

# The non-SUSY baryonic branch: soft supersymmetry breaking of $\mathcal{N} = 1$ gauge theories

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**ABSTRACT:** We study a non-supersymmetric deformation of the field theory dual to the baryonic branch of Klebanov-Strassler. Using a combination of analytical (series expansions) and numerical methods we construct non-supersymmetric backgrounds that smoothly interpolate between the desired UV and IR behaviors. We calculate various observables of the field theory and propose a picture of soft breaking by gaugino masses that is consistent with the various calculations on the string side.

**KEYWORDS:** Gauge-gravity correspondence, Supersymmetry Breaking, D-branes

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## 1 Introduction and summary

The Maldacena Conjecture [1–3] provides what is probably the most effective and controllable tool to study non-perturbative dynamics of a variety of field theories. A large variety of effects have been discovered or checked in field theories using a suitable string dual. Integrability, correlation functions of various interesting operators (protected or not by symmetries), aspects of lower dimensional systems, applications in condensed matter and QCD-like systems have been successfully studied using gauge/gravity duality.

The results above, while more numerous and spectacular in highly (super)symmetric theories, are not restricted to examples of this sort. As a matter of fact, there are many applications where black holes (and hence dual field theories at finite temperature) play a fundamental role. In these cases the dynamics is neither driven by SUSY nor by conformal symmetry.

As a result, an interesting problem is to construct backgrounds duals to field theories where supersymmetry has been broken in a soft way. These systems should conserve some of the dynamics of the SUSY case with the addition of the deformations by relevant operators that break the supersymmetry. The low energy dynamics should then be determined by a combination of SUSY and non-SUSY effects. This is an interesting problem, on which it seems feasible to make progress.

In this paper, we will construct duals to field theories in four dimensions where SUSY has been explicitly and softly broken by the addition of relevant operators to the Lagrangian. The original field theories will be those obtained by a twisted compactification of five branes wrapping a calibrated two cycle in the resolved conifold and those obtained by studying the dynamics of D3 and fractional D5 branes on the tip of a conifold. Both are non-conformal theories with interesting low energy dynamics (confinement, R-symmetry breaking, formation of domain walls, k-strings, etc.)

We will construct our non-SUSY backgrounds by finding an explicit solution of the Einstein, dilaton and RR-form equations of motion. We also impose that irrelevant operators are absent from the dynamics and that the string backgrounds are regular all along the space. We will concentrate on the case in which the SUSY breaking parameters are small compared the others already present in the system in the SUSY case.

These will then be examples of backgrounds dual to the strongly coupled dynamics of well understood SUSY field theories in which SUSY has been softly and controllably broken. Some examples of this sort have appeared in the past for deformations of well-known SUSY backgrounds, see for example [4–10].

Our paper is organized as follows. We start in section 2 by presenting the SUSY system. While the formalism summarized there does not apply to problem of interest, we do

give some details that are useful in attempting to construct the non-SUSY solutions (in particular large-radius asymptotics). In section 3, we will propose a SUSY breaking solution in series expansion for large (UV) and small (IR) values of the radial coordinate. We will carefully count the parameters that control our solutions and find numerical solutions interpolating in a smooth way between the desired UV and IR asymptotics. Section 4 gives some details of the numerical method. In section 5 we calculate the ADM Energy of the new solutions (with the SUSY solutions as reference backgrounds). In section 6 we perform a detailed study of various field theory quantities, whose strong-coupling result points us to an interpretation of the dual field theory being deformed by the insertion of relevant operators, like gaugino masses that break SUSY and may also influence VEVs. We close with some conclusions and possible interesting problems to be solved constructing on the results of this paper. The high technical nature of our work is clear from the outline above. For the benefit of readers, we have included explicit technical points in detailed appendices.

**Note added.** While this paper was close to completion, we were informed of the work by Dymarsky and Kuperstein, having interesting overlap with ours [63]. We thank Anatoly Dymarsky for letting us know about this work prior to publication and discussion on these topics.

## 2 Presentation of the SUSY system

In this section we summarize well established aspects of particular supersymmetric field theories and their dual backgrounds. This will be useful when introducing SUSY breaking deformations.

We start by considering two apparently different field theories. The first one, we refer to it as ‘theory I’ or ‘Type I theory’ (hoping not to cause confusion with the Type I string theory), is a quiver with gauge group  $SU(n + N_c) \times SU(n)$  and bifundamental matter multiplets  $A_i, B_\alpha$  with  $i, \alpha = 1, 2$ . The global symmetries are<sup>1</sup>

$$SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_R. \tag{2.1}$$

These bifundamentals transform under the local and global symmetries as

$$A_i = \left( n + N_c, \bar{n}, 2, 1, 1, \frac{1}{2} \right), \quad B_\alpha = \left( \bar{n} + \bar{N}_c, n, 1, 2, -1, \frac{1}{2} \right). \tag{2.2}$$

There is also a superpotential of the form  $W = \frac{1}{\mu} \epsilon_{ij} \epsilon_{\alpha\beta} tr[A_i B_\alpha A_j B_\beta]$ . The field theory is taken to be close to a strongly coupled fixed point. In that case one can show that the anomalous dimensions should be  $\gamma_{A,B} \sim -\frac{1}{2}$ . This field theory is well known to be the dual to the Klebanov-Strassler background [11] and its generalization to the baryonic branch [12].

The second field theory, that we will call ‘theory II’ (again not to be confused with the Type II string!) is obtained after a twisted compactification (to four dimensions) of six dimensional SUSY  $SU(N_c)$  Yang-Mills with 16 supercharges. This special compactification studied in [14–16] preserves four supercharges. In four dimensional language, the

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<sup>1</sup>The R-symmetry is anomalous, breaking  $U(1)_R \rightarrow Z_{2N_c}$ .

field content is a massless vector multiplet and a ‘Kaluza-Klein’ tower of massive chiral and massive vector multiplets. The Lagrangian, the weakly coupled mass spectrum and degeneracies are written in [15, 16]. The local and global symmetries are (the R-symmetry is anomalous, like in the theory I above),

$$SU(N_c) \times SU(2)_L \times SU(2)_R \times U(1)_R. \tag{2.3}$$

These two theories, apparently so different, can be connected as discussed in [17] and [18] via higgsing. Indeed, giving a particular (classical) baryonic VEV to the fields  $(A_i, B_\alpha)$  and expanding around it, the field content and degeneracies of [15, 16] is reproduced. This weakly coupled field theory connection has its counterpart in the type IIB solutions dual to each of the field theories. Indeed, it is possible to connect the dual backgrounds to field theories I and II, using U-duality [17]. This connection was further studied in [18–22].

We will now explain this connection among the explicit Type IIB string backgrounds. We start from a background describing the strong dynamics of the ‘field theory II’ (the twisted compactification of five branes). A quite generic configuration of this kind can be compactly written using the  $SU(2)$  left-invariant one-forms

$$\begin{aligned} \tilde{\omega}_1 &= \cos \psi d\tilde{\theta} + \sin \psi \sin \tilde{\theta} d\tilde{\varphi}, & \tilde{\omega}_2 &= -\sin \psi d\tilde{\theta} + \cos \psi \sin \tilde{\theta} d\tilde{\varphi} \\ \tilde{\omega}_3 &= d\psi + \cos \tilde{\theta} d\tilde{\varphi} \end{aligned} \tag{2.4}$$

and the vielbeins

$$\begin{aligned} E^{x_i} &= e^{\frac{\Phi}{4}} dx_i, & E^\rho &= e^{\frac{\Phi}{4}+k} d\rho, & E^\theta &= e^{\frac{\Phi}{4}+h} d\theta, & E^\varphi &= e^{\frac{\Phi}{4}+h} \sin \theta d\varphi, \\ E^1 &= \frac{1}{2} e^{\frac{\Phi}{4}+g} (\tilde{\omega}_1 + a d\theta), & E^2 &= \frac{1}{2} e^{\frac{\Phi}{4}+g} (\tilde{\omega}_2 - a \sin \theta d\varphi), \\ E^3 &= \frac{1}{2} e^{\frac{\Phi}{4}+k} (\tilde{\omega}_3 + \cos \theta d\varphi). \end{aligned} \tag{2.5}$$

In terms of these, the background and the RR three-form read (in the Einstein frame)

$$\begin{aligned} ds_E^2 &= \sum_{i=1}^{10} (E^i)^2, \\ F_3 &= e^{-\frac{3}{4}\Phi} \left[ f_1 E^{123} + f_2 E^{\theta\varphi 3} + f_3 (E^{\theta 23} + E^{\varphi 13}) + f_4 (E^{\rho 1\theta} + E^{\rho\varphi 2}) \right] \end{aligned} \tag{2.6}$$

where we defined

$$\begin{aligned} E^{ijk..l} &= E^i \wedge E^j \wedge E^k \wedge \dots \wedge E^l, \\ f_1 &= -2N_c e^{-k-2g}, & f_2 &= \frac{N_c}{2} e^{-k-2h} (a^2 - 2ab + 1), \\ f_3 &= N_c e^{-k-h-g} (a - b), & f_4 &= \frac{N_c}{2} e^{-k-h-g} b'. \end{aligned} \tag{2.7}$$

The dilaton, as usual, is a function of the radial coordinate  $\Phi(\rho)$  and we have set  $\alpha' g_s = 1$ .

The full background is then determined by solving the equations of motion for the functions  $(a, b, \Phi, g, h, k)$ . A system of BPS equations is derived using this Ansatz (see

appendix of reference [23]). These non-linear and coupled first order equations can be arranged in a convenient form, by rewriting the functions of the background in terms of a new basis of functions  $P(\rho)$ ,  $Q(\rho)$ ,  $Y(\rho)$ ,  $\tau(\rho)$ ,  $\sigma(\rho)$  that decouples the equations (as explained in [24, 25]–[26]). We quote this change of basis in our appendix A.

Using these new variables, one can manipulate the decoupled BPS equations, solving most of them and obtaining a single decoupled second order equation for  $P(\rho)$ . All other functions are obtained from  $P(\rho)$  — see [24, 25] and our appendix A for details. The second order equation mentioned above reads

$$P'' + P' \left( \frac{P' + Q'}{P - Q} + \frac{P' - Q'}{P + Q} - 4 \coth(2\rho - 2\rho_0) \right) = 0. \tag{2.8}$$

We will refer to eq. (2.8) as the *master equation*: this is the only equation that needs solving in order to generate the large classes of solutions of Type IIB dual to “field theory II” in different circumstances (vacua, insertion of operators in the Lagrangian, etc.).<sup>2</sup>

In this paper, we will not be concerned with SUSY solutions, but they will play an important guiding role. We summarize below the small and large  $\rho$  expansions of the function  $P(\rho)$ .

### 2.1 Aspects of the SUSY solutions

Let us start from the solution of the master equation (2.8) for large values of the radial coordinate (describing the UV of the field theory II). The SUSY solutions have an expansion for  $\rho \rightarrow \infty$  of the form,

$$P = e^{4\rho/3} \left[ c_+ + \frac{e^{-8\rho/3} N_c^2}{c_+} \left( 4\rho^2 - 4\rho + \frac{13}{4} \right) + e^{-4\rho} \left( c_- - \frac{8c_+}{3} \rho \right) + \frac{N_c^4 e^{-16\rho/3}}{c_+^3} \left( \frac{18567}{512} - \frac{2781}{32} \rho + \frac{27}{4} \rho^2 - 36\rho^3 \right) \right] \tag{2.9}$$

Notice that this expansion involves two integration constants,  $c_+ > 0$  and  $c_-$ . The background functions at large  $\rho$  are written in appendix A.

Regarding the IR expansion, we look for solutions with  $P \rightarrow 0$  as  $\rho \rightarrow 0$ , in which case we find

$$P = h_1 \rho + \frac{4h_1}{15} \left( 1 - \frac{4N_c^2}{h_1^2} \right) \rho^3 + \frac{16h_1}{525} \left( 1 - \frac{4N_c^2}{3h_1^2} - \frac{32N_c^4}{3h_1^4} \right) \rho^5 + \mathcal{O}(\rho^7), \tag{2.10}$$

where  $h_1$  is again an arbitrary constant, there is of course another integration constant, taken to zero here, to avoid singularities. This gives background functions that are quoted in appendix A. Of course, there is a smooth numerical interpolation between both expansions. However, there is then only one independent parameter; given a value for one of  $\{c_+, c_-, h_1\}$ , the requirement that the solution matches *both* expansions is sufficient to determine the values of the other two.

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<sup>2</sup>As an example, the solution  $P = 2N_c \rho$  gives the background of [14, 27]. This solution and those with the same large  $\rho$  asymptotics will not be the focus of this paper.

As explained in [18], this solution corresponds to a dual field theory II in the presence of a dimension-eight operator inserted in the Lagrangian which ultimately couples the field theory to gravity. This calls for a completion in the context of field theory. This is achieved with the U-duality of [17] (which we will sometimes refer to as the ‘rotation’).

After the U-duality described in [17] is applied, we define the new vielbein (which we use in the following),

$$\begin{aligned}
 e^{xi} &= e^{\frac{\Phi}{4}} \hat{h}^{-\frac{1}{4}} dx_i, & e^\rho &= e^{\frac{\Phi}{4}+k} \hat{h}^{\frac{1}{4}} d\rho, & e^\theta &= e^{\frac{\Phi}{4}+h} \hat{h}^{\frac{1}{4}} d\theta, & e^\varphi &= e^{\frac{\Phi}{4}+h} \hat{h}^{\frac{1}{4}} \sin\theta d\varphi, \\
 e^1 &= \frac{1}{2} e^{\frac{\Phi}{4}+g} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_1 + a d\theta), & e^2 &= \frac{1}{2} e^{\frac{\Phi}{4}+g} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_2 - a \sin\theta d\varphi), \\
 e^3 &= \frac{1}{2} e^{\frac{\Phi}{4}+k} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_3 + \cos\theta d\varphi).
 \end{aligned}
 \tag{2.11}$$

The newly generated metric, RR and NS fields are

$$\begin{aligned}
 ds_E^2 &= \sum_{i=1}^{10} (e^i)^2, \\
 F_3 &= \frac{e^{-\frac{3}{4}\Phi}}{\hat{h}^{3/4}} \left[ f_1 e^{123} + f_2 e^{\theta\varphi 3} + f_3 (e^{\theta 23} + e^{\varphi 13}) + f_4 (e^{\rho 1\theta} + e^{\rho\varphi 2}) \right] \\
 H_3 &= -\kappa \frac{e^{\frac{5}{4}\Phi}}{\hat{h}^{3/4}} \left[ -f_1 e^{\theta\varphi\rho} - f_2 e^{\rho 12} - f_3 (e^{\theta 2\rho} + e^{\varphi 1\rho}) + f_4 (e^{1\theta 3} + e^{\varphi 23}) \right] \\
 C_4 &= -\kappa \frac{e^{2\Phi}}{\hat{h}} dt \wedge dx_1 \wedge dx_2 \wedge dx_3, \\
 F_5 &= \kappa e^{-\frac{5}{4}\Phi - k} \hat{h}^{\frac{3}{4}} \partial_\rho \left( \frac{e^{2\Phi}}{\hat{h}} \right) \left[ e^{\theta\varphi 123} - e^{tx_1 x_2 x_3 \rho} \right].
 \end{aligned}
 \tag{2.12}$$

We have defined

$$\hat{h} = 1 - \kappa^2 e^{2\Phi},
 \tag{2.13}$$

where  $\kappa$  is a constant that we will choose to be  $\kappa = e^{-\Phi(\infty)}$ , forcing the dilaton to be bounded at large distances. The rationale for this choice is to obtain a dual QFT decoupled from gravity. Details of this were carefully discussed in [18, 22]. The tuning  $\kappa = e^{-\Phi(\infty)}$  (also chosen in [28], though in slightly different notation) is the geometric version of the fact that, in order to eliminate an irrelevant operator in the dual field theory I, we have to finely-tune the matter content and the gauge group with which we will UV-complete the theory II after un-higgsing from the single node to the quiver. See [18] for a complete explanation. We will now move to study SUSY breaking deformations.

