

# The Standard Model confronts CP violation in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$

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**ABSTRACT:** The recently measured direct CP asymmetries in the processes  $D^0 \rightarrow \pi^+\pi^-$  and  $D^0 \rightarrow K^+K^-$  show a significant deviation from the naive Standard Model expectation. Using a general parameterization of the decay amplitudes, we show that the measured branching ratios imply large SU(3) breaking and large violations of the naive  $1/N_c$  counting. Furthermore, rescattering constrains the  $I = 0$  amplitudes in the  $\pi\pi$  and  $KK$  channels. Combining all this information, we show that, with present errors, the observed asymmetries are marginally compatible with the Standard Model. Improving the experimental accuracy could lead to an indirect signal of new physics.

**KEYWORDS:** Rare Decays, Heavy Quark Physics, CP violation

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## 1 Introduction

The LHCb collaboration reported an interesting result based on  $0.62 \text{ fb}^{-1}$  of data at the Hadron Collider Physics Symposium 2011 [1]:

$$\Delta A_{\text{CP}} \equiv A_{\text{CP}}(K^+K^-) - A_{\text{CP}}(\pi^+\pi^-) = [-0.82 \pm 0.21 (\text{stat.}) \pm 0.11 (\text{sys.})] \%, \quad (1.1)$$

which deviates from zero at  $3.5\sigma$  level. Note that the effects from indirect CP violation cancel to a large extent in the sum, and a non-vanishing  $\Delta A_{\text{CP}}$  originates from the difference of the direct CP asymmetries, as explained below. In the above expression, the time-integrated CP asymmetry  $A_{\text{CP}}(f)$  may be written as follows due to the slow mixing of neutral  $D$  mesons [2]:

$$\begin{aligned} A_{\text{CP}}(f) &= \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow \bar{f})} \\ &\approx a_{\text{CP}}^{\text{dir}}(f) + a_{\text{CP}}^{\text{ind}} \int_0^\infty dt \frac{t}{\tau_{D^0}} D_f(t) = a_{\text{CP}}^{\text{dir}}(f) + \frac{\langle t \rangle_f}{\tau_{D^0}} a_{\text{CP}}^{\text{ind}}, \end{aligned} \quad (1.2)$$

Channel	$A_{CP}(\%)$	References
$D^0 \rightarrow K^+K^-$	$0.00 \pm 0.34 \pm 0.13$	[10]
$D^0 \rightarrow \pi^+\pi^-$	$-0.24 \pm 0.52 \pm 0.22$	[10]
$D^0 \rightarrow K^+K^-$	$-0.43 \pm 0.30 \pm 0.11$	[11]
$D^0 \rightarrow \pi^+\pi^-$	$0.43 \pm 0.52 \pm 0.12$	[11]
$D^0 \rightarrow K_S K_S$	$-23 \pm 19$	[12]
$D^0 \rightarrow \pi^0\pi^0$	$0 \pm 5$	[12]
$D^+ \rightarrow K^+K_S$	$-0.1 \pm 0.6$	[13–16]

**Table 1.** Experimental data on individual CP asymmetries in units of  $10^{-2}$ .

Channel	BR	References
$D^+ \rightarrow \pi^+\pi^0$	$(1.19 \pm 0.06) \times 10^{-3}$	[17–19]
$D^0 \rightarrow \pi^+\pi^-$	$(1.400 \pm 0.026) \times 10^{-3}$	[19–23]
$D^0 \rightarrow \pi^0\pi^0$	$(0.80 \pm 0.05) \times 10^{-3}$	[19, 24]
$D^+ \rightarrow K^+K_S$	$(2.83 \pm 0.16) \times 10^{-3}$	[17, 19, 25, 26]
$D^0 \rightarrow K^+K^-$	$(3.96 \pm 0.08) \times 10^{-3}$	[19–23, 27–31]
$D^0 \rightarrow K_S K_S$	$(0.173 \pm 0.029) \times 10^{-3}$	[26, 31–33]

**Table 2.** Experimental averages on BR’s from ref. [16].

where  $D_f(t)$  is the observed distribution of proper decay time and  $\tau_{D^0}$  is the lifetime of the neutral  $D$  mesons. The indirect CP-violation parameter is given in terms of the parameters  $x \equiv \Delta m_D/\Gamma_D$ ,  $y \equiv \Delta\Gamma_D/(2\Gamma_D)$ ,  $|q/p|$  and  $\phi \equiv \arg(q/p)$  [3]:

$$a_{CP}^{\text{ind}} = -A_\Gamma = -\frac{\eta_{CP}}{2} \left[ \left( \left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) y \cos \phi - \left( \left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) x \sin \phi \right], \quad (1.3)$$

where  $\eta_{CP} = +1$  is the CP parity of the final state considered here and  $\phi$  is the CP-violating phase. The HFAG average of the indirect CP asymmetry is  $A_\Gamma = (0.123 \pm 0.248)\%$  [4–7]. In addition, LHCb recently measured  $A_\Gamma = (-0.59 \pm 0.59 \pm 0.21)\%$  [8]. However, to exploit all available information, we use as input for the indirect CP asymmetry the result of a global fit to  $D$  mixing by the UTfit Collaboration,  $A_\Gamma = (0.12 \pm 0.12)\%$ . The difference of the two asymmetries is given by

$$\Delta A_{CP} = a_{CP}^{\text{dir}}(K^+K^-) - a_{CP}^{\text{dir}}(\pi^+\pi^-) + \frac{\Delta\langle t \rangle}{\tau_{D^0}} a_{CP}^{\text{ind}}, \quad (1.4)$$

where  $\Delta\langle t \rangle/\tau_{D^0} \equiv (\langle t \rangle_K - \langle t \rangle_\pi)/\tau_{D^0} = (9.83 \pm 0.22 \pm 0.19)\%$  at LHCb [1].

Very recently, the CDF collaboration reported an updated measurement of  $\Delta A_{CP}$  [9]:

$$\Delta A_{CP} = A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-0.62 \pm 0.21 \pm 0.10)\% \quad (1.5)$$

with  $\Delta\langle t \rangle/\tau_{D^0} = 0.26 \pm 0.01$ . The individual asymmetries are listed in table 1, while the relevant CP-averaged Branching Ratios (BR’s) are reported in table 2.

Combining the measurements in table 1 with the LHCb value in eq. (1.1), with the CDF one in eq. (1.5) and with  $A_\Gamma$  we obtain the following average for the CP asymmetries:

$$a_{\text{CP}}^{\text{dir}}(\pi^+\pi^-) = (0.45 \pm 0.26) \%, \quad a_{\text{CP}}^{\text{dir}}(K^+K^-) = (-0.21 \pm 0.24) \%, \quad (1.6)$$

$$\Delta a_{\text{CP}}^{\text{dir}} = a_{\text{CP}}^{\text{dir}}(K^+K^-) - a_{\text{CP}}^{\text{dir}}(\pi^+\pi^-) = (-0.66 \pm 0.16) \%.$$

U-spin would predict  $a_{\text{CP}}^{\text{dir}}(\pi^+\pi^-) = -a_{\text{CP}}^{\text{dir}}(K^+K^-)$ . We will comment on SU(3) breaking in the following.

