

# Small vacuum energy from small equivalence violation in scalar gravity

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**ABSTRACT:** The theory of scalar gravity proposed by Nordström, and refined by Einstein and Fokker, provides a striking analogy to general relativity. In its modern form, scalar gravity appears as the low-energy effective field theory of the spontaneous breaking of conformal symmetry within a CFT, and is AdS/CFT dual to the original Randall-Sundrum I model, but without a UV brane. Scalar gravity faithfully exhibits several qualitative features of the cosmological constant problem of standard gravity coupled to quantum matter, and the Weinberg no-go theorem can be extended to this case as well. Remarkably, a solution to the scalar gravity cosmological constant problem has been proposed, where the key is a very small violation of the scalar equivalence principle, which can be elegantly formulated as a particular type of deformation of the CFT. In the dual AdS picture this involves implementing Goldberger-Wise radion stabilization where the Goldberger-Wise field is a pseudo-Nambu Goldstone boson. In quantum gravity however, global symmetries protecting pNGBs are not expected to be fundamental. We provide a natural six-dimensional gauge theory origin for this global symmetry and show that the violation of the equivalence principle and the size of the vacuum energy seen by scalar gravity can naturally be exponentially small. Our solution may be of interest for study of non-supersymmetric CFTs in the spontaneously broken phase.

**KEYWORDS:** Field Theories in Higher Dimensions, AdS-CFT Correspondence, Conformal Field Theory

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## 1 Introduction

The cosmological constant problem (see [1, 2] for reviews) is notoriously challenging from the viewpoint of general relativity effective field theory as well as quantum gravity. Therefore, any simplified theory which replicates significant qualitative features of the problem is very valuable. In this paper, we will focus on such an analog [3], the theory of scalar gravity. This theory, first introduced by Nordström [4], was perfected and formulated as a theory of curved spacetime by Einstein and Fokker [5], and possesses a very faithful version

of the equivalence principle. In the modern era, this theory emerged as the low-energy effective field theory of the spontaneous breaking of conformal symmetry within a conformal field theory (CFT) [6, 7]. The associated Goldstone boson, the “dilaton”, mediates a scalar “gravitational” force and couples to the trace of the stress energy tensor of any other light degrees of freedom. The AdS/CFT dual of this appears in the original Randall-Sundrum I model [8], where the dilaton is dual to the radion, and light matter is present on the IR brane. More precisely, to have only scalar gravity present, the UV brane is removed.

Reference [3] studied the coupling of quantum matter to scalar gravity and showed that there was a very faithful version of the cosmological constant problem (CCP) in this theory. Within the grammar of scalar gravity the problem seems as robust as in general relativity (tensor gravity) and indeed the Weinberg no-go theorem [1] can be extended to this case as well. It is therefore of considerable interest that a solution to the scalar version of the CCP has been proposed [9–12]. The key to this proposal is a very small violation of the scalar equivalence principle, which is technically considerably easier to achieve and to deeply understand than such a violation in general relativity. Indeed, in the AdS/CFT dual picture, it just involves a particular implementation of the Goldberger-Wise (GW) radion stabilization mechanism [13] where the GW field is a pseudo-Nambu Goldstone Boson (pNGB). In quantum gravity, global symmetries such as those associated to pNGBs are not expected to be fundamental, and should be realized as accidental symmetries enforced by gauge structure. In this paper, we realize the mechanism of reference [9–11] in just such a manner, and show that the violation of the equivalence principle and the size of the cosmological constant can naturally be exponentially small. We will formulate our refined mechanism in the AdS-dual language, as a six-dimensional warped effective field theory.

To be self-contained, we will review scalar gravity, its CCP and the proposed solution, closely following the discussion of refs. [3, 12].

### 1.1 Scalar gravity

Even though scalar gravity is ruled out experimentally (for example, light does not bend in a scalar gravity field since the stress-energy tensor of a free Maxwell field is traceless), it serves as a useful analogy to general relativity. A key feature is that the scalar gravity theory respects the Strong Equivalence Principle. This implies that the inertial and gravitational mass of compact objects is the same, with the strong version including scenarios where the gravitational binding energy of the object is not negligible [14].

The covariance of scalar gravity is explicitly seen in the metric formulation,

$$g_{\mu\nu} = \frac{\varphi^2}{M_{\text{pl}}^2} \eta_{\mu\nu}, \tag{1.1}$$

where  $\varphi$  is the scalar graviton, and  $M_{\text{pl}} = \langle \varphi \rangle$  is the scalar gravity Planck scale. This equation does not seem covariant, but can be viewed as a diffeomorphism gauge-fixed version of the generally covariant constraint of the vanishing of the Weyl tensor,<sup>1</sup>

$$C_{abcd} = R_{abcd} - (g_{a[c}R_{d]b} - g_{b[c}R_{d]a}) + \frac{1}{3}Rg_{a[c}g_{d]b} = 0. \tag{1.2}$$

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<sup>1</sup>In quantum effective field theory, this constraint can be imposed at the level of the path integral using a Lagrange multiplier.

In  $d \geq 4$ , it is a necessary and sufficient condition for obtaining a conformally flat solution that the Weyl tensor vanish. (Here we restrict to  $d = 4$ .) Then, a modified Einstein-Hilbert action serves as the action for scalar gravity

$$S = \int d^4x \sqrt{g} \left( \frac{M_{\text{pl}}^2}{12} R + \Lambda + \mathcal{L}_{\text{matter}} \right). \tag{1.3}$$

Note, the sign of the Ricci scalar is opposite of that in the usual Einstein-Hilbert action (which is negative in the  $(+---)$  signature we use). This is a consequence of the fact that in GR the conformal mode has the “wrong-sign kinetic term”, which is not a problem since this mode is non-propagating. In the case of scalar gravity, however, this is precisely the only propagating mode allowed by the Weyl constraint and hence the sign of the Einstein-Hilbert action must be reversed. In particular, we can recover scalar gravity equation of motion in the Jordan frame,

$$R = \frac{12}{M_{\text{pl}}^2} T^\mu{}_\mu. \tag{1.4}$$

Notice that a cosmological constant term  $\Lambda$  naturally appears in equation (1.3) and will be renormalized by quantum matter.

In its modern incarnation, this scalar gravity theory can be viewed as the low-energy limit of a CFT with spontaneously broken conformal invariance. The subgroup of diffeomorphisms which preserve the form of the metric as in equation (1.1) are conformal transformations,

$$x \rightarrow x'(x), \tag{1.5}$$

$$g_{\mu\nu}(x) \rightarrow f^2(x) g_{\mu\nu}(x), \tag{1.6}$$

or

$$\varphi(x) \rightarrow f(x) \varphi(x). \tag{1.7}$$

Rewriting  $\varphi(x)$  as  $e^{\tau(x)}$ , for a constant scaling  $e^\lambda$  we get a shift,

$$\tau(x) \rightarrow \tau(\lambda x) + \lambda. \tag{1.8}$$

This subgroup of diffeomorphisms is precisely the conformal group,  $O(4,2)$  and the field  $\varphi$  transforms as the dilaton, the Goldstone boson of a spontaneously broken conformal symmetry [6]. The transformation differs from that of a Goldstone boson of an internal symmetry crucially in the spacetime argument of  $\tau$ . As a consequence, non-derivative interactions, and in particular a  $\varphi^4$  potential is allowed in the low energy theory.

Requiring non-linearly realized conformal invariance automatically matches a generally coordinate invariant form of the action, written in terms of equation (1.1), up to the inclusion of a Wess-Zumino term [12],

$$S_{CI}(\tau) = S(g) + S_{WZ}(\tau). \tag{1.9}$$

Therefore, the two theories are identical at the level of effective field theory. This leads to an important conclusion: we know what UV completes scalar gravity (equations (1.2) and (1.3))! It is a quantum CFT on a flat Minkowski background.

The appearance of a CFT opens up another connection. The AdS/CFT correspondence allows us to relate the spontaneously broken CFT with an AdS spacetime, cut off by a IR brane at high redshifts [15, 16]. The radion is identified with the dilaton in this case. Since the radion itself is a geometrical modulus giving the position of the IR brane, in this formulation it is not surprising that the curved spacetime of underlying scalar gravity is the curved spacetime of the IR brane, described by the radion field. Light matter sees this curved spacetime by being localized to this brane.

## 1.2 Cosmological constant problem in scalar gravity

### 1.2.1 Classical level

For general relativity, maximally symmetric solutions exist for all values of the cosmological constant, but the Poincaré invariant vacuum solution is only obtained at a single fine-tuned value of zero CC. This situation is replicated in scalar gravity.

Let us consider the “chiral Lagrangian” for scalar gravity in the Einstein frame on a Minkowski background. The low energy couplings of the dilaton are fixed by transformations under non-linearly realized conformal transformations, and can be simply written as,

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \lambda \varphi^4 + \mathcal{L}_{\text{matter}}, \tag{1.10}$$

where we have truncated the Lagrangian at two-derivative order, and  $\mathcal{L}_{\text{matter}}$  contains “standard model” fields.  $\varphi$  is coupled to the matter as a “compensator field” to ensure conformal invariance. We note again that the appearance of a non-derivative coupling for the Goldstone boson is a consequence of the unique transformation law shown in equation (1.8). In fact, the  $\lambda$ -term corresponds to  $\Lambda/M_{\text{pl}}^4$  in equation (1.3), the cosmological constant.

