Ghostbusters in $f(R)$ supergravity

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ABSTRACT: $f(R)$ supergravity is known to contain a ghost mode associated with higher-derivative terms if it contains R^n with n greater than two. We remove the ghost in $f(R)$ supergravity by introducing auxiliary gauge field to absorb the ghost. We dub this method as the ghostbuster mechanism [1]. We show that the mechanism removes the ghost super-multiplet but also terms including R^n with $n \geq 3$, after integrating out auxiliary degrees of freedom. For pure supergravity case, there appears an instability in the resultant scalar potential. We then show that the instability of the scalar potential can be cured by introducing matter couplings in such a way that the system has a stable potential.

Contents

1   Introduction

Higher-order derivative interactions naturally appear in effective field theories. In particular, in the system with gravity, we need to take into account such terms since various higher-order corrections can be relevant to the dynamics. However, higher-derivative interactions often lead to the so-called Ostrogradski instability [2, 3]: higher-derivative interactions give additional degrees of freedom which makes the Hamiltonian unbounded from below, and hence the system shows an instability. If such a ghost mode appears, one should regard the system as an effective theory which is valid only below the energy scale of the mass of the ghost mode, otherwise the system loses the unitarity. In a class of ghost-free higher-derivative interactions, one does not come across with such an instability problem. In the case of a system with a single scalar and a tensor, the Horndeski class [4, 5] of interactions are free from ghosts. In this class of interactions, the equations of motion (E.O.M) are at most the second order differential equations, and no additional degree of freedom shows up. In general, one may ask the following question: among many possible higher-order derivative terms, what kind of structure gives us ghost-free interactions? For example, in the so-called Galileon models [6], Galileon scalar fields can be understood as the Goldstone mode of translation symmetry in extra dimensions, and the action is made out of ghost-free derivative terms. Therefore, one can say that the hidden translation symmetry controls the higher-derivative interactions so that there appear no new degrees of

freedom. The absence of ghosts in supersymmetric Galileon model [7] can be also achieved by a spontaneously broken hidden SUSY [8].

Higher-derivative interactions are also studied in gravity theories. Despite the existence of fourth-order derivative interactions, the so-called Starobinsky model [9], which has a quadratic term of the Ricci scalar, does not have any ghost as well as the Horndeski class. This is because such a system is equivalent to the scalar-tensor system without higher-derivatives. As a cosmological application, the Starobinsky model predicts the spectral tilt of scalar curvature perturbation compatible with the latest CMB observation [10]. One can extend this model to the system with an arbitrary function of the Ricci scalar, called the $f(R)$-gravity model [11] (see also refs. [12, 13] for review), which is also dual to a scalar-tensor system, and therefore free from the ghost instability.

Higher-derivative interactions were also studied in supersymmetric (SUSY) theories, both for global SUSY and supergravity (SUGRA). In SUSY cases, there is another problem called the auxiliary field problem: space-time derivatives may act in general on SUSY auxiliary fields (F and D for chiral and vector multiplets, respectively) in the off-shell superfield formulation. Then, they become dynamical and so one cannot eliminate them by their E.O.M [14, 15]. The auxiliary field problem and the higher-derivative ghosts usually come up together [16, 17]. In four dimensional (4D) $\mathcal{N} = 1$ SUSY theories, a classification of higher-derivative terms free from ghosts and the auxiliary field problem was given for chiral superfields [18–21] as well as for vector superfields [22]. Such higher-derivative interactions of chiral superfields were applied to low-energy effective theory [23–26] (see also [27]), coupling to SUGRA [20, 28], Galileons [19], ghost condensation [21], a Dirac-Born-Infeld (DBI) inflation [29], flattening of the inflaton potential [30, 31], a (baby) Skyrme model [32–40], other BPS solitons [34, 35, 41, 42], and modulated vacua [43, 44], while higher-derivative interactions of vector superfields were applied to the DBI action [45–47], SUGRA coupling [45, 48–50], SUSY Euler-Heisenberg action [17, 28, 51, 52], and non-linear self-dual actions [48, 49, 53–55].

On the other hand, higher-derivative interaction of gravity multiplets were studied in 4D $\mathcal{N} = 1$ SUGRA. In ref. [56], Cecotti constructed the higher-order terms of the Ricci scalar in the old minimal supergravity formulation and showed that at least one ghost superfield appears if we have R^n ($n \geq 3$) terms in the system. It is possible to avoid the ghost by some modifications of the system. In [57], the so-called nilpotent constraint on the Ricci scalar multiplet, which removes a scalar field in the multiplet, is considered. Due to the absence of the scalar, the bosonic ghost is absent in the spectrum of the system. This mechanism has been applied to various higher-curvature models in SUGRA [58]. The nilpotent constraint $\mathcal{R}^2 = 0$, however, is an effective description of a broken-SUSY system. If the linearly realized SUSY is restored in a higher energy regime, the ghost mode would show up.[1] As another approach, in [59] the authors considered a deformation of the ghost kinetic term by introducing an additional Kähler potential term. It is shown that the resultant ghost-free system is equivalent to the matter coupled $f(R)$ SUGRA.

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[1]The nilpotent condition on a chiral superfield Φ has two solutions. A nontrivial solution is $\phi = \frac{\psi\psi}{F^\phi}$ where ϕ, ψ and F^ϕ are scalar, Weyl spinor, and auxiliary scalar components of Φ. Obviously, this solution is well-defined for $F^\phi \neq 0$, that is, SUSY should be spontaneously broken.

Meanwhile, in our previous work [1], we proposed a simple method to remove a ghost mode in 4D $\mathcal{N} = 1$ SUSY chiral multiplets [16, 17], which we dubbed "ghostbuster mechanism." We gauge a U(1) symmetry by introducing a non-dynamical gauge superfield without kinetic term to the higher-derivative system with assigning charges on chiral superfields properly in order for the gauge field to absorb the ghost. Namely, due to the gauge degree of freedom, the ghost in the system is removed by the U(1) gauge fixing. In this class of models, a hidden local symmetry plays a key role in the ghostbuster mechanism. Actually, before this work, essentially the same technique is used for superconformal symmetry in the conformal SUGRA formalism: the conformal SUGRA has one ghost-like degree of freedom called a compensator. Such a degree of freedom is removed by the superconformal gauge fixing, whereas in the ghostbuster mechanism, the hidden local U(1) gauge fixing removes the ghost associated with higher-derivatives. Therefore, in SUGRA models, one may understand the higher-derivative ghost as a second compensator for the system with the superconformal symmetry × hidden local U(1) symmetry.

In this paper, we apply the ghostbuster mechanism to remove the ghost in the $f(R)$ SUGRA system. Interestingly, the hidden U(1) symmetry required for the mechanism can be understood as the gauged R-symmetry, since the gravitational superfield should be gauged under the U(1) symmetry. The U(1) charge assignment is uniquely determined, and therefore, naively one cannot expect a ghost mode cancelation a priori. As we will show, a would-be ghost superfield has a gauge charge and can be nicely removed by the gauge fixing of the U(1) symmetry. As a price of this achievement, however, the resultant system generically has an unstable scalar potential in a pure SUGRA case. Such an unstable scalar potential can be cured by various modifications. As an example we propose a model with a matter chiral superfield. We will find that such a deformation leads to a healthy model of SUGRA without either ghosts or instabilities of the scalar potential.