For direct CP violation to occur, two terms with different weak and strong phases should contribute to the decay amplitude. For singly Cabibbo suppressed  $D$  decays such as  $D \rightarrow \pi\pi$  and  $D \rightarrow KK$ , the CP-violating part of the relevant weak Hamiltonian is numerically suppressed by the ratio  $r_{\text{CKM}} = \text{Im}(V_{cb}^*V_{ub})/(V_{cd}^*V_{ud}) \sim 6.4 \times 10^{-4}$ . Due to this suppression, the contribution of penguin operators is totally negligible. The possibility of direct CP violation then mainly rests on penguin contractions of current-current operators, which may be large due to Final State Interactions (FSI). Unfortunately, these long-distance effects are essentially uncalculable, making a prediction of  $a_{\text{CP}}^{\text{dir}}$  in these channels a formidable task. Previous efforts in this direction, both before and after the LHCb results, used either SU(3) [34–51] or (QCD) factorization [51–58] to predict  $a_{\text{CP}}^{\text{dir}}$ , or simply studied CP asymmetries as a function of the size of penguin matrix elements [49, 59]. We improve on previous analyses in several aspects. First, we do not assume SU(3) nor any kind of factorization, since SU(3) appears to be badly broken in the decays at hand and since factorization holds only in the  $m_c \rightarrow \infty$  limit, while for realistic values of  $m_c$  power corrections cannot be neglected (nor estimated). Second, we implement unitarity constraints in a consistent way, using the wealth of experimental data on  $\pi N$  scattering accumulated in the seventies [60–62], yielding information on  $\pi\pi \rightarrow \pi\pi, KK$  rescattering at energies close to the  $D$  mass scale. Third, we exploit the information coming from BR's to estimate the size of penguin contractions and other subleading contributions to the decay amplitude. Combining all information, we provide a detailed study of the compatibility of the Standard Model (SM) with the experimental data on CP violation. We conclude that, with present errors, the observed asymmetries are marginally compatible with the SM. Should the present central value be confirmed with smaller errors, it would require a factor of six (or larger) enhancement of the penguin amplitude with respect to all other topologies, well beyond our theoretical expectations. Thus, improving the experimental accuracy could lead to an indirect signal of new physics.

The paper is organized as follows. In section 2 we report the expression of the relevant decay amplitudes in terms of isospin reduced matrix elements and in terms of renormalization group invariant parameters, and give a dictionary between the two parametrizations. In section 3 we discuss the available information on rescattering and the way to implement this knowledge in  $D \rightarrow \pi\pi$  and  $D \rightarrow KK$  decays. In section 4 we discuss the implications of the measured BR's on the CP-conserving part of the amplitudes, and extrapolate this information to the CP-violating contributions. In section 5 we present our main results on the CP asymmetries, and discuss theoretical uncertainties. Finally, in section 6 we summarize our findings.

## 2 Isospin decomposition and parameterization of $D \rightarrow \pi\pi$ and $D \rightarrow KK$ decays

In this section, we write down the relevant decay amplitudes both in terms of isospin reduced matrix elements and in terms of renormalization group invariant (RGI) parameters, and discuss the relation between the two parameterizations. The isospin parameterization will prove useful to exploit the experimental information on final state interactions from  $\pi N$  scattering, while the RGI parameterization will allow us to give a dynamical interpretation to the results. Before dwelling into the analysis, we give a brief summary of the relevant literature.

Factorization approaches, such as the BSW model [52], have been used to calculate the decay amplitudes of  $D$  decays. The experimental data favor  $\xi = 1/N_c^{\text{eff}} \approx 0$  and demand significant FSI effects. Two-body hadronic  $D$  decays have also been analyzed in the diagrammatic approach with SU(3) flavor symmetry in refs. [38–47], while earlier studies can be found in refs. [34–37]. The global fits to experimental data suggest that the color-suppressed tree is comparable to the color-allowed tree in size with a large relative strong phase, the exchange amplitude is sizable with a large strong phase relative to the color-allowed tree, and significant SU(3) breaking effects are required in the exchange amplitude. It is expected that the large exchange contribution originates from FSI.

FSI effects on  $D$  decays have been considered in several ways: elastic and inelastic scatterings, resonance contributions, etc., for example, in refs. [40, 53–57, 63–71]. In refs. [53–57], Buccella et al. studied Cabibbo-allowed and Cabibbo-suppressed  $D$  decays based on a modified factorization approximation, in which the effective parameter  $\xi$  and annihilation and exchange contributions are fixed from the data, and rescattering effects are assumed to be dominated by resonant contributions. From global analyses of the data, they showed the significance of the annihilation and exchange contributions and large SU(3) violation, where the latter could be explained by the rescattering effects [56]. In ref. [70], Lai and Yang considered elastic SU(3) rescattering (see also [72]) together with the QCDF approach for the short-distance annihilation amplitudes. Moreover, in refs. [63–67], coupled-channel analyses of the  $\pi\pi$  and  $KK$  scatterings were considered for the FSI's in the  $D \rightarrow \pi\pi$  and  $KK$  decays.

In ref. [73], Golden and Grinstein pointed out that an enhancement of CP violation in  $D$  decays may occur due to an enhancement of the penguin-contraction contribution as in the case of the  $\Delta I = 1/2$  rule in kaon decays, where the  $\Delta I = 1/2$  contribution dominates over the  $\Delta I = 3/2$  one. One should note, however, that the  $D \rightarrow \pi\pi$  data show no enhancement of the  $\Delta I = 1/2$  over the  $\Delta I = 3/2$  amplitude. We will return to this point in detail below.

### 2.1 Isospin decomposition

The effective Hamiltonian for the Cabibbo-suppressed decays with  $\Delta C = 1$  and  $\Delta S = 0$  can be decomposed into  $\Delta I = 1/2$  and  $3/2$  components, where the  $\Delta I = 1/2$  component involves both the current-current and penguin operators, while the  $\Delta I = 3/2$  component involves only the current-current operator  $O_+ = [(\bar{d}_L \gamma_\mu c_L)(\bar{u}_L \gamma^\mu d_L) + (\bar{u}_L \gamma_\mu c_L)(\bar{d}_L \gamma^\mu d_L)]/2$ .

Namely, the  $\Delta I = 3/2$  contribution involves the CKM factor  $V_{cd}^* V_{ud}$ . Denoting the isospin reduced matrix elements of the CP-even (CP-odd) part of the weak Hamiltonian by  $\mathcal{A}(\mathcal{B})$ , and using the original KM phase choice in which  $V_{cd}^* V_{ud}$  is real, we write the decay amplitudes as follows:

$$\begin{aligned}
 A(D^+ \rightarrow \pi^+ \pi^0) &= \frac{\sqrt{3}}{2} \mathcal{A}_2^\pi, & (2.1) \\
 A(D^0 \rightarrow \pi^+ \pi^-) &= \frac{\mathcal{A}_2^\pi - \sqrt{2}(\mathcal{A}_0^\pi + ir_{\text{CKM}} \mathcal{B}_0^\pi)}{\sqrt{6}}, \\
 A(D^0 \rightarrow \pi^0 \pi^0) &= \frac{\sqrt{2} \mathcal{A}_2^\pi + \mathcal{A}_0^\pi + ir_{\text{CKM}} \mathcal{B}_0^\pi}{\sqrt{3}}, \\
 A(D^+ \rightarrow K^+ \bar{K}^0) &= \frac{\mathcal{A}_{13}^K}{2} + \mathcal{A}_{11}^K + ir_{\text{CKM}} \mathcal{B}_{11}^K, \\
 A(D^0 \rightarrow K^+ K^-) &= \frac{-\mathcal{A}_{13}^K + \mathcal{A}_{11}^K - \mathcal{A}_0^K + ir_{\text{CKM}} \mathcal{B}_{11}^K - ir_{\text{CKM}} \mathcal{B}_0^K}{2}, \\
 A(D^0 \rightarrow K^0 \bar{K}^0) &= \frac{-\mathcal{A}_{13}^K + \mathcal{A}_{11}^K + \mathcal{A}_0^K + ir_{\text{CKM}} \mathcal{B}_{11}^K + ir_{\text{CKM}} \mathcal{B}_0^K}{2}.
 \end{aligned}$$

The CP-conjugate amplitudes are obtained flipping the sign of the  $\mathcal{B}$  terms in eq. (2.1).

