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RG domain wall for the N=1 minimal superconformal models

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Abstract: We specify Gaiotto's proposal for the RG domain wall between some coset CFT models to the case of two minimal N=1 SCFT models SM_p and SM_{p-2} related by the RG flow initiated by the top component of the Neveu-Schwarz superfield $\Phi_{1,3}$. We explicitly calculate the mixing coefficients for several classes of fields and compare the results with the already known in literature results obtained through perturbative analysis. Our results exactly match with both leading and next to leading order perturbative calculations.

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Introduction. The existence of a RG flow between two CFT's suggests that this theories could be connected by a non-trivial interface which encodes the map from the UV observables to the IR ones [1, 2] In particular in [2] such an interface (RG domain wall) was constructed for the $N = 2$ superconformal models using matrix factorisation technique.

Later in [3] an algebraic construction of a RG domain wall for the unitary minimal CFT models was proposed and was shown that the results agree with those of the leading order perturbative analysis performed by A. Zamolodchikov in [4].

The leading order perturbative calculation of the mixing coefficients for the wider class of local fields including non-primary ones again is in an impressive agreement with the RG domain wall approach [5].

Higher order perturbative calculations [6, 7] further confirm the validity of this construction.

In the same paper [3] Gaiotto suggests that a similar construction should be valid also for more general coset CFT models. The $N = 1$ minimal superconformal CFT models [8–10], which are the main subject of this paper, are among these cosets.

The Renormalisation Group (RG) flow between minimal $N = 1$ superconformal models SM_p and SM_{p-2} initialised by the perturbation with the top component of the Neveu-Schwarz superfield $\Phi_{1,3}$ in leading order of the perturbation theory has been investigated in [11] (see also [12, 13]).

Recently, extending the technique developed in [6] for the minimal models to the supersymmetric case, in [14] the analysis of this RG flow has been sharpened even further by including also the next to leading order corrections.

In this paper we specialise Gaiotto's proposal to the case of the minimal N=1 SCFT models. The method we use is based directly on the current algebra construction and, in this sense, is more general than the one originally employed by Gaiotto for the case of minimal models. Namely he heavily exploited the fact that the product of successive minimal models can be alternatively represented as a product of $N = 1$ superconformal and

Ising models. We explicitly calculate the mixing coefficients for several classes of fields and compare the results with the perturbative analysis of [11, 14] finding a complete agreement.

The paper is organised as follows:

section 1 is a brief review of the 2d $N = 1$ superconformal filed theories.

Section 2 is devoted to the description of the coset construction of $N = 1$ SCFT. Of course everything here is well known; our purpose here is to fix notations and list the relevant formulae in a form, most convenient for the further calculations.

In section 3 we formulate Gaiotto's general proposal for a class of coset CFT models.

Section 4 is the main part of our paper. We explicitly calculate the mixing coefficients for the several classes of local fields in the case of the supersymmetric RG flow discussed above using RG domain wall proposal. Then we compare this with the perturbation theory results available in the literature finding a complete agreement.

1 N=1 superconformal field theory

In any conformal field theory the energy-momentum tensor has two nonzero components: the holomorphic field $T(z)$ with conformal dimension $(2, 0)$ and its anti-holomorphic counterpart $\bar{T}(\bar{z})$ with dimensions $(0, 2)$. In $N = 1$ superconformal field theories one has in addition superconformal currents $G(z)$ and $\bar{G}(\bar{z})$ with dimensions $(3/2, 0)$ and $(0, 3/2)$ respectively. These fields satisfy the OPE rules

$$T(z)T(0) = \frac{c}{2z^4} + \frac{2T(0)}{z^2} + \frac{T'(0)}{z} + \cdots, \tag{1.1}$$
$$T(z)G(0) = \frac{3G(0)}{2z^2} + \frac{G'(0)}{z} + \cdots, \tag{1.2}$$
$$G(z)G(0) = \frac{2c}{3z^3} + \frac{2T(0)}{z} + \cdots. \tag{1.3}$$

The corresponding expressions for the anti-chiral fields look exactly the same. One should simply substitute z by $\bar{z}$. Further on we'll mainly concentrate on the holomorphic part assuming similar expressions for anti-holomorphic quantities implicitly. We can expand $T(z)$ in Laurent series

$$T(z) = \sum_{n=-\infty}^{+\infty} \frac{L_n}{z^{n+2}}, \tag{1.4}$$

where L_n's are the Virasoro generators. Due to the fermionic nature of the super current, there are two distinct possibilities for its behavior under the rotation of the argument around 0 by the angle 2π

$$G\big(e^{2\pi i}z\big) = G(z) \qquad \text{Neveu-Schwarz sector (NS)}, \tag{1.5}$$
$$G\big(e^{2\pi i}z\big) = -G(z) \qquad \text{Ramond sector (R)}. \tag{1.6}$$

The space of fields $\mathcal{A}$ of the superconformal theory decomposes into a direct sum

$$\mathcal{A} = \{\text{NS}\} \oplus \{\text{R}\}, \tag{1.7}$$

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where the subspaces {NS} and {R} consist of the Neveu-Shwarz and the Ramond fields respectively. By definition, the monodromy of $G(z)$ around a Neveu-Schwarz field is trivial (the case of eq. (1.5)) and its monodromy around a Ramond field produces a minus sign (the case of eq. (1.6)). Because of these two possibilities the Laurent expansions for the super-current will be

$$G(z) = \sum_{k\in Z+1/2} \frac{G_k}{z^{k+3/2}} \qquad \text{Neveu-Schwarz sector (NS)}\,,$$
$$G(z) = \sum_{k\in Z} \frac{G_k}{z^{k+3/2}} \qquad \text{Ramond sector (R)}\,.$$

The OPE's (1.1), (1.2), (1.3) are equivalent to the Neveu-Schwarz-Ramond algebra relations

