

A hybrid setup for fundamental unknowns in neutrino oscillations using T2HK (ν) and μ -DAR ($\bar{\nu}$)

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ABSTRACT: Neutrino mass hierarchy, CP-violation, and octant of θ_{23} are the fundamental unknowns in neutrino oscillations. In order to address all these three unknowns, we study the physics reach of a setup, where we replace the antineutrino run of T2HK with antineutrinos from muon decay at rest (μ -DAR). This approach has the advantages of having higher statistics in both neutrino and antineutrino modes, and lower beam-on backgrounds for antineutrino run with reduced systematics. We find that a hybrid setup consisting of T2HK (ν) and μ -DAR ($\bar{\nu}$) in conjunction with full exposure from T2K and NO ν A can resolve the issue of mass hierarchy at greater than 3σ C.L. irrespective of the choices of hierarchy, δ_{CP} , and θ_{23} . This hybrid setup can also establish the CP-violation at 5σ C.L. for $\sim 55\%$ choices of δ_{CP} , whereas the same for conventional T2HK ($\nu + \bar{\nu}$) setup along with T2K and NO ν A is around 30%. As far as the octant of θ_{23} is concerned, this hybrid setup can exclude the wrong octant at 5σ C.L. if θ_{23} is at least 3° away from maximal mixing for any δ_{CP} .

KEYWORDS: Neutrino Physics, CP violation

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1 Introduction and motivation

The remarkable discovery of the Higgs boson at the Large Hadron Collider [1, 2] marked the confirmation of the last missing piece of the Standard Model (SM) of particle physics. The SM explains most of the observed fundamental phenomena in particle physics with unprecedented precision. However, there are strong reasons to suspect that this model is not a complete description of Nature. For instance, in the simplest form of the SM, neutrinos are massless fermions. On the other hand, the discovery of neutrino flavor oscillations by the Super-Kamiokande, SNO, and KamLAND experiments [3–5] implies that neutrinos have mass and they mix with each other, providing an evidence for physics beyond the SM. Several different models extending the SM have been suggested in the literature to explain the non-zero neutrino mass and mixing (for recent reviews, see [6–9]). High-precision measurements of the neutrino oscillation parameters can have a major impact on these models and can exclude a large subset of the parameter space of these models, providing crucial guidance towards our understanding of the physics of neutrino masses and mixing [6, 10–14].

In the three-flavor scheme, neutrino oscillation probabilities depend on six fundamental parameters. These are the mixing angles θ_{12} , θ_{13} , θ_{23} , the CP-violating phase δ_{CP} , and the mass-squared differences Δm_{21}^2 and Δm_{31}^2 ($\Delta m_{ij}^2 = m_i^2 - m_j^2$). According to the recent global fit of world neutrino data [15], the solar parameters $\sin^2 \theta_{12}$ and Δm_{21}^2 are known with a relative 1σ precision¹ of 5.8% and 2.3% respectively. The reactor angle $\sin^2 \theta_{13}$ has been measured very accurately with a precision of 4%. As far as the 2–3

¹Here, 1σ accuracy is defined as 1/6 of the $\pm 3\sigma$ range.

mixing angle is concerned, the 3σ allowed range of $\sin^2 \theta_{23}$ is 0.381–0.626, which suggests that θ_{23} can be maximal or non-maximal.² At present, the 3σ allowed range for $|\Delta m_{31}^2|$ is $2.446 \times 10^{-3} \text{ eV}^2 \rightarrow 2.686 \times 10^{-5} \text{ eV}^2$. The atmospheric mass splitting is allowed to be either positive (known as normal hierarchy or NH) or negative (labeled as inverted hierarchy or IH) by the present oscillation data. According to ref. [15], the current oscillation data show an overall preference for NH with respect to IH at the level of $\sim 2\sigma$, which mainly stems from the Super-Kamiokande atmospheric data. The complementarity among the various data sets provided by the accelerator and reactor experiments has already enabled us to probe the parameter space for the δ_{CP} phase [15, 16]. Currently, we have a hint in favor of δ_{CP} around -90° and this trend is getting strengthened day by day. Also, the values of CP phase around $\delta_{\text{CP}} \simeq 90^\circ$ (in the range $\sim 30^\circ$ to 135°) are disfavored at more than 3σ confidence level.

If in Nature, hierarchy is normal and δ_{CP} is around -90° , then the combined data from the presently running off-axis ν_e appearance experiments, T2K [17, 18] and NO ν A [19–21], can exclude the possibility of δ_{CP} being 0° and 180° (CP-conserving choices) at $\sim 2.5\sigma$ C.L. assuming θ_{23} to be maximal [22–29]. Under the same conditions, these experiments can reject the wrong hierarchy at $\sim 3.4\sigma$ confidence level. These experiments can also resolve the octant of θ_{23} at 2σ C.L. provided $\sin^2 \theta_{23} \leq 0.44$ or ≥ 0.57 [30, 31]. Hence, future facilities consisting of intense high-power beams and large smart detectors are inevitable to resolve these fundamental unknowns at high confidence level [32].

The proposed long-baseline neutrino experiment using the Hyper-Kamiokande detector and a powerful neutrino beam from the J-PARC proton synchrotron (commonly known as T2HK experiment) is one of the major candidates in the future neutrino road-map to attain the discovery for the above mentioned issues [33–42]. This superbeam facility heavily relies on the data from both $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ channels to measure CP-violation³ (CPV). However, dealing with the antineutrino run is very challenging for the following reasons. The antineutrino flux is lower compared to the neutrino flux, since the production rate of π^- is less than for π^+ . The charged current (CC) cross-section for antineutrino is lower than for neutrino. These two factors cause a substantial depletion in the event rate for antineutrinos [43]. Due to the larger contamination from wrong sign pions, the background event rates are higher in case of antineutrino run and the systematic uncertainties are also expected to be larger [36].

An elegant way of tackling this issue is to replace the antineutrino run of the superbeam experiment with the antineutrinos from muon decay at rest (μ -DAR) [43–45]. In a stopped pion source, the low energy protons of a few GeV energy are bombarded on a target to produce both π^+ and π^- . Then, with the help of a high-Z beam stop, these pions are brought to rest. The high-Z nuclei help to capture the parent π^- and also the daughter μ^- . The positively charged pions undergo the following cascade of decays at rest to produce

²In case of non-maximal θ_{23} , it can have two solutions: one $< 45^\circ$, termed as lower octant (LO), and other $> 45^\circ$, denoted as higher octant (HO).

³If δ_{CP} differs from both 0° and 180° .