## 2.2 Renormalization-group invariant parameterization

In ref. [74], a general and complete parameterization of two-body non-leptonic  $B$ -decay amplitudes was introduced based on the OPE in the weak effective Hamiltonian and on Wick contractions. The parameterization is independent of renormalization scale and scheme, and allows us to make phenomenological analyses including long-distance contributions unambiguously. We apply it to the amplitudes of the  $D \rightarrow \pi\pi$  and  $D \rightarrow KK$  decays:

$$\begin{aligned}
 A(D^+ \rightarrow \pi^+ \pi^0) &= -\frac{\lambda_d}{\sqrt{2}} [E_1(\pi) + E_2(\pi)], & (2.2) \\
 A(D^0 \rightarrow \pi^+ \pi^-) &= -\lambda_d [E_1(\pi) + A_2(\pi) - P_1^{\text{GIM}}(\pi) - P_3^{\text{GIM}}(\pi)] + \lambda_b [P_1(\pi) + P_3(\pi)], \\
 A(D^0 \rightarrow \pi^0 \pi^0) &= -\lambda_d [E_2(\pi) - A_2(\pi) + P_1^{\text{GIM}}(\pi) + P_3^{\text{GIM}}(\pi)] - \lambda_b [P_1(\pi) + P_3(\pi)], \\
 A(D^+ \rightarrow K^+ \bar{K}^0) &= \lambda_d [E_1(K) - A_1(K) + P_1^{\text{GIM}}(K)] + \lambda_b [E_1(K) + P_1(K)], \\
 A(D^0 \rightarrow K^+ K^-) &= \lambda_d [E_1(K) + A_2(s, q, s, K) + P_1^{\text{GIM}}(K) + P_3^{\text{GIM}}(K)] \\
 &\quad + \lambda_b [E_1(K) + A_2(s, q, s, K) + P_1(K) + P_3(K)], \\
 A(D^0 \rightarrow K^0 \bar{K}^0) &= -\lambda_d [A_2(s, q, s, K) - A_2(q, s, q, K) + P_3^{\text{GIM}}(K)] \\
 &\quad - \lambda_b [A_2(s, q, s, K) + P_3(K)],
 \end{aligned}$$

where  $\lambda_q = V_{cq}^* V_{uq}$  for  $q = d, b$ . From ref. [74, 75] we have the following counting in  $1/N_c$ :  $E_1$  and  $A_1$  are the leading amplitudes, all other amplitudes are suppressed by  $1/N_c$  except for  $P_3$  and  $P_3^{\text{GIM}}$ , which are suppressed by  $1/N_c^2$ . The amplitude for  $D^0 \rightarrow K^0 \bar{K}^0$  is  $1/N_c$  suppressed and originates from SU(3) breaking effects.

In terms of the RGI amplitudes, neglecting the contribution proportional to  $r_{\text{CKM}}$  to the  $\mathcal{A}$  terms, the isospin amplitudes can be written as

$$\begin{aligned}
\mathcal{A}_2^\pi &= -\sqrt{\frac{2}{3}} \lambda_d [E_1(\pi) + E_2(\pi)], \\
\mathcal{A}_0^\pi &= \frac{1}{\sqrt{3}} \lambda_d [2E_1(\pi) - E_2(\pi) + 3A_2(\pi) - 3P_1^{\text{GIM}}(\pi) - 3P_3^{\text{GIM}}(\pi)], \\
\mathcal{B}_0^\pi &= -\sqrt{3} \lambda_d [P_1(\pi) + P_3(\pi)], \\
\mathcal{A}_{13}^K &= -\frac{2}{3} \lambda_d [A_1(K) + A_2(q, s, q, K)], \\
\mathcal{A}_{11}^K &= \lambda_d \left[ E_1(K) - \frac{2}{3} A_1(K) + \frac{1}{3} A_2(q, s, q, K) + P_1^{\text{GIM}}(K) \right], \\
\mathcal{B}_{11}^K &= \lambda_d [E_1(K) + P_1(K)], \\
\mathcal{A}_0^K &= -\lambda_d [E_1(K) - A_2(q, s, q, K) + 2A_2(s, q, s, K) + P_1^{\text{GIM}}(K) + 2P_3^{\text{GIM}}(K)], \\
\mathcal{B}_0^K &= -\lambda_d [E_1(K) + 2A_2(s, q, s, K) + P_1(K) + 2P_3(K)].
\end{aligned} \tag{2.3}$$

Therefore, one expects  $\mathcal{B}_0^\pi$  to be  $1/N_c$ -suppressed with respect to  $\mathcal{A}_0^\pi$ . This suppression is partially compensated by the Clebsch-Gordan coefficients, so that the two amplitudes could be of the same size. Concerning the amplitudes with kaons in the final state, they are all leading in the  $1/N_c$  counting; however, a cancellation between the emission and annihilation parameters may occur in  $\mathcal{A}_{11}^K$ , possibly leading to an effective  $1/N_c$  suppression.

Neglecting the  $O(1/N_c^2)$  contributions, the combinations of the effective amplitudes for the  $\pi\pi$  modes are written in terms of the isospin amplitudes as

$$\begin{aligned}
E_1(\pi) + E_2(\pi) &= -\lambda_d^{-1} \sqrt{\frac{3}{2}} \mathcal{A}_2^\pi, \\
E_1(\pi) + A_2(\pi) - P_1^{\text{GIM}}(\pi) &= \lambda_d^{-1} \frac{1}{\sqrt{3}} \left( -\frac{\mathcal{A}_2^\pi}{\sqrt{2}} + \mathcal{A}_0^\pi \right), \\
E_2(\pi) - A_2(\pi) + P_1^{\text{GIM}}(\pi) &= -\lambda_d^{-1} \frac{1}{\sqrt{3}} \left( \sqrt{2} \mathcal{A}_2^\pi + \mathcal{A}_0^\pi \right), \\
P_1(\pi) &= -\lambda_d^{-1} \frac{1}{\sqrt{3}} \mathcal{B}_0^\pi,
\end{aligned} \tag{2.4}$$

while those for the  $KK$  modes are given by

$$\begin{aligned}
A_1(K) &= \lambda_d^{-1} \frac{1}{2} (-2 \mathcal{A}_{13}^K - \mathcal{A}_{11}^K + \mathcal{B}_{11}^K - \mathcal{A}_0^K + \mathcal{B}_0^K), \\
A_2(q, s, q, K) &= \lambda_d^{-1} \frac{1}{2} (-\mathcal{A}_{13}^K + \mathcal{A}_{11}^K - \mathcal{B}_{11}^K + \mathcal{A}_0^K - \mathcal{B}_0^K), \\
A_2(s, q, s, K) &= \lambda_d^{-1} \frac{1}{2} (-\mathcal{B}_{11}^K - \mathcal{B}_0^K), \\
E_1(K) + P_1(K) &= \lambda_d^{-1} \mathcal{B}_{11}^K, \\
E_1(K) + P_1^{\text{GIM}}(K) &= \lambda_d^{-1} \frac{1}{2} (-\mathcal{A}_{13}^K + \mathcal{A}_{11}^K + \mathcal{B}_{11}^K - \mathcal{A}_0^K + \mathcal{B}_0^K), \\
P_1(K) - P_1^{\text{GIM}}(K) &= \lambda_d^{-1} \frac{1}{2} (\mathcal{A}_{13}^K - \mathcal{A}_{11}^K + \mathcal{B}_{11}^K + \mathcal{A}_0^K - \mathcal{B}_0^K).
\end{aligned} \tag{2.5}$$

Before turning to the phenomenological analysis, we discuss the constraints implied by unitarity on the isospin amplitudes.

### 3 Rescattering and unitarity

Unitarity of the  $S$ -matrix implies constraints on weak decay matrix elements, provided that the strong  $S$  matrix at the relevant energy scale is experimentally accessible. As we discuss below, this is indeed the case for  $D \rightarrow \pi\pi$  and  $KK$  decays, leading to interesting constraints on the decay amplitudes.

Notice that any Wick contraction, as defined in refs. [74, 76], can be seen as an emission followed by rescattering [76, 77]. Thus, rescattering establishes a link between emissions and long-distance contributions to other subleading topologies such as penguins.

