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Scalar field as an intrinsic time measure in coupled dynamical matter-geometry systems. II. Electrically charged gravitational collapse

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Abstract: Investigating the dynamics of gravitational systems, especially in the regime of quantum gravity, poses a problem of measuring time during the evolution. One of the approaches to this issue is using one of the internal degrees of freedom as a time variable. The objective of our research was to check whether a scalar field or any other dynamical quantity being a part of a coupled multi-component matter-geometry system can be treated as a 'clock' during its evolution. We investigated a collapse of a self-gravitating electrically charged scalar field in the Einstein and Brans-Dicke theories using the 2+2 formalism. Our findings concentrated on the spacetime region of high curvature existing in the vicinity of the emerging singularity, which is essential for the quantum gravity applications. We investigated several values of the Brans-Dicke coupling constant and the coupling between the Brans-Dicke and the electrically charged scalar fields. It turned out that both evolving scalar fields and a function which measures the amount of electric charge within a sphere of a given radius can be used to quantify time nearby the singularity in the dynamical spacetime part, in which the apparent horizon surrounding the singularity is spacelike. Using them in this respect in the asymptotic spacetime region is possible only when both fields are present in the system and, moreover, they are coupled to each other. The only nonzero component of the Maxwell field four-potential cannot be used to quantify time during the considered process in the neighborhood of the whole central singularity. None of the investigated dynamical quantities is a good candidate for measuring time nearby the Cauchy horizon, which is also singular due to the mass inflation phenomenon.

Keywords: Black Holes, Classical Theories of Gravity

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Contents

1 Introduction

Time measuring in dynamical gravitational systems is an important and demanding issue, especially when one considers investigating them within the quantized canonical formulations of the theory of gravity. Any general notion of a time measurer which could be transferred from the classical to the quantum level has not yet been proposed. One of the ideas in this regard, which has been widely used in analyses within the fields of canonical gravity and cosmology, is to employ one of the internal degrees of freedom of a time-dependent system to act as a 'clock' [1]. However, arguments in favor of such a treatment are limited to certain cases and thus detailed investigations are still required. The current research addresses the problem of time quantification with the use of scalar fields and also other dynamical quantities present in evolving coupled multi-component matter-geometry systems. The studied evolution was a gravitational collapse of an electrically charged scalar field in the Einstein and Brans-Dicke theories.

The gravitational collapse of a self-interacting electrically charged scalar field is a toy-model of a more realistic collapse, which produces the rotating and neutral Kerr black hole. It leads to the formation of a dynamical Reissner-Nordström spacetime, which possesses a spacelike central singularity surrounded by the null Cauchy and event horizons [2–5]. The influence of pair creation in strong electric fields on the outcomes of the process was described in [6, 7]. Its course when the neutralization and the black hole evaporation due to the Hawking radiation emission are taken into account was studied in [8, 9]. The evolution of interest was also examined in the dilaton [10, 11], phantom [12] and Brans-Dicke [13, 14] theories of gravity. The course and results of the electrically charged gravitational collapse of a scalar field in the de Sitter spacetime were characterized in [15].

The current paper describes the continuation of the studies whose outcomes were presented in [16], which from now on will be referred to as paper I. The performed analyses dealt with the problem of a dynamical gravitational collapse of neutral coupled matter-geometry systems in the context of performing time measurements intrinsically during the process. A broad discussion on the following topics can be found in paper I:

- the existing approaches to intrinsic time measurements in dynamical gravitational systems and their specific implementations in quantum gravity and cosmology,
- a discussion on the arguments in favor of the above propositions and the justification of the undertaken studies in this context,
- a synopsis to the Brans-Dicke theory of gravity, its relations to experimental data, the Einstein theory of relativity and cosmology,
- a justification for choosing the Brans-Dicke setup for the conducted research and a brief summary of previous numerical achievements within the theory,
- specific arguments for choosing the particular values of the free evolution parameters (the Brans-Dicke coupling constant ω and the coupling between the Brans-Dicke and scalar fields β), which were used during the analyses.

Thus, in order to get acquainted with the above-listed essential issues related to the background and the core of our research, we refer the Reader to paper I.

As was pointed out at the beginning of the introduction, the issue of using a dynamical quantity present in the coupled matter-geometry system as an intrinsic 'clock' during inspecting its evolution is crucial for the spacetime regions of high curvature. For this reason, the discussion on the results will mainly concentrate on the neighborhood of spacetime singularities, which emerge during the gravitational collapse of matter. In order to treat the particular quantity as a time measurer, its constancy hypersurfaces must fulfill two conditions, at least within the spacetime regions of interest. First, the slices have to be spacelike in these areas. Second, their parametrization needs to remain monotonic during the whole evolution.

The current paper was constructed in the following way. Section 2 contains the description of the theoretical formulation of the investigated problem. The necessary details of numerical computations and the results presentation are placed in section 3. The first general aim of our analyses was to investigate the potential of measuring time with the use of dynamical quantities during the collapse of a self-gravitating electrically charged scalar field within the Einstein theory. The related results are presented in section 4. The second general goal was to address the above problem in the context of a dynamical gravitational evolution in the Brans-Dicke theory. The obtained outcomes are elaborated in section 5. The overall conclusions are gathered in section 6, while appendix A contains a comment on the numerical computations and the code tests.

2 Gravitational evolution of an electrically charged scalar field

2.1 Covariant form of the equations of motion

The action which describes an electrically charged scalar field in the Brans-Dicke theory with a nontrivial exponential coupling between the two scalar fields present within the system is

$$S_{\mathrm{BD}} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}\left(\Phi R - \frac{\omega}{\Phi}\Phi_{;\mu}\Phi_{;\nu}g^{\mu\nu}\right) + \Phi^{\beta}L^{\mathrm{EM}}\right], \tag{2.1}$$

where g denotes the determinant of the metric $g^{\mu\nu}$, R is the Ricci scalar, Φ and ω are the Brans-Dicke field function and coupling constant, respectively, while β is a constant which controls the coupling between the Brans-Dicke and electrically charged fields. The Lagrangian of the latter field has the usual form

$$L^{\mathrm{EM}} = -\frac{1}{2}\left(\phi_{;\mu} + ieA_{\mu}\phi\right)\left(\bar{\phi}_{;\nu} - ieA_{\nu}\bar{\phi}\right)g^{\mu\nu} - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}, \tag{2.2}$$

in which the complex field ϕ is charged under a U(1) gauge field, whose four-potential is denoted as A_{μ} and the coupling between the two is e. The quantity $F_{\mu\nu} \equiv A_{\nu;\mu} - A_{\mu;\nu}$ is the strength tensor of the gauge field, while i is the imaginary unit.

The equations of motion of the gravitational field resulting from the above theoretical setup can be written as follows:

$$G_{\mu\nu} = 8\pi\left(T^{\mathrm{BD}}_{\mu\nu} + \Phi^{\beta-1}T^{\mathrm{EM}}_{\mu\nu}\right) \equiv 8\pi T_{\mu\nu}. \tag{2.3}$$

The components of the Einstein tensor $G_{\mu\nu}$ are determined by the selected metric and their form will be presented in the next section. The stress-energy tensors of the Brans-Dicke and electrically charged fields are

$$T^{\mathrm{BD}}_{\mu\nu} = \frac{1}{8\pi\Phi}\left(\Phi_{;\mu\nu} - g_{\mu\nu}\Phi_{;\rho\sigma}g^{\rho\sigma}\right) + \frac{\omega}{8\pi\Phi^2}\left(\Phi_{;\mu}\Phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\Phi_{;\rho}\Phi_{;\sigma}g^{\rho\sigma}\right), \tag{2.4}$$

$$\begin{aligned} T^{\mathrm{EM}}_{\mu\nu} = {} & \frac{1}{2}\left(\phi_{;\mu}\bar{\phi}_{;\nu} + \bar{\phi}_{;\mu}\phi_{;\nu}\right) + \frac{1}{2}\left(\bar{\phi}_{;\mu}ieA_{\nu}\phi + \bar{\phi}_{;\nu}ieA_{\mu}\phi - \phi_{;\mu}ieA_{\nu}\bar{\phi} - \phi_{;\nu}ieA_{\mu}\bar{\phi}\right) + \\ & + \frac{1}{4\pi}F_{\mu\rho}F_{\nu}{}^{\rho} + e^2A_{\mu}A_{\nu}\phi\bar{\phi} + g_{\mu\nu}L^{\mathrm{EM}}. \end{aligned} \tag{2.5}$$

JHEP05(2016)155

The covariant forms of the equations of motion of the Brans-Dicke field, the electrically charged scalar field and its complex conjugate and the Maxwell field are the following:

$$\Phi_{;\mu\nu}g^{\mu\nu} - \frac{8\pi\Phi^{\beta}}{3+2\omega}\left(T^{\mathrm{EM}} - 2\beta L^{\mathrm{EM}}\right) = 0, \tag{2.6}$$

$$\phi_{;\mu\nu}g^{\mu\nu} + ieA^{\mu}\left(2\phi_{;\mu} + ieA_{\mu}\phi\right) + ieA_{\mu;\nu}g^{\mu\nu}\phi + \frac{\beta}{\Phi}\Phi_{;\mu}\left(\phi_{;\nu} + ieA_{\nu}\phi\right)g^{\mu\nu} = 0, \tag{2.7}$$

$$\bar{\phi}_{;\mu\nu}g^{\mu\nu} - ieA^{\mu}\left(2\bar{\phi}_{;\mu} - ieA_{\mu}\bar{\phi}\right) - ieA_{\mu;\nu}g^{\mu\nu}\bar{\phi} + \frac{\beta}{\Phi}\Phi_{;\mu}\left(\bar{\phi}_{;\nu} - ieA_{\nu}\bar{\phi}\right)g^{\mu\nu} = 0, \tag{2.8}$$

$$\frac{1}{2\pi}\left(F^{\nu}{}_{\mu;\nu} + \frac{\beta}{\Phi}F^{\nu}{}_{\mu}\Phi_{;\nu}\right) - ie\phi\left(\bar{\phi}_{;\mu} - ieA_{\mu}\bar{\phi}\right) + ie\bar{\phi}\left(\phi_{;\mu} + ieA_{\mu}\phi\right) = 0, \tag{2.9}$$

where T^{EM} denotes the trace of (2.5).

2.2 Dynamics in double null coordinates

The evolution will be traced in double null coordinates $(u,\, v,\, \theta,\, \varphi)$, in which a spherically symmetric line element is

$$ds^2 = -\alpha\,(u,v)^2\,dudv + r\,(u,v)^2\,d\Omega^2, \tag{2.10}$$

where u and v are retarded and advanced times, respectively, $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element of a unit sphere, while θ and φ denote angular coordinates. The arbitrary functions α and r depend on both the retarded and advanced time. Their dynamics reflects the evolution of spacetime in the investigated system.

