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Supermembrane in $D = 5$: component action

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Abstract: Based on the connection between partial breaking of global supersymmetry, coset approach, which realized the given pattern of supersymmetry breaking, and the Nambu-Goto actions for the extended objects, we have constructed on-shell component action for $N = 1, D = 5$ supermembrane and its dual cousins. We demonstrate that the proper choice of the components and the use of the covariant (with respect to broken supersymmetry) derivatives drastically simplify the action: it can be represented as a sum of four terms each having an explicit geometric meaning.

Keywords: p-branes, Supersymmetry Breaking, Extended Supersymmetry

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Contents

1 Introduction

The characteristic feature of the theories with a partial breaking of the global supersymmetries is the appearance of the Goldstone fermionic fields, associated with the broken supertranslations, as the components of Goldstone supermultiplets of unbroken supersymmetry. The natural description of such theories is achieved within the coset approach [1–4]. The usefulness of the coset approach in the applications to the theories with partial breaking of the supersymmetry have been demonstrated by many authors [5]–[25]. The presence of the unbroken supersymmetry makes quite reasonable the idea to choose the corresponding superfields as the basic ones and many interesting superspace actions describing different patterns of supersymmetry breaking have been constructed in such a way [9–12, 14, 16]. However, the standard methods of coset approach fail to construct the superfield action, because the superspace Lagrangian is weakly invariant with respect to supersymmetry - it is shifted by the full space-time or spinor derivatives under broken/unbroken supersymmetry transformations. Another, rather technical difficulty is to obtain the component action from the superspace one, which is written in terms of the superfields subjected to highly nonlinear constraints. Finally, in some cases the covariantization of the irreducibility constraints with respect to the broken supersymmetry is not evident, if at all possible. For example, it has been demonstrated in [9–11] that such constraints for the vector supermultiplet can be covariantized only together with the equations of motion.

It turned out that one can gain more information about component on shell-actions if attention is shifted to the broken supersymmetry. It was demonstrated in [25] that with a suitable choice of the parametrization of the coset, the θ-coordinates of unbroken supersymmetry and the physical bosonic components do not transform under broken supersymmetry. Moreover, the physical fermions transform as the Goldstino of the Volkov-Akulov model [26, 27] with respect to broken supersymmetry. Therefore, the physical fermions can enter the component on-shell action only i) through the determinant of the fermionic vielbein (to compensate the variation of the volume d^dx), ii) through the covariant space-time derivatives, or iii) through the Wess-Zumino term, if it exists. The first two ingredients can be easily constructed within the coset method, while the Wess-Zumino can be also constructed from Cartan forms following the recipe of ref. [28]. As a result, we will have the Ansatz for the action with several constant parameters, which have to be fixed by the invariance with respect to unbroken supersymmetry. The pleasant feature of such an approach is that the fermions are "hidden" inside covariant derivatives and determinant of the vielbein, making the whole action short, with the explicit geometric meaning of each term. In the present paper we apply this procedure to construct the action of $N = 1, D = 5$ supermembrane and its dual cousins.

2 Supermembrane in $D = 5$ space-time

In accordance with the general consideration presented in [25], to construct the component action for the supermembrane in $D = 5$ one has to carry out the following steps:

- Choosing the proper parametrization of the coset space element corresponding to the given pattern of the supersymmetry breaking; constructing the Cartan forms and finding the covariant derivatives,
- Imposing the kinematical and dynamical constraints,
- Finding the bosonic limit of the action and then generalizing it to the full supersymmetric case,
- Fixing the arbitrary constants in the supersymmetric action by imposing the invariance with respect to unbroken supersymmetry.

Let us perform this programme.

2.1 Coset space

In the present case we are dealing with the spontaneous breaking of $N = 1, D = 5$ Poincaré supersymmetry down to $N = 2, d = 3$ one. From the $d = 3$ standpoint the $N = 1, D = 5$

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supersymmetry algebra is a central-charges extended $N = 4$ Poincaré superalgebra with the following basic anticommutation relations:

$$\{Q_a, \overline{Q}_b\} = 2P_{ab},\ \{S_a, \overline{S}_b\} = 2P_{ab},\ \{Q_a, S_b\} = 2\epsilon_{ab}Z,\ \{\overline{Q}_a, \overline{S}_b\} = 2\epsilon_{ab}\overline{Z}. \tag{2.1}$$

The $d = 3$ translations generator P_{ab} and the central charge generators $Z, \overline{Z}$ form $D = 5$ translation generators. We will also split the generators of $D = 5$ Lorentz algebra $so(1, 4)$ into $d = 3$ Lorentz algebra generators M_{ab}, the generators K_{ab} and $\overline{K}_{ab}$ belonging to the coset $\mathrm{SO}(1, 4)/\mathrm{SO}(1, 2) \times \mathrm{U}(1)$ and the U(1) generator J. The full set of commutation relations can be found in the appendix A, (A.2).

Keeping $d = 3$ Lorentz and, commuting with it, U(1) subgroups of $D = 5$ Lorentz group $\mathrm{SO}(1, 4)$ linearly realized, we will choose the coset element as

$$g = e^{\mathrm{i}x^{ab}P_{ab}} e^{\theta^a Q_a + \bar{\theta}^a \overline{Q}_a} e^{\mathrm{i}(\mathbf{q}Z + \bar{\mathbf{q}}\overline{Z})} e^{\boldsymbol{\psi}^a S_a + \bar{\boldsymbol{\psi}}^a \overline{S}_a} e^{\mathrm{i}(\boldsymbol{\Lambda}^{ab} K_{ab} + \overline{\boldsymbol{\Lambda}}^{ab} \overline{K}_{ab})}. \tag{2.2}$$

Here, $\{x^{ab}, \theta^a, \bar{\theta}^a\}$ are $N = 2, d = 3$ superspace coordinates, while the remaining coset parameters are $N = 2$ Goldstone superfields. The whole $N = 1, D = 5$ super Poincaré group can be realized in this coset by the left acting on (2.2) of the different elements of the supergroup. The resulting transformation properties of the coordinates and superfields with respect to unbroken and broken supersymmetries are presented in (A.6), (A.7). The results of a pure technical calculation of the corresponding Cartan forms, semi-covariant derivatives and their algebra are summarized in the appendix A, (A.10), (A.15), (A.19).

2.2 Kinematical constraints and equations of motion

In accordance with the general theorem (Inverse Higgs phenomenon) formulated in [29], in order to reduce the number of independent superfields one has to impose the constraints

$$\Omega_Z = 0 \quad \Rightarrow \quad \begin{cases} \nabla_{ab}\mathbf{q} = -2\mathrm{i}\frac{(1+\boldsymbol{l}\cdot\bar{\boldsymbol{l}})\boldsymbol{l}_{ab} - \boldsymbol{l}^2\bar{\boldsymbol{l}}_{ab}}{(1+\boldsymbol{l}\cdot\bar{\boldsymbol{l}})^2 - \boldsymbol{l}^2\bar{\boldsymbol{l}}^2}, \\ \nabla_a\mathbf{q} = -2\mathrm{i}\boldsymbol{\psi}_a, \quad \overline{\nabla}_a\mathbf{q} = 0, \end{cases} \qquad \overline{\Omega}_Z = 0 \quad \Rightarrow \quad \begin{cases} \nabla_{ab}\bar{\mathbf{q}} = 2\mathrm{i}\frac{(1+\boldsymbol{l}\cdot\bar{\boldsymbol{l}})\bar{\boldsymbol{l}}_{ab} - \bar{\boldsymbol{l}}^2\boldsymbol{l}_{ab}}{(1+\boldsymbol{l}\cdot\bar{\boldsymbol{l}})^2 - \boldsymbol{l}^2\bar{\boldsymbol{l}}^2}, \\ \overline{\nabla}_a\bar{\mathbf{q}} = -2\mathrm{i}\bar{\boldsymbol{\psi}}_a, \quad \nabla_a\bar{\mathbf{q}} = 0. \end{cases} \tag{2.3}$$

