

RECEIVED: January 14, 2015

REVISED: April 19, 2015

ACCEPTED: April 27, 2015

PUBLISHED: May 15, 2015

Probing texture zeros with scaling ansatz in inverse seesaw

Ambar Ghosal and Rome Samanta

*Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Kolkata 700064, India*

E-mail: ambar.ghosal@saha.ac.in, rome.samanta@saha.ac.in

ABSTRACT: We investigate neutrino mass matrix phenomenology involving scaling ansatz and texture zeros adhering inverse seesaw mechanism. It is seen that four is the maximum number of zeros in m_D and μ to obtain viable phenomenology. Depending upon the generic nature of the effective neutrino mass matrices we classify all the emerged matrices in four categories. One of them is ruled out phenomenologically due to inappropriate value of reactor mixing angle after breaking of the scaling ansatz. The mass ordering is inverted in all cases. One of the distinguishable feature of all these categories is the vanishingly small value of CP violation measure J_{CP} due to small value of δ_{CP} . Thus those categories will be ruled out if CP violation is observed in the leptonic sector in future experiments.

KEYWORDS: Neutrino Physics, CP violation

ARXIV EPRINT: [1501.00916](https://arxiv.org/abs/1501.00916)

Contents

1	Introduction	1
2	Scaling property and texture zeros	2
3	4 zero texture	5
4	Parametrization	7
5	Phase rotation	8
6	Breaking of the scaling ansatz	9
6.1	Breaking in the Dirac sector	10
6.1.1	Category A,B	10
6.1.2	Category C	11
6.1.3	Category D	12
6.2	Numerical analysis	12
6.2.1	Category A,B	12
6.2.2	Category C	13
6.2.3	Category D	14
6.3	Breaking in Dirac+Majorana sector	15
6.3.1	Numerical results	15
7	Summary and conclusion	17

1 Introduction

Among the variants of the seesaw mechanism, inverse seesaw [1–13] stands out as an attractive one, due to its characteristic feature of generation of small neutrino mass without invoking high energy scale in the theory. Although to realize such feature one has to pay the price in terms of incorporation of additional singlet fermions, nevertheless, in different GUT models accommodation of such type of neutral fermions are natural. Furthermore, such mechanism appeals to the foreseeable collider experiments to be testified due to its unique signature. The 9×9 neutrino mass matrix in this mechanism is written as

$$m_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_{RS} \\ 0 & M_{RS}^T & \mu \end{pmatrix} \quad (1.1)$$

with the choice of basis (ν_L, ν_R^c, S_L) . The three matrices appear in m_ν are m_D , M_{RS} and μ among them m_D and M_{RS} are Dirac type whereas μ is Majorana type mass matrix. After diagonalization, the low energy effective neutrino mass comes out as

$$\begin{aligned} m_\nu &= m_D M_{RS}^{-1} \mu (m_D M_{RS}^{-1})^T \\ &= F \mu F^T \end{aligned} \tag{1.2}$$

where $F = m_D M_{RS}^{-1}$. Such definition resembles the above formula as a conventional type-I seesaw expression of m_ν . However, m_ν contains large number of parameters and it is possible to fit them with neutrino oscillation experimental data [14–16] (but the predictability is less). Our goal in this work is to find out a phenomenologically viable texture of m_D and μ with minimum number of parameters or equivalently maximum number of zeros. We bring together two theoretical ideas to find out a minimal texture and they are

- i) Scaling ansatz [17–27],
- ii) Texture Zeros [28–45].

At the outset of the analysis, we choose a basis where the charged lepton mass matrix (m_E) and M_{RS} are diagonal along with texture zeros in m_D and μ matrices. We also start by assuming the scaling property in the elements of m_D and μ to reduce the number of relevant matrices. Although, we are not addressing the explicit origin of such choice of matrices, however, qualitatively we can assume that this can be achieved due to some flavour symmetry [46] which is required to make certain that the texture zeros appear in m_D and μ are in the same basis in which m_E and M_{RS} are diagonal. We restrict ourselves within the frame work of $SU(2)_L \times U(1)_Y$ gauge group however, explicit realization of such scheme obviously more elusive which will be studied elsewhere.

2 Scaling property and texture zeros

We consider scaling property between the second and third row of m_D matrix and the same for μ matrix also. Explicitly the relationships are written as

$$\frac{(m_D)_{2i}}{(m_D)_{3i}} = k_1 \tag{2.1}$$

$$\frac{(\mu)_{2i}}{(\mu)_{3i}} = k_2 \tag{2.2}$$

where $i = 1, 2, 3$ is the column index. We would like to mention that although we have considered different scale factors for m_D and μ matrices, however, the effective m_ν is still scale invariant and leads to $\theta_{13} = 0$. Thus, it is obvious to further break the scaling ansatz. In order to generate nonzero θ_{13} it is necessary to break the ansatz in m_D since, breaking in μ does not affect the generation of nonzero θ_{13} although in some cases it provides $m_3 \neq 0$. In our scheme texture zero format is robust and it remains intact while the scaling ansatz is explicitly broken. Such a scenario can be realized by considering the scaling ansatz and texture zeros to have a different origin.

Another point is to be noted that, since the μ matrix is complex symmetric whereas m_D is asymmetric, the scale factor considered in μ matrix is different from that of m_D to keep the row wise invariance as dictated by eq. (2.1) (for m_D), and eq. (2.2) (for μ). Finally, since the texture of M_{RS} matrix is diagonal it is not possible to accommodate scaling ansatz considered in the present scheme.

Let us now turn to further constrain the matrices assuming zeros in different entries. Since, in our present scheme the matrix M_{RS} is diagonal, we constrain the other two matrices. We start with the maximal zero textures with scaling ansatz of general 3×3 matrices and list different cases systematically in table 1.

We consider all the matrices¹ listed in table 1 as the Dirac type matrices(m_D). As the lepton number violating mass matrix μ is complex symmetric, therefore, the maximal number of zeros with scaling invariance is 5. Therefore, only m_3^5 and m_5^5 type matrices can be made complex symmetric with the scaling property and are shown in table 2 where they are renamed as μ_1^5 and μ_2^5 with a different scale factor k_2 .

Now using eq. (1.2) we can construct m_ν and it is found that all the mass matrices constructed out of these matrices are not suitable to satisfy the neutrino oscillation data. The reason goes as follows:

Case A: m_D (7, 6 zero) + μ_1^5, μ_2^5 (5 zero):

We can not generate nonzero θ_{13} by breaking the scaling ansatz because in this case all the structures of m_D are scaling ansatz invariant. This can be understood in the following way: if we incorporate scaling ansatz breaking by $k'_1 \rightarrow k_1(1 + \epsilon)$ all the structures of m_D are still invariant and m_ν matrix will still give $\theta_{13} = 0$ as breaking of scaling in μ_1^5 and μ_2^5 play no role for the generation of nonzero value of θ_{13} . To generate nonzero θ_{13} it is necessary to break scaling ansatz in the Dirac sector.

Case B: m_D (5 zero) + μ_1^5, μ_2^5 (5 zero):

The matrices in the last three rows (m_4^5 to m_{12}^5) of the ‘**5 zero texture**’ part of table 1 are ruled out due to the same reason as mentioned in **Case A** while, the matrices in the first row i.e. m_1^5, m_2^5 and m_3^5 give rise to the structure of m_ν as

$$A_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \tag{2.3}$$

where ‘*’ represents some nonzero entries in m_ν . This structure leads to complete disappearance of one generation. Moreover it has been shown in ref. [28] that if the number of independent zeros in an effective neutrino mass matrix (m_ν) is ≥ 3 it doesn’t favour the oscillation data and hence, ‘ A_1 ’ type mass matrix is ruled out.

Case C: m_D (4 zero) + μ_1^5 (5 zero):

There are 12 m_D matrices with 4 zero texture and they are designated as m_1^4, \dots, m_{12}^4 in table 1. Due to the same reason as discussed in **Case A**, m_{10}^4, m_{11}^4 and m_{12}^4 are not

¹From now on we use m^n as a mass matrix where $n(= 4, 5, 6, 7)$ is the number of zeros in that matrix.