The rescaling of the complex field function $s \equiv \sqrt{4\pi}\phi$ simplifies the form of the equations of motion. The second-order differential equations were turned into the first-order ones via the substitutions

$$\begin{aligned} h &= \frac{\alpha_{,u}}{\alpha}, & d &= \frac{\alpha_{,v}}{\alpha}, & f &= r_{,u}, & g &= r_{,v}, \\ W &= \Phi_{,u}, & Z &= \Phi_{,v}, & w &= s_{,u}, & z &= s_{,v}. \end{aligned} \tag{2.11}$$

The components of the Einstein tensor related to the line element (2.10) are

$$G_{uu} = -\frac{2}{r}\left(f_{,u} - 2fh\right), \tag{2.12}$$

$$G_{vv} = -\frac{2}{r}\left(g_{,v} - 2gd\right), \tag{2.13}$$

$$G_{uv} = \frac{1}{2r^2}\left(4rf_{,v} + \alpha^2 + 4fg\right), \tag{2.14}$$

$$G_{\theta\theta} = \sin^{-2}\theta\; G_{\varphi\varphi} = -\frac{4r^2}{\alpha^2}\left(d_{,u} + \frac{f_{,v}}{r}\right), \tag{2.15}$$

JHEP05(2016)155

while the elements of the stress-energy tensors of the considered theory (2.4) and (2.5) are the following:

$$T_{uu}^{\mathrm{BD}} = \frac{1}{8\pi\Phi}\left(W_{,u} - 2hW\right) + \frac{\omega}{8\pi\Phi^2}W^2, \tag{2.16}$$

$$T_{vv}^{\mathrm{BD}} = \frac{1}{8\pi\Phi}\left(Z_{,v} - 2dZ\right) + \frac{\omega}{8\pi\Phi^2}Z^2, \tag{2.17}$$

$$T_{uv}^{\mathrm{BD}} = -\frac{Z_{,u}}{8\pi\Phi} - \frac{gW + fZ}{4\pi r\Phi}, \tag{2.18}$$

$$T_{\theta\theta}^{\mathrm{BD}} = \sin^{-2}\theta\, T_{\varphi\varphi}^{\mathrm{BD}} = \frac{r^2}{2\pi\alpha^2\Phi}Z_{,u} + \frac{r}{4\pi\alpha^2\Phi}\left(gW + fZ\right) + \frac{\omega r^2}{4\pi\Phi^2\alpha^2}WZ, \tag{2.19}$$

$$T_{uu}^{\mathrm{EM}} = \frac{1}{4\pi}\left[w\bar{w} + ieA_u\left(\bar{w}s - w\bar{s}\right) + e^2A_u^2 s\bar{s}\right], \tag{2.20}$$

$$T_{vv}^{\mathrm{EM}} = \frac{1}{4\pi}z\bar{z}, \tag{2.21}$$

$$T_{uv}^{\mathrm{EM}} = \frac{A_{u,v}^2}{4\pi\alpha^2}, \tag{2.22}$$

$$T_{\theta\theta}^{\mathrm{EM}} = \sin^{-2}\theta\, T_{\varphi\varphi}^{\mathrm{EM}} = \frac{r^2}{4\pi\alpha^2}\left[w\bar{z} + z\bar{w} + ieA_u\left(\bar{z}s - z\bar{s}\right) + \frac{2A_{u,v}^2}{\alpha^2}\right]. \tag{2.23}$$

Due to the gauge freedom fixing, the only nonzero component of the electromagnetic four-potential is A_u [5], which is a function of advanced and retarded times.

The $\theta\theta$ (or $\varphi\varphi$) and uv components of the Einstein equations (2.3) can be written together with the equation of the Brans-Dicke field (2.6) in a matrix form

$$\begin{pmatrix} 1 & \frac{1}{r} & \frac{1}{\Phi} \\ 0 & 1 & \frac{r}{2\Phi} \\ 0 & 0 & r \end{pmatrix} \begin{pmatrix} d_{,u} \\ f_{,v} \\ Z_{,u} \end{pmatrix} = \begin{pmatrix} \mathcal{A} \\ \mathcal{B} \\ \mathcal{C} \end{pmatrix}. \tag{2.24}$$

The elements of the right-hand side vector are

$$\mathcal{A} \equiv -\frac{2\pi\alpha^2}{r^2\Phi}\widetilde{T}_{\theta\theta}^{\mathrm{EM}} - \frac{1}{2r\Phi}\left(gW + fZ\right) - \frac{\omega}{2\Phi^2}WZ, \tag{2.25}$$

$$\mathcal{B} \equiv -\frac{\alpha^2}{4r} - \frac{fg}{r} + \frac{4\pi r}{\Phi}\widetilde{T}_{uv}^{\mathrm{EM}} - \frac{1}{\Phi}\left(gW + fZ\right), \tag{2.26}$$

$$\mathcal{C} \equiv -fZ - gW - \frac{2\pi r\alpha^2}{3 + 2\omega}\left(\widetilde{T}^{\mathrm{EM}} - 2\beta\widetilde{L}^{\mathrm{EM}}\right), \tag{2.27}$$

where $\widetilde{Q} \equiv \Phi^\beta Q$ for any quantity Q and

$$T^{\mathrm{EM}} = -\frac{4}{\alpha^2}T_{uv}^{\mathrm{EM}} + \frac{2}{r^2}T_{\theta\theta}^{\mathrm{EM}}, \tag{2.28}$$

$$L^{\mathrm{EM}} = \frac{1}{4\pi\alpha^2}\left(w\bar{z} + z\bar{w}\right) + \frac{ieA_u}{4\pi\alpha^2}\left(\bar{z}s - z\bar{s}\right) + \frac{A_{u,v}^2}{2\pi\alpha^4}. \tag{2.29}$$

An equivalent form of (2.24), appropriate for numerical computations, is

$$\begin{pmatrix} d_{,u} = h_{,v} \\ g_{,u} = f_{,v} \\ Z_{,u} = W_{,v} \end{pmatrix} = \frac{1}{r^2}\begin{pmatrix} r^2 & -r & -\frac{r}{2\Phi} \\ 0 & r^2 & -\frac{r^2}{2\Phi} \\ 0 & 0 & r \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \mathcal{B} \\ \mathcal{C} \end{pmatrix}. \tag{2.30}$$

The remaining of the Einstein equations (2.3), i.e., their uu and vv components, provide the constraints

$$f_{,u} = 2fh - \frac{r}{2\Phi}\left(W_{,u} - 2hW\right) - \frac{r\omega}{2\Phi^2}W^2 - \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{uu}, \tag{2.31}$$

$$g_{,v} = 2dg - \frac{r}{2\Phi}\left(Z_{,v} - 2dZ\right) - \frac{r\omega}{2\Phi^2}Z^2 - \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{vv}. \tag{2.32}$$

The evolution of the complex scalar field described by (2.7) and (2.8) is governed by the equations

$$z_{,u} = w_{,v} = -\frac{fz}{r} - \frac{gw}{r} - ieA_u z - \frac{ieA_u gs}{r} - \frac{ie}{4r^2}\alpha^2 qs - \frac{\beta}{2\Phi}\left(Wz + Zw + iesA_u Z\right), \tag{2.33}$$

$$\bar{z}_{,u} = \bar{w}_{,v} = -\frac{f\bar{z}}{r} - \frac{g\bar{w}}{r} + ieA_u \bar{z} + \frac{ieA_u g\bar{s}}{r} + \frac{ie}{4r^2}\alpha^2 q\bar{s} - \frac{\beta}{2\Phi}\left(W\bar{z} + Z\bar{w} - ie\bar{s}A_u Z\right), \tag{2.34}$$

where the function reflecting the amount of electric charge contained within a sphere of a radius $r\left(u,v\right)$ was defined as [5]

$$q\left(u,v\right) = \frac{2r^2 A_{u,v}}{\alpha^2}. \tag{2.35}$$

The Maxwell field dynamics (2.9) is described by the relations

$$A_{u,v} = \frac{\alpha^2 q}{2r^2}, \tag{2.36}$$

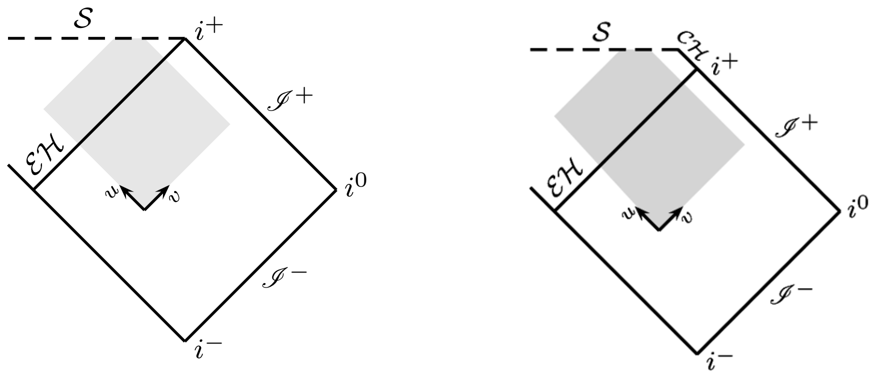
$$q_{,v} = -\frac{ier^2}{2}\left(\bar{s}z - s\bar{z}\right) - \beta q\frac{Z}{\Phi}. \tag{2.37}$$

3 Details of computer simulations and results analysis

The dynamics of the system containing an electrically charged scalar field described by (2.2) coupled with the Brans-Dicke field according to (2.1) is governed in double null coordinates by a set of equations (2.30), (2.33)–(2.37). The constraints, which were used to control the accuracy of numerical calculations, are provided by (2.31) and (2.32). The computational domain, within which the evolution was traced, is shown in figure 1. It is presented against the dynamical Schwarzschild and Reissner-Nordström spacetimes in the (vu)-plane. The respective diagrams are relevant to the cases, in which the Cauchy horizon does not and does form in the emerging spacetime. The only freely specifiable initial conditions of the studied process were the profiles of the complex scalar and Brans-Dicke fields posed on an arbitrarily chosen $u = \text{const.}$ hypersurface, which were the same as in paper I. The field functions were nonzero only within the interval $v \in [0, 20]$ and thus the spacetime region from within the specified range will be referred to as dynamical. The details of the numerical code and its consistency tests are presented in appendix A.

The investigated values of the evolution parameters (β and ω) and the justification for their choice were discussed in paper I. $\delta \in [0, 0.5]$ controls the character of the scalar field, which is either real or complex when the parameter is equal and not equal to zero, respectively. Its value was constant in all computations of the current paper and equal to 0.5,

JHEP05(2016)155



(a) (b)

Figure 1. The computational domain (marked gray) on the background of the Carter-Penrose diagram of the dynamical (a) Schwarzschild and (b) Reissner-Nordström spacetimes. The central singularity along $r = 0$, the event and Cauchy horizons are denoted as $\mathcal{S}$, $\mathcal{EH}$ and $\mathcal{CH}$, respectively. $\mathscr{I}^{\pm}$ and $i^{\pm}$ are null and timelike infinities, while i^0 is a spacelike infinity.

because obtaining a charged field requires a complex scalar coupled to a U(1) gauge field. The electric coupling constant was arbitrarily set as $e = 0.3$ in all conducted simulations, as changing its value (provided that it was not equal to zero) did not influence the outcomes qualitatively, and thus also the ultimate conclusions. Spacetimes obtained in the case of an uncoupled collapse, i.e., for $e = 0$, will be also presented for a convenient comparison.

The presentation of the results will be based on the Penrose spacetime diagrams ($r =$ const. lines in the (vu)-plane) and plots of constancy hypersurfaces of adequate dynamical quantities ($|\phi|$, Re ϕ, Im ϕ, A_u and q), also within the (vu)-plane. The singularities and future infinities are marked as thick black curves on the plots and signed, while the apparent horizons, situated along the hypersurfaces $r_{,v} = 0$ and $r_{,u} = 0$, are marked as red and blue lines, respectively. The areas, in which the constancy hypersurfaces of the particular quantity are spacelike are blue, and the regions with their timelike character are purple. In most cases, the constancy hypersurfaces were plotted only for the real part of the complex scalar field. For explanations, we refer the Reader to paper I. In order to confirm the fact that real and imaginary parts of the complex field behave analogously and to check additional quantities, which may be of interest during time measuring, the constancy hypersurfaces of the field modulus, the u-component of the Maxwell field four-potential and the charge function were plotted for selected cases. The modulus of the field function was calculated according to the definition from the values of its real and imaginary parts.

The values of the evolution parameters used to generate the particular outcome will be presented above each diagram. The ranges of the plotted quantities and the steps between adjacent lines representing their constant values are the following.

- On the diagrams of spacetime structures r differs from $r = 0$ to $r = 40$, and the range is divided into 40 steps.

- On the plots presenting the constancy hypersurfaces of the real and imaginary parts of the electrically charged scalar field, as well as its modulus, the ranges of Re ϕ, Im ϕ and $|\phi|$ from -1.75 to 0.5 are divided into 75 steps.

- On the plots presenting the constancy hypersurfaces of the Brans-Dicke field, its ranges, divided into 100 steps in each case, depend on the values of ω and β as follows.
  - For $\omega > -1.5$ and $\beta = 0$, the range is from $\Phi = 0$ to $\Phi = 1$.
  - For $\omega > -1.5$ and $\beta = 0.5$ and 1, Φ ranges from 0 to 10.
  - For $\omega < -1.5$ and $\beta = 0$, Φ ranges from 1 to 20.9.
  - For $\omega < -1.5$ and $\beta = 0.5$ and 1, the range is from $\Phi = 0$ to $\Phi = 2$.
- The plots of the constancy hypersurfaces of the quantities related to the electromagnetic field present A_u from -50 to 50 and q from 0 to 10, with both ranges divided into 100 steps.

