeta^I \otimes v_{\mp} \otimes \chi_2^I \quad (2.4)$$

where ζ^I and $\chi_{1,2}^I$ are 4 independent spinors on AdS_3 and M_7 respectively and v_{\pm} is an auxiliary 2 vector, that is always required when decomposing an even dimensional spinor in terms of 2 odd ones — \pm refers to chirality, so the upper/lower signs are taken in IIA/B. The spinors on AdS_3 are Killing, so they obey the equation

$$\nabla_a^{\text{AdS}_3} \zeta^I = \frac{\mu}{2} \gamma_a^{\text{AdS}_3} \zeta^I \quad (2.5)$$

where $\frac{\mu}{|\mu|} = \pm 1$ parametrise a spinor that is charged under the $\text{SL}(2)_{\text{L/R}}$ subgroup of $\text{SO}(2,2) \cong \text{SL}(2)_{\text{L}} \times \text{SL}(2)_{\text{R}}$ and is a singlet under $\text{SL}(2)_{\text{R/L}}$.

As we want an $\text{SO}(4)$ R-symmetry, $\chi_{1,2}^I$ should transform in fundamental of this group and solutions should admit a local description with $\text{SO}(4)$ realised geometrically. There are several ways to do this, but from the perspective of finding solutions, the simplest way to realise this R-symmetry is to decompose the internal space as a foliation of $S_1^3 \times S_2^3$ over an interval in which the physical fields have support only, i.e. we take the internal metric to be

$$ds^2(M_7) = e^{2k} dr^2 + e^{2C_1} ds^2(S_1^3) + e^{2C_2} ds^2(S_2^3) \quad (2.6)$$

where the functions $e^{2k}, e^{2C_1}, e^{2C_2}$ and also now e^A and Φ depend on r only and we impose that the fluxes depend on S_1^3 and S_2^3 through their respective volume forms only. This gives us an $\text{SO}(4)_1 \times \text{SO}(4)_2$ isometry on M_7 to work with allowing for enhancements to $\mathcal{N} = (4, 4)$ supersymmetry with $\text{SO}(4) \times \text{SO}(4)$ R-symmetry whenever the physical fields obey certain constraints we shall discuss at the end of the section. A Killing spinors on $S_{1,2}^3$, ξ , obeys the equations

$$\nabla_i^{S^3} \xi = \frac{i\nu}{2} \gamma_i^{S^3} \xi, \quad \nabla_i^{S^3} \xi^c = \frac{i\nu}{2} \gamma_i^{S^3} \xi^c \quad (2.7)$$

for ξ^c the Majorana conjugate of ξ . This time $\frac{\nu}{|\nu|} = \pm 1$ parametrise a spinor charged under the $\text{SU}(2)_{\text{L/R}}$ subgroup of $\text{SO}(4) \cong \text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}}$ that are singlets under the $\text{SU}(2)_{\text{R/L}}$. As shown in [34], in the Hopf fibration frame of S^3 (A.7), the doublets of $\text{SU}(2)_{\text{L/R}}$ are simply

$$\xi^a = \begin{pmatrix} \xi \\ \xi^c \end{pmatrix}^a, \quad (2.8)$$

and obey the S^3 Killing spinor equation (2.7) component by component. On S^3 there are two sets of one-forms that are charged under $\text{SU}(2)_{\text{L}}$ and $\text{SU}(2)_{\text{R}}$ that are dual to the corresponding $\text{SU}(2)_{\text{L/R}}$ Killing vectors. We parametrise these in a unified language as K_i such that

$$dK_i + \frac{\nu}{2} \epsilon_{ijk} K_j \wedge K_k = 0, \quad (2.9)$$

where the sign of ν determines the relevant $SU(2)$ as before — these are of course the $SU(2)_{R/L}$ invariant 1-forms. The spinorial Lie derivative⁶ of the $SU(2)_{L/R}$ doublet along the $SU(2)_{L/R}$ Killing vectors is

$$\mathcal{L}_{K_i}\xi^a = \frac{i\nu}{2}(\sigma_i)^a_b \xi^b, \tag{2.11}$$

where σ_i are the Pauli matrices, so it is the Lie algebra of $SU(2)$ appearing on the right hand side. Acting on the $SU(2)_L$ doublet with the $SU(2)_R$ Killing vector on the other hand, or vice-versa, yields zero. One can exploit (2.11) to form a spinor transforming in the fundamental of $SO(4)_{L/R} \cong SO(3)_{1L/R} \times SO(3)_{2L/R}$ and as a singlet under $SO(4)_{R/L}$ depending on the sign of ν . When one couples such an $SO(4)$ spinor to an AdS_3 spinor as in (2.4), the result will be a spinor realising the bosonic algebra

$$\mathfrak{sl}(2) \oplus \mathfrak{so}(4) \tag{2.12}$$

as required for large $\mathcal{N} = (4, 0)$ supersymmetry. Thus let us now construct such an $SO(4)$ spinor.

In [36] it was established how to form an $SO(3)$ triplet from products of two $SU(2)$ doublets — when the doublets are both formed from S^3 Killing spinors, there is only one such triplet (for each sign of ν), namely

$$\eta_i = (\sigma_2 \sigma_i)_{ab} \xi_1^a \otimes \xi_2^b, \tag{2.13}$$

it turns out that this is also Majorana. We define diagonal and anti-diagonal $SO(3)$ Killing vectors as

$$K_i^+ = K_i^1 + K_i^2, \quad K_i^- = K_i^1 - K_i^2, \tag{2.14}$$

then it is a simple exercise in Pauli matrix manipulations to establish that

$$\mathcal{L}_{K_i^+}\eta_j = \nu \epsilon_{ij}^k \eta_k, \quad \mathcal{L}_{K_i^-}\eta_j = 0, \tag{2.15}$$

so that K_i^+ realises the Lie algebra of $SO(3)$. We can parameterise a basis for the Lie algebra of $SO(4)$ in block form as

$$(T_i^+)_{ij} = \left(\begin{array}{c|c} \epsilon_{ijk} & 0 \\ \hline 0^T & 0 \end{array} \right), \quad (T_i^-) = \left(\begin{array}{c|c} 0_{3 \times 3} & c_i \\ \hline -c_i^T & 0 \end{array} \right), \tag{2.16}$$

where $c_1 = (1, 0, 0)^T$, $c_2 = (0, 1, 0)^T$, $c_3 = (0, 0, 1)^T$. It is then clear that 3 components of the $SO(4)$ spinor one wishes to construct are simply the $SO(3)$ triplet as these give rise to the top left blocks of (2.16) under K_i^+ and K_i^- . The 4th component should be a singlet

⁶In general, when taken along a Killing vector K , this is defined as

$$\mathcal{L}_K \psi = K^m \nabla_m \psi + \frac{1}{4} \nabla_m K_n \Gamma^{mn} \psi \tag{2.10}$$

for ψ an arbitrary spinor, Γ_m a basis curved gamma matrices, ∇_m the covariant derivative.

under the action of K_i^+ — such a spinor was also provided in [36], and for S^3 is once more unique and Majorana

$$\eta_4 = -i(\sigma_2)_{ab}\xi_1^a \otimes \xi_2^b. \tag{2.17}$$

So there is exactly one SO(4) spinor (for each sign of ν) we can define on $S^3 \times S^3$, namely

$$\eta^I = (\mathcal{M}_I)_{ab}\xi_1^a \otimes \xi_2^b, \quad \mathcal{M}_I = (\sigma_2\sigma_1, \sigma_2\sigma_2, \sigma_2\sigma_3, -i\sigma_2)_I. \tag{2.18}$$

