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Resonant CP violation in rare $\tau^{\pm}$ decays

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Abstract: In this work, we study the lepton number violating tau decays via intermediate on-shell Majorana neutrinos N_j into two scalar mesons and a lepton $\tau^{\pm} \to M_1{}^{\pm} N_j \to M_1^{\pm} M_2^{\pm} \ell^{\mp}$. We calculate the Branching ratios $Br(\tau^{\pm})$ and the CP asymmetry $(\Gamma(\tau^{+}) - \Gamma(\tau^{-}))/(\Gamma(\tau^{+}) + \Gamma(\tau^{-}))$ for such decays, in a scenario that contains at least two heavy Majorana neutrinos. The results show that the CP asymmetry is small, but becomes comparable with the branching ratio $\mathrm{Br}(\tau^{\pm})$ when their mass difference is similar with their decay width $\Delta M_N \sim \Gamma_N$. We also present regions of the heavy-light neutrino mixing elements, in which the CP asymmetry could be explored in future tau factories.

Keywords: CP violation, Neutrino Physics, Beyond Standard Model

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1 Introduction

During the last decades, neutrino experiments that have shown that neutrinos have non-zero masses [1, 2], also suggest that the first three mass eigenstates are very light with masses $\sim 1\,\mathrm{eV}$, and the mixing between flavour and mass eigenstates is characterized by the Pontecorvo-Maki-Nakagawa-Sakata Matrix, U_{PMNS} [3]. Therefore, if these light masses are produced by means of some see-saw mechanism [4, 5], the existence of one or more heavier neutrinos is needed. The current experimental uncertainties in the B_{PMNS} matrix elements allow introduce these new heavy neutral leptons called sterile neutrinos (SN) [6–10], however the small values of these uncertainties imply a strongly suppressed interaction between standard model (SM) particles and SN. In addition, due to the fact that neutrinos are massive particles, a fundamental question arises: are neutrinos Dirac or Majorana particles?, If neutrinos are Dirac particles, the reactions in which they participate must preserve the lepton number ($\Delta L = 0$). On the contrary, if neutrinos are Majorana particles, they are indistinguishable from their antiparticles, and the lepton number can be violated in two units ($\Delta L = 2$). On the other hand, Neutrino oscillations (NOs) experiments have confirmed that θ_{13} angle of B_{PMNS} is non zero [11, 12], thus, the possibility of CP violation in the light neutrino sector is still open; nevertheless, extra sources of CP violation are needed in order to explain Baryogenesis via Leptogenesis [13]. Recent studies explored the CP violation and the phenomenology of SN neutrinos in the context of rare meson decays [14–21], however, in this work we will focus in the phenomenology of the rare tau decays in the framework of tau factories, such as Super Charm-Tau Factory (CTF) in the Budker Institute of Nuclear Physics (Novosibirsk, Russia), [22, 23] making it possible

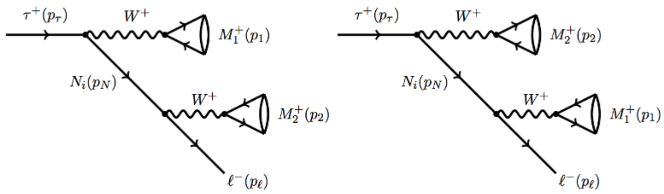

Figure 1. Feynmann diagrams for the process $\tau^+ \to M_1^+ M_2^+ \ell^-$. Left side: direct channel D. Right side: crossed channel C.

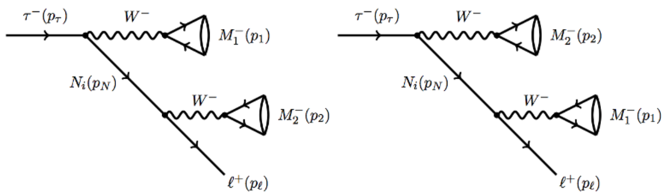

Figure 2. Feynmann diagrams for the process $\tau^- \to M_1^- M_2^- \ell^+$. Left side: direct channel D. Right side: crossed channel C.

to extend the SN searches to tau decay processes. In this letter we focus in the rare decays of tau leptons into two scalar mesons and one charged lepton ($\ell = e, \mu$), via two on-shell intermediate neutrinos N_j, and look for the possibility of detection of CP asymmetries in such decays. The relevant processes are the lepton number violating channels $\tau^\pm \to M_1^\pm M_2^\pm \ell^\mp$ where M_1,M_2= π,K and $\ell = e$,μ. We also show that the branching ratios are very small,[1] but could be appreciable enough and could be measured in future τ factories where huge numbers of taus will be produced [23, 24], if the heavy-light neutrino mixing elements are sufficiently large but still lower than the present upper bounds.

The program of this paper is the following: in section 2 we present the notation and formalism for the rare tau decay; in sections 3 we present the relevant expression for the branching ratio calculations; in sections 4 we present the relevant expression for the CP asymmetries calculations; in sections 5 we present the results of the relevant parameters for the future searches; finally, in section 6 we present the summary and conclusions.

2 Process and formalism

As we stated above, we are interested in studying the $\Delta L = 2$ rare tau decays mediated by two on-shell heavy ($0.140 \leq M_N \leq 1.638\,\text{GeV}$) Majorana neutrinos with the expectation of obtaining CP violating signal in the neutrino sector. The relevant Feynman diagrams of the studied processes are presented in figure 1 and figure 2 for $\tau^+ \to M_1^+ M_2^+ \ell^-$ and $\tau^- \to M_1^- M_2^- \ell^+$, respectively.

[1] Both the branching ratio as CP asymmetries are proportional to the product of square mixing elements $|B_{\tau N}|^2 |B_{\ell N}|^2$.

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In order to write down the amplitude and all the relevant quantities, we first define the neutrino flavor state as:

$$\nu_\ell = \sum_{i=1}^{3} B_{\ell i}\nu_i + \sum_{j=1}^{n} B_{\ell N_j} N_j \,, \tag{2.1}$$

where $B_{\ell N_j}$ are the elements of the $PMNS$ matrix[2] (heavy-light neutrino mixings elements) which are define as follow

$$B_{\ell N_j} = |B_{\ell N_j}| e^{i\phi_{\ell N_j}} \,, \tag{2.2}$$

the left side of eq. (2.1) stand for light neutrino sector and the right side for the heavy neutrino sector. The amplitude for a general process involving n sterile neutrinos is[3]

$$\begin{aligned} i\mathcal{M}_+ &\equiv i\mathcal{M}(\tau^+ \to M_1^+ M_2^+ \ell^-) = \mathcal{M}_+^D + \mathcal{M}_+^C \\ &= \underbrace{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B^*_{\tau N_j} P_j(D)\, \not{L}_+^D}_{\mathcal{M}_+^D} + \underbrace{G_F^2 f_{M_1} f_{M_2} V_{M_1} V_{M_2} B_{\ell N_j} B^*_{\tau N_j} P_j(C)\, \not{L}_+^C}_{\mathcal{M}_+^C} \,, \end{aligned} \tag{2.3a}$$

$$\begin{aligned} i\mathcal{M}_- &\equiv i\mathcal{M}(\tau^- \to M_1^- M_2^- \ell^+) = \mathcal{M}_-^D + \mathcal{M}_-^C \\ &= \underbrace{G_F^2 f_{M_1} f_{M_2} V^*_{M_1} V^*_{M_2} B^*_{\ell N_j} B_{\tau N_j} P_j(D)\, \not{L}_-^D}_{\mathcal{M}_-^D} + \underbrace{G_F^2 f_{M_1} f_{M_2} V^*_{M_1} V^*_{M_2} B^*_{\ell N_j} B_{\tau N_j} P_j(C) \not{L}_-^C}_{\mathcal{M}_-^C} \,, \end{aligned} \tag{2.3b}$$