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,0}\,, \\ [L_n, G_k] &= \frac{1}{2}(n-2k)G_{n+k}\,, \\ \{G_k, G_l\} &= 2L_{k+l} + \frac{c}{3}\left(k^2 - 1/4\right)\delta_{k+l,0}\,, \end{aligned} \tag{1.8}$$

where $\{,\}$ denotes the anticommutator. In this paper we'll deal with minimal super-conformal series denoted as SM_p ($p = 3, 4, 5\ldots$) corresponding to the choice of the central charge

$$c_p = \frac{3}{2}\left(1 - \frac{8}{p(p+2)}\right)\,. \tag{1.9}$$

The main distinctive mark of the minimal super-conformal theories is that they have finitely many super primary fields. These fields are numerated by two integers $n \in \{1, 2, \cdots, p-1\}$, $m \in \{1, 2, \cdots, p+1\}$ and will be denoted as $\phi_{n,m}$. It is assumed that $\phi_{p-n,p+2-m} \equiv \phi_{n,m}$, hence the number of super primaries is equal to $[p^2/2]$ ([x] is the integer part of x). $\phi_{p-1,p+1} \equiv \phi_{1,1}$ is the identity operator. For even (odd) $n-m$ the super-conformal classes $[\phi_{n,m}]$ form irreducible representations of the Neveu-Schwarz (Ramond) algebra. The fields $\phi_{n,m}$ have dimensions

$$h_{n,m} = \frac{((p+2)n - pm)^2 - 4}{8p(p+2)} + \frac{1}{32}(1 - (-)^{n-m})\,. \tag{1.10}$$

2 Current algebra and the coset construction

We will use the coset construction [16, 17] of super-minimal models in terms of $\widehat{SU}(2)_k$ WZNW models [18, 19].

Recall that WZNW models are endowed with spin one holomorphic currents. The OPE relations of these currents specified to the case of $\widehat{SU}(2)_k$ read:

$$\begin{aligned} J^0(z)J^0(0) &= \frac{k/2}{z^2} + \text{reg}\,, \\ J^0(z)J^\pm(0) &= \pm\frac{J^\pm(0)}{z} + \text{reg}\,, \\ J^+(z)J^-(0) &= \frac{k}{z^2} + \frac{2J^0(0)}{z} + \text{reg}\,, \end{aligned} \tag{2.1}$$

where k is the level. The isotopic indices $\pm, 0$ convenient for the later use are related to the usual Euclidean indices as:

$$J^0 \equiv J^3 \qquad \text{and} \qquad J^\pm \equiv J^1 \pm iJ^2 \,. \tag{2.2}$$

The Laurent expansion of the currents reads

$$J^a(z) = \sum_{n\in Z} \frac{J^a_n}{z^{n+1}} \tag{2.3}$$

and the OPE rules (2.1) imply that the current algebra generators are subject to the Kač − *Moody* algebra commutation relations

$$\begin{aligned}
\left[J^\pm_n, J^\pm_m\right] &= 0\,,\\
\left[J^+_n, J^-_m\right] &= kn\delta_{n+m,0} + 2J^0_{n+m}\,,\\
\left[J^0_n, J^\pm_m\right] &= \pm J^\pm_{n+m}\,,\\
\left[J^0_n, J^0_m\right] &= \frac{kn}{2}\delta_{n+m,0}\,.
\end{aligned} \tag{2.4}$$

Notice that the subalgebra generated by J^a_0 is simply the Lie algebra $su(2)$.
The energy momentum tensor can be expressed through the currents with the help of the Sugawara construction

$$T(z) = \frac{1}{k+2}\left(J^0J^0 + \frac{1}{2}J^+J^- + \frac{1}{2}J^-J^+\right)\,. \tag{2.5}$$

As it is custom in CFT above and in what follows we assume that any product of local fields taken at coinciding points is regularised subtracting singular parts of the respective OPE. The central charge of the Virasoro algebra can be easily computed using (2.5). The result is:

$$c_k = \frac{3k}{k+2}\,. \tag{2.6}$$

The primary fields of the theory $\phi_{j,m}$ and corresponding states $|j,m\rangle$ are labeled by the spin of the representation $j = 0, 1/2, 1, \ldots, k/2$ and its projection $m = -j, -j+1, \ldots, j$. The corresponding conformal dimensions are given by

$$h = \frac{j(j+1)}{k+2}\,. \tag{2.7}$$

The zero modes of the currents act on the states $|j,m\rangle$ as[1]

$$\begin{aligned}
J^\pm|j,m\rangle &= \sqrt{j(j+1)-m(m\pm 1)}\,|j,m\pm 1\rangle\,,\\
J^0|j,m\rangle &= m|j,m\rangle\,.
\end{aligned} \tag{2.8}$$

We'll need also the explicit form of the $su(2)$ WZNW modular matrices

$$S^{(k)}_{n,m} = \sqrt{\frac{2}{k+2}}\sin\frac{\pi nm}{k+2}\,. \tag{2.9}$$

[1]Note that a consistent with eq. (2.8) conjugation rule for the primary fields would be $\phi^\dagger_{j,m} = (-)^{j-m}\phi_{j,-m}$.

It is well known that the $N = 1$ super-minimal models can be represented as a coset [16, 17]

$$\mathcal{SM}_{k+2} = \frac{su(2)_k \times su(2)_2}{su(2)_{k+2}} .$$

In particular the energy momentum tensor of $\mathcal{SM}_{k+2}$ is given by

$$T_{(su(2)_k \times su(2)_2)/su(2)_{k+2}} = T_{su(2)_k} + T_{su(2)_2} - T_{su(2)_{k+2}} . \tag{2.10}$$

Indeed the combination of the central charges (2.6) corresponding to these three terms matches with the central charge of the super-minimal models (1.9).