One will easily find how the ghost supermultiplet is eliminated from the dual matter-coupled SUGRA viewpoint. We also address the same question in the higher-curvature SUGRA system. We find that, after integrating out the auxiliary vector superfield for the mechanism, the scalar curvature terms including R^n with $n \geq 3$ disappear, and the resultant system has linear and quadratic terms in R. However, the $R + R^2$ SUGRA system has couplings completely different from that proposed in [56]. This observation means that, despite the disappearance of higher scalar curvatures in the final form, the higher-curvature deformation in the original action gives a physical consequence even after applying the ghostbuster mechanism.

This paper is organized as follows. In section 2, we briefly review the higher-curvature SUGRA models and its dual description. In particular, one finds that once the SUSY version of the higher order Ricci scalar term R^n ($n \geq 3$) is included in the old minimal SUGRA formulation, there appears at least one ghost chiral superfield. We apply the ghostbuster mechanism to the higher-curvature SUGRA in section 3. We will see that although the ghost superfield can be removed by the mechanism, the resultant system has a scalar potential with an instability in the direction of a scalar field. Then, in section 4, we discuss a simple modification of the model by introducing an extra matter chiral superfield. We show an example which is stable and free from ghost as well. Finally, we conclude in section 6. Throughout this paper, we will use the notation of [60].

2 Higher-curvature terms in supergravity

In this section, we review the construction of higher-order terms of the Ricci scalar in 4D $\mathcal{N} = 1$ SUGRA [56].[2] In this paper, we use the conformal SUGRA formalism, in which there are conformal symmetry and its SUSY counterparts in addition to super-Poincaré symmetry [64–67]. In order to fix the extra gauge degree of freedom, we need to introduce an unphysical degrees of freedom called the conformal compensator, which should be in a superconformal multiplet. In this paper, we adopt a chiral superfield as a compensator superfield, which leads to the so-called old minimal SUGRA after superconformal gauge fixing. We show the components of supermultiplet, the density formulas, and identities in appendix A.

First, let us show the pure conformal SUGRA action,

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 \right]_D, \tag{2.1}$$

where S_0 is the chiral compensator with the charges $(w, n) = (1, 1)$ in conformal SUGRA (see Apeendix A for the definition of the charges), and $[\cdots]_D$ denotes the D-term density formula. Taking the pure SUGRA gauge, $S_0 = \bar{S}_0 = 1$, $b_\mu = 0$, we obtain an action whose bosonic part takes the form

$$S = \int d^4 x \sqrt{-g} \left(\frac{1}{2} R - 3|F^{S_0}|^2 + 3A_a A^a \right), \tag{2.2}$$

where R is the Ricci scalar, F^{S_0} is the F-term of S_0 and A_a is the gauge field of chiral U(1)$_A$ symmetry, which is a part of superconformal symmetry. The E.O.M. for the auxiliary fields F^{S_0} and A_a can be solved by setting $F^{S_0} = A_a = 0$, and then we find the pure SUGRA action. The action (2.1) can also be written as

$$S = \left[\frac{3}{2} S_0^2 \mathcal{R} \right]_F, \tag{2.3}$$

where $[\cdots]_F$ is the F-term density formula. Here we have used the identity given in (A.26). The chiral superfield $\mathcal{R}$ is the so-called scalar curvature superfield, defined by

$$\mathcal{R} \equiv \frac{\Sigma(\bar{S}_0)}{S_0}, \tag{2.4}$$

where Σ is the chiral projection operator. Its components in the pure SUGRA gauge are given by

$$\mathcal{R} = [\Phi, \, P_L \chi, \, F] = \left[-\bar{F}^{S_0}, \, \cdots, \, |F^{S_0}|^2 + \frac{1}{6} R + A_a A^a - i\partial_a A^a + \cdots \right], \tag{2.5}$$

where ellipses denote fermionic parts. From this expression, we find that the F-component of $\mathcal{R}$ contains the Ricci scalar.

It has been known that there is no ghost in the system involving R^2, which is realized as

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 + \frac{\alpha}{2} \mathcal{R} \bar{\mathcal{R}} \right]_D, \tag{2.6}$$

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[2]Cosmological application of SUSY Starobinsky model is discussed e.g. in [61–63].

where α is a real constant. The bosonic part of this action after the superconformal gauge fixing is

$$S|_B = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \frac{\alpha}{36} R^2 - 3|F^{S_0}|^2 - \alpha D_a F^{S_0} D^a \bar{F}^{S_0} + 3 A_a A^a + \alpha(\partial_a A^a)^2 \right.$$
$$\left. + \frac{\alpha R}{6} \left(|F^{S_0}|^2 + 2 A_a A^a \right) + \alpha \left(|F^{S_0}|^2 + A_a A^a \right)^2 \right], \quad (2.7)$$

where D_a represents the covariant derivative, $D_a S_0 = (\partial_a - iA_a)S_0 = -iA_a, D_a F^{S_0} = (\partial_a + 2iA_a)F^{S_0}$. The Lagrangian has the quadratic Ricci scalar term $\frac{\alpha}{36} R^2$ and also the non-minimal couplings between F^{S_0}, A_a and R. In this system, there exist four real massive modes φ_i with the common mass $m^2 = 3/\alpha$ in the fluctuations around the vacuum $g_{\mu\nu} = \eta_{\mu\nu}$ and $F^{S_0} = A_a = 0$:

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box} \right) \varphi_1, \quad A_\mu = \partial_\mu \varphi_2, \quad F^{S_0} = \varphi_3 + i\varphi_4. \quad (2.8)$$

We stress that, as is often the case with SUSY higher derivative models, the auxiliary fields have their kinetic terms and hence they are dynamical degrees of freedom in the presence of the higher-derivative term.

Next, let us consider a SUGRA system with R^n, $n \geq 3$ along the line of refs. [56, 57, 68]. As we discussed in the previous section, $\mathcal{R}$ superfield has the Ricci scalar in its F-component. Using the chiral projection operator Σ, one can obtain the superfield $\Sigma(\bar{\mathcal{R}})$ which has R in the lowest component:

$$\Sigma(\bar{\mathcal{R}}) = \left[-\frac{1}{6} R - |F^{S_0}|^2 - A_a A^a - i\partial_a A^a + \cdots, \cdots, \right.$$
$$\left. \frac{1}{6} R F^{S_0} + \left(\partial_a^2 + i\partial_a A^a - A_a A^a \right) F^{S_0} + \cdots \right], \quad (2.9)$$

where we have shown only the relevant part. With this superfield $\Sigma(\bar{\mathcal{R}})$, one can construct an action involving arbitrary functions of R, i.e. $f(R)$ gravity models in SUGRA. Here we consider the action of the form