where f_1 and f_2 are the meson decay constants of $M_1^\pm$ and $M_2^\pm$, and V_{M_1}, V_{M_2} are the mixings elements of CKM matrix corresponding to mesons M_1 and M_2, respectively. The factors $\not{L}_\pm^D$ and $\not{L}_\pm^C$ contain the information related to the kinematics and are given by

$$\not{L}_+^D = \bar{u}(p_\ell) \not{p}_2 \not{p}_1 P_j(D)(1+\gamma_5)u(p_\tau) \,; \quad \not{L}_+^C = \bar{u}(p_\ell) \not{p}_1 \not{p}_2 P_j(C)(1+\gamma_5)u(p_\tau) \,, \tag{2.4}$$

$$\not{L}_-^D = \bar{u}(p_\ell) \not{p}_2 \not{p}_1 P_j(D)(1+\gamma_5)u(p_\tau) \,; \quad \not{L}_-^C = \bar{u}(p_\ell) \not{p}_1 \not{p}_2 P_j(C)(1+\gamma_5)u(p_\tau) \,, \tag{2.5}$$

and finally the factors $P_j(D)$ and $P_j(C)$ are the heavy Majorana neutrino propagators

$$P_j(D) = \sum_{j=1}^{n} \frac{M_{N_j}}{(p_\tau - p_1)^2 - M_{N_j}^2 + i\Gamma_{N_j} M_{Nj}} \,; \quad P_j(C) = \sum_{j=1}^{n} \frac{M_{N_j}}{(p_\tau - p_2)^2 - M_{N_j}^2 + i\Gamma_{N_j} M_{Nj}} \,, \tag{2.6}$$

here Γ_{N_j} is the total decay width of the intermediate neutrinos, and can be approximated as follow

$$\Gamma_{N_j} \approx \mathcal{K}_j^{Ma} \, \frac{G_F^2 M_{N_j}^5}{96\pi^3} \,, \tag{2.7}$$

where

$$\mathcal{K}_j^{Ma} \equiv \mathcal{K}_j(M_{N_j}) = \mathcal{N}_{ej}\, |B_{eN_j}|^2 + \mathcal{N}_{\mu j}\, |B_{\mu N_j}|^2 + \mathcal{N}_{\tau j}\, |B_{\tau N_j}|^2 \,, \tag{2.8}$$

the factors $\mathcal{N}_{\ell j}$ being effective mixing coefficients and are presented in figure 3 for our mass range of interest.

[2] Experimental limits for $|B_{\ell N_j}|^2$ in our mass range of interest are presented in figure figure 7.

[3] The definitions $\mathcal{M}_\pm^D$ and $\mathcal{M}_\pm^C$ can be understood as the amplitude for the direct channel and for the crossed one, respectively. Furthermore, the squared amplitude probability for the process will be $|\mathcal{M}_\pm|^2 = |\mathcal{M}_\pm^D|^2 + |\mathcal{M}_\pm^C|^2 + \mathcal{M}_\pm^D \mathcal{M}_\pm^{C\dagger} + \mathcal{M}_\pm^{D\dagger} \mathcal{M}_\pm^C$.

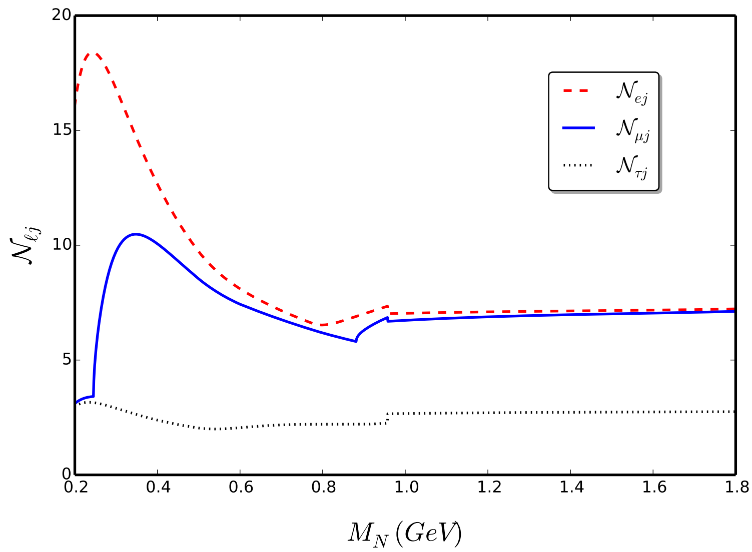

Figure 3. Effective mixing coefficients. The dashed line (online red) is for $\mathcal{N}_{ej}$, solid line (online blue) for $\mathcal{N}_{\mu j}$ and the dotted one (online black) for $\mathcal{N}_{\tau j}$.

The decay with of the process is given as follow

$$\Gamma(\tau^\pm \to M_1^\pm M_2^\pm \ell^\mp) \equiv \Gamma(\tau^\pm) = \frac{1}{2!}(2-\delta_{M_1 M_2})\frac{1}{2M_\tau}\int \overline{|\mathcal{M}_\pm|^2}\, d_3 \ , \tag{2.9}$$

where $\frac{1}{2!}(2-\delta_{M_1 M_2})$ is the symmetry factor that counts for identical particles in the final states, d_3 denotes the number of states available per unit of energy in the 3-body final state.[4]

$$d_3 \equiv \frac{d^3\vec{p}_1}{2E_1(\vec{p}_1)}\frac{d^3\vec{p}_2}{2E_2(\vec{p}_2)}\frac{d^3\vec{p}_\ell}{2E_\ell(\vec{p}_\ell)}\delta^{(4)}\left(p_\tau - p_1 - p_2 - p_\ell\right) \ , \tag{2.10}$$

here, p_1 and p_2 denote the momenta of M_1 and M_2 respectively, and p_ℓ the momentum of the charged lepton (see figure 1 and figure 2).

3 Branching ratio of $\tau^\pm \to M_1^\pm N_j \to M_1^\pm M_2^\pm \ell^\mp$ decays

In a scenario with $n = 2$ sterile neutrinos, the decay widths presented in eq. (2.9) can be written as the double sum of the contributions of N_i and N_j $(i, j = 1, 2)$, with the mixing elements factored out

$$\Gamma(\tau^\pm) = \frac{1}{2!}(2-\delta_{M_1 M_2})\sum_{i=1}^{2}\sum_{j=1}^{2} k_i^{(\pm)} k_j^{(\pm)*} \times \left[\widetilde{\Gamma}_\tau (DD^*)_{ij} + \widetilde{\Gamma}_\tau (CC^*)_{ij} + \widetilde{\Gamma}_{\tau\pm}(DC^*)_{ij} + \widetilde{\Gamma}_{\tau\pm}(CD^*)_{ij}\right] , \tag{3.1}$$

here $\widetilde{\Gamma}$'s are the canonical decay widths (without heavy-light explicit mixing), and $k_j^{(\pm)}$ are parameters which contain the corresponding mixing factors and are presented in eq. (3.2).

$$k_j^{(+)} = B_{\ell N_j} B^*_{\tau N_j} \ , \qquad k_j^{(-)} = (k_j^{(+)})^* \ . \tag{3.2}$$

[4]The decomposition of the 3-body phase space is presented in appendix B.

