

Goldstino superfields in $\mathcal{N} = 2$ supergravity

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ABSTRACT: We present off-shell $\mathcal{N} = 2$ supergravity actions, which exhibit spontaneously broken local supersymmetry and allow for de Sitter vacua for certain values of the parameters. They are obtained by coupling the standard $\mathcal{N} = 2$ supergravity-matter systems to the Goldstino superfields introduced in arXiv:1105.3001 and arXiv:1607.01277 in the rigid supersymmetric case. These $\mathcal{N} = 2$ Goldstino superfields include nilpotent chiral and linear supermultiplets. We also describe a new reducible $\mathcal{N} = 1$ Goldstino supermultiplet.

KEYWORDS: Extended Supersymmetry, Supergravity Models, Superspaces, Supersymmetry Breaking

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1 Introduction

Several years ago, two of us introduced a family of constrained Goldstino superfields [1], in terms of which the models for spontaneously broken $\mathcal{N} = 2$ Poincaré supersymmetry are formulated. Some of these Goldstino superfields have been generalised to the case of $\mathcal{N} = 2$ anti-de Sitter supersymmetry [2]. The common feature of the constrained $\mathcal{N} = 2$ superfields given in [1] is that their only independent component fields are the two Goldstini. Therefore, if such a Goldstino superfield is coupled to supergravity in order to describe spontaneously broken $\mathcal{N} = 2$ local supersymmetry, it does not bring in any new degrees of freedom, except for making the gravitini massive and generating a positive contribution to

the cosmological constant, in accordance with the super-Higgs effect [3–5]. In particular, the absence of scalars is an attractive feature for phenomenological applications.

Some of the $\mathcal{N} = 2$ superfield Goldstino models given in [1] have natural $\mathcal{N} = 1$ counterparts [6–10]. In particular, the constrained chiral scalar superfield, which will be reviewed in section 2.2, is the $\mathcal{N} = 2$ analogue of the $\mathcal{N} = 1$ chiral scalar superfield ϕ [6, 7], $\bar{D}_{\dot{\alpha}}\phi = 0$, which is subject to the constraints [6]:

$$\phi^2 = 0, \tag{1.1a}$$

$$f\phi = -\frac{1}{4}\phi\bar{D}^2\bar{\phi}, \tag{1.1b}$$

where f is a real parameter of mass dimension +2 which characterises the supersymmetry breaking scale. As was shown by Roček [6] (see also [11] for a recent review), the Goldstino, which may be identified with $D_{\alpha}\phi|_{\theta=0}$, is the only independent component field contained in ϕ . In the case of $\mathcal{N} = 1$ supersymmetry, there is an alternative superfield approach to describe the Goldstino dynamics, which was advocated in [12, 13]. It consists of getting rid of the nonlinear constraint (1.1b) and working with a chiral scalar \mathcal{X} , $\bar{D}_{\dot{\alpha}}\mathcal{X} = 0$, which is only constrained to be nilpotent,

$$\mathcal{X}^2 = 0. \tag{1.2}$$

Unlike ϕ , the chiral scalar \mathcal{X} contains an independent auxiliary field in addition to the Goldstino. Nevertheless, the former proves to be a function of the latter on the mass shell. In practice, the use of \mathcal{X} is somewhat simpler than that of ϕ from the point of view of its couplings to supergravity and supersymmetric matter. Conceptually, however, the two constrained superfield realisations ϕ and \mathcal{X} are completely equivalent [14] (as long as one deals with low-energy effective actions without higher-derivative terms). In particular, they lead to equivalent couplings to supergravity and supersymmetric matter, see [14] for the technical details. The $\mathcal{N} = 2$ superfield analogue of \mathcal{X} was given in [15], see section 2.3 below for a review. We will demonstrate that this realisation is equivalent to the $\mathcal{N} = 2$ chiral scalar Goldstino model of [1].

In this paper we propose models for spontaneously broken local $\mathcal{N} = 2$ supersymmetry, which are obtained by coupling the standard off-shell supergravity-matter systems to the Goldstino superfields introduced in [1, 15]. Recently, there has been much interest in $\mathcal{N} = 1$ supergravity coupled to nilpotent Goldstino superfields for several reasons. Firstly, such theories are interesting from the point of view of cosmology due to the possibility of describing inflation [16, 17]. Secondly, every Goldstino superfield coupled to supergravity provides a universal positive contribution to the cosmological constant [14, 18–21], unlike the supersymmetric cosmological term [22] which yields a negative contribution to the cosmological constant. The same property is true for the Goldstino brane [23]. Of course, the observation that the coupling of the Volkov-Akulov theory [24–26] to supergravity generates a model-independent positive contribution to the cosmological constant was made long ago in the frameworks of on-shell supergravity [5] and off-shell supergravity [18]. But it seems that at that time nobody was interested in generating a positive cosmological constant. Cosmological model building [16, 17] and the so-called de Sitter supergravity [19]

and its extensions [27, 28] have invigorated interest in the coupling of nonlinear supersymmetry to supergravity. Nonlinear supersymmetries are also intriguing in the context of amplitudes [29]. It is also worth recalling that $\mathcal{N} = 2$ supergravity [30] realised Einstein’s dream of unifying electromagnetism and gravity [31] by adding a massless complex gravitino to the photon and graviton. When $\mathcal{N} = 2$ supergravity is coupled to any of the Goldstino superfields introduced in [1], the resulting theory describes (in a unitary gauge) the Einstein-Maxwell system coupled to a massive complex gravitino. Integrating out the massive gravitino fields leads to a low-energy Einstein-Maxwell theory of purely supersymmetric origin.

This paper is organised as follows. In section 2 we review the Goldstino superfields introduced in [1, 15] and elaborate on their properties and the explicit relationships between them. In section 3 we couple the chiral scalar and analytic Goldstino superfields to supergravity and supersymmetric matter. Section 4 is devoted to the coupling of the spinor Goldstino superfield to supergravity. Several generalisations of our results are given in section 5. The main body of the paper is accompanied by four technical appendices. Appendix A describes the component content of the nilpotent chiral scalar superfield Φ . Appendix B gives a summary of the $SU(2)$ superspace [32], while appendix C briefly introduces $\mathcal{N} = 2$ conformal superspace [33]. Finally, appendix D discusses nilpotent $\mathcal{N} = 1$ supergravity following and extending [21]. Our two-component notation and conventions correspond to [34].

2 Goldstino superfields in Minkowski superspace

We start by reviewing some results of [1] and elaborating on them.

2.1 Spinor Goldstino superfields

The $\mathcal{N} = 2$ analogue of the nonlinear realisation for $\mathcal{N} = 1$ supersymmetry [7–9, 35, 36], in which there is a pair of Goldstone fields $\xi_i^\alpha(x)$ which mix only with themselves under supersymmetry transformation, is based on the coset parametrisation [1]

$$g(x, \xi_i(x), \bar{\psi}^i(x)) = e^{i(-x^a P_a + f^{-1} \xi_i^\alpha(x) Q_\alpha^i)} e^{if^{-1} \bar{\psi}_\alpha^i(x) \bar{Q}_i^\alpha} . \tag{2.1}$$

This yields the supersymmetry transformations

$$\delta \xi_i^\alpha = f \epsilon_i^\alpha - 2i f^{-1} \xi_j^\beta \bar{\epsilon}^{\dot{\beta}j} \partial_{\beta\dot{\beta}} \xi_i^\alpha , \tag{2.2a}$$

$$\delta \bar{\psi}_\alpha^i = f \bar{\epsilon}_\alpha^i - 2i f^{-1} \xi_j^\beta \bar{\epsilon}^{\dot{\beta}j} \partial_{\beta\dot{\beta}} \bar{\psi}_\alpha^i . \tag{2.2b}$$

The construction of $\mathcal{N} = 2$ superfields associated with these Goldstino fields proceeds as in the $\mathcal{N} = 1$ case, and the resulting superfields Ξ_i^α and $\bar{\Psi}_\alpha^i$ satisfy the following set of constraints involving the $\mathcal{N} = 2$ covariant derivatives $D_A = (\partial_a, D_\alpha^i, \bar{D}_{\dot{\alpha}i})$:

$$D_\beta^j \Xi_i^\alpha = f \delta_\beta^\alpha \delta_j^i , \tag{2.3a}$$

$$\bar{D}_{\dot{\beta}j} \Xi_i^\alpha = -2i f^{-1} \Xi_j^\beta \partial_{\beta\dot{\beta}} \Xi_i^\alpha , \tag{2.3b}$$

$$D_\beta^j \bar{\Psi}_\alpha^i = 0 , \tag{2.3c}$$

$$\bar{D}_{\dot{\beta}j} \bar{\Psi}_\alpha^i = f \varepsilon_{\dot{\beta}\alpha} \delta_j^i - 2i f^{-1} \Xi_j^\beta \partial_{\beta\dot{\beta}} \bar{\Psi}_\alpha^i . \tag{2.3d}$$

The constraints (2.3a) and (2.3b) were derived for the first time by Wess [37] as a generalisation of the $\mathcal{N} = 1$ construction [9]. The constraints (2.3) tell us that $\xi_i^\alpha = \Xi_i^\alpha|_{\theta=0}$ and $\bar{\psi}_\alpha^i = \bar{\Psi}_\alpha^i|_{\theta=0}$ are the only independent component fields contained in the Goldstino superfields introduced.

The spinor superfields Ξ_i^α and $\bar{\Psi}_\alpha^i$ provide equivalent descriptions of the Goldstini. It may be checked that the latter is expressed via the former as

$$\bar{\Psi}_\alpha^i = \frac{1}{f^4} D^4 (\bar{\Xi}_\alpha^i \Xi^4) = \bar{\Xi}_\alpha^i + O(\Xi^3), \quad \Xi^4 := \frac{1}{3} \Xi^{ij} \Xi_{ij} = -\frac{1}{3} \Xi^{\alpha\beta} \Xi_{\alpha\beta}, \quad (2.4)$$

which extends the $\mathcal{N} = 1$ result given in [14]. Here and below we make use of the following definitions¹

$$D^4 = \frac{1}{48} D^{ij} D_{ij} = -\frac{1}{48} D^{\alpha\beta} D_{\alpha\beta}, \quad D_{ij} = D_i^\alpha D_{\alpha j}, \quad D_{\alpha\beta} = D_\alpha^i D_{\beta i}, \quad (2.5a)$$

$$\bar{D}^4 = \frac{1}{48} \bar{D}^{ij} \bar{D}_{ij} = -\frac{1}{48} \bar{D}^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}\dot{\beta}}, \quad \bar{D}_{ij} = \bar{D}_{\dot{\alpha}i} \bar{D}_j^{\dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}\dot{\beta}} = \bar{D}_{\dot{\alpha}i} \bar{D}_{\dot{\beta}}^i. \quad (2.5b)$$

The composites Ξ_{ij} and $\Xi_{\alpha\beta}$ in eq. (2.4) are defined similarly. Note that eq. (2.4) implies that $\bar{\psi}_\alpha^i = \bar{\xi}_\alpha^i + \dots$, where the ellipsis stands for nonlinear terms in ξ_i^α and $\bar{\xi}_\alpha^i$.

Eq. (2.3c) means that the spinor superfields $\bar{\Psi}_\alpha^i$ are antichiral and their complex conjugates $\Psi_{\alpha i}$ are chiral, and so they provide ingredients for an action obtained by integration over the chiral subspace of $\mathcal{N} = 2$ Minkowski superspace:

$$S_{\text{Goldstino}} = -\frac{1}{2f^2} \int d^4x d^4\theta \Psi^4 - \frac{1}{2f^2} \int d^4x d^4\bar{\theta} \bar{\Psi}^4, \quad (2.6)$$

where $\Psi^4 := \frac{1}{3} \Psi^{ij} \Psi_{ij}$, $\Psi_{ij} := \Psi_i^\alpha \Psi_{\alpha j}$ and $\Psi^{ij} = \varepsilon^{ik} \varepsilon^{jl} \Psi_{kl}$. Making use of (2.4) allows us to reformulate the Goldstino action (2.6) in terms of Ξ_i^α and its conjugate:

$$S_{\text{Goldstino}} = -\frac{1}{f^6} \int d^4x d^4\theta d^4\bar{\theta} \Xi^4 \bar{\Xi}^4. \quad (2.7)$$

This action was given for the first time in ref. [38], which built on the earlier work [37]. At the component level, the functionals (2.6) and (2.7) lead to nonlinear actions, which prove to be equivalent to the $\mathcal{N} = 2$ supersymmetric Volkov-Akulov theory [24–26]. To quadratic order in Goldstini, the action (2.7) is

$$S_{\text{Goldstino}} = - \int d^4x \left(f^2 + i \xi_i^\alpha \overleftrightarrow{\partial}_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}i} \right) + O(\xi^4). \quad (2.8)$$

The constant term in the integrand (2.8) generates a positive (de Sitter) contribution to the cosmological constant when the Goldstino superfields Ξ_i^α are coupled to supergravity, see section 4.