The presence of the  $\lambda \neq 0$  does not admit a Poincaré invariant classical solution. We can see that depending upon  $\lambda$ , the solutions are [12]:

$$\lambda > 0 \quad \rightarrow \varphi \sim 1/z \quad \text{SO}(3,2) \equiv AdS_4, \tag{1.11}$$

$$\lambda < 0 \quad \rightarrow \varphi \sim 1/t \quad \text{SO}(4,1) \equiv dS_4, \tag{1.12}$$

$$\lambda = 0 \quad \rightarrow \varphi \sim \text{const.} \quad \text{ISO}(3,1) \equiv \text{Poincaré}. \tag{1.13}$$

Generically, the  $\text{SO}(4,2)$  group of the CFT is broken to  $\text{SO}(3,2)$  or to  $\text{SO}(4,1)$ , corresponding to a classical solution with the dilaton carrying a non-trivial spacetime dependence. Matter fields coupled to the dilaton effectively see an Anti-de Sitter or a de Sitter background. The form of the dilaton effective potential is fixed by conformal symmetries, and only admits a translationally invariant solution upon tuning. In particular, the hierarchy between the Planck scale and the cosmological constant in our universe is replicated in scalar gravity for  $\lambda \sim 10^{-120}$ . Strictly speaking, spontaneous conformal breaking is eliminated for  $\lambda \neq 0$ , however if  $\lambda$  is small enough it is a useful approximation. For example, solutions in equations (1.11) and (1.12) give a cosmological version of spontaneous breaking.

### 1.2.2 Quantum corrections

In equation (1.10), the cosmological constant appears as a marginal operator. One might then question whether the severity of the fine tuning here is similar to the cosmological constant problem in general relativity, where the operator is super-renormalizable. In principle, since there is no symmetry protecting the  $\varphi^4$  operator, we expect thresholds to generate the operator with  $\mathcal{O}(1)$  coefficient. An irreducible running contribution is generated from the (small) dimensionless couplings of the dilaton to the SM fields,  $(v_{\text{weak}}/M_{\text{pl}})^4$ . Numerically, we see that the tuning is as severe as in our universe, where the CC gets irreducible contributions of at least  $v_{\text{weak}}^4$ .

It turns out that the situation in this case actually parallels the GR case even more closely. In order to consistently regulate UV divergences arising from our effective Lagrangian, we need to ensure that the Ward identities associated with the conformal invariance are satisfied. A convenient way to ensure this is to have a  $\varphi$ -dependent renormalization scale,

$$\hat{\mu} \equiv \mu \frac{\varphi}{M_{\text{pl}}}, \tag{1.14}$$

such that vacuum bubbles, two-point functions and four-point functions (or more generally the Coleman-Weinberg Potential of the dilaton) appear as

$$V(\varphi) \sim \alpha \hat{\mu}^4 + \beta \hat{\mu}^2 \varphi^2 + \gamma \log[\hat{\mu}^2/\varphi^2] \varphi^4 \sim \frac{\mu^4}{M_{\text{pl}}^4} \varphi^4. \tag{1.15}$$

Thus, we see the same matter loop diagrams that contribute to the spin-2 cosmological constant indeed do contribute to  $\lambda$ .

### 1.2.3 Comparison with the cosmological constant problem in GR

It is interesting to ask whether the scalar CCP is on the same footing as the problem in GR. The no-go theorem derived by Weinberg [1] focuses on the trace of Einstein's equations, which is of course identical to the equation of motion for scalar gravity. The crucial point is that for translationally invariant solutions, when the matter field satisfy their equations of motion, general covariance forces the Lagrangian to have a very specific dependence on  $g_{\mu\nu}$ ,

$$\mathcal{L} = c\sqrt{-g}. \tag{1.16}$$

Similarly, for scalar gravity, conformal invariance forces the Lagrangian (again on the classical solutions for matter fields) to be,

$$\mathcal{L} = c\varphi^4. \tag{1.17}$$

We see that in either case, there is no non-trivial solution for the gravitational equation of motion. In order to dynamically relax the cosmological constant to zero, solutions of matter equations of motion should imply a solution to the (trace of) Einstein's equations. Weinberg has argued [1] that this is not possible without fine tuning.

Other features of the CCP [2] which make a solution hard are also reflected in the scalar gravity case. We briefly recall some challenges that any solution faces:

- Since binding energies and loop corrections to energy levels have been measured to gravitate, by the equivalence principle these loops should also contribute to the cosmological constant.
- Modification of gravity at short distances ( $\sim 100 \mu\text{m}$ ) [17, 18] does not help the situation, since the matter loops in question are not cut off at this scale, and the graviton momentum probing the CC is Hubble scale, nothing to do with the “compositeness scale” of gravity.
- Modification of gravity at very long distances, comparable to current Hubble scale runs into the problem that the short distance uncancelled cosmological constant prevents the universe from ever becoming large enough to probe the very long distance behavior.
- Mechanisms which involve gravitational dynamics solving the cosmological problem suffer from the problem that the CC only very recently became an appreciable contribution to the energy budget, so it would be impossible for a mechanism in the early universe to operate setting it to be so small.

All of these issues apply to the scalar gravity case as well as they do to spin-2 gravity. Some of these objections appeal to our cosmic history and some others merely to the particle physics. We will focus on the particle physics aspects of the fine tuning. The cosmological mechanisms that address the issues above are left for future work.

### 1.3 Solution: deformation of the CFT

We outline the solution here, which arises from considering deformations of the CFT [9–12] (see also more recent discussions [19–21]). If we add a relevant deformation, it explicitly breaks the CFT, giving a mass to the dilaton. In order to obtain a regime where the theory approximates scalar gravity, we would like to have a hierarchy between the scale of spontaneous CFT breaking (interpreted as the  $M_{\text{pl}}$  of scalar gravity) and the mass of the dilaton, the scale at which there is maximal equivalence principle violation and below which there is no long range gravitational force. We would ideally like this regime to be extremely large to reproduce qualitatively the exponential hierarchy of scales observed in our universe.

What sets the mass of the dilaton? The order parameter for CFT breaking is the non-conservation of the scale current, which is proportional to the  $\beta$ -function. For compact internal symmetries, we can usually ensure that the deformation is small in a controlled fashion, yielding a light pNGB naturally, as is the case for the pion and chiral symmetry in the SM. However, the present situation for the CFT is different. The presence of the Poincaré invariant solution requires that at the scale of spontaneous breaking, the dilaton potential contributions from the spontaneous breaking are balanced by those from the explicit breaking. Thus, we expect the deformation to be  $\mathcal{O}(1)$  at the breaking scale, generically implying an  $\mathcal{O}(1)$   $\beta$ -function.



The gauge coupling in QCD is an example of such a deformation, and as the above discussion illustrates, we do not expect a narrow, light resonance associated with the dilaton. The condition for obtaining a light dilaton is rather special: the  $\beta$ -function of the deformation should stay parametrically small over a range of values of the coupling. Thus, even when the deformation grows large, the amount of scale violation is parametrically small. The mass of the dilaton is suppressed by the small parameter. Since the deformation preserves the Lorentz subgroup of the CFT, while explicitly breaking scaling, the low-energy theory still has Poincaré invariance.

This dynamical requirement from the CFT point of view is somewhat mysterious. The AdS dual theory in five dimensions makes the situation much clearer. The tuning associated with the scalar gravity is nothing but the tuning of the IR brane tension required in the original Randall-Sundrum model (RS1 [8]). There is of course an additional tuning of the UV brane itself in that set up, which is associated with the tuning of the spin-2 cosmological constant. However, this tuning is decoupled from the IR tuning issue. In fact, in our analysis, we will always assume that the UV brane is absent, so that the dual theory runs to a UV fixed point. Indeed, this is the theory of scalar gravity, and the spin-2 graviton has been decoupled.

The above discussion suggests inclusion of a Goldberger-Wise stabilization mechanism in the gravity picture. The presence of a small  $\beta$ -function for the deformation corresponds to a suppressed bulk potential for the corresponding AdS scalar. Such a suppression can be protected naturally by an approximate shift symmetry, in turn realized if the GW scalar is a 5D pNGB of a global symmetry with a tiny explicit breaking. While technically natural, we expect all global symmetries to be at best emergent below the quantum gravity scale. The fact that the AdS gravity theory is expected to get strongly coupled not far from the curvature scale suggests that there may be unacceptably large violations of the global symmetry by quantum gravity effects. This is the aspect that we study and control in this paper.

A familiar solution to the problem is to use a gauge symmetry in a higher dimensions to obtain the shift symmetry [22]. A gauge field in one higher compactified dimension yields a scalar field in the low energy theory. As a result of residual gauge symmetry, the scalar possesses a global shift symmetry which can be robustly protected against quantum effects by higher-dimensional locality. Thus, we will obtain our Goldberger-Wise field in 5D from one higher dimension as the sixth component of a gauge field. The very small potential term is generated by non-local Aharonov-Bohm phases, which are exponentially suppressed if 6D charged particle masses are somewhat heavier than the inverse-size of the sixth dimension. Therefore, we can naturally obtain an exponentially small potential terms for the GW field.

For fluctuations about the stabilized radion, the (approximate) shift symmetry for the GW field translates into a shift symmetry for the dilaton, suppressing its potential. It is worth noting that the mechanism involves physics above  $M_{\text{pl}}$ , the scalar gravity Planck scale, which is also the scale of spontaneous conformal symmetry breaking. And yet, the mechanism robustly cancels contributions (from phase transitions or thresholds) far below this Planck scale.