The construction of the super-current G is more subtle; it involves the primary fields $\phi_{1,m}$ of the level $k = 2$ WZNW theory (we denote the currents of this theory as K^a and summation over the index $a = \pm, 0$ is assumed):

$$G(z) = C_a J^a(z)\phi_{1,-a}(z) + D_a K^a_{-1}\phi_{1,-a}(z) . \tag{2.11}$$

The coefficients C_a, D_a can be fixed requiring that the respective state be the highest weight state of the diagonal current algebra $J + K$. In other words both $J^+_0 + K^+_0$ and $J^+_1 + K^+_1$ annihilate the state

$$C_a J^a_{-1}|0\rangle|1, -a\rangle + D^a|0\rangle K^a_{-1}|1, -a\rangle . \tag{2.12}$$

Up to an overall constant κ we get

$$\begin{aligned}
&D_+ = \frac{\kappa}{\sqrt{2}} , && D_0 = \kappa , && D_- = -\frac{\kappa}{\sqrt{2}} , \\
&C_+ = -\frac{3\kappa\sqrt{2}}{k} , && C_0 = -\frac{6\kappa}{k} , && C_- = \frac{3\kappa\sqrt{2}}{k} .
\end{aligned} \tag{2.13}$$

The value of κ may be determined using the normalization condition of the the super-current fixed by the OPE (1.3)

$$\kappa = \sqrt{\frac{(k+2)(k+4)}{(k+6)(5k+54)}} , \tag{2.14}$$

but this won't be of importance for our goals.

3 Perturbative RG flows and domain walls

In a well known paper A. Zamolodchikov [4] has investigated the RG flow from minimal model $\mathcal{M}_p$ to $\mathcal{M}_{p-1}$ initiated by the relevant field $\phi_{1,3}$. Using leading order perturbation theory valid for $p \gg 1$, for the several classes of local fields he calculated the mixing coefficients specifying the UV-IR map.

It was shown in [11] that a similar RG trajectory connecting $\mathcal{N} = 1$ super-minimal models $\mathcal{SM}_p$ to $\mathcal{SM}_{p-2}$ exists. In this case the RG flow is initiated by the top component of the Neveu-Schwartz superfield $\Phi_{1,3}$. For us it will be important that also in this case a detailed analysis of some classes of fields has been carried out.

As it became clear later [12, 15], above two examples are just the first simplest cases of more general RG flows. A wide class of CFT coset models

$$\mathcal{T}_{\rm UV} = \frac{\hat{g}_l \times \hat{g}_m}{\hat{g}_{l+m}}\,, \qquad m > l \tag{3.1}$$

under perturbation by the relevant field $\phi = \phi_{1,1}^{Adj}$ [15] at the IR limit flow to the theories

$$\mathcal{T}_{\rm IR} = \frac{\hat{g}_l \times \hat{g}_{m-l}}{\hat{g}_m}\,. \tag{3.2}$$

Recently in [3] Gaiotto constructed a nontrivial conformal interface between successive minimal CFT models and made a striking proposal that this interface (RG domain wall) encodes the UV-IR map resulting through the RG flow discussed above. It was shown that the proposal agrees with the leading order perturbative analysis of [4].

Generalization of leading order calculations to a wider class of local fields [5] as well as next to leading order calculations [6, 7] further confirm the validity of this construction.

Actually in [3] Gaiotto suggests also a candidate for RG domain wall for the much more general RG flow between (3.1) and (3.2). Let us briefly recall the construction. Since a conformal interface between two CFT models is equivalent to some conformal boundary for the direct product of these theories (folding trick), it is natural to consider the product theory $\mathcal{T}_{\rm UV} \times \mathcal{T}_{\rm IR}$

$$\frac{\hat{g}_l \times \hat{g}_m}{\hat{g}_{m+l}} \times \frac{\hat{g}_l \times \hat{g}_{m-l}}{\hat{g}_m} \sim \frac{\hat{g}_{m-l} \times \hat{g}_l \times \hat{g}_l}{\hat{g}_{l+m}}\,. \tag{3.3}$$

Notice the appearance of two identical factors $\hat{g}_l$ so one has a natural $\mathbb{Z}_2$ automorphism. Essentially the proposal of Gaiotto boils down to the statement that the boundary of the theory

$$\mathcal{T}_B = \frac{\hat{g}_l \times \hat{g}_l \times \hat{g}_{m-l}}{\hat{g}_{l+m}}, \qquad m > l \tag{3.4}$$

acts as a $\mathbb{Z}_2$ twisting mirror. Explicitly the RG boundary condition is the image of the $\mathbb{Z}_2$ twisted $\mathcal{T}_B$ brane

$$|\tilde{B}\rangle = \sum_{s,t} \sqrt{S_{1,t}^{(m-l)} S_{1,s}^{(m+l)}} \sum_d |t, d, d, s; \mathcal{B}, Z_2\rangle\rangle, \tag{3.5}$$

where the indices t, d, s refer to the representations of $\hat{g}_{m-l}$, $\hat{g}_l$, $\hat{g}_{l+m}$ respectively and $S_{1,r}^{(k)}$ are the modular matrices of the $\hat{g}_k$ WZNW model.

In what follows we will examine in details the case of RG flow between $\mathcal{N} = 1$ super-minimal models. The method we apply directly explores the current algebra representation in contrary to the analysis in [3] where a specific representation applicable only for the unitary minimal series was used.