ν_μ , $\bar{\nu}_\mu$, and ν_e :

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ &\quad \downarrow \\ &\quad e^+ + \nu_e + \bar{\nu}_\mu. \end{aligned}$$

Due to oscillation, $\bar{\nu}_\mu$ can transform into $\bar{\nu}_e$ and these low energy electron antineutrinos can be efficiently detected and uniquely identified with the help of well known inverse beta decay (IBD⁴) process in a water Cerenkov detector.

The idea of combining a high energy ν_μ beam from long-baseline superbeam facility with a low energy $\bar{\nu}_\mu$ beam from short-baseline μ -DAR setup to measure θ_{13} , mass hierarchy, and leptonic CPV was first proposed in ref. [43]. The physics reach of similar setups⁵ to measure δ_{CP} was presented in refs. [44, 45]. In this work, we explore the physics reach of a setup, where we replace the antineutrino run of T2HK, with antineutrinos from muon decay at rest (μ -DAR). This setup has the benefits of having higher signal event rates in both neutrino and antineutrino channels, and less beam-on backgrounds for antineutrino signal events with reduced systematic uncertainties. We find that a hybrid setup consisting of neutrino run from T2HK and antineutrino run from μ -DAR in combination with full data sets provided by the T2K and NO ν A experiments can resolve all the three fundamental unknowns, namely, the neutrino mass hierarchy, leptonic CPV, and octant degeneracy of θ_{23} at high confidence level.

This paper is organized as follows. We describe the experimental setups that we have considered in section 2. In section 3, we discuss the parameter degeneracies and show how the combination of neutrino run from T2HK and antineutrino run from μ -DAR can elevate them at the probability and event levels. This is followed by the details of our analysis in section 4. In section 5, we show the results of our simulation and discuss the features of our findings before summarizing in section 6.

2 Description of experimental setups

T2K is a long-baseline experiment with the source of neutrinos at J-PARC in Tokai. The detector is a 22.5 kt water Cerenkov detector at Kamioka, 295 km away from the source. The total beam exposure is 7.8×10^{21} pot (protons on target) which is equally shared among neutrino and antineutrino modes. We use the same configuration of T2K as used in ref. [22]. We have taken an uncorrelated 2% (0.1%) overall normalization error on signal and 5% (0.1%) overall normalization error on background corresponding to appearance (disappearance) channel. The systematic errors for neutrinos and antineutrinos are same.

For T2HK, we use the configuration as described in ref. [34]. We have considered a 560 kt water Cerenkov detector with a total beam strength of 15.6×10^{21} pot either running

⁴At energies below 100 MeV, the IBD cross-section is very high and also well measured. Another important feature of this process is that it provides a very useful delayed coincidence tag between the prompt positron and the delayed neutrino capture.

⁵An entirely μ -DAR based experiment was considered to measure leptonic CPV in ref. [46].

in pure neutrino mode or running in equal neutrino-antineutrino mode.⁶ The systematic errors for T2HK is 3.3% for neutrino mode for both appearance and disappearance channels and 6.2% (4.5%) for appearance (disappearance) channel in antineutrino mode. Systematic errors for signal and background are same. In our analysis, we assume that the total runs of T2HK will be completed in twelve years and the μ -DAR setup will be build after the first six years of operation of T2HK. Thus, the runtime of the μ -DAR setup is taken to be six years. While μ -DAR is running, we assume that the J-PARC beam continues to run in neutrino mode, thus doubling the neutrino exposure.

For DAR, we consider exactly the same configuration as described in ref. [45]. A six year run of μ -DAR is considered which corresponds to a total integrated 1.1×10^{25} pot. The μ -DAR accelerator complex is assumed to be located at a distance of 15 km from the Super-Kamiokande (SK) detector and 23 km from the Hyper-Kamiokande (HK) detector. The main beam-off backgrounds for the μ -DAR setup are invisible muons⁷ and atmospheric IBD events. We have incorporated these backgrounds in our analysis following ref. [45]. A neutrino oscillation experiment with a μ -DAR source has much lower systematic errors, since the flux of neutrinos is exactly known from kinematics and the IBD cross-section is well measured in the MeV range. Here, we assume the systematic errors to be 5% for both signal and background. Energy resolution for this set up is taken as $\delta E/E = 40\% (60\%)/\sqrt{E/\text{MeV}}$ for the SK (HK) detector.

We also consider the prospective data from NO ν A experiment. NO ν A uses the NuMI beam at Fermilab with 6×10^{20} pot/year. We have considered 3 years running in neutrino mode and 3 years running in antineutrino mode. The detector is a 14 kt liquid scintillator detector located 810 km away. We have taken a systematic error of 5% (2.5%) for appearance (disappearance) channel corresponding to signal and 10% for both appearance and disappearance channels in case of background. The systematics error for neutrinos and antineutrinos are same. Ref. [24] gives the detailed experimental specifications for NO ν A that we have used in our simulation. The experimental specifications of all these setups are summarized in table 1.

3 Discussion at the probability and event levels

The $\nu_\mu \rightarrow \nu_e$ oscillation channel is sensitive to all the unknown oscillation parameters. The superbeam experiments T2K, NO ν A, T2HK, DUNE, etc. make use of this channel to determine these parameters. In the presence of matter and keeping terms only up to second order in the small quantities $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$ and $\sin\theta_{13}$, this oscillation probability can

⁶According to the latest report [47], the proposed detector volume for T2HK is 374 kt and total beam exposure is 27×10^{21} pot. Thus, if we compare the exposures in terms of (kt \times pot), then we see that exposures of both these configurations differ only by a factor of 1.2.

⁷The CC interactions of atmospheric muon neutrinos give rise to muons inside the detector. The muons which are below the Cerenkov threshold will not be visible inside the SK and HK detectors and from their decay, we will have electrons or positrons inside the detectors which will constitute the invisible muon background.

Experiment	Detector mass (kt)	Baseline (km)	Total pot	Systematic errors
Conventional T2HK	560	295	$7.8 \times 10^{21} (\nu)$ $+ 7.8 \times 10^{21} (\bar{\nu})$	ν : 3.3% for app and disap, $\bar{\nu}$: 6.2% (4.5%) for app (disap). Errors are same for sg and bg.
T2HK with μ -DAR	560	295	$15.6 \times 10^{21} (\nu)$, No $\bar{\nu}$	Same as above for ν .
μ -DAR	22.5 (SK) + 560 (HK)	15 (SK) + 23 (HK)	$1.1 \times 10^{25} (\bar{\nu})$ (same for both SK and HK)	5% for both sg and bg (same for both SK and HK).
T2K	22.5	295	$3.9 \times 10^{21} (\nu)$ $+ 3.9 \times 10^{21} (\bar{\nu})$	sg: 2% (0.1%) for app (disap), bg: 5% (0.1%) for app (disap). Errors are same for ν and $\bar{\nu}$.
NO ν A	14	810	$1.8 \times 10^{21} (\nu)$ $+ 1.8 \times 10^{21} (\bar{\nu})$	sg: 5% (2.5%) for app (disap), bg: 10% (10%) for app (disap). Errors are same for ν and $\bar{\nu}$.