Due to the fact that $|\not{L}_+^D|^2 = |\not{L}_-^D|^2$ and $|\not{L}_+^C|^2 = |\not{L}_-^C|^2$, we can omit the subscripts $\pm$ in the contribution terms $\widetilde{\Gamma}_\tau(DD^*)_{ij}$ and $\widetilde{\Gamma}_\tau(CC^*)_{ij}$ in eq. (3.1). The canonical decay widths $\widetilde{\Gamma}_{\tau\pm}(XY^*)_{ij}$, where X, Y stand for direct and crossed channel $(X, Y = C, D)$ and $(i, j = 1, 2)$, are given by

$$\widetilde{\Gamma}_{\tau\pm}(XY^*)_{ij} \equiv K_\tau^2 \, \frac{1}{2M_\tau} \int d_3 \; P_i(X) P_j(Y)^* \not{L}_\pm^X \not{L}_\pm^{Y\dagger} \,, \tag{3.3}$$

where

$$K_\tau^2 = G_F^4 f_{M_1}^2 f_{M_2}^2 V_{M_1}^2 V_{M_2}^2 \; . \tag{3.4}$$

From now on, we will pay our attention in a scenario where both mesons are equal, then $M_1 = M_2 \equiv M_M$ and the constant $K_\tau^2 \equiv K_M^2$ presented in eq. (3.4) becomes $K_\pi^2 = G_F^4 f_\pi^4 V_{u\bar{d}}^4$ when the mesons are pions and $K_K^2 = G_F^4 f_K^4 V_{u\bar{s}}^4$ when they are kaons. The canonical decay width has been evaluated numerically by means of *Monte-Carlo* integrations using *Vegas* algorithm [25].[5] Furthermore, the evaluation were implemented using small $\Gamma_{N_j} = 10^{-3}$ in the heavy neutrino propagators. The numerical results can be summarized as follows:

i) The contribution of $(DD^*)_{jj}$ and $(CC^*)_{jj}$ channels are approximately equal, thus $\widetilde{\Gamma}_\tau(DD^*)_{jj} \approx \widetilde{\Gamma}_\tau(CC^*)_{jj}$.

ii) The contribution of $(DC^*)_{ij}$ and $(CD^*)_{ij}$ channels are approximately equal, thus $\widetilde{\Gamma}_\tau(DC^*)_{ij} \approx \widetilde{\Gamma}_\tau(CD^*)_{ij}$.

iii) The terms $\widetilde{\Gamma}_\tau(DD^*)_{jj} \propto 1/\Gamma_{N_j}$,[6] while $\widetilde{\Gamma}_\tau(DC^*)_{jj}$ and $\widetilde{\Gamma}_\tau(DC^*)_{ij}$ are approximately independent of Γ_{N_j}.

iv) When $\Gamma_{N_i} = 10^{-3}$, the terms $\widetilde{\Gamma}_{\tau\pm}(DC^*)_{ii}$ and $\widetilde{\Gamma}_{\tau\pm}(CD^*)_{ii}$ are suppressed by a factor $\sim 10^{-3}$, besides taking into account the latter point iii), the terms $\widetilde{\Gamma}_{\tau\pm}(DC^*)_{jj}$ and $\widetilde{\Gamma}_{\tau\pm}(DC^*)_{ij}$ are negligible in all cases, in comparison with $\widetilde{\Gamma}_\tau(DD^*)_{jj}$ and $\widetilde{\Gamma}_\tau(CC^*)_{jj}$.

v) The contribution of $(DD^*)_{ij}$ and $(CC^*)_{ij}$ channels are approximately equal, and can reach the same order of magnitude than the $(DD^*)_{jj}$ and $(CC^*)_{jj}$ contributions.[7]

[5] The integration were performed in two different languages *Pyhton* and *Fortran* in order to reduce the uncertainties.

[6] It is important to note that the dependence $\widetilde{\Gamma}_\tau(DD^*)_{jj} \propto 1/\Gamma_{N_j}$ is in agreement with the fact that sterile neutrino are weakly interacting particles and therefore the narrow width approximation $\frac{M_{N_j}}{(p_N^2 - M_{N_j}^2)^2 + (M_{N_j}\Gamma_{N_j})^2} \to \frac{\pi}{\Gamma_{N_j}} \, \delta(p_N^2 - M_{N_j}^2)$ is valid.

[7] The effect of this kind of interference will be studied later in detail.

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Thus, under the above considerations and taking into account that $M_1 = M_2 = M_\pi, M_K$, we rewrite the eq. (3.1) only in terms of the dominant contributions, as follows

$$\Gamma(\tau^\pm) = \frac{1}{2!}\sum_{i=1}^{2}\sum_{j=1}^{2} k_i^{(\pm)} k_j^{(\pm)*} \times \left[\widetilde{\Gamma}_\tau(DD^*)_{ij} + \widetilde{\Gamma}_\tau(CC^*)_{ij}\right] \tag{3.5a}$$

$$\begin{aligned} &= |B_{\ell N_1}|^2 |B_{\tau N_1}|^2 \widetilde{\Gamma}_\tau(DD^*)_{11} + |B_{\ell N_2}|^2 |B_{\tau N_2}|^2 \widetilde{\Gamma}_\tau(DD^*)_{22} \\ &\quad + 2|B_{\ell N_1}||B_{\ell N_2}||B_{\tau N_1}||B_{\tau N_2}|\widetilde{\Gamma}_\tau(DD^*)_{11}\cos(\theta_{12})\delta_{12}\,, \\ &\quad \mp 2|B_{\ell N_1}||B_{\ell N_2}||B_{\tau N_1}||B_{\tau N_2}|\widetilde{\Gamma}_\tau(DD^*)_{11}\frac{\eta(y)}{y}\sin(\theta_{12})\,, \end{aligned} \tag{3.5b}$$

here $\delta_{12} \equiv \frac{\Re\left[\widetilde{\Gamma}_\tau(DD^*)_{12}\right]}{\widetilde{\Gamma}_\tau(DD^*)_{11}}$ measures the effect of $N_1 - N_2$ overlap,[8] the factor $\frac{\eta(y)}{y}$ will be discussed later, however, their values are presented in figure 4 and $\theta_{12} = \phi_{\ell N_1} - \phi_{\ell N_2} + \phi_{\tau N_2} - \phi_{\tau N_1}$. The diagonal canonical decay widths, presented in eq. (3.5b), can be implemented by means of the narrow width approximation

$$\widetilde{\Gamma}_\tau(DD^*)_{jj} = \frac{K_M^2}{128\pi^2 M_\tau^3 M_{N_j}\Gamma_{N_j}} \times \lambda^{1/2}\left(1, \frac{M_\ell^2}{M_N^2}, \frac{M_M^2}{M_N^2}\right) \times Z(M_\tau, M_{N_j}, M_M, M_\ell)\,, \tag{3.6}$$

where the functions $Z(a,b,c,d)$ and $\lambda(x,y,z)$ are kinematical functions, which are defined in appendix B. The branching ratio for the process $\tau^\pm \to M_1^\pm M_2^\pm \ell^\mp$ is

$$Br(\tau^\pm) = \frac{\Gamma(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})}\,, \tag{3.7}$$

where $\Gamma(\tau^\pm \to \text{all})$ is the total decay width for $\tau^\pm$ lepton and is given by

$$\Gamma(\tau^\pm \to \text{all}) = \frac{\mathrm{G_F^2 M_\tau^5}}{192\pi^3}\,. \tag{3.8}$$