4 Einstein gravity

The collapse of a self-gravitating electrically charged scalar field within the Einstein theory was realized by setting the evolution parameters β and ω as equal to 0 and 1000, respectively, because the Einstein limit of the Brans-Dicke theory is obtained for $\omega \to \infty$. It was also checked that the results are consistent with the ones obtained when the Brans-Dicke field is absent, i.e., the amplitude of its initial profile vanishes and the corresponding constants β and ω are equal to zero. The structure of a spacetime formed during the process of interest, as well as the plots of constant hypersurfaces of the modulus, the real and imaginary parts of the evolving field function, together with the u-component of the U(1) gauge field four-potential and the charge function, are presented in figure 2.

4.1 Spacetime structure

The formed spacetime contains a spacelike central singularity along $r = 0$ surrounded by a single apparent horizon at the hypersurface $r_{,v} = 0$. The horizon is spacelike for small values of advanced time, indicating the dynamical region of the spacetime, and becomes null as $v \to \infty$. It is situated along $u = $ const. hypersurface there and specifies the location of the event horizon in the spacetime, which remains in its stationary state after the dynamical collapse. The lines of constant r settle along null $u = $ const. hypersurfaces inside the apparent horizon. It means that there exists a Cauchy horizon in the spacetime, which is located at $v = \infty$ null hypersurface inside the event horizon (for clarification, see figure 1(b)). It is located outside of the computational domain, whose ranges in both null directions are finite. The obtained outcome is consistent with the results of previous investigations of the dynamical spacetimes emerging from the collapse of an electrically charged scalar field [5, 8, 9, 17]. The described structure is also formed during the process running in the presence of dark matter [18].

4.2 Dynamical quantities in the evolving spacetime

The dynamics of the complex scalar field is reflected in the behavior of the constancy hypersurfaces of the real and imaginary parts, as well as the modulus of the field function.

JHEP05(2016)155

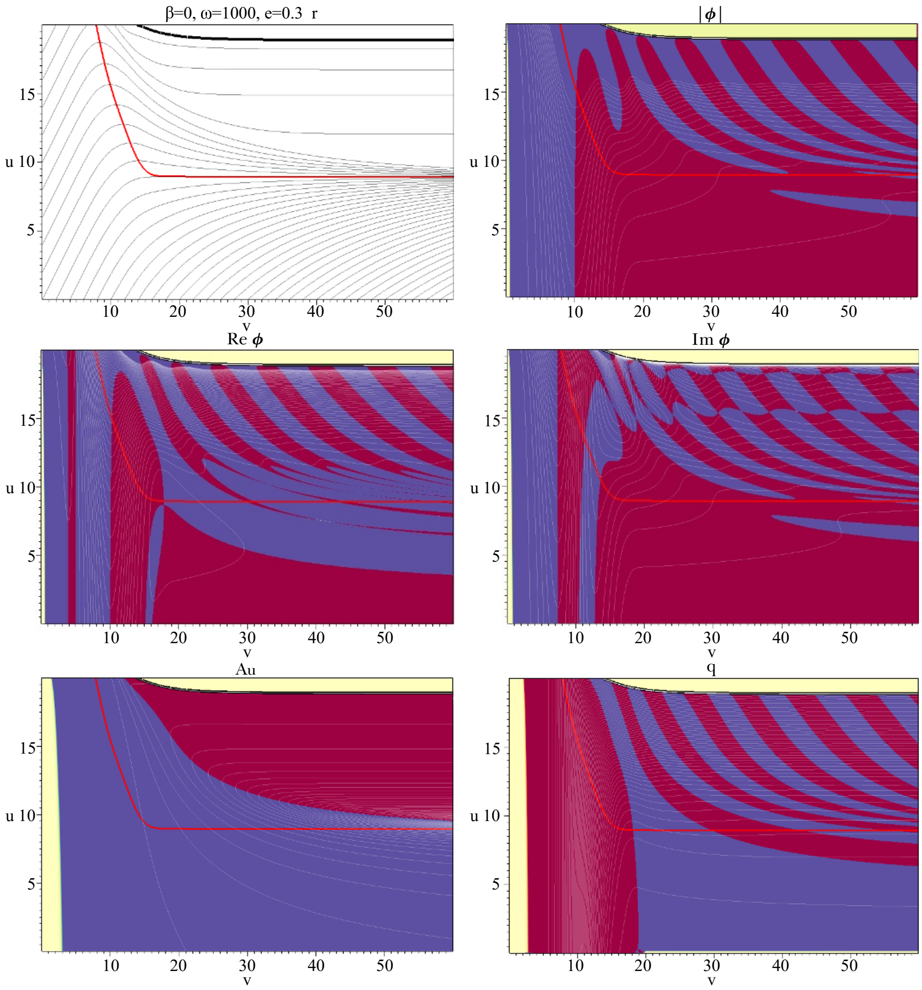


Figure 2. (Color online) The diagram of $r = \text{const.}$ lines in the (vu)-plane for spacetimes formed during the gravitational evolution of an electrically charged scalar field in the Einstein theory. The constancy hypersurfaces of the modulus, the real and imaginary parts of the electrically charged scalar field function ($|\phi|$, Re ϕ and Im ϕ, respectively), as well as the u-component of the U(1) gauge field four-potential (A_u) and the charge function (q).

Their values change most considerably in the dynamical region of the spacetime and beyond the horizon. Nearby the singularity of the former area, the constancy hypersurfaces of all the quantities characterizing the complex scalar field are spacelike and their changes are monotonic. Hence, they can be used as time measurers there. This result agrees with the conclusion for a self-gravitating neutral scalar field [19]. However, in contrast to the uncharged scalar field, the electrically charged one cannot serve as a 'clock' in the asymptotic spacetime region of high curvature. The constancy hypersurfaces of the real and imaginary parts and the modulus of the complex field function clearly have either spacelike or timelike character in the vicinity of the central singularity at large values of v.

The dynamics of the u-component of the Maxwell field four-potential is the biggest near the apparent horizon at large values of advanced time, while the dynamics of the charge function is the same as in the case of the above-mentioned characteristics of the complex

JHEP05(2016)155

scalar field. The constancy hypersurfaces of the quantity A_u are timelike along the whole singularity. For this reason, it cannot be used to quantify time there. Similarly to the quantities $|\phi|$, Re ϕ and Im ϕ described above, the function q can be employed as a 'clock' in the dynamical spacetime region of high curvature, because its constancy hypersurfaces are spacelike there and the monotonicicty of their parametrization is preserved. It is not a good candidate for a time measurer nearby the singularity in the asymptotic region, as its constancy hypersurfaces change their character between spacelike and timelike.

The Cauchy horizon is situated at the null hypersurface $v = \infty$ inside the event horizon. Although it exists outside of the computational domain, some conclusions can be drawn about using the dynamical quantities to measure time in its neighborhood, which is also a region of high curvature due to the mass inflation effect [3, 4, 14, 20–24]. In the region of large values of v beyond the event horizon, the constancy hypersurfaces of Re ϕ, Im ϕ and $|\phi|$ change their character between spacelike and timelike as u changes. The u-component of the U(1) gauge field four-potential is almost entirely timelike in the region of interest and the behavior of the charge function is similar to the behavior of the quantities characterizing the complex scalar field. For the above reasons, none of the dynamical quantities can be used as a 'clock' in the vicinity the Cauchy horizon.

5 Brans-Dicke theory

5.1 Spacetime structures

The structures of spacetimes resulting from the gravitational collapse of an electrically charged scalar field in the Brans-Dicke theory for β equal to 0, 0.5 and 1 are shown in figures 3, 4 and 5, respectively.

5.1.1 Uncoupled Brans-Dicke and electrically charged scalar fields

When β equals 0, each spacetime emerging from the studied collapse contains a spacelike singularity along $r = 0$, wholly surrounded by an apparent horizon $r_{,v} = 0$. The horizon is spacelike in the dynamical region of the spacetime and becomes null and coincides with the event horizon when the spacetime settles in its stationary state as $v \to \infty$. For ω equal to -1.4 and -1.6, between the two stages there also exists a v-range within which the horizon is timelike. For all values of the Brans-Dicke coupling constant, additional $r_{,v} = 0$ apparent horizons are visible nearby the singularity. When ω equals 0, -1 and -1.4, also the hypersurfaces of the apparent horizons $r_{,u} = 0$ exist in the spacetime for large retarded times. In all the forming spacetimes, Cauchy horizons are present at $v = \infty$, what can be inferred from the fact that beyond the event horizon the $r =$ const. lines settle along null $u =$ const. hypersurfaces at large advanced times. This fact, together with the existence of multiple apparent horizons at large u, distinguishes the collapse with $e \neq 0$ from the case of the vanishing e, which produces a typical Schwarzschild-type spacetime. Such a spacetime contains a central spacelike singularity surrounded by a single apparent horizon $r_{,v} = 0$, without a Cauchy horizon.

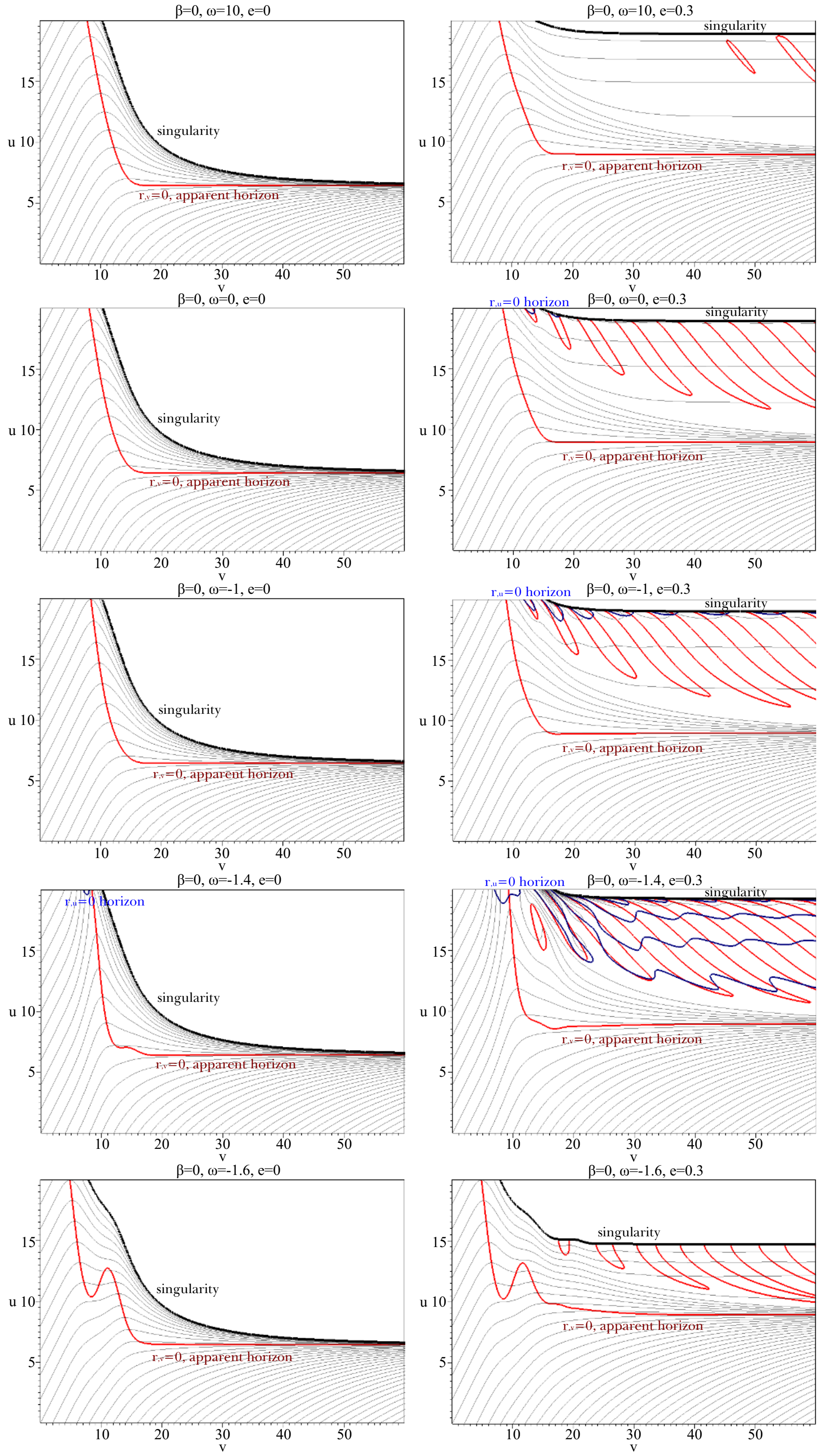


Figure 3. (Color online) The diagrams of $r =$ const. lines in the (vu)-plane for spacetimes formed during evolutions of a neutral complex scalar field (left column) and electrically charged scalar field (right column) in the Brans-Dicke theory for $\beta = 0$.