Table 1. Summary of experimental details assumed in our simulations. Where app = appearance channel, disap = disappearance channel, ν = neutrino, $\bar{\nu}$ = antineutrino, sg = signal and bg = background.

be expressed as [48–51]:

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) = P_{\mu e} = & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \\
 & + \alpha \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta + \delta_{\text{CP}}) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
 & + \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}. \tag{3.1}
 \end{aligned}$$

In the above equation, $\Delta = \Delta m_{31}^2 L/4E$ and $\hat{A} = A/\Delta m_{31}^2$, where $A = 2\sqrt{2}G_F N_e E$ is the matter potential expressed in terms of the electron number density inside the Earth, N_e , E is the neutrino energy, and L is the baseline. The vacuum oscillation probability which is applicable when neutrinos travel through very little or no matter (as in the case of very-short-baselines) can be obtained by taking the limit $A \rightarrow 0$.

Eq. (3.1) is very useful to understand the various features of neutrino oscillations, especially the parameter degeneracies, and the resulting sensitivity of the experiments. For the purposes of this discussion, we neglect the term proportional to α^2 , since it is very small. Under a flip of the mass hierarchy, both α and \hat{A} change their sign. Going from one octant to another affects the $\sin^2 \theta_{23}$ term but not the $\sin^2 2\theta_{23}$ term. Finally, the effect of δ_{CP} is only felt through the term $\cos(\Delta + \delta_{\text{CP}})$. As a direct consequence, we find that the oscillation probability for NH and $\delta_{\text{CP}} = +90^\circ$ can be matched by the probability for IH and $\delta_{\text{CP}} = -90^\circ$, leading to a degeneracy. The other two combinations are free of this hierarchy- δ_{CP} degeneracy. Now, going from neutrino to antineutrino oscillations changes the sign of both \hat{A} and δ_{CP} . Even if one looks at the antineutrino probability, the same

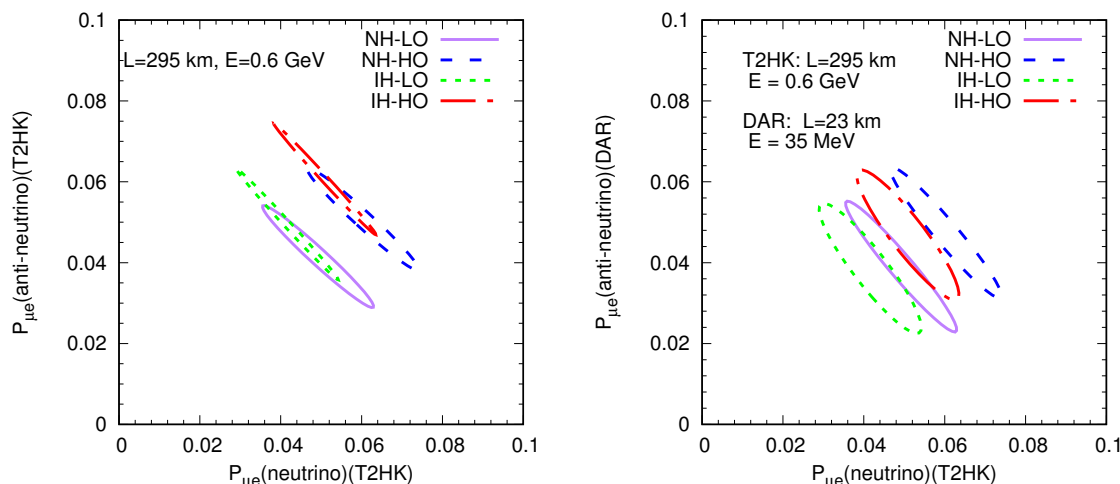


Figure 1. $P_{\mu e}$ bi-probability plots, with neutrinos at T2HK on the x -axis and antineutrinos from T2HK (μ -DAR) on the y -axis in the left (right) panel.

degeneracy remains. Therefore, one does not expect this degeneracy to be lifted by the addition of antineutrino data [23, 24]. Similarly, the probability for NH and LO and can be matched by IH and HO, leading to a degeneracy in the octant measurement. However for antineutrinos, the degeneracy is opposite. Thus a combination of neutrino and antineutrino data can lift the octant degeneracy [30].

In figure 1, we show the effect of degeneracy using the bi-probability plots. In the left panel, for the relevant baseline and peak energy of T2HK, we have the neutrino (antineutrino) probability on the x - (y -) axis. All four combinations of the hierarchy and octant — NH-LO, NH-HO, IH-LO and IH-HO are considered. Here, NH (IH) corresponds to $\Delta m_{31}^2 = +(-)2.4 \times 10^{-3} \text{ eV}^2$ and for LO (HO), we consider $\theta_{23} = 42^\circ(48^\circ)$. As δ_{CP} varies over its full range, the plot gives rise to an ellipse in this plane. It is easy to see the hierarchy- δ_{CP} degeneracy from the overlap of NH and IH ellipses. As the overlap points correspond to same value in both neutrino and antineutrino probabilities, adding antineutrino information from T2HK itself cannot resolve this degeneracy. If we use information from one of the channels between neutrino and antineutrino, then the octant degeneracy arises into the picture. Since, we have informations from both neutrino and antineutrino channels, we can exclude the wrong octant irrespective of the choice of hierarchy [30, 52–54]. In the right panel of figure 1, we have replaced the antineutrino probability of T2HK with that of μ -DAR setup with the relevant baseline and energy.⁸ The overlap between NH and IH ellipses for a given octant is less visible in the right panel as compared to the left panel. It happens due to the following reasons. First of all, the L/E informations are different in μ -DAR ($\bar{\nu}$) and T2HK (ν) setups. Secondly, the CP dependency for antineutrinos in μ -DAR is different as compared to neutrinos in T2HK. When we combine these two

⁸There are two relevant baselines for the μ -DAR setup — 15 km and 23 km. We have chosen the latter baseline in the plot since it corresponds to the HK detector which drives the physics sensitivity by virtue of higher statistics.