In sections 2 and 3 we review the solution originally proposed in an unpublished work by Contino, Pomarol, Rattazzi [9–11] and then discussed later in [12, 19–21]. The discussion of the mechanism in 4D in section 2 highlights the conditions required for the solution to work, and we show that we expect it to be robust as long as the  $\beta$  function stays parametrically small. We then present a simple example in 5D in section 3, where an approximate shift symmetry protecting the stabilizing GW field results in the small  $\beta$ -function in the 4D effective theory. The shift symmetry in 5D is the target for our solution in 6D presented in section 4, where we obtain naturally the exponentially small potential for the GW field. We show by way of an explicit computation that all other fields involved in our calculation decouple and we indeed reproduce the 5D EFT desired. We conclude in section 5.

## 2 The mechanism in four dimensions

The relaxation mechanism operates dynamically at low energies in the vacuum such that it robustly cancels various contributions arising from different scales. This means that we should be able to study this mechanism purely in terms of the dilaton effective potential. We consider the dilaton potential in the deep IR, after integrating out all matter fields.

Let us begin by considering the undeformed CFT. Then, the “SM” self-couplings, masses and couplings to the dilaton respect conformal invariance and the IR dilaton potential is given by,

$$V(\varphi) = \lambda\varphi^4. \tag{2.1}$$

This result holds exactly, following from symmetries of the dilaton in the non-linearly realized CFT. In a particular renormalization scheme, care has to be taken in order for the regulator to not introduce spurious scale dependence in the potential. Technically the terminology “spontaneous breaking of the CFT” implies that  $\varphi$  is a modulus, and hence is only applicable to the situation  $\lambda = 0$ . The above equation should be thought of as a (somewhat small  $\lambda$ ) deviation from this tuned limit.

We next add a weakly relevant deformation to the CFT,

$$\mathcal{L}(\mu) = \mathcal{L}_{\text{CFT}} + g(\mu)\mathcal{O}. \tag{2.2}$$

The scale dependence of the coupling constant now affects the low-energy effective potential. The  $\beta$ -function of  $g$  is the only source of explicit violation of the CFT. By treating the running coupling as a spurion [19], we can write the general form of the effective potential,

$$V(\varphi) = \kappa[g(\varphi)]\varphi^4, \tag{2.3}$$

where  $\kappa$  is a function determined by the strong dynamics of the CFT. This Coleman-Weinberg potential generates a mass for the dilaton, and can stabilize it at a non-zero value. The stable minimum for this potential can be calculated from

$$\left. \frac{\partial V}{\partial \varphi} \right|_{\varphi_{\text{min}}} = 4\varphi_{\text{min}}^3 \kappa[g(\varphi_{\text{min}})] + \kappa'[g(\varphi_{\text{min}})]\beta[g(\varphi_{\text{min}})]\varphi_{\text{min}}^3 = 0. \tag{2.4}$$

In order to balance an  $\mathcal{O}(1)$  contribution arising from the spontaneous CFT breaking, the marginal deformation must itself grow to be  $\mathcal{O}(1)$ . At this point generically conformal invariance is no longer an approximate symmetry, since for  $\mathcal{O}(1)$  couplings the  $\beta$  function itself is not small, which parametrizes the non-conservation of the scale current,

$$\partial_\mu S^\mu = T^\mu_\mu \propto \beta(g). \tag{2.5}$$

We see from above that the generic expectation in the absence of tuning is that  $m_\varphi \sim \varphi_{\min}$ , which is identified as the  $M_{\text{pl}}$  for scalar gravity. Thus, the scalar graviton mass is expected to be of the order  $M_{\text{pl}}$ , and we do not obtain a large hierarchy of scales between which we can approximate the theory as scalar gravity. This is analogous to the situation in QCD, where no light dilaton emerges.

Therefore, we would like to engineer a special situation, where even when the coupling grows to be  $\mathcal{O}(1)$ , the  $\beta$ -function stays robustly small. We *assume*

$$\beta(g) = \epsilon \bar{\beta}(g), \tag{2.6}$$

where  $\bar{\beta}(g)$  is a generic,  $\mathcal{O}(1)$  function of  $g$ , with the only restriction being that it does not have a zero for a finite range of  $g$  (except at  $g = 0$  where conformal invariance is restored, of course).

With a slowly varying  $\kappa$ , we see from equation (2.4) that the minimum of the dilaton potential lies parametrically close to the zero of  $\kappa$ . At zeroth order in  $\epsilon$ ,  $g(\varphi_{\min}) = g_*$ , where  $\kappa(g_*) = 0$ . Expanding around this point,

$$g(\varphi_{\min}) = g_* - \frac{\epsilon}{4} \bar{\beta}(g_*) + \mathcal{O}(\epsilon^2). \tag{2.7}$$

We can use this to find  $\varphi_{\min}$ ,

$$\varphi_{\min} = \Lambda \exp \left[ \frac{1}{\epsilon} \int_{g(\Lambda)}^{g_* - \frac{1}{4} \epsilon \bar{\beta}(g_*)} \frac{dg}{\bar{\beta}(g)} \right], \tag{2.8}$$

where  $\Lambda$  is a reference scale. We can expand the potential around the minimum for field fluctuations  $|\delta\varphi| \leq \varphi_{\min}$ ,

$$V(\varphi_{\min} + \delta\varphi) = \epsilon \frac{\partial \kappa}{\partial g} \left( -\frac{1}{4} \bar{\beta} + \bar{\beta} \log(1 + \delta\varphi/\varphi_{\min}) \right) [6\varphi_{\min}^2 \delta\varphi^2 + \delta\varphi^4]. \tag{2.9}$$

This shows rather explicitly that the dilaton mass and quartic are suppressed parametrically over a range of  $\delta\varphi$ , allowing a large separation of scale between  $m_\varphi$  and  $M_{\text{pl}}$ , as well as an expanding phase with a tiny scalar cosmological constant if  $\varphi$  is displaced from its potential.

Note that except for the smallness of the  $\beta$  function, we have made no other assumptions and the form of the effective potential is fixed by the symmetries. So, we expect the above conclusions are robust. It is instructive however to see an explicit example.

Let us first consider the limit  $\epsilon = 0$ , i.e. the limit of unbroken CFT, and study an effective Lagrangian for the dilaton coupling with a fermion,

$$\mathcal{L}_{\text{eff}} \ni \bar{\psi} i \not{\partial} \psi + y \varphi \bar{\psi} \psi + \kappa \varphi^4. \tag{2.10}$$

The fermion gets a mass proportional to its Yukawa coupling,  $m_\psi = \langle \varphi \rangle y$ , which is a manifestation of the equivalence principle. We can study the effective Lagrangian below this mass scale after integrating out this fermion,

$$\mathcal{L}_{\text{eff}} \ni \kappa' \varphi^4. \tag{2.11}$$

The form of the potential for the dilaton has unchanged, but the coefficient gets a correction from the threshold. Thus, even if we started from a zero dilaton quartic just below the scale of CFT breaking, we would still get contributions from all thresholds below that scale, similar to what happens for the spin-2 cosmological constant. Recall the discussion of equation (1.15) that we do not get the usual Coleman-Weinberg potential of the form  $\sim \varphi^4 \log(\varphi)$ , but only  $\sim \varphi^4$ .

In the presence of the running coupling, the explicit breaking appears in the effective Lagrangian as the spurion  $g(\mu)$ ,

$$\mathcal{L}_{\text{eff}} = \bar{\psi} i \not{\partial} \psi + y(g(\varphi)) \varphi \bar{\psi} \psi + \kappa(g(\varphi)) \varphi^4. \tag{2.12}$$

The spurion  $g(\varphi)$  leads to violation of scalar equivalence principle. At the linearized level, the coupling of  $\psi$  to  $\varphi$  is

$$y(g(\varphi)) + y'(g(\varphi)) \epsilon \bar{\beta}(g(\varphi)), \tag{2.13}$$

which is no longer tied to the mass of the fermion  $\psi$ , and we would observe deviations from the equivalence principle by measuring the gravitational couplings for multiple  $\psi$  species. We see that the violation of the equivalence principle is suppressed by  $\epsilon$ . As before, integrating out this fermion results in the Coleman-Weinberg potential modification of the function  $\kappa$ . However, the form of the potential is unchanged from that in equation (2.3), with scale dependence arising solely from  $g(\varphi)$ , the running of the coupling near the CFT breaking scale. Thus we see that our conclusions are robust to matter effects and phase transitions below the CFT breaking scale.

We have not yet justified the origin of the assumption,  $\beta = \epsilon \bar{\beta}$ . We next show that this can be achieved in a technically natural way in a variant of an RS model — a 5D model with an IR brane.

### 3 The mechanism in five dimensions

In this section we present a 5D realization of the 4D solution above. Here we focus on a simple realization in order to focus on the essential features and look at more general theories in the next section. The 5D model is a gravitational theory with AdS background, truncated by an IR brane (that is to say it is the RS1 model [8], but without the UV brane and hence extending all the way to  $\partial\text{AdS}$ ). We start by outlining the dictionary to translate between the AdS and CFT theories.