It is not hard to confirm that

$$\mathcal{L}_{K_i^\pm}\eta^I = \nu(T_i^\pm)^I{}_J\eta^J, \quad \eta^{I\dagger}\eta^J = \delta^{IJ}, \tag{2.19}$$

where in the latter we fix an arbitrary normalisation. So the spinorial Lie derivative of the SO(4) spinor along the SO(4) Killing vectors realise the associated Lie algebra. We can now write the explicit form of the SO(4) spinors on M_7 — given that they must be Majorana, and satisfy $|\chi_{1,2}^I|^2 = e^A$ component by component [26], the most general form these can take may be parametrised as

$$\chi_1^I = e^{\frac{A}{2}} \begin{pmatrix} \sin(\beta_1 + \beta_2) \\ i \cos(\beta_1 + \beta_2) \end{pmatrix} \otimes \eta^I, \quad \chi_2^I = e^{\frac{A}{2}} \begin{pmatrix} \sin(\beta_1 - \beta_2) \\ i \cos(\beta_1 - \beta_2) \end{pmatrix} \otimes \eta^I, \tag{2.20}$$

where β_1, β_2 are functions of r only. Now since each component of $\chi_{1,2}^I$ can be mapped into every other through the action of the R-symmetry (2.19), which we assume the physical fields also respect, we need only explicitly solve the supersymmetry conditions of an $\mathcal{N} = 1$ sub-sector to know that $\mathcal{N} = (4, 0)$ is preserved. There are several ways to see this but perhaps the most easy argument to follow comes from considering the conditions on (2.4) that follow from setting the gravitino and dilatino variations to zero directly — we defer the proof of this claim to appendix B.

Having established that solving for an $\mathcal{N} = 1$ sub-sector of our SO(4) spinor is sufficient to know that $\mathcal{N} = (4, 0)$ is realised, we shall now take this sub-sector to be,

$$\chi_1 = \chi_1^4, \quad \chi_2 = \chi_2^4, \tag{2.21}$$

but we stress that as long as the SO(4) R-symmetry is preserved by a solution, this choice is totally arbitrary.⁷

Finally, before moving on we should address the issue of a potential enhancement of supersymmetry. If one finds a solution with metric, dilaton and fluxes that do not depend on the signs of μ, ν then there exists a second independent 10 dimensional spinor of the form (2.4) charged under the second copies of SO(4) and SL(2) at our disposal. This enhances supersymmetry to large $\mathcal{N} = (4, 4)$ — as we shall see, this will indeed happen in some instances.

⁷This would no longer be the case if we were to break the R symmetry to some subgroup by for instance fibring one S^3 over the other as in [36] or with the fluxes — then different choices of $\chi_{1,2}$ would lead to different amounts of supersymmetry preserved.

3 Supersymmetric bi-spinors conditions

Historically one established whether a supergravity solution was supersymmetric by solving spinorial conditions that follow from setting the gravitino and dilatino variations to zero. A more modern approach is the bi-spinor formalism where one instead attempts to solve a set of (generalised) geometric constraints that are necessary and sufficient for supersymmetry. Geometric conditions for AdS₃ solutions in type II, i.e. those of the form (2.1), to preserve $\mathcal{N} = 1$ supersymmetry were recently presented in [26], where it was additionally found that a non trivial RR sector requires one to fix $h_0 = 0$.⁸

The fundamental object in the construction of [26] is the 7 dimensional bi-spinor $\chi_1 \otimes \chi_2^\dagger$ that is defined in terms of two 7 dimensional Majorana spinors $\chi_{1,2}$ defined on the internal space M₇ as

$$\chi_1 \otimes \chi_2^\dagger = \frac{1}{8} \sum_{n=0}^7 \frac{1}{n!} \chi_2^\dagger \gamma_{a_n \dots a_1} \chi_1 e^{a_1} \wedge \dots \wedge e^{a_n}, \quad |\chi_1|^2 = |\chi_2|^2 = e^A, \quad (3.1)$$

with γ_a a basis of the flat space gamma matrices in 7 dimensions and e^a is a vielbein on M₇ (ie (A.4)), as such the bi-spinor is a polyform. It is a general feature (in odd dimensions) that the bi-spinor can be decomposed as

$$e^{-A} \chi_1 \otimes \chi_2^\dagger = \Psi_+ + i\Psi_-, \quad (3.2)$$

for Ψ_\pm two real polyforms containing only even/odd forms. An AdS₃ solution in type II supergravity of the form (2.1) is guaranteed⁹ to satisfy $\mathcal{N} = 1$ supersymmetry if it obeys

$$\begin{aligned} (d - H \wedge)(e^{A-\Phi} \Psi_\mp) &= 0, \\ (d - H \wedge)(e^{2A-\Phi} \Psi_\pm) - 2\mu e^{A-\Phi} \Psi_\mp &= \pm \frac{e^{3A}}{8} \star_7 \lambda(f), \\ e^{-\Phi}(f, \Psi_\pm) - \frac{\mu}{2} \text{Vol}_7 &= 0, \end{aligned} \quad (3.3)$$

with the upper/lower signs taken in IIA/B and where (\cdot, \cdot) is the Mukai pairing in 7 dimensions defined as

$$(X, Y) = \left(\lambda(X) \wedge Y \right)_7, \quad (3.4)$$

with the operator λ defined in (2.2). The μ that appears is a constant defining the AdS radius as in (2.5), it can in fact be set to $\mu = \pm 1$ by rescaling e^A without loss of generality, but it will become useful to have kept track of it later.¹⁰

In the previous section an $\mathcal{N} = 1$ spinor was constructed, (2.21), on $M_7 = \mathbb{R} \times S^3 \times S^3$ that transforms under an SO(4) R-symmetry. When the physical fields respect this SO(4)

⁸One can make this assumption without loss of generality because, while solutions with $f = 0$ and $h_0 \neq 0$ do exist, they all lie in the common NS sector of type II supergravity. As such, when viewed as solutions in IIB, they are all S-dual to solutions with $f \neq 0$ and $h_0 = 0$.

⁹Strictly speaking the following statement is only true away from localised sources. When these are present supersymmetry also requires some additional constraints that we shall discuss when this issue arises.

¹⁰The same applies for ν and the two 3-sphere warp factor $e^{2C_{1,2}}$ appearing in (2.7) and elsewhere in the previous section.

the amount of supersymmetry preserved by any solution one can construct from this spinor will be at least $\mathcal{N} = (4, 0)$, with the remaining independent spinors generated through the action of the $\text{SO}(4)$ R-symmetry. Specifically one has

$$\chi_1 = -ie^{\frac{A}{2}}(\sigma_2)_{ab} \begin{pmatrix} \sin(\beta_1 + \beta_2) \\ i \cos(\beta_1 + \beta_2) \end{pmatrix} \otimes \xi_1^a \otimes \xi_2^b, \quad \chi_2 = -ie^{\frac{A}{2}}(\sigma_2)_{ab} \begin{pmatrix} \sin(\beta_1 - \beta_2) \\ i \cos(\beta_1 - \beta_2) \end{pmatrix} \otimes \xi_1^a \otimes \xi_2^b$$

as this is a tensor product of spinors in each factor of the foliated internal space (2.6) it should be clear from its definition (3.1) that the 7 dimensional bi-spinor can be expressed in terms of wedge products of bi-spinors on the interval and two 3-spheres — to this end it is useful to know the bi-spinors on $S_{1,2}^3$ as these are the only non trivial building blocks one requires. One can show, [34], that the matrix spinor¹¹ following from the two $\text{SU}(2)$ spinor doublets of the form (2.8) are