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 \Omega \left(\frac{\mathcal{R}}{S_0}, \frac{\bar{\mathcal{R}}}{\bar{S}_0}, \frac{\Sigma(\bar{\mathcal{R}})}{S_0^2}, \frac{\bar{\Sigma}(\mathcal{R})}{\bar{S}_0^2} \right) \right]_D + \left[S_0^3 \mathcal{F} \left(\frac{\mathcal{R}}{S_0}, \frac{\Sigma(\bar{\mathcal{R}})}{S_0^2} \right) \right]_F, \quad (2.10)$$

where Ω is an arbitrary real function and $\mathcal{F}$ is an arbitrary holomorphic function. If we chose $\Omega = 0, \mathcal{F}(S, X) = S(3 - \alpha X)/2$, then this action reduces to (2.6) since

$$\left[S_0^3 \mathcal{F} \left(\frac{\mathcal{R}}{S_0}, \frac{\Sigma(\bar{\mathcal{R}})}{S_0^2} \right) \right]_F = \left[\frac{3}{2} S_0^2 \mathcal{R} - \frac{\alpha}{2} \mathcal{R} \Sigma(\bar{\mathcal{R}}) \right]_F = \left[-\frac{3}{2} S_0 \bar{S}_0 + \frac{\alpha}{2} \mathcal{R} \bar{\mathcal{R}} \right]_D. \quad (2.11)$$

The bosonic part of the action contains the following terms including higher-order terms of Ricci scalar R

$$\int d^4x \sqrt{-g} \left\{ -\frac{R^2}{12} \Omega_{S\bar{S}}(S, \bar{S}, X, \bar{X}) + \frac{R}{6} \mathcal{F}_S(S, X) + \text{h.c.} \right\}_{S=-\bar{F}^{S_0}, X=-R/6}, \quad (2.12)$$

where the subscripts on the functions denote the differentiations with respect to the scalar fields.

Such SUSY higher-derivative terms have derivative interactions of auxiliary fields, and the interactions make the auxiliary fields dynamical as

$$\int d^4x\sqrt{-g}\left\{\frac{1}{12}g^{\mu\nu}\partial_\mu R\,\partial_\nu R\,\Omega_{X\bar{X}} + \left(\partial_\mu^2 F^{S_0}\mathcal{F}_X + \text{h.c.}\right) + \cdots\right\}_{S=-\bar{F}^{S_0},\,X=-R/6}. \tag{2.13}$$

In this system, in addition to the scalar degree of freedom from the derivative terms of the Ricci-curvature, the higher-derivative terms of the "dynamical" auxiliary field F^{S_0} give rise to multiple scalar degrees of freedom, some of which are ghost-like. If we choose $\Omega(S,\bar{S},X,\bar{X}) = S\bar{S}\tilde{\Omega}(X,\bar{X})$, $\mathcal{F}(S,X) = S\tilde{\mathcal{F}}(X)$, and set $F^{S_0} = 0$ identically as is done by imposing the nilpotent condition $\mathcal{R}^2 = 0$ in ref. [57], the above terms vanish and no ghost seems to appear. Without such a condition, however, the appearance of ghost is unavoidable as is clearly shown in the following.

The present system is also equivalent to a standard SUGRA model coupled to matter superfields. As in the previous section, we use Lagrange multiplier suerfields, and rewrite the action (2.10) as

$$S' = \left[-\frac{3}{2}S_0\bar{S}_0\,\Omega(S,\bar{S},X,\bar{X})\right]_D + \left[S_0^3\,\mathcal{F}(S,X)\right]_F$$
$$+ \left[3S_0^3\,T\left(\frac{\mathcal{R}}{S_0} - S\right)\right]_F + \left[3S_0^3\,Y\left(\frac{\Sigma(\bar{S}_0\bar{S})}{S_0^2} - X\right)\right]_F, \tag{2.14}$$

where T and Y are Lagrange multiplier superfields with $(w,n) = (0,0)$. The E.O.Ms of T and Y give the constraints which reproduce the original action (2.10). Instead, using the identity (A.26), we can also obtain the dual action

$$S' = \left[-\frac{3}{2}S_0\bar{S}_0\left(T + \bar{T} + Y\bar{S} + \bar{Y}S + \Omega(S,\bar{S},X,\bar{X})\right)\right]_D + \left[S_0^3\left(\mathcal{F}(S,X) - 3TS - 3XY\right)\right]_F. \tag{2.15}$$

This is a standard SUGRA system with the following Kähler and super-potentials,

$$K = -3\log\left(T + \bar{T} + Y\bar{S} + \bar{Y}S + \Omega(S,\bar{S},X,\bar{X})\right), \tag{2.16}$$
$$W = \mathcal{F}(S,X) - 3TS - 3XY. \tag{2.17}$$

Let us show the existence of a ghost mode. The Kähler metric of the $\{S,Y\}$ sector takes the form,

$$K_{I\bar{J}} = \begin{pmatrix} K_{S\bar{S}} & -\frac{1}{A} \\ -\frac{1}{A} & 0 \end{pmatrix}, \tag{2.18}$$

where $A = T + \bar{T} + Y\bar{S} + \bar{Y}S + \Omega(S,\bar{S},X,\bar{X})$. The determinant of this sub matrix has negative determinant, and this Kähler metric has one negative eigenvalue corresponding to a ghost. Thus, the $f(R)$ SUGRA model has one ghost mode in general.

Note that X becomes an auxiliary superfield if $\Omega = \Omega(S, \bar{S})$ is independent of X. Even in such a case, the system has higher-curvature terms in the $\mathcal{F}(S, X)$ term in (2.12). The reduced dual system is described by

$$
\begin{aligned}
K &= -3\log\left(T + \bar{T} + Y\bar{S} + \bar{Y}S + \Omega(S, \bar{S})\right), \\
W &= g(S, Y) - 3TS,
\end{aligned}
\tag{2.19}
$$

where $g(S, Y) = [\mathcal{F} - X\mathcal{F}_X]_{X=X(S,Y)}$ and $X(S, Y)$ is a solution of $\mathcal{F}_X - 3Y = 0$.[3] This reduction does not change the above discussion, and hence a ghost mode appears in this system as well.

3 Ghostbuster in $f(R)$ supergravity

In this section, we consider the elimination of the ghost superfield along the line of ref. [1]. To eliminate the ghost superfield, one needs to introduce a gauge redundancy, by which one of the degrees of freedom is removed. In the $f(R)$ SUGRA discussed above, all the superfields $\mathcal{R}, \Sigma(\bar{\mathcal{R}})$ are expressed in terms of S_0 with the SUSY derivative operators. Hence, once we introduce a vector superfield V_R for a U(1) gauge symmetry and assign the charge to S_0 so that it transforms as

$$
S_0 \to e^\Lambda S_0, \qquad V_R \to V_R - \Lambda - \bar{\Lambda},
\tag{3.1}
$$

the transformation law of $\mathcal{R}$ and $\Sigma(\bar{\mathcal{R}})$ are automatically determined as

$$
\mathcal{R}_g \equiv \frac{\Sigma(\bar{S}_0 e^{V_R})}{S_0} \to e^{-2\Lambda}\mathcal{R}_g, \quad \Sigma_g(\bar{\mathcal{R}}) \equiv \Sigma(\bar{\mathcal{R}}_g e^{-2V_R}) \to e^{2\Lambda}\Sigma_g(\bar{\mathcal{R}}),
\tag{3.2}
$$

where the chiral projection Σ needs to be modified so that the operations is covariant under the gauge symmetry. In the rest of this section, we omit the suffix g attached to $\mathcal{R}_g, \Sigma_g$. Interestingly, the U(1) gauge symmetry under which the compensator is charged becomes a gauged R-symmetry [69]. We call it a $U(1)_R$ symmetry in the following discussion. Here, however, we do not introduce a kinetic term for V_R and thus the vector superfield V_R is an auxiliary superfield, which should be written as a composite field consisting of curvature superfields $\mathcal{R}$ and $\Sigma(\bar{\mathcal{R}})$.