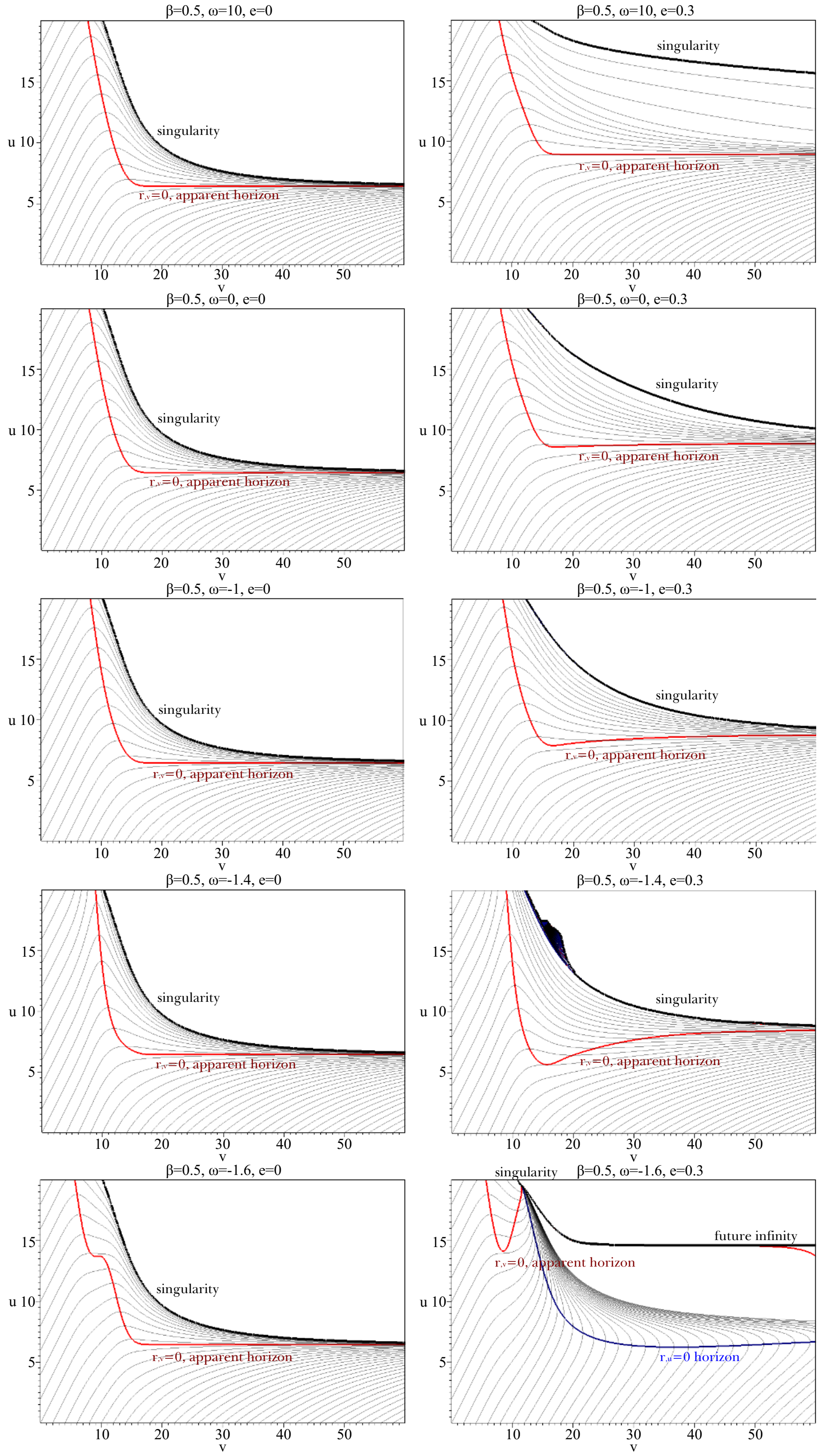


Figure 4. (Color online) The diagrams of $r=$ const. lines in the (vu)-plane for spacetimes formed during evolutions of a neutral complex scalar field (left column) and electrically charged scalar field (right column) in the Brans-Dicke theory for $\beta=0.5$.

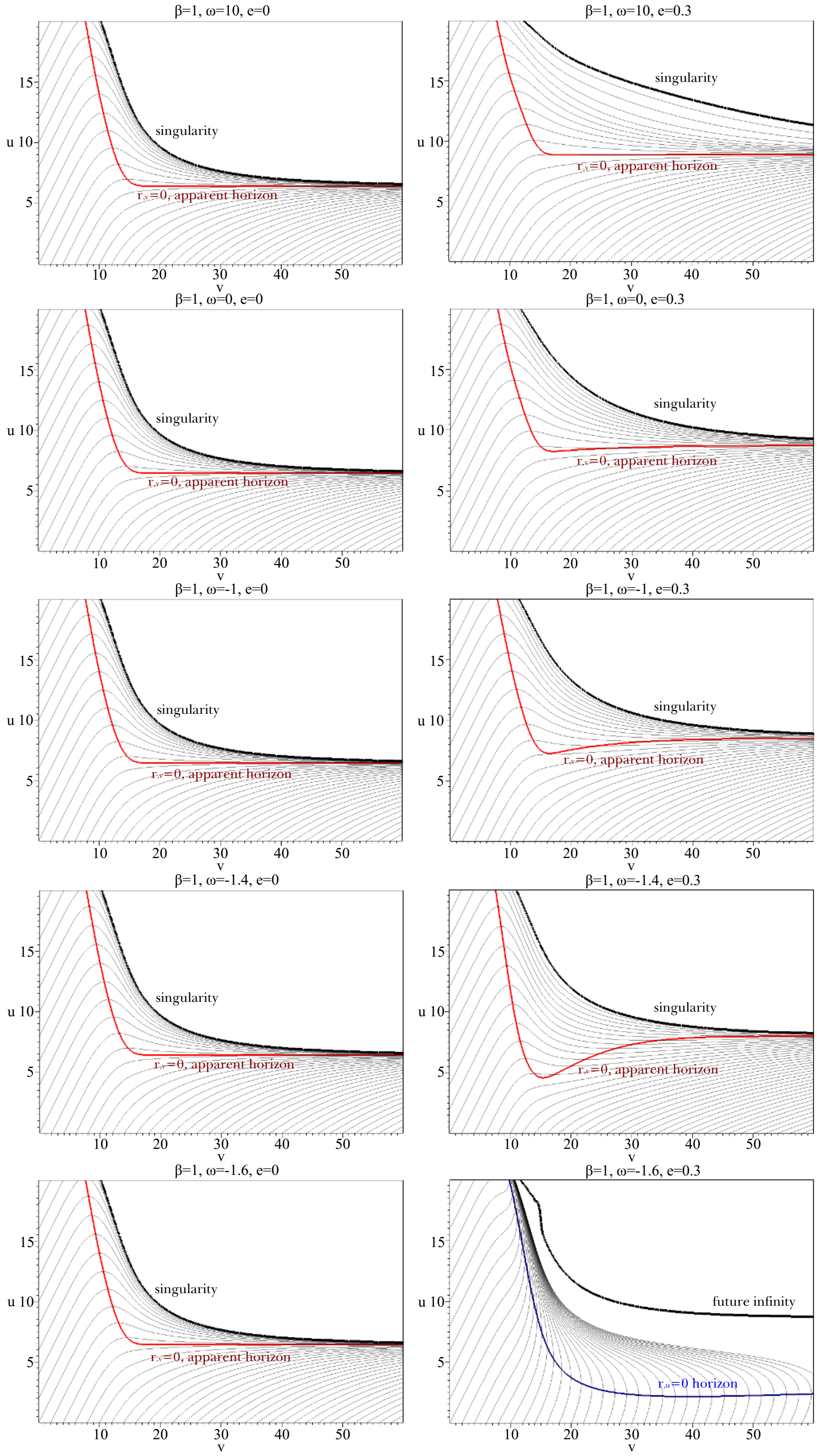


Figure 5. (Color online) The diagrams of $r = \text{const.}$ lines in the (vu)-plane for spacetimes formed during evolutions of a neutral complex scalar field (left column) and electrically charged scalar field (right column) in the Brans-Dicke theory for $\beta = 1$.

5.1.2 Coupled Brans-Dicke and electrically charged scalar fields

The spacetimes formed during the collapse of interest for β equal to 0.5 and 1 share many properties. For $\omega \geqslant -1.4$, in each of them there exists a central spacelike singularity at $r = 0$ surrounded by a single apparent horizon $r_{,v} = 0$. The horizon is spacelike in the dynamical part of the spacetime and tends towards a null direction as v increases, indicating the location of an event horizon. When the Brans-Dicke coupling constant is equal to -1 and -1.4, an intermediate part of the horizon hypersurface is timelike. The Cauchy horizon is absent in all spacetimes, analogously to the case with $e = 0$. A tendency towards the Cauchy horizon formation can be seen for the Brans-Dicke coupling equal to 10, as this large ω case signifies an approach to the Einstein limit of the theory.

A brief comment is required regarding a bump on the singularity in the case of $\beta = 0.5$, $\omega = -1.4$, $e = 0.3$. This region, which appears as a black area on the respective plot, covers the spacetime very close to the singularity, which remains spacelike, just as in the rest of the spacetime. The fields are highly dynamical there and many horizons seem to be folded. Since it is an area of extremely high curvature, its physical meaning is limited for the current studies, which are conducted within the scope of the classical theory of gravity.

Entirely distinct structures are observed for $\beta \neq 0$ and $\omega = -1.6$. When β equals 0.5, the central spacelike singularity does not extend to large values of advanced time. It is surrounded by the $r_{,v} = 0$ apparent horizon, which is spacelike and then timelike for larger values of v. At the point of their coincidence another apparent horizon, i.e., $r_{,u} = 0$ appears and extends to infinity, initially in a spacelike, and then in a null direction. The future infinity is situated beyond it. For $\beta = 1$, only the $r_{,u} = 0$ apparent horizon, which is spacelike in the dynamical spacetime region and becomes null as $v \to \infty$, is visible in the spacetime. Again, the future infinity is located within it. Such exotic structures were observed previously [12, 13] and they can be formed during the collapse with $\omega = -1.6$, because in the ghost limit the weak cosmic censorship can be violated.

5.1.3 Overall dependence on evolution parameters

Similarly to the case of an uncharged collapse studied in paper I, the variety and complexity of the emerging spacetime structures decrease as β increases. The exotic structures formed for $\omega = -1.6$ and $\beta \neq 0$ constitute an exception in this regard. It is worth emphasizing that in none of the cases the collapse proceeds similarly to the process running in the Einstein gravity, whose outcome was described in section 4.1. The Cauchy horizon forms when the Brans-Dicke field is not coupled with the complex scalar one and its emergence is prevented for the remaining values of β. The existence of the Cauchy horizon is always accompanied by the presence of multiple apparent horizon hypersurfaces within the event horizon. Negative values of the parameter ω favor an existence of a timelike part of the apparent horizon $r_{,v} = 0$ between its spacelike and null sections. A summary of causal structures of spacetimes obtained as a result of the examined process is presented in figure 6 in the form of Carter-Penrose diagrams.