Here, to simplify the expressions, we have passed to the some variant of the stereographic parametrization of the coset $\mathrm{SO}(1, 4)/\mathrm{SO}(1, 2) \times \mathrm{U}(1)$

$$\boldsymbol{l}_{ab} = \left(\frac{\tanh\sqrt{\boldsymbol{Y}}}{\sqrt{\boldsymbol{Y}}}\right)_{ab}^{cd} \boldsymbol{\Lambda}_{cd}, \quad \bar{\boldsymbol{l}}_{ab} = \left(\frac{\tanh\sqrt{\boldsymbol{Y}}}{\sqrt{\boldsymbol{Y}}}\right)_{ab}^{cd} \overline{\boldsymbol{\Lambda}}_{cd}. \tag{2.4}$$

The equations (2.3) allow us to express the superfields $\boldsymbol{\Lambda}_{ab}, \overline{\boldsymbol{\Lambda}}_{ab}$ and $\boldsymbol{\psi}^a, \bar{\boldsymbol{\psi}}^a$ through covariant derivatives of $\mathbf{q}(x, \theta, \bar{\theta})$ and $\bar{\mathbf{q}}(x, \theta, \bar{\theta})$. Thus, the bosonic superfields $\mathbf{q}(x, \theta, \bar{\theta}), \bar{\mathbf{q}}(x, \theta, \bar{\theta})$ are the only essential Goldstone superfields needed for this case of the partial breaking of the global supersymmetry. The constraints (2.3) are covariant under all symmetries, they do not imply any dynamics and leave $\mathbf{q}(x, \theta, \bar{\theta})$ and $\bar{\mathbf{q}}(x, \theta, \bar{\theta})$ off shell.

Within the coset approach we may also to write the covariant superfield equations of motion. It was shown in [15]–[21] that this can be achieved by imposing the proper constraint on the Cartan forms for broken supersymmetry. In the present case these constraints

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$$\Omega_S| = 0 \;\Rightarrow\; (a)\; \nabla_a \boldsymbol{\psi}_b = 0, \qquad (b)\; \bar\nabla_b \boldsymbol{\psi}^a = -\mathrm{i}\boldsymbol{\Lambda}_b{}^c \left(\frac{\tan 2\sqrt{\boldsymbol{T}}}{\sqrt{\boldsymbol{T}}}\right)_c^{\;a} \equiv -\mathrm{i}\boldsymbol{\lambda}_b^a$$

$$\overline{\Omega}_S| = 0 \;\Rightarrow\; (a)\; \bar\nabla_a \bar{\boldsymbol{\psi}}_b = 0, \qquad (b)\; \nabla_b \bar{\boldsymbol{\psi}}^a = \mathrm{i}\bar{\boldsymbol{\Lambda}}_b{}^c \left(\frac{\tan 2\sqrt{\boldsymbol{T}}}{\sqrt{\boldsymbol{T}}}\right)_c^{\;a} \equiv \mathrm{i}\bar{\boldsymbol{\lambda}}_b^a, \tag{2.5}$$

where $|$ means the $d\theta$-projection of the forms. These constraints are closely related with constraints of the super-embedding approach [30].

To conclude this subsection let us make a few comments:

- The easiest way to check that the equations (2.3), (2.5) put the theory on-shell is to consider these equations in the linearized form

$$\partial_{ab}\mathbf{q} = -2\mathrm{i}\boldsymbol{\Lambda}_{ab}\ (a), \qquad D_a\mathbf{q} = -2\mathrm{i}\boldsymbol{\psi}_a\ (b), \qquad \overline{D}_a\mathbf{q} = 0\ (c), \tag{2.6}$$

$$D_a\boldsymbol{\psi}_b = 0\ (a), \qquad \overline{D}_b\boldsymbol{\psi}^a = -2\mathrm{i}\boldsymbol{\Lambda}_b^a\ (b). \tag{2.7}$$

  Acting on eq. (2.6b) by $\overline{D}_b$ and using the eq. (2.6c) and the algebra of spinor derivatives (A.16) we immediately conclude that eq. (2.7b) follows from (2.6). In addition, the eq. (2.7a) means that the auxiliary component of the superfield $\mathbf{q}$ is zero and, therefore, our system is on-shell

$$D_a\boldsymbol{\psi}_b = 0 \;\Rightarrow\; D^2\mathbf{q} = 0 \;\Rightarrow \partial_{ab}D^b\mathbf{q} = 0 \;\Rightarrow \Box\mathbf{q} = 0. \tag{2.8}$$

- It turns out that the variables $\{\boldsymbol{\lambda}_a^b, \bar{\boldsymbol{\lambda}}_a^b\}$ defined in (2.5), are more suitable then $\{\boldsymbol{l}_{ab}, \bar{\boldsymbol{l}}_{ab}\}$ (2.4) one. Using the algebra of covariant derivatives (A.19) it is easy to find the following relations from (2.3) and (2.5)

$$\nabla_{ab}\mathbf{q} = -\mathrm{i}\frac{\boldsymbol{\lambda}_{ab} - \frac{1}{2}\boldsymbol{\lambda}^2\bar{\boldsymbol{\lambda}}_{ab}}{1 - \frac{1}{4}\boldsymbol{\lambda}^2\bar{\boldsymbol{\lambda}}^2}, \qquad \nabla_{ab}\bar{\mathbf{q}} = \mathrm{i}\frac{\bar{\boldsymbol{\lambda}}_{ab} - \frac{1}{2}\bar{\boldsymbol{\lambda}}^2\boldsymbol{\lambda}_{ab}}{1 - \frac{1}{4}\boldsymbol{\lambda}^2\bar{\boldsymbol{\lambda}}^2}. \tag{2.9}$$

  These equations play the same role as those in (2.3), relating the superfields $\{\boldsymbol{\lambda}_{ab}, \bar{\boldsymbol{\lambda}}_{ab}\}$ (and, therefore, the superfields $\{\boldsymbol{\Lambda}_{ab}, \overline{\boldsymbol{\Lambda}}_{ab}\}$) with the space-time derivatives of the superfields $\{\mathbf{q}, \bar{\mathbf{q}}\}$.

2.3 Bosonic part

In what follows we will mainly deal with the component approach. So, let us define the components of our superfields as

$$q = \mathbf{q}|_{\theta=0}, \quad \psi_a = \boldsymbol{\psi}_a|_{\theta=0}, \quad \lambda_{ab} = \boldsymbol{\lambda}_{ab}|_{\theta=0}, \quad \Lambda_{ab} = \boldsymbol{\Lambda}_{ab}|_{\theta=0}. \tag{2.10}$$

The basic idea of the approach outlined in [25] is to write the candidate for supersymmetric action as a proper supersymmetrization of the bosonic action. So, the crucial step is to construct the bosonic action. Keeping in the mind that the system of equations (2.3), (2.5) is invariant with respect to *all* $N = 1, D = 5$ super Poincaré group, we have to conclude that its bosonic sub-sector has to be invariant under the bosonic part of the super-Poincaré group, i.e. under ISO$(1,4)$ transformations. This information is enough to construct the bosonic action.

There are, at least, three equivalent ways to construct the bosonic action.

2.3.1 Bosonic coset

The simplest, straightforward way to construct the bosonic action is to consider the bosonic coset, i.e. the coset (2.2) with discarded θ's and all fermions

$$g_{\rm bos} = e^{{\rm i}x^{ab}P_{ab}} e^{{\rm i}(qZ+\bar{q}\overline{Z})} e^{{\rm i}(\Lambda^{ab}K_{ab}+\overline{\Lambda}^{ab}\overline{K}_{ab})}. \tag{2.11}$$

Clearly, the corresponding bosonic Cartan forms can be easily extracted from their superfields version (A.10). The bosonic version of the constraints (2.3) will result in the relations

$$\partial_{ab}q = -2{\rm i}\frac{(1+l\cdot\bar{l})l_{ab} - l^2\bar{l}_{ab}}{(1+l\cdot\bar{l})^2 - l^2\bar{l}^2}, \quad \partial_{ab}\bar{q} = 2{\rm i}\frac{(1+l\cdot\bar{l})\bar{l}_{ab} - \bar{l}^2 l_{ab}}{(1+l\cdot\bar{l})^2 - l^2\bar{l}^2}, \tag{2.12}$$

while the bosonic vielbein ${\mathcal B_{ab}}^{cd}$

$$\left(\Omega_P^{bos}\right) = dx^{ab}{\mathcal B_{ab}}^{cd}P_{cd} \tag{2.13}$$

acquires the form

$$\mathcal{B}_{ab}^{cd} = \delta_a^{(c}\delta_b^{d)} - \frac{2}{(1+l\cdot\bar{l})^2 - l^2\,\bar{l}^2}\left[(1+l\cdot\bar{l})\,\left(\bar{l}^{cd}l_{ab} + l^{cd}\bar{l}_{ab}\right) - \bar{l}^2\,l^{cd}l_{ab} - l^2\,\bar{l}^{cd}\bar{l}_{ab}\right],$$

Therefore, the simplest invariant bosonic action reads

$$S_{\rm bos} = \int d^3x \det B = \int d^3x \frac{(1-l\cdot\bar{l})^2 - l^2\,\bar{l}^2}{(1+l\cdot\bar{l})^2 - l^2\,\bar{l}^2}, \tag{2.14}$$

or in terms of $\{q,\bar{q}\}$

$$S_{\rm bos} = \int d^3x \sqrt{(1-\partial_{ab}q\,\partial^{ab}\bar{q})^2 - (\partial_{ab}q\,\partial^{ab}q)\,(\partial_{cd}\bar{q}\,\partial^{cd}\bar{q})}. \tag{2.15}$$

The latter is the static gauge Nambu-Goto action for the membrane in D=5.