In general, $\mathcal{N} = 2$ supersymmetric Goldstino actions contain terms to sixteenth order in the fields. The striking feature of the action (2.6) is that it is at most of eighth order

¹We point out that the second-order operators D_{ij} , $D_{\alpha\beta}$, \bar{D}_{ij} and $\bar{D}_{\dot{\alpha}\dot{\beta}}$ are symmetric in their indices.

in the fields ξ_i^α and $\bar{\psi}_\alpha^i$, as a consequence of the constraints (2.3). To quartic order in Goldstini, the component form of the action (2.6) is

$$\begin{aligned}
 S_{\text{Goldstino}} = & - \int d^4x \left(\frac{1}{2} f^2 + i \xi_i^\alpha \partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}i} - \frac{1}{4f^2} \xi^{ij} \partial_{\alpha\dot{\alpha}} \partial^{\dot{\alpha}\alpha} \bar{\psi}_{ij} - \frac{1}{4f^2} \xi^{\alpha\beta} \partial_{\alpha\dot{\alpha}} \partial_{\beta\dot{\beta}} \bar{\psi}^{\dot{\alpha}\dot{\beta}} \right. \\
 & + \frac{1}{f^2} \xi^{\alpha i} (\partial_{\alpha\dot{\alpha}} \xi^{\beta j}) \partial_{\beta\dot{\beta}} \bar{\psi}_{ij} - \frac{1}{f^2} \xi^{\alpha i} (\partial_{\alpha\dot{\alpha}} \xi_i^\beta) \partial_{\beta\dot{\beta}} \bar{\psi}^{\dot{\alpha}\dot{\beta}} - \frac{1}{4f^2} \xi^{\alpha\beta} \partial_{\beta\dot{\beta}} ((\partial_{\alpha\dot{\alpha}} \bar{\psi}_\beta^i) \bar{\psi}_i^{\dot{\beta}}) \\
 & \left. + \frac{1}{2f^2} \xi^{\alpha\beta} (\partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}i}) \partial_{\beta\dot{\beta}} \bar{\psi}_i^{\dot{\beta}} + \frac{1}{2f^2} \xi^{\alpha\beta} (\partial_{\alpha\dot{\alpha}} \partial_{\beta\dot{\beta}} \bar{\psi}^{\dot{\alpha}i}) \bar{\psi}_i^{\dot{\beta}} + \text{c.c.} \right) + \dots \quad (2.9)
 \end{aligned}$$

This action turns into (2.8) once ψ_i^α and $\bar{\psi}_\alpha^i$ are expressed in terms of ξ_i^α and $\bar{\xi}_\alpha^i$.

2.2 Chiral scalar Goldstino superfield

The chiral scalar superfield [1]

$$\Phi := \Psi^4, \quad \bar{D}^{\dot{\alpha}} \Phi = 0 \quad (2.10)$$

obeys the following nilpotency conditions

$$\Phi^2 = 0, \quad (2.11a)$$

$$\Phi D_A D_B \Phi = 0, \quad (2.11b)$$

$$\Phi D_A D_B D_C \Phi = 0, \quad (2.11c)$$

as well as the nonlinear relation

$$f\Phi = \Phi \bar{D}^4 \bar{\Phi}, \quad (2.12)$$

which is similar to Roček's constraint (1.1b). It follows from the definition of Φ , eq. (2.10), and from (2.3d) that $D^4\Phi$ is nowhere vanishing.

The chiral scalar Φ has been defined as the composite superfield (2.10) constructed from Ψ_i^α . It can also be realised as a different composite,

$$\Phi = \frac{1}{f^7} \bar{D}^4 (\bar{\Xi}^4 \Xi^4), \quad (2.13)$$

which is constructed from Ξ_i^α and its conjugate. In both realisations, the relations (2.11) and (2.12) hold identically. On the other hand, if we view Φ as a fundamental Goldstino superfield, then (2.11) and (2.12) must be imposed as constraints. In addition, it is necessary to require $D^4\Phi$ to be nowhere vanishing. These properties guarantee that the two Goldstini, which occur at order θ^3 , are the only independent component fields of Φ , see appendix A for the details. In this approach, the spinor Goldstino superfields can be realised as composite ones constructed from Φ and its conjugate. In particular, one finds

$$\Xi_i^\alpha = -\frac{f}{12} \frac{D^{\alpha j} D_{j i} \Phi}{D^4 \Phi}. \quad (2.14)$$

It is a constructive exercise to check that this composite superfield obeys the constraints (2.3a) and (2.3b).

The Goldstino action takes the form

$$S_{\text{Goldstino}} = -\frac{f}{2} \int d^4x d^4\theta \Phi - \frac{f}{2} \int d^4x d^4\bar{\theta} \bar{\Phi} = - \int d^4x d^4\theta d^4\bar{\theta} \bar{\Phi} \Phi. \quad (2.15)$$

2.3 Reducible chiral scalar Goldstino superfield

Instead of working with the Goldstino superfield Φ , which contains only two independent component fields — the Goldstini — one can follow a different path, in the spirit of the $\mathcal{N} = 1$ constructions advocated in [12, 13]. Specifically, one can consider a chiral scalar X , $\bar{D}_i^{\dot{\alpha}} X = 0$, which is only required to obey the nilpotency constraints

$$X^2 = 0, \quad (2.16a)$$

$$XD_A D_B X = 0, \quad (2.16b)$$

$$XD_A D_B D_C X = 0, \quad (2.16c)$$

in conjunction with the requirement that $D^4 X$ be nowhere vanishing, $D^4 X \neq 0$. This approach was pursued in [15].² The chiral superfield X contains two independent component fields that we identify with the lowest components of the descendants $\chi_i^\alpha := -\frac{1}{12} D^{\alpha j} D_{ij} X$ and $D^4 X$, which have the obvious, albeit useful, properties

$$X \chi_i^\alpha = 0, \quad (2.17a)$$

$$D_\beta^j \chi_i^\alpha = \delta_\beta^\alpha \delta_i^j D^4 X. \quad (2.17b)$$

The dynamics of this supermultiplet is governed by the action

$$\tilde{S}_{\text{Goldstino}} = \int d^4 x d^4 \theta d^4 \bar{\theta} \bar{X} X - f \int d^4 x d^4 \theta X - f \int d^4 x d^4 \bar{\theta} \bar{X}. \quad (2.18)$$

Making use of the constraints (2.16), it is not difficult to derive the following nonlinear representation³ for X [15]:

$$X = \frac{\chi^4}{(D^4 X)^3}, \quad \chi^4 = \frac{1}{3} \chi^{ij} \chi_{ij} = -\frac{1}{3} \chi^{\alpha\beta} \chi_{\alpha\beta}, \quad (2.19)$$

where $\chi^{ij} = \chi^{\alpha i} \chi_\alpha^j$ and $\chi_{\alpha\beta} := \chi_\alpha^i \chi_{\beta i}$. With this representation for X , the constraints (2.16) hold identically.

The Goldstino model (2.18) is equivalent to the one described by the action (2.15). The simplest way to prove this is by extending the $\mathcal{N} = 1$ analysis of [14] to the $\mathcal{N} = 2$ supersymmetric case. The starting point is to notice that if X obeys the constraints (2.16), then $e^{-\rho} X$ also obeys the same constraints, for every chiral scalar superfield ρ , $\bar{D}_i^{\dot{\alpha}} \rho = 0$. This freedom may be used to represent

$$X = e^\rho \Phi, \quad (2.20)$$

where Φ is the Goldstino superfield described in the previous subsection. The superfield ρ in (2.20) is defined modulo gauge transformations of the form

$$\rho \rightarrow \rho + \delta\rho, \quad \Phi \delta\rho = 0. \quad (2.21)$$

²The chiral scalar X was the only novel $\mathcal{N} = 2$ Goldstino superfield introduced in [15], the others had been given five years earlier in [1].

³Relation (2.19) has a natural counterpart in the case of $\mathcal{N} = 1$ supersymmetry. Given a nilpotent $\mathcal{N} = 1$ chiral superfield X , with the properties $\bar{D}_{\dot{\alpha}} X = 0$ and $X^2 = 0$, it can be represented as $X = -\chi^2 (D^2 X)^{-1}$, where $\chi^2 = \chi^\alpha \chi_\alpha$ and $\chi_\alpha = D_\alpha X$.

We now make use of the representation (2.20) and vary the action (2.18) with respect to ρ , which gives

$$X\bar{D}^4\bar{X} = fX . \tag{2.22}$$

We see that on the mass shell the nilpotent chiral superfield X , defined by (2.20), obeys the same constraint as Φ , eq. (2.12). This means that $\rho = 1$ modulo the gauge freedom (2.21).

It is worth giving an $\mathcal{N} = 2$ extension of one more important result from [14]. The point is that the representation (2.14) does not require Φ to obey the nonlinear constraint (2.12); only the nilpotency constraints (2.11) are essential. In other words, starting from X , it turns out that the composite spinor superfield

$$\Xi_i^\alpha = f \frac{X_i^\alpha}{D^4 X} \tag{2.23}$$

obeys the constraints (2.3a) and (2.3b) (see also [15]). Now, given Ξ_i^α , we know that eq. (2.13) defines the chiral scalar Goldstino superfield Φ subject to the constraints (2.11) and (2.12). Therefore, we can always represent

$$X = \Phi + \Upsilon , \quad \Phi = \frac{1}{f^7} \bar{D}^4(\bar{\Xi}^4 \Xi^4) , \tag{2.24}$$

for some chiral scalar Υ obeying the generalised nilpotency condition

$$2\Phi\Upsilon + \Upsilon^2 = 0 . \tag{2.25}$$

The two component fields of X now belong to the two different chiral superfields Φ and Υ , of which Φ contains the Goldstino and Υ the auxiliary field.

According to the terminology of [14], the $\mathcal{N} = 2$ Goldstino superfields described in sections 2.1 and 2.2 are irreducible in the sense that the Goldstini are the only independent component fields of such a superfield, while the other component fields are simply composites constructed from the Goldstini. There also exist reducible Goldstino superfields. They contain certain independent auxiliary fields in addition to the Goldstini. Any reducible Goldstino superfield may be represented as a sum of an irreducible Goldstino superfield and a ‘‘matter’’ superfield, which contains the auxiliary fields. The chiral scalar X is an example of a reducible Goldstino superfield. It is represented in the form (2.24), where Φ is the irreducible Goldstino superfield and Υ the matter one.

2.4 Analytic Goldstino superfields

The superfields Ξ_i^α , $\bar{\Psi}_\alpha^i$ and Φ , which we have described in sections 2.1 and 2.2, are not the only irreducible Goldstino superfields considered in [1]. Another Goldstino multiplet introduced in [1] is a complex linear superfield, H^{ij} , constructed originally as a composite of the spinor ones. Here we study its properties in more detail using an alternative realisation for H^{ij} as a descendant of Φ .

Our first observation is that the degrees of freedom of the chiral scalar Φ can be encoded in the following complex iso-triplet

$$H^{ij} := \frac{1}{4} D^{ij} \Phi \tag{2.26}$$

and its conjugate $\bar{H}_{ij} = \overline{H^{ij}} = \frac{1}{4}\bar{D}_{ij}\bar{\Phi}$. By construction, H^{ij} satisfies the analyticity constraints

$$D_{\alpha}^{(i}H^{jk)} = 0, \quad \bar{D}_{\dot{\alpha}}^{(i}H^{jk)} = 0, \quad (2.27)$$

which mean that H^{ij} is a $\mathcal{N} = 2$ linear multiplet [39–41]. Following the modern projective-superspace terminology [42, 43], one may also refer to H^{ij} as a complex $\mathcal{O}(2)$ multiplet.

As shown in appendix A, the chiral scalar Φ is expressed in terms of its descendants χ_i^{α} and F , defined by (A.1), according to (A.2d). These are given in terms of H^{ij} as follows:

$$\chi_i^{\alpha} = -\frac{1}{3}D^{\alpha j}H_{ij}, \quad F = \frac{1}{12}D^{ij}H_{ij}. \quad (2.28)$$

As a result, Φ turns into a composite superfield constructed from H^{ij} . In particular, the Goldstini $\chi_i^{\alpha}|_{\theta=0}$ can be read off from H^{ij} by taking its first spinor derivative. Making use of (A.2b), we observe that H^{ij} satisfies the nilpotency constraint

$$H^{(ij}H^{kl)} = 0 \iff H^{ij}H_{kl} = \frac{2}{3}\delta_{(k}^i\delta_{l)}^j H^2, \quad H^2 = \frac{1}{2}H^{ij}H_{ij}. \quad (2.29)$$

Moreover, it holds that

$$H^{i_1j_1}H^{i_2j_2}H^{i_3j_3} = 0. \quad (2.30)$$

It may be shown that H^{ij} obeys the following nonlinear constraints

$$\bar{D}_j^{\dot{\alpha}}H^{ij} = -4i\partial^{\alpha\dot{\alpha}}\frac{H^{ij}D_{\alpha}^kH_{jk}}{D \cdot H}, \quad D \cdot H = \frac{1}{2}D^{ij}H_{ij} \quad (2.31a)$$

and

$$fH^{ij} = \frac{1}{6}D^{ij}\left(\frac{\bar{D} \cdot \bar{H}}{D \cdot H}H^2\right), \quad (2.31b)$$

which complete the list of conditions H^{ij} has to obey in order to be an irreducible Goldstino superfield. The Goldstino action (2.15) turns into

$$S_{\text{Goldstino}} = -\frac{f}{24}\int d^4x \left(D^{ij}H_{ij} + \bar{D}^{ij}\bar{H}_{ij}\right). \quad (2.32)$$

This action is supersymmetric because it is a variant of the $\mathcal{N} = 2$ linear multiplet action proposed by Sohnius [44].

It follows from (2.31a) that the action (2.32) can be rewritten in the form

$$S_{\text{Goldstino}} = -\frac{f}{12}\int d^4x \left(D^{ij} + \bar{D}^{ij}\right)\mathbb{H}_{ij}, \quad \mathbb{H}^{ij} := \frac{1}{2}(H^{ij} + \bar{H}^{ij}). \quad (2.33)$$

One may see that the dynamics of the Goldstini can be described using the real linear multiplet \mathbb{H}^{ij} , which is an irreducible Goldstino superfield. It satisfies a nilpotency condition of degree 3,

$$\mathbb{H}^{(i_1i_2}\mathbb{H}^{i_3i_4}\mathbb{H}^{i_5i_6)} = 0. \quad (2.34)$$

If \mathbb{H}^{ij} is used as a fundamental Goldstino superfield, the Goldstini may be defined to be proportional to $D^{\alpha j} \mathbb{H}_{ij}|_{\theta=0}$. The nonlinear constraint (2.31b) may be recast in terms of \mathbb{H}^{ij} .