7 zero texture		
$m_1^7 = \begin{pmatrix} 0 & 0 & 0 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_2^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_3^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$
6 zero texture		
$m_1^6 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_2^6 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_3^6 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$
$m_4^6 = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_5^6 = \begin{pmatrix} 0 & d_2 & 0 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_6^6 = \begin{pmatrix} 0 & 0 & d_3 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$
$m_7^6 = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$	$m_8^6 = \begin{pmatrix} 0 & d_2 & 0 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$	$m_9^6 = \begin{pmatrix} 0 & 0 & d_3 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$
5 zero texture		
$m_1^5 = \begin{pmatrix} 0 & 0 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$	$m_2^5 = \begin{pmatrix} 0 & 0 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$	$m_3^5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_1 c_1 & k_1 c_3 \\ 0 & c_1 & c_3 \end{pmatrix}$
$m_4^5 = \begin{pmatrix} d_1 & d_2 & 0 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_5^5 = \begin{pmatrix} 0 & d_2 & d_3 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_6^5 = \begin{pmatrix} d_1 & 0 & d_3 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$
$m_7^5 = \begin{pmatrix} d_1 & d_2 & 0 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_8^5 = \begin{pmatrix} 0 & d_2 & d_3 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_9^5 = \begin{pmatrix} d_1 & 0 & d_3 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$
$m_{10}^5 = \begin{pmatrix} d_1 & d_2 & 0 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$	$m_{11}^5 = \begin{pmatrix} 0 & d_2 & d_3 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$	$m_{12}^5 = \begin{pmatrix} d_1 & 0 & d_3 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$
4 zero texture		
$m_1^4 = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & k_1 c_2 & k_1 c_3 \\ 0 & c_2 & c_3 \end{pmatrix}$	$m_2^4 = \begin{pmatrix} 0 & d_2 & 0 \\ 0 & k_1 c_2 & k_1 c_3 \\ 0 & c_2 & c_3 \end{pmatrix}$	$m_3^4 = \begin{pmatrix} 0 & 0 & d_3 \\ 0 & k_1 c_2 & k_1 c_3 \\ 0 & c_2 & c_3 \end{pmatrix}$
$m_4^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$	$m_5^4 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$	$m_6^4 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$
$m_7^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$	$m_8^4 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$	$m_9^4 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$
$m_{10}^4 = \begin{pmatrix} d_1 & d_2 & d_3 \\ k_1 c_1 & 0 & 0 \\ c_1 & 0 & 0 \end{pmatrix}$	$m_{11}^4 = \begin{pmatrix} d_1 & d_2 & d_3 \\ 0 & k_1 c_2 & 0 \\ 0 & c_2 & 0 \end{pmatrix}$	$m_{12}^4 = \begin{pmatrix} d_1 & d_2 & d_3 \\ 0 & 0 & k_1 c_3 \\ 0 & 0 & c_3 \end{pmatrix}$

Table 1. Texture zeros with scaling ansatz of a general 3×3 matrix.

$\mu_1^5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k_2^2 s_3 & k_2 s_3 \\ 0 & k_2 s_3 & s_3 \end{pmatrix}$	$\mu_2^5 = \begin{pmatrix} 0 & k_2 s_3 & s_3 \\ k_2 s_3 & 0 & 0 \\ s_3 & 0 & 0 \end{pmatrix}$
---	---

Table 2. Maximal zero texture of μ matrix.

considered. Furthermore, m_ν arises through m_1^4 , m_4^4 and m_7^4 also correspond to the ‘ A_1 ’ type matrix (shown in eq. (2.3)) and hence are also discarded. Finally, remaining six m_D matrices m_2^4 , m_3^4 , m_5^4 , m_6^4 , m_8^4 and m_9^4 lead to the structure of m_ν with two zero eigenvalues and obviously they are also neglected.

Case D: m_D (4 zero) + μ_2^5 (5 zero):

In this case, for m_2^4 and m_3^4 the low energy mass matrix m_ν comes out as a null matrix while for m_1^4 the structure of m_ν is given by

$$A_2 = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix} \tag{2.4}$$

which is also neglected since the number of independent zeros ≥ 3 . On the other hand rest of the m_D matrices (m_4^4 to m_9^4) correspond to the structure of m_ν as

$$A_3 = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}. \tag{2.5}$$

Interestingly, a priori we cannot rule out the matrices of type A_3 , however it is observed that m_ν of this type fails to generate θ_{13} within the present experimental bound (details are mentioned in section 6.2.3). It is also observed that in this scheme to generate viable neutrino oscillation data, four zero texture of both m_D and μ matrices are necessary. Therefore, now on we discuss extensively the four zero texture in both the sectors (Dirac as well as Majorana sector).

3 4 zero texture

There are 126 ways to choose 4 zeros out of 9 elements of a general 3×3 matrix. Hence there are 126 textures. Incorporation of scaling ansatz leads to a drastic reduction to only 12 textures as given in the table 1. In our chosen basis since M_{RS} is taken as diagonal, therefore, the structure of m_D leads to the same structure of F . On the other hand the lepton number violating mass matrix μ is complex symmetric and therefore from the matrices listed in table 1, only m_1^4 and m_{10}^4 type matrices are acceptable. We renamed those matrices as μ_1^4 and μ_2^4 and explicit structures of them are presented in table 3.

There are now $2 \times 12 = 24$ types of m_ν due to both the choices of μ matrices. We discriminate different types of m_D matrices in the following way:

$$\mu_1^4 = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & k_2^2 s_3 & k_2 s_3 \\ 0 & k_2 s_3 & s_3 \end{pmatrix} \quad \mu_2^4 = \begin{pmatrix} r_1 & k_2 s_3 & s_3 \\ k_2 s_3 & 0 & 0 \\ s_3 & 0 & 0 \end{pmatrix}$$

Table 3. Four zero texture of μ matrix.

	m_D											
μ	m_1^4	m_2^4	m_3^4	m_4^4	m_5^4	m_6^4	m_7^4	m_8^4	m_9^4	m_{10}^4	m_{11}^4	m_{12}^4
μ_1^4	×	×	×	✓	✓	✓	✓	✓	✓	×	×	×
μ_2^4	×	×	×	✓	✓	✓	✓	✓	✓	×	×	×

Table 4. Compositions of the discarded and survived structures of m_ν .

$m_{D1}^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$	$m_{D2}^4 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$	$m_{D3}^4 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & 0 & k_1 c_3 \\ c_1 & 0 & c_3 \end{pmatrix}$
$m_{D4}^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$	$m_{D5}^4 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$	$m_{D6}^4 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & k_1 c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}$

Table 5. Four zero textures of the Dirac mass matrices.

- i) First of all, the texture m_{10}^4 , m_{11}^4 and m_{12}^4 are always scaling ansatz invariant due to the same reason mentioned earlier in **Case A** and hence are all discarded.

Next the matrices m_1^4 , m_2^4 and m_3^4 are also ruled out due to the following:

- a) When μ_1^4 matrix is taken to generate m_ν along with m_1^4 , m_2^4 and m_3^4 as the Dirac matrices, then the structure of the effective m_ν appears such that, one generation is completely decoupled thus leading to two mixing angles zero for the matrix m_1^4 and two zero eigenvalues when we consider m_2^4 and m_3^4 matrices.
- b) In case of μ_2^4 matrix, the form of m_ν for m_1^4 comes out as

$$A_4 = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix} \quad (3.1)$$

which is phenomenologically ruled out and for other two matrices (m_2^4 and m_3^4) m_ν becomes a null matrix. For a compact view of the above analysis we present the ruled out and survived structures of m_ν symbolically in table 4.

Thus we are left with same six textures of m_D for both the choices of μ and they are renamed in table 5 as m_{D1}^4 , m_{D2}^4 , \dots , m_{D6}^4 .

Obviously, it is clear that the above analysis leads to altogether 12 effective m_ν matrices arising due to six m_D (m_{D1}^4 to m_{D6}^4) and two μ (μ_1^4 and μ_2^4) matrices.

	Category A		Category B			
Matrices	I_A	II_A	I_B	II_B	III_B	IV_B
m_D	m_{D2}^4	m_{D6}^4	m_{D1}^4	m_{D3}^4	m_{D4}^4	m_{D5}^4
μ	μ_1^4	μ_1^4	μ_1^4	μ_1^4	μ_1^4	μ_1^4

Table 6. Different Composition of m_D and μ_1 matrices to generate m_ν .