4 RG domain walls for super minimal models

In the case of the $\mathcal{N} = 1$ super-minimal models one should consider

$$\frac{\widehat{su}(2)_k \times \widehat{su}(2)_2}{\widehat{su}(2)_{k+2}} \times \frac{\widehat{su}(2)_{k-2} \times \widehat{su}(2)_2}{\widehat{su}(2)_k} \sim \frac{\widehat{su}(2)_{k-2} \times \widehat{su}(2)_2 \times \widehat{su}(2)_2}{\widehat{su}(2)_{k+2}}\,, \tag{4.1}$$

where the first coset on l.h.s. corresponds to the UV super conformal model $\mathcal{SM}_{k+2}$ and the second one to the IR theory $\mathcal{SM}_k$. We denote by $K(z)$ and $\widetilde{K}(z)$ the WZNW currents of $\widehat{su}(2)_2$ entering in the cosets of the IR and UV theories respectively. The current of $\widehat{su}(2)_{k-2}$ WZNW theory will be denoted as $J(z)$. Using (2.10) and the Sugawara construction, for the energy-momentum tensor of the IR theory (the second factor of the l.h.s. of (4.1)) we get

$$T_{\text{ir}}(z) = \frac{1}{k}J(z)J(z) + \frac{1}{4}K(z)K(z) - \frac{1}{k+2}(K(z)+J(z))^2,$$

which can be rewritten as

$$T_{\text{ir}}(z) = \frac{2}{2k+k^2}J(z)J(z) - \frac{2}{2+k}J(z)K(z) + \frac{k-2}{4(k+2)}K(z)K(z). \tag{4.2}$$

Similarly the energy-momentum tensor for the UV theory is equal to

$$\begin{aligned} T_{\text{uv}}(z) = & \frac{2}{(2+k)(4+k)}J(z)J(z) + \frac{2}{(2+k)(4+k)}K(z)K(z) \\ & -\frac{2}{4+k}K(z)\widetilde{K}(z) + \frac{k}{4(k+4)}\widetilde{K}(z)\widetilde{K}(z) \\ & +\frac{4}{(2+k)(4+k)}J(z)K(z) - \frac{2}{4+k}J(z)\widetilde{K}(z)\,. \end{aligned} \tag{4.3}$$

In order to get the one-point functions of the theory $\mathcal{SM}_{k+2}\times\mathcal{SM}_k$ in the presence of RG boundary, one needs explicit expressions of the states corresponding to fields $\phi^{\text{IR}}\phi^{\text{UV}}$ in terms of the states of the coset theory

$$\mathcal{T}_B = \frac{\widehat{su}(2)_{k-2}\times\widehat{su}(2)_2\times\widehat{su}(2)_2}{\widehat{su}(2)_{k+2}}\,. \tag{4.4}$$

Let us denote the highest weight representation spaces of the current algebras $J(z)$, $K(z)$ and $\widetilde{K}(z)$ as $V_j^{(J)}$, $V_k^{(K)}$ and $V_{\tilde{k}}^{(\widetilde{K})}$ respectively (the lower indices specify the spins of the highest weight states). It is convenient to fix a unique representative of a state of the coset $\mathcal{T}_B$ in the space $V_j^{(J)}\otimes V_k^{(K)}\otimes V_{\tilde{k}}^{(\widetilde{K})}$ requiring that the state under consideration be a highest weight state of the diagonal current $J+K+\widetilde{K}$. The simplest case to analyse are the states corresponding to $\phi^{\text{IR}}_{n,n}\phi^{\text{UV}}_{n,n}$. Since

$$\begin{aligned} h^{\text{ir}}_{n,n} &= \frac{n^2-1}{4k} - \frac{n^2-1}{4(k+2)}\,, \\ h^{\text{uv}}_{n,n} &= \frac{n^2-1}{4(k+2)} - \frac{n^2-1}{4(k+4)}\,, \end{aligned}$$

the total dimension of the product field is

$$h^{\text{ir}}_{n,n} + h^{\text{uv}}_{n,n} = \frac{n^2-1}{4k} - \frac{n^2-1}{4(k+4)}\,, \tag{4.5}$$

so that the corresponding state is readily identified with ($|j,m\rangle$ denotes a primary state of spin j and projection m)

$$|\frac{n-1}{2},\frac{n-1}{2}\rangle|0,0\rangle|0,0\rangle \in V^{(J)}_{\frac{n-1}{2}}\otimes V_0^{(K)}\otimes V_0^{(\widetilde{K})}\,. \tag{4.6}$$

Indeed, this is a spin $\frac{n-1}{2}$ highest weight state of the combined current $J+K+\widetilde{K}$ and its $\mathcal{T}_B$ dimension

$$h^{(J)}_{\frac{n-1}{2}} + h^{(K)}_0 + h^{(\widetilde{K})}_0 - h^{(J+K+\widetilde{K})}_{\frac{n-1}{2}}$$

coincides with (4.5). Notice that $\mathbb{Z}_2$ action (i.e. permutation of the second and third factors) on this state is trivial. Thus the overlap of this state with its $\mathbb{Z}_2$ image is equal to 1 and from (3.5)

$$\langle \phi^{\mathrm{IR}}_{n,n}\phi^{\mathrm{UV}}_{n,n}|\mathrm{RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}} \,. \tag{4.7}$$

For large k and for $n \sim O(1)$ this gives $1+3/k^2+O(1/k^3)$. We conclude that up to $1/k^2$ terms, the fields $\phi^{\mathrm{UV}}_{n,n}$ flow to $\phi^{\mathrm{IR}}_{n,n}$ without mixing with other fields, in complete agreement with both leading order [11] and next to leading order [14] perturbative calculations.

Next let us examine the more interesting case of Ramond fields $\phi^{\mathrm{UV}}_{n,n\pm1}$ which are expected to flow to certain combinations of the fields $\phi^{\mathrm{IR}}_{n\pm1,n}$ [11].