We now introduce one more Goldstino superfield that is a real $\mathcal{O}(4)$ multiplet associated with H^{ij} and \bar{H}^{ij} . It is defined by

$$L^{ijkl} := H^{(ij} \bar{H}^{kl)} = 2\mathbb{H}^{(ij} \mathbb{H}^{kl)} \tag{2.35}$$

and obeys the analyticity constraints

$$D_{\alpha}^{(i} L^{jklm)} = \bar{D}_{\dot{\alpha}}^{(i} L^{jklm)} = 0. \tag{2.36}$$

The second form of the Goldstino action (2.15) may be recast in the alternative form

$$S_{\text{Goldstino}} = -\frac{1}{5} \int d^4x D^{ijkl} L_{ijkl}, \quad D^{ijkl} := \frac{1}{16} D^{(ij} \bar{D}^{kl)}. \tag{2.37}$$

This action is $\mathcal{N} = 2$ supersymmetric because it is a variant of the $\mathcal{O}(4)$ multiplet action introduced for the first time by Sohnius, Stelle and West in [41].

To get further insights into the structure of the constrained superfield L^{ijkl} , which is, by construction, defined on $\mathcal{N} = 2$ Minkowski superspace $\mathbb{M}^{4|8}$, it is useful to (i) reformulate L^{ijkl} as a holomorphic superfield on a superspace with auxiliary bosonic dimensions, $\mathbb{M}^{4|8} \times \mathbb{C}P^1$, which is the most relevant superspace setting for off-shell $\mathcal{N} = 2$ supersymmetric theories; and (ii) make use of the modern projective-superspace notation [43].⁴

Let $v^i \in \mathbb{C}^2 \setminus \{0\}$ be homogeneous coordinates for $\mathbb{C}P^1$. Given a symmetric iso-spinor of rank n , $T^{i_1 \dots i_n} = T^{(i_1 \dots i_n)}$, we associate with it a holomorphic homogeneous polynomial $T^{(n)}(v) := v_{i_1} \dots v_{i_n} T^{i_1 \dots i_n}$, where the superscript “ n ” denotes the degree of homogeneity, that is $T^{(n)}(cv) = c^n T^{(n)}(v)$, with $c \in \mathbb{C} \setminus \{0\}$. It is clear that $T^{(n)}(v)$ defines a holomorphic tensor field on $\mathbb{C}P^1$. If $T^{i_1 \dots i_n}(z)$ is a superfield constrained by

$$D_{\alpha}^{(i_1} T^{i_2 \dots i_{n+1})} = 0, \quad \bar{D}_{\dot{\alpha}}^{(i_1} T^{i_2 \dots i_{n+1})} = 0, \tag{2.38}$$

it is called an $\mathcal{O}(n)$ multiplet.⁵ The holomorphic superfield $T^{(n)}(z, v)$ on $\mathbb{M}^{4|8} \times \mathbb{C}P^1$, which is associated with the $\mathcal{O}(n)$ multiplet, obeys the analyticity constraints

$$D_{\alpha}^{(1)} T^{(n)} = 0, \quad \bar{D}_{\dot{\alpha}}^{(1)} T^{(n)} = 0, \tag{2.39}$$

where we have introduced the first-order operators $D_{\alpha}^{(1)} := v_i D_{\alpha}^i$ and $\bar{D}_{\dot{\alpha}}^{(1)} := v_i \bar{D}_{\dot{\alpha}}^i$, which anticommute with each other. The $\mathcal{O}(n)$ multiplets are examples of the so-called projective multiplets [49, 50], see [43] for a modern review.

The constraints (2.27) and (2.36) tell us that the Goldstino superfields H^{ij} and L^{ijkl} are $\mathcal{O}(2)$ and $\mathcal{O}(4)$ multiplets, respectively. They can equivalently be described in terms of the projective superfields $H^{(2)}(v) = v_i v_j H^{ij}$ and $L^{(4)}(v) = v_i v_j v_k v_l L^{ijkl}$.

⁴The superspace $\mathbb{M}^{4|8} \times \mathbb{C}P^1$ was originally introduced by Rosly [45]. It is the superspace setting for both the harmonic [46, 47] and the projective [48–50] superspace approaches to $\mathcal{N} = 2$ supersymmetric theories in four dimensions. The precise relationship between these approaches is thoroughly discussed in [43, 51].

⁵In case n is even, $n = 2m$, one can consistently define real $\mathcal{O}(2m)$ multiplets which are subject to the reality condition $\overline{T^{i_1 \dots i_{2m}}} = T_{i_1 \dots i_{2m}} = \varepsilon_{i_1 j_1} \dots \varepsilon_{i_{2m} j_{2m}} T^{j_1 \dots j_{2m}}$.

The Goldstino superfield $L^{(4)}$ satisfies the nilpotency constraints

$$L^{(4)}L^{(4)} = 0, \tag{2.40a}$$

$$L^{(4)}D_A D_B L^{(4)} = 0, \tag{2.40b}$$

$$L^{(4)}D_A D_B D_C L^{(4)} = 0, \tag{2.40c}$$

as well as the nonlinear relation

$$f^2 L^{(4)} = \frac{1}{4!} L^{(4)} D^{(4)} (\partial^{(-2)})^4 L^{(4)}, \tag{2.41}$$

which follows from eq. (2.31b). Here we have introduced the operators

$$D^{(4)} := v_i v_j v_k v_l D^{ijkl}, \quad \partial^{(-2)} := \frac{1}{(v, u)} u^i \frac{\partial}{\partial v^i}, \tag{2.42}$$

where $(v, u) := v^i u_i$, and u_i is an isospinor constrained by the only requirement $(v, u) \neq 0$ (which means that v^i and u^i are linearly independent). It is not difficult to see that the right-hand side of (2.41) is independent of u_i . In what follows, given a symmetric iso-spinor $T^{i_1 \dots i_n}$, we will associate with it not only $T^{(n)} = v_{i_1} \dots v_{i_n} T^{i_1 \dots i_n}$, but also the following object

$$T^{(-n)} := \frac{1}{(v, u)^n} u_{i_1} \dots u_{i_n} T^{i_1 \dots i_n}. \tag{2.43}$$

With this notation, the constraint (2.41) turns into

$$f^2 L^{(4)} = L^{(4)} D^{(4)} L^{(-4)}. \tag{2.44}$$

It is worth pointing out that the constraints (2.40) and (2.41) are quite similar to (2.11) and (2.12).

It may be seen that $L^{(4)}$ is an irreducible Goldstino superfield. To demonstrate the equivalence of this description to those discussed earlier, we point out that the following composite real $\mathcal{O}(4)$ multiplet

$$L^{(4)} := \frac{1}{f^6} D^{(4)} (\Xi^4 \bar{\Xi}^4) \tag{2.45}$$

satisfies the nilpotency constraints (2.40) and (2.41). This relation may be inverted to express Ξ_i^α in terms of $L^{(4)}$.

We believe that the Goldstino superfields H^{ij} and L^{ijkl} can be generalised to describe spontaneously broken supersymmetry with eight supercharges in five and six dimensions where chiral superfields are not defined in the $SU(2)$ covariant formalism.

2.5 Reducible linear Goldstino superfield

The linear Goldstino superfield (2.26) is constructed from the irreducible chiral scalar Goldstino superfield Φ . Instead of using Φ , we can choose X to define another complex linear superfield,

$$H_X^{ij} := \frac{1}{4} D^{ij} X, \tag{2.46}$$

which is a reducible Goldstino superfield. It satisfies the same analyticity and nilpotency conditions, eqs. (2.27) and (2.29), that H^{ij} does. However, there is no constraint (2.31b) in the case of H_X^{ij} .

Within the harmonic superspace approach [46, 47], one deals with SU(2) harmonics u_i^+ and u_i^- defined by

$$u_i^- := \overline{u^{+i}}, \quad u^{+i} u_i^- = 1 \quad \iff \quad (u_i^-, u_i^+) \in \text{SU}(2). \quad (2.47)$$

They may be related to the isospinors v^i and u_i , which we have used in the previous subsection, as follows:

$$v^i \rightarrow u^{+i} := \frac{v^i}{\sqrt{v^\dagger v}}, \quad u_i := u_i^- = \frac{\bar{v}_i}{\sqrt{v^\dagger v}}, \quad (2.48)$$

with $\bar{v}_i := \overline{v^i}$. Associated with H_X^{ij} is the analytic superfield $H_X^{++} = u_i^+ u_j^+ H_X^{ij}$.

In terms of H_X^{++} and \bar{H}_X^{++} , the Goldstino action (2.18) turns into

$$\begin{aligned} \tilde{S}_{\text{Goldstino}} &= \int du \int d\zeta^{(-4)} L^{(+4)}, \\ L^{(+4)} &= \bar{H}_X^{++} H_X^{++} + f\left((\theta^+)^2 + (\bar{\theta}^+)^2\right) \left(H_X^{++} + \bar{H}_X^{++}\right), \end{aligned} \quad (2.49)$$

where the integration is over the analytic subspace of harmonic superspace,

$$d\zeta^{(-4)} := d^4x (D^-)^4, \quad (D^-)^4 := \frac{1}{16} (\bar{D}^-)^2 (D^-)^2, \quad (2.50)$$

and the u -integral denotes the integration over the group manifold SU(2) defined as in [46]. The second term in the analytic Lagrangian (2.49) involves naked Grassmann variables, however the action proves to be supersymmetric [52].

3 Chiral and analytic Goldstino superfields in supergravity

In this section we couple the chiral scalar (Φ and X) and the analytic (H^{ij}) Goldstino superfields, which have been described in the previous section, to $\mathcal{N} = 2$ supergravity and supersymmetric matter. Since the two chiral realisations have been shown to be equivalent, here we first provide the locally supersymmetric extension of X and then explain how to read off the curved analogue of Φ .

In this section we make use of the superspace formulation for $\mathcal{N} = 2$ conformal supergravity, which was developed in [53] and employed in [53, 54] to construct general off-shell supergravity-matter couplings.⁶ A brief summary of the corresponding curved superspace

⁶This formulation is often called SU(2) superspace, since the corresponding superspace structure group is $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)_R$, with $\text{SL}(2, \mathbb{C})$ being the universal cover of the Lorentz group $\text{SO}_0(3, 1)$. There exist two more superspace formulations for $\mathcal{N} = 2$ conformal supergravity, which are characterised by larger structure groups, specifically: (i) the U(2) superspace of [55] with the structure group $\text{SL}(2, \mathbb{C}) \times \text{U}(2)_R$, where $\text{U}(2)_R = \text{SU}(2)_R \times \text{U}(1)_R$ denotes the $\mathcal{N} = 2$ R -symmetry group; and (ii) the conformal superspace of [33], which naturally leads to the superconformal tensor calculus [56–58]. In the latter formulation, the entire $\mathcal{N} = 2$

geometry is given in appendix B. The reason this superspace geometry is suitable to describe $\mathcal{N} = 2$ conformal supergravity is that it is compatible with super-Weyl invariance. The point is that the algebra of covariant derivatives (B.3) preserves its functional form under the super-Weyl transformations [53]

$$\delta_\sigma \mathcal{D}_\alpha^i = \frac{1}{2} \bar{\sigma} \mathcal{D}_\alpha^i + \mathcal{D}^{\gamma i} \sigma M_{\gamma\alpha} - \mathcal{D}_{\alpha k} \sigma J^{ki}, \quad (3.1a)$$

$$\delta_\sigma \bar{\mathcal{D}}_{\dot{\alpha}i} = \frac{1}{2} \sigma \bar{\mathcal{D}}_{\dot{\alpha}i} + \bar{\mathcal{D}}_{\dot{i}}^{\dot{\gamma}} \bar{\sigma} \bar{M}_{\dot{\gamma}\dot{\alpha}} + \bar{\mathcal{D}}_{\dot{\alpha}}^k \bar{\sigma} J_{ki}, \quad (3.1b)$$

$$\delta_\sigma \mathcal{D}_a = \frac{1}{2} (\sigma + \bar{\sigma}) \mathcal{D}_a + \frac{i}{4} (\sigma_a)^\alpha{}_\beta \mathcal{D}_\alpha^k \sigma \bar{\mathcal{D}}_k^{\dot{\beta}} + \frac{i}{4} (\sigma_a)^\alpha{}_\beta \bar{\mathcal{D}}_k^{\dot{\beta}} \bar{\sigma} \mathcal{D}_\alpha^k - \frac{1}{2} \mathcal{D}^b (\sigma + \bar{\sigma}) M_{ab}, \quad (3.1c)$$

with the parameter σ being an arbitrary covariantly chiral superfield, $\bar{\mathcal{D}}_{\dot{\alpha}i} \sigma = 0$. The dimension-1 components of the torsion transform as follows:

$$\delta_\sigma S^{ij} = \bar{\sigma} S^{ij} - \frac{1}{4} \mathcal{D}^{\gamma(i} \mathcal{D}_{\gamma}^{j)} \sigma, \quad (3.2a)$$

$$\delta_\sigma Y_{\alpha\beta} = \bar{\sigma} Y_{\alpha\beta} - \frac{1}{4} \mathcal{D}_{(\alpha}^k \mathcal{D}_{\beta)k} \sigma, \quad (3.2b)$$

$$\delta_\sigma W_{\alpha\beta} = \sigma W_{\alpha\beta}, \quad (3.2c)$$

$$\delta_\sigma G_{\alpha\dot{\beta}} = \frac{1}{2} (\sigma + \bar{\sigma}) G_{\alpha\dot{\beta}} - \frac{i}{4} \mathcal{D}_{\alpha\dot{\beta}} (\sigma - \bar{\sigma}). \quad (3.2d)$$

As is seen from (3.2c), the covariantly chiral symmetric spinor $W_{\alpha\beta}$ transforms homogeneously, and therefore it is a superfield extension of the Weyl tensor, known as the $\mathcal{N} = 2$ super-Weyl tensor [55, 57, 61].