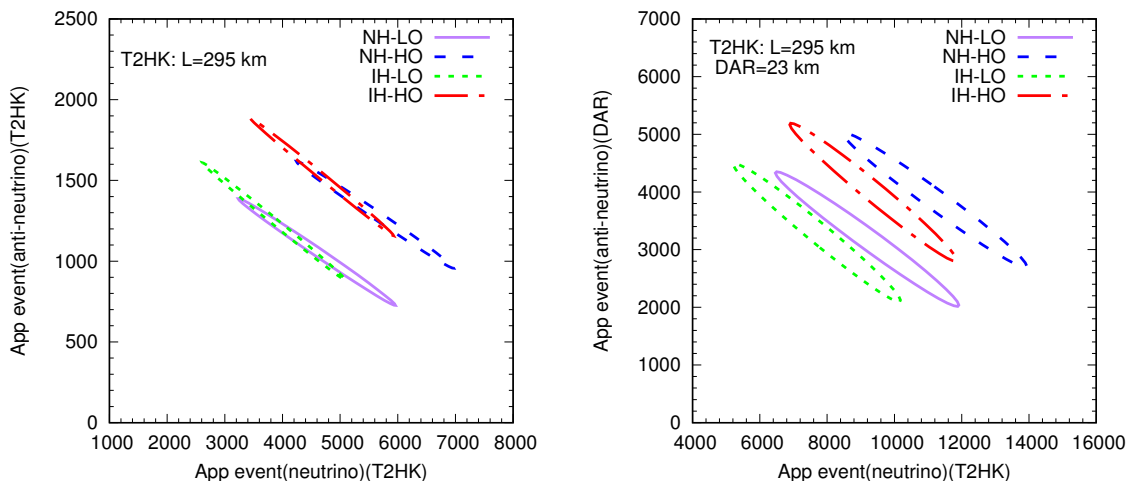


Figure 2. ν_e appearance bi-event plots, with neutrinos at T2HK on the x -axis and antineutrinos from T2HK (μ -DAR) on the y -axis in the left (right) panel. Note that in the hybrid setup, antineutrinos are provided by the μ -DAR source and the full J-PARC beam is run in the neutrino mode. That is why the neutrino events in the right panel are double those in the left panel.

setups, this fact helps to tackle the hierarchy- δ_{CP} degeneracy. But, it is also important to note that the ellipses corresponding to NH-LO and IH-HO combinations are closer in the right panel as compared to the left panel. Thus, we expect that the capability of T2HK (ν) + μ -DAR ($\bar{\nu}$) to resolve these hierarchy-octant solutions will be worse than conventional T2HK ($\nu + \bar{\nu}$).

Figure 2 is similar to figure 1, but at the level of (total) electron/positron appearance event rates. Once again in the left panel, we see the effect of hierarchy- δ_{CP} degeneracy for conventional T2HK. In the right panel, once we replace the antineutrinos of T2HK with antineutrinos from μ -DAR, we find that this degeneracy gets lifted. Note that the scales on the axes in the left and right panels are different. Due to the choice of short baseline and dominant IBD cross section at low energies, the antineutrino event rates are quite high for μ -DAR. Moreover, since T2HK does not need to run in antineutrino mode, it can run in the neutrino mode for the entire period, doubling the neutrino event rate. Consequently, the separation between the ellipses is more for T2HK+ μ -DAR setup.

4 Description of numerical analysis

For our numerical analysis, we use the GLoBES package [55, 56] along with its associated data files [57, 58]. GLoBES⁹ calculates the simulated ‘true’ event rates and the theoretically expected ‘test’ event rates to perform a χ^2 analysis. It takes into account the statistical χ^2 and includes the effect of systematics as well as priors on the parameter values. In our analysis we keep θ_{12} ($\sin^2 \theta_{12} = 0.312$), θ_{13} ($\sin^2 2\theta_{13} = 0.085$), Δm_{21}^2 ($= 7.5 \times 10^{-5} \text{ eV}^2$) and $|\Delta m_{31}^2|$ ($= 0.0024 \text{ eV}^2$) fixed in both the true and test spectra. This is because the

⁹GLoBES estimates the median sensitivity of the experiment without including statistical fluctuations. Thus, what we refer to as χ^2 in this work corresponds to $\Delta\chi^2$ in statistical terminology.

variation of these parameters in their currently allowed range does not affect our results much. Thus, the only relevant neutrino oscillation parameters in our analysis are θ_{23} and δ_{CP} .

Since the value of δ_{CP} in Nature is unknown, a good experimental setup must have the capability to determine the unknown parameters for any possible true value of δ_{CP} . That is why, most of our results are shown as a function of true δ_{CP} . In order to generate these results, we choose a specific value for true θ_{23} and δ_{CP} . These two parameters are allowed to vary in the test range, and the relevant χ^2 is evaluated between the true and test spectra. The minimum χ^2 over the whole range of test parameters is found. This process is then repeated for another value of true δ_{CP} and the corresponding minimum χ^2 is evaluated. In this manner, we construct the plot of χ^2 as a function of true δ_{CP} .

5 Our findings

In this section, we discuss the results of our numerical simulations. We have evaluated the sensitivity of our experimental setups towards determining the neutrino mass hierarchy, detecting CPV and determining the octant of θ_{23} . The discussion in the following three subsections highlight our findings.

5.1 Hierarchy discrimination

In this subsection, we discuss our results for determining the neutrino mass hierarchy. Figure 3 shows the χ^2 for excluding the wrong hierarchy as a function of true δ_{CP} . We assume the true hierarchy to be NH (IH) in the upper (lower) row. The true value of θ_{23} is taken to be $42^\circ/45^\circ/48^\circ$ in the left/middle/right column, while the test value of θ_{23} is allowed to vary in its full 3σ range. In each panel, we see that T2K+T2HK (violet, solid line) has good sensitivity in the favourable region of δ_{CP} , depending on the hierarchy. In fact for the the combination of $\delta_{\text{CP}} = -90^\circ$ and NH which has been hinted at by recent T2K and NO ν A results [59, 60], this setup can achieve well over 5σ exclusion of the wrong hierarchy.¹⁰ Including data from NO ν A (blue, dashed line) improves the sensitivity further. Now, if we replace the antineutrino run of T2HK with μ -DAR (green, dotted line), the following interesting features arise. In the unfavourable range of δ_{CP} , μ -DAR helps to lift the parameter degeneracy, which brings about an improvement in hierarchy exclusion. This is because the degenerate regions in hierarchy- δ_{CP} space for μ -DAR are different from those of T2HK, giving rise to a synergy between these experiments. Figure 3 shows that for the true {hierarchy, δ_{CP} , θ_{23} }-combinations of {NH, -90° , 42° } and {IH, 90° , 48° }, the hierarchy sensitivity is affected due to the octant degeneracy. The antineutrino run of μ -DAR is not sufficient to lift this degeneracy and therefore the sensitivity is lowered. Adding information from NO ν A (red, dot-dashed line) increases the sensitivity further. Finally, in all the panels, the hierarchy sensitivity for the hybrid setup, T2HK(ν)+ μ -DAR($\bar{\nu}$)+T2K+NO ν A is seen to be greater than 3σ irrespective of the true choices of hierarchy, δ_{CP} and θ_{23} .