2.3.2 Direct construction

Another way to derive the bosonic action is to use automorphism transformation laws. These laws (A.8) in the bosonic limit have the form

$$\delta x^{ab} = 2{\rm i}\left(\bar{a}^{ab}q - a^{ab}\bar{q}\right), \;\; \delta q = -2{\rm i}(ax), \;\; \delta\bar{q} = 2{\rm i}(\bar{a}x). \tag{2.16}$$

The active form of these transformations reads

$$\delta^\star q = -2{\rm i}(ax) - 2{\rm i}\partial_{ab}q\left(\bar{a}^{ab}q - a^{ab}\bar{q}\right), \;\; \delta^\star\bar{q} = 2{\rm i}(\bar{a}x) - 2{\rm i}\partial_{ab}\bar{q}\left(\bar{a}^{ab}q - a^{ab}\bar{q}\right). \tag{2.17}$$

Due to translations, U(1)-rotations and $d=3$ Lorentz invariance, the action may depend only on scalars ξ and $(\eta\bar{\eta})$, where

$$\xi = \partial_{ab}q\;\partial^{ab}\bar{q}, \quad \eta = \partial_{ab}q\;\partial^{ab}q, \;\; \bar{\eta} = \partial_{ab}\bar{q}\;\partial^{ab}\bar{q}. \tag{2.18}$$

Their transformation laws can be easily found to be

$$\delta^\star\xi = 2{\rm i}(\bar{a}\partial q) - 2{\rm i}(a\partial\bar{q}) - 2{\rm i}(\bar{a}^{ab}q - a^{ab}\bar{q})\partial_{ab}\xi - 2{\rm i}(\bar{a}\partial q)\xi + 2{\rm i}(a\partial\bar{q})\xi - 2{\rm i}(\bar{a}\partial\bar{q})\eta + 2{\rm i}(a\partial q)\bar{\eta},$$

$$\delta^\star(\eta\bar\eta) = 4\mathrm{i}(\bar a\partial\bar q)\eta - 4\mathrm{i}(a\partial q)\bar\eta - 2\mathrm{i}(\bar a^{kl}q - a^{kl}\bar q)\partial_{kl}(\eta\bar\eta) - 4\mathrm{i}(\bar a\partial q)\eta\bar\eta + 4\mathrm{i}(a\partial\bar q)\eta\bar\eta + \\ +4\mathrm{i}(a\partial q)\xi\bar\eta - 4\mathrm{i}(\bar a\partial\bar q)\xi\eta. \tag{2.19}$$

Therefore, the variation of the arbitrary function $F(\xi, \eta\bar\eta)$ reads

$$\frac{1}{2\mathrm{i}}\delta^\star F = [(a\partial q)\bar\eta - (\bar a\partial\bar q)\eta]\left(F_\xi + 2(\xi-1)F_{(\eta\bar\eta)}\right) + [(\bar a\partial q) - (a\partial\bar q)]\left(F + (1-\xi)F_\xi - 2\eta\bar\eta F_{(\eta\bar\eta)}\right) \\ -\partial_{ab}\left[\left(q\bar a^{ab} - \bar q a^{ab}\right)F\right]. \tag{2.20}$$

Thus, to achieve the invariance of the action one has impose the following restrictions on the function F:

$$F_\xi + 2(\xi-1)F_{(\eta\bar\eta)} = 0, \quad F + F_\xi(1-\xi) - 2(\eta\bar\eta)F_{(\eta\bar\eta)} = 0, \tag{2.21}$$

with the evident solution

$$F = \sqrt{(1-\xi)^2 - \eta\bar\eta}. \tag{2.22}$$

Therefore, the invariant action has the form

$$S = \int d^3x \sqrt{(1 - \partial_{ab}q\partial^{ab}\bar q)^2 - (\partial_{ab}q\partial^{ab}q)(\partial_{kl}\bar q\partial^{kl}\bar q)},$$

and thus, it coincides with the previously constructed one in (2.15), as it should be. Finally, one should note that the trivial action

$$S_0 = \alpha\int d^3x, \qquad \alpha = const \tag{2.23}$$

is also invariant under ISO(1, 4) transformations.

2.3.3 Using the equations of motion

This way is more involved, thus we just sketch main steps. The idea is to find the bosonic equations of motion for $\{q, \bar q\}$, which follow from (2.3), (2.5). These equations will explicitly contain $\{\lambda_{ab}, \bar\lambda_{ab}\}$, which have to be expressed through $\{\partial_{ab}q, \partial_{ab}\bar q\}$ from the bosonic version of the equations (2.9)

$$\partial_{ab}q = -\mathrm{i}\frac{\lambda_{ab} - \frac{1}{2}\lambda^2\bar\lambda_{ab}}{1 - \frac{1}{4}\lambda^2\bar\lambda^2}, \quad \partial_{ab}\bar q = \mathrm{i}\frac{\bar\lambda_{ab} - \frac{1}{2}\bar\lambda^2\lambda_{ab}}{1 - \frac{1}{4}\lambda^2\bar\lambda^2}. \tag{2.24}$$

Having at hands the equations of motion one may reconstruct the bosonic action, which of course, will again coincide with (2.15).

Clearly, in the present case the two previously discussed ways are simpler. Nevertheless, in the cases where some of the physical bosonic components have no Goldstone fields interpretation, this way is rather efficient, if not the simplest ones (see e.g. [22, 23]).

2.4 Adding supersymmetry

Now, we have at hands all ingredients to construct the full component action for the membrane which will be invariant under both, broken S and unbroken Q supersymmetries. In our approach, we are starting with the broken supersymmetry.

2.4.1 Broken S supersymmetry

In our parametrization of the coset (2.2) the superspace coordinates $\{\theta,\bar\theta\}$ do not transform under S supersymmetry. Therefore, each component of our superfields transforms independently and from (A.7) one may find that

$$\delta x^{ab} = \mathrm{i}\left(\varepsilon^{(a}\bar\psi^{b)} + \bar\varepsilon^{(a}\psi^{b)}\right), \quad \delta q = 0,\ \delta\bar q = 0, \quad \delta\psi^a = \varepsilon^a,\ \delta\bar\psi^a = \bar\varepsilon^a\,. \tag{2.25}$$

Then, one may easily check that the $\theta = 0$ projections of the covariant differential $\triangle x^{ab}$ (A.11)

$$\hat\triangle x^{ab} \equiv \triangle x^{ab}|_{\theta=0} = dx^{ab} - \mathrm{i}\left(\psi^{(a}d\bar\psi^{b)} + \bar\psi^{(a}d\psi^{b)}\right) \equiv \mathcal{E}^{ab}_{cd}\, dx^{cd}, \tag{2.26}$$

as well as the covariant derivatives constructed from them

$$\mathcal{D}_{ab} = \left(\mathcal{E}^{-1}\right)_{ab}^{cd}\,\partial_{cd} \tag{2.27}$$

are also invariant under S supersymmetry. From all these it immediately follows that the action possessing the proper bosonic limit (2.15) and invariant under broken supersymmetry reads

$$S_1 = \int d^3x\ \det\mathcal{E}\sqrt{(1-\mathcal{D}_{ab}q\mathcal{D}^{ab}\bar q)^2 - (\mathcal{D}_{ab}q\mathcal{D}^{ab}q)(\mathcal{D}_{cd}\bar q\mathcal{D}^{cd}\bar q)}. \tag{2.28}$$