3.1 Ghostbuster in pure $f(R)$ supergravity model

Let us introduce a $U(1)_R$ gauge symmetry under which S_0 has charge $c_{S_0} = 1$. Since the chiral superfield $\mathcal{R} = \Sigma(\bar{S}_0)/S_0$, the charge of $\mathcal{R}$ is determined as $c_{\mathcal{R}} = -2$. Analogously, we find that $c_{\Sigma(\bar{\mathcal{R}})} = 2$. Then the gauged extension of the system (2.10) with $\Omega = \Omega(S, \bar{S})$ is described by the action

$$
S = \left[-\frac{3}{2}\, S_0\, e^{V_R} \bar{S}_0\, \Omega\left(\frac{\mathcal{R}}{S_0}, \frac{\bar{\mathcal{R}}}{\bar{S}_0}e^{-3V_R}\right)\right]_D + \left[S_0^3\, \mathcal{F}\left(\frac{\mathcal{R}}{S_0}, \frac{\Sigma(\bar{\mathcal{R}})}{S_0^2}\right)\right]_F,
\tag{3.3}
$$

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[3]Here we assume that the equation $\mathcal{F}_X = \mathcal{F}_X(X)$ can be solved for X (e.g. $\mathcal{F} \propto SX^{n-1}$ with $n \geq 3$). Constant and linear terms in $\mathcal{F}$ merely rescale the R and R^2 terms respectively.

where Ω should be gauge invariant and $\mathcal{F}$ should have gauge charge $c_{\mathcal{F}} = -3$ in total. Hence $\mathcal{F}$ should take the form

$$\mathcal{F}\left(\frac{\mathcal{R}}{S_0}, \frac{\Sigma(\bar{\mathcal{R}})}{S_0^2}\right) = 3\tilde{\mathcal{F}}\left(\frac{\Sigma(\bar{\mathcal{R}})}{S_0^2}\right)\frac{\mathcal{R}}{S_0}. \tag{3.4}$$

To discuss the ghost elimination, it is useful to consider the dual system as in the non-gauged case (2.15). The dual system of the gauged model is described by

$$\begin{aligned} S' &= \left[-\frac{3}{2} S_0\, e^{V_R} \bar{S}_0\, \Omega(S\,, \bar{S}e^{-3V_R}) \right]_D + \left[3 S_0^3\, \tilde{\mathcal{F}}(X)S \right]_F \\ &\quad + \left[3S_0^3\, T\left(\frac{\mathcal{R}}{S_0} - S\right) \right]_F + \left[3S_0^3\, Y\left(\frac{\Sigma(\bar{S}_0\bar{S})}{S_0^2} - X\right) \right]_F, \end{aligned} \tag{3.5}$$

where the gauge charges of T, S, X, Y are $(c_T, c_S, c_X, c_Y) = (0, -3, 0, -3)$. Similarly to the non-gauged case, we can rewrite this action as

$$\begin{aligned} S' &= \left[-\frac{3}{2} S_0\, e^{V_R} \bar{S}_0 \left\{ T + \bar{T} + Y\bar{S}e^{-3V_R} + \bar{Y}e^{-3V_R}S + \Omega(S, \bar{S}e^{-3V_R}) \right\} \right]_D \\ &\quad + \left[3S_0^3 \left(\tilde{\mathcal{F}}(X)S - TS - XY \right) \right]_F. \end{aligned} \tag{3.6}$$

For simplicity, in the following discussion, we choose the function $\Omega = \gamma - hS\bar{S}e^{-3V_R}$, where γ is a real constant. Note that one can perform the following procedure with a more general form of Ω in a similar way. Then we obtain

$$\begin{aligned} S &= \left[-\frac{3}{2} S_0\, e^{V_R} \bar{S}_0 \left(\gamma + T + \bar{T} + (Y\bar{S} + \bar{Y}S - hS\bar{S})\, e^{-3V_R} \right) \right]_D \\ &\quad - \left[3S_0^3 \left(\tilde{\mathcal{F}}(X)S + TS + XY \right) \right]_F. \end{aligned} \tag{3.7}$$

We stress that the $U(1)_R$ charges of (S, Y) are automatically determined to be non-zero. This is a nontrivial and important nature of the $f(R)$ SUGRA model since the ghostbuster mechanism does not work if S and Y, either of which corresponds to the ghost mode, did not have the $U(1)_R$ charges.

The variation of V_R gives the following E.O.M for V_R

$$(\gamma + T + \bar{T})e^{V_R} - 2\left(Y\bar{S} + S\bar{Y} - hS\bar{S}\right)e^{-2V_R} = 0. \tag{3.8}$$

This equation can be algebraically solved in terms of V_R as

$$e^{-3V_R} = \frac{\gamma + T + \bar{T}}{2\left(Y\bar{S} + S\bar{Y} - hS\bar{S}\right)}. \tag{3.9}$$

Substituting this solution to the action, one finds

$$S = \left[-\frac{3}{2} S_0 \bar{S}_0 \left(\gamma + T + \bar{T} \right)^{\frac{2}{3}} \left(Y\bar{S} + S\bar{Y} - hS\bar{S} \right)^{\frac{1}{3}} \right]_D$$
$$- \left[\frac{2}{\sqrt{3}} S_0^3 \left(\tilde{\mathcal{F}}(X)S + TS + XY \right) \right]_F , \tag{3.10}$$

where we have rescaled S_0 as $S_0 \to 2^{1/3}/\sqrt{3}\, S_0$. Thus, starting from the modified higher-curvature action (3.3), we find the dual matter-coupled system (3.10). After partial gauge fixings of superconformal symmetry,[4] this system becomes Poincaré SUGRA with the following Kähler and superpotentials,

$$K = -2 \log \left(\gamma + T + \bar{T} \right) - \log \left(Y\bar{S} + S\bar{Y} - hS\bar{S} \right), \tag{3.11}$$
$$W = -\frac{2}{\sqrt{3}} \left(\tilde{\mathcal{F}}(X)S + TS + XY \right). \tag{3.12}$$

This system is invariant under the $U(1)_R$ gauge transformation $\{S, Y\} \to \{e^\Lambda S, e^\Lambda Y\}$. Therefore, if the lowest component of Y takes a non-zero value, we can fix the $U(1)_R$ gauge by setting $Y = 1$. Then, after a redefinition $S \to S + \frac{1}{h}$, we obtain

$$K = -2 \log(\gamma + T + \bar{T}) - \log(1 - h^2 S\bar{S}), \tag{3.13}$$
$$W = -\frac{2}{\sqrt{3}} \left(\tilde{\mathcal{F}}(X)(S + 1/h) + T(S + 1/h) + X \right). \tag{3.14}$$

If $S \neq 0$, we can also fix the gauge by setting $S = 1$. Then we find

$$K = -2 \log(\gamma + T + \bar{T}) - \log(Y + \bar{Y} - h), \tag{3.15}$$
$$W = -\frac{2}{\sqrt{3}} \left(\tilde{\mathcal{F}}(X) + T + XY \right). \tag{3.16}$$

Except for the two points $S = 0$ ($Y = \infty$), $Y = 0$ ($S = \infty$), the above two descriptions are equivalent and related by a coordinate transformation between S and Y. In both cases, all the eigenvalues of the Kähler metric are obviously positive. Therefore, we have shown that the ghost mode is eliminated by our ghostbuster mechanism. Note that X is an auxiliary field in this setup, and we need to solve the E.O.M for X to obtain the physical superpotential.