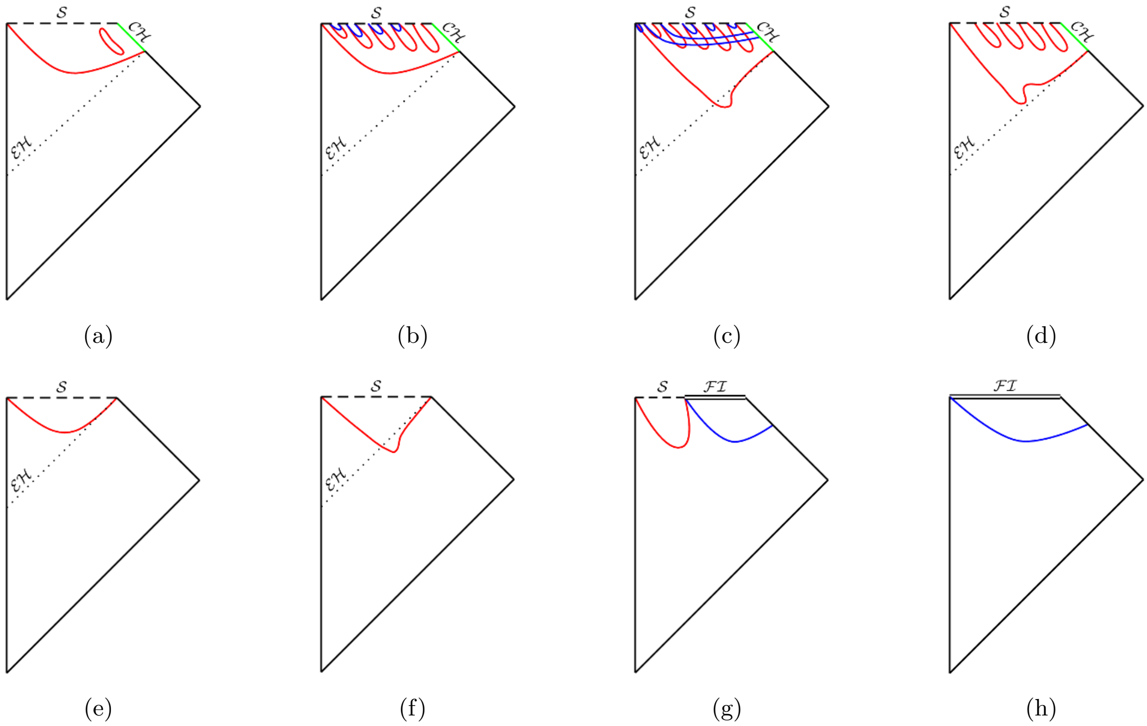


Figure 6. (Color online) The Carter-Penrose diagrams of spacetimes formed during evolutions of a scalar field in the Brans-Dicke theory, for the following sets of evolution parameters: (a) $\beta = 0$, $\omega = 10$, (b) $\beta = 0$, $\omega = 0, -1$, (c) $\beta = 0$, $\omega = -1.4$, (d) $\beta = 0$, $\omega = -1.6$, (e) $\beta \neq 0$, $\omega \geqslant 0$, (f) $\beta \neq 0$, $\omega = -1, -1.4$, (g) $\beta = 0.5$, $\omega = -1.6$ and (h) $\beta = 1$, $\omega = -1.6$. The central singularity along $r = 0$, the event and Cauchy horizons, as well as the future infinity are denoted as $\mathcal{S}$, $\mathcal{EH}$, $\mathcal{CH}$ and $\mathcal{FI}$, respectively.

5.2 Dynamical quantities in evolving spacetimes

The evolution of the Brans-Dicke and electrically charged scalar fields (2.6)–(2.7) in double null coordinates is governed by the following equations:

$$\Phi_{,uv} = -\frac{fZ + gW}{r} + \frac{\Phi^\beta \beta\, q^2 \alpha^2}{2r^4\,(3+2\omega)} - \frac{\Phi^\beta\,(1-\beta)}{3+2\omega}\left[w\bar{z} + z\bar{w} + ieA_u\,(\bar{z}s - z\bar{s})\right], \tag{5.1}$$

$$\phi_{,uv} = -\frac{fz + gw}{r} - ieA_u z - \frac{ieA_u gs}{r} - \frac{ie}{4r^2}\alpha^2 qs - \frac{\beta}{2\Phi}\left(Wz + Zw + iesA_u Z\right). \tag{5.2}$$

The evolution of the u-component of the U(1) gauge field four-potential and the charge function in the (vu)-plane is governed by equations (2.36) and (2.37), respectively. As may be inferred from (5.1), the case of $\omega = -1.5$ is a singular point of the evolution equation.

5.2.1 Type IIA model

The hypersurfaces of constant values of the Brans-Dicke field and the real part of the electrically charged scalar field for the evolutions proceeding with $\beta = 0$ are shown in figure 7. The Brans-Dicke and the electrically charged fields become more dynamical as the singularity is approached except the latter when ω equals -1.4, for which the opposite

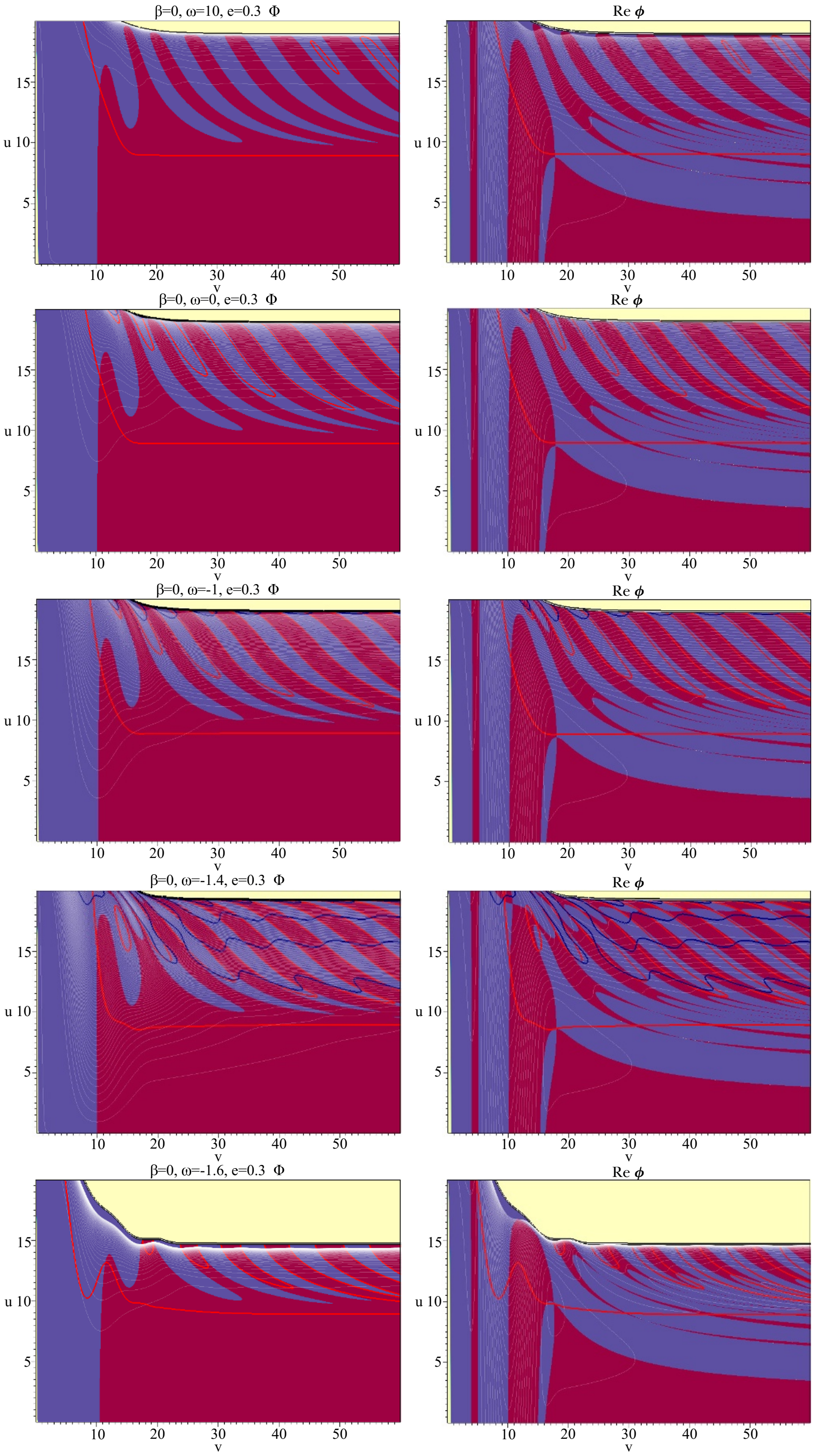


Figure 7. (Color online) The constancy hypersurfaces of the Brans-Dicke field (left column) and the real part of the electrically charged scalar field function (right column) for evolutions conducted within the Brans-Dicke theory for $\beta = 0$.

JHEP05(2016)155

tendency is observed. The Brans-Dicke field varies more substantially in the dynamical region of spacetime as ω decreases, except the case of $\omega = -1.6$, in which the changes of the field values are less considerable in comparison to, e.g., $\omega = -1.4$. The Brans-Dicke parameter does not influence the tendencies in the dynamics of the electrically charged scalar field, whose values vary in a similar manner within the dynamical area for all the studied values of ω.

In all the emerging spacetimes the constancy hypersurfaces of the Brans-Dicke field are spacelike nearby the central singularity in its dynamical part. It means that the field is a good candidate for a time measurer in this area, also because its values change monotonically there. For larger values of advanced time, the character of the hypersurfaces changes in the vicinity of the singularity and they are either spacelike or timelike there. Hence, the Brans-Dicke field cannot be used a 'clock' in the asymptotic spacetime region. This result is in general consistent with the outcomes obtained in paper I for the uncharged case. The only difference is that in the dynamical area, the Brans-Dicke field can be employed to quantify time further away from the singularity, as in the current case there are none nonspacelike hypersurfaces reaching $r = 0$, even at single points.

The constancy hypersurfaces of the real part of the electrically charged scalar field are spacelike in the dynamical spacetime region of high curvature. They also change monotonically there and thus, the field is a potential time measurer in this area. On the contrary, similarly to the above-mentioned case of the Brans-Dicke field, when v increases to the values at which the apparent horizon settles along a null hypersurface, the constancy hypersurfaces are either spacelike or timelike near the singularity. For this reason, the electrically charged scalar field cannot play a role of a 'clock' there. The conclusions are in agreement with the neutral case of paper I.

The constancy hypersurfaces of the Brans-Dicke field, as well as the modulus, the real and imaginary parts of the electrically charged scalar field function for the sample evolution with $\beta = 0$ and $\omega = -1.4$ are shown in figure 8. The behavior of the constancy hypersurfaces of the imaginary part of the complex field and its modulus is similar to their behavior for the real part of the field function. The field dynamics is most apparent in the dynamical spacetime region and beyond the event horizon, where it decreases as the singularity is approached. The constancy hypersurfaces are spacelike and change monotonically in the close neighborhood of the singularity for small v, and their character is either spacelike or timelike as $v \to \infty$. For these reasons, all the characteristics of the electrically charged scalar field can be used as 'clocks' in the highly curved dynamical area and are excluded in this regard for large values of advanced time.