The action S_1 reproduces the kinetic terms for the bosonic and fermionic components

$$S_1 = \int d^3x\ \left[-\mathrm{i}\left(\psi^a\partial_{ab}\bar\psi^b + \bar\psi^a\partial_{ab}\psi^b\right) - \partial_{ab}q\partial^{ab}\bar q + \ldots\right], \tag{2.29}$$

but the coefficient between these kinetic terms is strictly fixed. This could be not enough to maintain Q supersymmetry. So, one has to add to the action S_1 the purely fermionic action S_2

$$S_2 = \int d^3x\ \det\mathcal{E}, \tag{2.30}$$

which is trivially invariant under S supersymmetry. Finally, to have a proper limit

$$S_{q\to 0,\psi\to 0} = 0,$$

one has to involve into the game the trivial action S_0

$$S_0 = \int d^3x. \tag{2.31}$$

Thus, our anzatz for the supersymmetric action acquires the form

$$\begin{aligned} S &= (1+\alpha)\,S_0 - S_1 - \alpha S_2 = \\ &= (1+\alpha)\int d^3x - \int d^3x\,\det\mathcal{E}\left(\alpha + \sqrt{(1-\mathcal{D}_{ab}q\mathcal{D}^{ab}\bar q)^2 - (\mathcal{D}_{ab}q\mathcal{D}^{ab}q)(\mathcal{D}_{cd}\bar q\mathcal{D}^{cd}\bar q)}\right), \end{aligned} \tag{2.32}$$

where α is a constant that has to be defined.

In the cases previously considered within the present approach [25, 31], the Ansatz, similar to (2.32), was completely enough to maintain the second, unbroken supersymmetry.

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The careful analysis shows that in the present case there is one additional, Wess-Zumino term which has to be taken into account

$$S_{\rm WZ} = {\rm i} \int d^3x \;\det \mathcal{E}\; \left(\psi^m \mathcal{D}_{ab} \bar\psi_m - \bar\psi^m \mathcal{D}_{ab}\psi_m\right) \mathcal{D}^{ac}\, q\, \mathcal{D}_c{}^b\, \bar q. \tag{2.33}$$

The variation of the $S_{\rm WZ}$ under S supersymmetry reads (only the variations of $\psi, \bar\psi$ without derivatives play a role)

$$\delta S_{\rm WZ} = {\rm i} \int d^3x \;\det \mathcal{E}\; \left(\varepsilon^m \mathcal{D}_{ab} \bar\psi_m - \bar\varepsilon^m \mathcal{D}_{ab}\psi_m\right) \mathcal{D}^{ac}\, q\, \mathcal{D}_c{}^b\, \bar q. \tag{2.34}$$

The simplest way to check that $\delta S_{\rm WZ} = 0$ is to pass to the $d = 3$ vector notations.[1] Then we have

$$\begin{aligned} \delta S_{\rm WZ} \sim & \int d^3x \;\det \mathcal{E}\; \epsilon^{IJK} \left(\varepsilon^m \mathcal{D}_I \bar\psi_m - \bar\varepsilon^m \mathcal{D}_I \psi_m\right) \mathcal{D}_J\, q\, \mathcal{D}_K\, \bar q \sim \\ \sim & \int d^3x \;\det \mathcal{E}\; \det \mathcal{E}^{-1} \epsilon^{IJK} \left(\varepsilon^m \partial_I \bar\psi_m - \bar\varepsilon^m \partial_I \psi_m\right) \partial_J\, q\, \partial_K\, \bar q \sim \\ \sim & \int d^3x \;\partial_I \left[\epsilon^{IJK} \left(\varepsilon^m \bar\psi_m - \bar\varepsilon^m \psi_m\right) \partial_J\, q\, \partial_K\, \bar q\right] = 0. \end{aligned} \tag{2.35}$$

Thus, the action $S_{\rm WZ}$ (2.33) is invariant under S supersymmetry and our Ansatz for the membrane action extended to be

$$S = (1+\alpha)\, S_0 - S_1 - \alpha S_2 + \beta S_{\rm WZ}. \tag{2.36}$$

Let us stress, that after imposing broken supersymmetry, our component action (2.36) is fixed up to two constants α and β. No other terms or structures are admissible! Funny enough, the role of the unbroken supersymmetry is just to fix these constants.

2.4.2 Unbroken supersymmetry

To maintain the unbroken supersymmetry, firstly, one has to find the transformation properties of the components. Using the transformations of the super-space coordinates (A.6)

$$\delta\theta^a = \epsilon^a,\;\; \delta\bar\theta^a = \bar\epsilon^a, \quad \delta x^{ab} = {\rm i}(\epsilon^{(a}\bar\theta^{b)} + \bar\epsilon^{(a}\theta^{b)}),$$

one may easily find the transformations of the needed ingredients (we will explicitly present only ϵ-part of the transformations):

$$\begin{aligned} \delta\psi_a &= -\epsilon^b\, (D_b \boldsymbol{\psi}_a)|_{\theta=0} = \epsilon^b \psi^m \bar\lambda_b^n \partial_{mn}\psi_a, \\ \delta\mathcal{D}_{ab}\psi_c &= -\epsilon^d\, (D_d \nabla_{ab}\boldsymbol{\psi}_c)|_{\theta=0} = 2\epsilon^d \mathcal{D}_{ab}\psi^m \bar\lambda_d^n \mathcal{D}_{mn}\psi_b + \epsilon^d \psi^m \bar\lambda_d^n \partial_{mn}\mathcal{D}_{ab}\psi_c, \\ \delta\mathcal{D}_{ab} q &= -\epsilon^d\, (D_d \nabla_{ab} q)|_{\theta=0} = 2\epsilon^d \mathcal{D}_{ab}\psi^m \bar\lambda_d^n \mathcal{D}_{mn} q + 2{\rm i}\epsilon^d \mathcal{D}_{ab}\psi_d + \epsilon^d \psi^m \bar\lambda_d^n \partial_{mn}\mathcal{D}_{ab} q, \end{aligned} \tag{2.37}$$

and, as a consequence,

$$\delta \det \mathcal{E} = \partial_{mn}\left[\epsilon^d \psi^m \bar\lambda_d{}^n \det \mathcal{E}\right] - 2\epsilon^d \bar\lambda_d{}^n \mathcal{D}_{mn}\psi^n \det \mathcal{E}. \tag{2.38}$$

[1] Our conventions to pass to/from vector indices are summarized in the appendix A, (A.21).

To fix the parameter α one may consider just the kinetic terms in the action (2.36)

$$S_{\rm kin} = \int d^3x \left[-{\rm i}\,(\alpha+1)\left(\psi^a \partial_{ab}\bar\psi^b + \bar\psi^a \partial_{ab}\psi^b\right) + \partial_{ab} q \partial^{ab}\bar q \right], \tag{2.39}$$

which has to be invariant under linearized transformations (2.37)

$$\delta\bar\psi_a = -{\rm i}\epsilon^b \bar\lambda_{ba} \simeq -\epsilon^b \partial_{ba}\bar q, \quad \delta\partial_{ab} q = 2{\rm i}\epsilon^d \partial_{ab}\psi_d. \tag{2.40}$$

Varying the integrand in (2.39) and integrating by parts, we will get

$$\delta S_{\rm kin} = \int d^3x \left[2{\rm i}(\alpha+1)\epsilon^c \psi^a \partial_{ab}\partial_c{}^b \bar q - 2{\rm i}\epsilon^d \psi_d \Box \bar q \right] = \int d^3x \left[{\rm i}(\alpha+1)\epsilon^d \psi_d \Box\bar q - 2{\rm i}\epsilon^d\psi_d \Box \bar q\right]. \tag{2.41}$$