$$\xi_{1,2}^a \otimes \xi_{1,2}^{b\dagger} = \frac{1}{2} \left((1 - ie^{3C_{1,2}} \text{Vol}(S_{1,2}^3)) \delta^{ab} + \left(\frac{1}{2} e^{C_{1,2}} K_i^{1,2} - \frac{i}{8} e^{2C_{1,2}} \epsilon_{ijk} K_j^{1,2} \wedge K_k^{1,2} \right) (\sigma^i)^{ab} \right) \quad (3.5)$$

where $e^{2C_{1,2}}$ are the warp factor appearing in (2.6), and $K_i^{1,2}$ are the $\text{SU}(2)$ forms on the two 3-spheres that each obey (2.9). Given (3.5) it is now a relatively simple exercise to construct Ψ_{\pm} . These can be most succinctly written in terms of an $\text{SU}(3)$ -structure as

$$\Psi_+ = \frac{1}{8} \text{Re} \left[e^{i\beta_2} e^{-iJ} - e^k dr \wedge \Omega \right], \quad \Psi_- = \frac{1}{8} \text{Im} \left[-e^{i\beta_2} e^k dr \wedge e^{-iJ} + \Omega \right] \quad (3.6)$$

where the specific $\text{SU}(3)$ -forms are

$$J = \frac{1}{4} e^{C_1 + C_2} \left(K_1^1 \wedge K_1^2 + K_2^1 \wedge K_2^2 + K_3^1 \wedge K_3^2 \right),$$

$$\Omega = \frac{1}{8} e^{i\beta_1} \left(e^{C_1} K_1^1 + ie^{C_2} K_1^2 \right) \wedge \left(e^{C_1} K_2^1 + ie^{C_2} K_2^2 \right) \wedge \left(e^{C_1} K_3^1 + ie^{C_2} K_3^2 \right). \quad (3.7)$$

At this point, in principle, one could blindly plug (3.6) into (3.3) and find every solution that is consistent with the metric and spinor — but one needs to take a little more care if one wants to ensure that $\text{SO}(4) \times \text{SO}(4)$ symmetry and $\mathcal{N} = (4, 0)$ supersymmetry is preserved. As long as the dilaton and warp factors of the metric only depend on the interval the only remaining issue is the fluxes. Specifically (3.3) only assumes $\mathcal{N} = 1$ supersymmetry is unbroken so the second condition will generically define flux components that break some (super)symmetry. To mitigate this issue we demand that all fluxes must decompose in a basis of the invariant forms of $\text{SO}(4) \times \text{SO}(4)$, namely

$$dr, \quad \text{Vol}(S_1^3), \quad \text{Vol}(S_2^3), \quad (3.8)$$

and their wedge products, with functional support on the interval only — this greatly increases the number of independent conditions in (3.3) that give rise to purely geometric

¹¹Component by component this is defined as in (3.1), but with $n = 0, \dots, 3$ and weighted by $\frac{1}{2}$ rather than $\frac{1}{8}$ with the veilbein just the $e^{i1,2}$ parts of (A.4).

constraints and allows for the exact local form of all solutions consistent with an $SO(4) \times SO(4)$ isometry to be found in the following sections. We study type IIA in section 4 and type IIB in section 5, in both instances we fix the NS 3-form as

$$H = c_1 \text{Vol}(S_1^3) + c_2 \text{Vol}(S_2^3), \tag{3.9}$$

for constants c_i without loss of generality.

4 All local solutions in type IIA

In this section we find all $\mathcal{N} = (4, 0)$ solutions with an $SO(4) \times SO(4)$ isometry in type IIA supergravity. There are two independent forms of local solution we study in sections 4.1 and 4.2 that (generically) preserve $\mathcal{N} = (4, 0)$ and $\mathcal{N} = (4, 4)$ respectively. We show how the former can be used to construct new compact global solutions, and provide a hint as to how one might do the same with the latter.

Upon plugging the bi linears of (3.6) into (3.3) one quickly realises two zero form constraints

$$(\cos \beta_1 e^{C_1} - \sin \beta_1 e^{C_2}) = (\mu \cos \beta_1 e^{C_1} - \nu e^A \sin \beta_2) = 0 \tag{4.1}$$

these are very useful as they cannot be solved when any of $\cos \beta_1$, $\sin \beta_1$ or $\sin \beta_2$ are set to zero, as this would require us to do the same to one of the warp factors. We can then take (4.1) as general definitions for e^{C_i} in IIA and eliminate these factors from the rest of the supersymmetry constraints, after some work we find the additional conditions

$$\beta_1' = F_2 = F_0 \cos \beta_2 = 0, \tag{4.2}$$

$$c_1 \cos^4 \beta_1 + c_2 \sin^4 \beta_1 = \mu^2 c_2 \sin^3 \beta_1 + \nu^3 \cos \beta_1 e^{2A} \sin(2\beta_2) = 0, \tag{4.3}$$

$$(e^{5A-\Phi} \sin^3 \beta_2)' - 2\mu e^{4A+k-\Phi} \cos \beta_2 \sin^2 \beta_2 = 0, \tag{4.4}$$

$$(e^{3A} \sin^3 \beta_2)' - \frac{3\mu}{2} e^{2A+k} \cos \beta_2 + \frac{3}{4} e^{3A+k-\Phi} \sin^2 \beta_2 F_0 = 0, \tag{4.5}$$

where the last of these comes from imposing that F_0 is constant — the rest of the Bianchi identities then follow rather trivially. Clearly there are two cases, $F_0 = 0$ and $\cos \beta_2 = 0$.

4.1 Case I: compact solutions from D8/O8s back reacted on $AdS_3 \times S^3 \times S^3 \times S^1$

For Case I we set

$$\cos \beta_2 = 0, \quad \sin \beta_2 = s, \quad s = \pm 1, \tag{4.6}$$

then (4.3) implies also

$$c_1 = c_2 = 0, \tag{4.7}$$

so there is no NS flux turned on. We can solve (4.4) by fixing

$$e^{5A-\Phi} = qL^5, \tag{4.8}$$

where L and q are constants and we use diffeomorphism invariance to fix

$$e^{A+k} = qL. \tag{4.9}$$

We can use (4.1) to define e^{C_1}, e^{C_2} which leaves (4.5) to solve. This becomes simply $(L^4 e^{-4A})' = \nu F_0$, which is solved by

$$L^4 e^{-4A} = H_8, \quad H_8 = F_0 \nu r + c \tag{4.10}$$

for c another constant — i.e. the warp factor of a D8 brane or O8 hole depending on the sign of F_0 and ν . We then fix

$$s = \mu = \nu = \pm 1 \tag{4.11}$$

and find the following general form for local solutions in massive IIA

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H_8}} \left(L^2 ds^2(\text{AdS}_3) + \frac{L^2}{\cos^2 \beta_1} ds^2(S_1^3) + \frac{L^2}{\sin^2 \beta_1} ds^2(S_2^3) \right) + \sqrt{H_8} q^2 dr^2, \\ F_4 &= 2q^2 H_8 \left(L^2 \text{Vol}(\text{AdS}_3) + \frac{L^2}{\cos^2 \beta_1} \text{Vol}(S_1^3) + \frac{L^2}{\sin^2 \beta_1} \text{Vol}(S_2^3) \right) \wedge dr, \\ e^{-\Phi} &= q H_8^{\frac{5}{4}}. \end{aligned} \tag{4.12}$$

Clearly when $F_0 = 0$ we recover the standard solution on $AdS_3 \times S^3 \times S^3 \times S^1$ which preserves $\mathcal{N} = (4, 4)$ supersymmetry — in this limit none of the physical fields depend on $\mu = \nu = \pm 1$ confirming the enhanced supersymmetry. The generic local solution is D8 branes or O8 planes or both¹² back reacted on this. As the warp factor now depends on ν supersymmetry is broken to $\mathcal{N} = (4, 0)$ in the presence of the back reacted D8/O8 system — which is by no means a surprise.