### 3.1 AdS/CFT dictionary

The AdS/CFT dictionary provides a handy way to identify the corresponding physics in 5D. The dilaton is dual to the size of the extra dimension, parametrized by the radion field,

$$\varphi(x) \leftrightarrow z_{\text{IR}}(x), \tag{3.1}$$

where  $z_{\text{IR}}$  is the position of the IR brane. A deformation in the CFT is dual to an AdS scalar field, with the running coupling identified as (one mode of) the profile of the scalar,

$$g(\mu) \leftrightarrow \omega \left( z = \frac{1}{k} \log \mu \right), \tag{3.2}$$

where  $\omega$  is a scalar field which will play the role of a Goldberger-Wise stabilizing field [13],  $k$  is the AdS curvature and we have identified the warped extra dimension coordinate ( $z$ ) as the holographic renormalization scale. In the limit of a small potential for  $\omega$ , the evolution of  $\omega$  in  $z$  is given by a “slow-roll” approximation, such that  $\partial_z^2 \omega \ll k \partial_z \omega$ . The evolution of the scalar profile is then simply related to the potential  $\partial_z \omega \simeq \partial V / \partial \omega$ , yielding the following identification,

$$\beta(g(\mu)) \leftrightarrow \frac{\partial V}{\partial \omega}. \tag{3.3}$$

Let us consider the case where the field  $\omega$  is an exact Nambu-Goldstone boson of a global symmetry, say a U(1). This implies that there is a shift symmetry for  $\omega$ , setting  $V = 0$ . This is dual to a circle of fixed points, where  $\beta = 0$ . This is expected from the fact that the spontaneous breaking of U(1) leads to a set of degenerate AdS vacua, each of which corresponds to a CFT. Since U(1) SSB is robust in AdS effective field theory, an approximate shift symmetry for  $\omega$  can also be realized robustly, leading to a small potential, and hence a small  $\beta$  function.<sup>2</sup> In order to obtain a slightly relevant deformation we can add a small negative mass-squared for the scalar (which is stable on an AdS background). Once we turn on the small potential and source it at the  $\partial\text{AdS}$ , the scalar field backreacts on the metric taking it away from the pure AdS limit. This corresponds to a breaking of the CFT via running of the dual coupling.

The “SM matter” terms in the dilaton effective action (equation (1.10)) appear as brane localized terms in the 5D picture,

$$\mathcal{L}_{\text{matter}} \leftrightarrow \mathcal{L}_{\text{brane,IR}}. \tag{3.4}$$

The cosmological constant problem in scalar gravity is dual to the IR brane tension tuning in RS1 model,

$$\lambda \varphi^4 \leftrightarrow [\sqrt{g} \delta T]_{\text{IR}}, \tag{3.5}$$

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<sup>2</sup>In AdS quantum gravity some breaking of the U(1) global symmetry — and hence the shift symmetry of  $\omega$  — is to be expected. Indeed, it is to control this aspect of the problem that we present a 6D construction in section 4. Here we take the AdS effective field theory view that small explicit breaking is natural.

where  $\delta T_{\text{IR}}$  is the detuning of the brane tension from the value in RS1 required to tune the radion potential to zero. It includes the vacuum energy contributions from the SM fields living on the IR brane. Upon including the deformation, the correspondence becomes

$$\kappa(g(\varphi))\varphi^4 \leftrightarrow [\sqrt{g}f(\omega)]_{\text{IR}}, \tag{3.6}$$

where we have combined the brane tension detuning and couplings to  $\omega$  into a single function  $f(\omega)$ . We see that we want the Goldberger-Wise scalar to have a generic potential localized on the IR brane. Locality preserves the approximate global symmetry in the bulk even though it is broken badly on the IR brane. We discuss a simple realization of these features next.

### 3.2 5D classical solution

We review the simple version of a 5D model presented in [12]. For computational simplicity we work in the case where backreaction of the field is somewhat small everywhere. As shown in [12], this assumption is not needed. Since we are tracking a fine-tuning of  $\mathcal{O}(10^{-120})$ , taking some parameters to be small (but  $\mathcal{O}(1)$ ) to maintain perturbative control should be harmless.

The model is,

$$\frac{S}{M_5^3} = \int d^4x dz \sqrt{G} \left[ -\frac{1}{4}R + 3k^2 + \frac{1}{2}(\partial\omega)^2 + 2k^2\epsilon\omega^2 \right] - \frac{1}{2} \int_{\text{IR}} d^4x \sqrt{g} k [-3 + f(\omega)], \tag{3.7}$$

where  $\omega$  is a dimensionless, pNGB field (which is denoted  $\pi$  in [12]), and  $f(\omega)$  is a generic brane-localised potential for  $\omega$ . Note that we are working in the  $(+ - - - \dots)$  signature.

The model above is clearly not the most general Lagrangian consistent with the approximate shift symmetry of the Goldstone. However, it serves to illustrate the key features of the solution, and we consider robustness against generalizations, other deformations and quantum corrections later.

We first seek a 4D Poincaré invariant ground state within the domain of the EFT,

$$ds^2 = e^{2A(z)} dx^\mu dx^\nu \eta_{\mu\nu} - dz^2, \tag{3.8}$$

$$\omega(x, z) = \omega(z). \tag{3.9}$$

The equation of motions for  $A, \omega$  in the bulk are,

$$A'^2 - k^2 - \frac{2}{3}k^2\epsilon\omega^2 - \frac{1}{6}\omega'^2 = 0, \tag{3.10}$$

$$\omega'' + 4A'\omega' + 4k^2\epsilon\omega = 0, \tag{3.11}$$

$$A'' + \frac{2}{3}\omega'^2 = 0, \tag{3.12}$$

and the junction matching conditions on the IR brane are,

$$\omega'(z = z_{\text{IR}}) = -\frac{1}{2}k \frac{\partial f(\omega_{\text{IR}})}{\partial \omega}, \tag{3.13}$$

$$A'(z = z_{\text{IR}}) = -k \left[ 1 + \frac{1}{3}f(\omega_{\text{IR}}) \right]. \tag{3.14}$$

If we neglect the backreaction of the GW field on the metric, then we can write the solution for its equation of motion in a background AdS space

$$\omega = \omega_* e^{\Delta_- kz} + \hat{\omega} e^{\Delta_+ k(z-z_{\text{IR}})}, \tag{3.15}$$

where  $\Delta_{\pm} = 2(1 \pm \sqrt{1-\epsilon})$ . This is the familiar Goldberger-Wise scalar [13] profile used to stabilize the Randall-Sundrum branes.

When is our assumption of small backreaction justified? Since the potential  $V(\omega) \sim \mathcal{O}(\epsilon)$ , the dominant backreaction comes from the kinetic term for  $\omega$ . The contribution from the slowly varying  $e^{\Delta_- kz}$  term is small, so that the backreaction is determined by the size of  $e^{\Delta_+ k(z-z_{\text{IR}})}$  term. To ensure that this is small even as we get close to IR brane, we need  $\hat{\omega}$  to be parametrically small. Let us first consider if we can have  $\hat{\omega} = \mathcal{O}(\epsilon)$ . The junction conditions are then only satisfied if  $f(\omega_{\text{IR}}) = \partial f(\omega_{\text{IR}})/\partial \omega = \mathcal{O}(\epsilon)$ , which reintroduces the tuning of the brane tension on the IR brane. Let us allow a detuning of the brane tension away from this limit, with the detuning set by a moderately small parameter which we will call  $\eta$ . In this case the function  $f(\omega)$  is chosen such that exists a value  $\omega = \bar{\omega}$  where  $f(\bar{\omega}) \sim \partial f(\bar{\omega})/\partial \omega \sim \mathcal{O}(\eta)$ . This corresponds to a mild tuning of the IR brane tension with the 5D bulk CC. Note that this tuning applies to the fully renormalized brane tension / bulk CC, including all radiative corrections, so that there is just one tuning of  $\mathcal{O}(\eta)$ . We can treat the backreaction perturbatively in  $\eta$ .

With this mild tuning, the boundary matching conditions are satisfied by our solutions at zeroth order in detuning parameter  $\eta$ . The IR brane position is fixed at

$$z_{\text{IR}} \simeq \frac{1}{k\Delta_-} \log \left[ \frac{\omega_*}{\bar{\omega}} \right]. \tag{3.16}$$

This suggests that the radion has been stabilized. At higher orders in  $\eta$ ,  $\hat{\omega}$  is non-zero, and the GW profile backreacts on the metric. The solution can be self-consistently solved for order by order in  $\eta$ . We present this solution later in section 4, and now turn to the effective potential for the radion and the mass of the radion fluctuations.

### 3.3 Effective action for the radion

It is instructive to derive the 4D effective action for the radion  $r(x)$  by plugging in the solution above back into the 5D action. The metric with the radion fluctuation is conveniently parametrized as [23],

$$ds^2 = e^{-2k(z+r(x)e^{2kz})} dx^\mu dx^\nu \eta_{\mu\nu} - (1 + 2kr(x)e^{2kz}) dz^2. \tag{3.17}$$

The difference relative to the RS case is the behavior of the fields towards the AdS boundary (which is cut off by the UV brane in the usual Randall-Sundrum models). In the present case, we need to add a “regulator”, for which we introduce a boundary at  $z_{\text{UV}}$ . The presence of the boundary yields a finite action upon dimensional reduction, and the boundary terms are also required for a well-defined variational principle [24]. The boundary term is

$$S \supset -\frac{M_5^3}{2} \int_{\text{UV}} d^4x \sqrt{g} [3k + \Delta_- k \omega^2]. \tag{3.18}$$

This is analogous to UV regulating the 4D CFT. This boundary term ensures that our deformation is a free input, parametrized by  $\omega_*$ , the co-efficient of the near-boundary behavior of the scalar field. It also ensures that the near-boundary geometry is pure AdS.