Therefore, we have to fix

$$\alpha = 1. \tag{2.42}$$

Unfortunately, the fixation of the last parameter β is more involved. Using the transformation properties (2.37) one may find

$$\delta\mathcal{F} = 2\left(\epsilon^c \bar\lambda_c^n \mathcal{D}_{ab}\psi^m \mathcal{D}_{nm} q + {\rm i}\epsilon^c \mathcal{D}_{ab}\psi_c\right)\frac{\partial \mathcal{F}}{\partial \mathcal{D}_{ab} q} + 2\epsilon^c \bar\lambda_c^n \mathcal{D}_{ab}\psi^m \mathcal{D}_{mn}\bar q \frac{\partial\mathcal{F}}{\partial \mathcal{D}_{ab}\bar q} + \\ +\epsilon^c\bar\lambda_c^n \psi^m \partial_{mn}\mathcal{F}, \tag{2.43}$$

where

$$\mathcal{F} \equiv \sqrt{(1-\mathcal{D}_{ab}q\mathcal{D}^{ab}\bar q)^2 - (\mathcal{D}_{ab}q\mathcal{D}^{ab}q)(\mathcal{D}_{cd}\bar q\mathcal{D}^{cd}\bar q)}. \tag{2.44}$$

To avoid the appearance of the square roots, it is proved to be more convenient to use the following equalities

$$\frac{\partial\mathcal{F}}{\partial \mathcal{D}_{ab}q} = -{\rm i}\frac{\bar\lambda^{ab} + \frac12 \bar\lambda^2 \lambda^{ab}}{1-\frac14 \lambda^2\bar\lambda^2}, \quad \frac{\partial\mathcal{F}}{\partial \mathcal{D}_{ab}\bar q} = {\rm i}\frac{\lambda^{ab} + \frac12 \lambda^2 \bar\lambda^{ab}}{1-\frac14 \lambda^2\bar\lambda^2}. \tag{2.45}$$

After some straightforward calculations we get

$$\delta\left[-\det\mathcal{E}\,(1+\mathcal{F})\right] = 2{\rm i}\epsilon^c \det\mathcal{E}\left(\mathcal{D}_{ab}\psi_c\mathcal{D}^{ab}\bar q - 2\mathcal{D}_{am}\psi^m \mathcal{D}_c^a \bar q\right) - 2\epsilon^c\bar\lambda_{cm}\mathcal{D}_{ab}\psi^m\mathcal{D}^{ad}q\mathcal{D}_d^b\bar q\det\mathcal{E}. \tag{2.46}$$

Similarly, one may find the variation of the integrand of the action $S_{\rm WZ}$ (up to surface terms disappearing after integration over d^3x)

$$\delta\mathcal{L}_{\rm WZ} = -2\beta\det\mathcal{E}\left[\left(\psi^k\mathcal{D}_{ab}\bar\psi_k - \bar\psi^k \mathcal{D}_{ab}\psi_k\right)\epsilon^c \mathcal{D}^{ad}\psi_c \mathcal{D}_d^b \bar q - \epsilon^c\bar\lambda_{cm}\mathcal{D}_{ab}\psi^m\mathcal{D}^{ad}q\mathcal{D}_d^b\bar q\right]. \tag{2.47}$$

Now, it is a matter of quite lengthly, but again straightforward calculations, to check that the sum of variations (2.46) and (2.47) is a surface term if

$$\beta = 1. \tag{2.48}$$

Thus, we conclude that the action of the supermembrane in $D = 5$, which is invariant with respect to unbroken and broken supersymmetries, has the form

$$S = 2\int d^3x - \int d^3x \det\mathcal{E}\left(1+\sqrt{(1-\mathcal{D}_{ab}q\mathcal{D}^{ab}\bar q)^2 - (\mathcal{D}_{ab}q\mathcal{D}^{ab}q)(\mathcal{D}_{cd}\bar q\mathcal{D}^{cd}\bar q)}\right) + \\ +{\rm i}\int d^3x \,\det\mathcal{E}\,\left(\psi^m \mathcal{D}_{ab}\bar\psi_m - \bar\psi^m\mathcal{D}_{ab}\psi_m\right)\mathcal{D}^{ac}\, q\, \mathcal{D}_c{}^b\,\bar q. \tag{2.49}$$

3 Dualization of the scalars: vector and double vector supermultiplets

Due to the duality between scalar field, entering the action with the space-time derivatives only, and gauge field strength in $d = 3$, the actions for the vector (one scalar dualized) and the double vector (both scalars dualized) supermultiplets can be easily obtained within the coset approach. Before performing such dualizations, let us firstly rewrite our action (2.49) in the vector notations. If we introduce the quantity

$$\mathcal{G}_{ab} = \frac{1}{\sqrt{2}} \left(\psi^m \mathcal{D}_{ab} \bar{\psi}_m - \bar{\psi}^m \mathcal{D}_{ab} \psi_m\right), \tag{3.1}$$

then only vector indices show up in the action. Passing to the vector notation, we will get

$$S = 2\int d^3x - \int d^3x \det \mathcal{E} \left(1 + \sqrt{(1 - \mathcal{D}_I q \mathcal{D}_I \bar{q})^2 - (\mathcal{D}_I q \mathcal{D}_I q)(\mathcal{D}_J \bar{q} \mathcal{D}_J \bar{q})}\right) + \\ + \mathrm{i} \int d^3x \ \det \mathcal{E}\, \epsilon^{IJK} \mathcal{G}_I \mathcal{D}_J \, q \, \mathcal{D}_K \, \bar{q}, \tag{3.2}$$

where

$$\mathcal{D}_I \equiv \left(\mathcal{E}^{-1}\right)_I{}^J \partial_J, \quad \mathcal{E}_I{}^J = \delta_I^J - \frac{1}{\sqrt{2}} \left(\sigma^J\right)_{ab} \left(\psi^a \partial_I \bar{\psi}^b + \bar{\psi}^a \partial_I \psi^b\right). \tag{3.3}$$

3.1 Vector supermultiplet

The standard $N = 2, d = 3$ supermultiplet includes one scalar and one gauge fields (entering the action through the field strength) among the physical bosonic components. Thus, we have to dualize one of the scalar fields in the action (3.2). To perform dualization, firstly, one has to pass to the real bosonic fields $\{u, v\}$

$$q = \frac{1}{2}(u + iv), \quad \bar{q} = \frac{1}{2}(u - iv). \tag{3.4}$$

In terms of newly defined scalars, the action (3.2) reads

$$S = 2\int d^3x - \int d^3x \det \mathcal{E} \left[1 + \sqrt{\left(1 - \frac{1}{2}\mathcal{D}_I u \mathcal{D}_I u\right)\left(1 - \frac{1}{2}\mathcal{D}_J v \mathcal{D}_J v\right) - \frac{1}{4}\left(\mathcal{D}_I u \mathcal{D}_I v\right)^2}\right] + \\ + \frac{1}{2} \int d^3x \ \det \mathcal{E}\, \epsilon^{IJK} \mathcal{G}_I \mathcal{D}_J \, u \, \mathcal{D}_K \, v, \tag{3.5}$$

The equation of motion for bosonic field v has the form

$$\partial_I \left(\det \mathcal{E}\, \left(\mathcal{E}^{-1}\right)^I_J \, V_J\right) = 0, \quad V_I = \widetilde{V}_I + \frac{1}{2}\epsilon_{IJK} G_J \mathcal{D}_K u, \tag{3.6}$$

where

$$\widetilde{V}_I = \frac{\left(1 - \frac{1}{2}\mathcal{D}u \cdot \mathcal{D}u\right) \mathcal{D}_I v + \frac{1}{2}\, \mathcal{D}u \cdot \mathcal{D}v \, \mathcal{D}_I u}{2\sqrt{\left(1 - \frac{1}{2}\mathcal{D}u \cdot \mathcal{D}u\right)\left(1 - \frac{1}{2}\mathcal{D}v \cdot \mathcal{D}v\right) - \frac{1}{4}\left(\mathcal{D}u \cdot \mathcal{D}v\right)^2}}. \tag{3.7}$$

Then, one may find that

$$\mathcal{D}_I v = \frac{2\widetilde{V}_I - \widetilde{V} \cdot \mathcal{D}u \, \mathcal{D}_I u}{\sqrt{1 - \frac{1}{2}\mathcal{D}u \cdot \mathcal{D}u + 2\widetilde{V} \cdot \widetilde{V} - \left(\widetilde{V} \cdot \mathcal{D}u\right)^2}}. \tag{3.8}$$

Now, performing the Rauth transformation over bosonic field v, we will finally get

$$\tilde{S} = 2\int d^3x - \int d^3x \det \mathcal{E} \left(1 + \sqrt{1 - \frac{1}{2}\mathcal{D}u \cdot \mathcal{D}u + 2\widetilde{V} \cdot \widetilde{V} - \left(\widetilde{V} \cdot \mathcal{D}u \right)^2} \right). \tag{3.9}$$

This is the action for $N = 2, d = 3$ vector supermultiplet which possesses additional, spontaneously broken $N = 2$ supersymmetry.