When $F_0 \neq 0$ the internal space of (4.12) is non compact, if we assume $F_0 > 0$ and $\nu = 1$, then the warp factor does bound the interval from below at $r = -\frac{c}{F_0}$ where the behaviour is consistent with a D8/O8 system wrapped on $AdS_3 \times S^3 \times S^3$, however the interval is not bounded from above and $r \rightarrow \infty$ is at infinite proper distance. Before giving up though, one should remember that this is only a local solution — which is to say that all coordinate patches of a global solution can be expressed in the form (4.12). One can try to make a compact solution by gluing a second mirrored copy of (4.12) onto the first in the spirit of [38]. At the point where the local patches connect there should be a D8 brane defect where F_0 jumps, but the metric and dilaton are continuous. The simplest way to arrange for this is to glue the patches together at $r = 0$ and have F_0 flip from positive to negative as one crosses $r = 0$ from below, i.e. one takes the warp factor to be

$$H_8 = c + |F_0| r, \quad r < 0, \quad H_8 = c - |F_0| r, \quad r > 0, \tag{4.13}$$

so that the metric and dilaton are continuous without the need to further tune constants, and only F_0 jumps. This does indeed bound r to the interval \mathcal{I} between two D8/O8 systems at $r = \pm \frac{c}{|F_0|}$ and one is now able to quantise the fluxes without issue. In units where $g_s = \alpha' = 1$ one requires that the following charges are integer valued

$$n_0 = 2\pi F_0, \quad N_2 = -\frac{1}{(2\pi)^5} \int_{S_1^3 \times S_2^3} \star F_4, \quad N_4^i = \frac{1}{(2\pi)^3} \int_{S_i^3} \int_{r \in \mathcal{I}} F_4. \tag{4.14}$$

¹²The near horizon geometry of a D8 brane is indistinguishable from the geometry near an O8 plane.

This is not hard to achieve by tuning

$$\frac{2\pi F_0}{c_1^2 q^2 L^2} = \frac{1}{N_4^1} + \frac{1}{N_4^2}, \quad \frac{L^5 q}{4\pi \sin^3 \beta_1 \cos^3 \beta_1} = N_1, \quad \tan \beta_1 = \frac{N_4^2}{N_4^1}, \quad (4.15)$$

and the curvature of the solution is under parametric control. A standard computation¹³ then leads to the finite central charge

$$c = 6c_1 N_2 \frac{N_4^1 N_4^2}{N_4^1 + N_4^2} + \mathcal{O}(1) \quad (4.17)$$

where $\mathcal{O}(1)$ parametrises sub leading terms that supergravity is insensitive to (at least with the computation performed here). Note that this is independent of F_0 and when $c_1 = 1$ is actually the same central charge as that of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$. More generally it is consistent with (1.1), i.e. what one expects from a CFT with large superconformal algebra provided c_1 is integer — indeed one can identify

$$k_+ = c_1 N_2 N_4^1, \quad k_- = c_1 N_2 N_4^2, \quad \alpha = \tan \beta_1^2 = \frac{N_4^2}{N_4^1}, \quad (4.18)$$

with α the continuous parameter of $\mathfrak{d}(2, 1, \alpha)$.

This all sounds very positive, however to be sure this solution really exists, and preserves supersymmetry, we need to check the Bianchi identities at the D8 brane defect are satisfied and that the D8 brane is calibrated [42–44]. The Bianchi identities in the presence of the defect require

$$df = \frac{N_8}{2\pi} \delta(r) e^{2\pi f_g} \wedge dr \quad (4.19)$$

where f_g is a gauge field on the world volume of the D8 brane. We find ourselves in a far simpler scenario than [38], because there is no NS flux and the only RR flux that shifts across the defect is F_0 as

$$\Delta F_0 = 2|F_0|. \quad (4.20)$$

Comparing this with the integrated form of (4.19), we find that the Bianchi identity requires simply

$$N_8 = 4\pi|F_0| = 2|n_0|, \quad f_g = 0. \quad (4.21)$$

It is also not hard to confirm that the brane is supersymmetric — this is so whenever the DBI action of a given brane satisfies a so called calibration condition. Here the DBI action of the D8 should equal the integral of $e^{3A-\Phi} \text{Vol}(\text{AdS}_3) \wedge \Psi_6$ — a quick computation shows

¹³The holographic central charge for a warped AdS_3 solution in 10 dimensions at leading order is given for instance in [20], converting this reference to string frame and setting $\alpha' = g_s = \mu = 1$ gives

$$c = \frac{3}{2^4 \pi^6} \int_{M_7} e^{A-2\Phi} \text{Vol}(M_7). \quad (4.16)$$

Using this formula with $F_0 = 0$ (ie the $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ limit) one finds $c = 6N_1 \frac{N_4^1 N_4^2}{N_4^1 + N_4^2}$ which implies $k_+ = N_1 N_4^1$, $k_- = N_1 N_4^2$ and $\alpha = \tan^2 \beta_1 = \frac{N_4^2}{N_4^1}$.

this to indeed be the case. Thus we have constructed a bona fide $\mathcal{N} = (4, 0)$ solution in massive IIA.

The result of gluing the two local solutions together is essentially a global solution with an orientifold under which a circle parameterised by r becomes a segment. At the two ends there are two O8-planes with different charges and tensions. One can interpret this as two O8₋s with k and $16 - k$ D8s, or when $k = 8$ as an O8₋ and an O8₊, the later being similar to what appears in the recently constructed classical dS solutions in [40].

It would be interesting to find the local solution of (4.12) as a near horizon limit of some intersecting brane set-up. This should be in some sense a trivial extension of the realisation of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ in terms of D2 and D4 branes — however to the author’s knowledge this first step is currently absent from the literature (see [4] for every near horizon realisation of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ except D2-D4), and finding it is beyond the scope here.

Finally let us stress that there is not any particular need to place the D8 brane defect at $r = 0$ and so there is no restriction to gluing together an arbitrary number of local solutions of the form (4.12), with a D8 brane defect at each intersection in the vein of [41]. All one needs to ensure is that h_8 is continuous across each defect by tuning c_1, F_0 and the intersection points in each local patch, and that the interval has an upper and lower bound. One can construct infinitely many globally distinct compact solutions in this fashion, so it would be interesting to study this possibility in more detail. The dual CFT interpretation of this infinite class will presumably be adding various amounts of conformal matter to the CFT dual of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ in such a way that $\mathcal{N} = (4, 4)$ is broken to $\mathcal{N} = (4, 0)$ — but that remains to be seen.

4.2 Case II: a new local $\mathcal{N} = (4, 4)$ solution with O2 plane

For Case II we set

$$F_0 = 0. \tag{4.22}$$

To avoid falling into a sub-case of the previous section one must demand $\cos \beta_2 \neq 0$ which requires the same of c_1, c_2 without loss of generality. We can solve the first condition of (4.3) with

$$c_1 = c \sin^4 \beta_1, \quad c_2 = -c \cos^4 \beta_1 \tag{4.23}$$

and take the second condition as the definition of e^A . Using this, and by taking a linear combination of (4.4) and (4.5) such that e^k is eliminated one finds $(\tan \beta_2 e^{-2\Phi})' = 0$ which is solved by

$$\tan \beta_2 e^{-2\Phi} = q^2 \tag{4.24}$$

where q is a constant. At this point it is useful to use diffeomorphism invariance to fix e^k in terms of another arbitrary function $f(r)$ such that

$$32\nu^3 e^{A+k} = c\mu \sin^3 \beta_1 f' \tag{4.25}$$

the remaining conditions (4.4)–(4.5) both then reduce to $f' = (\sec \beta_2)'$ which one can solve without loss of generality with