¹⁰Here we use the ‘conventional’ terminology $n\sigma$ for statistical significance, where $n = \sqrt{\chi^2}$. For a detailed discussion on the statistical interpretation of oscillation experiments, we refer the reader to refs. [61–64].

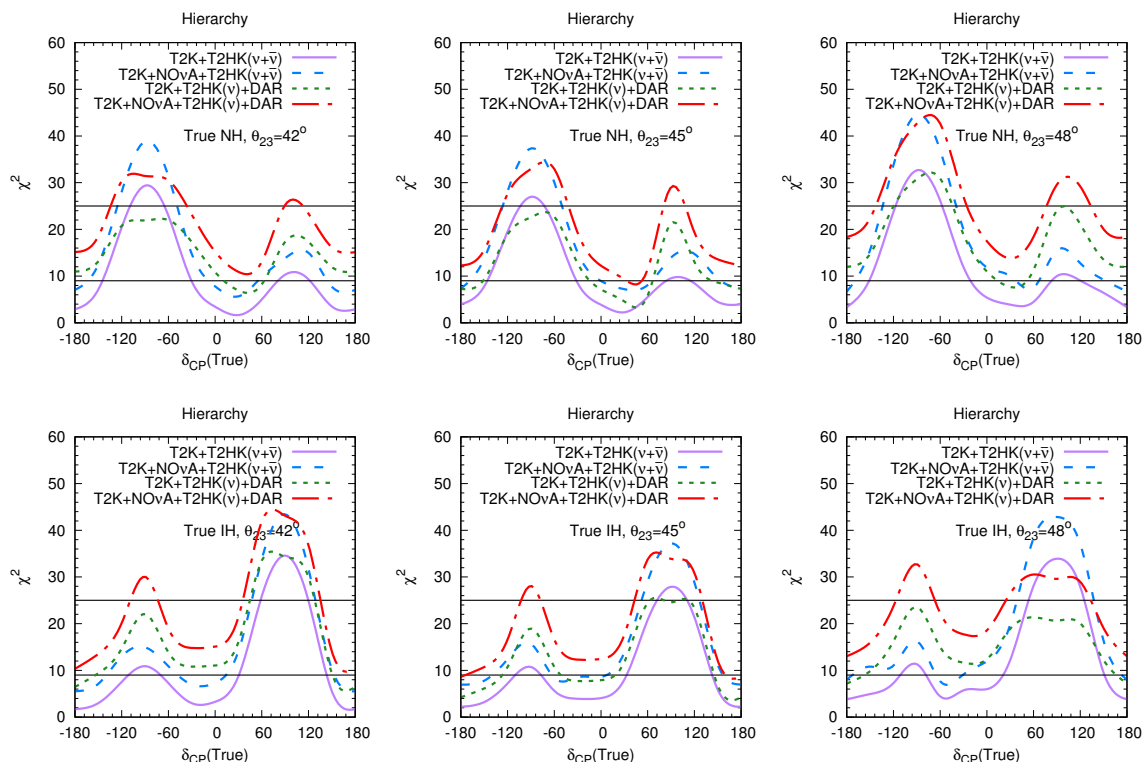


Figure 3. Exclusion of the wrong hierarchy, as a function of true δ_{CP} . The true hierarchy is NH (IH) in the upper (lower) row. The true value of θ_{23} is taken to be $42^\circ/45^\circ/48^\circ$ in the left/middle/right column. We assume that the octant is unknown, i.e. the test value of θ_{23} varies in both octants.

In order to explore the effect of octant degeneracy on our hierarchy results, we show in figure 4 the hierarchy exclusion results assuming the octant of θ_{23} is known. The two panels in this figure correspond to the true combinations NH-LO and IH-HO. Since the octant is assumed to be known, the test value of θ_{23} is only varied in the corresponding octant. Since the octant degeneracy is no longer relevant, we see that our hybrid setup $\text{T2HK}(\nu)+\mu\text{-DAR}(\bar{\nu})+\text{T2K}+\text{NO}\nu\text{A}$ gives better results than $\text{T2K}+\text{T2HK}$, as expected.

5.2 CP-violation discovery

Next, we discuss the CPV discovery sensitivity of the setups. In figure 5, we show the ability of the various setups to detect CPV in the neutrino sector. CP symmetry is said to be violated if the true value of δ_{CP} in Nature is different from 0° and 180° . Thus, the sensitivity of these setups is evaluated for various values of true δ_{CP} in the range $[-180^\circ, 180^\circ)$, while the test values of δ_{CP} are 0° and 180° . We select the minimum $\Delta\chi^2$ out of these two test choices. The six panels of this figure correspond to the same true hierarchy and θ_{23} values as in figure 3.

Once again, we see that depending on the true hierarchy, there are favourable and unfavourable regions of δ_{CP} for all the setups. Comparing the purple (solid) and green (dashed) lines, we can clearly see a substantial improvement in the sensitivity when the

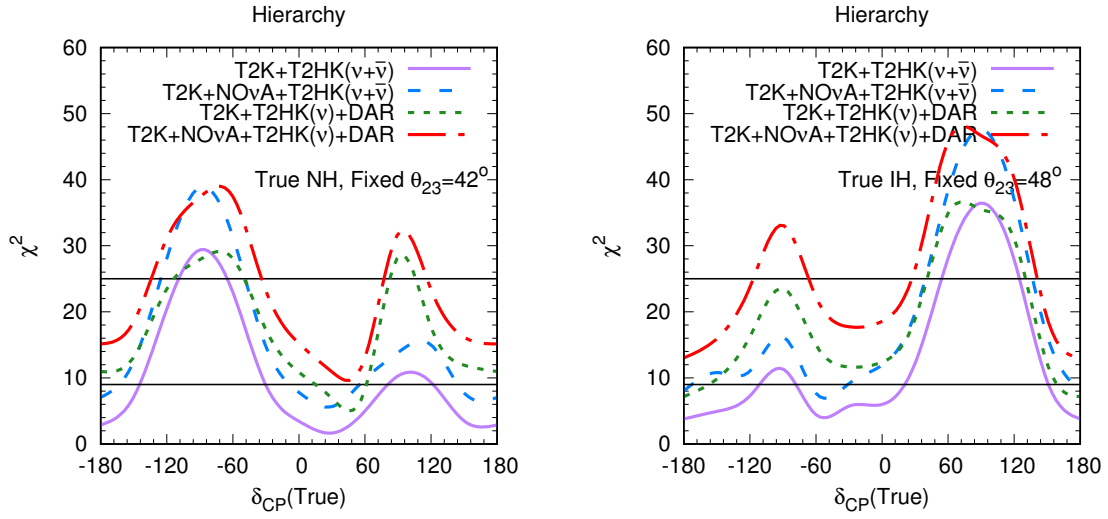


Figure 4. Exclusion of the wrong hierarchy, as a function of true δ_{CP} . The true hierarchy and θ_{23} are specified in the panels. We assume that the octant is known, i.e. the test value of θ_{23} is varied within the same octant as the true value.