#### 3.1 Coupled-channel unitarity

We split the effective Hamiltonian for weak charm decays into a CP-even  $H_R$  and a CP-odd  $H_I$  part. Then we can write

$$T_{fi} = \langle f|H|i\rangle = \langle f|H_R + iH_I|i\rangle = T_{fi}^R + iT_{fi}^I. \tag{3.1}$$

The  $S$  matrix can be written as

$$S = \left( \begin{array}{c|ccc} D \rightarrow D & D \rightarrow \pi\pi & D \rightarrow KK & \dots \\ \hline \pi\pi \rightarrow D & \pi\pi \rightarrow \pi\pi & \pi\pi \rightarrow KK & \dots \\ KK \rightarrow D & KK \rightarrow \pi\pi & KK \rightarrow KK & \dots \\ \vdots & \vdots & \vdots & \ddots \end{array} \right) \equiv \begin{pmatrix} 1 & -i(T)^T \\ -i\text{CP}(T) & S_S \end{pmatrix}, \tag{3.2}$$

where the time reversal of  $T$  is equal to the CP conjugate of  $T$ :  $T(T) = \text{CP}(T) = T^R - iT^I$ , and  $S_S$  is the strong interaction rescattering matrix. Unitarity of  $S$  (and of  $S_S$ ) implies, at lowest order in weak interactions,

$$T^R = S_S(T^R)^*, \quad T^I = S_S(T^I)^*, \tag{3.3}$$

where separate equalities hold for  $T^R$  and  $T^I$ . These equalities can be used to reduce the number of unknown hadronic parameters in the decay amplitudes, if  $S_S$  is known independently. The simplest case corresponds to decay channels where  $S_S$  can be approximated with a pure phase  $e^{2i\delta}$ . Then we obtain

$$T^R = |T^R|e^{i\delta}, \quad T^I = |T^I|e^{i\delta}, \tag{3.4}$$

where  $\delta + \pi$  is also possible. Notice that, even in this simple case, eq. (3.4) cannot be used to add FSI to factorized amplitudes, since the identification of factorized results with  $|T|$  (or  $\text{Re } T$ ) is ambiguous.

In principle, this could be the case for  $I = 2$  S-wave  $\pi\pi \rightarrow \pi\pi$  scattering, whose phase can be extracted from the data in ref. [62]:

$$\delta_{\pi\pi}^{I=2} = (-8 \pm 5)^\circ. \tag{3.5}$$



However, there is a sizable inelasticity in this channel at the  $D$  mass, so that the information above cannot be used (see the discussion below on multi-channel unitarity). This does not spoil the rescattering analysis since, as we show below, the  $D \rightarrow \pi\pi$  BR's fix the relative phase of  $\mathcal{A}_2^\pi$  and  $\mathcal{A}_0^\pi$  with an excellent accuracy.

Concerning the  $I = 1$   $KK$  rescattering, if it were elastic we would have  $\arg \mathcal{A}_{13}^K = \arg \mathcal{A}_{11}^K = \arg \mathcal{B}_{11}^K$  up to a  $\pi$  ambiguity, leading to the absence of direct CP violation in  $D^+ \rightarrow K^+ K_S$ . However, it is well conceivable that  $KK$  scattering at the  $D$  mass is inelastic, so that we do not impose the relation above.

The case of  $I = 0$  amplitudes is more involved. Experimental data on  $\pi\pi$  and  $KK$  final states have been collected in refs. [60] and [61] respectively. The data on  $\pi p \rightarrow K_S K_{S\bar{N}}$  show a strong suppression of the  $\pi\pi \rightarrow KK$  amplitude at energies close to the  $D$  mass, as can be seen for example in figure 6 of ref. [61]. Conversely, the extraction of isospin amplitudes from data on  $\pi\pi \rightarrow \pi\pi$  scattering at the  $D$  mass is ambiguous, leading to widely different results for the inelasticity. For example, ref. [78] provides four different amplitude fits corresponding to discrete ambiguities; one of them gives results compatible with the  $KK$  data close to the  $D$  mass, while the others point to violations of two-channel unitarity. The latter could be due to the four pion channel, see for example figure 3 of ref. [79]. Thus, two scenarios may be envisaged.

### 3.1.1 Two-channel analysis of $I = 0$ amplitudes

First, one can assume that the strong  $S$  matrix is well described by a two-channel analysis with  $\pi\pi$  and  $KK$  states only. Indeed, two-channel fits give a reasonable description of data in a wide range of energies (see for example figure 1 of ref. [80]). The corresponding two-by-two symmetric rescattering matrix can be parameterized as

$$S_S = \begin{pmatrix} \eta e^{2i\delta_1} & \pm i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ \pm i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}, \quad (3.6)$$

where  $\eta$  is the inelasticity parameter. We extract the  $I = 0$  S-wave scattering phases of  $\pi\pi \rightarrow \pi\pi$  and  $\pi\pi \rightarrow KK$  and the inelasticity parameter  $\eta$  at the  $D$  mass from the experimental data in refs. [60, 61]:

$$\delta_1 = (40 \pm 10)^\circ, \quad \delta_1 + \delta_2 = (360 \pm 60)^\circ, \quad \eta = 0.95 \pm 0.05, \quad (3.7)$$

where the last value has been estimated using  $KK$  data.

Unitarity implies the following equation for  $I = 0$  CP-even amplitudes:

$$\begin{pmatrix} \mathcal{A}_0^\pi \\ \mathcal{A}_0^K \end{pmatrix} = \begin{pmatrix} \eta e^{2i\delta_1} & \pm i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ \pm i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} \begin{pmatrix} (\mathcal{A}_0^\pi)^* \\ (\mathcal{A}_0^K)^* \end{pmatrix}, \quad (3.8)$$

and an identical equation holds for the CP-odd amplitudes  $\mathcal{B}_0^{\pi,K}$ . Defining  $\mathcal{A}_0^\pi = |\mathcal{A}_0^\pi| e^{i\varphi_0^\pi}$  and  $\mathcal{A}_0^K = |\mathcal{A}_0^K| e^{i\varphi_0^K}$ , eq. (3.8) can be written as

$$\begin{aligned} \cos(\varphi_0^K - \delta_2) &= \pm \left| \frac{\mathcal{A}_0^\pi}{\mathcal{A}_0^K} \right| \sqrt{\frac{1+\eta}{1-\eta}} \sin(\varphi_0^\pi - \delta_1), \\ \sin(\varphi_0^K - \delta_2) &= \pm \left| \frac{\mathcal{A}_0^\pi}{\mathcal{A}_0^K} \right| \sqrt{\frac{1-\eta}{1+\eta}} \cos(\varphi_0^\pi - \delta_1), \end{aligned} \quad (3.9)$$

where one can add  $\pi$  to the phases  $\varphi_0^\pi$  and  $\varphi_0^K$  simultaneously. From these equations, we find that the ratio  $|\mathcal{A}_0^K/\mathcal{A}_0^\pi|$  obeys the following constraints:

$$\frac{1-\eta}{1+\eta} \leq \left| \frac{\mathcal{A}_0^K}{\mathcal{A}_0^\pi} \right|^2 \leq \frac{1+\eta}{1-\eta}, \quad (3.10)$$

and the phase differences  $\varphi_0^\pi - \delta_1$  and  $\varphi_0^K - \delta_2$  are determined in terms of  $|\mathcal{A}_0^K/\mathcal{A}_0^\pi|$  and  $\eta$ .

In the limit of  $\eta \rightarrow 1$ , where the scatterings are elastic, eq. (3.8) can be written as

$$|\mathcal{A}_0^\pi| e^{i\varphi_0^\pi} = e^{2i\delta_1} |\mathcal{A}_0^\pi| e^{-i\varphi_0^\pi}, \quad |\mathcal{A}_0^K| e^{i\varphi_0^K} = e^{2i\delta_2} |\mathcal{A}_0^K| e^{-i\varphi_0^K}, \quad (3.11)$$

and the strong phases are then given by

$$\varphi_0^\pi = \delta_1 + n\pi, \quad \varphi_0^K = \delta_2 + m\pi, \quad (3.12)$$

where  $n$  and  $m$  are arbitrary integers. Similarly, the strong phases of the CP-odd amplitudes are given by  $\varphi_0^\pi = \delta_1 + n'\pi$  and  $\varphi_0^K = \delta_2 + m'\pi$ , where  $n'$  and  $m'$  could be different from  $n$  and  $m$ . In this case, CP violation cannot be generated from the interference of  $\mathcal{A}_0^\pi$  ( $\mathcal{A}_0^K$ ) and  $\mathcal{B}_0^\pi$  ( $\mathcal{B}_0^K$ ). Thus, in this scenario, given the small inelasticity of  $\pi\pi$  scattering, we expect that CP violation in  $D \rightarrow \pi\pi$  decays mainly arises through the interference of  $\mathcal{B}_0^\pi$  with  $\mathcal{A}_2^\pi$ .