One should stress, that the real field strength is defined in (3.6), but the action has a much more simple structure being written in terms of $\widetilde{V}_I$.

3.2 Double vector supermultiplet

Finally, one may dualize both scalars in the action (3.2). As the first step, one has to find the equations of motion for the scalar fields

$$\partial_I \left(\det \mathcal{E} \left(\mathcal{E}^{-1} \right)_J^I V^J \right) = 0, \quad \partial_I \left(\det \mathcal{E} \left(\mathcal{E}^{-1} \right)_J^I \overline{V}^J \right) = 0, \tag{3.10}$$

where

$$V_I = \widetilde{V}_I - \mathrm{i}\epsilon_{IJK} G_J \mathcal{D}_K \bar{q}, \quad \widetilde{V}_I = \frac{(1 - \mathcal{D}q \cdot \mathcal{D}\bar{q})\, \mathcal{D}_I \bar{q} + (\mathcal{D}\bar{q} \cdot \mathcal{D}\bar{q})\, \mathcal{D}_I q}{\sqrt{(1 - \mathcal{D}q \cdot \mathcal{D}\bar{q})^2 - (\mathcal{D}q \cdot \mathcal{D}q)(\mathcal{D}\bar{q} \cdot \mathcal{D}\bar{q})}}. \tag{3.11}$$

After a standard machinery with the Rauth transformations we will finally get the action

$$\widehat{S} = 2\int d^3x - \int d^3x \det \mathcal{E} \left[1 + \sqrt{\left(1 + \widetilde{V} \cdot \overline{\widetilde{V}} \right)^2 - \widetilde{V}^2\, \overline{\widetilde{V}}^2} - \mathrm{i}\epsilon_{IJK}\, G_I\, \widetilde{V}_J \overline{\widetilde{V}}_K \right]. \tag{3.12}$$

The bosonic sector of this action coincides with that constructed in [32]. Again, the simplest form of the action is achieved with the help $\widetilde{V}_I$ variables which are related with field strengths as in (3.10), (3.11).

4 Conclusion

In this paper, using a remarkable connection between partial breaking of global supersymmetry, coset approach, which realized the specific pattern of supersymmetry breaking, and the Nambu-Goto actions for the extended object, we have constructed the on-shell component action for $N = 1, D = 5$ supermembrane and for its dual cousins. Of course, such an action can be obtained by dimensional reduction from the superspace action of the 3-brane in $D = 6$ (see e.g., [9–12]) or from the action of ref. [7]. Nevertheless, if we pay more attention to the spontaneously broken supersymmetry and, thus, use the corresponding covariant derivatives, together with the proper choice of the components, the resulting action can be drastically simplified. So, the implications of our results are threefold:

- we demonstrated that the coset approach can be used far beyond the construction of the superfield equations of motion if we are interested in the component actions,
- we showed that there is a rather specific choice of the superfields and their components which drastically simplifies the component action,

- we argued that the broken supersymmetry fixed the on-shell component action up to some constants, while the role of the unbroken supersymmetry is just to fix these constants.

The application of our approach is not limited to the cases of P-branes only. Different types of D-branes could be also considered in a similar manner. However, once we are dealing with the field strengths, which never show up as the coordinates of the coset space, the proper choice of the components becomes very important. In particular, the Born-Infeld-Nambu-Goto action (3.5), we constructed by the dualization of one scalar field, has a nice, compact form in terms of the "covariant" field strength $\widetilde{V}_I$ which is related with the "genuine" field strength, obeying the Bianchi identity, in a rather complicated way (3.6). The same is also true for the Born-Infeld type action (3.12). In order to clarify the nature of these variables, one has to consider the corresponding patterns of the supersymmetry breaking (with one, or without central charges in the $N = 4, d = 3$ Poincaré superalgebra (A.2)) independently. In this respect, the detailed analysis of $N = 2 \to N = 1$ supersymmetry breaking in $d = 4$ seems to be much more interesting, being a preliminary step to the construction of $N = 4$ Born-Infeld action [22, 23, 33] and/or to the action describing partial breaking of $N = 1, D = 10$ supersymmetry with the hypermultiplet as the Goldstone superfield.

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A Superalgebra, coset space, transformations and Cartan forms

In this appendix we collected some formulas describing the nonlinear realization of $N = 1, D = 5$ Poincaré group in its coset over $d = 3$ Lorentz group SO(1,2).

In $d = 3$ notation the $N = 1, d = 5$ Poincaré superalgebra contains the following set of generators:

$$\text{N=4, d=3 SUSY} \quad \propto \quad \left\{P_{ab}, Q_a, \overline{Q}_a, S_a, \overline{S}_a, Z, \overline{Z}, M_{ab}, K_{ab}, \overline{K}_{ab}, J\right\}, \tag{A.1}$$

$a, b = 1, 2$ being the $d = 3$ $\mathrm{SL}(2, R)$ spinor indices.[2] Here, P_{ab}, Z and $\overline{Z}$ are $D = 5$ translation generators, $Q_a, \overline{Q}_a$ and $S_a, \overline{S}_a$ are the generators of super-translations, the generators M_{ab} form $d = 3$ Lorentz algebra $so(1,2)$, the generators K_{ab} and $\overline{K}_{ab}$ belong to the coset $\mathrm{SO}(1,4)/\mathrm{SO}(1,2)\times\mathrm{U}(1)$, while J span $u(1)$. The basic commutation relations read

$$[M_{ab}, M_{cd}] = \epsilon_{ad}M_{bc} + \epsilon_{ac}M_{bd} + \epsilon_{bc}M_{ad} + \epsilon_{bd}M_{ac} \equiv (M)_{ab,cd}\,,$$
$$[M_{ab}, P_{cd}] = (P)_{ab,cd}\,, \; [M_{ab}, K_{cd}] = (K)_{ab,cd}\,, \; \left[M_{ab}, \overline{K}_{cd}\right] = \left(\overline{K}\right)_{ab,cd}\,,$$

[2] The indices are raised and lowered as follows: $V^a = \epsilon^{ab}V_b$, $V_b = \epsilon_{bc}V^c$, $\quad \epsilon_{ab}\epsilon^{bc} = \delta_a^c$.

$$
\begin{aligned}
\left[K_{ab},\overline{K}_{cd}\right] &= \frac{1}{2}\left(M\right)_{ab,cd}+2\left(\epsilon_{ac}\epsilon_{bd}+\epsilon_{bc}\epsilon_{ad}\right)J,\\
\left[K_{ab},P_{cd}\right] &= -\left(\epsilon_{ac}\epsilon_{bd}+\epsilon_{bc}\epsilon_{ad}\right)Z,\ \left[\overline{K}_{ab},P_{cd}\right]=\left(\epsilon_{ac}\epsilon_{bd}+\epsilon_{bc}\epsilon_{ad}\right)\overline{Z},\\
\left[K_{ab},\overline{Z}\right] &= -2P_{ab},\ \left[\overline{K}_{ab},Z\right]=2P_{ab},\ \left[M_{ab},Q_c\right]=\epsilon_{ac}Q_b+\epsilon_{bc}Q_a\equiv\left(Q\right)_{ab,c},\\
\left[M_{ab},\overline{Q}_c\right] &= \left(\overline{Q}\right)_{ab,c},\ \left[M_{ab},S_c\right]=\left(S\right)_{ab,c},\ \left[M_{ab},\overline{S}_c\right]=\left(\overline{S}\right)_{ab,c},\\
\left[\overline{K}_{ab},Q_c\right] &= -\left(\overline{S}\right)_{ab,c},\ \left[K_{ab},\overline{Q}_c\right]=\left(S\right)_{ab,c},\ \left[\overline{K}_{ab},S_c\right]=\left(\overline{Q}\right)_{ab,c},\ \left[K_{ab},\overline{S}_c\right]=-\left(Q\right)_{ab,c},\\
\left[J,Q_a\right] &= -\frac{1}{2}Q_a,\ \left[J,\overline{Q}_a\right]=\frac{1}{2}\overline{Q}_a,\ \left[J,S_a\right]=-\frac{1}{2}S_a,\ \left[J,\overline{S}_a\right]=\frac{1}{2}\overline{S}_a,\\
\left[J,K_{ab}\right] &= -K_{ab},\ \left[J,\overline{K}_{ab}\right]=\overline{K}_{ab},\ \left[J,Z\right]=-Z,\ \left[J,\overline{Z}\right]=\overline{Z},\\
\left\{Q_a,\overline{Q}_b\right\} &= 2P_{ab},\ \left\{S_a,\overline{S}_b\right\}=2P_{ab},\ \left\{Q_a,S_b\right\}=2\epsilon_{ab}Z,\ \left\{\overline{Q}_a,\overline{S}_b\right\}=2\epsilon_{ab}\overline{Z}.
\end{aligned}\tag{A.2}
$$