We stress that the elimination of the ghost mode by the ghostbuster mechanism in this higher-curvature system is nontrivial since we do not have any choice of the charge assignment to the superfields. As we have seen above, the would-be ghost modes have charges under $U(1)_R$, which enables us to remove the ghost mode by the gauge degree of freedom.

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[4]More specifically, we fix dilatiation, chiral U(1) symmetry, S-SUSY, and conformal boost, so that Poincare SUSY remains in the resultant system. The detailed procedure of superconformal gauge fixing is discussed e.g. in [60].

3.2 Instability of scalar potential

In this section, we analyze the scalar potential of the ghost-free system derived in the previous section. The F-term scalar potential in the Poincaré SUGRA is given by

$$V = e^K \left[K^{A\bar{B}}(W_A + K_A W)(\bar{W}_{\bar{B}} + K_{\bar{B}}\bar{W}) - 3|W|^2 \right]. \tag{3.17}$$

If we choose the gauge fixing condition $S = 1$, $\operatorname{Im}T$ appears only in W due to the shift symmetry of $\operatorname{Im}T$ in the Kähler potential, and hence the mass of $\operatorname{Im}T$ is given by

$$m^2_{\operatorname{Im}T} \propto e^K (K^{A\bar{B}}K_A K_{\bar{B}} - 3). \tag{3.18}$$

The Kähler potential in eq. (3.15) has the property called the no-scale relation

$$K^{A\bar{B}}K_A K_{\bar{B}} = 3. \tag{3.19}$$

Since $W \propto T + XY$, the potential has the following linear term of $\operatorname{Im}T$

$$\operatorname{Im}(K_{\bar{B}}K^{\bar{B}A}W_A)\operatorname{Im}T. \tag{3.20}$$

To realize a stable vacuum at $\operatorname{Im}T = 0$, this quantity must vanish identically. By using the Kähler potential in eq. (3.15), we find that the coefficient of the linear term is given by

$$\operatorname{Im}(K_{\bar{B}}K^{\bar{B}A}W_A) = \frac{4}{\sqrt{3}}(Y + \bar{Y} - h)\operatorname{Im}X. \tag{3.21}$$

Note that the non-dynamical field X becomes a function of Y after solving its E.O.M. $\operatorname{Im}T$ has only a mass term $\sim \langle Y + \bar{Y} - h \rangle (X_Y Y + \bar{X}_Y \bar{Y})\operatorname{Im}T$, where $X_Y \equiv \langle \partial_Y(\operatorname{Im}X) \rangle$. Unfortunately, this "off-diagonal" contribution in the mass matrix leads to a tachyonic mode.[5] This instability cannot be cured by any higher-order terms since $\operatorname{Im}T$ appears only in the term (3.20). Therefore, $\operatorname{Im}X \neq 0$ makes $\operatorname{Im}T$ unstable and even if there is the local minimum in $\operatorname{Im}X = \operatorname{Im}T = 0$, that point cannot be a local minimum, but must be a saddle point. We conclude that although the instability caused by ghost mode is absent thanks to the ghostbuster mechanism, the pure higher-curvature action has an unstable scalar potential, which does not have any stable SUSY minimum. In the next section, we consider an extension of our model to improve this point.

4 Stable ghostbuster model with extra matter

4.1 Preliminary

As we discussed in the previous section, the scalar potential of our minimal model has no stable SUSY minimum. One may improve such a situation by various types of modifications. Here we take a relatively simple way; we introduce an additional matter field Z so that the coupling between the gravitational sector and the additional sector stabilizes

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[5]In general, $\langle Y + \bar{Y} - h \rangle$ should be nonzero since the Kähler potential has $-\log(Y + \bar{Y} - h)$ and diverges for $\langle Y + \bar{Y} - h \rangle = 0$.

the potential.[6] Let us assume that Z carries no $\mathrm{U}(1)_R$ charge so that the superpotential W contains TZ term in the $S = 1$ gauge. Then it is possible to introduce Z in the superpotential in such a way that the constraint for S is modified as

$$S = \frac{\mathcal{R}}{S_0} \quad \to \quad SZ = \frac{\mathcal{R}}{S_0}. \tag{4.1}$$

We can also change the definition of X as

$$X = \frac{\Sigma(\bar{S}_0 \bar{S})}{S_0^2} \quad \to \quad X = \frac{\Sigma\left(\bar{S}_0 \bar{S}\, \bar{k}(\bar{Z}, Z)\right)}{S_0^2}, \tag{4.2}$$

with an arbitrary function $k(Z, \bar{Z})$. Note that if we chose $k(Z, \bar{Z}) = Z$, then we obtain the same unstable model as in section 3 with the redefinition $S \to S' = SZ$. Therefore, $k(Z, \bar{Z})$ should have a constant term around the minimum of Z, i.e. $k(\langle Z \rangle, \langle \bar{Z} \rangle) \equiv c \neq 0$. Under this modification, the dual system is given by

$$S' = \left[-\frac{3}{2} S_0\, e^{V_R} \bar{S}_0\, \Omega(S, \bar{S}e^{-3V_R}, Z, \bar{Z}) \right]_D + \left[3 S_0^3\, T \left(\frac{\mathcal{R}}{S_0} - SZ \right) \right]_F$$
$$+ \left[S_0^3 S \tilde{\mathcal{F}}(X) \right]_F + \left[3 S_0^3\, Y \left(\frac{\Sigma(\bar{S}_0 \bar{S}\, \bar{k}(\bar{Z}))}{S_0^2} - X \right) \right]_F, \tag{4.3}$$

which can be rewritten as

$$S' = \left[-\frac{3}{2} S_0\, e^{V_R} \bar{S}_0 \left\{ T + \bar{T} + Y \bar{S} e^{-3V_R}\, \bar{k}(\bar{Z}) + \bar{Y} e^{-3V_R} S\, k(Z) + \Omega \right\} \right]_D$$
$$+ \left[S_0^3 \left(\tilde{\mathcal{F}}(X)S - 3TSZ - 3XY \right) \right]_F. \tag{4.4}$$

For simplicity, let us choose the function as

$$\Omega = \gamma - g(Z, \bar{Z}) - h(Z, \bar{Z})\, S\bar{S}\, e^{-3V_R}. \tag{4.5}$$

After solving the E.O.M for V_R, we find the following Kähler potential and superpotential

$$K = -2 \log\left[\gamma + T + \bar{T} - g(Z, \bar{Z}) \right] - \log\left[Y\bar{k}(\bar{Z}) + \bar{Y}k(Z) - h(Z, \bar{Z}) \right], \tag{4.6}$$

$$W = \frac{2}{\sqrt{3}} \left[\frac{1}{3} \tilde{\mathcal{F}}(X) - (TZ + XY) \right], \tag{4.7}$$

in the $S = 1$ gauge.