Consider the state corresponding to $\phi^{\mathrm{ir}}_{n-1,n}\phi^{\mathrm{uv}}_{n,n-1}$. From (1.10) we get

$$h^{\mathrm{ir}}_{n-1,n} = \frac{3}{16} + \frac{(n-1)^2-1}{4k} - \frac{n^2-1}{4(k+2)} \,, \tag{4.8}$$

$$h^{\mathrm{uv}}_{n,n-1} = \frac{3}{16} - \frac{(n-1)^2-1}{4(k+4)} + \frac{n^2-1}{4(k+2)} \,. \tag{4.9}$$

Hence the conformal dimension of this product field will be

$$h^{\mathrm{ir}}_{n-1,n} + h^{\mathrm{uv}}_{n,n-1} = \frac{3}{8} + \frac{(n-1)^2-1}{4k} - \frac{(n-1)^2-1}{4(k+4)} \,. \tag{4.10}$$

There are three primaries in $su(2)_2$ WZNW theory with $j=0,1,2$ representations and conformal dimensions 0, $\frac{3}{16}$ and $\frac{1}{2}$ respectively. So, to get the right dimension one should choose a combination of states $|\frac{n}{2}-1,m\rangle|\frac{1}{2},\alpha\rangle|\frac{1}{2},\beta\rangle$. In addition this combination must be the spin $\frac{n}{2}-1$ highest weight state of $J+K+\widetilde{K}$ (to match with the last, negative term of (4.10)). Thus we are lead to

$$C_{\alpha\beta}|\frac{n}{2}-1,\frac{n}{2}-1-\alpha-\beta\rangle|\frac{1}{2},\alpha\rangle|\frac{1}{2},\beta\rangle, \tag{4.11}$$

where a summation over the indices $\alpha,\beta=\pm1/2$ is assumed. The highest weight condition that the operator $J^+_0+K^+_0+\widetilde{K}_0$ annihilates this state, implies

$$\sqrt{n-2}C_{++}+C_{-+}+C_{+-}=0\,.$$

A further constraint

$$C_{++}-\sqrt{n-2}C_{-+}=0\,,$$

one obtains imposing the condition that this state should be an eigenstate of the Virasoro operator L^{IR}_0 constructed from the energy-momentum tensor T_{ir} (4.2) with eigenvalue $h^{\mathrm{ir}}_{n,n-1}$ (4.8). Thus we get

$$C_{++}=\sqrt{n-2}\,C_{-+}\,, \qquad C_{+-}=-(n-1)\,C_{-+}$$

(of course, the undefined overall multiplier could be fixed from the normalization condition). Taking (normalized) scalar product of the state (4.11) with its $\mathbb{Z}_2$ image we find

$$\langle \phi^{\rm ir}_{n-1,n}\phi^{\rm uv}_{n,n-1}|{\rm RG}\rangle = -\frac{1}{n-1}\frac{\sqrt{S^{(k-2)}_{1,n-1}S^{(k+2)}_{1,n-1}}}{S^k_{1,n}}\,. \tag{4.12}$$

Consideration of the product $\phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n+1}$ fields is quite similar and leads to the state

$$C_{\alpha\beta}|\frac{n}{2},\frac{n}{2}-\alpha-\beta\rangle|\frac{1}{2},\alpha\rangle|\frac{1}{2},\beta\rangle\,,$$

with the coefficients

$$C_{+-}=0\,,\qquad C_{++}=-\frac{1}{\sqrt{n}}C_{-+}\,.$$

So, in this case

$$\langle \phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n+1}|{\rm RG}\rangle = \frac{1}{n+1}\frac{\sqrt{S^{(k-2)}_{1,n+1}S^{(k+2)}_{1,n+1}}}{S^k_{1,n}}\,. \tag{4.13}$$

Constructing the states corresponding to $\phi^{\rm ir}_{n-1,n}\phi^{\rm uv}_{n,n+1}$ and $\phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n-1}$ is even simpler and one easily gets $|\frac{n}{2}-1,\frac{n}{2}-1\rangle|\frac{1}{2},\frac{1}{2}\rangle|\frac{1}{2},\frac{1}{2}\rangle$ and $|\frac{n}{2},\frac{n}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle|\frac{1}{2},-\frac{1}{2}\rangle$ respectively. In both cases the $\mathbb{Z}_2$ action is trivial, hence

$$\langle \phi^{\rm ir}_{n-1,n}\phi^{\rm uv}_{n,n+1}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n-1}S^{(k+2)}_{1,n+1}}}{S^k_{1,n}}\,, \tag{4.14}$$

$$\langle \phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n-1}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n+1}S^{(k+2)}_{1,n-1}}}{S^k_{1,n}}\,. \tag{4.15}$$

In the large k limit we get

$$\langle \phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n+1}|{\rm RG}\rangle = \frac{1}{n}+O(1/k^2)\,, \tag{4.16}$$

$$\langle \phi^{\rm ir}_{n+1,n}\phi^{\rm uv}_{n,n-1}|{\rm RG}\rangle = \frac{\sqrt{n^2-1}}{n}+O(1/k^2)\,, \tag{4.17}$$

$$\langle \phi^{\rm ir}_{n-1,n}\phi^{\rm uv}_{n,n+1}|{\rm RG}\rangle = \frac{\sqrt{n^2-1}}{n}+O(1/k^2)\,, \tag{4.18}$$

$$\langle \phi^{\rm ir}_{n-1,n}\phi^{\rm uv}_{n,n-1}|{\rm RG}\rangle = -\frac{1}{n}+O(1/k^2)\,, \tag{4.19}$$

in complete agreement with the second order perturbation theory results [14].