Since none of the above quantities can be used to measure time along the whole central singularity at $r = 0$, the u-component of the U(1) gauge field four-potential and the charge function were also tested in this respect. Their constancy hypersurfaces for the sample evolution with $\beta = 0$ and $\omega = -1.4$ are shown in figure 9. The former is mostly dynamical for large advanced times in the vicinity of the event horizon. The constancy hypersurfaces of A_u are timelike along the whole singularity, so it definitely cannot serve as a time quantifier in the regions of high curvature during the collapse. The values of the charge function change considerably in the dynamical region of the spacetime and the dynamics increases

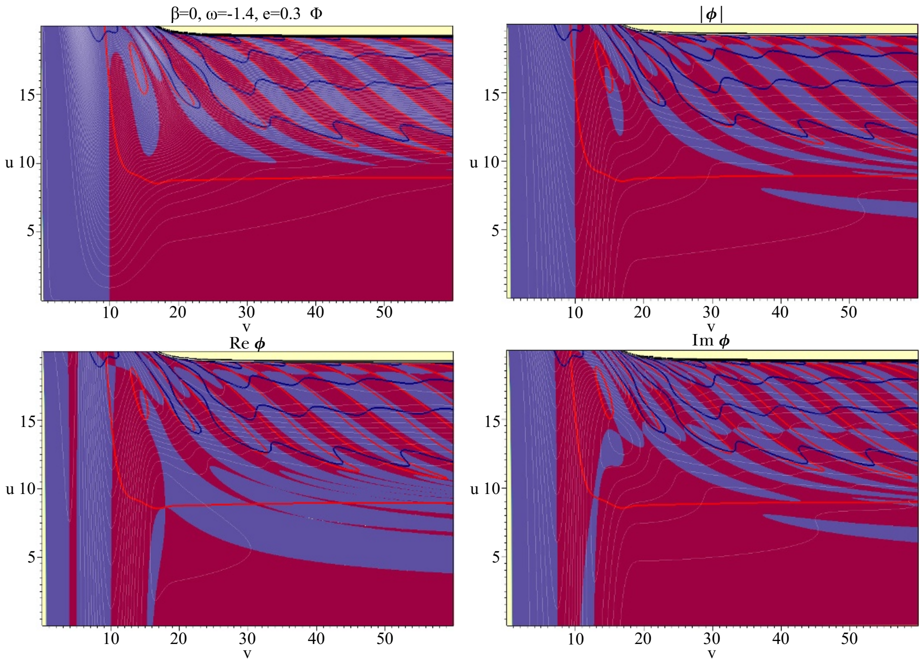


Figure 8. (Color online) The constancy hypersurfaces of the Brans-Dicke field Φ, as well as the modulus, the real and imaginary parts of the electrically charged scalar field function ($|\phi|$, Re ϕ and Im ϕ, respectively) for the evolution described by the parameters $\beta = 0$ and $\omega = -1.4$.

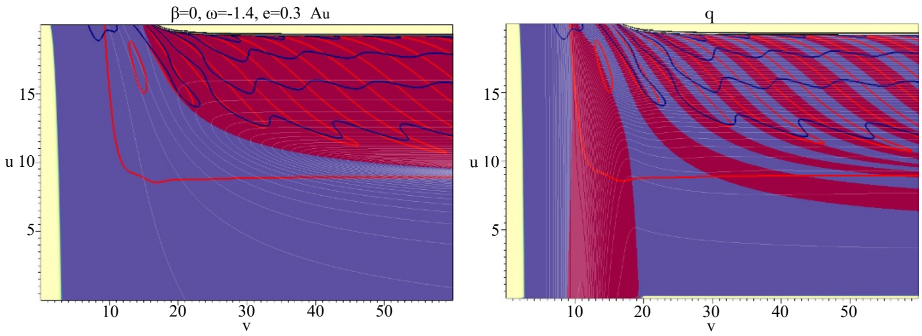


Figure 9. (Color online) The constancy hypersurfaces of the u-component of the U(1) gauge field four-potential (A_u) and the charge function (q) for the evolution described by the parameters $\beta = 0$ and $\omega = -1.4$.

for all v when the singularity is approached. The nature of the constancy hypersurfaces of q is either spacelike or timelike nearby the singularity and for this reason this quantity is not a good candidate for a 'clock' during the examined process.

As may be inferred from figures 7 and 8, as the Cauchy horizon is approached the nature of the hypersurfaces of constant Re ϕ, Im ϕ and $|\phi|$ change between spacelike and timelike as u changes. For this reason, these quantities cannot be used as time measurers nearby the Cauchy horizon. As can be seen in figure 9, the u-component of the U(1) gauge field four-potential is timelike in almost the whole region of interest and the behavior of the charge function is similar to the behavior of the quantities characterizing the complex

JHEP05(2016)155

scalar field. Thus, A_u and q are also excluded as time quantifiers as the Cauchy horizon is approached. This result obtained for the vicinity of the Cauchy horizon is the same as in the case of the collapse proceeding in the Einstein gravity, described in section 4.2.

5.2.2 Type I and heterotic models

The constancy hypersurfaces of the Brans-Dicke field and the real part of the electrically charged scalar field for the evolutions proceeding with β equal to 0.5 and 1 are shown in figures 10 and 11, respectively. The values of the Brans-Dicke field function vary more and more considerably as the central singularity is approached along both null directions. Its dynamics in the dynamical spacetime region is less apparent and it increases as ω decreases. In the case of $\omega = -1.6$, the Brans-Dicke field function values change dynamically nearby the point, at which the $r_{,v} = 0$ and $r_{,u} = 0$ apparent horizons meet the singularity and the future infinity. The electrically charged scalar field is most dynamical within the v-range, in which the $r_{,v} = 0$ apparent horizon is spacelike. The values of the real part of its field function change significantly in the dynamical spacetime region and this behavior does not depend on the parameter ω. The field becomes more dynamical as the singularity is approached and this phenomenon diminishes as ω decreases.

When both scalar fields present in the system are coupled, the constancy hypersurfaces of the Brans-Dicke field are spacelike nearby the whole singularity. The field function values vary monotonically in the region of high curvature. When β equals 0.5, there can exist a single point at the singularity, at which a nonspacelike constancy hypersurface reaches it. However, as was explained in paper I, such isolated points do not exclude the field from being a time quantifier. For the above reasons, the Brans-Dicke field can be employed to measure time in the neighborhood of the singular $r = 0$. The above outcomes obtained for the charged scalar field collapse are different from the results achieved for the neutral one in paper I. When a real or complex uncharged scalar field accompanied the Brans-Dicke field during the process, the latter could be a time measurer only in the dynamical spacetime region when β was equal to 0.5. Hence, the existence of electric charge in the spacetime enables the Brans-Dicke field to become a good candidate for a 'clock' in the entire vicinity of the central singularity for both examined values of $\beta \neq 0$.

The hypersurfaces of constant values of the real part of the electrically charged scalar field are spacelike along the singularities in all cases with a nonvanishing β, except $\omega = 10$ and $\beta = 1$. There can only exist separated points, at which a nonspacelike hypersurface can reach the singularity. These points, as was mentioned above, do not prevent the quantity from being a 'clock' nearby the singularity. Moreover, the field function values change monotonically when the singularity is approached. Hence, the charged scalar field can be treated as a time measurer along the central singularity for all the cases apart from $\omega = 10$ with $\beta = 1$, for which timelike constancy hypersurfaces reach the singularity at large values of advanced time. This exceptional behavior probably signals an approach of the Einstein limit of the theory, which appears for large values of the Brans-Dicke coupling.

The hypersurfaces of constant values of the Brans-Dicke field, as well as the modulus, the real and imaginary parts of the electrically charged scalar field function for the evolution with $\beta = 0.5$ and $\omega = -1$ are shown in figure 12. The same set of quantities for the collapse

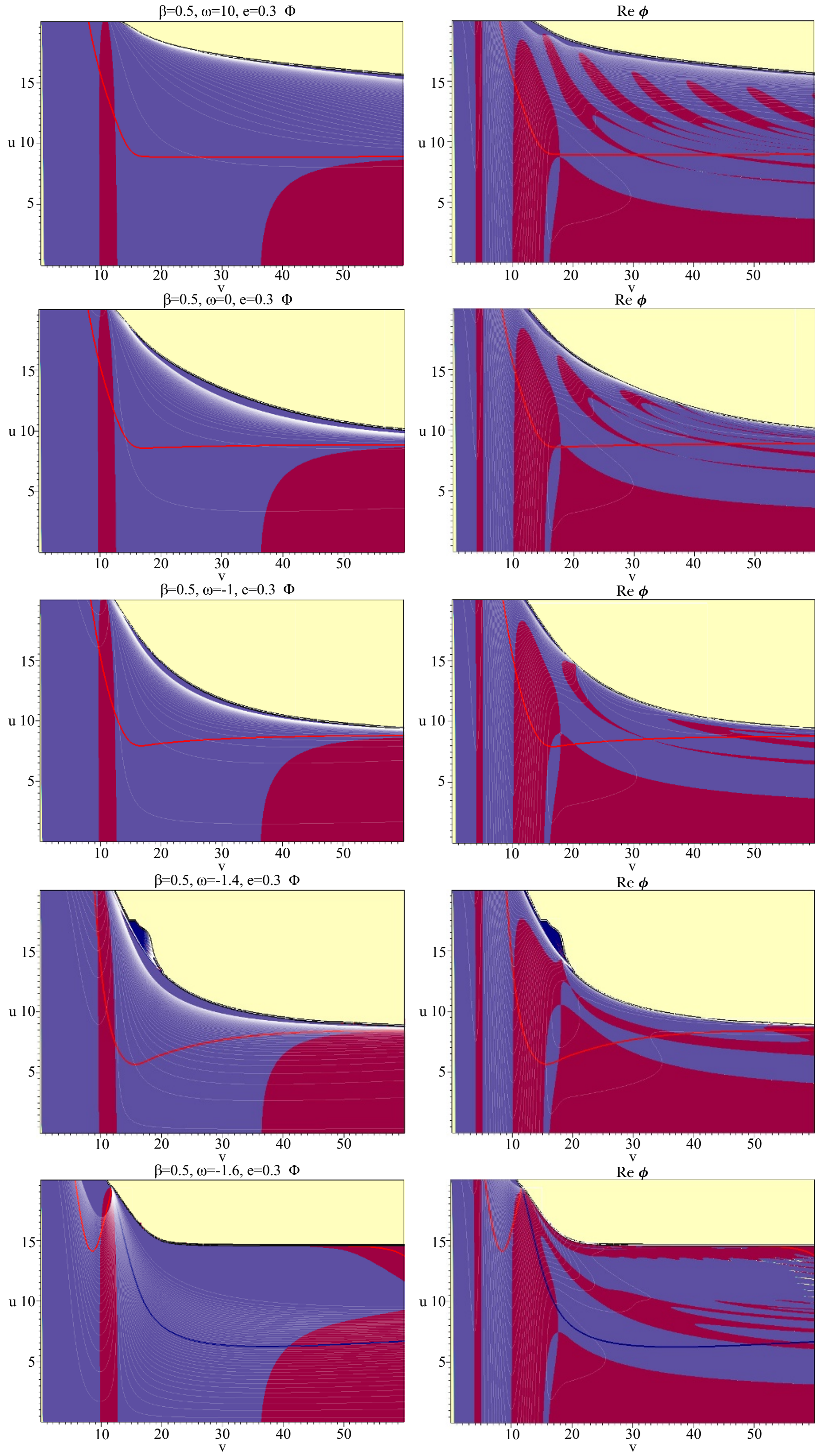


Figure 10. (Color online) The constancy hypersurfaces of the Brans-Dicke field (left column) and the real part of the electrically charged scalar field function (right column) for evolutions conducted within the Brans-Dicke theory for $\beta = 0.5$.