3.1 Two realisations for the chiral Goldstino superfield

The Goldstino superfield X is covariantly chiral,

$$\bar{\mathcal{D}}_{\dot{i}}^{\dot{\alpha}} X = 0, \quad (3.3)$$

and obeys the nilpotency constraints

$$X^2 = 0, \quad (3.4a)$$

$$X \mathcal{D}_A \mathcal{D}_B X = 0, \quad (3.4b)$$

$$X \mathcal{D}_A \mathcal{D}_B \mathcal{D}_C X = 0. \quad (3.4c)$$

We choose X to be inert under the super-Weyl transformations,

$$\delta_\sigma X = 0. \quad (3.5)$$

The constraints (3.4) are clearly super-Weyl invariant.

superconformal algebra is gauged in superspace. The three formulations prove to be equivalent, and they are also related to each other in the following sense: (i) SU(2) superspace is a gauge fixed version of U(2) superspace [59]; and (ii) U(2) superspace is a gauge fixed version of conformal superspace [33]. The most general off-shell $\mathcal{N} = 2$ supergravity-matter couplings were constructed in SU(2) superspace [53, 54], a few years before the conformal superspace was introduced. They can uniquely be lifted to U(2) superspace [59] and also to conformal superspace [60]. For certain applications, SU(2) superspace is the simplest formalism to deal with. We will use the conformal superspace setting in section 4.

As in the rigid supersymmetric case, the Goldstino superfield is subject to the additional requirement that ΔX is nowhere vanishing, $\Delta X \neq 0$, so that $(\Delta X)^{-1}$ is well defined. Here Δ denotes the complex conjugate of the $\mathcal{N} = 2$ chiral projection operator [62]

$$\begin{aligned}\bar{\Delta} &= \frac{1}{96} \left((\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij})\bar{\mathcal{D}}_{ij} - (\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}})\bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}} \right) \\ &= \frac{1}{96} \left(\bar{\mathcal{D}}_{ij}(\bar{\mathcal{D}}^{ij} + 16\bar{S}^{ij}) - \bar{\mathcal{D}}_{\dot{\alpha}\dot{\beta}}(\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} - 16\bar{Y}^{\dot{\alpha}\dot{\beta}}) \right),\end{aligned}\quad (3.6)$$

with $\bar{\mathcal{D}}^{\dot{\alpha}\dot{\beta}} := \bar{\mathcal{D}}_k^{(\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\beta})k}$. Its main properties can be summarised in terms of an arbitrary super-Weyl inert scalar superfield U as follows:

$$\bar{\mathcal{D}}_i^{\dot{\alpha}}\bar{\Delta}U = 0, \quad (3.7a)$$

$$\delta_\sigma U = 0 \quad \implies \quad \delta_\sigma \bar{\Delta}U = 2\sigma \bar{\Delta}U, \quad (3.7b)$$

$$\int d^4x d^4\theta d^4\bar{\theta} E U = \int d^4x d^4\theta \mathcal{E} \bar{\Delta}U. \quad (3.7c)$$

Here $E^{-1} = \text{Ber}(E_A{}^M)$ is the full superspace measure, and \mathcal{E} denotes the chiral density. The derivation of (3.7c) can be found in [63].

We postulate the Goldstino superfield action in curved superspace to be

$$\tilde{S}_{\text{Goldstino}} = \int d^4x d^4\theta d^4\bar{\theta} E \bar{X} X - \left\{ f \int d^4x d^4\theta \mathcal{E} W^2 X + \text{c.c.} \right\}, \quad (3.8)$$

as a natural curved-superspace extension of (2.18). Here W denotes the field strength of an Abelian vector multiplet. It is a covariantly chiral superfield, $\bar{\mathcal{D}}_i^{\dot{\alpha}}W = 0$, which is subject to the constraint⁷ [55, 64]

$$\Sigma^{ij} := \frac{1}{4} \left(\mathcal{D}^{ij} + 4S^{ij} \right) W = \frac{1}{4} \left(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij} \right) \bar{W}, \quad (3.9)$$

and is characterised by the super-Weyl transformation law

$$\delta_\sigma W = \sigma W. \quad (3.10)$$

It is assumed that W is nowhere vanishing, $W \neq 0$, and therefore it may be identified with one of the two supergravity compensators.⁸ The Goldstino action (3.8) is super-Weyl invariant.

The constraints (3.4) are preserved if X is locally rescaled,

$$X \rightarrow e^\tau X, \quad \bar{\mathcal{D}}_i^{\dot{\alpha}}\tau = 0, \quad (3.11)$$

for an arbitrary covariantly chiral scalar τ . Requiring the action (3.8) to be stationary under arbitrary displacements (3.11) gives

$$X = \Phi, \quad fW^2\Phi = \Phi\bar{\Delta}\bar{\Phi}. \quad (3.12)$$

⁷Every covariantly chiral superfield W under the additional reality condition (3.9) is called reduced chiral.

⁸The choice of the second compensator is not unique. Different choices lead to different off-shell formulations for $\mathcal{N} = 2$ supergravity [65].

The constraint on Φ is the curved-superspace generalisation of (2.12). Making use of (3.12), the Goldstino action (3.8) reduces to

$$S_{\text{Goldstino}} = -\frac{f}{2} \int d^4x d^4\theta \mathcal{E} W^2 \Phi + \text{c.c.} \quad (3.13)$$

3.2 Spontaneously broken supergravity

In this subsection we present two off-shell models for spontaneously broken $\mathcal{N} = 2$ supergravity. They are described by actions of the form

$$\tilde{S} = \tilde{S}_{\text{Goldstino}} + S_{\text{SUGRA}} . \quad (3.14a)$$

Here the Goldstino action is given by (3.8), and S_{SUGRA} stands for a pure supergravity action. Requiring this action to be stationary under arbitrary displacements (3.11) turns X into Φ defined by (3.12), and the action (3.14a) into

$$S = S_{\text{Goldstino}} + S_{\text{SUGRA}} , \quad (3.14b)$$

with $S_{\text{Goldstino}}$ being given by (3.13). Below we will consider two different off-shell formulations for $\mathcal{N} = 2$ supergravity.

Let us first consider the minimal formulation for $\mathcal{N} = 2$ supergravity with two compensators, the vector multiplet and the (improved) tensor multiplet, proposed in 1983 by de Wit, Philippe and Van Proeyen [65]. In superspace, the corresponding gauge-invariant supergravity action can be written in the form given in [66]

$$S_{\text{SUGRA}} = \frac{1}{\kappa^2} \int d^4x d^4\theta \mathcal{E} \left\{ \Psi \mathbb{W} - \frac{1}{4} W^2 + m \Psi W \right\} + \text{c.c.} \quad (3.15)$$

where κ is the gravitational constant, m the cosmological parameter, and \mathbb{W} denotes the following reduced chiral superfield⁹

$$\mathbb{W} := -\frac{G}{8} (\bar{\mathcal{D}}_{ij} + 4\bar{S}_{ij}) \left(\frac{G^{ij}}{G^2} \right) , \quad G = \sqrt{\frac{1}{2} G^{ij} G_{ij}} , \quad (3.16)$$

which is associated with the tensor multiplet. The tensor multiplet is usually described using its gauge invariant field strength G^{ij} , which is defined to be a real iso-triplet (that is, $G^{ij} = G^{ji}$ and $\bar{G}_{ij} := \overline{G^{ij}} = G_{ij}$) subject to the covariant constraints [39–41]

$$\mathcal{D}_\alpha^{(i} G^{jk)} = \bar{\mathcal{D}}_{\dot{\alpha}}^{(i} G^{jk)} = 0 , \quad (3.17)$$

with the super-Weyl transformation law

$$\delta_\sigma G^{ij} = (\sigma + \bar{\sigma}) G^{ij} . \quad (3.18)$$

The constraints (3.17) are solved [67–70] in terms of a covariantly chiral prepotential Ψ , $\bar{\mathcal{D}}_i^{\dot{\alpha}} \Psi = 0$, as follows:

$$G^{ij} = \frac{1}{4} (\mathcal{D}^{ij} + 4S^{ij}) \Psi + \frac{1}{4} (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}) \bar{\Psi} , \quad \bar{\mathcal{D}}_{\dot{\alpha}}^i \Psi = 0 . \quad (3.19)$$

⁹This multiplet was originally discovered in [65] using the superconformal tensor calculus. The regular procedure to derive \mathbb{W} within the superspace setting was given in [66].

The field strength G^{ij} is invariant under gauge transformations of the form

$$\delta_\Lambda \Psi = i\Lambda, \quad \bar{\mathcal{D}}_i^\alpha \Lambda = 0, \quad (\mathcal{D}^{ij} + 4S^{ij})\Lambda = (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\Lambda}, \quad (3.20)$$

with Λ being an arbitrary reduced chiral superfield. The action (3.15) is invariant under these gauge transformations, since both \mathbb{W} and W are reduced chiral superfields. The action (3.15) is also super-Weyl invariant, since the super-Weyl transformation laws of Ψ and \mathbb{W} are [54, 66]

$$\delta_\sigma \Psi = \sigma \Psi, \quad \delta_\sigma \mathbb{W} = \sigma \mathbb{W}. \quad (3.21)$$

Since the iso-vector superfield G^{ij} is one of the two supergravity compensators, its length G must be nowhere vanishing, $G \neq 0$.

To vary the action (3.14a) with respect to the vector multiplet, it is advantageous to represent W in the form [66]

$$W = \frac{1}{4} \bar{\Delta} (\mathcal{D}^{ij} + 4S^{ij}) V_{ij}, \quad (3.22)$$

where $\bar{\Delta}$ is the chiral projection operator (3.6). Here the unconstrained real iso-triplet $V_{ij} = V_{ji}$ is the curved-superspace extension of Mezincescu's prepotential [71] (see also [67]). The equation of motion for the vector multiplet is

$$\Sigma^{ij} - mG^{ij} = -2f\kappa^2 (H_X^{ij} + \bar{H}_X^{ij}), \quad H_X^{ij} := \frac{1}{4} (\mathcal{D}^{ij} + 4S^{ij})(WX). \quad (3.23)$$

In the limit $f \rightarrow 0$, this equation reduces to the one given in [66]. Since the tensor multiplet does not couple to the Goldstino superfield in (3.8), the equation of motion for the tensor multiplet is the same as in pure supergravity¹⁰ [66]

$$\mathbb{W} + mW = 0. \quad (3.24)$$

Making use of the nilpotency constraint (3.4a), from (3.23) we deduce

$$(\Sigma^{(2)} - mG^{(2)})^3 = 0, \quad (3.25)$$

where $\Sigma^{(2)}(v) = v_i v_j \Sigma^{ij}$ is the real $\mathcal{O}(2)$ multiplet associated with (3.9), and $G^{(2)}(v) = v_i v_j G^{ij}$. Eq. (3.25) is a nilpotency condition of degree 3. It tells us that we are dealing with nilpotent $\mathcal{N} = 2$ supergravity. The equation (3.23) is similar to that in spontaneously broken $\mathcal{N} = 1$ supergravity [21, 23], see appendix D for a review of the construction of [21].

The supergravity theory (3.15) possesses a dual formulation in which the tensor multiplet compensator is dualised into a polar hypermultiplet compensator [54]. To obtain the dual formulation, the first step is to recast the chiral action (3.15) as a projective action.

¹⁰In pure supergravity, the equation of motion for the $\mathcal{N} = 2$ gravitational superfield, which describes the Weyl multiplet, is $G - W\bar{W} = 0$, as demonstrated in [66]. This equation has a natural counterpart at the component level [65]. In the case of spontaneously broken supergravity described by the action (3.14a), this equation gets deformed by terms involving X and its conjugate, $G - W\bar{W}[1 + 2f\kappa^2(X + \bar{X})] = 0$.

Within the off-shell formulation for general supergravity-matter systems developed in [53], a universal locally supersymmetric action is given by

$$S = \frac{1}{2\pi} \oint (v, dv) \int d^4x d^4\theta d^4\bar{\theta} E \frac{W\bar{W}\mathcal{L}^{(2)}}{(\Sigma^{(2)})^2} . \quad (3.26)$$

The Lagrangian $\mathcal{L}^{(2)}(v)$ in (3.26) is a covariant projective multiplet of weight two, which is real with respect to the so-called smile conjugation, see [53] for the details. The projective Lagrangian corresponding to (3.26) was given in [54]. It is

$$\kappa^2 \mathcal{L}_{\text{SUGRA}}^{(2)} = G^{(2)} \ln \frac{G^{(2)}}{i\Upsilon^{(1)}\check{\Upsilon}^{(1)}} - \frac{1}{2} V \Sigma^{(2)} + m V G^{(2)} , \quad (3.27)$$

where $V(v)$ is the tropical prepotential for the vector multiplet, and $\Upsilon^{(1)}(v)$ is a weight-one arctic multiplet (both $\Upsilon^{(1)}$ and its smile-conjugate $\check{\Upsilon}^{(1)}$ are pure gauge degrees of freedom). The chiral field strength W is constructed in terms of the tropical prepotential as follows [72]:

$$W = \frac{1}{8\pi} \oint (v, dv) \left(\bar{\mathcal{D}}^{(-2)} + 4\bar{S}^{(-2)} \right) V(v) , \quad (3.28)$$

where we have used the notation defined in subsection 2.4. This field strength is invariant under gauge transformations of the form:

$$\delta_\lambda V = \lambda + \check{\lambda} , \quad (3.29)$$

with the gauge parameter $\lambda(v)$ being a covariant weight-zero arctic multiplet, and $\check{\lambda}$ its smile-conjugate.

Unlike the action (3.15), the tensor multiplet appears in (3.27) only via its gauge invariant field strength $G^{(2)}$. With reference to the vector multiplet, it appears in (3.15) only via its gauge invariant field strength, while the projective Lagrangian (3.27) involves the gauge prepotential V . The locally supersymmetric action generated by (3.27) is invariant under the gauge transformations (3.29).