### 3 The SUSY-breaking deformation

The goal is to find a non supersymmetric solution with the same symmetries and structure as the ones described above. We proceed as follows: we will find a non-SUSY generalization of the system in eq. (2.6). We will solve the equations corresponding to Einstein, Maxwell, dilaton and Bianchi equations of the system. The nice properties of the SUSY formalism just explained do not apply. We will then propose series expansions for the

individual background functions  $\Phi, h, g, k, a, b$ . With the experience gained in the SUSY example, especially keeping in mind the expansions quoted in eqs. (A.3), (A.5), we propose similar asymptotics.

### 3.1 Asymptotic expansions

In the UV (large values of  $\rho$ ) our expansions take the form,

$$\begin{aligned}
 e^{2h} &\sim \sum_{i=0}^{\infty} \sum_{j=0}^i H_{ij} \rho^j e^{4(1-i)\rho/3}, & \frac{e^{2g}}{4} &\sim \sum_{i=0}^{\infty} \sum_{j=0}^i G_{ij} \rho^j e^{4(1-i)\rho/3}, \\
 \frac{e^{2k}}{4} &\sim \sum_{i=0}^{\infty} \sum_{j=0}^i K_{ij} \rho^j e^{4(1-i)\rho/3}, & e^{4\Phi} &\sim \sum_{i=1}^{\infty} \sum_{j=0}^i \Phi_{ij} \rho^j e^{4(1-i)\rho/3}, \\
 b(\rho) &\sim \sum_{i=1}^{\infty} \sum_{j=0}^i V_{ij} \rho^j e^{2(1-i)\rho/3}, & a(\rho) &\sim \sum_{i=1}^{\infty} \sum_{j=0}^i W_{ij} \rho^j e^{2(1-i)\rho/3}.
 \end{aligned}
 \tag{3.1}$$

We have found that a generic solution of this sort can be written in terms of nine integration constants. These constants are free; all other coefficients in the series expansion can be written in terms of them. The independent constants are taken to be,

$$K_{00}, K_{30}, H_{10}, H_{11}, \Phi_{10}, \Phi_{30}, W_{20}, W_{40}, V_{40}.
 \tag{3.2}$$

Note that we have found the constants  $V_{21}, W_{21}$  must vanish for this to be a solution. Also, we imposed that terms that would spoil the UV behavior of the SUSY solution (corresponding to irrelevant operators in the dual QFT) are absent from our expansions.

Without loss of generality, we relabel the UV parameters in eq. (3.2) to make contact with the SUSY case (see appendix A):

$$\begin{aligned}
 W_{40} &= 2e^{\rho_o}, & K_{00} &= \frac{2c_+}{3}, & \Phi_{10} &= e^{4\Phi_\infty}, \\
 H_{10} &= \frac{Q_o}{4}, & K_{30} &= \frac{c_- - 64e^{4\rho_o} c_+^3}{48c_+^2}.
 \end{aligned}
 \tag{3.3}$$

The independent parameters are then

$$c_+, c_-, \Phi_\infty, Q_o, \rho_o, H_{11}, W_{20}, \Phi_{30}, V_{40},
 \tag{3.4}$$

and we can recover the SUSY case by setting

$$H_{11} = \frac{1}{2}, \quad W_{20} = 0, \quad \Phi_{30} = -\frac{3e^{4\Phi_\infty}}{4c_+^2} (3 + 4Q_o), \quad V_{40} = 2e^{2\rho_o} (1 + Q_o).
 \tag{3.5}$$

For small values of the radial coordinate (which we take to end at  $\rho = 0$ ) we will propose an expansion of the form (again, imposing regularity of the solution),

$$\begin{aligned}
 e^{2h} &\sim \sum_{j=2}^{\infty} h_j \rho^j, & \frac{e^{2g}}{4} &\sim \sum_{j=0}^{\infty} g_j \rho^j, & \frac{e^{2k}}{4} &\sim \sum_{j=0}^{\infty} k_j \rho^j, \\
 e^{4\Phi} &\sim \sum_{j=0}^{\infty} f_j \rho^j, & a(\rho) &\sim \sum_{j=0}^{\infty} w_j \rho^j, & b(\rho) &\sim \sum_{j=0}^{\infty} v_j \rho^j.
 \end{aligned}
 \tag{3.6}$$



In this case the free parameters are  $k_0, f_0, k_2, v_2$  and  $w_2$ . For any value of these numbers we find a solution. To make contact with the SUSY solution in eq. (A.5), we relabel  $k_0 = h_1/2$  and  $f_0 = e^{4\phi_0}$ . We then recover the SUSY solution if the remaining three parameters take the values

$$k_2 = \frac{2}{5h_1}(h_1^2 - 4), \quad v_2 = -\frac{2}{3}, \quad w_2 = \frac{8}{3h_1} - 2. \quad (3.7)$$

If we want to restrict our attention to those solutions which match both the UV *and* IR expansions described above, we expect to have fewer independent parameters. To see this, note that we can describe the solutions either by the IR boundary conditions, so that they are parameterised by the five IR parameters  $\{h_1, \phi_0, k_2, v_2, w_2\}$ , or by the UV boundary conditions, giving a parameterisation in terms of

$$\{c_+, c_-, \Phi_\infty, Q_o, \rho_o, H_{11}, W_{20}, \Phi_{30}, V_{40}\}.$$

If a solution exists which connects our IR and UV expansions, the functions resulting from these two parameterizations must be the same. We can formally express this as a system of twelve equations,<sup>3</sup>

$$\begin{aligned} g_{h_1 \dots w_2}(\rho) &= g_{c_+ \dots V_{40}}(\rho), & g'_{h_1 \dots w_2}(\rho) &= g'_{c_+ \dots V_{40}}(\rho), \\ h_{h_1 \dots w_2}(\rho) &= h_{c_+ \dots V_{40}}(\rho), & h'_{h_1 \dots w_2}(\rho) &= h'_{c_+ \dots V_{40}}(\rho), \\ &\vdots & &\vdots \\ b_{h_1 \dots w_2}(\rho) &= b_{c_+ \dots V_{40}}(\rho), & b'_{h_1 \dots w_2}(\rho) &= b'_{c_+ \dots V_{40}}(\rho). \end{aligned}$$

However, the derivative of one of the functions can be expressed in terms of the other derivatives and the functions themselves using the constraint, so only eleven of these equations can be independent. We therefore expect to be able to solve for eleven of the fourteen parameters, leaving only three independent.

We know that the dilaton can be shifted without otherwise affecting the solution, so one of these parameters must be either  $\phi_0$  or  $\Phi_\infty$ . Additionally, we know that we also need one of  $h_1, c_+$  or  $c_-$  to describe the class of SUSY solutions. The third parameter therefore breaks SUSY.

Note that there does not appear to be anything to stop us choosing, say,  $Q_o$  to parameterise the SUSY-breaking, despite the fact that UV expansions with  $Q_o \neq -N_c$  do not break SUSY. This is explained by the fact that SUSY solutions with  $Q_o \neq -N_c$  do not have a regular IR. If we simultaneously demand that  $Q_o \neq -N_c$  *and* the IR is of the form (C.13)–(C.24) we must therefore have a non-SUSY solution. However, it seems conceptually simpler to choose the third parameter to be one which *explicitly* breaks SUSY. For our three independent parameters, we might select (in the IR)  $\{h_1, \phi_0, w_2\}$ , or (in the UV)  $\{c_+, \Phi_\infty, W_{20}\}$ .

In the next section, we will study the challenging numerical problem of finding a solution that interpolates between the small and the large  $\rho$  expansions in eqs. (3.1)–(3.7).

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<sup>3</sup>We write the functions resulting from a given choice  $\{h_1, \phi_0, k_2, v_2, w_2\}$  of the IR parameters in the form  $g_{h_1, \phi_0, k_2, v_2, w_2}(\rho)$ . Similarly, the expressions of the form  $g_{c_+, c_-, \Phi_\infty, Q_o, \rho_o, H_{11}, W_{20}, \Phi_{30}, V_{40}}(\rho)$  refer to the functions resulting from a given choice of the UV parameters.

To summarize: we want to find a numerical solution for the functions above. This will provide us with a non-SUSY deformation of the background in eq. (2.6) dual to field theory of type II. Applying the U-duality in [17]–[22] we construct a non-SUSY background of the form given in eq. (2.12), dual to a non-SUSY version of the field theory I.

## 4 Numerical analysis

In this section we show that for some values of the free constants in the IR and in the UV we can connect the asymptotics numerically. We first briefly describe our method of relating the IR and UV parameters, the details of which are relegated to appendix D, before presenting a sample solution which smoothly connects the IR and UV asymptotics of the previous section.

Our approach is to solve the equations of motion (B.4)–(B.9) numerically, using the expansions (see appendix C) as boundary conditions. We start from the IR expansions (C.13)–(C.24), meaning that our numerical solutions are described by the three SUSY-breaking parameters  $\{k_2, v_2, w_2\}$ , in addition to those already present in the SUSY case  $h_1$  and  $\phi_0$ .

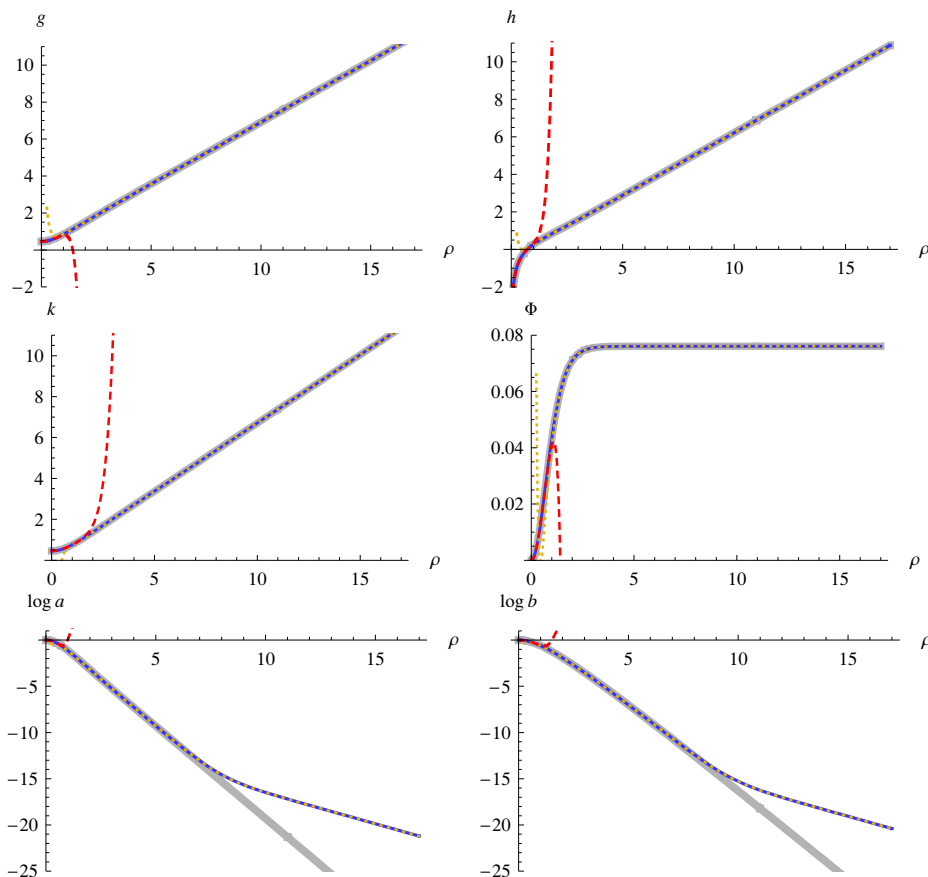
However, we have seen in section 3.1 that we expect only three independent parameters in total, one of which breaks SUSY. This would mean that in the non-SUSY case we cannot treat even the five IR parameters as independent. This is confirmed by our numerical analysis: a generic choice of the IR parameters yields UV behaviour of the form  $b \sim \pm e^{2\rho}$  and  $e^{2g} \sim e^{2h} \sim e^{2k} \sim e^{8\rho/3}$ , which is incompatible with our expansions (3.1). To obtain a solution connecting the IR and UV expansions, then, we have to determine both an appropriate combination of values for the IR parameters and the corresponding values for the UV parameters.

We achieve this using the method described in detail in appendix D. In outline, we start with a manual search of the IR parameter space, using Mathematica’s `NDSolve` to obtain numerical solutions. Having obtained a solution with the desired UV behaviour ( $b \sim e^{-2\rho/3}$ ,  $g \sim h \sim k \sim e^{4\rho/3}$ ) we optimise the match to the UV expansions (C.1)–(C.6) with respect to the parameters (3.4) using `NMinimize`.

An example of solution is shown in figure 1. As is expected from the expansions (C.1)–(C.6), the most significant modification with respect to the SUSY solution is the presence of the  $e^{-2\rho/3}$  behaviour in the UV of  $a$  and  $b$ . The size of this effect is controlled by the SUSY-breaking parameter  $W_{20}$ ; in the SUSY case we have  $W_{20} = 0$ , resulting in  $a \sim e^{-2\rho}$  and  $b \sim \rho e^{-2\rho}$ . The other functions are modified at higher order in (C.1)–(C.6), and as expected no effect is visible in figure 1. See appendix D for details of the numerical analysis.