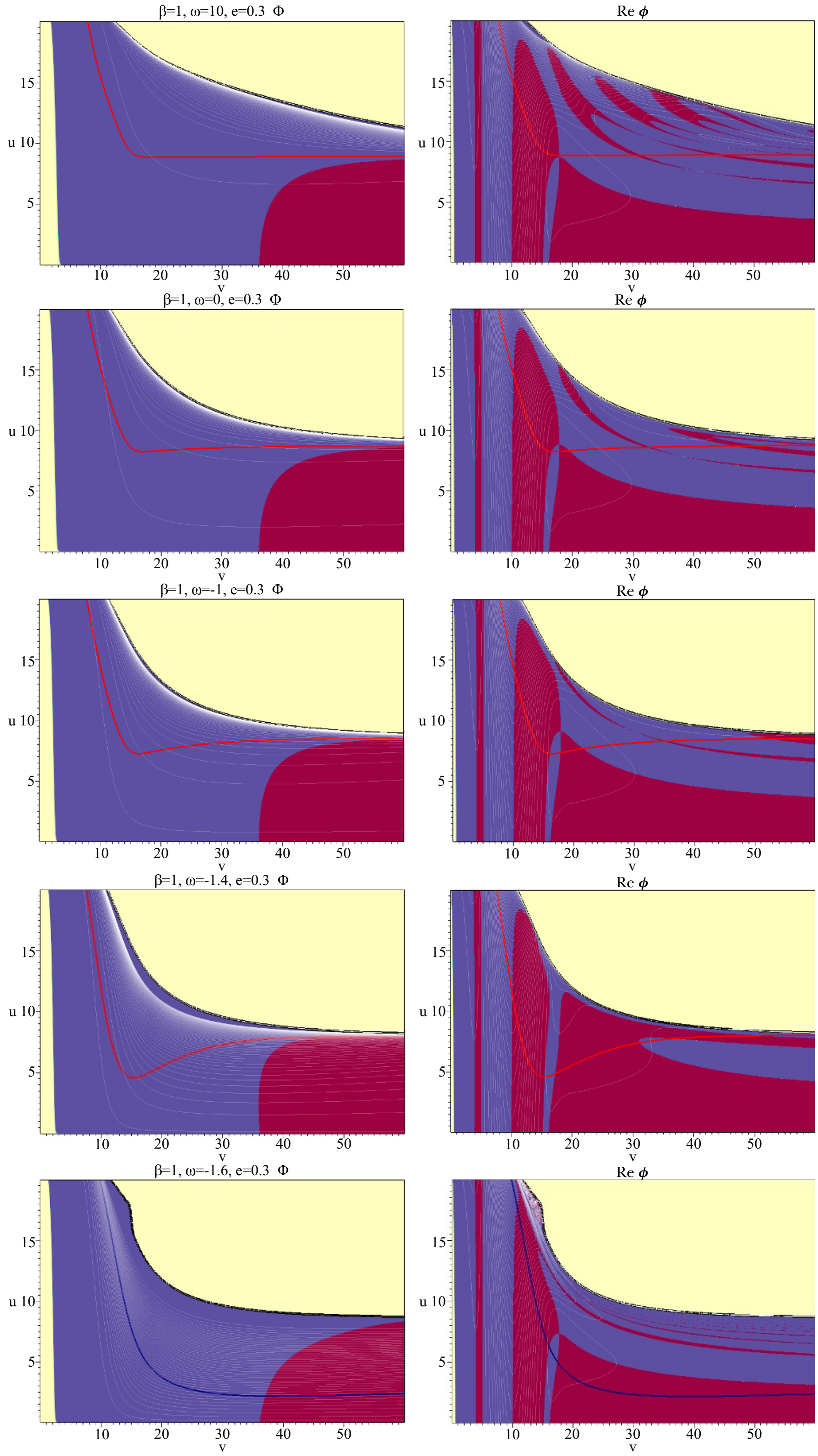


Figure 11. (Color online) The constancy hypersurfaces of the Brans-Dicke field (left column) and the real part of the electrically charged scalar field function (right column) for evolutions conducted within the Brans-Dicke theory for $\beta = 1$.

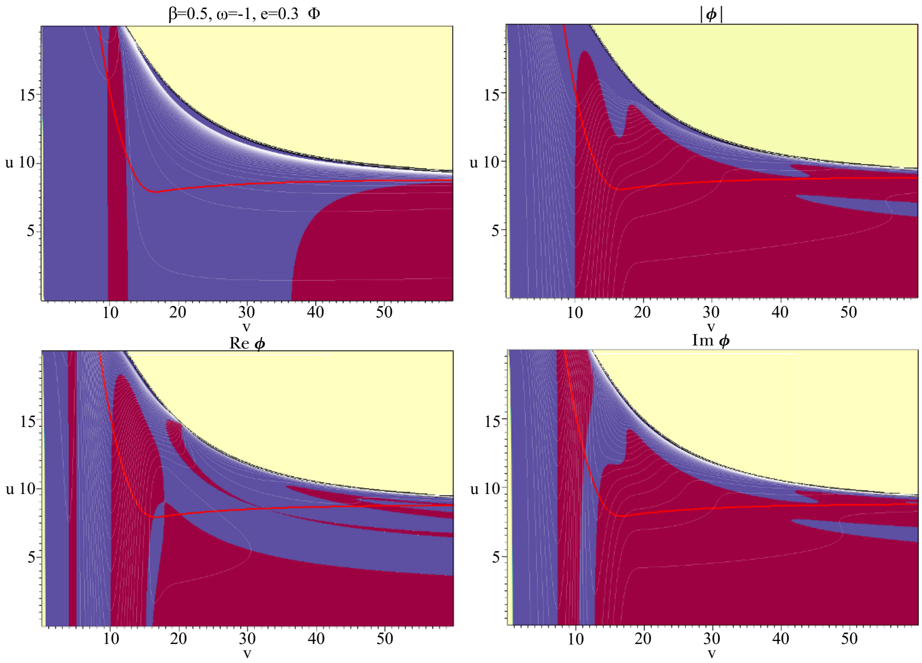


Figure 12. (Color online) The constancy hypersurfaces of the Brans-Dicke field Φ, as well as the modulus, the real and imaginary parts of the electrically charged scalar field function ($|\phi|$, Re ϕ and Im ϕ, respectively) for the evolution described by the parameters $\beta = 0.5$ and $\omega = -1$.

running for $\beta = 1$ and $\omega = -1.4$ is shown in figure 13. Both the imaginary part of the field function and its modulus behave analogously to the real part of the complex scalar field. They are spacelike nearby the central singularity and they change monotonically there. Although there can exist separated points, at which a nonspacelike hypersurface reaches the singularity, both of the quantities can be treated as 'clocks' in the whole region of high curvature. Since all the investigated field characteristics provide a time measure in the area of interest, there does not exist the need for examining other quantities in this respect.

5.2.3 Overall dependence on evolution parameters

The dynamics of the Brans-Dicke field is most evident as the central singularity is approached for all the investigated values of β and ω. The field function values change more significantly for smaller retarded times as ω decreases, with one exception of $\beta = 0$ and $\omega = -1.6$. The nature of the Brans-Dicke field constancy hypersurfaces strongly depends on whether the parameter β vanishes or not. Thus, it can be suspected that it is mainly related to the second term of the right-hand side of the equation (5.1). In the latter case, the hypersurfaces are spacelike in the whole region of high curvature, while in the former case they can be timelike nearby the singularity for large values of v. Whatever the value of β and ω, the real part of the electrically charged scalar field is most dynamical in the dynamical spacetime region and its values vary more significantly in the vicinity of the singularity, apart from $\beta = 0$ and $\omega = -1.4$, in which the latter tendency is reversed. Similarly to the Brans-Dicke field function, Re ϕ is always spacelike in the neighborhood of the singularity when $\beta \neq 0$ and can be timelike at large advanced times when β vanishes.

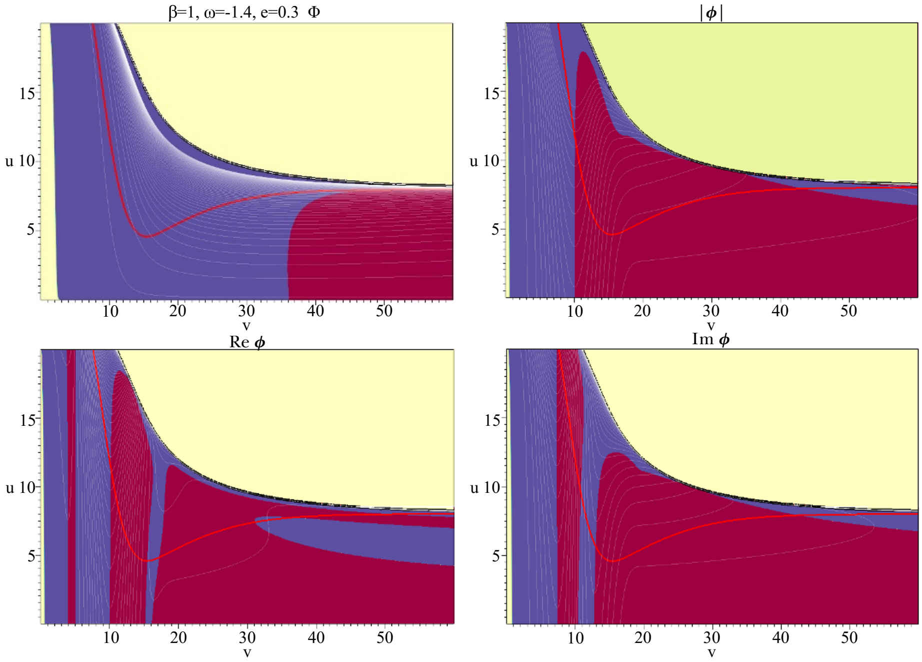


Figure 13. (Color online) The constancy hypersurfaces of the Brans-Dicke field Φ, as well as the modulus, the real and imaginary parts of the electrically charged scalar field function ($|\phi|$, Re ϕ and Im ϕ, respectively) for the evolution described by the parameters $\beta = 1$ and $\omega = -1.4$.

6 Conclusions

The dynamical collapse of a self-gravitating electrically charged scalar field was examined within the frameworks of the Einstein gravity and the Brans-Dicke theory. In the latter case, a set of values of the Brans-Dicke coupling constant and the coupling between the Brans-Dicke and the electrically charged scalar fields was considered. The assumed values of ω allowed us to investigate the large ω, $f(R)$, dilatonic, brane-world and ghost limits of the theory. The chosen values of the β parameter were motivated by the type IIA, type I and heterotic string theory-inspired models.

Apart from the emerging spacetime structures, which were described in each of the investigated cases, a possibility of measuring time with the use of dynamical quantities present in the system was assessed. The current paper broadens the scope of [19] and complements the analyses described in paper I, which referred to the neutral scalar field collapse in the general relativistic and Brans-Dicke regimes, respectively.

The outcome of the collapse of an electrically charged scalar field in the Einstein gravity is a dynamical Reissner-Nordström spacetime. The central spacelike singularity along $r = 0$ is surrounded by a single apparent horizon $r_{,v} = 0$, which settles along the event horizon of the spacetime in the region, where $v \to \infty$. The Cauchy horizon is situated at the null hypersurface $v = \infty$ beyond the event horizon.

The real and imaginary parts of the complex scalar field function, as well as its modulus and the charge function are most dynamical in the spacetime region, in which the apparent horizon is spacelike. Their values change more and more considerably as the singularity

JHEP05(2016)155

is approached. The variations of the values of the u-component of the Maxwell field four-potential are most significant nearby the horizon at large values of advanced time. Nearby the singularity, all the quantities characterizing the electrically charged scalar field and the charge function can be employed as 'clocks' in the dynamical spacetime region, because their constancy hypersurfaces are spacelike and their values change monotonically in the area. For larger values of v, measuring time with their use in the region of high curvature is impossible, because the character of the constancy hypersurfaces changes between spacelike and timelike. The quantity A_u cannot be used to quantify time, as the hypersurfaces of its constant values are timelike along the whole singularity. None of the above-mentioned quantities is a good candidate for a time measurer in the neighborhood of the Cauchy horizon, due to the possible timelike character of their constancy hypersurfaces as the horizon is approached.

In comparison to the neutral scalar field collapse described from the viewpoint of time quantification using a dynamical field in [19], the results obtained in the case of an electrically charged scalar field case support the conclusion that the evolving scalar field has a potential for measuring time in the dynamical spacetime region nearby the singularity. On the contrary, the existence of electric charge in the spacetime excludes the possibility of using the scalar field as a 'clock' in the asymptotic region of high curvature, i.e., for large values of advanced time in the vicinity of the singularity.

During the collapse proceeding in the Brans-Dicke theory, when β equals 0 each of the emerging spacetimes contains a central spacelike singularity surrounded by an apparent horizon $r_{,v} = 0$, which coincides with the event horizon as v tends to infinity. In the vicinity of the singular $r = 0$ line, there exist several additional hypersurfaces of $r_{,v} = 0$ and $r_{,u} = 0$. The Cauchy horizon is present in each spacetime within the event horizon at $v = \infty$. For the nonvanishing β parameter, each of the spacetimes forming during the collapse with $\omega \geqslant -1$ consists of a spacelike singularity along $r = 0$ surrounded by a single apparent horizon $r_{,v} = 0$, whose location for large values of advanced time indicates the event horizon position in the spacetime. Neither additional apparent horizons nor the Cauchy horizon exist in the spacetimes. In the ghost limit of $\beta \neq 0$, i.e., for $\omega = -1.6$, the future infinity is situated at large u and surrounded by an apparent horizon $r_{,u} = 0$. Additionally, when $\beta = 0.5$, from the point of coincidence of the two a spacelike central singularity extends along $r = 0$ towards smaller values of v and larger retarded times. The singularity is fully surrounded by an $r_{,v} = 0$ apparent horizon.