Since the tensor multiplet compensator appears in the Lagrangian (3.27) only via the gauge invariant field strength, $G^{(2)}$, the tensor multiplet can be dualised into an off-shell polar hypermultiplet following the scheme described in [54]. The dual supergravity Lagrangian is

$$\kappa^2 \mathcal{L}_{\text{SUGRA,dual}}^{(2)} = -\frac{1}{2} V \Sigma^{(2)} - i \check{\Upsilon}^{(1)} e^{mV} \Upsilon^{(1)} . \quad (3.30)$$

Under the gauge transformation (3.29), the hypermultiplet varies as $\delta_\lambda \Upsilon^{(1)} = -m\lambda \Upsilon^{(1)}$ such that the supergravity action is gauge invariant.

3.3 Matter couplings

General matter couplings for the Goldstini are obtained by replacing the Goldstino action (3.8) with

$$\tilde{S}_{\text{Goldstino}} \longrightarrow \int d^4x d^4\theta d^4\bar{\theta} E \mathcal{N} \bar{X} X - \left\{ \int d^4x d^4\theta \mathcal{E} \mathcal{Z} X + \text{c.c.} \right\} . \quad (3.31)$$

Here \mathcal{N} is a super-Weyl invariant real scalar, $\delta_\sigma \mathcal{N} = 0$, while \mathcal{Z} is a covariantly chiral scalar, $\bar{\mathcal{D}}_i^{\dot{\alpha}} \mathcal{Z} = 0$, with the super-Weyl transformation law $\delta_\sigma \mathcal{Z} = 2\sigma \mathcal{Z}$. If the supersymmetric matter consists of a set of Abelian vector multiplets described by covariantly chiral field strengths W_I , then $\mathcal{N} = \mathcal{N}(W_I, \bar{W}_J)$ and $\mathcal{Z} = \mathcal{Z}(W_I)$. In order to guarantee the required super-Weyl transformation laws, the composites $\mathcal{N}(W_I, \bar{W}_J)$ and $\mathcal{Z}(W_I)$ must be assigned certain homogeneity properties. Some of the reduced chiral superfields W_I may be composite. For instance, we may choose

$$\mathcal{Z} = W \mathfrak{W}, \quad \mathfrak{W} = \frac{1}{8\pi} \oint(v, dv) \left(\bar{\mathcal{D}}^{(-2)} + 4\bar{S}^{(-2)} \right) K(\Upsilon^a, \check{\Upsilon}^{\bar{b}}), \quad (3.32)$$

where W is the chiral compensator, and $K(\varphi^a, \bar{\varphi}^{\bar{b}})$ is the Kähler potential of a real analytic Kähler manifold, with $a, \bar{b} = 1, \dots, n$. The hypermultiplet variables $\Upsilon^a(v)$ in (3.32) are covariant weight-zero arctic multiplets, and $\check{\Upsilon}^{\bar{b}}$ denotes the smile conjugate of Υ^b , see [53] for the technical details. The reduced chiral superfield \mathfrak{W} is invariant under Kähler transformations [72]

$$K(\Upsilon, \check{\Upsilon}) \rightarrow K(\Upsilon, \check{\Upsilon}) + \Lambda(\Upsilon) + \bar{\Lambda}(\check{\Upsilon}), \quad (3.33)$$

with $\Lambda(\varphi)$ being a holomorphic function. The action (3.31) with \mathcal{Z} given by (3.32) describes the off-shell coupling of the Goldstino superfield to hypermultiplets.

Requiring the action (3.31) to be stationary under arbitrary displacements (3.11) gives

$$X = \Phi, \quad \mathcal{Z}\Phi = \Phi \bar{\Delta}(\mathcal{N}\bar{\Phi}), \quad (3.34)$$

which is a consistent deformation of the constraint (3.12).

3.4 Analytic Goldstino superfields

We now recast the Goldstino action (3.13) in terms of a linear Goldstino superfield. This is achieved by recalling the observation [54, 63] that the $\mathcal{N} = 2$ locally supersymmetric chiral action

$$S_{\text{chiral}} = \int d^4x d^4\theta \mathcal{E} \mathcal{L}_c + \text{c.c.}, \quad \bar{\mathcal{D}}_i^{\dot{\alpha}} \mathcal{L}_c = 0, \quad (3.35)$$

can be realised as a projective superspace action:

$$S_{\text{chiral}} = \frac{1}{2\pi} \oint(v, dv) \int d^4x d^4\theta d^4\bar{\theta} E \frac{W\bar{W} \mathcal{L}_c^{(2)}}{(\Sigma^{(2)})^2},$$

$$\mathcal{L}_c^{(2)} = \frac{1}{4} V \left\{ (\mathcal{D}^{(2)} + 4S^{(2)}) \frac{\mathcal{L}_c}{W} + (\bar{\mathcal{D}}^{(2)} + 4\bar{S}^{(2)}) \frac{\bar{\mathcal{L}}_c}{\bar{W}} \right\}, \quad (3.36)$$

where V is the tropical prepotential for the vector multiplet with field strength W . Applying this general result to the Goldstino action (3.13) gives the projective Lagrangian

$$\mathcal{L}_{\text{Goldstino}}^{(2)} = -\frac{f}{2} V(H^{(2)} + \bar{H}^{(2)}) \equiv -fV\mathbb{H}^{(2)}, \quad (3.37)$$

where we have introduced the complex linear Goldstino superfield

$$H^{(2)} = v_i v_j H^{ij}, \quad H^{ij} = \frac{1}{4}(\mathcal{D}^{ij} + 4S^{ij})(W\Phi). \quad (3.38)$$

It obeys the analyticity constraints

$$\mathcal{D}_\alpha^{(1)} H^{(2)} = \bar{\mathcal{D}}_\alpha^{(1)} H^{(2)} = 0 \iff \mathcal{D}_\alpha^{(i)} H^{jk} = \bar{\mathcal{D}}_\alpha^{(i)} H^{jk} = 0, \quad (3.39)$$

as well as the nilpotency condition

$$H^{(ij} H^{kl)} = 0 \iff (H^{(2)})^2 = 0. \quad (3.40)$$

In terms of the real linear superfield $\mathbb{H}^{(2)}$, the nilpotency condition is

$$(\mathbb{H}^{(2)})^3 = 0. \quad (3.41)$$

We now show how the spontaneously broken supergravity (3.14a) can be reformulated as a nilpotent $\mathcal{N} = 2$ supergravity theory. Varying the action (3.14a) with respect to the tensor multiplet compensator leads to the equation (3.23). We then require the action (3.14a) to be stationary under arbitrary displacements (3.11), which implies that $X = \Phi$ and the action (3.14a) turns into (3.14b). Then, the equation (3.23) takes the form

$$\Sigma^{(2)} - mG^{(2)} = -4f\kappa^2 \mathbb{H}^{(2)}. \quad (3.42)$$

As a result, the projective Lagrangian for the theory (3.14b) can be written as

$$\kappa^2 \mathcal{L}^{(2)} = G^{(2)} \ln \frac{G^{(2)}}{i\Upsilon^{(1)} \bar{\Upsilon}^{(1)}} - \frac{1}{4} V \Sigma^{(2)} + \frac{3}{4} m V G^{(2)}. \quad (3.43)$$

This has the form of the supergravity Lagrangian (3.27) with rescaled parameters. The two conformal compensators have to obey the nilpotency condition (3.25) as well as curved-superspace analogues of the nonlinear constraints (2.31).

4 Spinor Goldstino superfields in supergravity

In this section, we provide a curved-superspace extension of the spinor Goldstino superfield Ξ_i^α defined by the constraints (2.3a) and (2.3b). It is known that some superfield representations of Poincaré supersymmetry cannot be lifted to the locally supersymmetric case. In particular, covariantly chiral $\mathcal{N} = 2$ superfields with Lorentz and $SU(2)$ indices cannot be defined in general. This means that there is no straightforward curved-superspace generalisation of the spinor Goldstino superfield $\bar{\Psi}_\alpha^i$ defined by the constraints (2.3c) and (2.3d). Fortunately, the constraints on Ξ_i^α allow for a supergravity analogue.

In order to lift Ξ_i^α to supergravity, it is advantageous to employ the superspace formulation for $\mathcal{N} = 2$ conformal supergravity developed by Butter¹¹ [33] and further elaborated

¹¹Conformal superspace was originally constructed for 4D $\mathcal{N} = 1$ supergravity [73] and then extended to the 4D $\mathcal{N} = 2$ [33] case, 3D \mathcal{N} -extended conformal supergravity [74], 5D conformal supergravity [75] and recently to the 6D $\mathcal{N} = (1, 0)$ case [76].

in [77]. We denote by $\nabla_A = (\nabla_a, \nabla_\alpha^i, \bar{\nabla}_i^{\dot{\alpha}})$ the corresponding superspace covariant derivatives. Throughout this section we use the notation and various results from ref. [77]. Appendix C includes those technical details on conformal superspace which are relevant for our analysis.

We proceed by lifting the Goldstino superfield X , which has so far been defined in $SU(2)$ superspace, to conformal superspace. Such a reformulation is unique if X is required to be primary, in addition to being covariantly chiral. Then the superconformal properties of X are:

$$\mathbb{D}X = 0, \quad YX = 0, \quad K^A X = 0, \quad \bar{\nabla}_i^{\dot{\alpha}} X = 0, \quad (4.1)$$

where \mathbb{D} , Y and $K^A = (K^a, S_i^\alpha, \bar{S}_\alpha^i)$ are respectively the dilatation, $U(1)_R$, special conformal and S -supersymmetry generators of the $\mathcal{N} = 2$ superconformal algebra. The nilpotency constraints (3.4) turn into

$$X^2 = 0, \quad (4.2a)$$

$$X \nabla_A \nabla_B X = 0, \quad (4.2b)$$

$$X \nabla_A \nabla_B \nabla_C X = 0. \quad (4.2c)$$

As in $SU(2)$ superspace, ΔX is required to be nowhere vanishing, $\Delta X \neq 0$, where the covariantly antichiral projection operator is

$$\Delta := \frac{1}{48} \nabla^{ij} \nabla_{ij} = -\frac{1}{48} \nabla^{\alpha\beta} \nabla_{\alpha\beta}, \quad \nabla_{ij} := \nabla_{(i}^{\gamma} \nabla_{\gamma j)}, \quad \nabla_{\alpha\beta} := \nabla_{(\alpha}^k \nabla_{\beta)k}, \quad (4.3)$$

and this expression for Δ is much simpler than the same operator in $SU(2)$ superspace, eq. (3.6). For every primary superfield U with the properties $\mathbb{D}U = 0$, $YU = 0$ and $K^A U = 0$, ΔU proves to be an antichiral primary superfield of dimension 2, that is: $\mathbb{D}\Delta U = 2\Delta U$, $Y\Delta U = 4\Delta U$, $K^A \Delta U = 0$ and $\nabla_\alpha^i \Delta U = 0$. An example of a superfield U is provided by X . The Goldstino action (3.8) is uniquely lifted to conformal superspace to describe the dynamics of X .

As in the flat case, the nilpotent chiral superfield X contains two independent component fields which can be identified with the θ -independent components of the descendants $\chi_i^\alpha := -\frac{1}{12} \nabla^{\alpha j} \nabla_{ij} X$ and ΔX . Unlike X and ΔX , the spinor superfield χ_i^α is not primary. It has two obvious properties:

$$X \chi_i^\alpha = 0, \quad (4.4a)$$

$$\nabla_\beta^j \chi_i^\alpha = \delta_\beta^\alpha \delta_i^j \Delta X. \quad (4.4b)$$

It is much more difficult to derive the following relation

$$\begin{aligned} \bar{\nabla}_j^{\dot{\beta}} \chi_i^\alpha &= -i(\Delta X)^{-1} \left(\varepsilon_{ij} \chi^{\alpha k} \nabla^{\gamma \dot{\beta}} \chi_{\gamma k} + \varepsilon_{ij} \chi_\gamma^k \nabla^{\gamma \dot{\beta}} \chi_k^\alpha + 2\chi_{(i}^\gamma \nabla^{\alpha \dot{\beta}} \chi_{\gamma j)} \right) \\ &+ (\Delta X)^{-2} \left(i\chi_{ij} \nabla^{\alpha \dot{\beta}} \Delta X - i\varepsilon_{ij} \chi^{\alpha \gamma} \nabla_\gamma^{\dot{\beta}} \Delta X + \frac{2}{3} \varepsilon_{ij} (\bar{\nabla}_\gamma^k \bar{W}^{\dot{\beta} \gamma}) \chi_{kl} \chi^{\alpha l} \right) \\ &- \frac{4i}{3} \varepsilon_{ij} (\Delta X)^{-3} \bar{W}^{\dot{\beta} \gamma} \chi_{kl} \chi^{\gamma k} \nabla_\gamma^\alpha \chi_\gamma^l + 3i\varepsilon_{ij} (\Delta X)^{-4} \bar{W}^{\dot{\beta} \gamma} \chi^4 \nabla_\gamma^\alpha \Delta X, \end{aligned} \quad (4.4c)$$

which involves the super-Weyl tensor. To completely specify the properties of χ_i^α , we also need its S -supersymmetry transformations

$$\bar{S}_{\dot{\beta}}^j \chi_i^\alpha = 0, \quad S_j^\beta \chi_i^\alpha = 2(\Delta X)^{-1} \left(\varepsilon^{\alpha\beta} \chi_{ij} + \varepsilon_{ij} \chi^{\alpha\beta} \right). \quad (4.5)$$

As in the flat-superspace case, we make use of the definitions: $\chi^{ij} := \chi^{\alpha i} \chi_\alpha^j$, $\chi_{\alpha\beta} := \chi_\alpha^k \chi_{k\beta}$ and $\chi^4 := \frac{1}{3} \chi^{ij} \chi_{ij} = -\frac{1}{3} \chi^{\alpha\beta} \chi_{\alpha\beta}$.