## 5 Energy

In this section we study the energy of the non-SUSY solutions found above. For any stationary spacetime admitting foliations by a spacelike hypersurface  $\Sigma_t$ , the free energy and the energy are related via the thermodynamic relation  $F = E - TS$ . Here we are



**Figure 1.** Plots of the functions  $g$ ,  $h$ ,  $k$ ,  $\Phi$ ,  $\log a$  and  $\log b$ , obtained numerically (solid blue), together with the IR (dotted red) and UV (dashed orange) expansions (appendix C), with small deviations from the SUSY values of the parameters. The SUSY solution (grey) is included for comparison.

considering  $T = 0$  backgrounds and so we expect  $F = E$ . In this section we will first calculate the ADM energy  $E$ , for the solutions before the U-duality — we will refer to the U-duality of reference [17] as ‘rotation’. We will then repeat this calculation for the solutions after rotation and show that the energies before and after rotation are equal. As a check of our results, in appendix E we obtain the free energy using the on-shell action method and show that  $F = E$ .

### 5.1 ADM energy

Consider a non-asymptotically flat 10-dimensional background. Let  $\Sigma_t$  be a 9-dimensional constant-time slice whose 8 dimensional boundary is a constant-radius surface  $S_t^\infty$ . The regularized internal energy  $E$  is defined as [29],

$$E = -\frac{1}{8\pi} \int_{S_t^\infty} [N_t ({}^8K - {}^8K_0) + N_t^\mu p_{\mu\nu} n^\nu] dS_t^\infty. \tag{5.1}$$

$N_t$  is the lapse function,  $N_t^\mu$  is the shift vector,  $p_{\mu\nu}$  the momentum conjugate to the time derivative in the constant time-slice,  ${}^8K$  and  ${}^8K_0$  are the extrinsic curvatures of the

8 dimensional boundary  $S_t^\infty$ , for the background under consideration and the reference background respectively. Finally  $n^\nu$  is the spatial unit vector normal to the constant radius-surface  $S_t^\infty$ . It is required that both geometries induce the same metric on  $S_t^\infty$ . The matter fields should also agree at  $S_t^\infty$  or at least the difference should tend to zero as  $S_t^\infty$  goes to infinity. We will choose a SUSY background as a reference geometry.

For the metrics before rotation (2.5)–(2.6) we have  $N_t^\mu = 0$ ,  $N_t = \sqrt{|g_{00}|} = e^{\Phi/4}$ ,  $dS_t^\infty = \frac{1}{8}e^{2(\Phi+g+h)+k}$ ,  $n^\mu = \sqrt{g^{r\bar{r}}}\delta_r^\mu = e^{-\Phi/4}e^{-k}$ . The extrinsic curvature is

$${}^8K = \nabla_\mu n^\mu = \frac{1}{\sqrt{g_9}}\partial_\mu(\sqrt{g_9}n^\mu) = e^{-\Phi/4-k}\left[2(\Phi' + g' + h') + k'\right], \quad (5.2)$$

where  $g_9$  denotes the determinant of the 9-dimensional constant time slice  $\Sigma_t$ . The requirement that the induced metrics on  $S_t^\infty$  agree at the boundary implies,<sup>4</sup>

$$e^{\frac{\Phi_{ns}}{2}}e^{2g_{ns}} = e^{\frac{\Phi_{su}}{2}}e^{2g_{su}}, \quad e^{\frac{\Phi_{ns}}{2}}e^{2h_{ns}} = e^{\frac{\Phi_{su}}{2}}e^{2h_{su}}, \quad e^{\frac{\Phi_{ns}}{2}}e^{2k_{ns}} = e^{\frac{\Phi_{su}}{2}}e^{2k_{su}}, \quad (5.3)$$

and the  $g_{00}$  component agrees if

$$e^{-\frac{\Phi_{ns}}{2}} = e^{-\frac{\Phi_{su}}{2}}. \quad (5.4)$$

All the quantities in (5.3) and (5.4) are evaluated at some large but finite  $r_c$  that acts as a cutoff. Using (5.1), the energy is

$$E = -\frac{vol_8}{64\pi} \lim_{r_c \rightarrow \infty} \left\{ e^{-k_{ns}} \left( e^{2\Phi_{ns}+2g_{ns}+2h_{ns}+k_{ns}} \right)' - e^{-k_s} \left( e^{2\Phi_s+2g_s+2h_s+k_s} \right)' \right\}. \quad (5.5)$$

Before evaluating (5.5) we have to satisfy the matching conditions at the boundary, (5.3) and (5.4). In order to do this we have to use the most general asymptotics of a supersymmetric solution. As discussed in eq. (3.5), analyzing the BPS equations (see appendix A in reference [23]) we see that the most general supersymmetric UV asymptotics is obtained by replacing

$$W_{20} \rightarrow 0, \quad V_{40} \rightarrow 2e^{2\rho_0}(1 + Q_0), \quad H_{11} \rightarrow 1/2, \quad \Phi_{30} \rightarrow -3\frac{(3 + 4Q_0)}{4c_+^2}e^{4\Phi_\infty} \quad (5.6)$$

in the non-supersymmetric expansion (3.1). Notice that this substitution restores the integration constants  $Q_0$ ,  $\rho_0$  and  $e^{\Phi_\infty}$  that are usually set to  $-1$ ,  $0$  and  $1$  respectively [24, 25]. Reintroducing the integration constants is equivalent to using the shift invariance of the  $r$  coordinate (encoded in  $Q_0$  and  $\rho_0$ ) and the dilaton [30]. Adjusting these constants will allow us to satisfy the matching conditions at the boundary and cancel divergences in the energy. Given the complexity of the UV expansions the matching procedure is cumbersome but straightforward. Working to linear order in  $W_{20}$  we obtain,

$$E = \frac{1}{24\pi}c_+^2e^{2\rho_0+2\Phi_\infty}W_{20}. \quad (5.7)$$

After the duality transformations the UV asymptotics changes drastically. In this case we have  $N_t^\mu = 0$ ,  $N_t = \sqrt{|g_{00}|} = \frac{e^{-\Phi/4}}{H^{1/4}}$ ,  $dS_t^\infty = \frac{1}{8}e^{3\Phi+2g+2h+k}H^{1/2}$ ,  $n^\mu = e^{-3\Phi/4}e^{-k}H^{-1/4}$

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<sup>4</sup>The subscripts *ns* and *su* stand for non-supersymmetric and supersymmetric respectively.

and

$${}^8K = \nabla_\mu n^\mu = \frac{1}{\sqrt{g_9}} \partial_\mu (\sqrt{g_9} n^\mu) = \frac{e^{-\frac{3\Phi}{4}-k}}{2H^{5/4}} [H' + 2H(3\Phi' + 2g' + 2h' + k')]. \quad (5.8)$$

Note that here we defined  $H = e^{-2\Phi} - e^{-2\Phi_\infty}$ . The regularized energy after the rotation is

$$E = -\frac{vol_8}{64\pi^2} \lim_{r \rightarrow \infty} \{\Delta_{ns} - \Delta_{su}\} \quad (5.9)$$

where

$$\Delta \equiv \frac{e^{-\Phi-k}}{\sqrt{H}} \left( \sqrt{H} e^{3\Phi+2g+2h+k} \right)'. \quad (5.10)$$

The matching conditions now read,

$$\begin{aligned} H_{ns}^{1/2} e^{\frac{3\Phi_{ns}}{2}+2g_{ns}} &= H_{su}^{1/2} e^{\frac{3\Phi_{su}}{2}+2g_{su}}, & H_{ns}^{1/2} e^{\frac{3\Phi_{ns}}{2}+2h_{ns}} &= H_{su}^{1/2} e^{\frac{3\Phi_{su}}{2}+2h_{su}}, \\ H_{ns}^{1/2} e^{\frac{3\Phi_{ns}}{2}+2k_{ns}} &= H_{su}^{1/2} e^{\frac{3\Phi_{su}}{2}+2k_{su}}, & H_{ns}^{-\frac{1}{2}} e^{-\frac{\Phi_{ns}}{2}} &= H_{su}^{-1/2} e^{-\frac{\Phi_{su}}{2}}. \end{aligned} \quad (5.11)$$

Note that

$$\begin{aligned} \Delta &= e^{-k} \left( e^{2\Phi+2g+2h+k} \right)' + \frac{e^{\Phi+2g+2h}}{\sqrt{H}} \left( e^\Phi \sqrt{H} \right)' \\ &= \Delta^{\text{before}} + \Delta^{\text{extra}} \end{aligned} \quad (5.12)$$

where  $\Delta^{\text{before}} \equiv e^{-k} \left( e^{2\Phi+2g+2h+k} \right)'$  and  $\Delta^{\text{extra}} \equiv \frac{e^{\Phi+2g+2h}}{\sqrt{H}} \left( e^\Phi \sqrt{H} \right)'$ . We have

$$E = -\frac{vol_8}{64\pi^2} \lim_{r_c \rightarrow \infty} \left\{ (\Delta_{ns}^{\text{before}} - \Delta_s^{\text{before}}) - (\Delta_{ns}^{\text{extra}} - \Delta_s^{\text{extra}}) \right\}, \quad (5.13)$$

where all the functions are evaluated at some large but finite cutoff  $r_c$ . After adjusting the parameters to ensure that the induced metrics at the boundaries are the same, as required in (5.11), we take the cutoff to infinity. The first two terms in (5.13) are the same as in the energy before rotation (5.5). We find that — to first order in  $W_{20}$  — the matching conditions are satisfied using the same set of integration constants as before the rotation. Thus, the first two terms in (5.13) give exactly the energy before rotation. Any difference in energies will come from the *extra* terms (5.13). However, it can be shown that using the integration constants necessary to satisfy (5.11),

$$\lim_{r_c \rightarrow \infty} \left\{ (\Delta_{ns}^{\text{extra}} - \Delta_s^{\text{extra}}) \right\} = 0. \quad (5.14)$$

Thus the energy before and after rotation are the same.<sup>5</sup> Indeed, plugging in the UV expansions directly in (5.9) we obtain,

$$E = \frac{1}{24\pi} c_+^2 e^{2\rho_0+2\Phi_\infty} W_{20}. \quad (5.15)$$

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<sup>5</sup>This suggests that the ADM Energy is ‘uncharged’ under the U-duality, like probably are also uncharged various thermodynamical quantities.

A couple of comments are in order. First, note that the overall constant that appears in the energy can be changed by shifting the value of the dilaton at infinity. Thus, the physically meaningful statement is that the energies before and after rotation have the same functional dependence on the parameters,

$$E_{\text{before}} \sim E_{\text{after}} \sim c_+^2 e^{2\rho_0 + 2\Phi_\infty} W_{20}. \quad (5.16)$$

Second, this calculation can be carried out to higher order in the SUSY breaking parameter  $W_{20}$ . The divergences in the energy can be cancelled by subtracting an appropriate SUSY background. However, at higher orders there will always be a discrepancy of order  $W_{20}^2$  of the metrics at the boundary. This clearly indicates that the treatment presented in this section is valid only for soft supersymmetry breaking with small breaking parameter,  $W_{20}$ . Had we not expanded around  $W_{20} \sim 0$  the mismatch at the boundary could be arbitrarily large indicating that the non-supersymmetric solution does not approach the SUSY solution fast enough for the energy to be finite, indicating that one should find another reference background to subtract. Note that this substantiates the smallness of  $W_{20}$  seen numerically in the previous section where the solutions found have  $W_{20} \sim \mathcal{O}(10^{-5})$ .

## 6 Field theory aspects

In this section we will analyze various field theory aspects of a non-SUSY version of the quiver that we called field theory I and described below eq. (2.1) To this end, we will use the non-SUSY background one obtains when plugging our numerical solutions in section 4 in the background of eq. (2.12) dual to the field theory I.

To begin with, notice that in eq. (2.12) we did not specify the NS potential  $B_2$ . Since this will be useful below, we discuss it here (the result is different from the SUSY one).

Following the intuition gained in the SUSY example, we propose a  $B_2$  of the form

$$B_2 = b_1(\rho)e^{\rho^3} + b_2(\rho)e^{\theta\varphi} + b_3(\rho)e^{12} + b_4(\rho)e^{\theta^2} + b_5(\rho)e^{\varphi^1}, \quad (6.1)$$

by imposing that  $dB_2 = H_3$  and that the Page charge vanishes  $Q_{\text{Page, D3}} = 0$  (see below) we obtain — all details are discussed in appendix F —

$$\begin{aligned} b_1 &= \frac{e^{2g-2k}}{4\hat{h}} \left[ 2b_3\Phi' - 3\hat{h}b_3\Phi' - 4\hat{h}b_3g' - 2\hat{h}b_3' + \kappa N_c e^{\frac{3\Phi}{2} - 2h} \hat{h}^{\frac{1}{2}} (a^2 - 2ab + 1) \right] \\ b_2 &= \frac{e^{-2h}}{4\hat{h}^{1/2}} \left\{ e^{2g} \hat{h}^{\frac{1}{2}} (1 - a^2) b_3 - \frac{\kappa}{N_c} e^{\frac{3\Phi}{2}} \left[ N_c^2 (a - b)b' + 4e^{2(g+h)} \Phi' \right] \right\} \\ b_4 = b_5 &= -\frac{1}{2} e^{g-h} ab_3 - \frac{\kappa N_c e^{\frac{3\Phi}{2} - g - h} b'}{4\hat{h}^{1/2}}, \end{aligned} \quad (6.2)$$

with  $b_3(\rho)$  an undetermined function. This freedom corresponds to a gauge transformation. A general  $B_2$  can be expressed as

$$B_2 = (B_2)_{b_3=0} - \frac{1}{2} d \left( e^{2g-k+\Phi/4} \hat{h}^{1/4} b_3 e^3 \right). \quad (6.3)$$

Before computing various observables of the strongly coupled non-SUSY field theory I, we will quote another quantity that will appear frequently in the analysis. This is a periodic quantity in the string theory. Given the two cycle defined as,

$$\Sigma_2 = [\theta = \tilde{\theta}, \varphi = 2\pi - \tilde{\varphi}, \psi = \psi_0], \tag{6.4}$$

we define

$$b_0 = \frac{1}{4\pi^2} \int_{\Sigma_2} B_2. \tag{6.5}$$

When computed explicitly using the form of the  $B_2$  potential in eqs. (6.1)–(6.2), we obtain

$$b_0(\psi_0) = \frac{\kappa N_c}{4\pi} e^{2\Phi} b'(b + \cos \psi_0) - \frac{\kappa}{\pi N_c} e^{2\Phi+2h+2g} \Phi' \tag{6.6}$$

These quantities together with those appearing in the background of eq. (2.12) will be important in the study of the non-perturbative field theory dynamics.

### 6.1 Calculation of observables

We now move into the calculation of observables that will help us understand the field theory interpretation of our solution.