In order to have a more realistic discussion, we must consider the acceptance factor, which is defined as the probability of the neutrino N_j decay inside of a detector of length L

$$P_{N_j} \approx \frac{L}{\gamma_{N_j}\tau_{N_j}\beta_{N_j}} \approx \frac{L\,\Gamma_{N_j}}{\gamma_{N_j}\beta_{N_j}} \tag{3.9}$$

where γ_{N_j} is the Lorentz time dilation factor in the Laboratory frame and β is the neutrino speed.[9] Therefore, the effective branching ratio[10] is

$$\mathrm{Br}^{\text{eff}}(\tau^\pm) = P_{N_j}\mathrm{Br}(\tau^\pm) = \frac{\Gamma^{\text{eff}}(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})} = P_{N_j}\frac{\Gamma(\tau^\pm)}{\Gamma(\tau^\pm \to \text{all})}\,. \tag{3.10}$$

[8] $\Re$ stand for the real part.

[9] In this work, we will provide $\gamma_{N_j} \sim 2$, $\beta \sim 1$ and $L = 1$ mts.

[10] The $Br^{\text{eff}}(\tau^\pm)$ correspond to the real branching ratio, while $\Gamma^{\text{eff}}(\tau^\pm)$ correspond to the effective decay with, whose can be measured in an experiment.

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4 CP asymmetry of $\tau^\pm \to M_1^\pm N_j \to M_1^\pm M_2^\pm \ell^\mp$ decays

In this section we will calculate the size of CP asymmetry A_{CP}, which is defined as follows

$$A_{CP} = \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{\Gamma(\tau^+) + \Gamma(\tau^-)}\,, \tag{4.1}$$

The CP violation comes from the complex phases in the transition amplitudes eq. (2.3a), and the observable effects only arise due to interference of at least two amplitudes. The CP-odd phases are those that come from the Lagrangian of the theory, in other words from the heavy-light mixing elements ($B_{\ell N}$); these phases change sign between a process and its conjugate. On the other hand, the CP-even phases appear as absorptive parts in the propagators eq. (2.6) and do not change sign for the conjugate process. In order to have a more phenomenological discussion about CP violation, it is useful define a new quantity $A_{CP}Br^{\text{eff}}(\tau^+)$ which is the corresponding branching ratio for the CP-violating asymmetry[11]

$$A_{CP}\ Br^{\text{eff}}(\tau^+) = \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{\Gamma(\tau^+) + \Gamma(\tau^-)}\, Br^{\text{eff}}(\tau^+) \approx P_{N_j}\, \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{2\Gamma(\tau^+ \to all)} \tag{4.2}$$

The CP-violating difference $\Gamma(\tau^+) - \Gamma(\tau^-)$ is proportional to the imaginary part of $\widetilde{\Gamma}_\tau(DD^*)_{12}$ and can be written as[12]

$$\Gamma(\tau^+) - \Gamma(\tau^-) \approx 4|B_{\ell N_1}||B_{\ell N_2}||B_{\tau N_1}||B_{\tau N_2}|\ \sin\theta_{12}\ \Im\Big[\widetilde{\Gamma}_\tau(DD^*)_{12}\Big] \tag{4.3}$$

where we have neglected all the (DC^*) and (CD^*) interference contributions , due to fact that numerical simulation shows that they are strongly suppressed in comparison with (DD^*) and (CC^*). The imaginary part of eq. (4.3) correspond to the imaginary part of the off-diagonal elements in eq. (3.5)

$$\Im\Big[\widetilde{\Gamma}_\tau(DD^*)_{12}\Big] = \frac{1}{2M_\tau}\int d_3\ \Im\Big[P_1(D)P_2(D)^*\Big]|\not{L}_+^D|^2\,. \tag{4.4}$$

The imaginary part of the product of propagators (see eq. (A.7b) in appendix. A) can be expressed using the narrow width approximation as

$$\begin{aligned}
\text{Im}\,(P_1(D)P_2(D)^*) &= \frac{(p_N^2 - M_{N_1}^2)\,\Gamma_{N_2}M_{N_2} - \Gamma_{N_1}M_{N_1}\,(p_N^2 - M_{N_2}^2)}{\Big[\Big(p_N^2 - M_{N_1}^2\Big)^2 + \Gamma_{N_1}^2 M_{N_1}^2\Big]\Big[\Big(p_N^2 - M_{N_2}^2\Big)^2 + \Gamma_{N_2}^2 M_{N_2}^2\Big]} &\text{(4.5a)}\\
&\approx \frac{\pi}{M_{N_2}^2 - M_{N_1}^2}\,\big[\delta(p_N^2 - M_{N_2}^2) + \delta(p_N^2 - M_{N_1}^2)\big]\ ; &\text{(4.5b)}
\end{aligned}$$

the validity of eq. (4.5b) strongly depends on the assumption $\Gamma_{N_j} \ll |\Delta M_N| \equiv M_{N_2} - M_{N_1}$. However, it is useful introduce the parameter $\eta(y)$ where $y \equiv \frac{\Delta M_N}{\Gamma_N} = \frac{\Delta M_N}{\frac{1}{2}(\Gamma_{N_1}+\Gamma_{N_2})}$, which

[11] In eq. (4.2) we have used $\Gamma(\tau^+) + \Gamma(\tau^-) \approx 2\Gamma(\tau^+)$.

[12] Here we assumed the fact that $\Im\Big[\widetilde{\Gamma}_\tau(DD^*)_{12}\Big] \approx \Im\Big[\widetilde{\Gamma}_\tau(CC^*)_{12}\Big]$.

parametrizes any deviation of eq. (4.5a) when $\Gamma_{N_j} \not\ll |\Delta M_N|$

$$\eta(y) = \frac{\Im\left[\widetilde{\Gamma}_\tau (DD^*)_{12}\right]_{\rm NWA}}{\Im\left[\widetilde{\Gamma}_\tau (DD^*)_{12}\right]_{\rm NUM}} \tag{4.6}$$

In eq. (4.6) the subscripts NWA and NUM stand for "Narrow Width Approximation" and "Numerical", respectively. The values of $\eta(y)$ were evaluated numerically using finite ΔM_N and their values are presented in figure 4 as a function of $y \equiv \Delta M_N/\Gamma_N$. The general expression of eq. (4.4) including the $\eta(y)$ parameter and under the assumptions $M_{N_1} + M_{N_2} \approx 2M_N$ is given by[13]

$$\Im\left[\widetilde{\Gamma}_\tau (DD^*)_{12}\right] \approx \eta(y)\, \frac{K_M^2}{128\pi^2 M_\tau^3 M_N \Delta M_N} \times \lambda^{1/2}\left(1, \frac{M_\ell^2}{M_N^2}, \frac{M_M^2}{M_N^2}\right) \times Z(M_\tau, M_N, M_M, M_\ell) \tag{4.7}$$

finally, the CP-violating difference becomes

$$\Gamma(\tau^+) - \Gamma(\tau^-) \approx \eta(y)\, \frac{K_M^2 |B_{\ell N_1}||B_{\ell N_2}||B_{\tau N_1}||B_{\tau N_2}|}{32\pi^2 M_\tau^3 M_N \Delta M_N} \sin\theta_{12} \times \lambda^{1/2}\left(1, \frac{M_\ell^2}{M_N^2}, \frac{M_M^2}{M_N^2}\right) \times Z(M_\tau, M_N, M_M, M_\ell) \tag{4.8}$$