We have analysed also the more complicated case of mixing of the primary Neveu-Schwartz superfields $\Phi_{n,n\pm2}$ and the descendant superfield $\mathbf{D}\bar{\mathbf{D}}\Phi_{n,n}$ (here $\mathbf{D}$ and $\bar{\mathbf{D}}$ are the super-derivatives). The details of calculations are presented in the appendix. Here are the final results:

$$\langle \psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{(n+1)(n+2)}\frac{\sqrt{S^{(k-2)}_{1,n+2}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,, \tag{4.20}$$

$$\langle \phi^{\rm ir}_{n+2,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = \frac{2}{n+1}\frac{\sqrt{S^{(k-2)}_{1,n+2}S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}}\,, \tag{4.21}$$

$$\langle \psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n+2}S^{(k+2)}_{1,n-2}}}{S^{(k)}_{1,n}}\,, \tag{4.22}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{n+1}\frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,, \tag{4.23}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = \frac{n^2-5}{n^2-1}\frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}}\,, \tag{4.24}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = -\frac{2}{n-1}\frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n-2}}}{S^{(k)}_{1,n}}\,, \tag{4.25}$$

$$\langle \psi^{\rm ir}_{n-2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n-2}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,, \tag{4.26}$$

$$\langle \phi^{\rm ir}_{n-2,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = -\frac{2}{n-1}\frac{\sqrt{S^{(k-2)}_{1,n-2}S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}}\,, \tag{4.27}$$

$$\langle \phi^{\rm ir}_{n-2,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{2}{(n-1)(n-2)}\frac{\sqrt{S^{(k-2)}_{1,n-2}S^{(k+2)}_{1,n-2}}}{S^{k}_{1,n}}\,. \tag{4.28}$$

At the large k limit we get

$$\langle \psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{n(n+1)} + O\big(1/k^2\big)\,, \tag{4.29}$$

$$\langle \phi^{\rm ir}_{n+2,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = \frac{2}{n+1}\sqrt{\frac{n+2}{n}} + O\big(1/k^2\big)\,, \tag{4.30}$$

$$\langle \psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{\sqrt{n^2-4}}{n} + O\big(1/k^2\big)\,, \tag{4.31}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{n+1}\sqrt{\frac{n+2}{n}} + O\big(1/k^2\big)\,, \tag{4.32}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = \frac{n^2-5}{n^2-1} + O\big(1/k^2\big)\,, \tag{4.33}$$

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = -\frac{2}{n-1}\sqrt{\frac{n-2}{n}} + O\big(1/k^2\big)\,, \tag{4.34}$$

$$\langle \psi^{\rm ir}_{n-2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{\sqrt{n^2-4}}{n} + O\big(1/k^2\big)\,, \tag{4.35}$$

$$\langle \phi^{\rm ir}_{n-2,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}|{\rm RG}\rangle = -\frac{2}{n-1}\sqrt{\frac{n-2}{n}} + O\big(1/k^2\big)\,, \tag{4.36}$$

$$\langle \phi^{\rm ir}_{n-2,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{2}{n(n-1)} + O\big(1/k^2\big)\,. \tag{4.37}$$

Again, the results are in complete agreement with the next to leading order perturbative calculations of [14].

It is interesting to note that, though the mixing coefficients computed here in the large k limit coincide with the respective cases of the $\phi_{1,3}$ perturbed minimal models, the exact k dependence in supersymmetric case enters solely through the modular matrices, in contrary to the quite complicated k dependence of the non supersymmetric case.

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A Mixing of the fields $\Phi_{n,n\pm 2}$ and the descendant $D\bar{D}\Phi_{n,n}$

Let us start with the product field $\phi^{\text{ir}}_{n-2,n}\phi^{\text{uv}}_{n,n-2}$. The corresponding dimensions are

$$h^{\text{ir}}_{n-2,n} = \frac{1}{2} + \frac{(n-2)^2-1}{4k} - \frac{n^2-1}{4(k+2)}\,, \tag{A.1}$$

$$h^{\text{uv}}_{n,n-2} = \frac{1}{2} - \frac{(n-2)^2-1}{4(4+k)} + \frac{n^2-1}{4(k+2)}\,, \tag{A.2}$$

hence

$$h^{\text{ir}}_{n-2,n} + h^{\text{uv}}_{n,n-2} = 1 + \frac{(n-2)^2-1}{4k} - \frac{(n-2)^2-1}{4(4+k)}\,. \tag{A.3}$$

A careful examination shows that the required state should be chosen among the combinations

$$\sum_{\alpha,\beta\in\{-1,0,1\}} C_{\alpha,\beta}\Big|\frac{n-3}{2},\frac{n-3}{2}-\alpha-\beta\Big\rangle|1,\alpha\rangle|1,\beta\rangle\,. \tag{A.4}$$

Indeed the other candidates such as $J^a_{-1}|\frac{n-3}{2},\frac{n-3}{2}-a\rangle|0\rangle|0\rangle$, $K^a_{-1}|\frac{n-3}{2},\frac{n-3}{2}-a\rangle|0\rangle|0\rangle$ or $\widetilde{K}^{\alpha}_{-1}|\frac{n-3}{2},\frac{n-3}{2}-a\rangle|0\rangle|0\rangle$ though have a correct total dimension, can not be combined to get the required IR dimension (A.1). This can be easily seen by examining the zero mode of the IR current

$$T^{\text{ir}} = \frac{1}{k}J^2 - \frac{1}{k+2}(J+K)^2 + \frac{1}{4}K^2\,. \tag{A.5}$$

The only way to get the term 1/2 of (A.1) is to choose $j=1$ representation of the current K (see the last term of (A.5)).