### 3.1.2 Three-channel unitarity

In the second scenario, instead, we allow for a third (effective) channel to give a sizable contribution, thus reconciling the large inelasticity solutions of  $\pi\pi \rightarrow \pi\pi$  amplitude fits with the  $KK$  data. This corresponds to a three by three  $S_S$  matrix in which the  $KK$  channel is almost decoupled, leading to a situation similar to the one described above but with  $\pi\pi$  coupled to the third effective channel with a large inelasticity (small  $\eta$ ). If the  $KK$  channel is decoupled, unitarity fixes the phase of  $\mathcal{A}_0^K$  and  $\mathcal{B}_0^K$  to be equal to  $\delta_2 + n\pi$  [81]. Conversely, since the  $\pi\pi$  channel has a large inelasticity, the solutions of two-channel unitarity discussed above give essentially no constraint on absolute value and phase of  $\mathcal{A}_0^\pi$  and  $\mathcal{B}_0^\pi$ . Thus, in this case CP violation in  $D \rightarrow \pi\pi$  can also arise from interference between  $\mathcal{A}_0^\pi$  and  $\mathcal{B}_0^\pi$ .

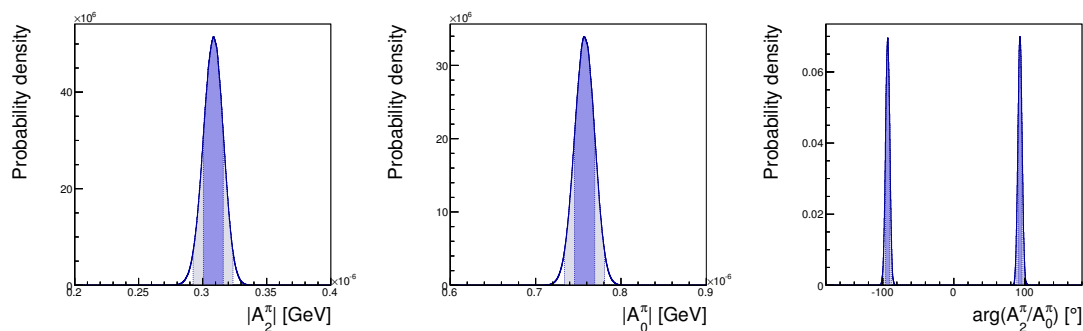
We have checked numerically that the results obtained in the three-channel scenario are essentially identical to the ones obtained in the most general case, where more than three channels contribute to the rescattering so that no significant constraint can be obtained from unitarity.

## 4 Branching ratios and CP-even contributions

The decay width of the process  $D \rightarrow PP$  is given by

$$\Gamma(D \rightarrow PP) = \frac{p_c}{8\pi m_D^2} |A(D \rightarrow PP)|^2, \quad (4.1)$$

where  $p_c = \sqrt{m_D^2 - 4m_P^2}/2$  is the center-of-mass momentum of the mesons in the final state, and an extra factor 1/2 must be added in the case of  $D^0 \rightarrow \pi^0\pi^0$ . We adopt



**Figure 1.** From left to right, p.d.f. for  $|\mathcal{A}_2^\pi|$ ,  $|\mathcal{A}_0^\pi|$  and  $\arg(\mathcal{A}_2^\pi/\mathcal{A}_0^\pi)$  in the three-channel scenario. Here and in the following, darker (lighter) areas correspond to 68% (95%) probability ranges.

$m_K = 0.498$  GeV,  $m_\pi = 0.135$  GeV,  $m_D = 1.865$  GeV,  $\tau_{D^0} = 410.1 \times 10^{-15}$  sec and  $\tau_{D^\pm} = 1040 \times 10^{-15}$  sec in numerical analyses. In this section, we discuss the determination of CP-even amplitude parameters from the measured BR's reported in table 2. Here and in the following, we follow the inferential framework outlined in ref. [82]. In particular, we obtain the 68% and 95% probability regions by integrating the posterior p.d.f. around the most probable value(s).

#### 4.1 $\pi\pi$ isospin amplitudes

In the case of  $D \rightarrow \pi\pi$ , the BR's are sufficient to determine  $|\mathcal{A}_{0,2}^\pi|$  and the relative phase. The magnitude of the  $I = 2$  CP-even  $\pi\pi$  amplitude  $\mathcal{A}_2^\pi$  can be extracted from  $BR(D^\pm \rightarrow \pi^\pm\pi^0)$ :

$$|\mathcal{A}_2^\pi| = \sqrt{\frac{4 BR(D^\pm \rightarrow \pi^\pm\pi^0)}{3 \tau_{D^\pm}} \frac{16\pi m_D^2}{\sqrt{m_D^2 - 4m_\pi^2}}}, \quad (4.2)$$

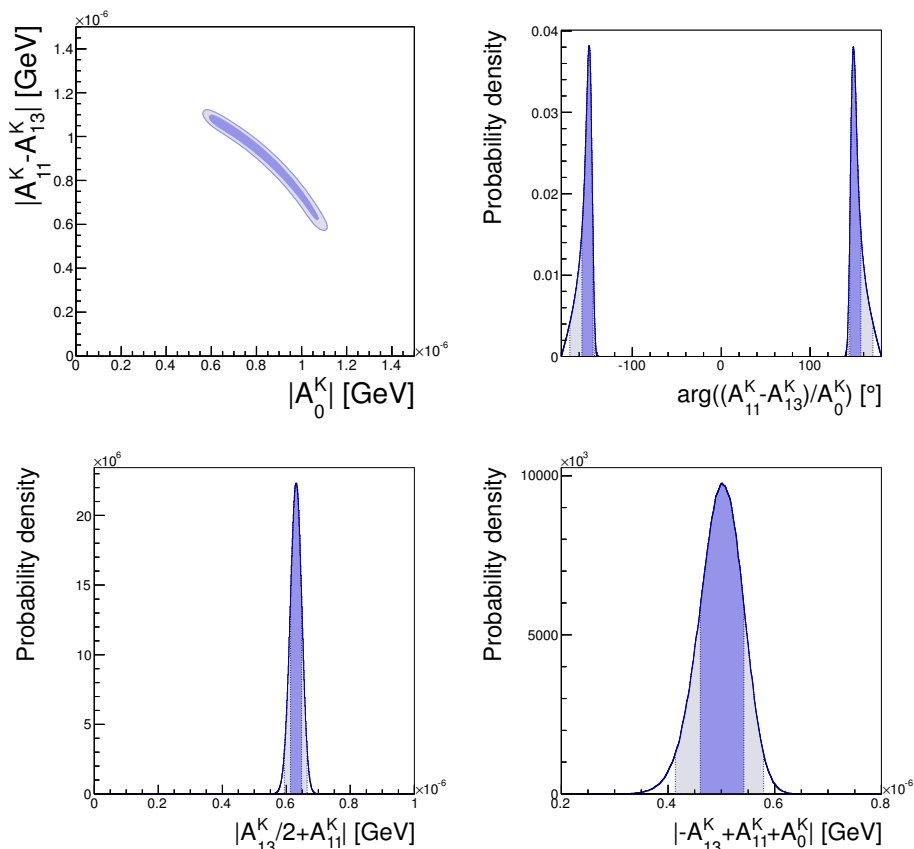
and then  $\mathcal{A}_0^\pi$  and the relative phase can be obtained from  $BR(D^0 \rightarrow \pi^0\pi^0)$  and  $BR(D^0 \rightarrow \pi^+\pi^-)$ . From the probability density function (p.d.f.) in figure 1 we obtain

$$\begin{aligned} |\mathcal{A}_2^\pi| &= (3.08 \pm 0.08) \times 10^{-7} \text{ GeV}, \\ |\mathcal{A}_0^\pi| &= (7.6 \pm 0.1) \times 10^{-7} \text{ GeV}, \\ \arg(\mathcal{A}_2^\pi/\mathcal{A}_0^\pi) &= (\pm 93 \pm 3)^\circ. \end{aligned} \quad (4.3)$$

Notice that the results in eq. (4.3) exclude order-of-magnitude enhancements of the  $I = 0$  amplitude. The quality of the fit to the BR's is excellent.