	Category C		Category D			
Matrices	I_C	II_C	I_D	II_D	III_D	IV_D
m_D	m_{D1}^4	m_{D4}^4	m_{D2}^4	m_{D3}^4	m_{D5}^4	m_{D6}^4
μ	μ_2^4	μ_2^4	μ_2^4	μ_2^4	μ_2^4	μ_2^4

Table 7. Different Composition of m_D and μ_2 matrices to generate m_ν .

4 Parametrization

Depending upon the composition of m_D and μ we subdivided those 12 m_ν matrices in four broad categories and each category is again separated in few cases and the decomposition is presented in table 6 and table 7.

Throughout our analysis we consider the matrix M_{RS} as

$$M_{RS} = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}. \tag{4.1}$$

Following eq. (1.2), the m_ν matrix arises in Category A and Category B can be written in a generic way as

$$m_\nu^{AB} = m_0 \begin{pmatrix} 1 & k_1 p & p \\ k_1 p & k_1^2 (q^2 + p^2) & k_1 (q^2 + p^2) \\ p & k_1 (q^2 + p^2) & (q^2 + p^2) \end{pmatrix} \tag{4.2}$$

with the definition of parameters as following

$$\begin{aligned} \text{Set } I_A : m'_0 &= \frac{d_3^2 s_3}{p_3^2}, \quad p' = \frac{p_3 c_2}{p_2 d_3}, \quad q' = \frac{c_1 p_3}{d_3 p_1} \sqrt{\frac{r_1}{s_3}}, \quad m_0 = m'_0, \quad p = k_2 p', \quad q = q' \\ \text{Set } II_A : m'_0 &= \frac{d_2^2 s_3}{p_2^2}, \quad p' = \frac{p_2 c_2}{p_3 d_2}, \quad q' = \frac{c_1 p_2}{d_2 p_1} \sqrt{\frac{r_1}{s_1}}, \quad m_0 = m'_0 k_2^2, \quad p = \frac{p'}{k_2}, \quad q = \frac{q'}{k_2} \\ \text{Set } I_B : m'_0 &= \frac{d_1^2 r_1}{p_1^2}, \quad p' = \frac{c_1}{d_1}, \quad q' = \frac{c_3 p_1}{d_1 p_3} \sqrt{\frac{s_3}{r_1}}, \quad m_0 = m'_0, \quad p = p', \quad q = q' \\ \text{Set } II_B : m'_0 &= \frac{d_3^2 s_3}{p_3^2}, \quad p' = \frac{c_3}{d_3}, \quad q' = \frac{c_1 p_3}{d_3 p_1} \sqrt{\frac{r_1}{s_1}}, \quad m_0 = m'_0, \quad p = p', \quad q = q' \\ \text{Set } III_B : m'_0 &= \frac{d_1^2 r_1}{p_1^2}, \quad p' = \frac{c_1}{d_1}, \quad q' = \frac{c_2 p_1}{d_1 p_2} \sqrt{\frac{s_3}{r_1}}, \quad m_0 = m'_0, \quad p = p', \quad q = k_2 q' \\ \text{Set } IV_B : m'_0 &= \frac{d_2^2 s_3}{p_2^2}, \quad p' = \frac{c_2}{d_2}, \quad q' = \frac{c_1 p_2}{d_2 p_1} \sqrt{\frac{r_1}{s_1}}, \quad m_0 = m'_0 k_2^2, \quad p = p', \quad q = \frac{q'}{k_2}. \end{aligned} \tag{4.3}$$

Similarly the m_ν matrix arises in Category C can be written as

$$m_\nu^C = m_0 \begin{pmatrix} 1 & k_1(p+q) & p+q \\ k_1(p+q) & k_1^2(2pq+p^2) & k_1(2pq+p^2) \\ p+q & k_1(2pq+p^2) & (2pq+p^2) \end{pmatrix} \quad (4.4)$$

with the following choice of parameters

$$\begin{aligned} \text{Set } I_C : m'_0 &= \frac{d_1^2 r_1}{p_1^2}, \quad p' = \frac{c_1}{d_1}, \quad q' = \frac{c_2 p_1}{d_1 p_2} \sqrt{\frac{s_3}{r_1}}, \quad m_0 = m'_0, \quad p = p', \quad q = k_2 q' \\ \text{Set } II_C : m'_0 &= \frac{d_1^2 r_1}{p_1^2}, \quad p' = \frac{c_1}{d_1}, \quad q' = \frac{c_3 p_1}{d_1 p_3} \sqrt{\frac{s_3}{r_1}}, \quad m_0 = m'_0, \quad p = p', \quad q = q'. \end{aligned} \quad (4.5)$$

For Category D the effective m_ν comes out as

$$m_\nu^D = m_0 \begin{pmatrix} 0 & k_1 p & p \\ k_1 p & k_1^2(q^2 + 2rp) & k_1(q^2 + 2rp) \\ p & k_1(q^2 + 2rp) & (q^2 + 2rp) \end{pmatrix} \quad (4.6)$$

with the definition of parameters as

$$\begin{aligned} \text{Set } I_D : m'_0 &= \frac{d_2^2 r_1}{p_1^2}, \quad p' = \frac{c_1 p_1 s_3}{d_2 p_2 r_1}, \quad q' = \frac{c_1}{d_2}, \quad r' = \frac{c_3}{d_2}, \quad m_0 = m'_0, \quad p = k_2 p', \quad q = q', \quad r = r' \\ \text{Set } II_D : m'_0 &= \frac{d_3^2 r_1}{p_1^2}, \quad p' = \frac{c_1 p_1 s_3}{d_3 p_3 r_1}, \quad q' = \frac{c_1}{d_3}, \quad r' = \frac{c_2}{d_3}, \quad m_0 = m'_0, \quad p = p', \quad q = q', \quad r = k_2 r' \\ \text{Set } III_D : m'_0 &= \frac{c_1 p_1 s_3}{d_3 p_3 r_1}, \quad p' = \frac{c_1}{d_1}, \quad q' = \frac{c_1}{d_3}, \quad r' = \frac{c_3}{d_3}, \quad m_0 = m'_0, \quad p = p', \quad q = k_2 q', \quad r = r' \\ \text{Set } IV_D : m'_0 &= \frac{d_2^2 r_1}{p_1^2}, \quad p' = \frac{c_1 p_1 s_3}{d_2 p_2 r_1}, \quad q' = \frac{c_1}{d_2}, \quad r' = \frac{c_2}{d_2}, \quad m_0 = m'_0, \quad p = k_2 p', \quad q = q', \quad r = r' \end{aligned} \quad (4.7)$$

and in general, we consider all the parameters m_0, k_1, p, r and q are complex.

5 Phase rotation

As mentioned earlier, all the parameters of m_ν are complex and therefore we can rephase m_ν by a phase rotation to remove the redundant phases. Here, we systematically study the phase rotation for each category.

Category A,B. The Majorana type mass matrix m_ν can be rotated in phase space through

$$m_\nu'^{AB} = P^T m_\nu^{AB} P \quad (5.1)$$

where P is a diagonal phase matrix and is given by $P = \text{diag}(e^{i\Phi_1}, e^{i\Phi_2}, e^{i\Phi_3})$.

Redefining the parameters of m_ν as

$$m_0 \rightarrow m_0 e^{i\alpha_m}, \quad p \rightarrow p e^{i\theta_p}, \quad q \rightarrow q e^{i\theta_q}, \quad k_1 \rightarrow k_1 e^{i\theta_1} \quad (5.2)$$

with

$$\Phi_1 = -\frac{\alpha_m}{2}, \Phi_2 = -\left(\theta_1 + \theta_p + \frac{\alpha_m}{2}\right), \Phi_3 = -\left(\theta_p + \frac{\alpha_m}{2}\right) \quad (5.3)$$

the phase rotated $m_\nu'^{AB}$ appears as

$$m_\nu'^{AB} = m_0 \begin{pmatrix} 1 & k_1 p & p \\ k_1 p & k_1^2 (q^2 e^{i\theta} + p^2) & k_1 (q^2 e^{i\theta} + p^2) \\ p & k_1 (q^2 e^{i\theta} + p^2) & (q^2 e^{i\theta} + p^2) \end{pmatrix} \quad (5.4)$$

where $\theta = 2(\theta_q - \theta_p)$ and all the parameters m_0, p, q and k_1 are real. Thus there is only a single phase parameter in $m_\nu'^{AB}$.

Category C.