From eq. (4.1), eq. (4.8) and figure 4 we can conclude that the best scenario for simultaneous maximization of ACP and $Br(\tau)$, occurs when $y = 1$. From now on, we will focus in a scenario where heavy neutrinos are almost degenerate $\Delta M_N \sim \Gamma_N$; within this context we have assumed $|B_{\ell N_1}| \approx |B_{\ell N_2}| \equiv |B_{\ell N}|$, where $\ell = e, \mu, \tau$ and the mixing elements are $\mathcal{K}_1^{Ma} \approx \mathcal{K}_2^{Ma} \equiv \mathcal{K}^{Ma}$, therefore, the CP asymmetry becomes

$$A_{CP} \approx \eta(y)\, \frac{\Gamma_N}{\Delta M_N}\, \frac{\sin\theta_{12}}{1 + \delta_{12}\cos\theta_{12}} \equiv \frac{\eta(y)}{y}\, \frac{\sin\theta_{12}}{1 + \delta_{12}\cos\theta_{12}}\,, \tag{4.9}$$

consequently

$$A_{CP}\; Br^{\rm eff}(\tau^+) \approx \frac{\eta(y)}{y}\, \frac{L}{\gamma_N}\, |B_{\ell N}|^2 |B_{\tau N}|^2 \sin\theta_{12}\, \frac{3\pi K_M^2}{2G_F^2 M_\tau^8 M_N} \times \lambda^{1/2}\left(1, \frac{M_\ell^2}{M_N^2}, \frac{M_M^2}{M_N^2}\right) \times Z(M_\tau, M_N, M_M, M_\ell)\,. \tag{4.10}$$

There is just one caveat in the expressions above: we have disregarded the effect of $N_1 - N_2$ oscillation, these type of oscillations have been studied in detail in ref. [19] and it

[13]Due to the fact that $\Gamma_N \sim \mathcal{K}_j^{Ma} \sim |B_{\ell N}|^2$ the mass difference becomes $\Delta M_N \ll 1$, hence the assumption $M_{N_1} + M_{N_2} \approx 2M_N$ is reasonable In eq. (4.7).

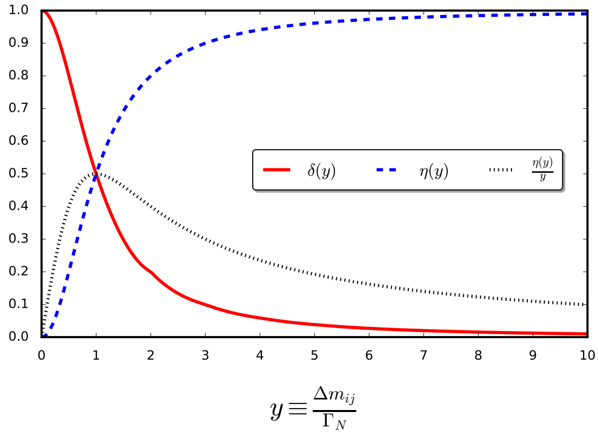

Figure 4. Solid line (online red) overlap function δ_{12}. Dashed line (online blue) $\eta(y)$ function. Dotted line (online black) $\eta(y)/y$ function.

is straightforward to show that the L dependent effective differential decay width is[14]

$$\frac{d}{dL}\Gamma^{(\mathrm{osc})}_{\mathrm{eff}}(\tau^+ \to \pi^+\pi^+\mu^-; L) \approx \frac{1}{\gamma_N \beta_N}\overline{\Gamma}(\tau^+ \to \pi^+ N)\overline{\Gamma}(N \to \pi^+\mu^-)$$
$$\times \left\{ \sum_{j=1}^{2} |B_{\mu N_j}|^2 |B_{\tau N_j}|^2 + 2|B_{\mu N_1}||B_{\tau N_1}||B_{\mu N_2}||B_{\tau N_2}| \cos\left(L\frac{\Delta M_N}{\beta_N \gamma_N} + \theta_{12}\right) \right\} \tag{4.11}$$

where $\overline{\Gamma}(\tau^+ \to \pi^+ N)$ and $\overline{\Gamma}(N \to \pi^+\mu^-)$ are kinematical functions presented in appendix A. In eq. (4.11) it is also possible to notice that the oscillation length is $L_{\mathrm{osc}} = \frac{2\pi\beta_N\gamma_N}{\Delta M_N}$. Then, the argument of cosine in eq. (4.11) can be written as $2\pi\frac{L}{L_{\mathrm{osc}}} + \theta_{12}$, therefore, in order to integrate out there are two possible scenarios:

1. $L \gg L_{\mathrm{osc}}$: in this regime we recover the main contributions of the L-independent effective decay width (eq. (3.10)), because the oscillation term $\sim \cos\big(f(L) + \theta_{12}\big)$ gives a relatively negligible contribution when integrated over several L_{osc}.

2. $L \nsim L_{\mathrm{osc}}$: in this scenario the integration of expression (4.11) is

$$\Gamma^{(\mathrm{osc})}_{\mathrm{eff}}(\tau^+ \to \pi^+\pi^+\mu^-; L) \approx \frac{L}{\gamma_N \beta_N}\overline{\Gamma}(\tau^+ \to \pi^+ N)\overline{\Gamma}(N \to \pi^+\mu^-) \times \left[\sum_{j=1}^{2} |B_{\mu N_j}|^2 |B_{\tau N_j}|^2 \right.$$
$$\left. + \frac{L_{\mathrm{osc}}}{\pi L}|B_{\mu N_1}||B_{\tau N_1}||B_{\mu N_2}||B_{\tau N_2}| \left(\sin\left(2\pi\frac{L}{L_{\mathrm{osc}}} + \theta_{12}\right) - \sin\left(\theta_{12}\right)\right)\right], \tag{4.12}$$

in (4.12) we can see, immediately, that when $L_{\mathrm{osc}} \gg L$ and $L_{\mathrm{osc}} = L$ the oscillation effect disappear and we recover the L-independent main contributions of the eq. (3.10). On the other hand, when $L \sim L_{\mathrm{osc}}$ neutrinos have traveled enough to

[14]In eq. (4.11) L is the distance between production vertex and detector; the quantities γ_N and β_N are: $\gamma_N = \frac{1}{2}(\gamma_{N_1} + \gamma_{N_2})$ and $\beta_N + \frac{1}{2}(\beta_{N_1} + \beta_{N_2})$, respectively.