#### 6.1.1 Interesting asymptotic behaviors

We start by studying some combination of fields that have a particular behavior. For this it is convenient to reduce the system to five dimensions as was done in [18]. Once in five dimensions, the paper [18] shows that some of the 5-d fields are invariants under the rotation. These fields are the dilaton  $\Phi$  and the combinations

$$M_1 = 1 + a^2 + 4e^{2h-2g}, \quad M_2 = e^{2h+2g-4k} \tag{6.7}$$

The corresponding UV expansions are (see section 4 and appendix C for the notation)

$$\begin{aligned} M_1 &= 2 + (8H_{11}\rho + 3c_+ W_{20}^2 + 2Q_o) \frac{e^{-4\rho/3}}{c_+} + O(e^{-8\rho/3}), \\ M_2 &= \frac{9}{16} - \frac{27}{16} W_{20}^2 e^{-4\rho/3} + O(e^{-8\rho/3}). \end{aligned} \tag{6.8}$$

Now, suppose that we define a variable  $z = e^{-2\rho/3}$ . Any field  $\mathcal{M}$  that for  $z \rightarrow 0$  scales like  $\mathcal{M} \sim z^\Delta$  indicates either the insertion of a relevant/marginal operator or the VEV for an operator of dimension  $\Delta$  (if  $\Delta > 0$  or  $\Delta = 0$ ). On the other hand, if  $\Delta < 0$ , it indicates the insertion in the Lagrangian of an irrelevant operator of dimension  $(4 - \Delta)$ .

Using the UV expansion of the dilaton (see appendix C), we see that the dilaton corresponds to a marginal operator of dimension  $\Delta = 4$  (this is identified with a combination of gauge couplings  $g_+^2$  discussed below). The expansion of the function  $b(\rho)$  — see again appendix C — indicates that the SUSY breaking constant  $W_{20}$  corresponds to the insertion of an operator of dimension three in the lagrangian. We associate this operator with the mass for the gaugino and in an analogous way, the constant  $e^{2\rho_o}$  is associated with the

VEV for the gaugino. The association is not exact, in the sense that once SUSY is broken there could be a contribution of  $W_{20}$  to the gaugino VEV. Then, schematically we have

$$W_{20} \rightarrow m\lambda\lambda, \quad e^{2\rho_0} \rightarrow \langle \lambda\lambda \rangle \sim \Lambda_{YM}^3. \quad (6.9)$$

Following this logic, the expansion of the field  $M_1 \sim z^2$  is interpreted as the VEV for a dimension two operator [28],

$$\mathcal{U} \sim \text{tr}[AA^\dagger - B^\dagger B]. \quad (6.10)$$

This same operator gets a VEV in the SUSY case and is the one that allows us to explore the baryonic branch. Notice that the SUSY breaking coefficient  $W_{20}$  contributes to this VEV.

In the theory of type I, that has two gauge groups, we should expect two independent gaugino masses. Here, the solution is obtained by U-duality applied on a background dual to a Theory of type II, with only one gauge group. We seem to have only one integration constant associated with gaugino mass, that is  $W_{20}$ . As emphasized below eq. (3.2) the numbers  $V_{21}, W_{21}$  corresponding to behavior of the functions  $a \sim b \sim \rho e^{-2\rho/3}$ , which could be associated with this second mass parameter, turn out to vanish in our particular solution.

### 6.1.2 Energy

We take the expressions for the ADM Energy of the non-SUSY backgrounds as derived in eqs. (5.7) and (5.16) and we use the map described in eq. (6.9), we obtain that

$$E_{\text{ADM}} \sim c_+^2 e^{2\Phi(\infty)} e^{2\rho_0} W_{20} \sim m\Lambda_{YM}^3. \quad (6.11)$$

Then the energy is proportional to the gaugino mass and the strong coupling scale, as expected. The result in eq. (6.11) was first obtained in [61, 62].

### 6.1.3 Charges

We will define the Maxwell and Page Charges

$$\begin{aligned} Q_{\text{Maxwell, D3}} &= \frac{1}{16\pi^4} \int_{X_5} F_5, & Q_{\text{Maxwell, D5}} &= \frac{1}{4\pi^2} \int_{X_3} F_3, \\ Q_{\text{Page, D3}} &= \frac{1}{16\pi^4} \int_{X_5} F_5 - B_2 \wedge F_3, \end{aligned} \quad (6.12)$$

where the manifold  $X_5 = [\theta, \varphi, \tilde{\theta}, \tilde{\varphi}, \psi]$  and  $X_3 = [\tilde{\theta}, \tilde{\varphi}, \psi]$ . As in the SUSY case we have that

$$Q_{\text{Maxwell, D3}} = \frac{\kappa}{\pi} e^{2g+2h+2\Phi} \Phi', \quad Q_{\text{Maxwell, D5}} = N_c. \quad (6.13)$$

We have also imposed that  $Q_{\text{Page, D3}} = 0$  in determining the  $B_2$  field of eq. (6.1) — see appendix F for details. The vanishing of the D3-Page charge is a feature of the SUSY non-singular solutions; this is the reason why we imposed it here. It would be interesting to see if one can obtain a regular non-SUSY solution in the presence of sources indicated by a non-vanishing Page charge. Using the UV expansions, the Maxwell charge for D3 branes is

$$\begin{aligned} Q_{\text{Maxwell, D3}} &= \frac{e^{\Phi_\infty}}{\pi} \rho - \frac{1}{24\pi} (9e^{\Phi_\infty} + 4c_+^2 e^{-3\Phi_\infty} \Phi_{30}) \\ &\quad + \frac{33e^{\Phi_\infty} W_{20}^2}{32\pi} e^{-4\rho/3} + O(e^{-8\rho/3}). \end{aligned} \quad (6.14)$$



So, we see that  $W_{20}$ , the same number that determines the mass of the gaugino according to the discussion above, changes the large energy value of the Maxwell charge (correspondingly of the c-function — see below) in a subleading way, as expected.

### 6.1.4 Gauge couplings and beta functions

Let us review briefly what happens in the SUSY case. In the  $SU(N_c + n) \times SU(n)$  SUSY quiver, we have two couplings  $g_1, g_2$ . Close to the Klebanov-Witten conformal point (in the UV), the anomalous dimensions are  $\gamma_{A,B} \sim -\frac{1}{2}$ . This implies that the beta functions for the diagonal combinations

$$\beta_{\frac{8\pi^2}{g_-^2}} = \beta_{\frac{8\pi^2}{g_1^2}} - \beta_{\frac{8\pi^2}{g_2^2}} = 6N_c, \quad \beta_{\frac{8\pi^2}{g_+^2}} = \beta_{\frac{8\pi^2}{g_1^2}} + \beta_{\frac{8\pi^2}{g_2^2}} = 0. \tag{6.15}$$

As in the SUSY case, we will adopt the definitions<sup>6</sup>

$$\frac{4\pi^2}{g_+^2} = \pi e^{-\Phi}, \quad \frac{4\pi^2}{g_-^2} = 2\pi e^{-\Phi} [1 - b_0(\pi)] \tag{6.16}$$

where  $b_0(\psi_0)$  is defined in eq. (6.5)–(6.6). We obtain

$$\frac{4\pi^2}{g_-^2} = 2e^{-\Phi} \left( \pi + \frac{\kappa}{N_c} e^{2g+2h+2\Phi} \Phi' \right) - \frac{\kappa N_c}{2} e^{\Phi} (b-1)b', \tag{6.17}$$

Notice that the result is independent of the gauge artifact function  $b_3(\rho)$ . In the UV, these formulas are typically trustable. The explicit expansions are

$$\frac{4\pi^2}{g_+^2} = e^{-\Phi_\infty} \pi + \left( \frac{3e^{-\Phi_\infty} \pi}{2c_+^2} \rho - \frac{1}{4} e^{-5\Phi_\infty} \pi \Phi_{30} \right) e^{-8\rho/3} + O(W_{20}^2 e^{-4\rho}) \tag{6.18}$$

and

$$\frac{4\pi^2}{g_-^2} = \left( 2\rho - \frac{1}{3} c_+^2 \Phi_{30} e^{-4\Phi_\infty} + 2\pi e^{-\Phi_\infty} - \frac{3}{4} \right) - \frac{3}{4} W_{20} e^{-2\rho/3} + O(e^{-4\rho/3}). \tag{6.19}$$

Let us now compute the beta functions as read from the geometry. We will use the radius/energy relation

$$r = e^{2\rho/3} = \frac{\mu}{\Lambda} \tag{6.20}$$

where  $\mu$  is the energy scale at which we probe the process and  $\Lambda$  the reference or strong coupling scale of the given gauge group. Notice that this choice is arbitrary, just reflecting the possibility of choosing a scheme. Other monotonic relations  $\rho(\mu)$  would express the beta function in other schemes. To calculate the beta functions we perform

$$\begin{aligned} \beta_{\frac{8\pi^2}{g_-^2}} &= \frac{d}{d\rho} \left( \frac{8\pi^2}{g_-^2} \right) \frac{d\rho}{d \log(\mu/\Lambda)} = 6N_c + W_{20} N_c \frac{\Lambda}{\mu}, \\ \beta_{\frac{8\pi^2}{g_+^2}} &= \frac{d}{d\rho} \left( \frac{8\pi^2}{g_+^2} \right) \frac{d\rho}{d \log(\mu/\Lambda)} = O \left( \log \left( \frac{\Lambda}{\mu} \right) \frac{\Lambda^4}{\mu^4} \right). \end{aligned} \tag{6.21}$$

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<sup>6</sup>These are strictly correct in the  $N = 2$  examples and the KW fixed point. We adopt the definition here to get a handle on the non-SUSY dynamics.

We have reinstated the factor of  $N_c$  in the expansions. With a naive use of the NSVZ expression for the Wilsonian beta functions one may have interpreted this result for  $\beta_-$  as the SUSY breaking parameter  $W_{20}$  changing slightly the value of the anomalous dimensions  $\gamma_{A,B} \sim -\frac{1}{2} + O\left(W_{20}\frac{\Lambda}{\mu}\right)$ . But this is not matching with the analogous calculation for  $\beta_+$ . Hence this solution does not respect the NSVZ expression (as expected). Also, notice that while in the SUSY case, the beta functions receive corrections  $O\left(\frac{\Lambda^3}{\mu^3}\right)$ , we have here an example where the SUSY breaking parameters produce lower order corrections  $O\left(\frac{\Lambda}{\mu}\right)$ . Let us move now to IR observables.

### 6.1.5 K-strings

We will follow the treatment in [31, 32]. We need to evaluate the action for a D3 brane that extends on the manifold  $\Sigma = [t, x_1, \theta = \tilde{\theta}, \varphi = 2\pi - \tilde{\varphi}]$ . The D3 brane is sitting at  $\rho = 0$  but can move on the angle  $\psi$ , so that it will minimize its energy. The (string frame) metric seen by such a D3 brane is

$$ds_{ind}^2 = \frac{e^{\phi_0}}{\sqrt{\hat{h}_0}} \left\{ dx_{1,1}^2 + N_c \hat{h}_0 \frac{h_1}{2} [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)] \right\} \quad (6.22)$$

where we have written  $\psi = 2\chi + \pi$ , and used the values of the functions at  $\rho = 0$ :

$$e^{2g(0)} = e^{2k(0)} = \frac{h_1}{2}, \quad e^{2h(0)} = 0, \quad \Phi(0) = \phi_0, \quad a(0) = 1. \quad (6.23)$$

We have additionally written  $\hat{h}_0 \equiv \hat{h}(0) = 1 - e^{2\phi_0 - 2\Phi_\infty}$ .

The RR field and its potential are,

$$F_3|_\Sigma = 2N_c \sin^2 \chi \Omega_2 \wedge d\chi, \quad C_2|_\Sigma = N_c \left( \chi - \frac{\sin 2\chi}{2} \right) \Omega_2, \quad (6.24)$$

$$\Omega_2 = \sin \theta \, d\theta \wedge d\varphi.$$

Using eqs. (6.1)–(6.2) and the fact that  $b'(0) = \Phi'(0) = 0$  we find that the NS potential  $B_2$  vanishes.

We will turn on an electric field  $F_2 = F_{tx} \, dt \wedge dx$  in the space-time directions. Then the Born-Infeld-Wess-Zumino action gives an effective one dimensional lagrangian,

$$L_{\text{eff}} = -4\pi T_{D3} N_c \left[ \frac{h_1}{2} \sqrt{e^{2\phi_0} - \hat{h}_0 \frac{F_{tx}^2}{4}} \sin^2 \chi - \left( \chi - \frac{\sin 2\chi}{2} \right) F_{tx} \right]. \quad (6.25)$$

This is equivalent to eq. (9.8) of [31, 32], with modifications which result from the U-duality,

$$\beta = \frac{h_1}{2} \rightarrow \frac{h_1}{2} \sqrt{\hat{h}_0}, \quad e^{2\Phi(0)} \rightarrow \frac{e^{2\phi_0}}{\hat{h}_0}. \quad (6.26)$$

The rest of the discussion then proceeds as in [31, 32]. We impose the equation of motion for  $F_{tx}$  and quantize it to be an integer multiple of the tension of the fundamental string,  $\frac{\partial L_{\text{eff}}}{\partial F_{tx}} = \frac{kT_f}{2}$ . The resulting tension follows an *approximate* sine-law, as in the whole baryonic branch, including the KS solution. This also happens for D5 solutions in section 8 of reference [23].

The influence from the SUSY breaking parameters enters only through the modifications (6.26).

### 6.1.6 The non-SUSY Seiberg-like duality

We will follow the treatment in the SUSY case, as developed in [33]. The basic idea is go back to the quantity  $b_0(\psi)$ , computed as specified around eq. (6.6) and compare with what occurs in the SUSY case. The Seiberg duality is identified with a large gauge transformation such that  $b_0 \rightarrow b_0 \pm 1$  and the charge of D3 branes changes by  $\pm N_c$ .

Consider the Page charge of section 6.1.3; a large gauge transformation on  $B_2$  will change  $b_0$  by one unit. This translates in the change of  $N_c$  units in the Page charge. This works exactly as in [33].