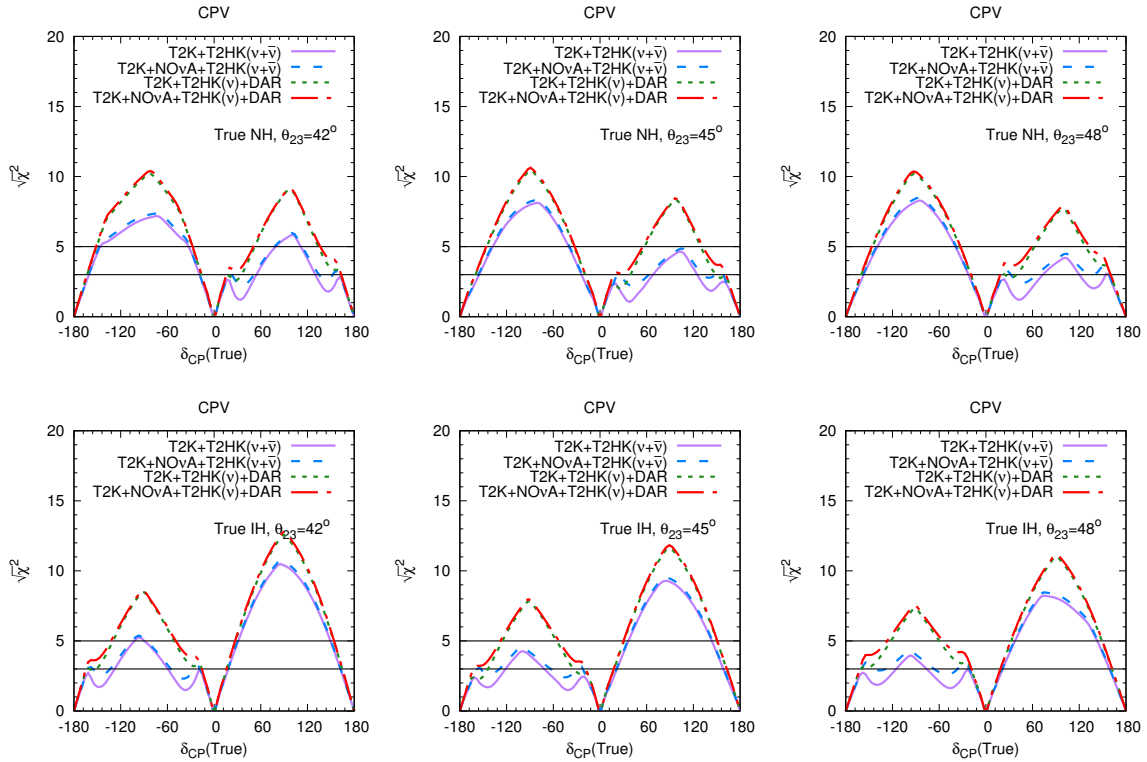


Figure 5. Exclusion of the CP conserving cases $\delta_{CP} = 0^\circ$ and $\delta_{CP} = 180^\circ$, as a function of true δ_{CP} . The true hierarchy is NH (IH) in the upper (lower) row. The true value of θ_{23} is taken to be $42^\circ/45^\circ/48^\circ$ in the left/middle/right column.

antineutrino run of T2HK is replaced by antineutrinos from μ -DAR, and the entire runtime of T2HK is dedicated to the neutrino run. This holds for all possible combinations¹¹ of the true hierarchy, octant and δ_{CP} . This improvement mainly stems from the following reasons: (a) the neutrino statistics for T2HK(ν) combined with antineutrinos from μ -DAR is twice compared to stand-alone T2HK($\nu + \bar{\nu}$) setup, (b) the antineutrino event rate for μ -DAR is larger compared to antineutrinos coming from the T2HK($\nu + \bar{\nu}$) setup, (c) the antineutrinos from μ -DAR are essentially free from any beam-related background, whereas the antineutrino beam in T2HK has substantial intrinsic beam contamination coming from wrong-sign mesons, and (d) for energies below 100 MeV, IBD provides the largest cross-section and comes with a useful delayed coincidence tag between the prompt positron and delayed neutron capture which helps in identifying $\bar{\nu}_e$ cleanly with high background rejection efficiency. Thus, antineutrinos from μ -DAR along with neutrinos from T2HK help to increase the fraction of δ_{CP} values for which CPV can be detected at a given confidence level. Comparing the green (dotted) and red (dot-dashed) lines in figure 5, we see that adding NO ν A as a part of the hybrid setup doesn't help much due to its low statistics compared to T2HK.

5.3 Resolution of θ_{23} octant

Finally, we discuss the ability of our setups to determine the octant of θ_{23} . In figure 6, we show the octant sensitivity as a function of the true value of δ_{CP} considering four different true hierarchy- θ_{23} combinations. Here, since we are interested in excluding the wrong octant, the test value of θ_{23} is varied only in the opposite octant compared to the true choice. We vary test δ_{CP} freely throughout its entire range, and also consider both hierarchies in the test spectrum, i.e. we assume that the hierarchy is unknown in the fit. For true NH-HO (top right panel) and IH-LO (bottom left panel), replacing the antineutrino run of T2HK (purple, solid line) by antineutrinos from μ -DAR (green, dotted line) increases the χ^2 for octant determination. Addition of data from NO ν A helps to improve the situation further, as in the case of the hybrid setup (red, dot-dashed line).

But the situation is quite different in the top left and bottom right panels. In these panels, the hierarchy- δ_{CP} -octant degeneracy crops up in the picture, as discussed in section 3 and antineutrinos from the μ -DAR source do not have enough matter effect to lift this degeneracy by determining the correct hierarchy. If we fix the hierarchy in the fit, this deterioration in the sensitivity does not occur, as demonstrated in figure 7 for the unfavourable hierarchy-octant combinations. Once again, the hybrid setup (red, dot-dashed line) is seen to outperform all the other setups if the hierarchy is known.

Now, it would be interesting to see how the octant sensitivity of these setups varies with the different true choices of θ_{23} . We present these results in figure 8 assuming δ_{CP} true = -90° , which is close to the best-fit value as hinted by present world neutrino data [15, 16]. If the hierarchy is unknown (known), the top (bottom) panels of this figure portray the range of true θ_{23} for which the wrong octant can be excluded at a given confidence level.

¹¹Note that unlike hierarchy and octant, CP sensitivity does not require matter effects. The lack of matter effect for the μ -DAR setup along with its intrinsic sensitivity to δ_{CP} helps in measuring CPV.