The kinetic piece of the radion action is given by,

$$\mathcal{L}_{\text{kin}} = \frac{3}{4}kM_5^3 \left( \exp \left[ 2k(z_{\text{IR}} - e^{2kz_{\text{IR}}}r(x)) \right] - \exp \left[ 2k(z_{\text{UV}} - e^{2kz_{\text{UV}}}r(x)) \right] \right) \partial_\mu r \partial^\mu r. \quad (3.19)$$

For the calculation of the potential, it is sufficient to look at the limit where the fluctuation  $r(x) = 0$ . In the bulk,

$$\begin{aligned} -\frac{V_{\text{bulk}}}{M_5^3} &= \frac{k}{2}e^{-4kz_{\text{IR}}} \left[ (1 - e^{-4k(z_{\text{UV}}-z_{\text{IR}})}) - \Delta_+ \hat{\omega}^2 (1 - e^{(\Delta_+ - \Delta_-)k(z_{\text{UV}}-z_{\text{IR}})}) \right] \\ &\quad - \frac{k\Delta_-}{2} \omega_*^2 (e^{-(\Delta_+ - \Delta_-)kz_{\text{IR}}} - e^{-(\Delta_+ - \Delta_-)kz_{\text{UV}}}). \end{aligned} \quad (3.20)$$

We see that there are bulk contributions which diverge as  $z_{\text{UV}} \rightarrow -\infty$ . There is an extra contribution from the extrinsic curvature of the UV and IR branes,

$$R|_{|z_{\text{IR}}|} = 8k\delta(z - z_{\text{IR}}), \quad R|_{|z_{\text{UV}}|} = -8k\delta(z - z_{\text{UV}}). \quad (3.21)$$

If the branes are thought of as orbifold fixed points, this contribution arises as the contribution of the kink to the curvature. Thus, the brane contributions to the potential are,

$$-\frac{V_{\text{brane,IR}}}{M_5^3} = -\frac{1}{2}e^{-4kz_{\text{IR}}} [k + kf(\omega(z_{\text{IR}}))], \quad (3.22)$$

$$-\frac{V_{\text{brane,UV}}}{M_5^3} = -\frac{1}{2}e^{-4kz_{\text{UV}}} [-k + k\Delta_- \omega(z_{\text{UV}})^2]. \quad (3.23)$$

The total potential is given by

$$\begin{aligned} -\frac{V}{M_5^3} &= -\frac{k}{2}e^{-4kz_{\text{IR}}} \left[ \Delta_+ \hat{\omega}^2 (1 - e^{(\Delta_+ - \Delta_-)k(z_{\text{UV}}-z_{\text{IR}})}) + f(\omega(z_{\text{IR}})) \right] \\ &\quad - \frac{k\Delta_-}{2} \left[ \omega_*^2 e^{(\Delta_- - \Delta_+)kz_{\text{IR}}} + 2\omega_* \hat{\omega} e^{-\Delta_+ kz_{\text{IR}}} + \hat{\omega}^2 e^{(\Delta_+ - \Delta_-)kz_{\text{UV}}} e^{-2\Delta_+ z_{\text{IR}}} \right]. \end{aligned} \quad (3.24)$$

We see that the regulator cancels the potentially dangerous terms which would blow up as  $z_{\text{UV}} \rightarrow \infty$ , and so we can proceed to that limit,

$$\frac{V}{M_5^3} = \frac{k}{2}e^{-4kz_{\text{IR}}} (\Delta_+ \hat{\omega}^2 + f(\omega(z_{\text{IR}}))) + \frac{k\Delta_-}{2} e^{-\Delta_+ kz_{\text{IR}}} \left( \omega_*^2 e^{\Delta_- kz_{\text{IR}}} + 2\omega_* \hat{\omega} \right). \quad (3.25)$$

Matching conditions are given by,

$$\omega'(z_{\text{IR}}) = -\frac{1}{2} \frac{\partial f(\omega(z_{\text{IR}}))}{\partial \omega}, \quad (3.26)$$

implying,

$$\hat{\omega} = -\frac{1}{2k\Delta_+} \frac{\partial f(\omega_{\text{IR}})}{\partial \omega} - \frac{\Delta_-}{\Delta_+} \omega_* e^{\Delta_- kz_{\text{IR}}}. \quad (3.27)$$



This also implies that

$$\omega(z_{\text{IR}}) = -\frac{1}{2k\Delta_+} \frac{\partial f}{\partial \omega} - \frac{\Delta_+ - \Delta_-}{\Delta_+} \omega_* e^{\Delta_- k z_{\text{IR}}} \equiv \sigma(\omega_* e^{\Delta_- k z_{\text{IR}}}). \quad (3.28)$$

Identifying the canonical radion,

$$\varphi(x) = f e^{-k(z_{\text{IR}} + r(x) \exp(2kz_{\text{IR}}))}, \quad (3.29)$$

where  $f^2 = \frac{3}{2} M_5^3/k$ , we get the canonically normalized kinetic term for  $\varphi$  from  $\mathcal{L}_{\text{kin}}$ . The potential can be calculated by taking the fluctuation  $r(x) = 0$ , as before. We also identify  $\omega_* e^{\Delta_- k z_{\text{IR}}} = g(\varphi)$  as the running coupling, which is weakly varying ( $\sim \varphi^{-\epsilon}$ ). Thus the effective Lagrangian (ignoring terms suppressed by powers of  $\epsilon$ ,

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{M_5^3 k}{2f^4} \varphi^4 \left[ \frac{1}{16} \left[ \frac{\partial f(\sigma(g(\varphi)))}{\partial g} \right]^2 + f(\sigma(g(\varphi))) \right]. \quad (3.30)$$

This matches the result of [12] for small backreaction. The term in the square brackets is a function  $\kappa(\varphi)$ , which is slowly varying by virtue of the running coupling being near marginal. Thus it is of the desired form in equation (2.3).

We have only considered a mass term for the  $\omega$  field, and adding other potential terms makes it challenging to obtain an analytical solution. The higher order terms in the  $\omega$  potential are subdominant if we additionally assume  $\omega_*$  to be small. That is, we assume that the deformation stabilizes CFT breaking at weak coupling. In this case, we can safely ignore higher order terms in the potential for  $\omega$ , and only keep the  $\epsilon k^2$  mass term. However, as we will see in the next section this approximation can be relaxed while preserving the qualitative mechanism.

The assumptions made in this section highlight the checks we need to perform. Since we have been working to leading order in  $\eta$ , we need to ensure that higher order terms in  $\eta$  do not spoil our mechanism. Crucially, we need the global symmetry of the  $\omega$  field to be only broken by  $\epsilon$ , even at the level of non-renormalizable interactions in 5D. This issue certainly needs addressing in presence of a quantum-gravitational threshold which is expected to generate shift-symmetry violating corrections to the  $\omega$  potential. From the AdS/CFT point of view, it is not clear what this global symmetry in the bulk corresponds to in the CFT. We have also not yet included other higher dimensional operators consistent with the shift symmetry, or studied carefully the effect of quantum corrections. We address these issues next in our 6D construction, where  $\omega$  is identified as the sixth component of 6D gauge field.

#### 4 Six dimensions: Goldberger-Wise field as $A_6$

The solution outlined in the previous section sets the stage for the 6D solution. In the 6D effective field theory, all aspects of our mechanism are robustly treated in the infrared, insensitive to further UV completion. We first consider an action in 6D, and write down an explicit classical solution. This serves to identify the heavy fields which can be integrated

out in the 5D effective theory. As mentioned before, once we ensure that the shift symmetry for Goldberger-Wise field is of high quality, subleading corrections to the simple solution above will not affect the basic mechanism. Our goal in this section will be to show that we obtain the simple 5D theory presented above with only subleading corrections, while focusing on protecting the approximate shift symmetry for GW.

### 4.1 Action

We parametrize the action in six dimensions as

$$\begin{aligned} \frac{S}{M_6^4} = & \int d^4x dz d\vartheta \sqrt{G} \left[ -\frac{1}{4}R + 4k_5^2 + \mathcal{L}_{\text{axion}} + \mathcal{L}_{\text{gauge}} \right] \\ & + \frac{1}{2} \int d^4x dz d\vartheta \sqrt{\bar{g}} \delta(z - z_{\text{IR}}) \mathcal{L}_{\text{brane}}. \end{aligned} \tag{4.1}$$

The 6D radius is stabilized by an axion wrapped around the sixth dimension,

$$\mathcal{L}_{\text{axion}} = \partial_a \chi^\dagger \partial_b \chi G^{ab}. \tag{4.2}$$

We treat the axion in a non-linear sigma model,  $\chi = v e^{i\sigma}$ . The value  $v$  on the brane can differ from the one in bulk in general without affecting our argument. Note that while our simple Lagrangian for the axion respects a global U(1) symmetry, this symmetry is not crucial for our solution and is merely retained for simplicity. In particular, there are no light 5D fields associated with this symmetry (appendix C), and hence U(1) symmetry breaking deformations do not affect the form of the 5D effective action we obtain. The 6D bulk contains a gauge field and a heavy charged scalar