In a similar way, the mass matrix of Category C can be rephased as

$$m_\nu'^C = m_0 \begin{pmatrix} 1 & k_1(p + qe^{i\theta}) & p + qe^{i\theta} \\ k_1(p + qe^{i\theta}) & k_1^2(2pqe^{i\theta} + p^2) & k_1(2pqe^{i\theta} + p^2) \\ p + qe^{i\theta} & k_1(2pqe^{i\theta} + p^2) & (2pqe^{i\theta} + p^2) \end{pmatrix} \quad (5.5)$$

with the same set of redefined parameters as mentioned in eq. (5.2) and (5.3) and the diagonal phase matrix mentioned in the previous case with $\theta = \theta_q - \theta_p$.

Category D.

For this category the rephased mass matrix comes out as

$$m_\nu'^D = m_0 \begin{pmatrix} 0 & k_1 p & p \\ k_1 p & k_1^2 (q^2 e^{i\alpha} + 2rpe^{i\beta}) & k_1 (q^2 e^{i\alpha} + 2rpe^{i\beta}) \\ p & k_1 (q^2 e^{i\alpha} + 2rpe^{i\beta}) & (q^2 e^{i\alpha} + 2rpe^{i\beta}) \end{pmatrix} \quad (5.6)$$

with $r \rightarrow re^{i\theta_r}$, $\alpha = 2(\theta_q - \theta_p)$, $\beta = (\theta_r - \theta_p)$ and the rest of the parameters are already defined in eq. (5.2) and eq. (5.3).

6 Breaking of the scaling ansatz

Since the neutrino mass matrix obtained in eq. (5.4), (5.5) and (5.6) are all invariant under scaling ansatz and thereby give rise to $\theta_{13} = 0$ as well as $m_3 = 0$. Although vanishing value of m_3 is yet not ruled out however, the former, $\theta_{13} = 0$ is refuted by the reactor experimental results. Popular paradigm is to consider $\theta_{13} = 0$ at the leading order and by further perturbation nonzero value of θ_{13} is generated. We follow the same way to produce nonzero θ_{13} through small breaking of scaling ansatz. It is to be noted in our scheme, generation of nonzero θ_{13} necessarily needs breaking in m_D . To generate nonzero m_3 breaking in μ matrix is also necessary along with m_D , however, in Category B since $\det(m_D = 0)$ even after breaking in the μ matrix m_ν still gives one of the eigenvalue equal to zero. On the other hand for Category C and Category D, μ_2^4 has always zero determinant because of being scaling ansatz invariant and therefore, leads to one zero eigenvalue as that of Category B. It is the Category A for which we get nonzero θ_{13} as well as nonzero m_3 after breaking the scaling ansatz in both the matrices (m_D and μ).

In the following, we invoke breaking of scaling ansatz in all four categories through

- i) breaking in the Dirac sector ($\theta_{13} \neq 0, m_3 = 0$)
- ii) breaking in the Dirac sector as well as Majorana sector ($\theta_{13} \neq 0, m_3 \neq 0$) and later we discuss separately both the cases.

6.1 Breaking in the Dirac sector

6.1.1 Category A,B

We consider minimal breaking of the scaling ansatz through a dimensionless real parameter ϵ in a single term of different m_D matrices of those categories as

$$m_{D2}^4 = \begin{pmatrix} 0 & d_2 & 0 \\ k_1(1+\epsilon)c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix}, \quad m_{D6}^4 = \begin{pmatrix} 0 & 0 & d_3 \\ k_1(1+\epsilon)c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \quad (6.1)$$

for Category A and

$$\begin{aligned} m_{D1}^4 &= \begin{pmatrix} d_1 & 0 & 0 \\ k_1c_1 & 0 & k_1(1+\epsilon)c_3 \\ c_1 & 0 & c_3 \end{pmatrix}, & m_{D3}^4 &= \begin{pmatrix} 0 & 0 & d_3 \\ k_1(1+\epsilon)c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \\ m_{D4}^4 &= \begin{pmatrix} d_1 & 0 & 0 \\ k_1c_1 & k_1(1+\epsilon)c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}, & m_{D5}^4 &= \begin{pmatrix} 0 & d_2 & 0 \\ k_1(1+\epsilon)c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix} \end{aligned} \quad (6.2)$$

for Category B. We further want to mention that breaking considered in any element of the second row are all equivalent. For example, if we consider breaking in the ‘23’ element of m_{D2}^4 it is equivalent to as considered in eq. (6.1). Neglecting the ϵ^2 and higher order terms, the effective m_ν matrix comes out as

$$m_\nu'^{AB\epsilon} = m_0 \begin{pmatrix} 1 & k_1p & p \\ k_1p & k_1^2(q^2e^{i\theta} + p^2) & k_1(q^2e^{i\theta} + p^2) \\ p & k_1(q^2e^{i\theta} + p^2) & (q^2e^{i\theta} + p^2) \end{pmatrix} + m_0\epsilon \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2k_1^2q^2e^{i\theta} & k_1q^2e^{i\theta} \\ 0 & k_1q^2e^{i\theta} & 0 \end{pmatrix}. \quad (6.3)$$

As mentioned earlier, that for Category B, $\det(m_D) = 0$ and it is not possible to generate $m_3 \neq 0$ even if we consider breaking in the μ matrices. On the other hand, the matrices in Category A possess $\det(m_D) \neq 0$ and thereby give rise to $m_3 \neq 0$.

Now to calculate the eigenvalues, mixing angles, J_{CP} , the Dirac and Majorana phases we utilize the results obtained in ref. [47], for a general complex matrix. We should mention that the formula obtained in ref. [47], for Majorana phases is valid when all three eigenvalues are nonzero. However, when one of the eigenvalue is zero (in this case $m_3 = 0$) one has to utilize the methodology given in ref. [18], which shows, a general Majorana type mass matrix m_ν can be diagonalized as

$$U^\dagger m_\nu U^* = \text{diag}(m_1, m_2, m_3) \quad (6.4)$$

or alternely,

$$m_\nu = U \text{diag}(m_1, m_2, m_3) U^T \quad (6.5)$$

where

$$U = U_{CKM} P_M. \quad (6.6)$$

The mixing matrix U_{CKM} is given by (following PDG [48]) convention)

$$U_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \quad (6.7)$$

with $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and δ_{CP} is the Dirac CP phase. The diagonal phase matrix P_M is parametrized as

$$P_M = \text{diag}(1, e^{i\alpha_M}, e^{i(\beta_M + \delta_{CP})}) \quad (6.8)$$

with α_M and $\beta_M + \delta_{CP}$ are the Majorana phases.

Writing eq. (6.5) explicitly with $m_3 = 0$ we can have expressions for six independent elements of m_ν in terms of the mixing angles, two eigenvalues and the Dirac CP phase, from which the m_{11} element can be expressed as

$$m_{11} = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{2i\alpha_M} \quad (6.9)$$

and therefore the Majorana phase α_M comes out as

$$\alpha_M = \frac{1}{2} \cos^{-1} \left\{ \frac{|m_{11}|^2}{2c_{12}^2 s_{12}^2 c_{13}^4 m_1 m_2} - \frac{(c_{12}^4 m_1^2 + s_{12}^4 m_2^2)}{2c_{12}^2 s_{12}^2 m_1 m_2} \right\}. \quad (6.10)$$

The Jarlskog measure of CP violation J_{CP} is defined in usual way as

$$J_{CP} = \frac{\text{Im}(h_{12}h_{23}h_{31})}{(\Delta m_{21}^2)(\Delta m_{32}^2)(\Delta m_{31}^2)} \quad (6.11)$$

where h is a hermitian matrix constructed out of m_ν as $h = m_\nu m_\nu^\dagger$.