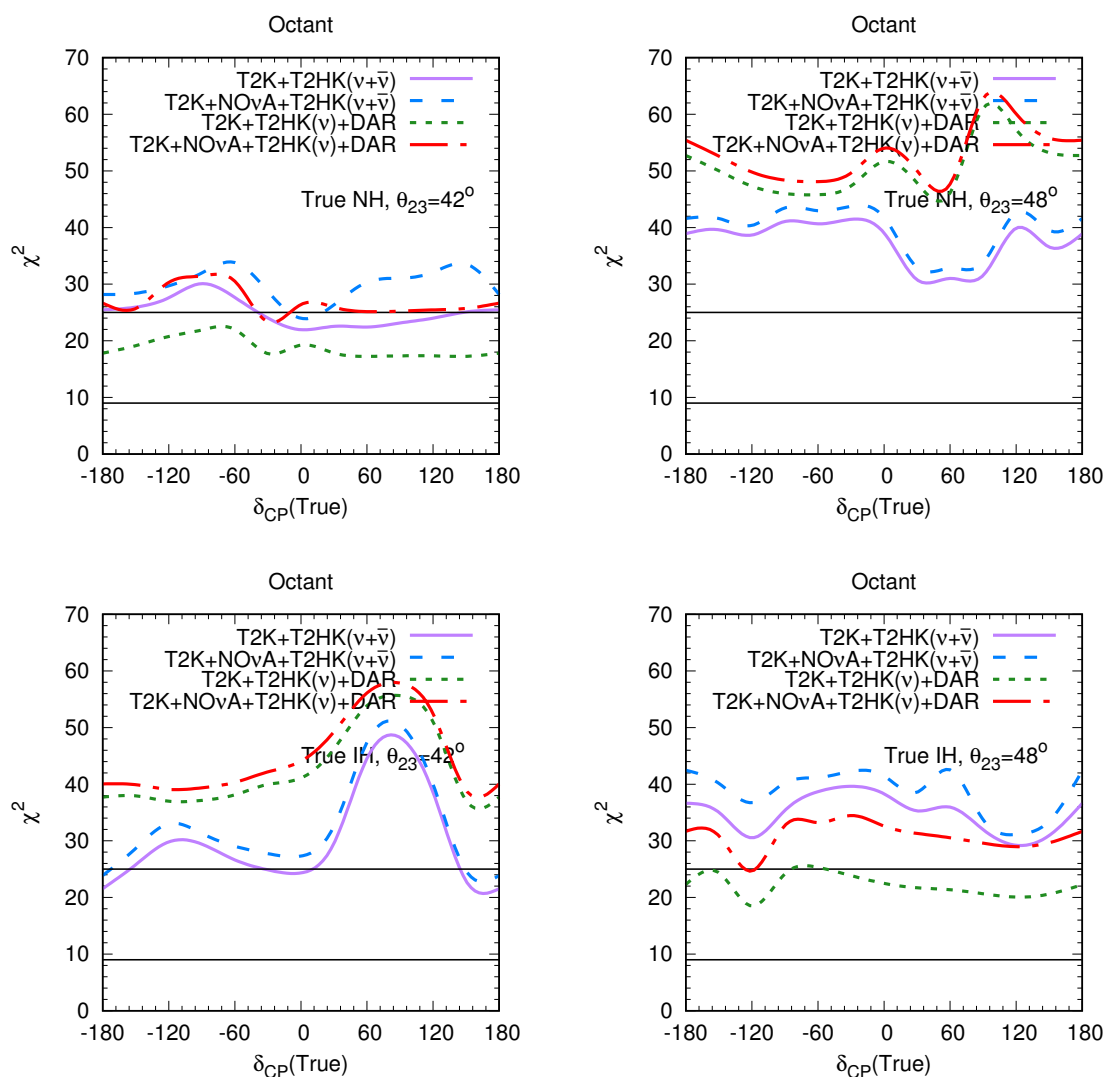


Figure 6. Exclusion of the wrong octant, as a function of true δ_{CP} . The true hierarchy is NH (IH) in the top (bottom) row. The true value of θ_{23} is taken to be 42° (48°) in the left (right) column. The true hierarchy is NH (IH) in the left (right) column. In all four panels, we assume that the hierarchy is unknown, i.e. the test hierarchy can be either NH or IH.

All the four panels suggest that these setups can resolve the octant of θ_{23} at 5σ confidence level as long as true θ_{23} is at least 3° away from maximal mixing. Again, the bottom panels confirm the fact that if the hierarchy is known, the hybrid setup provides the best octant sensitivity for all the true choices of true θ_{23} . Note, we have explicitly checked that at 5σ C.L., the ranges of true θ_{23} for which the octant can be resolved remain almost the same for various possible choices of true δ_{CP} .

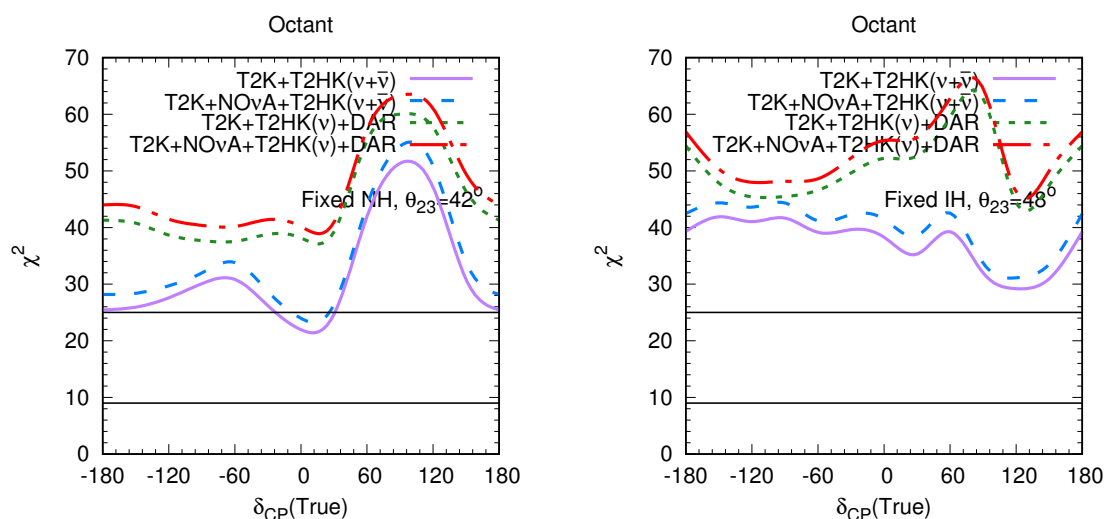


Figure 7. Exclusion of the wrong octant, as a function of true δ_{CP} . The true hierarchy and θ_{23} are specified in the panels. In both panels, we assume that the hierarchy is known, i.e. the test and true hierarchies are the same.