Making use of the constraints (4.2), it is possible to prove that X is a composite superfield constructed from χ_i^α ,

$$X = \frac{\chi^4}{(\Delta X)^3}, \quad \Delta X = \frac{1}{4} \nabla_\alpha^i \chi_i^\alpha. \quad (4.6)$$

Using this representation, the nilpotency conditions (4.2) are satisfied identically.

In the super-Poincaré case, the spinor Goldstino superfield Ξ_i^α was constructed from X according to (2.23). In the supergravity framework, we make use of a similar definition,

$$\Xi_i^\alpha := f \frac{\chi_i^\alpha}{\Delta X}, \quad (4.7)$$

with χ_i^α and ΔX given above. The superfield (4.7) proves to satisfy the following constraints:

$$\nabla_\beta^j \Xi_i^\alpha = f \delta_\beta^\alpha \delta_i^j, \quad (4.8a)$$

$$\begin{aligned} \bar{\nabla}_j^{\dot{\beta}} \Xi_i^\alpha &= 2i f^{-1} \Xi_{\gamma j} \nabla^{\gamma \dot{\beta}} \Xi_i^\alpha - i \varepsilon_{ij} f^{-3} (\nabla_{\dot{\gamma}}^\alpha \bar{W}^{\dot{\beta} \dot{\gamma}}) \Xi^4 - i \varepsilon_{ij} f^{-3} \bar{W}^{\dot{\beta} \dot{\gamma}} \nabla_{\dot{\gamma}}^\alpha \Xi^4 \\ &\quad - 2i \varepsilon_{ij} f^{-3} \bar{W}^{\dot{\beta} \dot{\gamma}} \Xi_{kl} \Xi^{\alpha k} \nabla_{\dot{\gamma}} \Xi^{\gamma l} - \frac{4i}{3} f^{-3} \bar{W}^{\dot{\beta} \dot{\gamma}} \Xi_{k(i} \Xi^{\alpha k} \nabla_{\dot{\gamma}} \Xi_{j)}^\gamma \\ &\quad - \frac{1}{3} \varepsilon_{ij} f^{-2} (\bar{\nabla}_{\dot{\gamma}}^k \bar{W}^{\dot{\beta} \dot{\gamma}}) \Xi_{kl} \Xi^{\alpha l} - \frac{2}{3} f^{-2} (\bar{\nabla}_{\dot{\gamma}(i} \bar{W}^{\dot{\beta} \dot{\gamma}}) \Xi_{j)k} \Xi^{\alpha k}, \end{aligned} \quad (4.8b)$$

which are the curved-superspace generalisation of the constraints (2.3a) and (2.3b).

As with χ_i^α , Ξ_i^α is not primary. Its superconformal properties are determined by the relations

$$S_j^\beta \Xi_i^\alpha = 2\varepsilon^{\alpha\beta} f^{-1} \Xi_{ij} + 2\varepsilon_{ij} f^{-1} \Xi^{\alpha\beta}, \quad \bar{S}_{\dot{\beta}}^j \Xi_i^\alpha = 0. \quad (4.9)$$

On the other hand, the composites χ^4 and Ξ^4 turn out to be primary superfields,

$$K^A \chi^4 = 0, \quad K^A \Xi^4 = 0. \quad (4.10)$$

An important property of Ξ^4 is

$$\bar{\nabla}_i^{\dot{\alpha}} \Xi^4 = -2i \Xi^4 \nabla^{\gamma \dot{\alpha}} \Xi_{\gamma i}. \quad (4.11)$$

This relation can be used to check that $X = \Xi^4 \Delta X$ is chiral. The same relation is useful to show that the dimensionless primary chiral scalar

$$\Phi := \frac{1}{f^7} W^{-2} \bar{\Delta} (W^2 \bar{W}^2 \Xi^4 \bar{\Xi}^4) = \frac{1}{f^7} \bar{\Delta} (\bar{W}^2 \Xi^4 \bar{\Xi}^4) \quad (4.12)$$

has the following properties:

$$\Phi^2 = 0, \quad \Phi \nabla_A \nabla_B \Phi = 0, \quad \Phi \nabla_A \nabla_B \nabla_C \Phi = 0, \quad (4.13a)$$

$$f\Phi = W^{-2} \Phi \bar{\Delta} \bar{\Phi}. \quad (4.13b)$$

This primary chiral scalar is the unique extension of the irreducible Goldstino superfield (3.12) to conformal superspace. It is worth pointing out that (4.11) implies that Φ defined by (4.12) is proportional to Ξ^4 .

The action for the Goldstino superfield Ξ_i^α coupled to supergravity is given by

$$S_{\text{Goldstino}} = -\frac{1}{f^6} \int d^4x d^4\theta d^4\bar{\theta} E W^2 \bar{W}^2 \Xi^4 \bar{\Xi}^4. \quad (4.14)$$

It can be recast in the form (3.13) if we make use of (4.12).

It is important to observe that the constraints (4.8) allow for the following unitary gauge condition

$$\Xi_i^\alpha|_{\theta=0} = 0, \quad (4.15)$$

which completely fixes the local Q -supersymmetry invariance. We now evaluate the Goldstino action (4.14) in this gauge and show that it generates a positive contribution to the cosmological constant upon imposing standard superconformal gauge conditions.

First of all, we recall that any action given by an integral over the full superspace can equivalently be represented as an integral over the chiral subspace,

$$\int d^4x d^4\theta d^4\bar{\theta} E \mathcal{L} = \int d^4x d^4\theta \mathcal{E} \bar{\Delta} \mathcal{L}. \quad (4.16)$$

Next, reducing the chiral action to components gives

$$S = \int d^4x e \left(\Delta + \dots \right) \bar{\Delta} \mathcal{L} \Big|_{\theta=0}. \quad (4.17)$$

Here the ellipsis denotes terms involving supergravity fields and at most three spinor derivatives (see [33, 77] for the complete expression). In the *unitary gauge*, it is easy to see that the component reduction of the action (4.14) is

$$S_{\text{Goldstino}} = -\frac{1}{f^6} \int d^4x e W^2 \bar{W}^2 (\Delta \Xi^4) (\bar{\Delta} \bar{\Xi}^4) \Big|_{\theta=0} = -f^2 \int d^4x e W^2 \bar{W}^2 \Big|_{\theta=0}. \quad (4.18)$$

Here we have used the fact that in the unitary gauge we have

$$\nabla_\beta^j \Xi_i^\alpha|_{\theta=0} = f \delta_\beta^\alpha \delta_i^j, \quad \bar{\nabla}_{\dot{j}} \Xi_i^\alpha|_{\theta=0} = 0, \quad (4.19)$$

together with $\Delta \Xi^4|_{\theta=0} = f^4$. We also have to fix the local dilatation, $U(1)_R$ and superconformal (K^A) symmetries in a standard way [65] in order to end up with the canonically normalised Einstein-Hilbert action. In superspace this requires choosing the gauge $W = 1$. The final expression for the cosmological constant proves to be

$$\Lambda = f^2 - 3 \frac{m^2}{\kappa^2}. \quad (4.20)$$

Here the second term on the right comes from the supersymmetric cosmological term in the supergravity action (3.15).

Let us conclude this section with a few comments. As mentioned above, Ξ_i^α is not a primary superfield. However, with the aid, for instance, of the chiral compensator W , a primary extension of Ξ_i^α can be constructed. It turns out that the superfield

$$\Xi_i^\alpha = -\frac{1}{12W\Delta X} \left(\nabla^{\alpha j} - 3W^{-1}(\nabla^{\alpha j}W) \right) \nabla_{ij}(WX) \quad (4.21)$$

is primary. Its evaluation gives

$$\begin{aligned} \Xi_i^\alpha &= \Xi_i^\alpha - \frac{1}{2W}(\nabla^{\alpha j}W)\Xi_{ij} - \frac{1}{2W}(\nabla_{\beta i}W)\Xi^{\alpha\beta} \\ &\quad - \frac{1}{6W} \left((\nabla_{ij}W) - W^{-1}(\nabla_{(i}^\gamma W)(\nabla_{\gamma j)}W) \right) \Xi^{jk}\Xi_k^\alpha \\ &\quad + \frac{1}{6W} \left((\nabla^{\alpha\beta}W) - 3W^{-1}(\nabla^{(\alpha k}W)(\nabla_k^{\beta)}W) \right) \Xi_{ij}\Xi_\beta^j \\ &\quad - \frac{1}{12W} \left((\nabla^{\alpha j}\nabla_{ij}W) - 3W^{-2}(\nabla^{\alpha j}W)(\nabla_{ij}W) \right) \Xi^4. \end{aligned} \quad (4.22)$$

In the derivation of (4.22), we have only used the fact that W is a chiral primary superfield. The Bianchi identity $\nabla^{ij}W = \bar{\nabla}^{ij}\bar{W}$ has not been used at all, and therefore the above construction does not require W to be the field strength of a vector multiplet.

Multiplying (4.22) by ΔX gives a primary extension of χ_i^α ,

$$\chi_i^\alpha = f^{-1}\Xi_i^\alpha\Delta X. \quad (4.23)$$

It holds that

$$\Xi^4 = \Xi^4, \quad \chi^4 = \chi^4. \quad (4.24)$$

For this reason the field redefinition $\Xi_i^\alpha \rightarrow \Xi_i^\alpha$ does not affect any models constructed in terms of X or Ξ^4 .

5 Generalisations

In conclusion we consider two generalisations inspired by the discussion in this paper.

5.1 \mathcal{N} -extended case

Whilst the results in equations (2.3) were derived in [1] with the case of $\mathcal{N} = 2$ supersymmetry in mind, they apply for arbitrary \mathcal{N} -extended supersymmetry in four spacetime dimensions.¹² This is because they are derived from the coset parametrisation (2.1) using only the anti-commutator

$$\{Q_\alpha^i, \bar{Q}_{\dot{\alpha}j}\} = 2P_{\alpha\dot{\alpha}}\delta_j^i, \quad i, j = 1, \dots, \mathcal{N} \quad (5.1)$$

¹²The $\mathcal{N} = 1$ case was also considered in [1].

and the conjugation rule $Q_\alpha^{i\dagger} = \bar{Q}_{\dot{\alpha}i}$, which are still applicable regardless of the range of the index i . However, the chiral action (2.6) is specific to the $\mathcal{N} = 2$ case, and in the general \mathcal{N} -extended case it must be replaced with

$$S = -\frac{1}{2f^{2(\mathcal{N}-1)}} \int d^4x d^{2\mathcal{N}}\theta \Psi^{2\mathcal{N}} - \frac{1}{2f^{2(\mathcal{N}-1)}} \int d^4x d^{2\mathcal{N}}\bar{\theta} \bar{\Psi}^{2\mathcal{N}}, \quad (5.2)$$

where we have introduced the chiral scalar

$$\Psi^{2\mathcal{N}} = \frac{2^\mathcal{N}}{\mathcal{N}!(\mathcal{N}+1)!} \Psi_{i_1 j_1} \dots \Psi_{i_\mathcal{N} j_\mathcal{N}} \varepsilon^{i_1 \dots i_\mathcal{N}} \varepsilon^{j_1 \dots j_\mathcal{N}} = \frac{2^\mathcal{N}}{(\mathcal{N}+1)!} \det(\Psi_{ij}), \quad (5.3)$$

and as earlier, $\Psi_{ij} := \Psi_i^\alpha \Psi_{\alpha j}$. The normalisation of the composite superfield $\Psi^{2\mathcal{N}}$ is chosen so that $\Psi^{2\mathcal{N}} = \Psi_{11} \Psi_{22} \dots \Psi_{\mathcal{N}\mathcal{N}}$. The determinant form of the \mathcal{N} -extended chiral Lagrangian, eq. (5.3), makes it analogous to the Volkov-Akulov theory [24–26].

5.2 Generalisation of the Lindström-Roček construction

Lindström and Roček [18] proposed to describe the Goldstino using a real scalar superfield V , which is nilpotent, $V^2 = 0$, and obeys the nonlinear constraint

$$f\bar{S}_0 S_0 V = \frac{1}{16} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R) \mathcal{D}_\alpha V, \quad (5.4)$$

with S_0 the chiral compensator, $\bar{\mathcal{D}}_\alpha S_0 = 0$, for the old minimal formulation for $\mathcal{N} = 1$ supergravity [78–80].¹³ Actually, V was realised in [18] only as a composite superfield,

$$f\bar{S}_0 S_0 V = \bar{\phi} \phi, \quad (5.5)$$

constructed from the covariantly chiral scalar Goldstino superfield ϕ (which is the curved-superspace extension of Roček’s nilpotent superfield [6]) constrained by

$$\bar{\mathcal{D}}_\alpha \phi = 0, \quad \phi^2 = 0, \quad f S_0^2 \phi = -\frac{1}{4} \phi (\bar{\mathcal{D}}^2 - 4R) \bar{\phi}, \quad (5.6)$$

compare with (1.1). If instead V is viewed as a fundamental Goldstino superfield, then it has been shown [14] that one has to impose the three nilpotency constraints

$$V^2 = 0, \quad (5.7a)$$

$$V \mathcal{D}_A \mathcal{D}_B V = 0, \quad (5.7b)$$

$$V \mathcal{D}_A \mathcal{D}_B \mathcal{D}_C V = 0, \quad (5.7c)$$

in addition to (5.4). It is also necessary to require that the descendant $\mathcal{D}^\alpha W_\alpha$ is nowhere vanishing, where

$$W_\alpha = -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4R) \mathcal{D}_\alpha V. \quad (5.8)$$

¹³Here we use the notation S_0 for the chiral compensator following [81, 82]. In the superspace literature reviewed in [83], it is usually denoted Φ . The super-Weyl gauge $S_0 = 1$ was used in [18].