6.1.2 Category C

In this case breaking is considered in m_D as

$$m_{D1}^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1(1+\epsilon)c_1 & k_1c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}, \quad m_{D4}^4 = \begin{pmatrix} d_1 & 0 & 0 \\ k_1(1+\epsilon)c_1 & 0 & k_1c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \quad (6.12)$$

and the scaling ansatz broken m_ν appears as

$$m_\nu^{C\epsilon} = m_0 \begin{pmatrix} 1 & k_1(p + qe^{i\theta}) & p + qe^{i\theta} \\ k_1(p + qe^{i\theta}) & k_1^2(2pqe^{i\theta} + p^2) & k_1(2pqe^{i\theta} + p^2) \\ p + qe^{i\theta} & k_1(2pqe^{i\theta} + p^2) & (2pqe^{i\theta} + p^2) \end{pmatrix} \\ + m_0 \epsilon \begin{pmatrix} 0 & k_1qe^{i\theta} & 0 \\ k_1qe^{i\theta} & 2k_1^2pqe^{i\theta} & k_1pqe^{i\theta} \\ 0 & k_1pqe^{i\theta} & 0 \end{pmatrix}. \quad (6.13)$$

Quantity	3σ ranges
$ \Delta m_{31}^2 \text{ N}$	$2.31 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.74$
$ \Delta m_{31}^2 \text{ I}$	$2.21 < \Delta m_{31}^2 (10^3 \text{ eV}^{-2}) < 2.64$
Δm_{21}^2	$7.21 < \Delta m_{21}^2 (10^5 \text{ eV}^{-2}) < 8.20$
θ_{12}	$31.3^\circ < \theta_{12} < 37.46^\circ$
θ_{23}	$36.86^\circ < \theta_{23} < 55.55^\circ$
θ_{13}	$7.49^\circ < \theta_{13} < 10.46^\circ$

Table 8. Input experimental values [16].

6.1.3 Category D

Breaking in m_D in this case is incorporated through

$$\begin{aligned}
 m_{D2}^4 &= \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & 0 & k_1(1+\epsilon)c_3 \\ c_1 & 0 & c_3 \end{pmatrix}, & m_{D3}^4 &= \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & 0 & k_1(1+\epsilon)c_3 \\ c_1 & 0 & c_3 \end{pmatrix} \\
 m_{D5}^4 &= \begin{pmatrix} 0 & d_2 & 0 \\ k_1 c_1 & k_1(1+\epsilon)c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}, & m_{D6}^4 &= \begin{pmatrix} 0 & 0 & d_3 \\ k_1 c_1 & k_1(1+\epsilon)c_2 & 0 \\ c_1 & c_2 & 0 \end{pmatrix}
 \end{aligned} \tag{6.14}$$

and the corresponding m_ν comes out as

$$\begin{aligned}
 m_\nu'^{D\epsilon} &= m_0 \begin{pmatrix} 0 & k_1 p & p \\ k_1 p & k_1^2(q^2 e^{i\alpha} + 2rpe^{i\beta}) & k_1(q^2 e^{i\alpha} + 2rpe^{i\beta}) \\ p & k_1(q^2 e^{i\alpha} + 2rpe^{i\beta}) & (q^2 e^{i\alpha} + 2rpe^{i\beta}) \end{pmatrix} \\
 &+ m_0 \epsilon \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2k_1^2 rpe^{i\beta} & k_1 rpe^{i\beta} \\ 0 & k_1 rpe^{i\beta} & 0 \end{pmatrix}.
 \end{aligned} \tag{6.15}$$

6.2 Numerical analysis

In order to perform the numerical analysis to obtain allowed parameter space we utilize the neutrino oscillation data obtained from global fit shown in table 8.

6.2.1 Category A,B

We first consider Category A,B for which the neutrino mass matrix is given in eq. (6.3). The parameter ϵ is varied freely to fit the extant data and it is constrained as $0.04 < \epsilon < 0.7$. However, to keep the ansatz breaking effect small we restrict the value of ϵ only upto 0.1. For this range of ϵ ($0 < \epsilon < 0.1$) under consideration the parameter spaces are obtained as $1.78 < p < 3.40$, $1.76 < q < 3.42$ and $0.66 < k_1 < 1.3$. It is interesting to note a typical feature of this category is that the Dirac CP phase δ_{CP} comes out too tiny and thereby generating almost vanishing value of J_{CP} ($\approx 10^{-6}$) while the range of the only Majorana phase in this category is obtained as $77^\circ < \alpha_M < 90^\circ$.

As one of the eigenvalue $m_3 = 0$ therefore, the hierarchy of the masses is clearly inverted in this category. The sum of the three neutrino masses $\Sigma_i m_i (= m_1 + m_2 + m_3)$

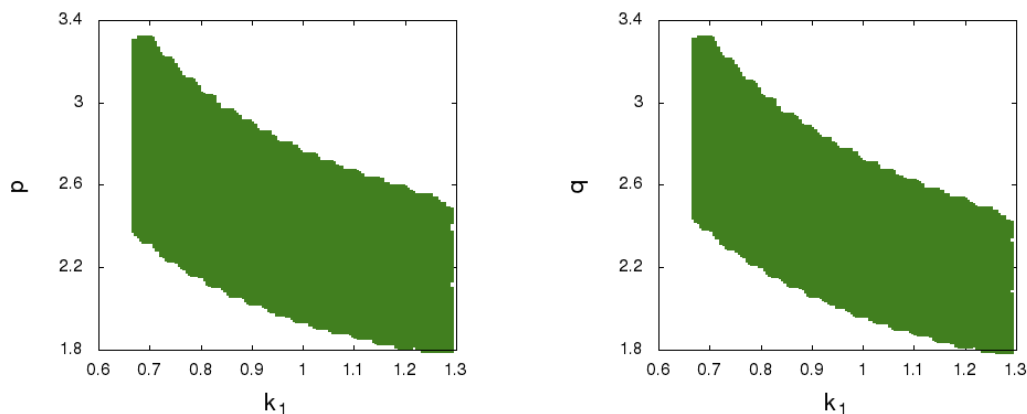


Figure 1. Plot of p vs k_1 (left), q vs k_1 (right) for the Category A,B with $\epsilon = 0.1$.

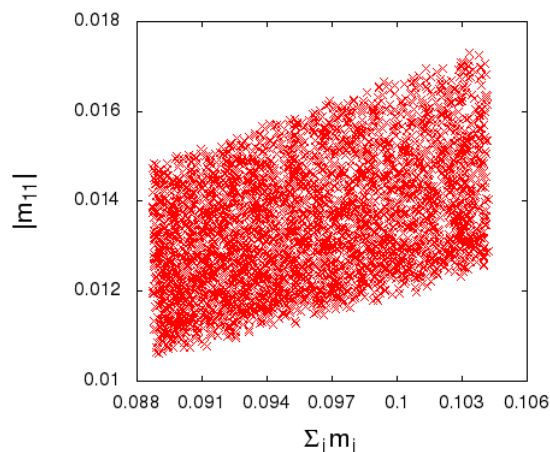


Figure 2. Plot of $|m_{11}|$ vs $\Sigma_i m_i$ for Category A,B with $\epsilon = 0.1$.

and $|m_{11}|$ are obtained as $0.088 \text{ eV} < \Sigma_i m_i < 0.104 \text{ eV}$ and $0.0102 \text{ eV} < |m_{11}| < 0.0181 \text{ eV}$ which predict the value of the two quantities below the present experimental upper bounds. To illustrate the nature of variation, in figure 1 we plot p vs k_1 and q vs k_1 while in figure 2 a correlation plot of $\Sigma_i m_i$ with $|m_{11}|$ is shown for $\epsilon = 0.1$ and it is also seen from figure 1 and 2 that the ranges of the parameters do not differ much compare to the values obtained for the whole range of ϵ parameter.

In brief, distinguishable characteristics of this category are i) tiny J_{CP} and δ_{CP} ii) inverted hierarchy of the neutrino masses. At the end of this section we will further discuss the experimental testability of these quantities for all the categories.

6.2.2 Category C

In this case it is found that a small breaking of ϵ ($0.02 < \epsilon < 0.09$) is sufficient to accommodate all the oscillation data. We explore the parameter space and the ranges obtained as $3.42 < p < 6.07$, $1.68 < q < 3.02$ and $0.7 < k_1 < 1.32$. The hierarchy obtained in this

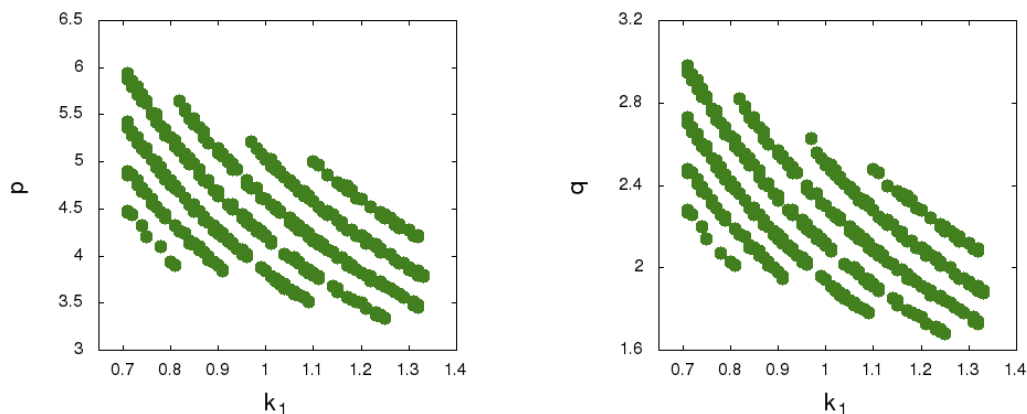


Figure 3. Plot of p vs k_1 (left), q vs k_1 (right) for the Category C with $\epsilon = 0.09$.