Let us now study how the Maxwell charge ‘sees’ the Seiberg duality. We will focus on the UV part of the background, where the cascade is known to work in the SUSY case. Following the steps described in appendix G, we have

$$b_0 = \frac{\hat{h}^{1/2} e^{\Phi/2}}{\pi} [b_2 e^{2h} - b_4 (a + \cos \psi_0) e^{h+g}] = \frac{N_c}{\pi} [(f + \tilde{k}) + (\tilde{k} - f) \cos \psi_0] \quad (6.27)$$

with (using the explicit values for  $b_2, b_4$ )

$$\begin{aligned} f &= \frac{e^{\Phi/2} \hat{h}^{1/2}}{2N_c} [b_2 e^{2h} - b_4 e^{g+h} (a - 1)] = \kappa \frac{e^{2\Phi}}{8} \left[ b'(b - 1) - \frac{4}{N_c^2} e^{2g+2h} \Phi' \right], \\ \tilde{k} &= \frac{e^{\Phi/2} \hat{h}^{1/2}}{2N_c} [b_2 e^{2h} - b_4 e^{g+h} (a + 1)] = \kappa \frac{e^{2\Phi}}{8} \left[ b'(b + 1) - \frac{4}{N_c^2} e^{2g+2h} \Phi' \right], \\ \rightarrow b_0 &= \frac{\kappa N_c e^{2\Phi}}{4\pi} b'(b + \cos \psi_0) - \frac{\kappa e^{2\Phi+2h+2g}}{\pi N_c} \Phi'. \end{aligned} \quad (6.28)$$

Now, it is interesting to notice that — far in the UV — the Maxwell charge

$$Q_{\text{Max}, D3} = \frac{\kappa}{\pi} e^{2g+2h+2\Phi} \Phi' = \frac{\kappa N_c^2 e^{2\Phi}}{4\pi} b'(b + \cos \psi_0) - N_c b_0 \quad (6.29)$$

changes under a change in  $b_0$  as,

$$b_0 \sim b_0 \pm 1 \rightarrow Q_{\text{Max}, D3} \sim Q_{\text{Max}, D3} \mp N_c. \quad (6.30)$$

Specially, notice that for large values of  $\rho$  the ‘correction-term’  $b'(b + \cos \psi_0)$  is quite suppressed. This ‘correction’ is more suppressed in the SUSY case, where  $b' \sim e^{-2\rho}$ , in contrast to our non-SUSY solutions, where  $b' \sim e^{-2\rho/3}$ . The ‘Seiberg duality’, associated with a large gauge transformation of index  $k$  that changes the Maxwell charge in  $kN_c$  units is better approximated in the SUSY than in the non-SUSY case. Nevertheless, in both cases, the transformation is good at leading order.

So, as expected, far in the UV we could think that the decrease in the Maxwell charge is interpreted as a non-SUSY version of Seiberg duality that is at work here.

### 6.1.7 Domain walls

Let us compute the tension of a domain wall as the effective tension of a five brane that sits at  $\rho = 0$  and is extended along  $\Sigma_6 = [t, x_1, x_2, \tilde{\theta}, \tilde{\varphi}, \psi]$ . Before the U-duality for field

theories of type II, we use the background in eq. (2.6) and obtain that the induced metric on such five brane is (in string frame)

$$ds_{\text{ind}}^2 = e^\Phi \left[ dx_{1,2}^2 + \frac{e^{2g}}{4} (d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\varphi}^2) + \frac{e^{2k}}{4} (d\psi + \cos \tilde{\theta} d\tilde{\varphi})^2 \right] \quad (6.31)$$

The induced tension on the three dimensional wall is

$$T_{\text{eff}} = 2\pi^2 T_{D5} e^{2\Phi+2g+k} \Big|_{\rho=0} = \frac{\pi^2 T_{D5} e^{2\phi_0} h_1^{3/2}}{\sqrt{2}}, \quad (6.32)$$

which is unchanged from the SUSY result.

After the U-duality, in the background of eq. (2.12), we place a similar five brane, the induced metric is,

$$ds_{\text{ind}}^2 = e^\Phi \left[ \frac{1}{\sqrt{\hat{h}}} dx_{1,2}^2 + \sqrt{\hat{h}} \left( \frac{e^{2g}}{4} (d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\varphi}^2) + \frac{e^{2k}}{4} (d\psi + \cos \tilde{\theta} d\tilde{\varphi})^2 \right) \right]. \quad (6.33)$$

There is also an induced  $B_2$  field,

$$B_2 = \frac{1}{4} \sqrt{\hat{h}} e^{2g+\Phi/2} b_3(\rho) \sin \tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi}. \quad (6.34)$$

In order to have a gauge invariant Born-Infeld Action, we must add the  $F_2$  field on the world-volume of the brane. Indeed, the change due to a gauge transformation of the  $B_2$  field is cancelled by a (non-gauge)-transformation on  $F_2$ ,<sup>7</sup>

$$B_2 \rightarrow B_2 + d\Lambda_1, \quad F_2 \rightarrow F_2 - d\Lambda_1. \quad (6.35)$$

Hence, we need to turn on gauge field strength on the world-volume of the brane,

$$F_{\tilde{\theta}\tilde{\varphi}} = -\frac{\sqrt{\hat{h}}}{4} e^{2g+\Phi/2} b_3(\rho) \sin \tilde{\theta}. \quad (6.36)$$

This implies that the BIWZ action will be

$$S = -T_{D5} (4\pi)^2 \frac{e^{2g+k+2\Phi}}{8} \int d^{2+1}x \quad (6.37)$$

which gives the same effective tension as in eq. (6.32) and the same as in the SUSY case.<sup>8</sup> Then the tension before and after the U-duality is the same. As a side remark, one may wonder if it is possible to fix the value of  $b_3$  at  $\rho = 0$  using some physical criterium. Though it is not an invariant quantity, the small  $\rho$  expansion of

$$B_{\mu\nu} B^{\mu\nu} \sim \frac{b_3(0)^2}{\rho^2} + \dots \quad (6.39)$$

suggests that we should take  $b_3(0) = 0$  as in the SUSY case.

<sup>7</sup>One can also add a field strength  $F_2$  such that aside from cancelling the gauge-variance of  $B_2$  adds a kind of ‘magnetic charge’ to the domain wall or a Maxwell-like term in the Minkowski directions. We will not consider the addition of these extra components of  $F_2$  as they will typically raise the energy of the wall.

<sup>8</sup>Notice, that in the SUSY case we have (using eq. (F.1))

$$b_3 = -\kappa e^{3\Phi/2} \hat{h}^{-1/2} \cos \alpha, \quad (6.38)$$

which vanishes for  $\rho = 0$ .

### 6.1.8 Central charge

We will calculate the central charge of this non-susy solution. We should follow the usual procedure of [34, 35], that requires a reduction to five dimensions. However, an equivalent treatment presented in [36] indicates that for any string-frame metric of the form

$$ds^2 \sim \alpha(\rho) dx_{1,d}^2 + \alpha(\rho)\beta(\rho)d\rho^2 + g_{ij}(\rho, y)dy^i dy^j \quad (6.40)$$

we define

$$V_{\text{int}} = \int d\vec{y} \sqrt{\det g_{ij}}, \quad H = e^{-4\Phi} \alpha^d V_{\text{int}}^2. \quad (6.41)$$

and the central charge (for  $d = 3$ ) is given by  $c \sim \frac{\beta^{3/2} H^{7/2}}{(H')^3}$ , in our case

$$c \sim \frac{e^{2h+2g+2\Phi+4k} \hat{h}^2}{(2h' + 2g' + 2\Phi' + k' + \frac{\hat{h}'}{2\hat{h}})^3} \quad (6.42)$$

In the IR, the explicit expansion for the central charge is

$$\begin{aligned} c \sim & e^{2\phi_0-4\Phi_\infty} \left( e^{2\Phi_\infty} - e^{2\phi_0} \right)^2 h_1^4 \rho^5 \\ & + \frac{1}{9} e^{2\phi_0-4\Phi_\infty} \left( e^{2\Phi_\infty} - e^{2\phi_0} \right) h_1^2 \left[ e^{2\Phi_\infty} (-16 - 15h_1 k_2 + 12h_1^2) \right. \\ & \left. + e^{2\phi_0} (28 + 15h_1 k_2 - 12h_1^2 + 9v_2^2) \right] \rho^7 + O(\rho^9), \end{aligned} \quad (6.43)$$

and in the UV we have

$$c \sim e^{2\Phi_\infty} \rho^2 - \left( \frac{3}{4} e^{2\Phi_\infty} + \frac{1}{3} c_+^2 e^{-2\Phi_\infty} \Phi_{30} \right) \rho + O\left(\frac{1}{\rho}\right). \quad (6.44)$$

It is immediately clear that the SUSY-breaking parameters have no effect at the leading order in the UV. However, in the IR the question is more subtle. Although none of the explicit SUSY-breaking parameters appear in the leading term there is an effect. This is because in the SUSY case there are only two independent parameters, so that fixing  $h_1$  and  $\phi_0$  is sufficient to determine  $\Phi_\infty$ . In the non-SUSY case the discussion of sections 3.1 and 4 suggests that there is one more parameter, which breaks SUSY. This means that even with fixed  $h_1$  and  $\phi_0$  we can expect that  $\Phi_\infty$  varies as a function of the SUSY-breaking parameter. Indeed, when in appendix D we compare SUSY and non-SUSY numerical solutions with the same  $h_1$  and  $\phi_0$ , we find that  $\Phi_\infty$  changes.

### 6.1.9 Force on a probe D3-brane

We will now consider a D3 probe brane that extends in the Minkowski directions and is free to move in the radial direction as suggested in [28],

$$D3 : [t, x_1, x_2, x_3], \quad \rho(t). \quad (6.45)$$

the induced metric and RR four form field are obtained from the string frame version of eq. (2.12),

$$\begin{aligned} ds_{\text{ind}}^2 &= e^\Phi \hat{h}^{-1/2} \left[ -dt^2 (1 - \hat{h} e^{2k} \rho'^2) + dx_1^2 + dx_2^2 + dx_3^2 \right], \\ C_4 &= -\kappa \frac{e^{2\Phi}}{\hat{h}} dt \wedge dx_1 \wedge dx_2 \wedge dx_3. \end{aligned} \quad (6.46)$$

this gives an action for the D3 brane,<sup>9</sup>

$$S_{\text{BIWZ}} = -T_{D3}V_3 \int dt \left( \frac{e^\Phi}{\hat{h}} \sqrt{1 - \hat{h}e^{2k}\rho'^2} - \kappa \frac{e^{2\Phi}}{\hat{h}} \right). \quad (6.47)$$

We then approximate this for small velocities and change to the variable  $dr = e^{k+\Phi/2}d\rho$  and get

$$S = T_{D3}V_3 \int dt \left( \frac{r'^2}{2} - \frac{e^\Phi}{1 + \kappa e^\Phi} \right). \quad (6.48)$$

the force on this probe is then

$$f = \frac{e^{\Phi/2-k}}{(1 + \kappa e^\Phi)^2} \Phi'. \quad (6.49)$$

In the IR, the explicit expansion for this force is

$$f = \frac{2\sqrt{2} e^{\frac{\phi_0}{2} + 2\Phi_\infty} (4 + 3v_2^2)}{3 (e^{\phi_0} + e^{\Phi_\infty})^2 h_1^{5/2}} \rho + O(\rho^2), \quad (6.50)$$

and in the UV we have

$$f = \left[ \sqrt{\frac{3}{2}} \frac{e^{\frac{\Phi_\infty}{2}}}{c_+^{5/2}} \rho - \frac{e^{-\frac{7\Phi_\infty}{2}} (9e^{4\Phi_\infty} + 4c_+^2 \Phi_{30})}{8\sqrt{6}c_+^{5/2}} \right] e^{-10\rho/3} + O(e^{-14\rho/3}). \quad (6.51)$$

As expected, the force vanishes quickly in the far UV, where the solution approaches the KS background. Also, notice that in the radial coordinate  $r \sim e^{-2\rho/3}$ , the force is  $f \sim \frac{\log r}{r^5}$  as obtained in [37]. The SUSY breaking parameters do not enter explicitly in the expression for this (small) force at leading order. In the other hand, the breaking of SUSY explicitly changes the value of the force in the IR, as expected.

## 6.2 Field theory comments

This section relies on the ideas of [38]–[39], but most fundamentally on the analysis of the paper [39]. Similar ideas that may be useful in thinking about our string backgrounds have been put forward for example in [40, 41]. This paper studies non-SUSY deformations of  $\mathcal{N} = 1$  SQCD. We use this to analyze the quiver field theory of type I. This is as we discussed, a non-SUSY deformation of the KS-quiver. In the SUSY case, the KS-field theory can be understood as  $\mathcal{N} = 1$  SQCD with gauged flavor group and a quartic superpotential (see for example [43]) and due to this, the results of [39] are important to us. The qualitative results of the paper [39] become quantitatively accurate once we take the SUSY breaking parameters much smaller than the relevant scale of the problem, namely  $\Lambda_{\text{SQCD}}$ .<sup>10</sup> In our case, this is reflected in the small size of the coefficient  $W_{20}$ .

In this case, lots of the structure of Seiberg’s SQCD [44] remains. Particularly interesting to us is the fact that for  $\text{SU}(N_c)$  SQCD with  $N_f$  flavors and with  $N_f = N_c$ , there

<sup>9</sup>Notice that in the way things have been defined, the action for a D3 has the WZ term with the same sign as the BI term. See eq. (2.13) in the paper [19].

<sup>10</sup>Ofer Aharony explained to us that the soft breaking mass terms for squarks could have different signs under a Seiberg Duality, see [42]. This technical subtlety seems to play no role in our analysis.

exists a vacuum which breaks spontaneously the U(1)-baryonic symmetry and this vacuum persists in the non-SUSY analysis of [39]. This will be relevant for us as the case  $N_f = N_c$  is associated in the SUSY case with the last step of the cascade. We then argue that our geometry describes a situation where SUSY is broken by gaugino masses and other VEV's and the baryonic symmetry is broken by the vacuum state.

In a bit more detail, the authors of [39], added to the SQCD lagrangian a term of the form

$$L = L_{\text{SQCD}} + \Delta L, \quad \Delta L \sim \int d^4\theta M_Q(Q^\dagger e^V Q + \tilde{Q}^\dagger e^{-V} \tilde{Q}) + \int d^2\theta M_g S, \quad (6.52)$$

where  $S$  is the superfield  $S = \text{Tr}[W_\alpha W^\alpha]$ ,  $M_Q$  is a vector multiplet whose D-component equals the mass of the squarks ( $-m_q^2$ ) and  $M_g$  is a chiral multiplet whose F-component is the mass of the gluino. The authors of [39] argued that to leading order in the SUSY breaking parameters  $M_Q, M_g$  one can write an effective lagrangian in terms of mesons  $\hat{M}$ , baryons ( $B, \tilde{B}$ ) and  $S$ ,

$$\Delta L \sim \int d^4\theta B_M M_Q \text{tr}[\hat{M}^\dagger \hat{M}] + B_b M_Q (B^\dagger B + \tilde{B}^\dagger \tilde{B}) + \int d^2\theta M_g S + \dots \quad (6.53)$$

The idea is then that one should supplement the usual actions and superpotentials discussed in the SUSY case with the SUSY breaking terms above. In particular, in the case  $N_f = N_c$  we will need to minimize the potential term coming from eq. (6.53) together with the potential coming from the SUSY superpotential

$$W = W_{\text{tree}} + W_{\text{quant}} = \kappa \text{Tr}(\hat{M}^\dagger \hat{M}) + \xi (\det M - \tilde{B}B - \Lambda^{2N_c}). \quad (6.54)$$

Therefore, the vacua of the theory are those that minimize the potential coming from the tree level superpotential, together with that from the SUSY breaking term, all subject to the constraint in  $W_{\text{quant}}$ . The result is that in the non-SUSY case, one finds one vacuum state where the baryons get a VEV and the mesons are at the origin of the moduli space,  $\hat{M} = 0$ .