The constraints (5.4) and (5.7) guarantee that V contains a single independent component field — the Goldstino, which is the lowest (θ -independent) component of W_α . The Goldstino action is

$$S_{\text{Goldstino}} = -f \int d^4x d^2\theta d^2\bar{\theta} E \bar{S}_0 S_0 V. \quad (5.9)$$

The constraints (5.4) and (5.7), as well as the action (5.9), are invariant under super-Weyl transformations [84] of the form

$$\delta_\sigma \mathcal{D}_\alpha = \left(\bar{\sigma} - \frac{1}{2} \sigma \right) \mathcal{D}_\alpha + (\mathcal{D}^\beta \sigma) M_{\alpha\beta}, \quad (5.10a)$$

$$\delta_\sigma \bar{\mathcal{D}}_{\dot{\alpha}} = \left(\sigma - \frac{1}{2} \bar{\sigma} \right) \bar{\mathcal{D}}_{\dot{\alpha}} + (\bar{\mathcal{D}}^{\dot{\beta}} \bar{\sigma}) \bar{M}_{\dot{\alpha}\dot{\beta}}, \quad (5.10b)$$

$$\begin{aligned} \delta_\sigma \mathcal{D}_{\alpha\dot{\alpha}} &= \frac{1}{2} (\sigma + \bar{\sigma}) \mathcal{D}_{\alpha\dot{\alpha}} + \frac{i}{2} (\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\sigma}) \mathcal{D}_\alpha + \frac{i}{2} (\mathcal{D}_\alpha \sigma) \bar{\mathcal{D}}_{\dot{\alpha}} \\ &\quad + (\mathcal{D}^\beta{}_{\dot{\alpha}} \sigma) M_{\alpha\beta} + (\mathcal{D}_\alpha{}^{\dot{\beta}} \bar{\sigma}) \bar{M}_{\dot{\alpha}\dot{\beta}}, \end{aligned} \quad (5.10c)$$

where σ is an arbitrary covariantly chiral scalar superfield, $\bar{\mathcal{D}}_{\dot{\alpha}} \sigma = 0$. It is assumed that V is super-Weyl inert, while S_0 transforms as $\delta_\sigma S_0 = \sigma S_0$.

It is possible to follow a different path than the one just discussed, in the spirit of the nilpotent $\mathcal{N} = 1$ chiral construction of [12, 13]. Specifically, we consider a Goldstino superfield V which only obeys the nilpotency constraints (5.7), in conjunction with the requirement that $\mathcal{D}^\alpha W_\alpha$ is nowhere vanishing. It has two independent component fields, the Goldstino $W_\alpha|_{\theta=0}$ and the auxiliary scalar $\mathcal{D}^\alpha W_\alpha|_{\theta=0}$. One may show that the constraints (5.7) imply the representation¹⁴

$$V = -4 \frac{W^2 \bar{W}^2}{(\mathcal{D}^\alpha W_\alpha)^3}, \quad W^2 = W^\alpha W_\alpha, \quad (5.11)$$

which ensures (5.7) is identically satisfied. The relation (5.11) is super-Weyl invariant, since W_α and $\mathcal{D}^\alpha W_\alpha$ transform as super-Weyl primary superfields, $\delta_\sigma W_\alpha = \frac{3}{2} \sigma W_\alpha$ and $\delta_\sigma (\mathcal{D}^\alpha W_\alpha) = (\sigma + \bar{\sigma}) \mathcal{D}^\alpha W_\alpha$.

The constraints (5.7) are invariant under local re-scalings of V

$$V \rightarrow e^\tau V, \quad (5.12)$$

with τ an arbitrary real scalar superfield. The dynamics of this supermultiplet is governed by the action

$$\tilde{S}_{\text{Goldstino}} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \frac{1}{16} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R) \mathcal{D}_\alpha V - 2f \bar{S}_0 S_0 V \right\}. \quad (5.13)$$

Varying the Goldstino superfield according to $\delta V = \tau V$, with τ being arbitrary, gives the constraint (5.4) as the corresponding equation of motion. Then the action (5.13) reduces to (5.9).

¹⁴In the case that V obeys the constraint (5.4), the relation (5.11) reduces to $(f \bar{S}_0 S_0)^3 V = 16 W^2 \bar{W}^2$, which was derived in [14] in the flat-superspace limit.

Within the new minimal formulation for $\mathcal{N} = 1$ supergravity [85], the compensator is a real covariantly linear scalar superfield,

$$(\bar{\mathcal{D}}^2 - 4R)\mathbb{L} = 0, \quad \bar{\mathbb{L}} = \mathbb{L}, \quad (5.14)$$

with the super-Weyl transformation $\delta_\sigma \mathbb{L} = (\sigma + \bar{\sigma})\mathbb{L}$, see [34, 82, 83] for reviews. The action for supergravity coupled to the Goldstino superfield V is

$$S = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \frac{3}{\kappa^2} \mathbb{L} \ln \frac{\mathbb{L}}{|S_0|^2} + \frac{1}{16} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R) \mathcal{D}_\alpha V - 2f\mathbb{L}V \right\}, \quad (5.15)$$

where now S_0 is a purely gauge degree of freedom.

If V is a real unconstrained superfield, the action (5.15) describes new minimal supergravity coupled to a massless vector supermultiplet with a Fayet-Iliopoulos term (see, e.g., [82]). The action is invariant under $U(1)$ gauge transformations $\delta V = \lambda + \bar{\lambda}$, with the gauge parameter λ being chiral, $\bar{\mathcal{D}}_{\dot{\alpha}} \lambda = 0$. However, in our case V is subject to the nilpotency conditions (5.7), which are incompatible with the gauge invariance. These nilpotency conditions guarantee that the Goldstino and the auxiliary field are the only independent component fields of V .

An important feature of unbroken new minimal supergravity is that it does not allow any supersymmetric cosmological term [86, 87].¹⁵ Our action for spontaneously broken supergravity (5.15) leads, at the component level, to a positive cosmological constant which is generated by the Goldstino superfield. The cosmological constant in (5.15) is strictly positive since there is no supersymmetric cosmological term producing a negative contribution.

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A The component content of Φ

In this appendix we elaborate on the component content of the chiral scalar Goldstino superfield defined by the constraints (2.11) and (2.12). For this it is relevant to introduce the two descendants of Φ :

$$\chi_i^\alpha := -\frac{1}{12} D^{\alpha j} D_{ij} \Phi = -\frac{1}{12} D_{\beta i} D^{\alpha \beta} \Phi, \quad F := D^4 \Phi. \quad (A.1)$$

¹⁵Among the known off-shell formulations for $\mathcal{N} = 1$ (see [34, 83] for reviews), supersymmetric cosmological terms exist only for the old minimal supergravity [88, 89] and the $n = -1$ non-minimal supergravity as formulated in [90].

The Goldstini may be identified with $\chi_i^\alpha|_{\theta=0}$. By assumption, the field $F|_{\theta=0}$ is nowhere vanishing. The constraints (2.11) prove to imply the relations:

$$D_\alpha^i \Phi = \frac{4}{3} \chi_{\alpha j} \chi^{ij} F^{-2} = \frac{4}{3} \chi^{\beta i} \chi_{\alpha \beta} F^{-2}, \quad (\text{A.2a})$$

$$D^{ij} \Phi = -4 \chi^{ij} F^{-1}, \quad (\text{A.2b})$$

$$D_{\alpha \beta} \Phi = 4 \chi_{\alpha \beta} F^{-1}, \quad (\text{A.2c})$$

$$\Phi = \chi^4 F^{-3}, \quad (\text{A.2d})$$

where we have introduced the composites

$$\chi^{ij} = \chi^{\alpha i} \chi_\alpha^j = \chi^{ji}, \quad \chi_{\alpha \beta} = \chi_\alpha^i \chi_{\beta i} = \chi_{\beta \alpha}, \quad \chi^4 := \frac{1}{3} \chi^{ij} \chi_{ij} = -\frac{1}{3} \chi^{\alpha \beta} \chi_{\alpha \beta}. \quad (\text{A.3})$$

The relations (A.2) imply that all the components of Φ are expressed in terms of $\chi_i^\alpha|_{\theta=0}$ and $F|_{\theta=0}$. Furthermore, by applying the operator D^4 to both sides of (2.12) one can derive the following nonlinear equation on F and its conjugate

$$\begin{aligned} fF = & -2i \chi_i^\alpha \partial_{\alpha \dot{\alpha}} \bar{\chi}^{\dot{\alpha} i} + F \bar{F} + \frac{\chi^{ij}}{F} \square \frac{\bar{\chi}_{ij}}{F} - \frac{\chi^{\alpha \beta}}{F} \partial_{\alpha \dot{\alpha}} \partial_{\beta \dot{\beta}} \frac{\bar{\chi}^{\dot{\alpha} \dot{\beta}}}{F} \\ & - \frac{8i}{9} \frac{\chi^{ij} \chi_j^\alpha}{F^2} \square \partial_{\alpha \dot{\alpha}} \frac{\bar{\chi}_{ik} \bar{\chi}^{\dot{\alpha} k}}{F^2} + \frac{\chi^4}{F^3} \square^2 \frac{\bar{\chi}^4}{F^3}. \end{aligned} \quad (\text{A.4})$$

This equation can be uniquely solved by iteration in order to express F in terms of χ_i^α and its complex conjugate $\bar{\chi}^{\dot{\alpha} i} := \overline{(\chi_i^\alpha)}$,

$$F = f - \frac{2i}{f} (\partial_{\alpha \dot{\alpha}} \chi_i^\alpha) \bar{\chi}^{\dot{\alpha} i} + O(\chi^4). \quad (\text{A.5})$$

The series terminates since χ_i^α and $\bar{\chi}^{\dot{\alpha} i}$ are anti-commuting.

B SU(2) superspace

This appendix contains a summary of the formulation for $\mathcal{N} = 2$ conformal supergravity [53] in SU(2) superspace [32]. A curved $\mathcal{N} = 2$ superspace is parametrised by local coordinates $z^M = (x^m, \theta_i^\mu, \bar{\theta}_{\dot{\mu}}^i = \overline{\theta_{\mu i}})$, where $m = 0, 1, 2, 3$ and $\mu, \dot{\mu}, \nu = 1, 2$. The superspace structure group is chosen to be $\text{SL}(2, \mathbb{C}) \times \text{SU}(2)$, and the covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\dot{\alpha} i})$ read

$$\begin{aligned} \mathcal{D}_A = & E_A + \Phi_A^{kl} J_{kl} + \frac{1}{2} \Omega_A^{bc} M_{bc} \\ = & E_A + \Phi_A^{kl} J_{kl} + \Omega_A^{\beta \gamma} M_{\beta \gamma} + \bar{\Omega}_A^{\dot{\beta} \dot{\gamma}} \bar{M}_{\dot{\beta} \dot{\gamma}}. \end{aligned} \quad (\text{B.1})$$

Here $E_A = E_A^M \partial_M$, M_{cd} and J_{kl} are the generators of the Lorentz and SU(2) groups respectively, and Ω_A^{bc} and Φ_A^{kl} the corresponding connections. The action of the generators on the covariant derivatives are defined as:

$$[M_{\alpha \beta}, \mathcal{D}_\gamma^i] = \varepsilon_{\gamma(\alpha} \mathcal{D}_{\beta)}^i, \quad [J_{kl}, \mathcal{D}_\alpha^i] = -\delta_{(k}^i \mathcal{D}_{\alpha l)}, \quad (\text{B.2})$$

together with their complex conjugates.

The algebra of covariant derivatives is [53]

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = 4S^{ij}M_{\alpha\beta} + 2\varepsilon^{ij}\varepsilon_{\alpha\beta}Y^{\gamma\delta}M_{\gamma\delta} + 2\varepsilon^{ij}\varepsilon_{\alpha\beta}\bar{W}^{\gamma\delta}\bar{M}_{\gamma\delta} \\ + 2\varepsilon_{\alpha\beta}\varepsilon^{ij}S^{kl}J_{kl} + 4Y_{\alpha\beta}J^{ij}, \quad (\text{B.3a})$$

$$\{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_j^{\dot{\beta}}\} = -2i\delta_j^i(\sigma^c)_\alpha{}^{\dot{\beta}}\mathcal{D}_c + 4\delta_j^iG^{\delta\dot{\beta}}M_{\alpha\delta} + 4\delta_j^iG_{\alpha\dot{\gamma}}\bar{M}^{\dot{\gamma}\dot{\beta}} + 8G_\alpha{}^{\dot{\beta}}J^i{}_j, \quad (\text{B.3b})$$

together with the complex conjugate of (B.3a), see [53] for the explicit expressions for the commutators $[\mathcal{D}_a, \mathcal{D}_\beta^j]$ and $[\mathcal{D}_a, \bar{\mathcal{D}}_j^{\dot{\beta}}]$. Here the real four-vector $G_{\alpha\dot{\alpha}}$, the complex symmetric tensors $S^{ij} = S^{ji}$, $W_{\alpha\beta} = W_{\beta\alpha}$, $Y_{\alpha\beta} = Y_{\beta\alpha}$ and their complex conjugates $\bar{S}_{ij} := \overline{S^{ij}}$, $\bar{W}_{\dot{\alpha}\dot{\beta}} := \overline{W_{\alpha\beta}}$, $\bar{Y}_{\dot{\alpha}\dot{\beta}} := \overline{Y_{\alpha\beta}}$ are constrained by the Bianchi identities [32, 53]. The latter comprise the dimension-3/2 identities

$$\mathcal{D}_\alpha^i S^{jk} = \bar{\mathcal{D}}_\alpha^i S^{jk} = 0, \quad \bar{\mathcal{D}}_i^{\dot{\alpha}} W_{\beta\gamma} = 0, \quad \mathcal{D}_{(\alpha}^i Y_{\beta\gamma)} = 0, \quad \mathcal{D}_\alpha^i S_{ij} + \mathcal{D}_j^\beta Y_{\beta\alpha} = 0, \quad (\text{B.4a})$$

$$\mathcal{D}_\alpha^i G_{\beta\dot{\beta}} = -\frac{1}{4}\bar{\mathcal{D}}_\beta^i Y_{\alpha\beta} + \frac{1}{12}\varepsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot{\beta}j} S^{ij} - \frac{1}{4}\varepsilon_{\alpha\beta}\bar{\mathcal{D}}^{\dot{\gamma}i}\bar{W}_{\dot{\beta}\dot{\gamma}}, \quad (\text{B.4b})$$

together with their complex conjugates as well as the dimension-2 relation

$$(\mathcal{D}_{(\alpha}^i \mathcal{D}_{\beta)i} - 4Y_{\alpha\beta})W^{\alpha\beta} = (\bar{\mathcal{D}}_i^{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\beta}i} - 4\bar{Y}^{\dot{\alpha}\dot{\beta}})\bar{W}_{\dot{\alpha}\dot{\beta}}. \quad (\text{B.5})$$