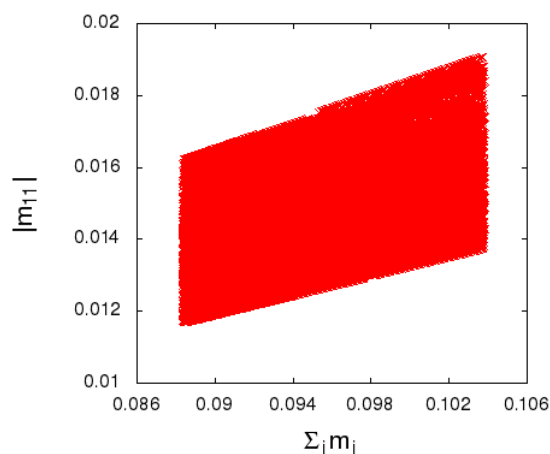


Figure 4. Plot of $|m_{11}|$ vs $\Sigma_i m_i$ for Category C with $\epsilon = 0.09$.

case is also inverted due to the vanishing value of m_3 . The other two quantities $\Sigma_i m_i$ and $|m_{11}|$ come out as $0.0118 \text{ eV} < |m_{11}| < 0.019 \text{ eV}$ and $0.088 \text{ eV} < \Sigma_i m_i < 0.105 \text{ eV}$. Similar to the previous category J_{CP} is vanishingly small due to low value of δ_{CP} . The range of the Majorana phase α_M is obtained as $81^\circ < \alpha_M < 89^\circ$. In figure 3 we plot k_1 vs p and k_1 vs q for $\epsilon = 0.09$ that predicts almost the same ranges of the parameters (p , q and k_1) and all other quantities ($|m_{11}|$, $\Sigma_i m_i$, α_M and J_{CP}) as obtained from the whole range of ϵ . We present a correlation plot of $\Sigma_i m_i$ with $|m_{11}|$ in figure 4.

6.2.3 Category D

In case of Category D, although a priori it is not possible to rule out $m_\nu^{D\epsilon}$ without going into the detailed numerical analysis, however in this case even if with $\epsilon = 1$ it is not possible to accommodate the neutrino oscillation data. Specifically, the value of θ_{13} is always beyond the reach of the parameter space. Exactly for the same reason the m_ν matrix of type A_3 in eq. (2.5) is phenomenologically ruled out.

6.3 Breaking in Dirac+Majorana sector

In this section we focus on the phenomenology of the neutrino mass matrix where the scaling ansatz is broken in both the sectors. This type of breaking is only relevant for Category A since in this case m_D is nonsingular after breaking of the ansatz and the resultant m_ν gives rise to nonzero θ_{13} along with $m_3 \neq 0$. In all the other categories due to the singular nature of m_D , inclusion of symmetry breaking in the Majorana sector will not generate $m_3 \neq 0$. Thus we consider only Category A under this scheme.

We consider the breaking in m_D as mentioned in eq. (6.1) and the ansatz broken texture of μ_1^4 matrix is given by

$$\mu_1^4 = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & k_2^2 s_3 & k_2(1 + \epsilon') s_3 \\ 0 & k_2(1 + \epsilon') s_3 & s_3 \end{pmatrix} \quad (6.16)$$

where ϵ' is a dimensionless real parameter. The effective neutrino mass matrix m_ν comes out as

$$m_{\nu\epsilon'}^{A\epsilon} = m_0 \begin{pmatrix} 1 & k_1 p & p \\ k_1 p & k_1^2(q^2 e^{i\theta} + p^2) & k_1(q^2 e^{i\theta} + p^2) \\ p & k_1(q^2 e^{i\theta} + p^2) & (q^2 e^{i\theta} + p^2) \end{pmatrix} + m_0 \epsilon \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2k_1^2 q^2 e^{i\theta} & k_1 q^2 e^{i\theta} \\ 0 & k_1 q^2 e^{i\theta} & 0 \end{pmatrix} \\ + m_0 \epsilon' \begin{pmatrix} 0 & k_1 p & p \\ k_1 p & 0 & 0 \\ p & 0 & 0 \end{pmatrix}. \quad (6.17)$$

6.3.1 Numerical results

As mentioned above, $\epsilon' = 0$ leads to inverted hierarchy with $m_3 = 0$ and thus to generate nonzero m_3 a small value of ϵ' is needed. Similar to the previous cases two breaking parameters ϵ and ϵ' can be varied freely through the ranges that are sensitive to the oscillation data and are obtained as $0.06 < \epsilon < 0.68$ and $0 < \epsilon' < 1$. It is to be noted that although the ϵ parameter is restricted due to θ_{13} value, ϵ' is almost insensitive to θ_{13} and it can vary within a wide range as $0 < \epsilon' < 1$. A correlation plot of ϵ with ϵ' is shown in figure 5. However, as mentioned earlier, the effect of the breaking term should be smaller than the unbroken one, therefore, to obtain the parameter space for this category we consider breaking of the scaling ansatz in both the sectors only upto 10 % and consequently for all combinatorial values of ϵ and ϵ' the parameters p , q and k_1 vary within the ranges as $1.07 < p < 3.10$, $1.03 < q < 3.12$ and $0.67 < k_1 < 1.31$. Interestingly, although all the eigenvalues are nonzero in this case, the hierarchy is still inverted. J_{CP} is found to be tiny ($\approx 10^{-6}$) again due to small value of δ_{CP} . The Majorana phases are obtained as $-96^\circ < \alpha_M < 74^\circ$ and $-100^\circ < \beta_M + \delta_{CP} < 102^\circ$ followed by the bounds on $\Sigma_i m_i$ and $|m_{11}|$ as $0.088 \text{ eV} < \Sigma_i m_i < 0.11 \text{ eV}$ and $0.010 \text{ eV} < |m_{11}| < 0.022 \text{ eV}$ which are well below the present experimental upper bounds. In figure 6 we demonstrate the above predictions for $\epsilon = \epsilon' = 0.1$. In the left panel of figure 6 the inverted hierarchical nature is shown and in the right panel variation of the Majorana phases is demonstrated. Some comments are in order regarding predictions of the present scheme:

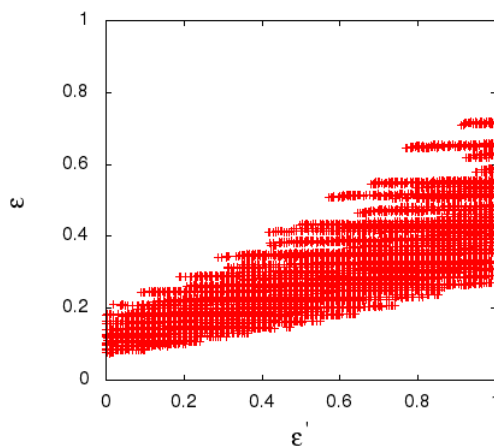


Figure 5. Correlated plot of ϵ with ϵ' .