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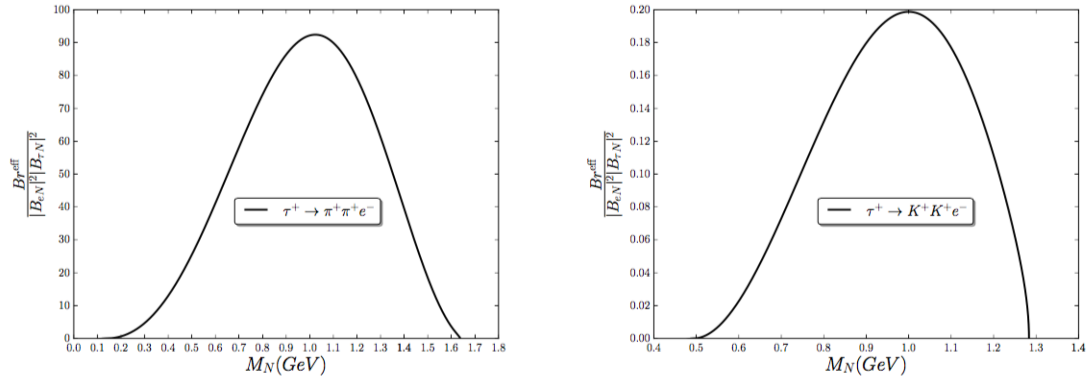

Figure 5. Effective branching ratios per unit of $|B_{eN}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\cos\theta_{12} = 1/\sqrt{2}$, overlap factor $\delta_{12} = 0.5$, detector length $L = 1$ mts, neutrino speed $\beta = 1$ and Lorentz factor $\gamma_N = 2$.

have a well-defined oscillation, which means that neutrinos have not decayed yet (i.e. $P_N \ll 1$). Moreover, $L \sim L_{\rm osc}$ means $y \equiv \frac{\Delta M_N}{\Gamma_N} \approx \frac{2\pi}{P_N} \gg 1$ and then from figure 4 we notice that $y \gg 1$ destroy the effect of resonant CP violation. Therefore, the fact that disregard the $N_1 - N_2$ oscillation when we have chosen $\eta(y) \sim 1$ is valid.

It is important to note that the oscillation effect is present when $L \sim L_{\rm osc}$, therefore, in general CP violating scenarios (i.e. when we are off CP resonant region) this must be taken into account.

5 Results

In this section the main results obtained in this work will be applied in order to provide a clue for future searches in tau factories. The result for the effective branching ratios presented in eq. (3.10) are shown in figure 5 and figure 6.

The difference between the cases with $M_M = \pi$ and $M_M = K$ in the final states is mainly due to the elements of CKM matrix, whereas for pions $V_\pi \approx 0.97$ and $V_K \approx 0.22$, respectively. Moreover, the values of meson decay constant are $f_\pi \approx 0.13\,\text{GeV}$ and $f_K \approx 0.15\,\text{GeV}$, therefore $K_\pi^2/K_K^2 \approx 2 \times 10^2$. In order to estimate the region of heavy-light mixings elements $|B_{\ell N}|^2|B_{\tau N}|^2$ which can be explored in future experiment[15] we define the following relation

$$A_{CP} Br^{\rm eff}(\tau^+) \times N_\tau \geq 1 \quad \Rightarrow \quad |B_{\ell N}|^2|B_{\tau N}|^2 \geq \frac{\gamma_N}{L N_\tau \sin\theta_{12} \overline{S}(M_N)}\,, \tag{5.1}$$

here N_τ is the number of τ lepton produced in an experiment and $\overline{S}(M_N)$ is given by

$$\overline{S}(M_N) = \frac{3\pi K_M^2}{4 G_F M_\tau^8 M_N}\, \lambda^{1/2}\left(1, \frac{M_\ell^2}{M_N^2}, \frac{M_M^2}{M_N^2}\right) \times Z(M_\tau, M_N, M_M, M_\ell)\,. \tag{5.2}$$

[15]The eq. (5.1) is presented in order to detect at least 1 event of difference between $Br(\tau^+)$ and $Br(\tau^-)$, here we have chosen $\eta(y)/y = 1/2$.

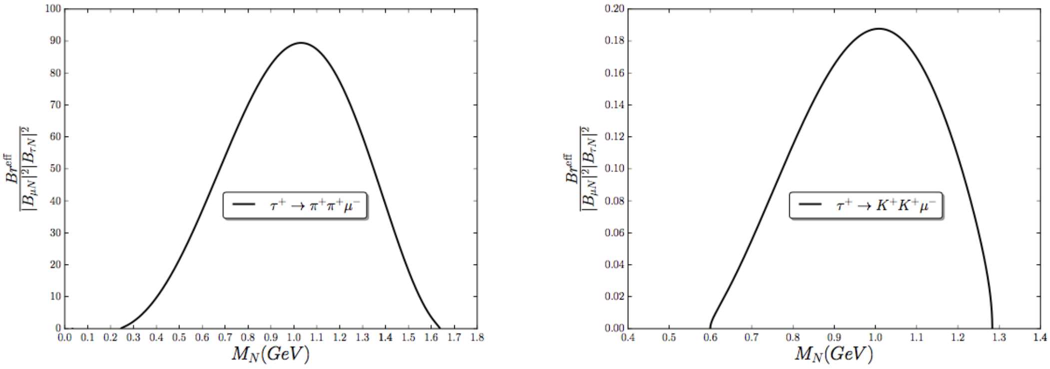

Figure 6. Effective branching ratios per unit of $|B_{\mu N}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\cos\theta_{12} = 1/\sqrt{2}$, overlap factor $\delta_{12} = 0.5$, detector length $L = 1$ mts, neutrino speed $\beta = 1$ and Lorentz factor $\gamma_N = 2$.

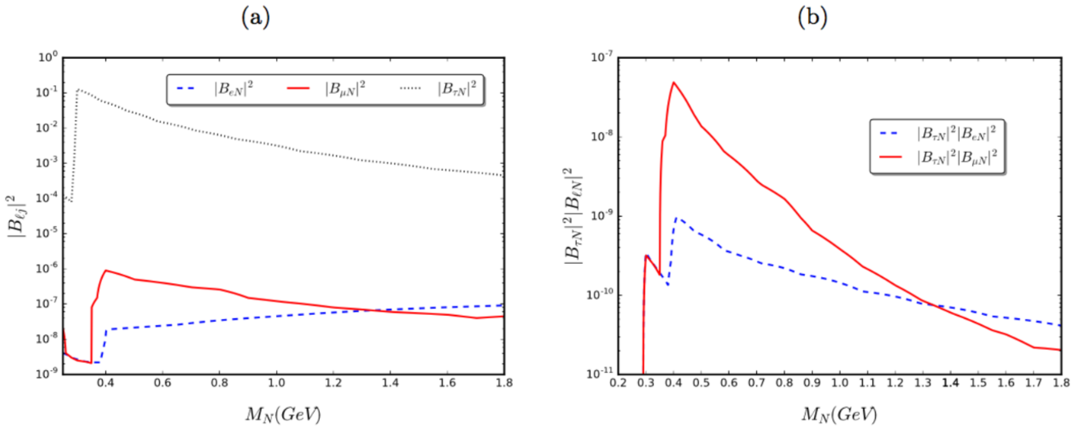

Figure 7. (a) Exclusion regions of $|B_{\ell N}|^2$ taken from [26]. The dotted line (online black) stand for $|B_{\tau N}|^2$, solid line (online red) stand for $|B_{\mu N}|^2$ and the dashed one (online blue) for $|B_{eN}|^2$. (b): exclusion regions for the product of heavy-light mixings $|B_{\tau N}|^2|B_{\ell N}|^2$. The dashed line (online blue) stand for $|B_{\tau N}|^2|B_{eN}|^2$ and the solid one (online red) for $|B_{\tau N}|^2|B_{\mu N}|^2$.

The actual experimental limits for heavy-light mixing elements are given in ref. [26], and we have summarized them in figure 7(a) for the range of mass of interest. On the other hand, and due to the fact that our results depend on $|B_{\tau N}|^2|B_{\ell N}|^2$, we present in figure 7(b) the product of the experimental limits of interest.