$$\cos \beta_2 = \frac{1}{\sqrt{f}}. \tag{4.26}$$

As the left hand side of this expression is bounded between 0 and 1, we must have that $1 < f < \infty$, a sensible choice is then

$$f = \frac{1}{\cos^2 r}, \tag{4.27}$$

so that we simply have $\beta_2 = r$. This leads to a completely determined local solution of the form

$$\begin{aligned} ds^2 &= L^2 \left[\frac{1}{\cos r \sin r} ds^2(\text{AdS}_3) + \frac{\sin^3 r}{\cos^5 r} dr^2 + \frac{\sin r}{\cos r} \left(\frac{1}{\cos^2 \beta} ds^2(S_1^3) + \frac{1}{\sin^2 \beta} ds^2(S_2^3) \right) \right], \\ H &= 2L^2 \left(\frac{\tan \beta}{\cos^2 \beta} \text{Vol}(S_1^3) - \frac{\cot \beta}{\sin^2 \beta} \text{Vol}(S_2^3) \right), \quad qe^\Phi = \sqrt{\tan r}, \\ F_4 &= 2L^3 \left[4 \frac{\sin 4r}{\sin^2 2r} \text{Vol}(\text{AdS}_3) + q \frac{\tan r}{\cos^2 r} \left(\frac{1}{\cos^2 \beta} \text{Vol}(S_1^3) + \frac{1}{\sin^2 \beta} \text{Vol}(S_2^3) \right) \right] \wedge dr, \end{aligned} \tag{4.28}$$

where we have introduced

$$L^2 = \frac{c}{2} \cos^3 \beta \sin^3 \beta, \tag{4.29}$$

and fixed

$$\sin \beta_1 = \nu \sin \beta, \quad \mu = \nu = \pm 1, \tag{4.30}$$

to simplify expressions. Notice that none of the physical fields depend on $\nu = \pm 1$ — so this solution experiences an enhancement of supersymmetry to large $\mathcal{N} = (4, 4)$.

The internal radial coordinate is bounded as $0 < r < \frac{\pi}{2}$, with the lower bound a singularity of the metric. The behaviour close to $r = 0$ is intriguing, indeed after redefining $r = \sqrt{y}$ the behaviour is that of O2 planes at the base of a cone over $S^3 \times S^3$, which is rather novel. More disappointing is the behaviour close to $r = \frac{\pi}{2}$ where the metric is actually regular but the dilaton is infinite, which does not appear to be physical behaviour. Worst still perhaps, is that $r = \frac{\pi}{2}$ is at infinite proper distance, so the internal space is non-compact. One way to see this is with the central charge which goes like

$$c \sim \lim_{r \rightarrow \frac{\pi}{2}} \tan^4 r \tag{4.31}$$

which is clearly divergent. Thus any putative CFT dual will have a continuous operator spectrum, a sign that it is sick.

One might be able to cure this issue as before by gluing two copies of (4.28) together. As $F_0 = 0$, one can no longer achieve this with D8 branes. However, since this sort of gluing does work with D8 branes, T-duality and S-duality informs us that at the very least, it should be possible to glue solutions together with other types of branes when they are smeared over all but one of their co-dimensions — the options here are D2 and NS5 branes. As this may be a way of constructing new holographic duals to well defined CFTs with large $\mathcal{N} = (4, 4)$ supersymmetry it would certainly be interesting to peruse this possibility in future.

Finally, since this solution has no Romans mass turned on, it can be lifted to M-theory. As such this solution fits within the classification of [10–12]. At first sight this sheds doubt

on the possibility of constructing compact solutions by gluing copies of (4.28) together with defects, as [12] claims that the only AdS₃ solutions in M theory with compact internal space are locally AdS₃ × S³ × S³ × T². This follows from ruling out the possibility of compact solutions with localised sources on the boundary of the Riemann surface orthogonal to AdS₃ × S³ × S³, however scenarios with additional defects on the interior do not appear to have been considered in [12]. Thus if compact solutions can be realised from (4.28) it is then possible that a broader class of $\mathcal{N} = (4, 4)$ solutions with defects may also exist in M-theory — but at this point this is just speculation.

5 All local solution in type IIB

In this section we find the local form of all solutions preserving at least an SO(4) R-symmetry on S³ × S³. In section 5.1 we find a new compact solution containing D5s and O5s which actually preserves small $\mathcal{N} = (4, 0)$ supersymmetry, while in section 5.2 we find a solution that back reacts D5s on AdS₃ × S³ × S³ × S¹, but is non compact.

Once again we begin by plugging the bi linears of (3.6) into (3.3). It is immediate to establish the following zero form constraints

$$c_1 \cos \beta_2 = c_2 \cos \beta_2 = \mu \sin \beta_2 = 0 \tag{5.1}$$

which means that for AdS₃ solutions we must set

$$c_1 = c_2 = \sin \beta_2 = 0 \tag{5.2}$$

and so all flux components but f_3 and f_7 are set to zero. We will thus parametrise the 3-form in terms of two constants c_3, c_4 as

$$f_3 = c_3 \text{Vol}(S_1^3) + c_4 \text{Vol}(S_2^3). \tag{5.3}$$

Given this, and after some massaging of expressions, it is possible to extract the following algebraic constraints

$$\cos \beta_2 = s, \tag{5.4}$$

$$\nu e^A (\cos \beta_1 e^{C_1} - \sin \beta_1 e^{C_2}) - s \mu e^{C_1+C_2} = 0, \tag{5.5}$$

$$e^A (c_3 \sin \beta_1 e^{3C_2} - c_4 \cos \beta_1 e^{3C_1}) + 2\mu e^{3C_1+3C_2-\Phi} = 0, \tag{5.6}$$

where $s = \pm 1$. Using these we can simplify the differential constraints to

$$(\beta_1)' = \cos \beta_1 \sin \beta_1 (e^{C_1-C_2})' = 0, \tag{5.7}$$

$$(e^{2A+2C_1+2C_2-\Phi})' = 2s\nu e^{2A+C_1+C_2+k-\Phi} (\cos \beta_1 e^{C_2} + \sin \beta_1 e^{C_1}), \tag{5.8}$$

$$(e^{3A+2C_1+C_2-\Phi} \cos \beta_1)' = 2e^{2A+C_1+C_2+k-\Phi} (s\nu e^A + \mu \sin \beta_1 e^{C_1}), \tag{5.9}$$

$$(e^{3A+C_1+2C_2-\Phi} \sin \beta_1)' = 2e^{2A+C_1+C_2+k-\Phi} (s\nu e^A - \mu \cos \beta_1 e^{C_2}), \tag{5.10}$$

$$(e^{3A+3C_2-\Phi} \cos \beta_1)' - 2\mu e^{2A+3C_2+k-\Phi} \sin \beta_1 = c_3 e^{3A-3C_1+3C_2+k}, \tag{5.11}$$

$$(e^{3A+3C_1-\Phi} \sin \beta_1)' + 2\mu e^{2A+3C_1+k-\Phi} \cos \beta_1 = c_4 e^{3A+3C_1-3C_2+k}, \tag{5.12}$$

which are not all independent, but this form makes finding a solution easier.

It appears that there are 3 cases, $\cos \beta_1 = 0$, $\sin \beta_1 = 0$ and $(e^{C_1-C_2})' = 0$, however there is no physical difference between the first two of these as one is mapped to the other by relabelling the spheres — thus there are two physically distinct cases.