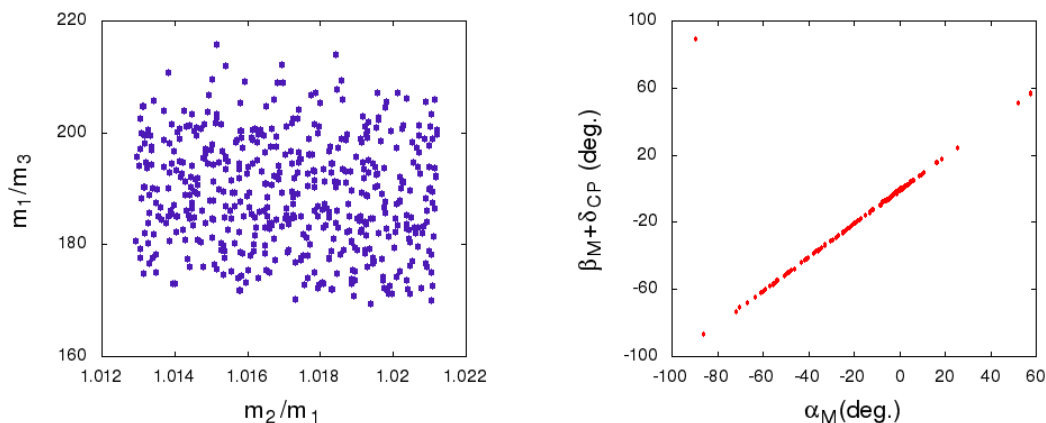


Figure 6. Plot of (m_1/m_3) vs (m_2/m_1) (left) and $\beta_M + \delta_{CP}$ vs α_M (right) after breaking of the scaling ansatz in both the sectors of Category A for a representative value of $\epsilon = \epsilon' = 0.1$.

1. After precise determination of θ_{13} taking full account of reactor neutrino experimental data, it is shown that the hierarchy of the light neutrino masses can be probed through combined utilization of NO ν A and T2K [49] neutrino oscillation experimental results in near future. Thus the speculation of hierarchy in the present scheme will be clearly verified. Moreover, taking the difference of probabilities between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ information on the value of J_{CP} can be obtained using neutrino and anti neutrino beams.
2. More precise estimation of the sum of the three light neutrino masses will be obtained utilizing a combined analysis with PLANCK data [50] and other cosmological and astrophysical experiments [51] such as, Baryon oscillation spectroscopic survey, The Dark energy survey, Large Synoptic Survey Telescope or the Euclid satellite data [52] etc. Such type of analysis will push $\Sigma_i m_i \sim 0.1$ eV (at the 4σ level for inverted order-

ing) and $\Sigma_i m_i \sim 0.05 \text{ eV}$ (at the 2σ level for normal ordering). Thus the prediction of the value of $\Sigma_i m_i$ in the different categories discussed in the present work will also be tested in the near future. Furthermore, the NEXT-100 [53] will probe the value of $|m_{11}|$ up to 0.1 eV which is a more precise estimation than the EXO-200 [54] experimental range ($0.14\text{-}0.38 \text{ eV}$).

7 Summary and conclusion

In this work we explore the phenomenology of neutrino mass matrix obtained due to inverse seesaw mechanism adhering i) Scaling ansatz, ii) Texture zeros within the framework of $SU(2)_L \times U(1)_Y$ model with three right handed neutrinos and three left chiral singlet fermions. Throughout our analysis we choose a basis in which the charged lepton mass matrix (m_E) and the M_{RS} matrix (appeared in inverse seesaw mechanism due to the coupling of ν_R and S_L) are diagonal. It is found that four is the maximum number of zeros that can be allowed in m_D and μ matrices to obtain viable phenomenology. We classify different four zero textures in four different categories depending upon their generic form. Since scaling ansatz invariance always gives rise to $\theta_{13} = 0$, we have to break such ansatz. We consider breaking in m_D and also in μ matrices. We explore the parameter space and it is seen that one category (Category D) is ruled out phenomenologically. The hierarchy obtained in all the cases are inverted and it is interesting to note that all such categories give rise to tiny CP violation measure J_{CP} due to small value of δ_{CP} . In conclusion, further observation of hierarchy of neutrino masses and CP violation in the leptonic sector in the forthcoming experiments will conclusively refute or admit all these categories obtained in the present scheme.

Acknowledgments

We thank Mainak Chakraborty for discussion and computational help.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] R.N. Mohapatra and J.W.F. Valle, *Neutrino Mass and Baryon Number Nonconservation in Superstring Models*, *Phys. Rev. D* **34** (1986) 1642 [[INSPIRE](#)].
- [2] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez and J.W.F. Valle, *Lepton Flavor Nonconservation at High-Energies in a Superstring Inspired Standard Model*, *Phys. Lett. B* **187** (1987) 303 [[INSPIRE](#)].
- [3] R.N. Mohapatra, *Mechanism for Understanding Small Neutrino Mass in Superstring Theories*, *Phys. Rev. Lett.* **56** (1986) 561 [[INSPIRE](#)].
- [4] J. Schechter and J.W.F. Valle, *Neutrino Decay and Spontaneous Violation of Lepton Number*, *Phys. Rev. D* **25** (1982) 774 [[INSPIRE](#)].

- [5] A. Palcu, *A straightforward realization of a quasi-inverse seesaw mechanism at TeV scale*, [arXiv:1408.6518](#) [INSPIRE].
- [6] J. Schechter and J.W.F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev. D* **22** (1980) 2227 [INSPIRE].
- [7] S. Fraser, E. Ma and O. Popov, *Scotogenic Inverse Seesaw Model of Neutrino Mass*, *Phys. Lett. B* **737** (2014) 280 [[arXiv:1408.4785](#)] [INSPIRE].
- [8] H. Hettmansperger, M. Lindner and W. Rodejohann, *Phenomenological Consequences of sub-leading Terms in See-Saw Formulas*, *JHEP* **04** (2011) 123 [[arXiv:1102.3432](#)] [INSPIRE].
- [9] B. Adhikary, A. Ghosal and P. Roy, *Maximal zero textures of the inverse seesaw with broken $\mu\tau$ symmetry*, *Indian J. Phys.* **88** (2014) 979 [[arXiv:1311.6746](#)] [INSPIRE].
- [10] S.S.C. Law and K.L. McDonald, *Generalized inverse seesaw mechanisms*, *Phys. Rev. D* **87** (2013) 113003 [[arXiv:1303.4887](#)] [INSPIRE].
- [11] P.S.B. Dev and A. Pilaftsis, *Minimal Radiative Neutrino Mass Mechanism for Inverse Seesaw Models*, *Phys. Rev. D* **86** (2012) 113001 [[arXiv:1209.4051](#)] [INSPIRE].
- [12] M. Malinsky, *Non-unitarity effects in the minimal inverse seesaw model*, *PoS(EPS-HEP 2009)288* [[arXiv:0909.1953](#)] [INSPIRE].
- [13] M. Hirsch, S. Morisi and J.W.F. Valle, *A_4 -based tri-bimaximal mixing within inverse and linear seesaw schemes*, *Phys. Lett. B* **679** (2009) 454 [[arXiv:0905.3056](#)] [INSPIRE].
- [14] D.V. Forero, M. Tortola and J.W.F. Valle, *Neutrino oscillations refitted*, *Phys. Rev. D* **90** (2014) 093006 [[arXiv:1405.7540](#)] [INSPIRE].
- [15] M.C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, *Global fit to three neutrino mixing: critical look at present precision*, *JHEP* **12** (2012) 123 [[arXiv:1209.3023](#)] [INSPIRE].
- [16] D.V. Forero, M. Tortola and J.W.F. Valle, *Global status of neutrino oscillation parameters after Neutrino-2012*, *Phys. Rev. D* **86** (2012) 073012 [[arXiv:1205.4018](#)] [INSPIRE].
- [17] A.S. Joshipura and W. Rodejohann, *Scaling in the Neutrino Mass Matrix, μ -tau Symmetry and the See-Saw Mechanism*, *Phys. Lett. B* **678** (2009) 276 [[arXiv:0905.2126](#)] [INSPIRE].
- [18] R.N. Mohapatra and W. Rodejohann, *Scaling in the neutrino mass matrix*, *Phys. Lett. B* **644** (2007) 59 [[hep-ph/0608111](#)] [INSPIRE].
- [19] A. Blum, R.N. Mohapatra and W. Rodejohann, *Inverted mass hierarchy from scaling in the neutrino mass matrix: low and high energy phenomenology*, *Phys. Rev. D* **76** (2007) 053003 [[arXiv:0706.3801](#)] [INSPIRE].
- [20] M. Obara, *The Possible Textures in the Seesaw Realization of the Strong Scaling Ansatz and the Implications for Thermal Leptogenesis*, [arXiv:0712.2628](#) [INSPIRE].
- [21] A. Damanik, M. Satriawan, Muslim and P. Anggraita, *Neutrino mass matrix from seesaw mechanism subjected to texture zero and invariant under a cyclic permutation*, [arXiv:0705.3290](#) [INSPIRE].
- [22] S. Goswami and A. Watanabe, *Minimal Seesaw Textures with Two Heavy Neutrinos*, *Phys. Rev. D* **79** (2009) 033004 [[arXiv:0807.3438](#)] [INSPIRE].
- [23] W. Grimus and L. Lavoura, *Softly broken lepton number $L_e - L_\mu - L_\tau$ with non-maximal solar neutrino mixing*, *J. Phys. G* **31** (2005) 683 [[hep-ph/0410279](#)] [INSPIRE].