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[6]Even in the R^2 model, the deformation of scalar potential of T corresponding to the scalaron superfield requires an additional degree of freedom in the dual higher-curvature SUGRA action [70].

4.2 Example of matter coupled $f(R)$ supergravity

Let us discuss a simple example by setting the functions as

$$k(Z) = c + Z, \qquad \Omega = \gamma + (\beta - bZ\bar{Z}) \, S\bar{S} \, e^{-3V_R}. \tag{4.8}$$

The corresponding Kähler potential is given by

$$K = -2\log \omega_1 - \log \omega_2, \tag{4.9}$$

$$\omega_1 \equiv \gamma + T + \bar{T}, \tag{4.10}$$

$$\omega_2 \equiv \beta + \left(\bar{Y}(c+Z) + \text{c.c.} \right) - bZ\bar{Z}, \tag{4.11}$$

where both ω_1 and ω_2 are required to be positive so that there exists a solution of the E.O.M. for V_R and the condition $e^K > 0$. The eigenvalues $\{\lambda_i \,|\, i = 1, 2, 3\}$ of the Kähler metric $K_{A\bar{B}}$ are given by

$$\lambda_1 = \frac{2}{\omega_1^2}, \quad \lambda_2 + \lambda_3 = \frac{|\partial_Y \omega_2|^2 + |\partial_Z \omega_2|^2 + b\,\omega_2}{\omega_2^2}, \quad \lambda_2 \lambda_3 = \frac{b|c|^2 - \beta}{\omega_2^3}. \tag{4.12}$$

Furthermore, by choosing the function $\tilde{\mathcal{F}}$ so that $\tilde{\mathcal{F}}(0) = 0$, $\tilde{\mathcal{F}}'(0) = 0$, we find a SUSY vacuum satisfying $W_A = W = 0$ at $X = Y = T = S = 0$, which is guaranteed to be stable. Therefore, there exists the SUSY vacuum with a positive definite metric if and only if

$$\gamma = \omega_1|_{\text{vac}} > 0, \quad \beta = \omega_2|_{\text{vac}} > 0, \quad b > \frac{\beta}{|c|^2}, \quad c \neq 0. \tag{4.13}$$

When these conditions are satisfied, there exist no ghost anywhere in the region $\mathcal{M} = \{T, Y, Z \,|\, \omega_1 > 0 \,, \omega_2 > 0\}$ and the boundary $\partial\mathcal{M}$ is geodesically infinitely far away from the SUSY vacuum.

5 Ghostbuster mechanism from higher-curvature SUGRA viewpoint

In this section, we discuss how the ghostbuster mechanism works in the higher-curvature frame. As we have seen in previous two sections, the ghost supermultiplet is eliminated in both pure and matter-coupled higher-curvature systems.

Let us consider the original action for $f(R)$ gravity before taking the dual transformation. For concreteness of the discussion, we take the simplest model with an additional matter superfield in eq. (4.8). The same conclusion follows even in the absence of an additional matter. The higher-curvature action can be obtained by solving E.O.M. for T and Y and imposing the constraints for S and X. Here we introduce $S_1 \equiv cS_0 S + \mathcal{R}_g$ as an extra matter and solve the modified constraint (4.1) for Z. After introducing the quadratic term of X, the original action takes the form

$$S' = \left[-\frac{3}{2}\gamma |S_0|^2 e^{V_R} - \frac{3\beta}{2|c|^2}|S_1 - \mathcal{R}_g|^2 e^{-2V_R} + \frac{3}{2}a|S_0 X|^2 e^{V_R} + \frac{3}{2}b|\mathcal{R}_g|^2 e^{-2V_R} \right]_D$$

$$+ \left[S_0^2 (S_1 - \mathcal{R}_g) X \mathcal{G}(X) \right]_F \tag{5.1}$$

with $\tilde{\mathcal{F}}(X) \equiv cX\mathcal{G}(X)$ and

$$\mathcal{R}_g = \frac{\Sigma(\bar{S}_0 e^{V_R})}{S_0}, \qquad X = \frac{\Sigma(\bar{S}_1 e^{-2V_R})}{S_0^2}, \tag{5.2}$$

where a and b are real (positive) parameters. Note that X now does not have the Ricci scalar in the lowest component but a higher-derivative superfield made out of S_1. This means that the higher-derivative term of $\mathcal{R}_g$ is now replaced by that of S_1, and hence the higher-curvature term does not show up. By expanding the action explicitly, one can check that this action has Ricci scalar terms up to the quadratic order. We note that, however, this does not lead to the conclusion that the ghost is removed by the additional matter: since there still exist higher-derivative terms of S_1, the ghost mode can arise from such terms. One may also confirm that the absence of the higher curvature terms R^n ($n \geq 3$) is not an artifact of field redefinition. We can show that in this specific matter coupled model, the higher-curvature terms exist only in the off-shell action before substituting the solution of the E.O.M for the auxiliary field in V_R.

We stress that this conclusion does not mean that the higher-curvature modification is removed by the ghostbuster mechanism. As we claimed above, the resultant system has scalar curvature terms only up to the quadratic order, as the simplest Cecotti model does [56]. However, the coupling of the resultant system is completely different from the Cecotti model. In our dual matter coupled system in section 4.2, Kähler potential (4.9) takes the form

$$K \sim -2\log(T + \bar{T}) - \log(Y + \bar{Y} + \cdots), \tag{5.3}$$

whereas, in the Cecotti model, it can be written as

$$K = -3\log(T_c + \bar{T}_c + \cdots), \tag{5.4}$$

where T, Y and T_c are chiral superfields. The difference of the Kähler potentials leads to a different moduli space geometry. Interestingly, all T, Y and T_c have the hyperbolic geometry structure, which is applicable to the so-called inflationary α-attractors [71, 72]. In the α-attractor inflation, we take the moduli space $K = -3\alpha\log(\Phi + \bar{\Phi})$ for an inflaton superfield Φ, and the value of the parameter α has a relation to the tensor to scalar ratio r as $r = \frac{12\alpha}{N^2}$, where N is the number of e-foldings at the horizon exit. In our model, we have $\alpha = \frac{1}{3}$ and $\frac{2}{3}$, whereas the Cecotti model has $\alpha = 1$. If we apply our model to inflation, we would find a value of tensor to scalar ratio r different from that of the Cecotti model. Therefore, the higher-curvature modification has physical consequences even though the higher-order scalar curvature terms seem to disappear after the ghostbuster mechanism. Since the construction of the inflation model is beyond the scope of this paper, we leave it as future work.

6 Conclusion

We have applied the ghost buster method to a higher-curvature system of SUGRA. It has been known that once we introduce a higher scalar curvature multiplet $\Sigma(\bar{\mathcal{R}})$, a ghost mode

generically shows up in the system as we reviewed in section 2. The ghostbuster method requires a nontrivial U(1) gauge symmetry with a non-propagating gauge superfield. It turned out that the required U(1) symmetry should be the gauged R-symmetry in the case of the higher-curvature system, since the ghost arises from the gravitational superfield. Due to the uniqueness of the gauge charge assignment, it is nontrivial that if the ghostbuster method is applicable to remove the ghost. As we have shown in section 3, thanks to the nonzero U(1) charge of "would-be" ghost mode, we can eliminate the ghost mode and obtain a ghost-free action. However, the resultant ghost-free system turned out to be unstable because of the scalar potential instability. Such an instability is easily cured by introducing matter fields, which would be necessary for realistic models. Additional matter superfields can stabilize the scalar potential if we choose proper couplings between gravity and matter multiplets.