5.1 Case I: a new compact solution with D5s and O5s back reacted on $\text{AdS}_3 \times S^3 \times \mathbb{R}^4$

For Case I we set

$$\cos \beta_1 = t = \pm 1 \tag{5.13}$$

and take (5.5)–(5.6) to define the AdS warp factor and dilaton. Substituting this back into (5.7)–(5.12) one is left with just two independent conditions

$$\nu e^{C_2+k} - t (e^{C_1+C_2})' = 0, \quad \nu c_3 e^k + c_4 s t e^{3C_1} (e^{-2C_2})' = 0. \tag{5.14}$$

We use diffeomorphism invariance to fix

$$\nu e^{k+C_2} = s t, \tag{5.15}$$

which trivialises the first ODE of (5.14), and so allows us to integrate both as

$$e^{C_1+C_2} = L r, \quad L^2 e^{-4C_2} = h_5 = a + \frac{c_3}{c_4 r^2}, \tag{5.16}$$

for a an arbitrary constant and where we now fix $s = t = \mu = \nu = \pm 1$. The resulting solution then takes the form

$$ds^2 = L^2 \left[\frac{1}{\sqrt{h_5}} \left(ds^2(\text{AdS}_3) + ds^2(S_2^3) \right) + \sqrt{h_5} \left(dr^2 + r^2 ds^2(S_1^3) \right) \right], \quad e^{-\Phi} = \frac{c_4}{2L} \sqrt{h_5},$$

$$F_3 = c_4 \left(\text{Vol}(\text{AdS}_3) + \text{Vol}(S_2^3) \right) + \nu c_3 \text{Vol}(S_1^3). \tag{5.17}$$

When $a = 1$ and $\frac{c_3}{c_4} > 0$, h_5 is the warp factor of a D5 brane so the solution is D5 branes back reacted on $\text{AdS}_3 \times S^3 \times \mathbb{R}^4$, which is non compact. When $c_3 = 0$ the solution no longer depends on ν and so supersymmetry is enhanced to $\mathcal{N} = (4, 4)$, consistent with the fact that the solution is locally $\text{AdS}_3 \times S^3 \times T^4$ in this limit.

It may be possible to construct a globally compact solution by gluing copies of (5.17) together with D5 brane defects in a similar fashion to section 4.1, however for this case there is an easier way to achieve this. The previous discussion depended on tuning a, c_3, c_4 in a certain fashion, but there is no requirement to do this — indeed if one assumes $a < 1$ then r becomes bounded to the interval $[0, \sqrt{\frac{c_3}{c_4|a|}}]$ and the solution is compact. To see this more clearly one can perform the coordinate transformation and redefinition

$$r \rightarrow \sqrt{\frac{c_3}{|a|c_4}} \cos r, \quad L^2 \rightarrow |a|L^2, \tag{5.18}$$

which modifies the metric and dilaton as

$$ds^2 = L^2 \left[\frac{\cos r}{\sin r} \left(ds^2(\text{AdS}_3) + ds^2(S_2^3) \right) + \frac{c_3 \sin r}{c_4 \cos r} \left(\sin^2 r dr^2 + \cos^2 r ds^2(S_1^3) \right) \right],$$

$$F_3 = c_4 \left(\text{Vol}(\text{AdS}_3) + \text{Vol}(S_2^3) \right) + \nu c_3 \text{Vol}(S_1^3), \quad e^{-\Phi} = \frac{c_4}{2L^2} \tan r, \tag{5.19}$$

and leaves the flux unchanged. Clearly c_3 can no longer be set to zero so the ν dependence of the flux means that only $\mathcal{N} = (4, 0)$ supersymmetry is preserved in general. The internal radius is now bounded as $0 < r < \frac{\pi}{2}$ and at the end points there are singularities, however these have an obvious physical origin. It should not be hard to see that close to $r = 0$ the metric becomes that of O5 planes wrapped on $\text{AdS}_3 \times S_2^3$, while at $r = \frac{\pi}{2}$ it behaves as D5s wrapped on AdS_3 and either of the two 3-spheres.

Flux quantisation requires that the following charges are integer

$$N_5^i = \frac{1}{(2\pi)^2} \int_{S^3} F_3, \quad N_1 = -\frac{1}{(2\pi)^6} \int_{\mathbb{R} \times S_1^3 \times S_2^3} \star F_3. \quad (5.20)$$

This can be simply achieved by tuning

$$c_{i+2} = 2N_5^i, \quad \frac{c_3^2 L^4}{64c_4 \pi^2} = N_1, \quad (5.21)$$

and the radius about the singularities for which the supergravity approximation does not hold can be made arbitrarily small by making L (and so the D1 charge) large. We once more compute the holographic central charge at leading order and find that it is finite with the following form

$$c = 6N_1 N_5^2 + O(1). \quad (5.22)$$

This behaviour is markedly different from that of $\text{AdS}_3 \times S^3 \times S^3 \times S^1$, indeed the form the central charge takes is consistent with small $\mathcal{N} = (4, 0)$ superconformal algebra with level $k = N_1 N_5^2$. Since this solution is D5s and O5s back reacted on $\text{AdS}_3 \times S^3 \times \mathbb{R}^4$, which itself preserves (two copies of) small $\mathcal{N} = (4, 0)$, this should not be surprising. It is of course well known that small $\mathcal{N} = (4, 0)$ comes equipped with only an $\text{SU}(2)$ R-symmetry which at first sight may appear at conflict with the $\text{SO}(4)$ R-symmetry preserving spinors from which this solution is constructed. In fact this apparent conflict exists for $\text{AdS}_3 \times S^3 \times \mathbb{T}^4$ as well and the resolution is the same for both cases. The $\text{SO}(4)$ R symmetry of the geometry is actually realising both the $\text{SU}(2)$ R-symmetry of the dual CFT and an $\text{SU}(2)$ outer automorphism symmetry¹⁴ and the dual CFT indeed has small superconformal algebra. Such solutions lie within the $\alpha \rightarrow 0$ limit of $\mathfrak{d}(2, 1, \alpha)$ superconformal symmetry which is isomorphic to $\mathfrak{psu}(2, 1, \alpha) \rtimes \mathfrak{su}(2)$, so (5.19) lies within a degenerate limit of large $\mathcal{N} = (4, 0)$.

This solution is a well defined AdS dual to an as yet to be determined CFT_2 with small $\mathcal{N} = (4, 0)$ superconformal symmetry so is well deserving of further detailed study — but this is beyond the scope here.

5.2 Case II: D5s back reacted on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$

In case II we assume

$$0 < \sin \beta_1 < 1, \quad s = \mu = \nu = \pm 1. \quad (5.23)$$

Due to (5.7) this means that the two 3-sphere warp factors can only differ by a constant, thus we introduce a new function, H and constants b_1, b_2 such that

$$e^{C_i} = b_i H. \quad (5.24)$$

¹⁴I thank the reviewer for clarification on this point.

We again use (5.5)–(5.6) as definitions for e^A, e^Φ and substitute for these quantities in (5.7)–(5.12) — they once more reduce to just two conditions that may be easily solved with

$$b_1 = b\sqrt{c_3}, \quad b_2 = b\sqrt{c_4}, \quad \nu e^{C+k} = bq, \quad H = c + \nu\lambda_1 r, \quad (5.25)$$

where we introduce

$$\lambda_1 = \frac{q \cos \beta_1}{\sqrt{c_3}} - \frac{q \sin \beta_1}{\sqrt{c_4}}, \quad \lambda_2 = \frac{\cos \beta_1}{\sqrt{c_4}} + \frac{\sin \beta_1}{\sqrt{c_3}}, \quad (5.26)$$

to ease notation and b, c are constants. The general local form of solutions can then be written as

$$ds^2 = L^2 H \left(ds^2(\text{AdS}_3) + c_3 \lambda_2 ds^2(S_1^3) + c_4 \lambda_2 ds^2(S_2^3) \right) + \frac{L^2 q^2 \lambda_2^2}{H} dr^2, \\ F_3 = \frac{1}{\lambda_2^2} \text{Vol}(\text{AdS}_3) + c_3 \text{Vol}(S_1^3) + c_4 \text{Vol}(S_2^3), \quad e^\Phi = 2L^2 \lambda_2^2 H, \quad b = L\lambda_2 \quad (5.27)$$

The warp factor H depends on ν so this solution generically experiences no enhancement beyond $\mathcal{N} = (4, 0)$. Similar to section 4.1 however, when one sets $\lambda_1 = 0$, ν drops out of all expressions and supersymmetry is enhanced to $\mathcal{N} = (4, 4)$ — this is because the solution becomes $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ in this limit. The generic solution has D5 branes back reacted on this. The attentive reader will note that H is not the warp factor of a D5 brane, however, if one assumes $\nu = 1$, then the interval is bounded from below at

$$r = -\frac{c}{\lambda_1}, \quad (5.28)$$

where the near horizon geometry of a D5 brane wrapped on either S^3 is recovered. The interval is not however bounded from above and $r = \infty$ is at infinite proper distance, so the metric is non compact.