- [24] M.S. Berger and S. Santana, *Combined flavor symmetry violation and lepton number violation in neutrino physics*, *Phys. Rev. D* **74** (2006) 113007 [[hep-ph/0609176](#)] [[INSPIRE](#)].
- [25] S. Goswami, S. Khan and W. Rodejohann, *Minimal Textures in Seesaw Mass Matrices and their low and high Energy Phenomenology*, *Phys. Lett. B* **680** (2009) 255 [[arXiv:0905.2739](#)] [[INSPIRE](#)].
- [26] S. Dev, R.R. Gautam and L. Singh, *Charged Lepton Corrections to Scaling Neutrino Mixing*, *Phys. Rev. D* **89** (2014) 013006 [[arXiv:1309.4219](#)] [[INSPIRE](#)].
- [27] B. Adhikary, M. Chakraborty and A. Ghosal, *Scaling ansatz, four zero Yukawa textures and large θ_{13}* , *Phys. Rev. D* **86** (2012) 013015 [[arXiv:1205.1355](#)] [[INSPIRE](#)].
- [28] P.H. Frampton, S.L. Glashow and D. Marfatia, *Zeros of the neutrino mass matrix*, *Phys. Lett. B* **536** (2002) 79 [[hep-ph/0201008](#)] [[INSPIRE](#)].
- [29] K. Whisnant, J. Liao and D. Marfatia, *Constraints on texture zero and cofactor zero models for neutrino mass*, *AIP Conf. Proc.* **1604** (2014) 273 [[INSPIRE](#)].
- [30] P.O. Ludl and W. Grimus, *A complete survey of texture zeros in the lepton mass matrices*, *JHEP* **07** (2014) 090 [[arXiv:1406.3546](#)] [[INSPIRE](#)].
- [31] W. Grimus and P.O. Ludl, *Correlations and textures in the neutrino mass matrix*, *PoS(EPS-HEP 2013)075* [[arXiv:1309.7883](#)] [[INSPIRE](#)].
- [32] J. Liao, D. Marfatia and K. Whisnant, *Texture and Cofactor Zeros of the Neutrino Mass Matrix*, *JHEP* **09** (2014) 013 [[arXiv:1311.2639](#)] [[INSPIRE](#)].
- [33] H. Fritzsch, Z.-z. Xing and S. Zhou, *Two-zero Textures of the Majorana Neutrino Mass Matrix and Current Experimental Tests*, *JHEP* **09** (2011) 083 [[arXiv:1108.4534](#)] [[INSPIRE](#)].
- [34] A. Merle and W. Rodejohann, *The elements of the neutrino mass matrix: allowed ranges and implications of texture zeros*, *Phys. Rev. D* **73** (2006) 073012 [[hep-ph/0603111](#)] [[INSPIRE](#)].
- [35] W. Wang, *Neutrino mass textures with one vanishing minor and two equal cofactors*, *Eur. Phys. J. C* **73** (2013) 2551 [[arXiv:1306.3556](#)] [[INSPIRE](#)].
- [36] W. Wang, *Parallel Texture Structures with Cofactor Zeros in Lepton Sector*, *Phys. Lett. B* **733** (2014) 320 [*Erratum ibid.* **738** (2014) 524] [[arXiv:1401.3949](#)] [[INSPIRE](#)].
- [37] L. Lavoura, *Zeros of the inverted neutrino mass matrix*, *Phys. Lett. B* **609** (2005) 317 [[hep-ph/0411232](#)] [[INSPIRE](#)].
- [38] A. Kageyama, S. Kaneko, N. Shimoyama and M. Tanimoto, *Seesaw realization of the texture zeros in the neutrino mass matrix*, *Phys. Lett. B* **538** (2002) 96 [[hep-ph/0204291](#)] [[INSPIRE](#)].
- [39] W. Wang, *Parallel Lepton Mass Matrices with Texture and Cofactor Zeros*, *Phys. Rev. D* **90** (2014) 033014 [[arXiv:1402.6808](#)] [[INSPIRE](#)].
- [40] G.C. Branco, D. Emmanuel-Costa, M.N. Rebelo and P. Roy, *Four Zero Neutrino Yukawa Textures in the Minimal Seesaw Framework*, *Phys. Rev. D* **77** (2008) 053011 [[arXiv:0712.0774](#)] [[INSPIRE](#)].
- [41] S. Choubey, W. Rodejohann and P. Roy, *Phenomenological consequences of four zero neutrino Yukawa textures*, *Nucl. Phys. B* **808** (2009) 272 [[arXiv:0807.4289](#)] [[INSPIRE](#)].
- [42] M. Chakraborty, H.Z. Devi and A. Ghosal, *Scaling ansatz with texture zeros in linear seesaw*, *Phys. Lett. B* **741** (2015) 210 [[arXiv:1410.3276](#)] [[INSPIRE](#)].

- [43] B. Adhikary, A. Ghosal and P. Roy, $\mu \tau$ symmetry, tribimaximal mixing and four zero neutrino Yukawa textures, *JHEP* **10** (2009) 040 [[arXiv:0908.2686](#)] [[INSPIRE](#)].
- [44] B. Adhikary, A. Ghosal and P. Roy, Baryon asymmetry from leptogenesis with four zero neutrino Yukawa textures, *JCAP* **01** (2011) 025 [[arXiv:1009.2635](#)] [[INSPIRE](#)].
- [45] B. Adhikary, A. Ghosal and P. Roy, Neutrino masses, cosmological bound and four zero Yukawa textures, *Mod. Phys. Lett. A* **26** (2011) 2427 [[arXiv:1103.0665](#)] [[INSPIRE](#)].
- [46] W. Grimus, A.S. Joshipura, L. Lavoura and M. Tanimoto, Symmetry realization of texture zeros, *Eur. Phys. J. C* **36** (2004) 227 [[hep-ph/0405016](#)] [[INSPIRE](#)].
- [47] B. Adhikary, M. Chakraborty and A. Ghosal, Masses, mixing angles and phases of general Majorana neutrino mass matrix, *JHEP* **10** (2013) 043 [Erratum *ibid.* **09** (2014) 180] [[arXiv:1307.0988](#)] [[INSPIRE](#)].
- [48] PARTICLE DATA GROUP collaboration, J. Beringer et al., *Review of Particle Physics (RPP)*, *Phys. Rev. D* **86** (2012) 010001 [[INSPIRE](#)].
- [49] K. Ieki, *Observation of $\nu_\mu \rightarrow \nu_e$ oscillation in the T2K experiment*, Ph.D. Thesis, Kyoto University, Kyoto Japan (2014), T2K-THESIS-040.
- [50] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XVI. Cosmological parameters*, *Astron. Astrophys.* **571** (2014) A16 [[arXiv:1303.5076](#)] [[INSPIRE](#)].
- [51] J. Lesgourgues and S. Pastor, *Neutrino cosmology and Planck*, *New J. Phys.* **16** (2014) 065002 [[arXiv:1404.1740](#)] [[INSPIRE](#)].
- [52] B. Audren et al., *Neutrino masses and cosmological parameters from a Euclid-like survey: Markov Chain Monte Carlo forecasts including theoretical errors*, *JCAP* **01** (2013) 026 [[arXiv:1210.2194](#)] [[INSPIRE](#)].
- [53] NEXT collaboration, D. Lorca, *The Hunt for neutrinoless double beta decay with the NEXT experiment*, [arXiv:1411.0475](#) [[INSPIRE](#)].
- [54] EXO collaboration, M. Auger et al., *Search for Neutrinoless Double-Beta Decay in ^{136}Xe with EXO-200*, *Phys. Rev. Lett.* **109** (2012) 032505 [[arXiv:1205.5608](#)] [[INSPIRE](#)].