In this way, we have argued that our solution, which breaks SUSY due to masses for the gauginos has very similar behavior to the KS-cascade (actually to the baryonic branch in [12]). We found that many non-perturbative aspects behave very similarly as the SUSY case: the expression of the domain wall tension is basically the same as in the SUSY case. Of course, numbers will differ as the functions in the IR pick the influence of the SUSY breaking terms. The tensions for k-strings gives an approximate sine-law, again with the SUSY breaking entering the value of the tension. In the UV, the beta function for the gauge couplings of the quiver and the leading order of the central charge behave at leading order in the UV like their SUSY counterpart, but in the case of the beta functions, the first correction is purely coming from SUSY breaking contributions. The Seiberg duality (identified here with the change of the Maxwell charge under large gauge transformations of the NS B-field) behaves very approximately as in the SUSY case. One can probably make an argument for self-similarity as presented in [43].

Obviously, what happens is that the SUSY breaking terms, for example the gaugino mass indicated by the quantity  $W_{20}$ , are not important at high energies. They enter some IR observables, correcting but not changing the qualitative behavior expected from the SUSY example. This suggests that we need to think that our SUSY breaking scales are smaller than our strong coupling scale. Hence, the phenomena are the same as in the SUSY models, but numerically there will be differences. All this is in line with the analysis of [38]–[39].

### 6.3 Some words on (meta)-stability

We will briefly comment here on the stability of our solutions. The way in which perturbative stability can be directly checked is to consistently fluctuate our solutions and find the presence of tachyons. Of course, it may be the case that the precise fluctuations that we study are not those leading to instabilities, while on the other hand, finding a tachyonic mode will ensure the instability of the solution. We will not make an exhaustive analysis of fluctuations here, but postpone it for future work. We will content ourselves with presenting here some arguments in favour of the stability of the backgrounds of this paper.

To begin with, we notice the close parallels between our backgrounds and the analogous SUSY (baryonic branch) solutions. Indeed, we are deforming the backgrounds by the presence of the coefficients  $v_2$  or  $w_2$  that break supersymmetry. These coefficients, as we insisted throughout the paper, are taken to be small. This was a technical requirement to ensure a good normalization of the energy functional — see section 5. The parametric smallness of the SUSY breaking parameters implies a good mapping with the field theory results of the paper [39]. In that paper, it was shown, using field theory techniques, that the generated potential has a minimum which we believe — due to the analogous behavior — is the vacuum of the field theory that our background is dual to. This would imply that tachyons are perturbatively absent from the spectrum in analogy with the results of [39].

One may of course be worried about non-perturbative instabilities. In this case, one may appeal to an argument very similar to that presented in [63]. Indeed, in our case, the tension of domain walls is not modified by the SUSY breaking parameters — see section 6.1.7 — we can then estimate the action of a vacuum bubble to be  $S_{\text{bubble}} \sim N_c(\Lambda/m_\lambda)^3$ , where  $m_\lambda$  is the mass of the gauginos and  $\Lambda$  the strong coupling scale. As in the rest of the paper  $\Lambda$  is taken to be hierarchically bigger than the SUSY breaking scale. In this way, for these solutions quite close to the SUSY ones, the probability of decay is very small and the tunneling rate to the SUSY solutions is suppressed as  $e^{-S_{\text{bubble}}}$ .

Of course, the ideal would be to make an argument similar to the ones made in ‘fake supergravity’ [64], but in this case, to construct a fake superpotential seems a formidable task, if possible at all.

## 7 Conclusions

In this paper we have used analytical (UV and IR series expansions) and numerical methods to construct smooth backgrounds dual to particular non-SUSY field theories. The field theories in question can be thought of as softly-broken-SUSY versions of the field theories appearing in twisted D5 branes and Klebanov-Strassler quivers.



We presented some details of the derivations, involving a U-duality, a careful numerical procedure and a detailed study of many observables at low and high energies. All this supports the field theory interpretation discussed towards the end of section 6. In other words, the dynamics is basically the SUSY one, but with interesting details and deviations coming from the soft-breaking terms.

Various things come to mind that would be nice to study using the backgrounds presented here. Before the U-duality, it would be nice to study the effect on k-string tensions, domain walls and the confining behavior of the Wilson loop, as there exist in the bibliography various results for  $\mathcal{N} = 1$  Super-Yang-Mills being softly broken. Also, it would be nice to study the effect of the mass terms responsible for the breaking, on the glueball spectrum. It would also be interesting to see if our solutions can be of any help for the interesting line of metastable broken SUSY, in the sense of providing a good set of UV boundary conditions that break SUSY. This may be used to get ideas on the singular IR behaviors obtained in [37, 53–60].

It would be interesting to calculate the mass spectrum and compare it with the result of the analogous glueballs in the KS/baryonic branch solution [45–50]. Also, it would be nice to find numerically the expression for the massless glueball corresponding to the spontaneous breaking of the baryonic symmetry. The spectrum of mesons is also of interest. In particular, comparing the masses of the lightest scalar meson and the lightest vector meson we could learn if the non-SUSY background presented here provides a plausible holographic dual of nuclear forces [51, 52].

Another problem that we are not addressing here: it is known that taking the integration constant  $c_+ \rightarrow \infty$  leads, in the SUSY case, to the Klebanov-Strassler background (see [19]). It is interesting to see this working numerically and to compare the solutions found here — in the limit — with those found in the past by first order fluctuations of the KS system (see [61, 62]). In this line, a nice problem would be to find the recent solution by Dymarsky and Kuperstein [63] as a scaling of ours.

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## A Technical aspects of the SUSY background

We write in this appendix various technical aspects of the supersymmetric backgrounds. As explained in section 2 one changes the basis of functions from  $\Phi, h, g, k, a, b$  into  $P, Q, Y, \hat{\Phi}, \tau, \sigma$  in order to decouple the non-linear system of BPS equations. As explained

in [24, 25], the change of basis functions is

$$\begin{aligned}
 4e^{2h} &= \frac{P^2 - Q^2}{P \cosh \tau - Q}, & e^{2g} &= P \cosh \tau - Q, & e^{2k} &= 4Y, \\
 a &= \frac{P \sinh \tau}{P \cosh \tau - Q}, & N_c b &= \sigma.
 \end{aligned}
 \tag{A.1}$$

Using the relations above, one can solve for the decoupled BPS equations,

$$\begin{aligned}
 Q(\rho) &= (Q_0 + N_c) \cosh \tau + N_c(2\rho \cosh \tau - 1), \\
 \sinh \tau(\rho) &= \frac{1}{\sinh(2\rho - 2\rho_0)}, & \cosh \tau(\rho) &= \coth(2\rho - 2\rho_0), \\
 Y(\rho) &= \frac{P'}{8}, & e^{4\Phi} &= \frac{e^{4\Phi_0} \cosh(2\rho_0)^2}{(P^2 - Q^2)Y \sinh^2 \tau}, \\
 \sigma &= \tanh \tau(Q + N_c) = \frac{(2N_c\rho + Q_0 + N_c)}{\sinh(2\rho - 2\rho_0)}.
 \end{aligned}
 \tag{A.2}$$

and the master equation (2.8). Solving the master equation in the UV (2.9) and plugging back into eqs. (A.1)–(A.2) the background functions read at large  $\rho$ ,

$$\begin{aligned}
 e^{2h} &\sim \left[ \frac{c_+ e^{4\rho/3}}{4} + \frac{N_c}{4}(2\rho - 1) + \frac{N_c^2 e^{-4\rho/3}}{16c_+} (16\rho^2 - 16\rho + 13) + \frac{e^{-8\rho/3}}{4} \left( c_- - c_+ \left( 2 + \frac{8\rho}{3} \right) \right) \right] \\
 \frac{e^{2g}}{4} &\sim \left[ \frac{c_+ e^{4\rho/3}}{4} - \frac{N_c}{4}(2\rho - 1) + \frac{N_c^2 e^{-4\rho/3}}{16c_+} (16\rho^2 - 16\rho + 13) + \frac{e^{-8\rho/3}}{4} \left( c_- + c_+ \left( 2 - \frac{8\rho}{3} \right) \right) \right] \\
 \frac{e^{2k}}{4} &\sim \left[ \frac{c_+ e^{4\rho/3}}{6} - \frac{N_c^2 e^{-4\rho/3}}{24c_+} (4\rho - 5)^2 + \frac{e^{-8\rho/3}}{3} \left( c_+ \left( \frac{8\rho}{3} - 1 \right) - c_- \right) \right] \\
 e^{4\Phi - 4\Phi_0} &\sim \left[ 1 + \frac{3N_c^2 e^{-8\rho/3}}{4c_+^2} (1 - 8\rho) + \frac{3N_c^4 e^{-16\rho/3}}{512c_+^4} (2048\rho^3 + 1152\rho^2 + 2352\rho - 775) \right] \\
 a &\sim 2e^{-2\rho} + \frac{2N_c}{c_+} (2\rho - 1)e^{-10\rho/3} + \frac{2N_c^2}{c_+^2} (2\rho - 1)^2 e^{-14\rho/3} \\
 b &= \frac{2\rho}{\sinh(2\rho)} \sim 4\rho e^{-\rho} + 4\rho e^{-6\rho}
 \end{aligned}
 \tag{A.3}$$

The geometry in eq. (2.6) asymptotes to the conifold after using the expansions above. In the IR we have using eq. (2.10) and (A.1),

$$\begin{aligned}
 e^{2h} &\sim \frac{h_1 \rho^2}{2} + \frac{4}{45} \left( -6h_1 + 15N_c - \frac{16N_c^2}{h_1} \right) \rho^4 + \mathcal{O}(\rho^6), \\
 \frac{e^{2g}}{4} &\sim \frac{h_1}{8} + \frac{1}{15} \left( 3h_1 - 5N_c - \frac{2N_c^2}{h_1} \right) \rho^2 + \frac{2(3h_1^4 + 70h_1^3 N_c - 144h_1^2 N_c^2 - 32N_c^4) \rho^4}{1575h_1^3} \\
 &\quad + \mathcal{O}(\rho^6), \\
 \frac{e^{2k}}{4} &\sim \frac{h_1}{8} + \frac{(h_1^2 - 4N_c^2) \rho^2}{10h_1} + \frac{(6h_1^4 - 8h_1^2 N_c^2 - 64N_c^4) \rho^4}{315h_1^3} + \mathcal{O}(\rho^6),
 \end{aligned}$$

$$e^{4(\Phi-\Phi_0)} \sim 1 + \frac{64N_c^2\rho^2}{9h_1^2} + \frac{128N_c^2(-15h_1^2 + 124N_c^2)\rho^4}{405h_1^4} + \mathcal{O}(\rho^6), \quad (\text{A.4})$$

$$a \sim 1 + \left(-2 + \frac{8N_c}{3h_1}\right)\rho^2 + \frac{2(75h_1^3 - 232h_1^2N_c + 160h_1N_c^2 + 64N_c^3)\rho^4}{45h_1^3} + \mathcal{O}(\rho^6),$$

$$b = \frac{2\rho}{\sinh(2\rho)} \sim 1 - \frac{2}{3}\rho^2 + \frac{14}{45}\rho^4 + \mathcal{O}(\rho^6). \quad (\text{A.5})$$

This space is free of singularities as can be checked by computing invariants.

## B Euler-Lagrange equations of motion

Here we write the full equations of motion, for reference. We start with the effective Lagrangian and the constraint and then write the equations of motion. We set  $N_c = 1$  for simplicity.

The effective Lagrangian is  $L = T - U$ , with

$$T = -\frac{1}{128}e^{2\Phi} \left\{ e^{4g} (a')^2 + (b')^2 N_c^2 - 8e^{2(g+h)} \left[ 2g' (2h' + k' + 2\Phi') + (g')^2 + 2h' (k' + 2\Phi') + (h')^2 + 2\Phi' (k' + \Phi') \right] \right\}, \quad (\text{B.1})$$

$$U = \frac{1}{256}e^{-2(g+h-\Phi)} \left[ a^4 e^{4g} (N_c^2 + e^{4k}) - 4a^3 b e^{4g} N_c^2 + 2a^2 e^{2g} (2b^2 e^{2g} N_c^2 + e^{2g} N_c^2 + 4e^{2h} N_c^2 - 8e^{2(g+h+k)} + 4e^{4g+2h} - e^{2g+4k} + 4e^{2h+4k}) - 4abe^{2g} N_c^2 (e^{2g} + 4e^{2h}) + 8b^2 N_c^2 e^{2(g+h)} + e^{4g} N_c^2 + 16e^{4h} N_c^2 - 16e^{2(2g+h+k)} - 64e^{2(g+2h+k)} + e^{4(g+k)} + 16e^{4(h+k)} \right]. \quad (\text{B.2})$$

The constraint is

$$\begin{aligned} 0 &= T + U \\ &= e^{-2(g+h-\Phi)} \left[ -2(a')^2 e^{6g+2h} + a^4 e^{4g} (e^{4k} + 1) - 4a^3 b e^{4g} + 2a^2 e^{2g} (2b^2 e^{2g} - 8e^{2(g+h+k)} + 4e^{4g+2h} - e^{2g+4k} + e^{2g} + 4e^{2h+4k} + 4e^{2h}) - 4abe^{2g} (e^{2g} + 4e^{2h}) - 2(b')^2 e^{2(g+h)} + 64e^{4(g+h)} g' h' + 32e^{4(g+h)} g' k' + 64e^{4(g+h)} g' \Phi' + 16e^{4(g+h)} (g')^2 + 8b^2 e^{2(g+h)} + 32e^{4(g+h)} h' k' + 64e^{4(g+h)} h' \Phi' + 16e^{4(g+h)} (h')^2 + 32e^{4(g+h)} k' \Phi' - 16e^{2(2g+h+k)} - 64e^{2(g+2h+k)} + 32e^{4(g+h)} (\Phi')^2 + e^{4(g+k)} + e^{4g} + 16e^{4(h+k)} + 16e^{4h} \right]. \quad (\text{B.3}) \end{aligned}$$