#### 4.2 $KK$ isospin amplitudes

In the case of  $D \rightarrow KK$  decays, the BR's are not sufficient to determine all isospin amplitudes. Given a value of  $\mathcal{A}_0^K$  that satisfies the unitarity constraints, we solve for  $|\mathcal{A}_{13}^K/2 + \mathcal{A}_{11}^K|$ ,  $|\mathcal{A}_{11}^K - \mathcal{A}_{13}^K|$  and  $\arg((\mathcal{A}_{11}^K - \mathcal{A}_{13}^K)/\mathcal{A}_0^K)$  using the three BR's. The p.d.f. for  $|\mathcal{A}_{13}^K/2 + \mathcal{A}_{11}^K|$ ,  $|\mathcal{A}_{11}^K - \mathcal{A}_{13}^K|$  vs  $|\mathcal{A}_0^K|$  and  $\arg((\mathcal{A}_{11}^K - \mathcal{A}_{13}^K)/\mathcal{A}_0^K)$  are reported in figure 2. In



**Figure 2.** From left to right and from top to bottom, p.d.f. for  $|\mathcal{A}_{11}^K - \mathcal{A}_{13}^K|$  vs  $|\mathcal{A}_0^K|$ ,  $\arg((\mathcal{A}_{11}^K - \mathcal{A}_{13}^K)/\mathcal{A}_0^K)$ ,  $|\mathcal{A}_{13}^K/2 + \mathcal{A}_{11}^K|$  and  $|\mathcal{A}_{13}^K + \mathcal{A}_{11}^K + \mathcal{A}_0^K|$  in the two-channel scenario. In the three-channel scenario one obtains essentially identical results.

order to reproduce the CP asymmetries, the degeneracy in  $\arg((\mathcal{A}_{11}^K - \mathcal{A}_{13}^K)/\mathcal{A}_0^K)$  is broken, with a mild preference for the negative solution.

An interesting result is given by the CP-conserving contribution to  $BR(D^0 \rightarrow K^0 \bar{K}^0)$ , which should vanish in the SU(3) limit. We obtain instead a result comparable to all other amplitudes in the  $KK$  channels (see figure 2):

$$|\mathcal{A}_{13}^K - \mathcal{A}_{11}^K - \mathcal{A}_0^K| = (5.0 \pm 0.4) \times 10^{-7} \text{ GeV}, \quad (4.4)$$

showing explicitly a breaking of  $\mathcal{O}(1)$  of the SU(3) flavour symmetry. Also in this case, we obtain an excellent fit of the BR's.

### 4.3 RGI parameters for CP conserving contributions

From eqs. (2.4) we obtain the following results for the pion RGI parameters in the three-channel scenario:

$$\begin{aligned} E_1(\pi) + E_2(\pi) &= (1.72 \pm 0.04) \times 10^{-6} e^{i\delta} \text{ GeV}, \\ E_1(\pi) + A_2(\pi) - P_1^{\text{GIM}}(\pi) &= (2.10 \pm 0.02) \times 10^{-6} e^{i(\delta \pm (71 \pm 3)^\circ)} \text{ GeV}, \\ E_2(\pi) - A_2(\pi) + P_1^{\text{GIM}}(\pi) &= (2.25 \pm 0.07) \times 10^{-6} e^{i(\delta \mp (62 \pm 2)^\circ)} \text{ GeV}, \end{aligned} \quad (4.5)$$

with a two-fold ambiguity and generic  $\delta$ . These results show that the  $E_1(\pi)$  parameter does not dominate the decay amplitude, and that  $1/N_c$ -suppressed topologies are comparable to  $E_1(\pi)$  with a large strong phase difference (this is evident by comparing the second and third lines of eq. (4.5)). This also shows that power-suppressed amplitudes in the  $m_c \rightarrow \infty$  limit are of the same size of leading ones.

Let us now turn to the  $KK$  channels. The result in eq. (4.4) implies, using eq. (2.2), that the SU(3)-suppressed combination of subleading amplitudes  $A_2(s, q, s, K) - A_2(q, s, q, K) + P_3^{\text{GIM}}(K)$  is of the same order of the leading contribution  $E_1(K)$ .

We conclude from the analysis of  $D \rightarrow \pi\pi$  and  $D \rightarrow KK$  BR's that subleading topologies are of the same order of leading ones, with a breaking of SU(3) of  $\mathcal{O}(1)$ . This is the starting point for our study of CP-violating asymmetries in the next section.

## 5 CP asymmetries

We turn to the main point of this work, namely the attempt to estimate the possible size of CP asymmetries in the SM and to quantify the agreement of the SM with experimental data.

Before dwelling in the analysis, we remark a few relevant points:

- present experimental data point to a larger CP asymmetry in the  $\pi^+\pi^-$  channel with respect to the  $K^+K^-$  one (indeed, the latter is compatible with zero at less than  $1\sigma$ );
- CP violation is always proportional to subleading contributions; in the case at hand, CP asymmetries in the  $K^+K^-$  and  $\pi^+\pi^-$  channels are due to penguin contractions of current-current operators, while in  $K^0\bar{K}^0$  also annihilations contribute;
- in the two-channel scenario, one has to a good accuracy  $\arg \mathcal{B}_0^\pi = \arg \mathcal{A}_0^\pi$  and CP violation can occur only through the interference of  $\mathcal{B}_0^\pi$  with  $\mathcal{A}_2^\pi$ , leading to a suppression of the CP asymmetry with respect to the three-channel scenario;
- given our phase convention for the CKM matrix, CP violation in the  $\pi^+\pi^-$  channel is signaled by  $\mathcal{B}_0^\pi \neq 0$ , while in the  $K^+K^-$  channel one must have  $\mathcal{B}_{11}^K \neq \mathcal{A}_{11}^K - \mathcal{A}_{13}^K$  or  $\mathcal{B}_0^K \neq \mathcal{A}_0^K$ .

Thus, to estimate CP asymmetries we need to estimate the size of subleading amplitudes. From the analysis of the BR's presented above, we do not see any evident suppression of subleading terms, so that we impose generically

$$\begin{aligned}
 |\mathcal{B}_0^\pi| &< \kappa |\mathcal{A}_0^\pi|, \\
 |\mathcal{B}_0^K - \mathcal{A}_0^K| &< \kappa |\mathcal{A}_0^K|, \\
 |\mathcal{B}_{11}^K - (\mathcal{A}_{11}^K - \mathcal{A}_{13}^K)| &< \kappa |\mathcal{A}_{11}^K - \mathcal{A}_{13}^K|,
 \end{aligned}
 \tag{5.1}$$

where  $\kappa$  parameterizes the size of the subleading terms. In terms of RGI parameters, this amounts to

$$\begin{aligned}
 |P_1(\pi)| &\leq \kappa \left| \frac{2}{3}E_1(\pi) - \frac{1}{3}E_2(\pi) + A_2(\pi) - P_1^{\text{GIM}}(\pi) \right|, \quad (5.2) \\
 |P_1(K) - P_1^{\text{GIM}}(K) + A_2(q, s, q, K)| &\leq \\
 &\quad \kappa |E_1(K) - A_2(q, s, q, K) + 2A_2(s, q, s, K) + P_1^{\text{GIM}}(K)|, \\
 |P_1(K) - P_1^{\text{GIM}}(K) - A_2(q, s, q, K)| &\leq \kappa |E_1(K) + A_2(q, s, q, K) + P_1^{\text{GIM}}(K)|,
 \end{aligned}$$

where  $P_3$  and  $P_3^{\text{GIM}}$  have been neglected.

Let us first present the results for  $1 \leq \kappa \leq 8$  and then comment on the values of  $\kappa$  that we consider acceptable. We can follow two different avenues. The first possibility is to give a prediction of the CP asymmetries as a function of  $\kappa$  and compare it with experimental data. The second option is to fit the measured CP asymmetries as a function of  $\kappa$ . In this case we can also study the values of the subleading topologies selected by the fit and compare them with our (albeit vague) theoretical expectations.