We have also discussed how the ghostbuster mechanism can be seen in the higher-curvature system in section 5. We have found that the higher-order scalar curvature terms R^n with $n \geq 3$ are eliminated in using the mechanism, and the resultant system has the scalar curvature up to the quadratic order. However, the higher-curvature modification is not completely eliminated by the mechanism. We find moduli space geometry different from the known $R + R^2$ supergravity [56]. Therefore, despite the absence of $f(R)$ type interactions in the final form, the SUSY higher-order curvature corrections give physical differences. In particular, the difference of the moduli space structure might be useful for constructing inflationary models.

In this work, we did not discuss the elimination of ghosts originated from higher-derivative terms of matter superfields. It is a straightforward extension of our previous work [1] for global SUSY to SUGRA and is much easier than the higher-curvature model discussed in this paper, since the U(1) charge assignment is not unique for matter higher-derivative models. Since the higher-derivatives of matter fields in SUGRA requires the compensator S_0, it would be interesting to assign the U(1) charge to the compensator as well, i.e. we can use U(1) R-symmetry for the ghostbuster mechanism as with the higher-curvature case, which is only possible for the SUGRA case.

Let us mention the applicability of our mechanism to the other SUGRA formulations, where the auxiliary fields in the gravity multiplet are different. Our mechanism is not applicable for the so-called new minimal SUGRA formulation [73], since the compensator is a real linear superfield, which cannot have any U(1) charge. For the non-minimal SUGRA case, it would be possible to assign a nontrivial U(1) charge to complex linear compensator. In addition, it is known that the R^2 model of non-minimal SUGRA has a ghost mode in the spectrum, so it is interesting to see if the ghost can be removed by our mechanism.

Acknowledgments

A   Superconformal tensor calculus

Here we give a brief summary of the superconformal formulation. We use the convention $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$ for the Minkowski metric.

In 4D $\mathcal{N} = 1$ conformal SUGRA, we have the super-Poincaré generators $\{P_a, M_{ab}Q_\alpha\}$, and the additional superconformal generators, $\{\mathbf{D}, \mathbf{A}, S_\alpha, K_a\}$. They correspond to the translation P_a, the Lorentz rotation M_{ab}, the SUSY Q_α, the dilatation $\mathbf{D}$, the chiral U(1) $\mathbf{A}$, the S-SUSY S_α and the conformal boost K_a, respectively. Such additional gauge degrees of freedom are technically useful for the construction of the SUGRA action. In conformal SUGRA, a supermultiplet is characterized by the charges under $\mathbf{D}$ and $\mathbf{A}$ denoted by w and n, respectively. We introduce one particular supermultiplet called the compensator, whose components are auxiliary fields or removed by the superconformal gauge fixing. In this paper, we use a chiral superfield as the compensator, which gives the so-called old-minimal SUGRA after the superconformal gauge fixing.

In the following, we summarize the component expressions of supermultiplets, the chiral projection operation, the invariant formulae and some identities.

General multiplet.   The components of a general multiplet with charges (w, n) are given by

$$\mathcal{C} = (C, \zeta, H, K, B_a, \lambda, D) \in \mathcal{G}_{(w,n)}, \tag{A.1}$$

whose charge conjugate $\bar{\mathcal{C}}$ is

$$\bar{\mathcal{C}} = (C^*, \zeta^c, H^*, K^*, B_a^*, \lambda^c, D^*) \in \mathcal{G}_{(w,-n)}, \tag{A.2}$$

where ζ^c and λ^c are the charge conjugates of ζ and λ, respectively. Note that $\bar{\mathcal{C}}$ has the $\mathbf{D}$ and $\mathbf{A}$ charges $(w, -n)$.

Multiplication law.   Here We show the multiplication rule of supermultiplets. Suppose $\mathcal{C}^I \in \mathcal{G}_{(w_I, n_I)}$ and consider a function $f(\mathcal{C}^I) \in \mathcal{G}_{(w,n)}$. The component of $f(\mathcal{C}^I)$ is given by

$$f(\mathcal{C}^I) = \Big[f(C^I),\, f_I \zeta^I,\, f_I H^I + \cdots,\, f_I K^I + \cdots,\, f_I B_a^I + \cdots,\, f_I \lambda^I + \cdots,$$
$$f_I D^I + \frac{1}{2} f_{IJ} \left(H^I H^J + K^I K^J - B_a^I B^{Ja} - D_a C^I D^a C^J \right) + \cdots \Big], \tag{A.3}$$

where ellipses denote terms containing fermions and f_I, f_{IJ} are derivatives defined as

$$f_I \equiv \frac{\partial f(C)}{\partial C^I}, \quad f_{IJ} \equiv \frac{\partial^2 f(C)}{\partial C^I \partial C^J}, \tag{A.4}$$

and the covariant derivative of C^I is given by

$$D_a C = e_a{}^\mu (\partial_\mu - \omega b_\mu - i n A_\mu) C + \cdots. \tag{A.5}$$

Note that since (w, n) are additive quantum numbers, the following relations are satisfied,

$$\sum_I w_I f_I C^I = w f(C), \quad \sum_I n_I f_I C^I = n f(C), \quad \sum_J w_J f_{IJ} C^J = (w - w_I) f_I(C). \tag{A.6}$$

Chiral multiplet and chiral projection. The components of a chiral superfield are given by

$$\Phi = (\phi, P_L\chi, F) \in \Sigma_w, \tag{A.7}$$

where $P_L = \frac{1+\gamma_5}{2}$ is the chirality projection operator. A chiral (anti-chiral) multiplet satisfies the constraint $w = n(-n)$ and can be embedded into a general multiplet (A.1) as

$$\Phi \;\rightarrow\; \mathcal{C}(\Phi) = \left(\phi, -\sqrt{2}iP_L\chi, -F, iF, iD_a\phi, 0, 0\right) \in \mathcal{G}_{(w,w)}, \tag{A.8}$$

whereas an anti-chiral multiplet $\bar{\Phi} = (\phi^*, P_R\chi, F^*) \in \bar{\Sigma}_w$ can be embedded as

$$\bar{\Phi} \;\rightarrow\; \mathcal{C}(\bar{\Phi}) = \left(\bar{\phi}, \sqrt{2}iP_R\chi, -\bar{F}, -i\bar{F}, -iD_a\bar{\phi}, 0, 0\right) \in \mathcal{G}_{(w,-w)}. \tag{A.9}$$