The CTF in Novosibirsk, Russia is expected to collect 10^{10} pairs of $\tau^{\pm}$ leptons after few years of operation [23], therefore under the latter considerations we can estimate the mixing region that can be explored in such experiment, this region is presented in figure 8.

It is important to point out that due to the *CKM* elements suppresion only channels with pions in the final state offer real possibilities to constrain the heavy-light mixings parameters.

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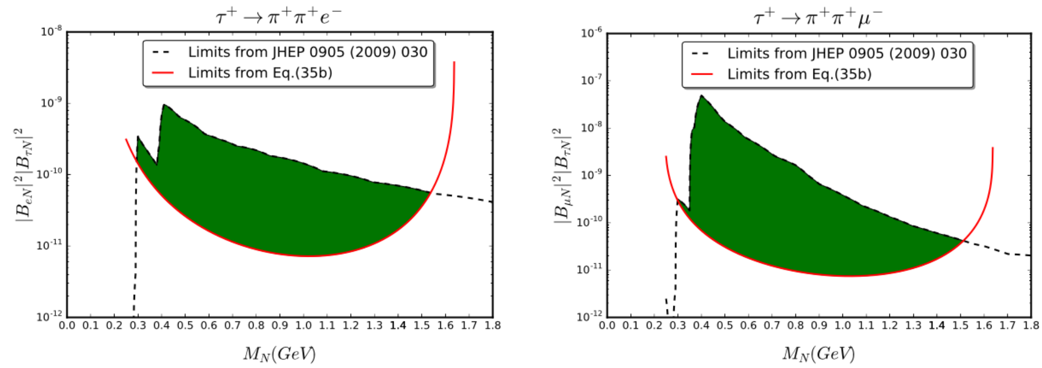

Figure 8. The shaded region (online green) show the limits over the mixings parameter which could be reached in the future $\tau^\pm$ factory [23]. Right side: limits for $|B_{eN}|^2|B_{\tau N}|^2$. Left side: limits for $|B_{\mu N}|^2|B_{\tau N}|^2$. Here we use the following input parameters: $\eta(y)/y = 0.5$, $N_\tau = 10^{10}$, $\cos\theta_{12} = 1/\sqrt{2}$, $L = 1$ mts, $\beta = 1$ and $\gamma_N = 2$.

6 Summary and conclusions

In this letter we studied the ($\Delta L = 2$) rare tau decays $\tau^\pm \to M_1^\pm M_2^\pm \ell^\mp$, where M_1 and M_2 are pseudo scalar mesons ($M_1,\ M_2 = \pi, K$) and the charged lepton can be $\ell = e, \mu$, also we studied the possibility of CP violation detection in future tau factories. We have assumed that the decays occur via the exchange of two on-shell sterile neutrinos N_j at tree level, and we have shown that the amplitude of these processes is suppressed by the mixing elements of the PMNS matrix $|B_{\tau N}|^2|B_{\ell N}|^2$. The aforementioned CP violation effects come from the interference between the N_1 and N_2 propagators and the complex phases (CP-odd phases $\phi_{\ell N_j}$, see eq. (2.2)) in the PMNS mixing matrix. Our results shows that these signals of CP violation could be detected in future tau factories for $\tau^\pm \to \pi^\pm\pi^\pm\ell^\mp$ tau decays, where $\ell = e, \mu$ if there exist, at least, two sterile neutrinos in the on-shell mass range, their masses are almost degenerate $\Delta M_N \sim \Gamma_N$, the CP odd phases $\sin\theta_{12} \not\ll 1$ and the mixing parameters are in the allowed region of figure 7. In such a case, the CP-violating difference $\Gamma(\tau^+) - \Gamma(\tau^-)$ becomes large and comparable with $\Gamma(\tau^+) + \Gamma(\tau^-)$ and the corresponding CP asymmetry A_{CP} becomes $A_{CP} \sim 1$. In addition, there exist several models with quasi-degeneracy $\Delta M_N \sim \Gamma_N$, between them it is worth to mention the well-know νMSM model [27, 28], where the quasi-degeneracy of the two heavy neutrinos (with mass $M_{N_j} \sim 1\,\mathrm{GeV}$) is fundamental in order to get a successful dark matter candidate. However, our results can be framed in the context of the νMSM model or more general models [21, 29] with at least two quasi-degenerate neutrinos.

Acknowledgments

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A Amplitude and kinematic relations for $\tau^{\pm} \to M_1^{\pm} M_2^{\pm} \ell^{\mp}$

The amplitude for the process via two on-shell intermediate heavy neutrino is

$$\begin{aligned}
\overline{|\mathcal{M}_+|^2} = K_\tau^2 \Big[& |B_{\ell 1}|^2 |B_{\tau 1}|^2 \Big(|P_1(D)|^2 \ |\not{L}_+^D|^2 + |P_1(C)|^2 \ |\not{L}_+^C|^2 \Big) \\
& + |B_{\ell 2}|^2 |B_{\tau 2}|^2 \Big(|P_2(D)|^2 \ |\not{L}_+^D|^2 + |P_2(C)|^2 \ |\not{L}_+^C|^2 \Big) \\
& + 2|B_{\ell 1}||B_{\tau 1}||B_{\ell 2}||B_{\tau 2}| \cos\theta_{21} \Big(\Re\big[P_1(D)P_2(D)^*\big] |\not{L}_+^D|^2 + \Re\big[P_1(C)P_2(C)^*\big] |\not{L}_+^C|^2 \Big) \\
& + \Big(2|B_{\ell 1}|^2 |B_{\tau 1}|^2 \Re\big[P_1(D)P_1(C)^*\big] + 2|B_{\ell 2}|^2 |B_{\tau 2}|^2 \Re\big[P_2(D)P_2(C)^*\big] \not{L}_+^D \not{L}_+^{C\dagger} \\
& + B_{\ell 1} B_{\tau 1}^* B_{\ell 2}^* B_{\tau 2} \Big(P_1(D)P_2(C)^* \ \not{L}_+^D \not{L}_+^{C\dagger} + P_1(C)P_2(D)^* \ \not{L}_+^C \not{L}_+^{D\dagger} \Big) \\
& + B_{\ell 1}^* B_{\tau 1} B_{\ell 2} B_{\tau 2}^* \Big(P_2(D)P_1(C)^* \ \not{L}_+^D \not{L}_+^{C\dagger} + P_2(C)P_1(D)^* \ \not{L}_+^C \not{L}_+^{D\dagger} \Big) \Big]
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
\overline{|\mathcal{M}_-|^2} = K_\tau^2 \Big[& |B_{\ell 1}|^2 |B_{\tau 1}|^2 \Big(|P_1(D)|^2 \ |\not{L}_-^D|^2 + |P_1(C)|^2 \ |\not{L}_-^C|^2 \Big) \\
& + |B_{\ell 2}|^2 |B_{\tau 2}|^2 \Big(|P_2(D)|^2 \ |\not{L}_-^D|^2 + |P_2(C)|^2 \ |\not{L}_-^C|^2 \Big) \\
& + 2|B_{\ell 1}||B_{\tau 1}||B_{\ell 2}||B_{\tau 2}| \cos\theta_{21} \Big(\Re\big[P_1(D)P_2(D)^*\big] |\not{L}_-^D|^2 + \Re\big[P_1(C)P_2(C)^*\big] |\not{L}_-^C|^2 \Big) \\
& + \Big(2|B_{\ell 1}|^2 |B_{\tau 1}|^2 \Re\big[P_1(D)P_1(C)^*\big] + 2|B_{\ell 2}|^2 |B_{\tau 2}|^2 \Re\big[P_2(D)P_2(C)^*\big] \not{L}_-^D \not{L}_-^{C\dagger} \\
& + B_{\ell 1}^* B_{\tau 1} B_{\ell 2} B_{\tau 2}^* \Big(P_1(D)P_2(C)^* \ \not{L}_-^D \not{L}_-^{C\dagger} + P_1(C)P_2(D)^* \ \not{L}_-^C \not{L}_-^{D\dagger} \Big) \\
& + B_{\ell 1} B_{\tau 1}^* B_{\ell 2}^* B_{\tau 2} \Big(P_2(D)P_1(C)^* \ \not{L}_-^D \not{L}_-^{C\dagger} + P_2(C)P_1(D)^* \ \not{L}_-^C \not{L}_-^{D\dagger} \Big) \Big].
\end{aligned} \tag{A.2}$$