One might wonder about the possibility of making the solution compact by gluing two copies of (5.27) together with D5 branes smeared on S^3 — the issues are essentially the same as for Case II in IIA.

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A Conventions for sections 2 and 3

In this section we detail a set of conventions that can be used to perform the calculations in sections 2 and 3 — one should understand however that the result of these sections do not strictly depend on this choice, which is why they do not appear in the main text.

A.1 Spinors and gamma matrices on $\text{AdS}_3 \times \text{M}_7$

We decomposing the 10 dimensional gamma matrices as in [45]

$$\Gamma_\mu = \gamma_\mu^{\text{AdS}_3} \otimes \sigma_3 \otimes \mathbb{I}, \quad \Gamma_a = \mathbb{I} \otimes \sigma_1 \otimes \gamma_a, \quad B^{10} = \mathbb{I} \otimes \sigma_3 \otimes B, \quad \hat{\Gamma} = -\mathbb{I} \otimes \sigma_2 \otimes \mathbb{I}, \quad (\text{A.1})$$

where $\gamma_\mu^{\text{AdS}_3}$ are a real basis of flat space gamma matrices on AdS_3 , γ_a a 7 dimensional basis such that $B^{-1}\gamma_a B = -\gamma_a^*$ and $BB^* = \mathbb{I}$, $\sigma_{1,2,3}$ are the Pauli matrices and $\hat{\Gamma}$ is the chirality matrix. A 10 dimensional spinor of \pm chirality on $\text{AdS}_3 \times \text{M}_7$ can then be decomposed in terms of this basis as

$$\epsilon = \zeta \otimes v_\pm \otimes \chi, \quad v_\pm = \begin{pmatrix} 1 \\ \mp i \end{pmatrix}, \quad (\text{A.2})$$

with ζ a real Killing spinor on AdS_3 , χ a spinor on M_7 and v_\pm an auxiliary 2 vector which takes care of 10 dimensional chirality as $-\sigma_2 v_\pm = \pm v_\pm$ and is required to make a representation of the gamma matrices on $\text{AdS}_3 \times \text{M}_7$ 32 dimensional. If ϵ is one of the Killing spinor for type II supergravity it should be Majorana, here this just requires imposing that χ is Majorana, ie

$$\chi^c = B\chi^* = \chi, \quad (\text{A.3})$$

as ζ are real and $v_\pm = \sigma_3 v_\pm^*$. This is what leads to the form of the 10 dimensional Killing spinors taken in (2.4).

A.2 Vielbein and gamma matrices in on M_7

The internal space M_7 is a foliation of $S^3 \times S^3$ over an interval as in (2.6), we can define a veilbein on this space as

$$e^r = e^k dr, \quad e^{i_{1,2}} = e^{C_{1,2}} \hat{e}^{i_{1,2}}, \quad \hat{e}^{i_{1,2}} = \frac{1}{4}(d\theta_{1,2}, \sin\theta_{1,2}d\phi_{1,2}, d\psi_{1,2} + \cos\theta_{1,2}d\phi_{1,2})^{i_{1,2}}, \quad (\text{A.4})$$

where $\hat{e}^{i_{1,2}}$ are in the Hopf fibration frame mentioned before (2.8) in the main text. A suitable basis of 7 dimensional gamma matrices are then

$$\gamma_r = e^k \sigma_1 \otimes \mathbb{I} \otimes \mathbb{I}, \quad \gamma_{i_1} = e^{C_1} \sigma_2 \otimes \gamma_{i_1}^{S_1^3} \otimes \mathbb{I}, \quad \gamma_{i_2} = e^{C_2} \sigma_3 \otimes \mathbb{I} \otimes \gamma_{i_2}^{S_2^3}, \quad B = \sigma_1 \otimes \sigma_2 \otimes \sigma_2, \quad (\text{A.5})$$

where $\gamma_{i_1}^{S_1^3}$ are the unwarped gamma matrices of $S_{1,2}^3$ that are flat with respect to $\hat{e}^{i_{1,2}}$. Euclidean gamma matrices in 3 dimensions are always the Pauli matrices, up to signs and ordering, so we can in fact take

$$\gamma_{i_1}^{S_1^3} = \sigma_{i_1}, \quad \gamma_{i_2}^{S_2^3} = \sigma_{i_2} \quad (\text{A.6})$$

without loss of generality.

A.3 Conventions on S^3

The $\text{SO}(4)$ spinors we construct in (2.20) are a certain product of Killing spinors on $S_{1,2}^3$ that transform as (2.19) along the sum and difference of the $\text{SU}(2)_{L/R}$ Killing vectors of each 3-sphere. Here we will be more explicit about the form these Killing spinors/vectors

in terms of (A.4)–(A.6). To this end, let us here label an objects dependence on $SU(2)_{L/R}$ explicitly — we will focus on an arbitrary unwarped 3-sphere of unit radius in the Hopf fibration frame

$$\hat{e}^i = \frac{1}{4}(d\theta, \sin\theta d\phi, d\psi + \cos\theta d\phi)^i, \tag{A.7}$$

which can be related to the main text by simply adding an index where appropriate. The 3-sphere Killing spinor equations (2.7) then becomes

$$\nabla_i \xi^L = \frac{i}{2} \sigma_i \xi^L, \quad \nabla_i^{S^3} \xi^R = -\frac{i}{2} \sigma_i \xi^R, \tag{A.8}$$

which are solved in general by

$$\xi^L = e^{\frac{i}{2}\theta\sigma_1} e^{\frac{i}{2}\phi\sigma_3} \xi^0, \quad \xi^R = e^{-\frac{i}{2}\psi\sigma_3} \xi^0, \tag{A.9}$$

for ξ^0 a constant spinor. We fix $\sigma_3 \xi^0 = \xi^0$ without loss of generality. The $SU(2)_{L/R}$ Killing vectors are given by

$$K_1^R + iK_2^R = e^{i\psi}(i\partial_\theta + \csc\theta\partial_\phi - \cot\theta\partial_\psi), \quad K_3^R = \partial_\psi, \tag{A.10}$$

$$K_1^L + iK_2^L = e^{-i\phi}(i\partial_\theta + \cot\theta\partial_\phi - \csc\theta\partial_\psi), \quad K_3^L = \partial_\phi. \tag{A.11}$$

Viewed as one forms these obey

$$dK_i^L + \frac{1}{2}\epsilon_{ijk}K_j^L \wedge K_k^L = dK_i^R - \frac{1}{2}\epsilon_{ijk}K_j^R \wedge K_k^R = 0, \tag{A.12}$$

and are given by

$$K_1^R + iK_2^R = e^{i\psi}(id\theta + \sin\theta d\phi), \quad K_3^R = d\psi + \cos\theta d\phi, \tag{A.13}$$

$$K_1^L + iK_2^L = e^{-i\phi}(id\theta - \sin\theta d\psi), \quad K_3^L = d\phi + \cos\theta d\psi. \tag{A.14}$$

These expressions can be repackaged as

$$K_i^L = -i\text{Tr}[\sigma_i dgg^{-1}], \quad K_i^R = -i\text{Tr}[\sigma_i g^{-1}dg], \quad g = e^{\frac{i}{2}\sigma_3\phi} e^{\frac{i}{2}\sigma_2\theta} e^{\frac{i}{2}\sigma_3\psi}, \tag{A.15}$$

which makes clear that the $SU(2)_{L/R}$ charged forms are $SU(2)_{R/L}$ invariant.