One can make a chiral multiplet out of a general multiplet satisfying $w - n = 2$, and we refer to this operation as the chiral projection Σ

$$\Sigma: \; \mathcal{C} \in \mathcal{G}_{(w,w-2)} \;\rightarrow\; \Sigma(\mathcal{C}) \in \Sigma_{w+1}, \tag{A.10}$$

whose components are given by

$$\Sigma(\mathcal{C}) = \left[\frac{1}{2}(H - iK), \frac{i}{\sqrt{2}}P_L(\lambda + \gamma^a D_a\zeta), -\frac{1}{2}(D + D^a D_a C + iD^a B_a)\right]. \tag{A.11}$$

where

$$D^a D_a C = e^{\mu a}\left(\partial_\mu - (w+1)b_\mu - inA_\mu\right)D_a C - \omega_a{}^{ab}D_b C + 2w f_a{}^a C + \cdots, \tag{A.12}$$

$$D^a B_a = e^{a\mu}(\partial_\mu - (w+1)b_\mu - inA_\mu)B_a - \omega_a{}^{ab}B_b + 2in f_a^a C + \cdots. \tag{A.13}$$

Here the ellipses denote terms containing fermions, which we do not focus on in this paper. In particular, for $\mathcal{C} \in \mathcal{G}_{(2,0)}$, we find that

$$D^a D_a C + iD^a B_a + \text{c.c.} = -\frac{1}{3}R(C + \bar{C}) + e^{-1}\partial_\mu\left(ee^{\mu a}(D_a C + iB_a + \text{c.c.})\right) + \cdots, \tag{A.14}$$

where we have used

$$\omega_b{}^{ba} = -3b^a - e^{-1}\partial_\mu(ee^{\mu a}) + \cdots, \quad f_a{}^a = -\frac{1}{12}R + \cdots. \tag{A.15}$$

For instance, we can construct a chiral superfield out of a chiral and an anti-chiral superfield:

$$\Phi \in \Sigma_0, \quad S_0 \in \Sigma_1, \quad \rightarrow \quad \Phi' = \frac{\Sigma(\bar{S}_0\bar{\Phi})}{S_0^2} \in \Sigma_0. \tag{A.16}$$

Note that the chiral projection does not act on a chiral multiplet, i.e. for $\mathcal{C} \in \mathcal{G}_{n,n-2}$, $\Phi \in \Sigma_m$, we find that

$$\Sigma(\mathcal{C}\Phi) = \Sigma(\mathcal{C})\Phi \in \Sigma_{n+m+1}. \tag{A.17}$$

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Vector multiplet $\mathcal{V} \in \mathcal{G}_{(0,0)}$ and gauge transformation. We define a gauge vector superfield as

$$V \in \mathcal{V}: \quad V \in \mathcal{G}_{(0,0)}, \quad V = \bar{V} \tag{A.18}$$

The composite supermultiplet $\bar{\Phi} e^{2gV} \Phi$ is invariant under the SUSY gauge transformation with $\Lambda \in \Sigma_0$,

$$e^{2gV} \rightarrow \ e^{2gV'} = e^{-g\bar{\Lambda}} e^{2gV} e^{-g\Lambda}, \quad \Phi \rightarrow \Phi' = e^{g\Lambda}\Phi, \quad \bar{\Phi} \rightarrow \bar{\Phi}' = \bar{\Phi}e^{g\bar{\Lambda}}. \tag{A.19}$$

Under this transformation, a chiral supermultiplet $\tilde{\Phi} \equiv \Sigma(\bar{\Phi}e^{2gV})$ transforms as

$$\tilde{\Phi} \quad \rightarrow \quad \tilde{\Phi}' = \Sigma\left(\bar{\Phi}e^{2gV}e^{-g\Lambda}\right) = \tilde{\Phi}e^{-g\Lambda}. \tag{A.20}$$

We take the Wess-Zumino gauge, in which the components of V are given by

$$V|_{\mathrm{WZ}} = [B_\mu^g, \lambda^g, D^g], \quad \mathcal{C}(V|_{\mathrm{WZ}}) = [0, 0, 0, 0, B_a^g, \lambda^g, D^g]. \tag{A.21}$$

Here the ordinary gauge transformation with $\Lambda = [i\theta, 0, 0]$ is given by

$$B_\mu'^g = B_\mu^g + \partial_\mu\theta, \quad \Phi' = e^{ig\theta}\Phi. \tag{A.22}$$

Invariant action formula. The superconformal invariant actions are given by the D- and F-term density formulas: the D-term invariant formula with $\mathcal{C} \in \mathcal{G}_{(2,0)}$ and $\bar{\mathcal{C}} = \mathcal{C}$ is given by

$$[\mathcal{C}]_D \equiv \int d^4x\, e \left(D - \frac{1}{3}RC + \cdots \right), \tag{A.23}$$

and the F-term invariant formula with $\Phi \in \Sigma_3$ is

$$[\Phi]_F \equiv \int d^4x\, e(F + \cdots) + \mathrm{c.c.}. \tag{A.24}$$

There is a useful identity between these two invariant formulas

$$[\Sigma(\mathcal{C})]_F = -\frac{1}{2}[\mathcal{C} + \bar{\mathcal{C}}]_D \quad \text{for } \mathcal{C} \in \mathcal{G}_{(2,0)}. \tag{A.25}$$

Then, for $T, X \in \Sigma_1$,

$$\left[\Sigma(\bar{X})T\right]_F = \left[\Sigma(\bar{X}T)\right]_F = -\frac{1}{2}\left[\bar{X}T + \bar{T}X\right]_D. \tag{A.26}$$

In addition, for U(1) charged chiral multiplets $\Phi, \tilde{\Phi} \in \Sigma_1$, we find that

$$\left[\Sigma\left(\bar{\Phi}e^{2gV}\right)\tilde{\Phi}\right]_F = -\frac{1}{2}\left[\bar{\Phi}e^{2gV}\tilde{\Phi} + \mathrm{c.c.}\right]_D. \tag{A.27}$$

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Composite supermultiplets. We finally show the components of composite superfields. With $\bar{\Phi} \in \bar{\Sigma}_1$, we can make a chiral superfield with $w = n = 2$ as

$$\Sigma(\bar{\Phi}) = \left[-\bar{F} \,,\, P_L \gamma^a D_a \chi \,,\, -D_a D^a \bar{\phi} \right] \in \Sigma_2 . \tag{A.28}$$

The composite anti-chiral superfield $\bar{\Phi} \, e^{2cV} \in \bar{\Sigma}_2$ can be embedded into a general multiplet as

$$\mathcal{C}(\bar{\Phi} \, e^{2cV}) \big|_{\mathrm{WZ}} = \Big[\bar{\phi} \,,\, \sqrt{2} i P_R \chi \,,\, -\bar{F} \,,\, -i\bar{F} \,,\, -i D_a \bar{\phi} + 2c B_a^g \bar{\phi} + \cdots \,,$$
$$\cdots \,,\, 2c D^g \bar{\phi} + 2i c B_a^g D^a \bar{\phi} - 2\bar{\phi} c^2 B_a^g B^{ga} \Big] . \tag{A.29}$$

Note that c denotes a gauge charge of Φ under V. In addition, the projected composite superfield takes the form

$$\Sigma(\bar{\Phi} \, e^{2cV}) \big|_{\mathrm{WZ}} = \left[-\bar{F} \,,\, \cdots \,,\, -c D^g \bar{\phi} - \mathcal{D}^a \mathcal{D}_a \bar{\phi} \right] , \tag{A.30}$$

with

$$\mathcal{D}_\mu \bar{\phi} = D_\mu \bar{\phi} + i c B_\mu^g \bar{\phi} . \tag{A.31}$$

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