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{mn} F_{ab} G^{ma} G^{nb} + D_a \xi^\dagger D_b \xi G^{ab} - m_\xi^2 \xi^\dagger \xi. \tag{4.3}$$

where  $D_a = \partial_a - ieA_a$ . Note that the gauge coupling has dimension  $[e] = 1$  due to our choice of normalization. The non-local Wilson loop along a curve  $\gamma$  that winds around the compactified sixth dimension,  $\oint_\gamma eA_6$ , matches on to the Goldberger-Wise scalar in 5D. Specifically we identify,

$$\omega(z) = \frac{1}{2\pi\rho(z)e} \oint_\gamma eA_6 = A_6(z). \tag{4.4}$$

Here,  $\rho$  is the radius of the sixth-dimension which has  $z$ -dependence in general, and we are working in the “almost-axial” gauge where  $A_6$  does not depend on  $\vartheta$ . This Aharonov-Bohm phase can only be detected by loops of charged matter that wind around the circle. This follows from the 6D gauge invariance which can be used to set  $A_6$  to zero everywhere except at one point on the circle. The leading dependence in the 6D effective action on the Wilson loop will arise from such loops of the lightest charged particles, and will match on the 5D GW potential for  $\omega$ . In the bulk, the only charged field is a heavy (compared to the 6D KK scale) scalar  $\xi$ . The propagators of  $\xi$  that wind around the sixth dimension are suppressed exponentially by a Yukawa factor (see appendix A),

$$V_{\text{bulk}}(\rho, \omega) \simeq \epsilon M_6^2 \cos(2\pi e\omega), \tag{4.5}$$

where the small parameter  $\epsilon$  is defined as

$$\epsilon = \frac{1}{2\pi\rho M_6^6} \left( \frac{m_\xi}{4\pi^2\rho} \right)^{5/2} e^{-2\pi\rho m_\xi}. \quad (4.6)$$

The action on the IR brane is assumed to contain light fields that appear as SM matter fields in the 4D effective theory. It also contains some light fields charged under the gauge group. The exact nature of the lights fields will not matter, only that they give rise to a generic brane potential for the sixth component,  $A_6$ , of the gauge field. Assuming that these light fields have masses  $m \lesssim 1/\rho(z_{\text{IR}})$ , this potential is unsuppressed. In general, for multiple light fields with different U(1) charges and masses we get a sum of terms, and thus can obtain a generic dependence on  $\omega$  at the brane (see appendix A). The brane potential is given by

$$V_{\text{brane}}(\rho, \omega) = \tau - \frac{\sqrt{2}}{3(2\pi)^3\rho^5 M_6^4} \cos(2\pi\epsilon\omega) + \eta f(v, \rho). \quad (4.7)$$

We note that at this stage upon the 6D compactification we also obtain a 5D gauge field,  $A_M$ . This corresponds to a conserved global current in the dual CFT. Depending on the boundary conditions on the IR brane, this global current may or may not be spontaneously broken. For simplicity, we choose Neumann boundary condition for the gauge field  $A_\mu$ , such that the U(1) global current is conserved. Note that there is no normalizable zero mode for  $A_\mu$  so the low energy 4D effective theory does not have this gauge boson. The zero mode for  $A_5$  (the would-be Goldstone boson if we chose to break the U(1) spontaneously) is projected out by this boundary condition.

## 4.2 Equations of motion

We first assume circular symmetry in the  $S_1$  direction and 4D Poincaré invariance. The ansatz parametrizing this is,

$$ds^2 = e^{2A(z)}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2 - \rho(z)^2 d\vartheta^2, \quad (4.8)$$

$$\chi = ve^{i\vartheta}, \quad \xi = 0, \quad A_M = 0, \quad A_6 = \omega(z). \quad (4.9)$$

The gravity equations of motion (after some algebra) read

$$-\frac{5}{2}A'^2 - A'' + \frac{1}{4}\frac{\omega'^2}{\rho^2} + \frac{v^2}{2\rho^2} - \frac{1}{2}\rho\frac{\partial V}{\partial\rho} + \frac{1}{2}V(\omega, \rho) = \frac{1}{4}\delta(z - z_{\text{IR}}) \left[ V_{\text{brane}} + \rho\frac{\partial V_{\text{brane}}}{\partial\rho} \right], \quad (4.10)$$

$$-\frac{3}{2}A'^2 - \frac{A'\rho'}{\rho} + \frac{1}{4}\frac{\omega'^2}{\rho^2} - \frac{v^2}{2\rho^2} + \frac{1}{2}V(\omega, \rho) = 0, \quad (4.11)$$

$$\frac{\rho''}{\rho} + \frac{4A'\rho'}{\rho} + \frac{3}{2}\frac{\omega'^2}{\rho^2} + \frac{4v^2}{\rho^2} - \frac{3\rho}{2}\frac{\partial V}{\partial\rho} - V(\omega, \rho) = -\frac{1}{4}\delta(z - z_{\text{IR}}) \left[ V_{\text{brane}} - 3\rho\frac{\partial V_{\text{brane}}}{\partial\rho} \right], \quad (4.12)$$

where for brevity we have redefined,

$$(4k_5^2 - V_{\text{bulk}}(\omega, \rho)) = V(\omega, \rho). \quad (4.13)$$

The equation of motion for  $\omega(z)$ ,

$$\omega'' + 4A'\omega' - \frac{\omega'\rho'}{\rho} - \rho^2\frac{\partial V}{\partial\omega} = \frac{\rho^2}{2}\delta(z - z_{\text{IR}})\frac{\partial V_{\text{brane}}}{\partial\omega}. \quad (4.14)$$

### 4.3 Solution at $\epsilon = 0$

We first work in the limit  $V_{\text{bulk}}(\omega, \rho) = 0$ . This is the limit of exact scalar gravity, so we will find a massless scalar graviton solution after tuning the radion potential to zero. We work perturbatively in the backreaction, parametrized by  $\eta$ . We parametrize the solutions as,

$$\rho(z) = \bar{\rho} + \eta\rho_1(z) + \dots \tag{4.15}$$

$$\omega(z) = \bar{\omega} + \eta\omega_1(z) + \dots \tag{4.16}$$

$$A'(z) = -k_5 + \eta A'_1(z) + \dots \tag{4.17}$$

The limit of negligible backreaction requires the brane terms to be tuned such that the geometry does not deviate from AdS all the way to the IR brane. Thus, there exists a solution with  $\omega(z), \rho(z) = \text{const}$ . Matching this solution at the boundary will provide us with the fine tuning we need to perform at zeroth order in  $\eta$ . This ansatz leads to the following equations of motion in the bulk,

$$-\frac{3}{2}A'^2 + \frac{1}{2}V(\bar{\omega}, \bar{\rho}) - \frac{v^2}{2\bar{\rho}^2} = 0, \tag{4.18}$$

$$-\frac{5}{2}A'^2 + \frac{1}{2}V(\bar{\omega}, \bar{\rho}) + \frac{v^2}{2\bar{\rho}^2} = 0, \tag{4.19}$$

$$-V(\bar{\omega}, \bar{\rho}) + \frac{4v^2}{\bar{\rho}^2} = 0. \tag{4.20}$$

The  $\omega(z)$  equation is satisfied trivially for  $\epsilon = 0$  for any constant  $\omega(z)$ .

The solution for  $A'$  and  $\bar{\rho}$  is,

$$A'^2 = k_5^2, \tag{4.21}$$

$$\frac{v^2}{\bar{\rho}^2} = k_5^2. \tag{4.22}$$

We see that this results in an AdS<sub>5</sub> space, and the 6D radius has been stabilized. The boundary conditions are given by,

$$\frac{\partial V_{\text{brane}}(\bar{\omega}, \bar{\rho})}{\partial \omega} = 0, \tag{4.23}$$

$$V_{\text{brane}}(\bar{\omega}, \bar{\rho}) + \bar{\rho} \frac{\partial V_{\text{brane}}}{\partial \rho} = -4k_5, \tag{4.24}$$

$$V_{\text{brane}}(\bar{\omega}, \bar{\rho}) - 3\bar{\rho} \frac{\partial V_{\text{brane}}}{\partial \rho} = 0. \tag{4.25}$$

Since the bulk solution already fixes all integration constants, we see that the boundary conditions are satisfied to this order by a tuning of  $\mathcal{O}(\eta)$ . The brane potential terms need to be tuned in order to be consistent with the ansatz that  $\omega(z)$  and  $\rho(z)$  are constant. The fact that the asymptotic value of the deformation,  $\bar{\omega}$ , is fixed by the junction matching conditions, will persist at ( $\epsilon = 0$ ) at every order in  $\eta$ .

Let us now go on to first order in  $\eta$ . For the first order terms, the GR equations in the bulk are

$$\frac{\rho_1''}{\bar{\rho}} - 4k_5 \frac{\rho_1'}{\bar{\rho}} - \frac{4v^2}{\bar{\rho}^3} \rho_1 = 0, \tag{4.26}$$

$$-A_1''(z) + 5k_5 A_1'(z) - \frac{v^2}{\bar{\rho}^3} \rho_1 = 0, \tag{4.27}$$

$$k_5 \frac{\rho_1'}{\bar{\rho}} + 3k_5 A_1' + \frac{v^2}{\bar{\rho}^3} \rho_1 = 0. \tag{4.28}$$

The matter equation of motion for  $\omega$  is,

$$\omega_1'' - 4k_5 \omega_1' = 0. \tag{4.29}$$

As usual, there is some redundancy in these equations. In particular, the overall constant in  $A_1(z)$  (say  $A_1(0)$ ) is unphysical, and we set it to zero. Equation (4.28) shows that  $A_1'(z)$  is determined algebraically once  $\omega_1(z)$  and  $\rho_1(z)$  are fixed. Thus, there are four unknown constants of integration, two each associated with second order differential equations for  $\rho_1(z)$  and  $\omega_1(z)$ .