The dynamics of both Brans-Dicke and electrically charged scalar fields becomes more significant when the singularity is approached, apart from the case of $\beta = 0$ and $\omega = -1.4$. The variations of the field functions values are also considerable in the dynamical area of the spacetime, which is more apparent in the case of a complex scalar field. The u-component of the electromagnetic field four-potential is most dynamical at large v nearby the event horizon, while the charge function dynamics is analogous to the complex scalar field behavior described above. In all the investigated cases, both the Brans-Dicke and the electrically charged scalar fields can act as time measurers in dynamical spacetime regions of high curvature, as their constancy hypersurfaces are spacelike and their values changes are monotonic there. The existence of separated points, at which single nonspacelike hypersurfaces of constant field functions can reach the singularity does not spoil this possibility. The feasibility of treating the dynamical quantities as 'clocks' in the asymptotic spacetime regions as the values of v increase mainly depends on the value of β. When the parameter vanishes, the Brans-Dicke field, as well as the real and imaginary parts of the complex scalar field function, along with its modulus and the charge function, are either spacelike or timelike near the singularity for all values of ω. Hence, all these quantities are excluded from being time quantifiers. Moreover, A_u is timelike along the whole singularity, so it also cannot be used to measure time there. On the opposite, when $\beta \neq 0$ the constancy hypersurfaces of Φ, Re ϕ, Im ϕ and $|\phi|$ are spacelike in the neighborhood of the singular $r = 0$ and they change monotonically there, except the case of $\beta = 1$ and $\omega = 10$. This means that all these quantities can be used as 'clocks' in the vicinity of the singularity as $v \to \infty$.

The conducted analyses revealed that there does not exist a good dynamical time measure in the spacetime region neighboring the Cauchy horizon, because all the examined quantities may be timelike in this area. The conclusions regarding the Cauchy horizon, which is present in the spacetimes formed during the Einstein collapse and the Brans-Dicke collapse with $\beta = 0$, are similar to the ones drawn in the case of a neutral scalar field collapse in the ghost regime with the vanishing β. In this case presented in paper I, measuring time with the use of dynamical quantities was also excluded nearby the emerging Cauchy horizon.

The dynamical quantities which can be used to quantify time in evolving gravitational systems are gathered in table 1, which summarizes the investigations of potential time measurements nearby the singularities emerging during the collapse within the Einstein and Brans-Dicke theories. The outcomes of the research concerning the charged collapse in the Brans-Dicke theory support the conclusions about using dynamical quantities as time variables within the evolving spacetimes during dynamical gravitational evolutions of coupled matter-geometry systems, which were formulated on the basis of investigating the neutral collapse in paper I. First, the spacelike character of the constancy hypersurfaces of the quantities and the monotonicity of their parametrization are not retained within the whole spacetime. Second, there does not exist a good time measure for the areas nearby the Cauchy horizons. Third, in attempts to use dynamical quantities as 'clocks', special attention should be paid to the values of the free evolution parameters, as they can strongly influence this possibility.

In general, in comparison to the collapse of a neutral scalar field, the presence of the electric charge in the spacetime modifies the feasibility of time quantification using the evolving quantities in the following way. In the case of uncoupled Brans-Dicke and complex scalar fields, i.e., for $\beta = 0$, the charge spoils the possibility of measuring time with their use, while when the fields are coupled, that is β is not equal to zero, the charge enhances it. In the studied charged collapse, there also exist two more potential time measures, i.e., the charge function and the nonzero component of the Maxwell field four-potential. In the context of time quantification, the former behaves analogously to the complex scalar field dynamical characteristics. On the other hand, the latter does not provide a good time measure, as its constancy hypersurfaces are timelike in the regions of high curvature.

During the gravitational collapse of an electrically charged scalar field, the mass inflation phenomenon can appear in the spacetime. If so, a super-Planckian surface develops

	scalar field collapse in the Einstein theory			
	neutral [19]		charged	
	$\mathcal{D}$	$\mathcal{A}$	$\mathcal{D}$	$\mathcal{A}$
	ϕ		ϕ, q	–
	scalar field collapse in the Brans-Dicke theory			
	neutral [16]		charged	
	$\mathcal{D}$	$\mathcal{A}$	$\mathcal{D}$	$\mathcal{A}$
$\beta = 0$	Φ, ϕ $^{(1)}$		Φ, ϕ, q	–
$\beta = 0.5$	Φ, ϕ		Φ, ϕ, q	
$\beta = 1$	ϕ		Φ, ϕ, q $^{(2)}$	

Table 1. Feasibility of using dynamical quantities of a coupled matter-geometry system to quantify time nearby the singularity emerging during the gravitational evolution. The dynamical ($v \leqslant 20$) and asymptotic ($v \to \infty$) spacetime regions are denoted as $\mathcal{D}$ and $\mathcal{A}$, respectively. The scalar and Brans-Dicke fields are ϕ and Φ, respectively, while q denotes the charge function (2.35). Notes: $^{(1)}$except the vicinity of the Cauchy horizon for $\omega = -1.6$, $^{(2)}$except the asymptotic region for $\omega = 10$.

outside the true singularity [9]. Within the region encompassed by it the spacetime curvature reaches values excluding the usage of a classical theory of gravity. Hence, quantized gravity should be applied not around the singularity in this case, but around the mass inflation super-Planckian surface. It is possible that in the vicinity of this hypersurface one of the quantities discussed above or their combination could provide a good time measure. However, since the proposed construction depends on the cutoff of the super-Planckian region, it requires more detailed studies on the region itself at first, as the determination of its boundaries could be only arbitrary without any specific analyses. For this reason, we leave the announced topic for future researches.

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A Numerical computations

The solution of the evolution equations (2.30), (2.33)–(2.37) was provided numerically with the use of an enhanced version of the code prepared for the needs of paper I. The modules governing the evolution of the gravitational, Brans-Dicke and scalar fields were modified in order to account for the presence of an electrically charged scalar field instead of a neutral one. The program was also supplied with a module governing the evolution of the Maxwell field. The quantities A_u and q were set as equal to zero along the initial

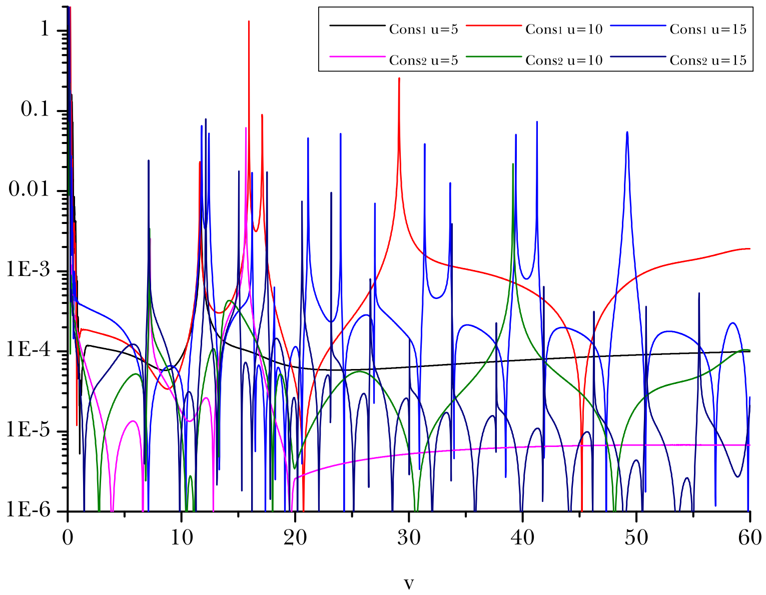

Figure 14. (Color online) Monitoring of the constraints. The values of the equations (A.1) and (A.2) were calculated along three null hypersurfaces of constant u equal to 5, 10 and 15.

$v = 0$ hypersurface, because due to the form of the evolving field, the center of the shell was not affected by matter. The u-component of the electromagnetic field four-potential and the charge function along the initial $u = 0$ hypersurface were calculated according to (2.36) and (2.37).

The accuracy checks of the numerical code will be presented for a sample evolution running for the parameters $\beta = 0$, $\omega = -1.4$ and $e = 0.3$. The consistency of the computations was monitored during all evolutions using the constraints (2.31) and (2.32). Figure 14 presents the descendant quantities

$$\mathrm{Cons}_1 \equiv \frac{2\left|r_{,uu} - 2fh + \frac{r}{2\Phi}\left(W_{,u} - 2hW\right) + \frac{r\omega}{2\Phi^2}W^2 + \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{uu}\right|}{\left|r_{,uu}\right| + \left|2fh - \frac{r}{2\Phi}\left(W_{,u} - 2hW\right) - \frac{r\omega}{2\Phi^2}W^2 - \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{uu}\right|}, \tag{A.1}$$

$$\mathrm{Cons}_2 \equiv \frac{2\left|r_{,vv} - 2dg + \frac{r}{2\Phi}\left(Z_{,v} - 2dZ\right) + \frac{r\omega}{2\Phi^2}Z^2 + \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{vv}\right|}{\left|r_{,vv}\right| + \left|2dg - \frac{r}{2\Phi}\left(Z_{,v} - 2dZ\right) - \frac{r\omega}{2\Phi^2}Z^2 - \frac{4\pi r}{\Phi}\widetilde{T}^{\mathrm{EM}}_{vv}\right|}, \tag{A.2}$$

calculated along three arbitrary null hypersurfaces for the evolution with parameters specified above. The values of Cons_1 and Cons_2 ought to be smaller than 2 in order to satisfy the constraints well (except the case when $r_{,uu}$ or $r_{,vv}$ vanishes). As can be inferred from the plot, the error is less than 1% within almost the whole integration domain. Since the constraint equations are stable, the simulations are consistent.

The outcome of the convergence check of the numerical code for the sample evolution is presented in figure 15. The values of a quantity constructed from the r function obtained on two grids with a quotient of integration steps equal to 2 were calculated at three arbitrary $u = \mathrm{const.}$ hypersurfaces. An overlap between two profiles of the defined quantity at each

JHEP05(2016)155

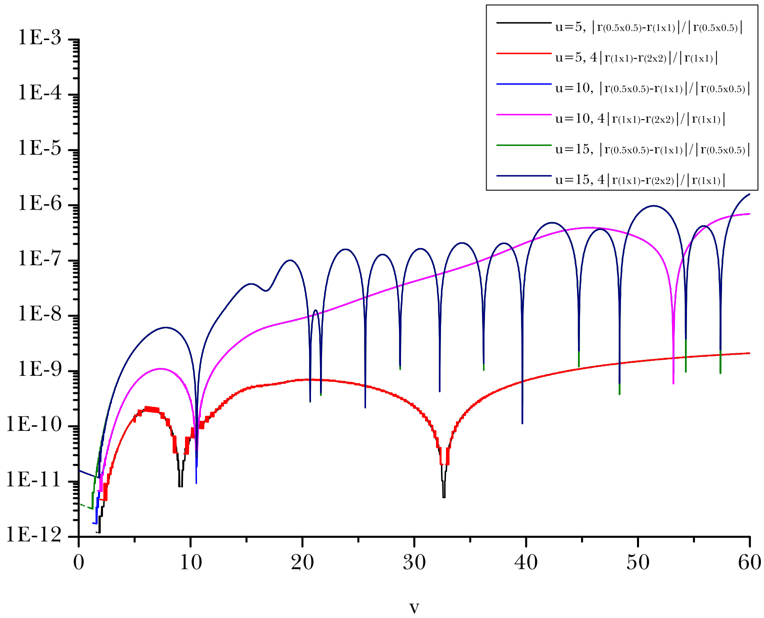

Figure 15. (Color online) The convergence of the code presented through the prism of the values of the quantity $|r_{(k\times k)} - r_{(2k\times 2k)}|/|r_{(k\times k)}|$ with $k = 0.5, 1$ calculated at the same hypersurfaces of constant u as in figure 14. $(k \times k)$ denotes the resolution of the numerical grid, on which the computations were conducted.

$u = \text{const.}$ was obtained when the result from finer grids was multiplied by 4. Thus, the code displays a second order convergence. The discrepancy between each two profiles at each constant u hypersurface is less than $10^{-4}\%$ except a close vicinity of the singularity.

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JHEP05(2016)155