The equations of motion are:

$$g'' = \frac{1}{8}e^{-4g-2h} \left[ e^{6g} (a')^2 - 4a^2 e^{2g+4k} - 4a^2 e^{2g} + 4a^2 e^{6g} + 8abe^{2g} - e^{2g} (b')^2 - 4b^2 e^{2g} - 16e^{4g+2h} g' h' - 16e^{4g+2h} g' \Phi' - 16e^{4g+2h} (g')^2 + 32e^{2g+2h+2k} - 16e^{2h+4k} - 16e^{2h} \right] \quad (\text{B.4})$$

$$h'' = -\frac{1}{8}e^{-2g-4h} \left[ (a')^2 e^{4g+2h} + a^4 e^{2g+4k} + a^4 e^{2g} - 4a^3 b e^{2g} + 4a^2 b^2 e^{2g} - 8a^2 e^{2g+2h+2k} + 4a^2 e^{4g+2h} - 2a^2 e^{2g+4k} + 2a^2 e^{2g} + 4a^2 e^{2h+4k} + 4a^2 e^{2h} - 4abe^{2g} - 8abe^{2h} + e^{2h} (b')^2 + 4b^2 e^{2h} + 16e^{2g+4h} g' h' + 16e^{2g+4h} h' \Phi' + 16e^{2g+4h} (h')^2 - 8e^{2g+2h+2k} + e^{2g+4k} + e^{2g} \right] \quad (\text{B.5})$$

$$k'' = \frac{1}{8}e^{-4g-4h} \left( a^4 e^{4g+4k} - a^4 e^{4g} + 4a^3 b e^{4g} - 4a^2 b^2 e^{4g} + 8a^2 e^{2g+2h+4k} - 8a^2 e^{2g+2h} - 8a^2 e^{6g+2h} - 2a^2 e^{4g+4k} - 2a^2 e^{4g} + 16abe^{2g+2h} + 4abe^{4g} - 8b^2 e^{2g+2h} - 16e^{4g+4h} g' k' - 16e^{4g+4h} h' k' - 16e^{4g+4h} k' \Phi' + e^{4g+4k} - e^{4g} + 16e^{4h+4k} - 16e^{4h} \right) \quad (\text{B.6})$$

$$\Phi'' = \frac{1}{8}e^{-4g-4h} \left[ a^4 e^{4g} - 4a^3 b e^{4g} + 4a^2 b^2 e^{4g} + 8a^2 e^{2g+2h} - 16abe^{2g+2h} + 2a^2 e^{4g} - 4abe^{4g} + 2 (b')^2 e^{2g+2h} + 8b^2 e^{2g+2h} - 16e^{4g+4h} g' \Phi' - 16e^{4g+4h} h' \Phi' - 16e^{4g+4h} (\Phi')^2 + e^{4g} + 16e^{4h} \right] \quad (\text{B.7})$$

$$a'' = e^{-4g-2h} \left( -4a' e^{4g+2h} g' - 2a' e^{4g+2h} \Phi' + a^3 e^{2g+4k} + a^3 e^{2g} - 3a^2 b e^{2g} + 2ab^2 e^{2g} - 8ae^{2g+2h+2k} + 4ae^{4g+2h} - ae^{2g+4k} + ae^{2g} + 4ae^{2h+4k} + 4ae^{2h} - be^{2g} - 4be^{2h} \right) \quad (\text{B.8})$$

$$b'' = -e^{-2h} \left( a^3 e^{2g} - 2a^2 b e^{2g} + ae^{2g} + 4ae^{2h} + 2e^{2h} b' \Phi' - 4be^{2h} \right) \quad (\text{B.9})$$

## C Explicit expansion of the functions

Here we include the explicit solutions for the expansions (3.1) and (3.7). In this section we again set  $N_c = 1$ .

### C.1 UV

$$e^{2g} = c_+ e^{4\rho/3} - (2c_+ W_{20}^2 + 4H_{11}\rho + Q_o) - \frac{1}{48c_+} \left\{ -12H_{11} [(32\rho - 6)Q_o - c_+ W_{20}^2(8\rho + 93)] - 12c_+ W_{20}^2 Q_o + 72c_+^2 W_{20} e^{2\rho_o} + 120c_+^2 W_{20}^4 \rho - 26c_+^2 W_{20}^4 + 12c_+^2 \Phi_{30} e^{-4\Phi_\infty} - 72\rho - 24H_{11}^2 (32\rho^2 - 12\rho + 15) - 48Q_o^2 + 9 \right\} e^{-4\rho/3} + O(e^{-8\rho/3}) \quad (\text{C.1})$$

$$\begin{aligned}
 e^{2h} &= \frac{c_+}{4} e^{4\rho/3} + \left( H_{11}\rho + \frac{Q_o}{4} \right) \\
 &\quad - \frac{1}{192c_+} \left\{ -12H_{11} [c_+ W_{20}^2 (88\rho + 75) + (32\rho - 6)Q_o] - 396c_+ W_{20}^2 Q_o \right. \\
 &\quad + 264c_+^2 W_{20} e^{2\rho_o} + 440c_+^2 W_{20}^4 \rho - 626c_+^2 W_{20}^4 + 12c_+^2 \Phi_{30} e^{-4\Phi_\infty} - 72\rho \\
 &\quad \left. - 24H_{11}^2 (32\rho^2 - 12\rho + 15) - 48Q_o^2 + 9 \right\} e^{-4\rho/3} + O(e^{-8\rho/3}) \tag{C.2}
 \end{aligned}$$

$$\begin{aligned}
 e^{2k} &= \frac{2c_+}{3} e^{4\rho/3} + \frac{c_+ W_{20}^2}{3} - \frac{1}{24c_+} \left[ 4H_{11}(16\rho - 9) (3c_+ W_{20}^2 + 2Q_o) + 16Q_o^2 \right. \\
 &\quad + 84c_+ W_{20}^2 Q_o - 72c_+^2 W_{20} e^{2\rho_o} - 120c_+^2 W_{20}^4 \rho + 190c_+^2 W_{20}^4 - 24\rho - 9 \\
 &\quad \left. + 4c_+^2 \Phi_{30} e^{-4\Phi_\infty} + 8H_{11}^2 (32\rho^2 - 36\rho + 27) \right] e^{-4\rho/3} + O(e^{-8\rho/3}) \tag{C.3}
 \end{aligned}$$

$$\begin{aligned}
 e^{4\Phi} &= e^{4\Phi_\infty} + \left( \Phi_{30} - \frac{6\rho e^{4\Phi_\infty}}{c_+^2} \right) e^{-8\rho/3} \\
 &\quad + \frac{W_{20}^2 (32c_+^2 \Phi_{30} - 3(64\rho + 25)e^{4\Phi_\infty})}{24c_+^2} e^{-4\rho} + O(e^{-16\rho/3}) \tag{C.4}
 \end{aligned}$$

$$\begin{aligned}
 a &= W_{20} e^{-2\rho/3} + \left( \frac{4H_{11} W_{20} \rho}{c_+} + 2e^{2\rho_o} + \frac{10W_{20}^3 \rho}{3} \right) e^{-2\rho} \\
 &\quad + \frac{1}{48c_+^2} \left( 4c_+ \left\{ H_{11} [96\rho e^{2\rho_o} + W_{20}^3 (160\rho^2 + 552\rho + 495)] + 2Q_o [12e^{2\rho_o} \right. \right. \\
 &\quad \left. \left. + W_{20}^3 (20\rho + 87)] \right\} + c_+^2 [W_{20}^5 (391 - 480\rho) - 288W_{20}^2 e^{2\rho_o}] \right. \\
 &\quad + 12W_{20} [2H_{11} (40\rho + 27) Q_o \\
 &\quad \left. + 6H_{11}^2 (32\rho^2 + 36\rho + 43) + 8Q_o^2 - 15] \right) e^{-10\rho/3} + O(e^{-4\rho}) \tag{C.5}
 \end{aligned}$$

$$\begin{aligned}
 b &= \frac{9}{4} W_{20} e^{-2\rho/3} + \left\{ \left[ 4e^{2\rho_o} + \frac{W_{20}^3}{6} (20\rho - 23) - \frac{12W_{20} Q_o}{6c_+} \right] \rho + V_{40} \right\} e^{-2\rho} \\
 &\quad + \frac{3W_{20} e^{-4\Phi_\infty}}{512c_+^2} \left\{ e^{4\Phi_\infty} \left[ -3456H_{11} (3c_+ W_{20}^2 + 2Q_o) - 3240c_+ W_{20}^2 Q_o \right. \right. \\
 &\quad + 192c_+^2 V_{40} W_{20} + 2819c_+^2 W_{20}^4 - 35712H_{11}^2 - 576Q_o^2 + 882] \\
 &\quad - 16\rho e^{4\Phi_\infty} [144H_{11} (3c_+ W_{20}^2 + 2Q_o) + 24c_+ W_{20}^2 Q_o \\
 &\quad - c_+^2 W_{20} (48e^{2\rho_o} + 269W_{20}^3) + 1728H_{11}^2 - 18] + 3024c_+^2 W_{20} e^{2\rho_o + 4\Phi_\infty} \\
 &\quad \left. - 128\rho^2 e^{4\Phi_\infty} (72H_{11}^2 - 5c_+^2 W_{20}^4) - 48c_+^2 \Phi_{30} \right\} e^{-10\rho/3} + O(e^{-4\rho}) \tag{C.6}
 \end{aligned}$$

We can see the effect of the SUSY-breaking parameters by looking at functions of the form  $\Delta(e^{2g}) = e^{2g} - e^{2g_{\text{SUSY}}}$ , where  $g$  corresponds to the full solution and  $g_{\text{SUSY}}$  corresponds to the SUSY case with  $Q_o = -N_c$  and  $\rho_o = 0$ . Note that in general only one of the two solutions will go to the regular IR — if we start with a SUSY solution and turn on one of

the SUSY-breaking parameters while keeping e.g.  $c_+$  fixed, we will have to change  $c_-$  to recover the regular IR. Those SUSY-breaking parameters which have non-zero values in the SUSY case are expressed here in terms of e.g.  $\Delta H_{11} = H_{11} - H_{11}^{\text{SUSY}}$ .

$$\begin{aligned}
 \Delta(e^{2g}) &= (-2c_+ W_{20}^2 - \Delta Q_o - 4\Delta H_{11}\rho) \\
 &+ \frac{e^{-4\Phi_\infty}}{24c_+} \left\{ -3c_+ W_{20}^2 e^{4\Phi_\infty} \left[ -2\Delta Q_o + 2\Delta H_{11}(8\rho + 93) + 8\rho + 95 \right] \right. \\
 &- c_+^2 \left[ 36W_{20}e^{2\rho_o+4\Phi_\infty} + W_{20}^4(60\rho - 13)e^{4\Phi_\infty} + 6\Delta\Phi_{30} \right] \\
 &+ 6e^{4\Phi_\infty} \left[ \Delta H_{11} \left( (32\rho - 6)\Delta Q_o + 64\rho^2 - 56\rho + 36 \right) + \Delta Q_o(4\Delta Q_o + 16\rho - 11) \right. \\
 &\left. \left. + \Delta H_{11}^2(64\rho^2 - 24\rho + 30) \right] \right\} e^{-4\rho/3} + O(e^{-8\rho/3}) \tag{C.7}
 \end{aligned}$$

$$\begin{aligned}
 \Delta(e^{2h}) &= \left( \frac{\Delta Q_o}{4} + \Delta H_{11}\rho \right) + \frac{e^{-4\Phi_\infty}}{96c_+} \left( 3c_+ W_{20}^2 e^{4\Phi_\infty} \left[ 66\Delta Q_o + 2\Delta H_{11}(88\rho + 75) + 88\rho + 9 \right] \right. \\
 &- c_+^2 \left[ 132W_{20}e^{2\rho_o+4\Phi_\infty} + W_{20}^4(220\rho - 313)e^{4\Phi_\infty} + 6\Delta\Phi_{30} \right] \\
 &+ 6e^{4\Phi_\infty} \left\{ \Delta H_{11} \left[ (32\rho - 6)\Delta Q_o + 64\rho^2 - 56\rho + 36 \right] + \Delta Q_o(4\Delta Q_o + 16\rho - 11) \right. \\
 &\left. \left. + \Delta H_{11}^2(64\rho^2 - 24\rho + 30) \right\} \right) e^{-4\rho/3} + O(e^{-8\rho/3}) \tag{C.8}
 \end{aligned}$$

$$\begin{aligned}
 \Delta(e^{2k}) &= \frac{1}{3}c_+ W_{20}^2 + \frac{e^{-4\Phi_\infty}}{12c_+} \left( -3c_+ W_{20}^2 e^{4\Phi_\infty} \left[ 14\Delta Q_o + 2\Delta H_{11}(16\rho - 9) + 16\rho - 23 \right] \right. \\
 &+ c_+^2 \left[ 36W_{20}e^{2\rho_o+4\Phi_\infty} + 5W_{20}^4(12\rho - 19)e^{4\Phi_\infty} - 2\Delta\Phi_{30} \right] \\
 &- 2e^{4\Phi_\infty} \left\{ 2\Delta H_{11} \left[ (16\rho - 9)\Delta Q_o + 4(8\rho^2 - 13\rho + 9) \right] \right. \\
 &+ \Delta Q_o(4\Delta Q_o + 16\rho - 17) \\
 &\left. \left. + \Delta H_{11}^2(64\rho^2 - 72\rho + 54) \right\} \right) e^{-4\rho/3} + O(e^{-8\rho/3}) \tag{C.9}
 \end{aligned}$$

$$\Delta(e^{4\Phi}) = \Delta\Phi_{30}e^{-8\rho/3} + \frac{W_{20}^2}{24c_+^2} \left[ 32c_+^2\Delta\Phi_{30} - 3(64\rho + 17)e^{4\Phi_\infty} \right] e^{-4\rho} + O(e^{-16\rho/3}) \tag{C.10}$$

$$\begin{aligned}
 \Delta a &= W_{20}e^{-2\rho/3} + \frac{1}{c_+} \left[ \frac{2}{3}c_+ (3e^{2\rho_o} + 5W_{20}^3\rho - 3) + 2W_{20}(2\Delta H_{11} + 1)\rho \right] e^{-2\rho} \\
 &+ \frac{1}{48c_+^2} \left( 2c_+ \left\{ W_{20}^3 \left[ 4(20\rho + 87)\Delta Q_o + 2\Delta H_{11}(160\rho^2 + 552\rho + 495) \right. \right. \right. \\
 &\left. \left. \left. + 160\rho^2 + 472\rho + 147 \right] + 48 \left[ 4\Delta H_{11}\rho e^{2\rho_o} + \Delta Q_o e^{2\rho_o} + (2\rho - 1)(e^{2\rho_o} - 1) \right] \right\} \right. \\
 &+ c_+^2 \left[ W_{20}^5(391 - 480\rho) - 288W_{20}^2 e^{2\rho_o} \right] + 6W_{20} \left\{ + (80\rho + 22)\Delta Q_o \right. \\
 &+ 4\Delta H_{11} \left[ (40\rho + 27)\Delta Q_o + 96\rho^2 + 68\rho + 102 \right] + 16\Delta Q_o^2 + 12\Delta H_{11}^2(32\rho^2 + 36\rho + 43) \\
 &\left. \left. + 96\rho^2 + 28\rho + 61 \right\} \right) e^{-10\rho/3} + O(e^{-14\rho/3}) \tag{C.11}
 \end{aligned}$$

$$\begin{aligned}
\Delta b = & \frac{9}{4}W_{20}e^{-2\rho/3} + \left[ c_+ \{ \rho [24 (e^{2\rho_0} - 1) + W_{20}^3(20\rho - 23)] + 6V_{40} \} \right. \\
& - 12W_{20}\rho (\Delta Q_o - 1) \left. \right] \frac{e^{-2\rho}}{6c_+} + \frac{3W_{20}e^{-4\Phi_\infty}}{512c_+^2} \left( c_+^2 \left\{ 48W_{20}e^{4\Phi_\infty} [(16\rho + 63)e^{2\rho_0} + 4V_{40}] \right. \right. \\
& + W_{20}^4 (640\rho^2 + 4304\rho + 2819) e^{4\Phi_\infty} - 48\Delta\Phi_{30} \left. \left. \right\} \right. \\
& - 24c_+W_{20}^2e^{4\Phi_\infty} [(16\rho + 135)\Delta Q_o + 144\Delta H_{11}(2\rho + 3) + 128\rho + 81] \\
& - 18e^{4\Phi_\infty} \left\{ 64\Delta H_{11} [(4\rho + 6)\Delta Q_o + 8\rho^2 + 20\rho + 25] \right. \\
& + 128(\rho + 1)\Delta Q_o + 32\Delta Q_o^2 + 64\Delta H_{11}^2 (8\rho^2 + 24\rho + 31) \\
& \left. \left. + 128\rho^2 + 240\rho + 289 \right\} \right) e^{-10\rho/3} + O(e^{-14\rho/3}) \tag{C.12}
\end{aligned}$$