The boundary conditions at this order are given by,

$$-\eta \omega_1'(z_{\text{IR}}) = \frac{\bar{\rho}^2}{2} \frac{\partial V_{\text{brane}}}{\partial \omega}, \tag{4.30}$$

$$\eta A_1'(z_{\text{IR}}) = \frac{1}{4} \left[ V_{\text{brane}} + \rho \frac{\partial V_{\text{brane}}}{\partial \rho} \right], \tag{4.31}$$

$$-\eta \rho_1'(z_{\text{IR}}) = -\frac{1}{4} \left[ V_{\text{brane}} - 3\rho \frac{\partial V_{\text{brane}}}{\partial \rho} \right]. \tag{4.32}$$

Recall that the brane terms were tuned to  $\mathcal{O}(\eta)$ , so they appear as generic  $\mathcal{O}(1)$  brane terms at this order, fixing 3 integration constants. The other undetermined constant is fixed by requiring a finite energy solution, which implies that  $\rho(z)|_{z \rightarrow -\infty} = \text{const}$ .

The solutions are of the form,

$$\rho_1(z) = c_1 e^{k_5 \Delta_\rho (z - z_{\text{IR}})}, \tag{4.33}$$

$$\omega_1(z) = c_2 + c_3 e^{4k_5 (z - z_{\text{IR}})}, \tag{4.34}$$

$$A_1'(z) = -c_1 \frac{k_5 (\Delta_\rho + 1)}{3\bar{\rho}} e^{\Delta_\rho (z - z_{\text{IR}})}, \tag{4.35}$$

where  $\Delta_\rho \simeq 2(1 + \sqrt{2})$  is the scaling dimension of the operator corresponding to the  $\rho$  deformation. The three constants  $c_i$  are fixed by the three junction matching conditions. Higher order terms in  $\eta$  all have a similar exponential behavior turning on near the IR boundary. From the holographic dictionary it is clear that the  $z$  dependence of  $\rho$  and  $\omega$  is dual to these deformations picking up vevs after spontaneous symmetry breaking. Note that currently the IR brane position ( $z_{\text{IR}}$ ) is not fixed, since a translation of the brane does not change the asymptotic solution as  $z \rightarrow -\infty$ . This is again consistent with the fact that we only have a spontaneous breaking of the CFT, leading to no potential for the dilaton.

We get a Poincaré invariant solution, so what happened to the quartic term in the dilaton potential? We see that all the undetermined constants are fixed by the boundary conditions. In particular the boundary value,  $\omega_1(z)|_{z \rightarrow -\infty}$  is fixed by the IR boundary conditions. Thus, at every order in  $\eta$ , we need to perform one tuning of the boundary value of  $\omega$  in order to obtain a Poincaré invariant ground state.

#### 4.4 Turning on $\omega$ potential in the bulk

We now take the quantum correction to the potential of  $\omega$  into account generated by the loops of charged matter as in equation (4.5).

Note that the fine tuning in the  $\epsilon = 0$  solution above arises because the IR boundary condition essentially fixes the UV boundary condition (at the AdS boundary, or any other convenient  $z \ll 0$ ) for  $\omega$ . Once we introduce a potential for  $\omega$  in the bulk, there is a slowly varying profile for  $\omega$  in the bulk, effectively scanning over different values of  $\omega(z)$  with varying  $z$ . Thus, we expect the modulus to be stabilized close to where the value of  $\omega(z)$  approaches  $\bar{\omega}$ .

Unlike the 5D example in section 3, an analytical solution is generically not feasible to obtain, even for small backreaction. However, an approximate solution can be obtained using singular perturbation theory [12, 20, 21]. There are two separate qualitative regions, which can be matched at  $\bar{z} \sim z_{\text{IR}} - \log \epsilon$ . For  $z > \bar{z}$ , close to the IR brane, the  $\epsilon$  perturbation is subdominant to the contribution from derivatives of  $\omega, \rho, \pi, A$  etc. Thus, the solution found above applies at leading order in  $\epsilon$ . In particular, this solution requires that  $\omega(z) = \bar{\omega} + \mathcal{O}(\epsilon)$  for  $z \sim \bar{z}$ .

The  $\epsilon = 0$  solutions enter their asymptotic behavior for  $z \lesssim \bar{z}$ . In this asymptotic region, the backreaction from  $\omega$  on the metric is small. In fact, in this region we can robustly say,

$$\rho(z) = \bar{\rho} + \mathcal{O}(\epsilon), \tag{4.36}$$

$$A'(z) = -k_5 + \mathcal{O}(\epsilon). \tag{4.37}$$

Additionally, the variation in  $z$  is controlled by the potential, so that each derivative is suppressed by additional powers of  $\epsilon$ . Thus, the leading order effect of  $\epsilon$  for these fields is to change their matching condition at  $z = \bar{z}$  by  $\epsilon$ .

The dominant effect is of course on the profile of  $\omega$  itself, where it can now slowly evolve to zero at the AdS boundary. Inspecting the equations of motion, we see that to leading order in  $\epsilon$ , the solution in this region is given by a first order differential equation for  $\omega$ .

$$\omega' = -\frac{\bar{\rho}^2}{4k_5} \frac{\partial V}{\partial \omega} = \frac{2\pi e \epsilon}{k_5} \bar{\rho}^2 M_6^2 \sin(2\pi e \omega). \tag{4.38}$$

The above equation is easily solved as is. For the sake of making contact with the discussion in section 3, let us consider the limit  $2\pi e \omega \ll 1$ , so that we can write,

$$\omega' \simeq \frac{4\pi^2 e^2}{k_5} \epsilon \bar{\rho}^2 M_6^2 \omega(z), \quad \Rightarrow \omega(z) = \omega_* e^{\frac{4\pi^2 e^2}{k_5} \bar{\rho}^2 M_6^2 \epsilon z}. \tag{4.39}$$

The IR brane requires a specific value for the asymptotic value of  $\omega$  (say  $\omega(\bar{z}) = \bar{\omega}$ ). We match our solution to this value at  $\bar{z}$ . The matching condition yields,

$$\bar{z} = \frac{k_5}{4\pi^2 e^2 \bar{\rho}^2 M_5^2 \epsilon} \log \frac{\omega_*}{\bar{\omega}}. \tag{4.40}$$

which leads to the familiar result we obtained above,  $\bar{z} \sim \frac{1}{\epsilon}$ . (Note that  $z = 0$  corresponds to the reference scale where our deformation coupling  $\omega_*$  is defined). The brane is stabilized at  $z_{\text{IR}} \sim \bar{z} + \log \epsilon$ . We see that we have now traded the parameter  $\omega_*$  for  $z_{\text{IR}}$ ; we are free to choose any asymptotic value for the deformation (defined at a reference scale), and that fixes the value of  $z_{\text{IR}}$ , the location where the IR brane is stabilized.

The only light fields in the bulk we have are  $\omega$  (see appendix C) and  $A_M$ , in addition to light fields on the brane. Therefore, we can move to a 5D effective description, integrating out physics above the scale  $1/\bar{\rho}$ .

### 4.5 General solution in 5D

In order to make connection with the 5D example earlier, we dimensionally reduce our 6D theory to 5D. The scale  $1/\bar{\rho}$  serves as the heavy threshold. The low energy degrees of freedom are the pseudo-Nambu Goldstone,  $\omega$ , the U(1) gauge field  $A_M$  (which will not play a role here) and other light matter fields on the IR brane. Therefore, the 5D effective Lagrangian looks like,

$$\begin{aligned} \mathcal{L} = M_5^3 \int d^4x dz \sqrt{G} & \left[ -\frac{1}{4}R + 3k_{\text{new}}^2 + \frac{1}{2}\partial_a\omega\partial_b\omega G^{ab} + V(\omega) + \mathcal{L}_{\text{hd}} \right] \\ & + \frac{1}{2}M_5^3 \int d^4x dz \sqrt{g}\delta(z - z_{\text{IR}}) [\tau - V_{\text{brane}}(\omega) + \mathcal{L}_{\text{brane,hd}}], \end{aligned} \tag{4.41}$$

where  $\mathcal{L}_{\text{hd}}$  are higher-dimensional operators, suppressed by the scale  $\sim 1/\bar{\rho}$ . The pNGB nature of  $\omega$  ensures that  $\omega \rightarrow \omega + a$  symmetry is only broken by terms suppressed by  $\epsilon$ . Therefore, for  $\omega = \text{constant}$ , the contributions to the bulk action  $V(\omega) \sim \mathcal{O}(\epsilon)$ , as well as  $\mathcal{L}_{\text{hd}}(\omega) \sim \mathcal{O}(\epsilon)$ . There is no such restriction on the brane terms (except the  $\eta$ -tuning on the brane that ensures we can work in perturbation theory with small backreaction).

Note that this form of the 5D effective action relies only on the symmetries in the 5D theory, and therefore is valid for a general 6D action respecting those symmetries. In particular, the simplifying assumption of a circularly symmetric solution in 6D is not necessary, and adding deformations away from the circular symmetry do not affect the low energy effective theory. This is expected from the fact that there are no light degrees of freedom associated with this circular symmetry (appendix C).

We have already derived the effective potential for the dilaton at leading order in  $\epsilon$  for this Lagrangian in section 3. The only difference in this case is the presence of generic potential and higher dimensional operators. However, these merely correct the form of the leading potential, without affecting the  $\epsilon$ -suppression of the GW field potential, and hence the  $\beta$ -function. Since our 4D solution was general, we conclude that these subleading effects cannot affect the conclusion.



Higher derivative operators need to be treated with some care in the presence of branes [25]. Using equations of motion, bulk higher derivative operators can be cast in a form such that any  $\omega$  only has at most one derivative acting on it. Such operators yield well-defined expressions in perturbation theory in  $\eta$ . The brane terms need classical renormalization, but the stabilization of the 5D radius does not depend on the details of the renormalization.