Note, that the generators obey the following conjugation rules:

$$
\begin{aligned}
&(P_{ab})^\dagger = P_{ab},\ (K_{ab})^\dagger=\overline{K}_{ab},\ (M_{ab})^\dagger=-M_{ab},\ J^\dagger=J,\ Z^\dagger=\overline{Z},\\
&(Q_a)^\dagger=\overline{Q}_a,\ (S_a)^\dagger=\overline{S}_a.
\end{aligned}\tag{A.3}
$$

We define the coset element as follows

$$
g=e^{\mathrm{i}x^{ab}P_{ab}}e^{\theta^aQ_a+\bar\theta^a\overline{Q}_a}e^{\mathrm{i}(\mathbf{q}Z+\bar{\mathbf{q}}\overline{Z})}e^{\boldsymbol{\psi}^aS_a+\bar{\boldsymbol{\psi}}^a\overline{S}_a}e^{\mathrm{i}(\boldsymbol{\Lambda}^{ab}K_{ab}+\overline{\boldsymbol{\Lambda}}^{ab}\overline{K}_{ab})}. \tag{A.4}
$$

Here, $\left\{x^{ab},\theta^a,\bar\theta^a\right\}$ are $N=2,d=3$ superspace coordinates, while the remaining coset parameters are Goldstone superfields, $\boldsymbol{\psi}^a\equiv\boldsymbol{\psi}^a(x,\theta,\bar\theta)$, $\bar{\boldsymbol{\psi}}^a\equiv\bar{\boldsymbol{\psi}}^a(x,\theta,\bar\theta)$, $\mathbf{q}\equiv\mathbf{q}(x,\theta,\bar\theta)$, $\bar{\mathbf{q}}\equiv\bar{\mathbf{q}}(x,\theta,\bar\theta)$, $\boldsymbol{\Lambda}^{ab}\equiv\boldsymbol{\Lambda}^{ab}(x,\theta,\bar\theta)$, $\overline{\boldsymbol{\Lambda}}^{ab}\equiv\overline{\boldsymbol{\Lambda}}^{ab}(x,\theta,\bar\theta)$. These $N=2$ superfields obey the following conjugation rules

$$
\left(x^{ab}\right)^\dagger=x^{ab},\ (\theta^a)^\dagger=\bar\theta^a,\quad \mathbf{q}^\dagger=\bar{\mathbf{q}},\ (\boldsymbol{\psi}^a)^\dagger=\bar{\boldsymbol{\psi}}^a,\ \left(\boldsymbol{\Lambda}^{ab}\right)^\dagger=\overline{\boldsymbol{\Lambda}}^{ab}. \tag{A.5}
$$

The transformation properties of the coordinates and superfields with respect to all symmetries can be found by acting from the left on the coset element g (A.4) by the different elements of $N=1, D=5$ Poincaré supergroup. In what follows, we will need the explicit form only for the broken $(S,\overline{S})$ and unbroken $(Q,\overline{Q})$ supersymmetries, and $(K,\overline{K})$ automorphism transformations which read:

- Unbroken (Q) supersymmetry $(g_0=\exp\left(\epsilon^aQ_a+\bar\epsilon^a\overline{Q}_a\right))$

$$
\delta x^{ab}=\mathrm{i}\left(\epsilon^{(a}\bar\theta^{b)}+\bar\epsilon^{(a}\theta^{b)}\right),\quad \delta\theta^a=\epsilon^a\ ,\delta\bar\theta^a=\bar\epsilon^a\ . \tag{A.6}
$$

- Broken (S) supersymmetry $(g_0=\exp\ \left(\varepsilon^aS_a+\bar\varepsilon^a\overline{S}_a\right))$

$$
\delta x^{ab}=\mathrm{i}\left(\varepsilon^{(a}\bar{\boldsymbol{\psi}}^{b)}+\bar\varepsilon^{(a}\boldsymbol{\psi}^{b)}\right),\quad \delta\mathbf{q}=2\mathrm{i}\varepsilon_a\theta^a,\ \delta\bar{\mathbf{q}}=2\mathrm{i}\bar\varepsilon_a\bar\theta^a,\quad \delta\boldsymbol{\psi}^a=\varepsilon^a,\ \delta\bar{\boldsymbol{\psi}}^a=\bar\varepsilon^a\ . \tag{A.7}
$$

- Automorphism $(K,\overline{K})$ transformations $(g_0=\exp\,\mathrm{i}\left(a^{ab}K_{ab}+\bar a^{ab}\overline{K}_{ab}\right))$

$$
\begin{aligned}
&\delta x^{ab}=-2\mathrm{i}\left(a^{ab}\mathbf{q}-\bar a^{ab}\bar{\mathbf{q}}\right)-2\theta^c\boldsymbol{\psi}_c\bar a^{ab}+2\bar\theta^c\bar{\boldsymbol{\psi}}_c a^{ab},\quad \delta\theta^a=-2\mathrm{i}a^{ab}\bar{\boldsymbol{\psi}}_b,\ \delta\bar\theta^a=2\mathrm{i}\bar a^{ab}\boldsymbol{\psi}_b,\\
&\delta\mathbf{q}=-2\mathrm{i}a^{ab}x_{ab}-2a^{ab}\left(\theta_a\bar\theta_b-\boldsymbol{\psi}_a\bar{\boldsymbol{\psi}}_b\right),\qquad \delta\boldsymbol{\psi}^a=2\mathrm{i}a^{ab}\bar\theta_b,\\
&\delta\bar{\mathbf{q}}=2\mathrm{i}\bar a^{ab}x_{ab}-2\bar a^{ab}\left(\theta_a\bar\theta_b-\boldsymbol{\psi}_a\bar{\boldsymbol{\psi}}_b\right),\qquad \delta\bar{\boldsymbol{\psi}}^a=-2\mathrm{i}\bar a^{ab}\theta_b.
\end{aligned}\tag{A.8}
$$

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As the next step of the coset formalism, one construct the Cartan forms

$$g^{-1}dg = \Omega_P + \Omega_Q + \overline{\Omega}_Q + \Omega_Z + \overline{\Omega}_Z + \Omega_S + \overline{\Omega}_S + \dots\ . \tag{A.9}$$

In what follows we will need only the forms $\Omega_P, \Omega_Q, \Omega_Z$ and Ω_S which explicitly read

$$\begin{aligned}
\Omega_P &= \left\{ \left(\cosh 2\sqrt{\boldsymbol{Y}}\right)_{ab}^{\ cd} \triangle x^{ab} - \mathrm{i}\left(\overline{\boldsymbol{\Lambda}}^{ab}\triangle\mathbf{q} - \boldsymbol{\Lambda}^{ab}\triangle\bar{\mathbf{q}}\right)\left(\frac{\sinh 2\sqrt{\boldsymbol{Y}}}{\sqrt{\boldsymbol{Y}}}\right)_{ab}^{\ cd} \right\} P_{cd}, \\
\Omega_Q &= \left\{ d\theta^b \left(\cos 2\sqrt{\overline{\boldsymbol{T}}}\right)_b^{\ c} - \mathrm{i}\, d\bar{\boldsymbol{\psi}}^b\, \boldsymbol{\Lambda}_b{}^a \left(\frac{\sin 2\sqrt{\overline{\boldsymbol{T}}}}{\sqrt{\overline{\boldsymbol{T}}}}\right)_a^{\ c} \right\} Q_c, \\
\Omega_Z &= \left\{ \triangle\mathbf{q} + \left(\overline{\boldsymbol{\Lambda}}^{ab}\triangle\mathbf{q} - \boldsymbol{\Lambda}^{ab}\triangle\bar{\mathbf{q}}\right)\left(\frac{\cosh 2\sqrt{\boldsymbol{Y}} - 1}{\boldsymbol{Y}}\right)_{ab}^{\ cd} \boldsymbol{\Lambda}_{cd} + \mathrm{i}\triangle^{ab}\left(\frac{\sinh 2\sqrt{\boldsymbol{Y}}}{\sqrt{\boldsymbol{Y}}}\right)_{ab}^{\ cd} \boldsymbol{\Lambda}_{cd} \right\} Z, \\
\Omega_S &= \left\{ d\boldsymbol{\psi}^b \left(\cos 2\sqrt{\overline{\boldsymbol{T}}}\right)_b^{\ c} + \mathrm{i}\, d\bar{\theta}^b\, \boldsymbol{\Lambda}_b{}^a \left(\frac{\sin 2\sqrt{\overline{\boldsymbol{T}}}}{\sqrt{\overline{\boldsymbol{T}}}}\right)_a^{\ c} \right\} S_c,
\end{aligned} \tag{A.10}$$