To get correct IR dimension one should impose the condition that the zero mode of $(J+K)^2$ on the state (A.4) must acquire the eigenvalue $\frac{n-1}{2}\frac{n+1}{2}$. Together with our usual requirement of being a highest weight state of the $J+K+\widetilde{K}$ algebra this fixes the coefficients up to an overall multiplier

$$C_{+0} = \sqrt{\frac{n-3}{2}}C_{00}\,, \qquad C_{++} = -\sqrt{\frac{n-3}{2}}\frac{\sqrt{n-4}}{n-2}C_{00}\,,$$

$$C_{+-} = \frac{1-n}{2}C_{00}\,, \qquad C_{0+} = -\frac{2}{n-2}\sqrt{\frac{n-3}{2}}C_{00}\,,$$

$$C_{-+} = -\frac{1}{n-2}C_{00}\,, \qquad C_{-0} = C_{0-} = C_{--} = 0\,.$$

This leads to the one point function

$$\langle\phi^{\rm ir}_{n-2,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{2}{(n-1)(n-2)}\frac{\sqrt{S^{(k-2)}_{1,n-2}S^{(k+2)}_{1,n-2}}}{S^{k}_{1,n}}\,. \tag{A.6}$$

In the same way we construct the state corresponding to $\phi^{\rm ir}_{n+2,n}\phi^{\rm uv}_{n,n+2}$

$$C_{\alpha\beta}|\frac{n+1}{2},\frac{n+1}{2}-\alpha-\beta\rangle|1,\alpha\rangle|1,\beta\rangle\,,$$

where

$$C_{++} = -\frac{1}{\sqrt{n}}C_{00},\quad C_{-+} = -\sqrt{\frac{n+1}{2}}\,C_{00},\qquad C_{0+} = C_{00} \tag{A.7}$$

(all other $C_{\alpha\beta}$ vanish) and

$$\langle\psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{(n+1)(n+2)}\frac{\sqrt{S^{(k-2)}_{1,n+2}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,. \tag{A.8}$$

The state corresponding to $\psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n-2}$ is simply $|\frac{n+1}{2},\frac{n+1}{2}\rangle|1,-1\rangle|1,-1\rangle$ and

$$\langle\psi^{\rm ir}_{n+2,n}\psi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n+2}S^{(k+2)}_{1,n-2}}}{S^{(k)}_{1,n}}\,. \tag{A.9}$$

Similarly for $\psi^{\rm ir}_{n-2,n}\psi^{\rm uv}_{n,n+2}$ the state is $|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,1\rangle$ and

$$\langle\psi^{\rm ir}_{n-2,n}\psi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{\sqrt{S^{(k-2)}_{1,n-2}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,. \tag{A.10}$$

Let us now consider states corresponding to the descendant field $G^{\rm ir}_{-1/2}\psi^{\rm ir}_{n,n}\psi^{\rm uv}_{n,n+2}$. Partial dimensions of the field $\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n+2}$ are

$$\begin{aligned} h^{\rm ir}_{n,n} &= \frac{n^2-1}{4k}-\frac{n^2-1}{4(k+2)}\,,\\ h^{\rm uv}_{n,n+2} &= \frac{1}{2}+\frac{n^2-1}{4(k+2)}-\frac{(n+2)^2-1}{4(k+4)}\,,\\ h^{\rm ir}_{n,n}+h^{\rm uv}_{n,n+2} &= \frac{1}{2}+\frac{n^2-1}{4k}-\frac{(n+2)^2-1}{4(k+4)}\,. \end{aligned}$$

Evidently the correct representative of the respective state is

$$|\frac{n-1}{2},\frac{n-1}{2}\rangle|0\rangle|1,1\rangle\,. \tag{A.11}$$

Using the expression (2.11) its is straightforward to find the result of the action of the super-current mode $G^{\rm ir}_{-1/2}$ on this state:

$$\begin{aligned} G^{\rm ir}_{-\frac{1}{2}}|\frac{n-1}{2},\frac{n-1}{2}\rangle|0\rangle|1,1\rangle &= C_aJ^a_0|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,-a\rangle|1,1\rangle\\ &\quad + D_aK^a_0|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,-a\rangle|1,1\rangle\,, \end{aligned} \tag{A.12}$$

where the coefficients C_a, D_a are given by (2.13) (one should replace k by $k-2$). The final result is:

$$G^{\rm ir}_{-\frac{1}{2}}|\frac{n-1}{2},\frac{n-1}{2}\rangle|0\rangle|1,1\rangle = -\frac{3(n-1)}{k-2}|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,0\rangle|1,1\rangle + \frac{6}{k-2}\sqrt{\frac{n-1}{2}}|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,1\rangle\,. \tag{A.13}$$

Thus for the one-point function we get

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n+2}|{\rm RG}\rangle = \frac{2}{n+1}\frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n+2}}}{S^{(k)}_{1,n}}\,. \tag{A.14}$$

Consideration of the remaining cases do not involve new ingredients and we will simply list the results.

- The state corresponding to $\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n-2}$ is:

$$-\frac{1}{\sqrt{n-2}}|\frac{n-1}{2},\frac{n-5}{2}\rangle|0\rangle|1,1\rangle + |\frac{n-1}{2},\frac{n-3}{2}\rangle|0\rangle|1,0\rangle - \sqrt{\frac{n-1}{2}}|\frac{n-1}{2},\frac{n-1}{2}\rangle|0\rangle|1,-1\rangle \quad .$$

The result of $G^{\rm ir}_{-\frac{1}{2}}$ action on this state looks ugly:

$$\begin{aligned}
&|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,-1\rangle|1,1\rangle + \frac{n-5}{2\sqrt{n-2}}|\frac{n-1}{2},\frac{n-5}{2}\rangle|1,0\rangle|1,1\rangle\\
&-\sqrt{\frac{3n-9}{2n-4}}|\frac{n-1}{2},\frac{n-7}{2}\rangle|1,1\rangle|1,1\rangle - \sqrt{\frac{n-1}{2}}|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,-1\rangle|1,0\rangle\\
&-\frac{n-3}{2}|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,0\rangle|1,0\rangle + \sqrt{n-2}|\frac{n-1}{2},\frac{n-5}{2}\rangle|1,1\rangle|1,0\rangle\\
&+\Big(\frac{n-1}{2}\Big)^{\frac{3}{2}}|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,0\rangle|1,-1\rangle - \frac{n-1}{2}|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,-1\rangle
\end{aligned}$$

multiplied by an overall factor $\frac{6}{k-2}$. The corresponding one-point function simply is:

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}\phi^{\rm uv}_{n,n-2}|{\rm RG}\rangle = -\frac{2}{n-1}\frac{\sqrt{S^{(k-2)}_{1,n}S^{(k+2)}_{1,n-2}}}{S^{(k)}_{1,n}}\,. \tag{A.15}$$

- In the $\phi^{\rm ir}_{n-2,n}\phi^{\rm uv}_{n,n}$ case the corresponding state is

$$|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,1\rangle|0\rangle\,. \tag{A.16}$$

Now we must act on this state by the operator $G^{\rm uv}_{-1/2}$

$$\begin{aligned}
G^{\rm uv}_{-1/2}|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,1\rangle|0\rangle &= \Big(C_a(K^a_0+J^a_0)+D_a\widetilde{K}^a_0\Big)|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,-a\rangle|0\rangle\\
&= -\frac{3(n-1)}{k}|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,0\rangle + \frac{6}{k}|\frac{n-3}{2},\frac{n-3}{2}\rangle|1,0\rangle|1,1\rangle\\
&\quad+\frac{6}{k}\sqrt{\frac{n-3}{2}}|\frac{n-3}{2},\frac{n-5}{2}\rangle|1,1\rangle|1,1\rangle\,.
\end{aligned}$$

The one point function:

$$\langle \phi^{\rm ir}_{n-2,n} G^{\rm uv}_{-\frac{1}{2}} \phi^{\rm uv}_{n,n} |{\rm RG}\rangle = -\frac{2}{n-1} \frac{\sqrt{S^{(k-2)}_{1,n-2} S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}} . \tag{A.17}$$

- The state corresponding to the field $\phi^{\rm ir}_{n+2,n}\phi^{\rm uv}_{n,n}$ is

$$\begin{aligned} -\frac{1}{\sqrt{n}} |\frac{n+1}{2}\frac{n-3}{2}\rangle|1,1\rangle|0\rangle + |\frac{n+1}{2},\frac{n-1}{2}\rangle|1,0\rangle|0\rangle \\ -\sqrt{\frac{n+1}{2}}|\frac{n+1}{2},\frac{n+1}{2}\rangle|1,-1\rangle|0\rangle \quad . \end{aligned} \tag{A.18}$$

Acting by $G^{\rm uv}_{-1/2}$ on this state we get

$$\begin{aligned} \frac{n-1}{2\sqrt{n}}|\frac{n+1}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,0\rangle + \sqrt{\frac{n+1}{2}}\left(\frac{n-1}{2}\right)|\frac{n+1}{2},\frac{n+1}{2}\rangle|1,-1\rangle|1,0\rangle \\ -\sqrt{\frac{3n-3}{2n}}|\frac{n+1}{2},\frac{n-5}{2}\rangle|1,1\rangle|1,1\rangle + \frac{n-1}{\sqrt{n}}|\frac{n+1}{2},\frac{n-3}{2}\rangle|1,0\rangle|1,1\rangle \\ -\frac{n-1}{2}|\frac{n+1}{2},\frac{n-1}{2}\rangle|1,0\rangle|1,0\rangle - \frac{n-1}{2}|\frac{n+1}{2},\frac{n-1}{2}\rangle|1,-1\rangle|1,1\rangle \end{aligned}$$

multiplied by $\frac{6}{k}$. The result for one-point function:

$$\langle \phi^{\rm ir}_{n+2,n} G^{\rm uv}_{-\frac{1}{2}} \phi^{\rm uv}_{n,n} |{\rm RG}\rangle = \frac{2}{n+1} \frac{\sqrt{S^{(k-2)}_{1,n+2} S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}} . \tag{A.19}$$

- Finally, the state corresponding to the field $G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n}G^{\rm uv}_{-\frac{1}{2}}\phi^{\rm uv}_{n,n}$ is

$$(C_a J^a_0 + D_a K^a_0)\Big(C_b\left(K^b_0 + J^b_0\right) + D_b \widetilde{K}^b_0\Big)|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,-a\rangle|1,-b\rangle \tag{A.20}$$

which after some algebra becomes

$$\begin{aligned} \left(\frac{n-1}{2}\right)^2|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,0\rangle|1,0\rangle - \sqrt{\frac{n-1}{2}}\frac{n-1}{2}|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,0\rangle|1,1\rangle \\ -\frac{n-1}{2}|\frac{n-1}{2},\frac{n-1}{2}\rangle|1,1\rangle|1,-1\rangle - \sqrt{\frac{n-1}{2}}\frac{n-3}{2}|\frac{n-1}{2},\frac{n-3}{2}\rangle|1,1\rangle|1,0\rangle \\ +\sqrt{\frac{n-1}{2}}\sqrt{n-2}|\frac{n-1}{2},\frac{n-5}{2}\rangle|1,1\rangle|1,1\rangle \end{aligned}$$

multiplied by $\frac{36}{k(k+2)}$. The respective one-point function is equal to

$$\langle G^{\rm ir}_{-\frac{1}{2}}\phi^{\rm ir}_{n,n} G^{\rm uv}_{-\frac{1}{2}} \phi^{\rm uv}_{n,n} |{\rm RG}\rangle = \frac{n^2-5}{n^2-1} \frac{\sqrt{S^{(k-2)}_{1,n} S^{(k+2)}_{1,n}}}{S^{(k)}_{1,n}} . \tag{A.21}$$

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