The radius stabilization calculation then proceeds as before. In particular, as we can see from the 6D solution that one can work perturbatively in  $\eta$ . At  $\epsilon = 0$ , we recover pure AdS solution away from the IR brane, indicating that the CFT is not explicitly broken and the radion potential needs to be fine tuned. This is a consequence of the fact the  $\omega = \text{const.}$  is a solution to the e.o.m in the bulk. Then, a maximally symmetric solution of 5D exists, i.e. the bulk is AdS.

If we turn on  $\epsilon$ , the solution close to the IR brane is still dominated by the  $\epsilon = 0$  solution. The  $\beta$  function depends on the potential, which is always suppressed by  $\epsilon$ . Away from the brane,  $\omega$  evolves slowly since its potential is small. Consequently, higher derivative operators are suppressed with even higher powers of the  $\epsilon$ . Therefore, the solution for  $\omega$  found explicitly in the 6D case holds more generally.

As noted in [12, 20, 25], in the presence of higher dimensional operators, the perturbation theory in  $\eta$  we used to derive the solution is not valid for  $\eta > \epsilon$ . Crucially, one needs to invoke a shift symmetry for the GW field in order to ensure that higher order corrections in  $\eta$  are also higher order in  $\epsilon$ . The general form of the perturbative expansion in 5D can be found in [25], where it was shown in detail that in the presence of a shift symmetry for the GW field, higher order corrections in both  $\epsilon$  and  $\eta$  are subdominant. Therefore, the dilaton potential derived in section 3 is the leading contribution.

## 5 Conclusion and discussion

It is very surprising to find a non-supersymmetric CFT in a spontaneously broken phase, and is usually associated with tuning. This tuning can be identified with a tuning of the cosmological constant in a scalar theory of gravity. While spontaneous breaking is non-generic, if there exists a deformation with specific properties, then it is possible to obtain a low-energy theory that has an approximate scale symmetry. With such an appropriately chosen deformation, the dilaton has an approximate shift symmetry around its minimum. The condition required to achieve a light dilaton are stated most simply in the 5D formulation of the theory. In order to get a light dilaton, one requires an approximate shift symmetry for the stabilizing Goldberger-Wise field, making its potential extremely small. In this paper we provide a partial UV completion to obtain the shift symmetry from a higher dimensional gauge field, which consequently can be completely consistent with quantum gravity expectations. It will be interesting to see if our solution can be embedded in to a full string theory UV completion. This is beyond the scope of the current paper.

It would be interesting to translate our full  $A_6$  mechanism into 4D QFT language, and identify what is the dual to the mechanism that protects the dilaton potential. That could

lead to explicit constructions in 4D language which would be very interesting for study of non-supersymmetric CFTs.

In this paper we have studied the vacuum structure of the scalar gravity cosmological constant problem. A full solution to the problem must also address cosmological evolution including obtaining a hot big bang. We will study this in future work. The interplay of cosmological evolution and naturalness also appears in the relaxation proposal for solving the hierarchy problem [26], although it takes a somewhat different form.

Needless to say, it would be very interesting if some sort of modified spin-2 gravity with very small violations of the equivalence principle could yield a naturally small cosmological constant in a manner analogous to the spin-0 mechanism (references [12, 21] explored ideas in this direction). This will require something more dramatic since Lorentz symmetry guarantees the equivalence principle for a massless spin-2 particle.

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### A Aharonov-Bohm potential

The Casimir energy contribution from a charged boson for a  $d$ -dimensional theory with 1 dimension compactified on a circle of radius  $\rho$  is, (following e.g. [27]),

$$V(A_6) = -2\pi\rho \sum_{n=1}^{\infty} \frac{2m^d}{(2\pi)^{d/2}} \frac{\cos(n\theta)}{(2\pi\rho mn)^{d/2}} K_{d/2}(2\pi\rho mn), \tag{A.1}$$

where  $\theta = \oint_{S_1} eA_6 = 2\pi eA_6$  and  $m$  is the mass of the corresponding charged field. The fermionic contribution has an additional overall negative sign, and anti-periodic boundary conditions yield a  $n$ -dependent negative sign.

For the bulk potential in our case, this becomes

$$V_{\text{bulk}}(A_6) = -\frac{2\pi\rho}{M_6^4} \sum_{n=1}^{\infty} \frac{2m_\xi^6}{(2\pi)^3} \frac{\cos(2\pi neA_6)}{(2\pi\rho m_\xi n)^3} K_3(2\pi\rho m_\xi n), \tag{A.2}$$

where we have accounted for our normalization of the action by  $M_6^4$ . Since we are interested in the limit  $2\pi\rho m_\xi \gg 1$ , we can use the asymptotic behavior of the Bessel function,

$$K_\nu(x) \xrightarrow{x \rightarrow \infty} \sqrt{\frac{\pi}{2x}} e^{-x}. \tag{A.3}$$

Therefore, the dominant contribution comes from  $n = 1$ ,

$$V_{\text{bulk}}(A_6) \simeq \frac{1}{M_6^4} \left( \frac{m_\xi}{4\pi^2\rho} \right)^{5/2} e^{-2\pi\rho m_\xi} \cos(2\pi eA_6). \tag{A.4}$$

Since this is an exponentially small number, one worry is if non-perturbative effects can overcome this suppression. This issue was considered in e.g. [27], with the conclusion that non-perturbative effects are also similarly exponentially suppressed.

For the brane potential, we can approximate the potential in the massless limit  $m\rho \ll 1$ ,

$$V_{\text{brane}} = -\frac{1}{3\sqrt{2}(2\pi)^2\rho^4M_6^4} \sum_k \frac{e^{2\pi i k e A_6}}{k^5} + h.c. \tag{A.5}$$

Ignoring the higher harmonics which are suppressed,

$$V_{\text{brane}} = -\frac{\sqrt{2}}{3(2\pi)^2\rho^4M_6^4} \cos(2\pi e A_6). \tag{A.6}$$

Multiple charged fields will give rise to a sum of such terms, resulting in a generic brane potential.

## B Relevant scales

In this section we outline the (mild) hierarchy of scales that we have assumed. In order to have a well-defined 6D effective field theory, we want the 6D Planck scale  $M_6$  to be somewhat larger than the inverse size of the extra-dimension. Further, since we want a moderately heavy charged particle in the 6D theory  $m_\xi$ , this should be captured within the EFT as well,

$$M_6 > m_\xi > \frac{1}{\bar{\rho}}. \tag{B.1}$$

Similarly, for the 5D effective field theory, we require,

$$M_5 > k_5. \tag{B.2}$$

Thus, the hierarchy of scales is,

$$M_5 > M_6 > m_\xi > \frac{1}{\bar{\rho}} > k_5 \gg M_{\text{pl}} \tag{B.3}$$

where  $M_{\text{pl}}$  is the 4D scalar gravity Planck scale. We can write these inequalities in terms of Lagrangian parameters in the 6D theory,  $\{k_5, v, m_\xi\}$  using

$$\frac{1}{\bar{\rho}} = \frac{k_5}{v} \tag{B.4}$$

$$M_5 = (2\pi\bar{\rho}M_6^4)^{\frac{1}{3}}M_6 = \left(\frac{2\pi v M_6}{k_5}\right)^{\frac{1}{3}}M_6. \tag{B.5}$$

We can check that these inequalities are satisfied for

$$m_\xi < M_6, \quad v < 1, \quad k_5 < \frac{M_6}{\mathcal{N}}, \tag{B.6}$$

where  $\mathcal{N} = \bar{\rho}m_\xi \sim 100$  in order to get  $\epsilon \sim 10^{-120}$ . We see that the 6D theory has only mild hierarchies.

## C Circular symmetry in 6D

In this section we show that the  $U(1)$  symmetry used for simplifying calculations in section 4 does not have any light degrees of freedom associated with it and hence does not appear in the 5D effective theory. Consequently, departure from the circularly symmetric solution in 6D will not modify the general form of the 5D effective theory.

There are potentially two gravitational massless excitations: the Kaluza-Klein  $U(1)$  gauge field associated with the  $S_1$  compactification, and the excitation  $\sigma(x, z, \vartheta) = \vartheta + \tilde{\sigma}(x, z)$ . Clearly, any configuration  $\tilde{\sigma}(x, z)$  can be absorbed by a  $(x, z)$ -dependent coordinate rotation in the sixth dimension  $\vartheta$ . In other words,  $\tilde{\sigma}(x, z) = 0$  defines the unitary gauge condition for the KK  $U(1)$  gauge boson. Thus, KK  $U(1)$  is spontaneously broken, with the gauge boson acquiring a mass. The symmetry breaking pattern is  $U(1)_{\text{KK}} \times U(1)_{\text{global}, \chi} \rightarrow U(1)_{\text{global}, \chi}$ , and hence no massless Goldstone appears.

This can be seen by substituting the following ansatz back into the action,

$$ds^2 = e^{-2k_5 z} dx^\mu dx^\nu \eta_{\mu\nu} - dz^2 - (\rho d\vartheta - \sqrt{2} V_M dx^M)^2, \quad \chi = v e^{i\vartheta}, \quad (\text{C.1})$$

yielding

$$S = M_6^4 \int d^4x dz \sqrt{g} (2\pi\rho) \left[ -\frac{1}{4} R[g] + 4k_5^2 - \frac{1}{4} V_{MN} V_{AB} g^{MA} g^{NB} + \frac{v^2}{\rho^2} (-1 + 2V_A V_B g^{AB}) \right]. \quad (\text{C.2})$$

We see that the KK  $U(1)$  gauge boson does obtain a mass of order the 5D curvature scale.

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