The kinematical factors presented in eq. (2.3a), eq. (A.1) and eq. (A.2) are given by

$$\begin{aligned}
|\not{L}_+^D|^2 = |\not{L}_-^D|^2 = {} & 32(p_1 \cdot p_2)(p_2 \cdot p_\ell)(p_1 \cdot p_\tau) - 16 M_2^2 (p_1 \cdot p_\tau)(p_1 \cdot p_\ell) \\
& - 16 M_1^2 (p_2 \cdot p_\tau)(p_2 \cdot p_\ell) + 8 M_1^2 M_2^2 (p_\ell \cdot p_\tau)
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
|\not{L}_+^C|^2 = |\not{L}_-^C|^2 = {} & 32(p_1 \cdot p_2)(p_1 \cdot p_\ell)(p_2 \cdot p_\tau) - 16 M_1^2 (p_2 \cdot p_\tau)(p_2 \cdot p_\ell) \\
& - 16 M_2^2 (p_1 \cdot p_\tau)(p_1 \cdot p_\ell) + 8 M_1^2 M_2^2 (p_\ell \cdot p_\tau)
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\not{L}_\pm^D \not{L}_\pm^{C\dagger} = {} & \mp 16 i \epsilon_{p1,p2,p_\ell,p_\tau} (p_1 \cdot p_2) + 16 M_2^2 (p_1 \cdot p_\tau)(p_1 \cdot p_\ell) + 16 M_1^2 (p_2 \cdot p_\tau)(p_2 \cdot p_\ell) \\
& + 16 (p_1 \cdot p_2)^2 (p_\ell \cdot p_\tau) - 16 (p_1 \cdot p_2)(p_2 \cdot p_\ell)(p_1 \cdot p_\tau) - 16 (p_1 \cdot p_2)(p_1 \cdot p_\ell)(p_2 \cdot p_\tau)
\end{aligned} \tag{A.5}$$

$$\not{L}_\pm^{D\dagger} \not{L}_\pm^C = \Big(\not{L}_\pm^D \not{L}_\pm^{C\dagger} \Big)^* \tag{A.6}$$

The product of propagators $P_1(X)P_2(X)^*$ (where $X = D, C$) can be expressed as the

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sum of the real and imaginary parts

$$P_1(X)P_2(X)^* = \underbrace{M_{N_1}M_{N_2}\frac{(P_N^2(X)-M_{N_1}^2)(P_N^2(X)-M_{N_2}^2)+\Gamma_{N_1}\Gamma_{N_2}M_{N_1}M_{N_2}}{\Big((P_N^2(X)-M_{N_1}^2)^2+\Gamma_{N_1}^2M_{N_1}^2\Big)\Big((P_N^2(X)-M_{N_2}^2)^2+\Gamma_{N_2}^2M_{N_2}^2\Big)}}_{\text{Rear part}} \tag{A.7a}$$

$$-\,i\,\underbrace{M_{N_1}M_{N_2}\frac{(P_N^2(X)-M_{N_2}^2)M_{N_1}\Gamma_{N_1}-(P_N^2(X)-M_{N_1}^2)M_{N_2}\Gamma_{N_2}}{\Big((P_N^2(X)-M_{N_1}^2)^2+\Gamma_{N_1}^2M_{N_1}^2\Big)\Big((P_N^2(X)-M_{N_2}^2)^2+\Gamma_{N_2}^2M_{N_2}^2\Big)}}_{\text{Imaginary part}} \tag{A.7b}$$

The partial decay widths presented in eq. (4.11) are:

$$\begin{aligned}\overline{\Gamma}(\tau^\pm\to\pi^\pm N) &= \frac{1}{8\pi}G_F^2 f_\pi^2|V_\pi|^2\frac{1}{M_\tau}\,\lambda^{1/2}\left(1,\frac{M_\pi^2}{M_\tau^2},\frac{M_N^2}{M_\tau^2}\right)\times\\ &\quad\left[\left(M_\tau^2-M_N^2\right)^2-M_\pi^2\left(M_\tau^2+M_N^2\right)\right], \qquad (A.8a)\\ \overline{\Gamma}(N\to\mu^\pm\pi^\mp) &= \frac{1}{16\pi}G_F^2 f_\pi^2|V_\pi|^2\frac{1}{M_N}\,\lambda^{1/2}\left(1,\frac{M_\pi^2}{M_N^2},\frac{M_e^2}{M_N^2}\right)\times\\ &\quad\left[\left(M_N^2+M_e^2\right)\left(M_N^2-M_\pi^2+M_e^2\right)-4M_N^2M_e^2\right]. \qquad (A.8b)\end{aligned}$$

B Phase space relations

The integration presented in eq. (2.9) can be performed in the following way:

$$\Gamma(\tau^\pm) = \frac{1}{2!}(2-\delta_{M_1M_2})\frac{1}{64\pi^3 M_\tau}\int\overline{|\mathcal{M}_\pm|^2}\,dE_1\,dE_2\,; \tag{B.1}$$

the integration limits over E_2 and E_1 for the (DD^*) channel are

$$E_2 \geq \frac{1}{2m_{23}^2}\left((M_\tau-E_1)(m_{23}^2+M_2^2-M_3^2)-\sqrt{(E_1^2-M_1^2)\lambda(m_{23}^2,M_2^2,M_3^2)}\right), \tag{B.2}$$

$$E_2 \leq \frac{1}{2m_{23}^2}\left((M_\tau-E_1)(m_{23}^2+M_2^2-M_3^2)+\sqrt{(E_1^2-M_1^2)\lambda(m_{23}^2,M_2^2,M_3^2)}\right), \tag{B.3}$$

$$M_1 \leq E_1 \leq \frac{M_\tau^2+M_1^2-(M_2+M_3)^2}{2M_\tau}, \tag{B.4}$$

where

$$m_{23}^2 = M_\tau^2+M_1^2-2M_\tau E_1\,. \tag{B.5}$$

Finally, the kinematical functions $\lambda(x,y,z)$ and $Z(a,b,c,d)$ are

$$\lambda(x,y,z) = x^2+y^2+z^2-2xy-2xz-2yz \tag{B.6}$$

$$\begin{aligned}Z(a,b,c,d) &= \Big((b^2-d^2)^2-c^2(d^2+b^2)\Big)\Big((a^2-b^2)^2-c^2(b^2+a^2)\Big)\\ &\quad\times\sqrt{\Big(a^2-(b-c)^2\Big)\Big(a^2-(b+c)^2\Big)}\end{aligned} \tag{B.7}$$

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