6 Conclusions and outlook

We can explain all the current neutrino data¹² quite successfully in terms of three-flavor neutrino oscillations and can identify the remaining fundamental unknowns, in particular, the type of the neutrino mass hierarchy, the possible presence of a CP-violating phase, and octant ambiguity of 2–3 mixing angle. In this paper, we have shown that a combination of low energy $\bar{\nu}_\mu$ from muon decay at rest (μ -DAR) facility and high energy ν_μ from T2HK superbeam experiment observed at the same Hyper-Kamiokande detector can address all these three pressing issues at high confidence level. This novel idea of replacing the antineutrino run of T2HK with antineutrinos from μ -DAR yields higher statistics in both neutrino and antineutrino modes, reduces the beam-on backgrounds for antineutrino events significantly, and also curtails the systematic uncertainties. The low energy muon antineutrinos from short-baseline μ -DAR setup oscillate into electron antineutrinos which can be efficiently detected and uniquely identified using the dominant and well known IBD process in water Cerenkov detector.

Our simulation shows that a hybrid setup consisting of T2HK (ν) and μ -DAR ($\bar{\nu}$) in conjunction with full exposure from T2K and NO ν A can settle the issue of mass hierarchy at greater than 3σ C.L. irrespective of the choices of hierarchy, δ_{CP} , and θ_{23} . This hybrid setup provides a vastly improved discovery reach for CPV. Using this hybrid setup, we can confirm the CPV at 5σ C.L. for almost 55% choices of true δ_{CP} assuming true NH and maximal mixing for true θ_{23} , whereas the same for conventional T2HK ($\nu + \bar{\nu}$) setup along

¹²We have obtained few anomalous results at very-short-baseline experiments which seem to point towards high $\Delta m^2 \sim 0.1\text{--}10\text{ eV}^2$ oscillations, involving a hypothetical fourth mass eigenstate, which must be essentially sterile without having any coupling to W and Z bosons [65–68].

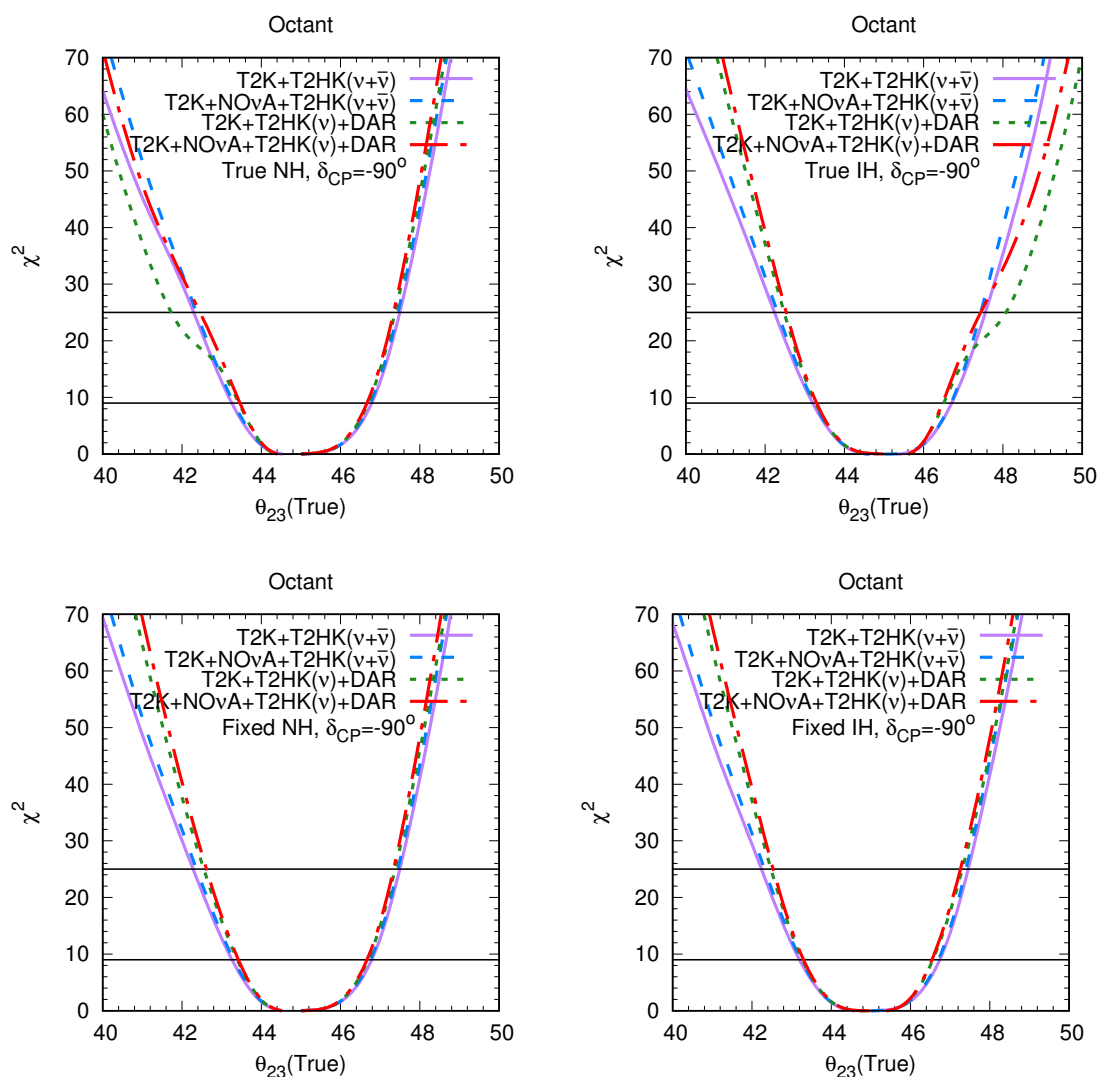


Figure 8. Exclusion of the wrong octant, as a function of true θ_{23} , for $\delta_{CP} = -90^\circ$. The true hierarchy is NH (IH) in the left (right) column. We assume that the hierarchy is unknown (known) in the top (bottom) panels.

with T2K and $\text{NO}\nu\text{A}$ is around 30%. The octant resolution capability of this hybrid setup is also quite good. This hybrid setup can reject the wrong octant at 5σ C.L. if θ_{23} is at least 3° away from maximal mixing for any δ_{CP} . However, the practicality of this hybrid setup relies critically on the ongoing feasibility study of low-cost, high-intensity stopped pion sources. We hope that the results presented in this paper in favor of this hybrid setup will boost these R&D activities and play an important role in optimizing the configuration of the proposed T2HK experiment to have the best possible sensitivity towards determining *all* the remaining unknown neutrino oscillation parameters.

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