With these definitions it is not hard to confirm that (2.8) obeys (2.11), which is the fundamental relations one needs to construct the $SO(4)$ spinors.

B Proof that $\mathcal{N} = 1$ implies $\mathcal{N} = 4$ for $SO(4)$ spinors

In section 2 it is claimed that if an $\mathcal{N} = 1$ sub-sector of the 7 dimensional $SO(4)$ spinors of (2.20) solves the supersymmetry conditions, the entire $\mathcal{N} = 4$ spinor also does. In this appendix we prove this claim using an argument based on studying (in abstract) the conditions (2.4) must satisfy to have vanishing gravitation and dilaton variation — see for example (2.10)–(2.14) of [15] for their explicit expressions in type IIA and IIB supergravity.

Since the solutions we consider respect the isometries of AdS_3 the 10 supersymmetry conditions that (2.4) must satisfy are implied by 7 dimensional conditions on $\chi_{1,2}^I$ only. For

clarity let us assume that $\chi_1^I = \chi_2^I = \chi^I$ — the proof of the general case is in essence the same, just more cumbersome to describe. Schematically the independent 7 dimensional spinorial conditions for $\mathcal{N} = 4$ then take one of two forms¹⁵

$$\Delta_1 \chi^I = 0, \tag{B.1a}$$

$$(\nabla_\mu + \Delta_2 \gamma_\mu) \chi^I = 0, \tag{B.1b}$$

for $I = 1, \dots, 4$, where $\Delta_{1,2}$ are 8×8 matrices containing combinations of the physical fields (dilaton, metric and RR and NS fluxes) and their derivatives contracted with the 7 dimensional gamma-matrices γ_μ and where ∇_μ is the covariant derivative. If one now decomposes

$$\chi^I = (\chi^i, \chi^4), \quad i = 1, 2, 3 \tag{B.2}$$

it follows from (2.19) that

$$\chi^i = \frac{1}{2} \mathcal{L}_{K_i^1} \chi^4 = \frac{1}{2} \mathcal{L}_{K_i^2} \chi^4 = \mathcal{L}_{K_i^+} \chi^4. \tag{B.3}$$

It should then be clear that if one assumes that just the 4th component of (B.1a)–(B.1b) is solved then one generates the other 3 automatically by acting with the spinorial derivative along the Killing vectors provided

$$\left[\mathcal{L}_{K_i^{1,2}}, \Delta_1 \right] \chi^4 = \left[\mathcal{L}_{K_i^{1,2}}, (\nabla_\mu + \Delta_2 \gamma_\mu) \right] \chi^4 = 0. \tag{B.4}$$

The first condition holds trivially whenever Δ_1 is an $SO(4)$ singlet, which is true whenever one imposes this condition on the physical fields as we are. The second condition is a little trickier — while Δ_2 commutes with the spinorial Lie derivative for the same reason as Δ_1 the individual μ indexed terms in (B.1b) do not in general commute by themselves. One can proceed by rewriting the 4th component of (B.1b) in the equivalent form

$$(\nabla_r + \Delta_2 \gamma_r) \chi^4 = 0, \tag{B.5a}$$

$$\left(\nabla_{K_i^1} + \Delta_2 \mathcal{K}_i^1 \right) \chi^4 = 0, \tag{B.5b}$$

$$\left(\nabla_{K_i^2} + \Delta_2 \mathcal{K}_i^2 \right) \chi^4 = 0, \tag{B.5c}$$

where $\nabla_K = K^\mu \nabla_\mu$ and $\mathcal{K} = K^\mu \gamma_\mu$. The form of the metric (2.6) ensures that one can always choose a frame where $\nabla_r = \partial_r$ so that (B.5a) commutes with $\mathcal{L}_{K_i^{1,2}}$ trivially. The proofs that (B.5b) and (B.5c) commute are essentially the same so we focus on the former: the 7 dimensional derivative term in (B.5b) decompose as

$$\nabla_{K_i^1} = K_i^{1\mu} \nabla_\mu^{S_1^3} - \frac{1}{2} \partial_{\mathcal{C}_1} \mathcal{K}_i^1 \tag{B.6}$$

¹⁵The 10 dimensional dilatino and gravitino conditions along AdS_3 give rise to 7 dimensional conditions of the form (B.1a), while the gravitino condition along the internal space give rise to a condition of the form (B.1a).

where $\nabla_\mu^{S^3}$ obeys the equation (2.7) when it acts on the components of the ξ_1^a factor of χ^4 — as such we can bring (B.5b) into the form

$$\Delta_3 \mathbb{K}_i^1 \chi^4 = 0, \tag{B.7}$$

for Δ_3 a new 8×8 matrix that is an $SO(4)$ singlet. This is close to the required result, which now follows if one can commute $\mathcal{L}_{K_j^1}$ past \mathbb{K}_i^1 . To achieve this, one more piece of information about the spinors on S^3 is required, namely that

$$\mathbb{K}_i^{1,2} \xi_{1,2}^a = \frac{1}{2} (\sigma_i)^a{}_b \xi_{1,2}^a, \tag{B.8}$$

which is easy to verify, for instance one can read it off from (3.5). Given (B.8) and (2.11) it is quick to establish that

$$\mathcal{L}_{K_j^{1,2}} \mathbb{K}_j^{1,2} \xi_{1,2}^a = \mathbb{K}_i^{1,2} \mathcal{L}_{K_j^{1,2}} \xi_{1,2}^a. \tag{B.9}$$

Thus if one acts on (B.7) with $\mathcal{L}_{K_j^1}$ one finds

$$\mathcal{L}_{K_j^1} (\Delta_3 \mathbb{K}_i^1 \chi^4) = \Delta_3 \mathbb{K}_j^1 \mathcal{L}_{K_j^1} \chi^4 = 2 \Delta_3 \mathbb{K}_j^1 \chi^i = 0, \tag{B.10}$$

where the first equality follows from (B.9) and because Δ_3 is an $SO(4)$ singlet. Repeating the same steps for (B.5c) one establishes that

$$(\nabla_r + \Delta_2 \gamma_r) \chi^i = 0, \tag{B.11a}$$

$$\left(\nabla_{K_j^1} + \Delta_2 \mathbb{K}_i^1 \right) \chi^i = 0, \tag{B.11b}$$

$$\left(\nabla_{K_j^2} + \Delta_2 \mathbb{K}_i^2 \right) \chi^i = 0, \tag{B.11c}$$

are implied by (B.5), from which it follows that the 4th component of (B.1b) implies the other 3, which completes the proof.

One can extend this argument to the case where $\chi_1^I \neq \chi_2^I$ without difficulty, so solving the supersymmetry constraints for an $\mathcal{N} = 1$ sub-sector of (2.20) implies that all components of $\chi_{1,2}^I$ also solve these constraints, provided the physical fields are $SO(4)$ singlets. Thus solving an $\mathcal{N} = 1$ sub-sector is sufficient to know that $\mathcal{N} = 4$ supersymmetry and an $SO(4)$ R-symmetry is preserved by all solutions consistent with $\chi_{1,2}^I$.

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