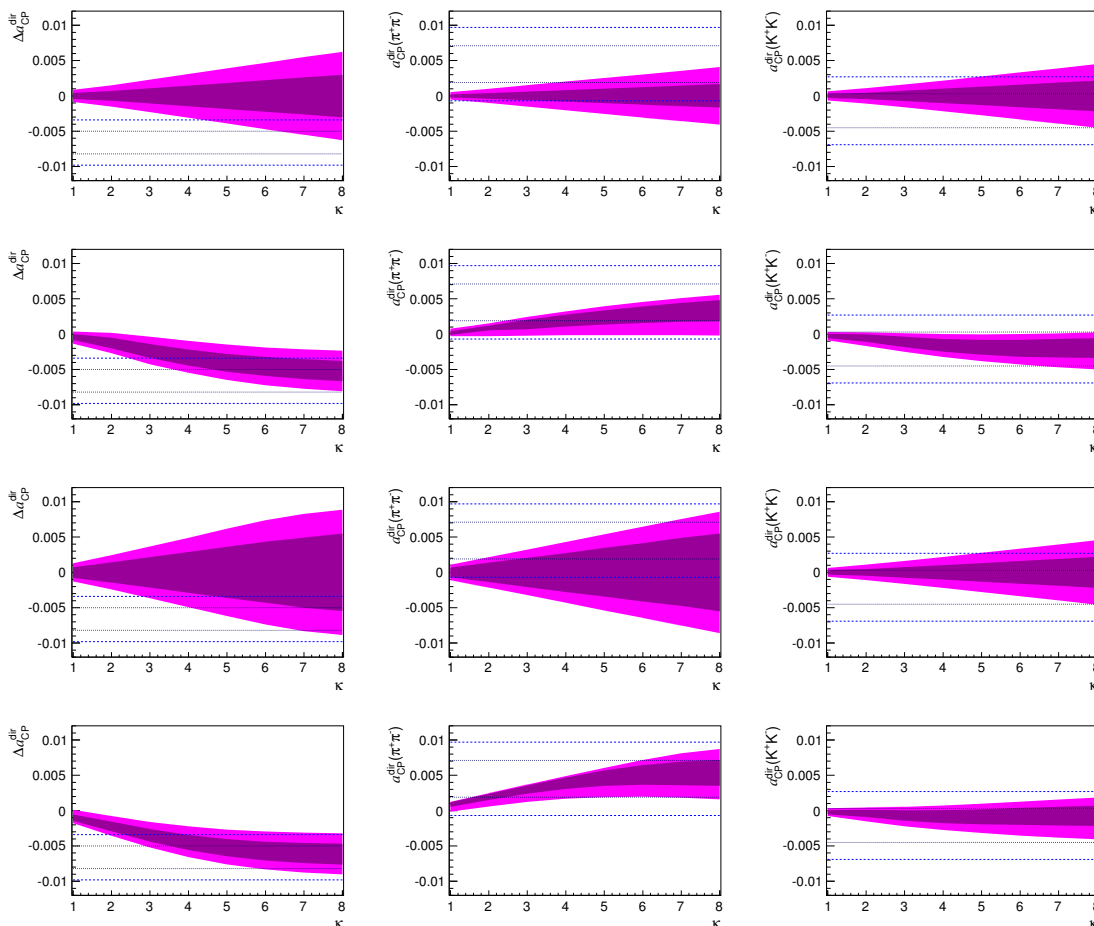
In the upper part of figure 3 we present the predictions and fit results for  $\Delta a_{\text{CP}}^{\text{dir}}$ ,  $a_{\text{CP}}^{\text{dir}}(\pi^+\pi^-)$  and  $a_{\text{CP}}^{\text{dir}}(K^+K^-)$  in the two-channel scenario. We see that the generic prediction would give much smaller asymmetries, and that the prediction does not reach the present experimental value within  $2\sigma$  for values of  $\kappa \leq 8$ . In the three-channel scenario, instead, we obtain the results in the lower part of figure 3. Since in this case the pion amplitudes are less constrained by unitarity, the predicted asymmetries are larger than in the two-channel scenario, and the present experimental value can be reached within  $2\sigma$  for  $\kappa \gtrsim 5$ , but even for  $\kappa = 8$  the prediction is still  $1\sigma$  from the experimental result. The p.d.f. for  $\Delta a_{\text{CP}}^{\text{dir}}$  for different values of  $\kappa$  can be found in figure 4.

To assess the compatibility of the experimental result with the SM, we can compare the distribution for  $P_1(\pi)$  obtained from the fit for different values of  $\kappa$  with the distribution of the pion amplitude parameters obtained from the BR's in eq. (4.5). To this aim, we report in figure 5 the p.d.f. for the absolute values of the parameters in eq. (4.5) and for  $P_1(\pi)$  for different values of  $\kappa$  in the two scenarios. We notice that in the three-channel scenario the preferred value for  $|P_1(\pi)|$ , corresponding to the central value of the measured  $\Delta a_{\text{CP}}^{\text{dir}}$ , is around  $1.3 \times 10^{-5}$  GeV, about 6 times larger than the RGI parameter combinations obtained from the BR's. In the two-channel scenario, instead, even for  $\kappa = 8$  the fit is still pulling  $|P_1(\pi)|$  to the upper edge of the allowed range, showing that the present central value cannot be reasonably accommodated in this scenario.

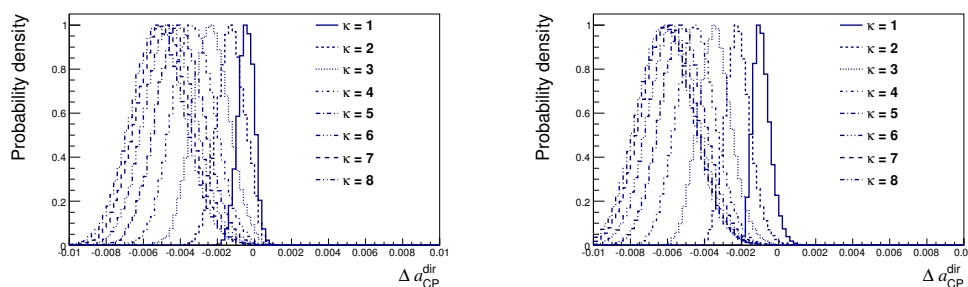
For the sake of completeness, we report in figures 6 and 7 the p.d.f.'s for the fitted CP asymmetries for different values of  $\kappa$ .

## 6 Summary

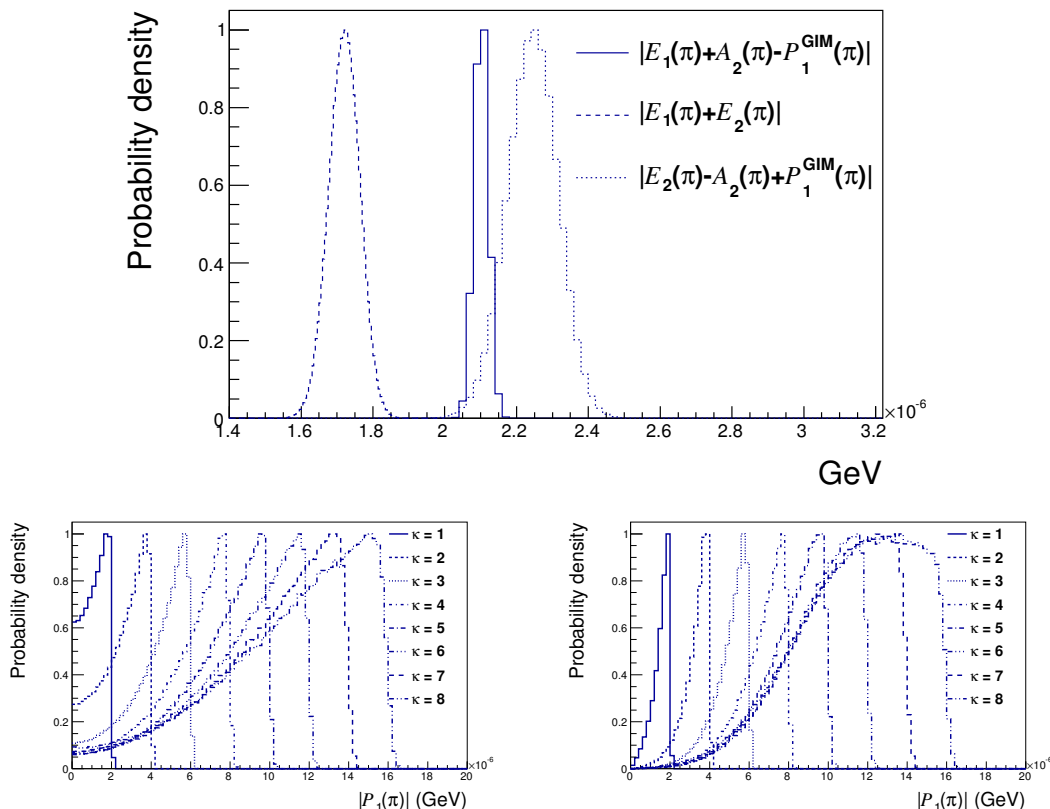
We have analyzed the  $D \rightarrow KK$  and  $D \rightarrow \pi\pi$  decays within the SM, assuming only isospin and using the information from  $\pi\pi$  scattering and unitarity. We have considered two possible scenarios for the strong  $S$  matrix (two- and three-channel unitarity). We have