$$\triangle x^{ab} = dx^{ab} - \mathrm{i}\left(\theta^{(a} d\bar{\theta}^{b)} + \bar{\theta}^{(a} d\theta^{b)} + \boldsymbol{\psi}^{(a} d\bar{\boldsymbol{\psi}}^{b)} + \bar{\boldsymbol{\psi}}^{(a} d\boldsymbol{\psi}^{b)}\right), \tag{A.11}$$

$$\triangle\mathbf{q} = d\mathbf{q} - 2\mathrm{i}\boldsymbol{\psi}_a d\theta^a,\ \triangle\bar{\mathbf{q}} = d\bar{\mathbf{q}} - 2\mathrm{i}\bar{\boldsymbol{\psi}}_a d\bar{\theta}^a. \tag{A.12}$$

Here, we defined matrix-valued functions $\boldsymbol{Y}_{ab}{}^{cd}, \boldsymbol{T}_a{}^b$ and $\overline{\boldsymbol{T}}_a{}^b$ as

$$\boldsymbol{Y}_{ab}{}^{cd} = \boldsymbol{\Lambda}_{ab}\overline{\boldsymbol{\Lambda}}^{cd} + \overline{\boldsymbol{\Lambda}}_{ab}\boldsymbol{\Lambda}^{cd}, \quad \boldsymbol{T}_a{}^b = \boldsymbol{\Lambda}_a{}^c\overline{\boldsymbol{\Lambda}}_c{}^b,\ \overline{\boldsymbol{T}}_a{}^b = \overline{\boldsymbol{\Lambda}}_a{}^c\boldsymbol{\Lambda}_c{}^b. \tag{A.13}$$

Note, that all these Cartan forms transform homogeneously under all symmetries.

Having at hands the Cartan forms, one may construct the "semi-covariant" (covariant with respect to $d = 3$ Lorentz, unbroken and broken supersymmetries only) derivatives as

$$\triangle x^{ab}\nabla_{ab} + d\theta^a\nabla_a + d\bar{\theta}^a\overline{\nabla}_a = dx^{ab}\frac{\partial}{\partial x^{ab}} + d\theta^a\frac{\partial}{\partial\theta^a} + d\bar{\theta}^a\frac{\partial}{\partial\bar{\theta}^a}. \tag{A.14}$$

Explicitly, they read

$$\begin{aligned}
\nabla_{ab} &= (E^{-1})_{ab}{}^{cd}\partial_{cd}, \\
\nabla_a &= D_a - \mathrm{i}\left(\boldsymbol{\psi}^b D_a\bar{\boldsymbol{\psi}}^c + \bar{\boldsymbol{\psi}}^b D_a\boldsymbol{\psi}^c\right)\nabla_{bc} = D_a - \mathrm{i}\left(\boldsymbol{\psi}^b\nabla_a\bar{\boldsymbol{\psi}}^c + \bar{\boldsymbol{\psi}}^b\nabla_a\boldsymbol{\psi}^c\right)\partial_{bc},
\end{aligned} \tag{A.15}$$

where

$$D_a = \frac{\partial}{\partial\theta^a} - \mathrm{i}\,\bar{\theta}^b\,\partial_{ab},\ \overline{D}_a = \frac{\partial}{\partial\bar{\theta}^a} - \mathrm{i}\,\theta^b\,\partial_{ab}, \quad \left\{D_a, \overline{D}_b\right\} = -2\mathrm{i}\partial_{ab}, \tag{A.16}$$

$$E_{ab}{}^{cd} = \delta_a^{(c}\delta_b^{d)} - \mathrm{i}\left(\boldsymbol{\psi}^{(c}\partial_{ab}\bar{\boldsymbol{\psi}}^{d)} + \bar{\boldsymbol{\psi}}^{(c}\partial_{ab}\boldsymbol{\psi}^{d)}\right), \tag{A.17}$$

$$(E^{-1})_{ab}{}^{cd} = \delta_a^{(c}\delta_b^{d)} + \mathrm{i}\left(\boldsymbol{\psi}^{(c}\nabla_{ab}\bar{\boldsymbol{\psi}}^{d)} + \bar{\boldsymbol{\psi}}^{(c}\nabla_{ab}\boldsymbol{\psi}^{d)}\right). \tag{A.18}$$

These derivatives obey the following algebra:

$$\begin{aligned}
\{\nabla_a, \nabla_b\} &= -2\mathrm{i}\left(\nabla_a\boldsymbol{\psi}^c\nabla_b\bar{\boldsymbol{\psi}}^d + \nabla_a\bar{\boldsymbol{\psi}}^c\nabla_b\boldsymbol{\psi}^d\right)\nabla_{cd}, \\
\left\{\nabla_a, \overline{\nabla}_b\right\} &= -2\mathrm{i}\,\nabla_{ab} - 2\mathrm{i}\left(\nabla_a\boldsymbol{\psi}^c\overline{\nabla}_b\bar{\boldsymbol{\psi}}^d + \nabla_a\bar{\boldsymbol{\psi}}^c\overline{\nabla}_b\boldsymbol{\psi}^d\right)\nabla_{cd},
\end{aligned}$$

$$
\begin{aligned}
[\nabla_{ab}, \nabla_c] &= -2\mathrm{i} \left(\nabla_{ab}\psi^d \nabla_c \bar{\psi}^f + \nabla_{ab}\bar{\psi}^d \nabla_c \psi^f \right) \nabla_{df}, \\
[\nabla_{ab}, \nabla_{cd}] &= 2\mathrm{i} \left(\nabla_{ab}\psi^m \nabla_{cd} \bar{\psi}^n - \nabla_{cd}\psi^m \nabla_{ab}\bar{\psi}^n \right) \nabla_{mn}.
\end{aligned} \tag{A.19}
$$

To complete this rather technical appendix, we will also define the $d = 3$ volume form in a standard manner as

$$
d^3x \equiv \epsilon_{IJK} dx^I \wedge dx^J \wedge dx^K \quad \Rightarrow \quad dx^I \wedge dx^J \wedge dx^K = \frac{1}{6}\epsilon^{IJK} d^3x. \tag{A.20}
$$

The translation to the vectors is defined as

$$
V^I \equiv \frac{\mathrm{i}}{\sqrt{2}} \left(\sigma^I\right)_a{}^b V_b{}^a \quad \Rightarrow \quad V_a{}^b = -\frac{\mathrm{i}}{\sqrt{2}} V^I \left(\sigma^I\right)_a{}^b, \qquad V^{ab}V_{ab} = V^I V^I. \tag{A.21}
$$

Here we are using the standard set of σ^I matrices

$$
\sigma^I \sigma^J = \mathrm{i}\epsilon^{IJK}\sigma^K + \eta^{IJ}E, \quad \left(\sigma^I\right)_a{}^b \left(\sigma^I\right)_c{}^d = 2\delta_a{}^d\delta_c{}^b - \delta_a{}^b\delta_c{}^d, \tag{A.22}
$$

were ϵ^{IJK} obeys relations

$$
\epsilon^{IJK}\epsilon_{IMN} = \delta^J_M\delta^K_N - \delta^J_N\delta^K_M, \quad \epsilon^{IJK}\epsilon_{IJN} = 2\delta^K_N, \quad \epsilon^{IJK}\epsilon_{IJK} = 6. \tag{A.23}
$$

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