C Conformal superspace

This appendix contains a summary of the formulation for $\mathcal{N} = 2$ conformal supergravity in conformal superspace [33] employed in section 4. We use the notations of [77] which are consistent with those of [53] and appendix B and review the results necessary for deriving results in section 4. The structure group of $\mathcal{N} = 2$ conformal superspace is chosen to be $\text{SU}(2, 2|2)$ and the covariant derivatives $\nabla_A = (\nabla_a, \nabla_\alpha^i, \bar{\nabla}_i^{\dot{\alpha}})$ have the form

$$\nabla_A = E_A + \frac{1}{2}\Omega_A{}^{ab}M_{ab} + \Phi_A{}^{ij}J_{ij} + i\Phi_A Y + B_A\mathbb{D} + \mathfrak{F}_A{}^B K_B \\ = E_A + \Omega_A{}^{\beta\gamma}M_{\beta\gamma} + \bar{\Omega}_A{}^{\dot{\beta}\dot{\gamma}}\bar{M}_{\dot{\beta}\dot{\gamma}} + \Phi_A{}^{ij}J_{ij} + i\Phi_A Y + B_A\mathbb{D} + \mathfrak{F}_A{}^B K_B. \quad (\text{C.1})$$

Here, as in $\text{SU}(2)$ superspace, $E_A = E_A{}^M\partial_M$, M_{cd} and J_{kl} are the generators of the Lorentz and $\text{SU}(2)$ R -symmetry groups respectively, and $\Omega_A{}^{bc}$ and $\Phi_A{}^{kl}$ the corresponding connections. The remaining generators and corresponding connections are: Y and Φ_A for the $\text{U}(1)$ R -symmetry group; \mathbb{D} and B_A for the dilatations; $K^A = (K^a, S_\alpha^a, \bar{S}_a^{\dot{\alpha}})$ and $\mathfrak{F}_A{}^B$ for the special superconformal generators.

The Lorentz and $\text{SU}(2)$ generators act on ∇_A as in the $\text{SU}(2)$ superspace case, see eq. (B.2). The $\text{U}(1)_R$ and dilatation generators obey

$$[Y, \nabla_\alpha^i] = \nabla_\alpha^i, \quad [Y, \bar{\nabla}_i^{\dot{\alpha}}] = -\bar{\nabla}_i^{\dot{\alpha}}, \quad (\text{C.2a})$$

$$[\mathbb{D}, \nabla_a] = \nabla_a, \quad [\mathbb{D}, \nabla_\alpha^i] = \frac{1}{2}\nabla_\alpha^i, \quad [\mathbb{D}, \bar{\nabla}_i^{\dot{\alpha}}] = \frac{1}{2}\bar{\nabla}_i^{\dot{\alpha}}. \quad (\text{C.2b})$$

The special superconformal generators K^A transform under Lorentz and $\text{SU}(2)$ as

$$[M_{ab}, K_c] = 2\eta_{c[a}K_{b]}, \quad [M_{\alpha\beta}, S_\gamma^i] = \delta_{(\alpha}^i S_{\beta)\gamma}, \quad [J_{ij}, S_k^\gamma] = -\varepsilon_{k(i}S_{j)}^\gamma, \quad (\text{C.3})$$

together with their complex conjugates, while their transformation under $U(1)$ and dilations is:

$$\begin{aligned} [Y, S_i^\alpha] &= -S_i^\alpha, & [Y, \bar{S}_\alpha^i] &= \bar{S}_\alpha^i, \\ [\mathbb{D}, K_a] &= -K_a, & [\mathbb{D}, S_i^\alpha] &= -\frac{1}{2}S_i^\alpha, & [\mathbb{D}, \bar{S}_\alpha^i] &= -\frac{1}{2}\bar{S}_\alpha^i. \end{aligned} \quad (\text{C.4a})$$

The generators K^A obey

$$\{S_i^\alpha, \bar{S}_\alpha^j\} = 2i\delta_i^j(\sigma^a)^\alpha{}_\alpha K_a, \quad (\text{C.5})$$

while the nontrivial (anti-)commutators of the algebra of K^A with ∇_B are given by

$$\begin{aligned} [K^a, \nabla_b] &= 2\delta_b^a \mathbb{D} + 2M^a{}_b, \\ \{S_i^\alpha, \nabla_\beta^j\} &= 2\delta_i^j \delta_\beta^\alpha \mathbb{D} - 4\delta_i^j M^\alpha{}_\beta - \delta_i^j \delta_\beta^\alpha Y + 4\delta_\beta^\alpha J_i{}^j, \\ [K^a, \nabla_\beta^j] &= -i(\sigma^a)_{\beta}{}^{\dot{\beta}} \bar{S}_{\dot{\beta}}^j, & [S_i^\alpha, \nabla_b] &= i(\sigma_b)^\alpha{}_{\dot{\beta}} \bar{\nabla}_i^{\dot{\beta}}, \end{aligned} \quad (\text{C.6a})$$

together with complex conjugates.

The (anti-)commutation relations of the covariant derivatives ∇_A [33, 77] relevant for calculations in this paper are

$$\{\nabla_\alpha^i, \nabla_\beta^j\} = 2\varepsilon^{ij}\varepsilon_{\alpha\beta} \bar{W}_{\dot{\gamma}\dot{\delta}} \bar{M}^{\dot{\gamma}\dot{\delta}} + \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta} \bar{\nabla}_{\dot{\gamma}k} \bar{W}^{\dot{\gamma}\dot{\delta}} \bar{S}_\delta^k - \frac{1}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta} \nabla_{\gamma\dot{\delta}} \bar{W}^{\dot{\delta}}{}_{\dot{\gamma}} K^{\gamma\dot{\gamma}}, \quad (\text{C.7a})$$

$$\{\nabla_\alpha^i, \bar{\nabla}_j^{\dot{\beta}}\} = -2i\delta_j^i \nabla_\alpha^{\dot{\beta}}, \quad (\text{C.7b})$$

$$\begin{aligned} [\nabla_{\alpha\dot{\alpha}}, \nabla_\beta^i] &= -i\varepsilon_{\alpha\beta} \bar{W}_{\dot{\alpha}\dot{\beta}} \bar{\nabla}^{\dot{\beta}i} - \frac{i}{2}\varepsilon_{\alpha\beta} \bar{\nabla}^{\dot{\beta}i} \bar{W}_{\dot{\alpha}\dot{\beta}} \mathbb{D} - \frac{i}{4}\varepsilon_{\alpha\beta} \bar{\nabla}^{\dot{\beta}i} \bar{W}_{\dot{\alpha}\dot{\beta}} Y + i\varepsilon_{\alpha\beta} \bar{\nabla}_j^{\dot{\beta}} \bar{W}_{\dot{\alpha}\dot{\beta}} J^{ij} \\ &\quad - i\varepsilon_{\alpha\beta} \bar{\nabla}_\beta^i \bar{W}_{\dot{\gamma}\dot{\alpha}} \bar{M}^{\dot{\beta}\dot{\gamma}} - \frac{i}{4}\varepsilon_{\alpha\beta} \bar{\nabla}_\alpha^i \bar{\nabla}_k^{\dot{\beta}} \bar{W}_{\dot{\beta}\dot{\gamma}} \bar{S}^{\dot{\gamma}k} + \frac{1}{2}\varepsilon_{\alpha\beta} \nabla^{\gamma\dot{\beta}} \bar{W}_{\dot{\alpha}\dot{\beta}} S_\gamma^i \\ &\quad + \frac{i}{4}\varepsilon_{\alpha\beta} \bar{\nabla}_\alpha^i \nabla_\gamma^{\dot{\beta}} \bar{W}^{\dot{\gamma}\dot{\beta}} K_{\gamma\dot{\beta}}, \end{aligned} \quad (\text{C.7c})$$

together with complex conjugates. The superfield $W_{\alpha\beta} = W_{\beta\alpha}$ and its complex conjugate $\bar{W}_{\dot{\alpha}\dot{\beta}} := \overline{W_{\alpha\beta}}$ are dimension one conformal primaries, $K_A W_{\alpha\beta} = 0$, and obey the additional constraints

$$\bar{\nabla}_i^{\dot{\alpha}} W_{\beta\gamma} = 0, \quad \nabla_\alpha^k \nabla_{\beta k} W^{\alpha\beta} = \bar{\nabla}_k^{\dot{\alpha}} \bar{\nabla}^{\dot{\beta}k} \bar{W}_{\dot{\alpha}\dot{\beta}}. \quad (\text{C.8})$$

D Nilpotent $\mathcal{N} = 1$ supergravity

Consider $\mathcal{N} = 1$ supergravity coupled to a covariantly chiral scalar \mathcal{X} , $\bar{D}_\alpha \mathcal{X} = 0$, subject to the nilpotency condition

$$\mathcal{X}^2 = 0. \quad (\text{D.1})$$

The complete action, which is equivalent to the action used in [19, 20], is

$$S = \int d^4x d^2\theta d^2\bar{\theta} E \left(-\frac{3}{\kappa^2} \bar{S}_0 S_0 + \bar{\mathcal{X}} \mathcal{X} \right) + \left\{ \int d^4x d^2\theta \mathcal{E} S_0^3 \left(\frac{\mu}{\kappa^2} - f \frac{\mathcal{X}}{S_0} \right) + \text{c.c.} \right\}. \quad (\text{D.2})$$

Under the super-Weyl transformation (5.10), \mathcal{X} transforms as a primary dimension-1 superfield, $\delta_\sigma \mathcal{X} = \sigma \mathcal{X}$.

Varying (D.2) with respect to the chiral compensator S_0 gives the equation

$$\mathbb{R} - \mu = -\frac{2}{3} f \kappa^2 \frac{\mathcal{X}}{S_0}, \quad (\text{D.3})$$

where we have introduced the super-Weyl invariant chiral scalar

$$\mathbb{R} = -\frac{1}{4} S_0^{-2} (\bar{\mathcal{D}}^2 - 4R) \bar{S}_0. \quad (\text{D.4})$$

Eq. (D.3) allows for two interpretations. Firstly, it is the equation of motion for S_0 , with \mathcal{X} being a spectator superfield. Secondly, it allows us to express \mathcal{X} as a function of the supergravity fields. The nilpotency constraint (D.1) and the equation of motion (D.3) imply that the chiral curvature becomes nilpotent [21, 23],

$$(\mathbb{R} - \mu)^2 = 0. \quad (\text{D.5})$$

Making use of (D.3) once more, the functional (D.2) can be rewritten as a higher-derivative supergravity action [21]

$$S = \left(\frac{3}{2f\kappa^2} \right)^2 \int d^4x d^2\theta d^2\bar{\theta} E \bar{S}_0 S_0 |\mathbb{R} - \mu|^2 - \left\{ \frac{1}{2} \frac{\mu}{\kappa^2} \int d^4x d^2\theta \mathcal{E} S_0^3 + \text{c.c.} \right\}, \quad (\text{D.6})$$

where \mathbb{R} is subject to the constraint (D.5). This action does not involve the Goldstino superfield explicitly.

The nilpotency condition (D.1) is preserved if \mathcal{X} is locally rescaled,

$$\mathcal{X} \rightarrow e^\tau \mathcal{X}, \quad \bar{\mathcal{D}}_{\dot{\alpha}} \tau = 0. \quad (\text{D.7})$$

Requiring the action (D.2) to be stationary under such re-scalings of \mathcal{X} gives

$$\mathcal{X} = \phi, \quad (\text{D.8})$$

where ϕ is the Lindström-Roček chiral scalar defined by (5.6). If the compensator satisfies its equation of motion (D.3), then the chiral curvature obeys the nonlinear constraint

$$\frac{2}{3} (f\kappa)^2 S_0^2 (\mathbb{R} - \mu) = \frac{1}{4} (\mathbb{R} - \mu) (\bar{\mathcal{D}}^2 - 4R) [\bar{S}_0 (\bar{\mathbb{R}} - \mu)], \quad (\text{D.9})$$

in addition to the nilpotency condition (D.5). Making use of (D.9) turns the action (D.6) into

$$S = -\frac{3}{2\kappa^2} \int d^4x d^2\theta d^2\bar{\theta} E \bar{S}_0 S_0 + \left\{ \frac{\mu}{4\kappa^2} \int d^4x d^2\theta \mathcal{E} S_0^3 + \text{c.c.} \right\}. \quad (\text{D.10})$$

This is a pure supergravity action with rescaled Newton's constant and cosmological parameter, $\kappa^2 \rightarrow 2\kappa^2$ and $\mu \rightarrow \frac{1}{2}\mu$. The chiral compensator S_0 in (D.10) obeys the nilpotency condition (D.5) and the nonlinear constraint (D.9). In a super-Weyl gauge $S_0 = 1$, these conditions turn into

$$(R - \mu)^2 = 0, \quad (\text{D.11a})$$

$$\frac{2}{3} (f\kappa)^2 (R - \mu) = \frac{1}{4} (R - \mu) (\bar{\mathcal{D}}^2 - 4R) (\bar{R} - \mu). \quad (\text{D.11b})$$

Other approaches to nilpotent $\mathcal{N} = 1$ supergravity were developed in [16, 23, 91–93].

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