**Figure 3.** From left to right, p.d.f. for the prediction (first row) and fit (second row) of  $\Delta a_{\text{CP}}^{\text{dir}}$ ,  $a_{\text{CP}}^{\text{dir}}(\pi^+\pi^-)$  and  $a_{\text{CP}}^{\text{dir}}(K^+K^-)$  in the two-channel scenario. Third and fourth rows: same as the first two rows in the three-channel scenario. Darker (lighter) areas correspond to 68% (95%) probability ranges. The dotted (dashed) lines correspond to 68% (95%) experimental ranges from eq. (1.6).



**Figure 4.** From left to right, p.d.f. for the fitted  $\Delta a_{\text{CP}}^{\text{dir}}$  for different values of  $\kappa$  in the two- and three-channel scenario. All the p.d.f.'s have been scaled to fit in the same plot.



**Figure 5.** First row: p.d.f. for the parameters in eq. (4.5). Second row: p.d.f. for  $|P_1(\pi)|$  in the two- and three-channel scenario. All the p.d.f.'s have been scaled to fit in the same plot.

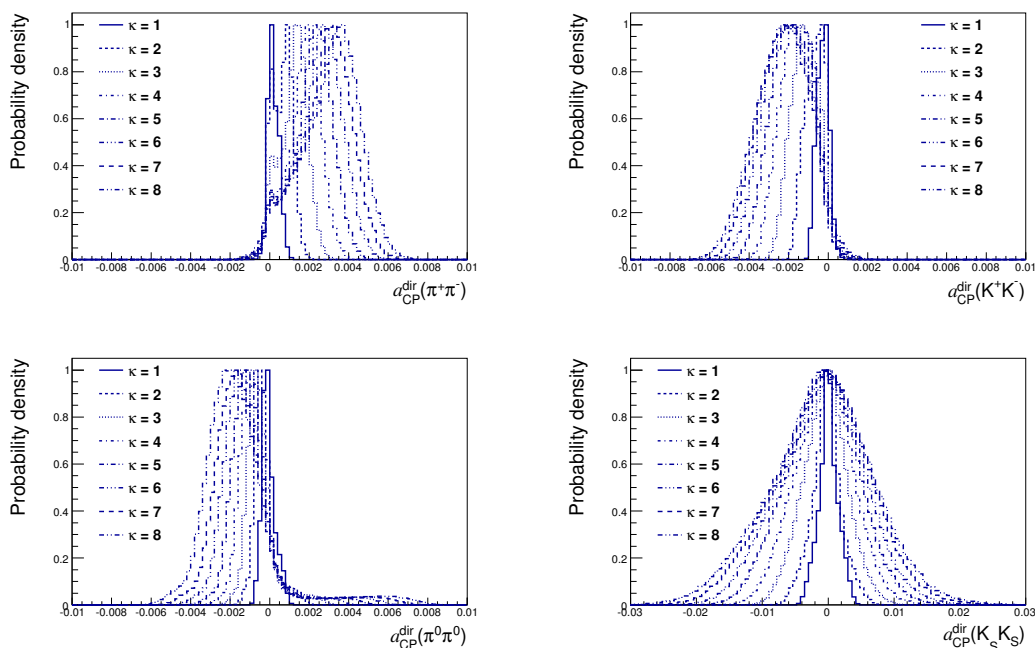
performed a fit of the CP conserving contributions from the CP-averaged BR's, obtaining information on isospin amplitudes and RGI parameters. We have predicted and fitted the CP asymmetries in the two scenarios.

Considering the more conservative three-channel scenario, we conclude that, with present errors, the observed asymmetries are marginally compatible with the SM. This conclusion holds also for the most general scenario with even more coupled channels in the  $I = 0$  rescattering, where no significant constraints arise from unitarity. Should the present central value be confirmed with smaller errors, it would require a factor of six (or larger) enhancement of the penguin amplitude with respect to all other topologies, well beyond our theoretical expectations. Thus, improving the experimental accuracy could lead to an indirect signal of new physics.

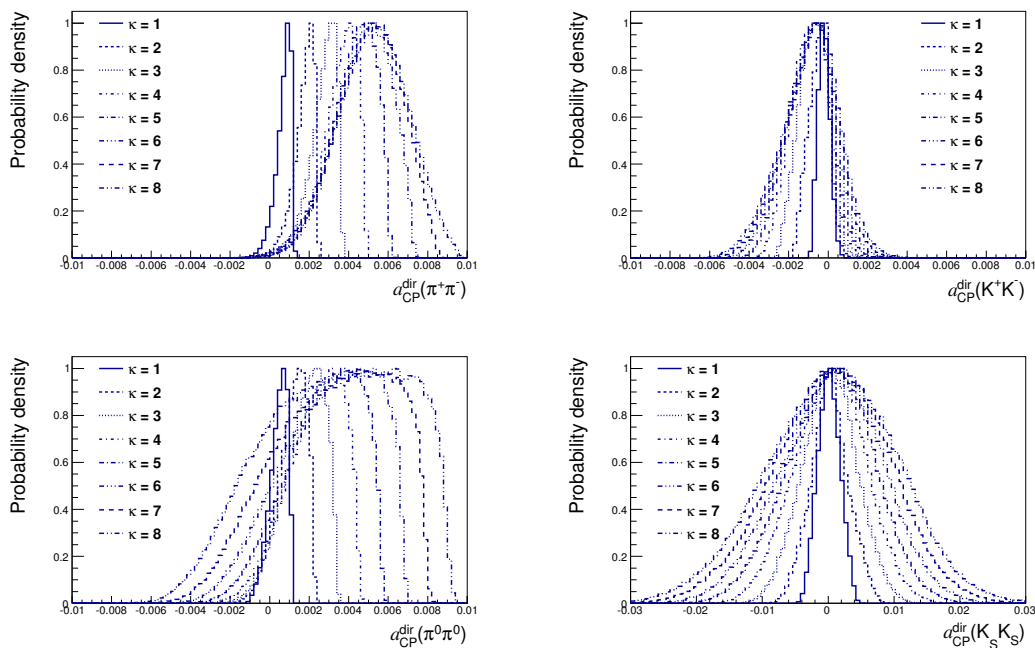
### Acknowledgments

The authors are associated to the Dipartimento di Fisica, Università di Roma “La Sapienza”. We acknowledge partial support from ERC Ideas Starting Grant n. 279972 “NPFflavour” and ERC Ideas Advanced Grant n. 267985 “DaMeSyFla”. It is a pleasure to thank Gilberto Colangelo for informative discussions on rescattering and Marco Ciuchini for his constant criticisms about this work.





**Figure 6.** P.d.f. for the CP asymmetries in the two-channel scenario for different values of  $\kappa$ . All the p.d.f.'s have been scaled to fit in the same plot.



**Figure 7.** P.d.f. for the CP asymmetries in the three-channel scenario for different values of  $\kappa$ . All the p.d.f.'s have been scaled to fit in the same plot.

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