**C.2 IR**

$$\begin{aligned}
e^{2g} = & \frac{h_1}{2} - \frac{1}{8} \left[ 4k_2 - h_1 (w_2^2 + 4) + \frac{4}{h_1} (v_2^2 + 4) \right] \rho^2 + \frac{1}{20160h_1^3} \left\{ 1600h_1k_2 (3v_2^2 + 8) \right. \\
& - 8h_1^2 [210k_2^2 + 144v_2 (w_2 - 3) w_2 + 3v_2^2 (105w_2^2 + 168w_2 + 580) \\
& + 4 (55w_2^2 + 360w_2 + 652)] - 480h_1^3k_2 (w_2^2 + 18w_2 + 18) \\
& + 3h_1^4 (75w_2^4 + 432w_2^3 + 488w_2^2 + 960w_2 + 1520) \\
& \left. + 16 (405v_2^4 + 1592v_2^2 + 656) \right\} \rho^4 + O(\rho^6) \tag{C.13}
\end{aligned}$$

$$\begin{aligned}
e^{2h} = & \frac{h_1}{2}\rho^2 + \frac{1}{72} \left[ 24k_2 - 3h_1 (3w_2^2 + 4) - \frac{4}{h_1} (9v_2^2 - 4) \right] \rho^4 \\
& + \frac{1}{907200h_1^3} \left\{ 24h_1^2 [5250k_2^2 + 432v_2w_2 (23w_2 + 57) \right. \\
& - 9v_2^2 (189w_2^2 + 2016w_2 + 940) - 20940w_2^2 + 8640w_2 + 5680] \\
& + 480h_1k_2 (9v_2^2 - 172) - 360h_1^3k_2 (627w_2^2 - 432w_2 + 44) \\
& + 9h_1^4 (2007w_2^4 - 6624w_2^3 + 11352w_2^2 - 5760w_2 + 1520) \\
& \left. + 16 (19359v_2^4 + 27288v_2^2 - 22672) \right\} \rho^6 + O(\rho^8) \tag{C.14}
\end{aligned}$$

$$\begin{aligned}
e^{2k} = & \frac{h_1}{2} + k_2\rho^2 + \frac{2}{315h_1^3} \left\{ 5h_1k_2 (27v_2^2 + 2) + \frac{15}{4}h_1^3k_2 (9w_2^2 + 36w_2 + 22) \right. \\
& + \frac{3}{2}h_1^2 [175k_2^2 + 12v_2w_2 (w_2 + 4) - 30v_2^2 + 30w_2^2 + 120w_2 + 208] \\
& - \frac{9}{16}h_1^4 (w_2^4 + 8w_2^3 + 36w_2^2 + 80w_2 + 80) \\
& \left. + 9v_2^4 + 36v_2^2 - 528 \right\} \rho^4 + O(\rho^6) \tag{C.15}
\end{aligned}$$

$$\begin{aligned}
 e^{4\Phi} = & e^{4\phi_0} + \frac{4e^{4\phi_0}}{3h_1^2} (3v_2^2 + 4) \rho^2 + \frac{e^{4\phi_0}}{135h_1^4} \left\{ -60h_1k_2 (3v_2^2 - 8) \right. \\
 & + 3h_1^2 \left[ 3v_2^2 (9w_2^2 + 36w_2 + 40) - 36v_2w_2 (w_2 + 4) - 176 \right] \\
 & \left. + 4 (243v_2^4 + 672v_2^2 + 944) \right\} \rho^4 + O(\rho^6)
 \end{aligned} \tag{C.16}$$

$$\begin{aligned}
 a = & 1 + w_2\rho^2 + \frac{1}{90h_1^2} \left\{ w_2 \left[ 150h_1k_2 - 3h_1^2 (6w_2^2 - 9w_2 + 28) + 400 \right] \right. \\
 & \left. + 36v_2^2 (w_2 + 2) - 72v_2 (w_2 + 2) \right\} \rho^4 + O(\rho^6)
 \end{aligned} \tag{C.17}$$

$$\begin{aligned}
 b = & 1 + v_2\rho^2 - \frac{1}{90h_1^2} \left\{ v_2 \left[ 30h_1k_2 - h_1^2 (9w_2^2 + 36w_2 + 60) + 176 \right] \right. \\
 & \left. + 9h_1^2 w_2 (w_2 + 4) + 72v_2^3 \right\} \rho^4 + O(\rho^6)
 \end{aligned} \tag{C.18}$$

The effect of the SUSY-breaking parameters can be seen from the differences:

$$\begin{aligned}
 \Delta(e^{2g}) = & \frac{1}{24h_1} \left[ -4h_1 (3\Delta k_2 - 4\Delta w_2) + 3h_1^2 (\Delta w_2 - 4) \Delta w_2 + 4 (4 - 3\Delta v_2) \Delta v_2 \right] \rho^2 \\
 & + \frac{1}{60480h_1^3} \left\{ -24h_1^2 \left[ 320\Delta k_2 (\Delta w_2 + 7) + 210\Delta k_2^2 - 4\Delta v_2 (69\Delta w_2^2 + 224) \right. \right. \\
 & + 3\Delta v_2^2 (105\Delta w_2^2 - 252\Delta w_2 + 584) + 8 (112 - 129\Delta w_2) \Delta w_2 \left. \right] \\
 & + 16\Delta v_2 (1215\Delta v_2^3 - 3240\Delta v_2^2 + 3216\Delta v_2 - 2944) \\
 & + 9h_1^4 \Delta w_2 (75\Delta w_2^3 - 168\Delta w_2^2 - 368\Delta w_2 + 896) \\
 & - 96h_1^3 \left[ 3\Delta k_2 (5\Delta w_2^2 + 70\Delta w_2 - 56) + \Delta w_2 (-75\Delta w_2^2 + 126\Delta w_2 + 184) \right] \\
 & + 64h_1 \left[ 3\Delta k_2 (75\Delta v_2^2 - 100\Delta v_2 + 264) - 126\Delta v_2^2 (5\Delta w_2 - 6) \right. \\
 & \left. + 552\Delta v_2 \Delta w_2 + 464\Delta w_2 \right] \left. \right\} \rho^4 + O(\rho^6)
 \end{aligned} \tag{C.19}$$

$$\begin{aligned}
 \Delta(e^{2h}) = & \frac{1}{24h_1} \left[ 8h_1 (\Delta k_2 - 2\Delta w_2) - 3h_1^2 (\Delta w_2 - 4) \Delta w_2 + 4 (4 - 3\Delta v_2) \Delta v_2 \right] \rho^4 \\
 & + \frac{1}{302400h_1^3} \left\{ 24h_1^2 \left[ -80\Delta k_2 (209\Delta w_2 - 490) + 1750\Delta k_2^2 \right. \right. \\
 & + \Delta v_2^2 (-567\Delta w_2^2 - 3780\Delta w_2 + 7032) + 4\Delta v_2 (1017\Delta w_2^2 - 3136) \\
 & + 40\Delta w_2 (157\Delta w_2 - 1288) \left. \right] + 32h_1 \left[ 5\Delta k_2 (9\Delta v_2^2 - 12\Delta v_2 - 4352) \right. \\
 & - 4 (189\Delta v_2^2 (3\Delta w_2 + 10) - 4068\Delta v_2 \Delta w_2 + 856\Delta w_2) \left. \right] \\
 & - 24h_1^3 \left[ 5\Delta k_2 (627\Delta w_2^2 - 2940\Delta w_2 + 3136) \right. \\
 & - 4\Delta w_2 (669\Delta w_2^2 - 5670\Delta w_2 + 14872) \left. \right] \\
 & + 9h_1^4 \Delta w_2 (669\Delta w_2^3 - 7560\Delta w_2^2 + 29744\Delta w_2 - 49280) \\
 & \left. + 48\Delta v_2 (2151\Delta v_2^3 - 5736\Delta v_2^2 + 6704\Delta v_2 + 7936) \right\} \rho^6 + O(\rho^8)
 \end{aligned} \tag{C.20}$$



$$\begin{aligned}
\Delta(e^{2k}) = & \Delta k_2 \rho^2 + \frac{1}{7560 h_1^3} \left\{ 36 h_1^3 \left[ 15 \Delta k_2 (3 \Delta w_2^2 + 14) - 8 \Delta v_2 (\Delta w_2^2 - 6) \right] \right. \\
& + 72 h_1^2 (120 \Delta k_2 \Delta w_2 + 175 \Delta k_2^2 + 12 \Delta v_2 \Delta w_2^2 + 6 \Delta v_2^2 - 56 \Delta v_2 - 30 \Delta w_2^2) \\
& + 16 h_1 \left[ 15 \Delta k_2 (27 \Delta v_2^2 - 36 \Delta v_2 - 106) + 16 (18 \Delta v_2 - 29) \Delta w_2 \right] \\
& + 16 \Delta v_2 (27 \Delta v_2^3 - 72 \Delta v_2^2 - 468 \Delta v_2 + 1072) \\
& \left. - 27 h_1^4 \Delta w_2^2 (\Delta w_2^2 - 12) \right\} \rho^4 + O(\rho^6) \tag{C.21}
\end{aligned}$$

$$\begin{aligned}
\Delta(e^{4\Phi}) = & \frac{4e^{4\phi_0} \Delta v_2}{3h_1^2} (3\Delta v_2 - 4) \rho^2 + \frac{e^{4\phi_0}}{135h_1^4} \left\{ 4h_1 \left[ \Delta k_2 (-45\Delta v_2^2 + 60\Delta v_2 + 100) \right. \right. \\
& + 36 (3\Delta v_2^2 - 8\Delta v_2 + 4) \Delta w_2 \left. \right] + 3h_1^2 \left[ 3\Delta v_2^2 (9\Delta w_2^2 - 4) \right. \\
& - 8\Delta v_2 (9\Delta w_2^2 - 20) + 36\Delta w_2^2 \left. \right] + 4\Delta v_2 (243\Delta v_2^3 - 648\Delta v_2^2 \\
& \left. + 1536\Delta v_2 - 1664) \right\} \rho^4 + O(\rho^6) \tag{C.22}
\end{aligned}$$

$$\begin{aligned}
\Delta a = & \Delta w_2 \rho^2 + \frac{1}{90h_1^3} \left\{ 4h_1 \left[ 100\Delta k_2 + (9\Delta v_2^2 - 30\Delta v_2 - 40) \Delta w_2 \right] \right. \\
& + 6h_1^2 \left[ 25\Delta k_2 (\Delta w_2 - 2) - 24 (\Delta w_2 - 5) \Delta w_2 \right] + 32\Delta v_2 (3\Delta v_2 - 10) \\
& \left. - 3h_1^3 \Delta w_2 (6\Delta w_2^2 - 45\Delta w_2 + 116) \right\} \rho^4 + O(\rho^6) \tag{C.23}
\end{aligned}$$

$$\begin{aligned}
\Delta b = & \Delta v_2 \rho^2 + \frac{1}{90h_1^2} \left\{ h_1 \left[ \Delta k_2 (20 - 30\Delta v_2) + 16 (3\Delta v_2 - 5) \Delta w_2 \right] \right. \\
& + 3h_1^2 \left[ \Delta v_2 (3\Delta w_2^2 + 4) - 5\Delta w_2^2 \right] \\
& \left. - 8\Delta v_2 (9\Delta v_2^2 - 18\Delta v_2 + 20) \right\} \rho^4 + O(\rho^6) \tag{C.24}
\end{aligned}$$

## D Details of the numerical analysis

Here we shall discuss in more detail our approach to connecting the given IR and UV expansions numerically. We start by noting that we have chosen to solve the equations of motion (B.4)–(B.9) starting from the IR boundary conditions. As the IR parameter space is much smaller than that of the UV, this makes a search for solutions with the correct behaviour much less computationally expensive than if we started from the UV.

We use as our boundary conditions the IR expansions (C.13)–(C.24), extended up to order  $\rho^8$ . Using `NDSolve` in Mathematica 7 we then are able to generate numerical solutions which extend into the UV. We start at  $\rho_{IR} = 10^{-4}$  as we found that in the SUSY case this gives approximately optimal accuracy. We use 40-digit `WorkingPrecision` in `NDSolve`.

Comparing the numerical solutions obtained by this method, with the known behaviour in the SUSY case, suggests that the results are trustable up to  $\rho \sim 11$ . This is supported by the observation that the constraint (B.3) is almost completely satisfied over this range.

More explicitly, we find  $T + U \lesssim 10^{-8}$  throughout this range. In fact it appears that the numerical solutions only fail when  $b$  decreases past  $\sim 10^{-9}$ . In the SUSY case (in which  $b \sim e^{-2\rho}$ ) this does correspond to  $\rho \sim 11$ , but in the non-SUSY case (with  $b \sim e^{2\rho/3}$ ) it occurs further into the UV.

In the IR we have five parameters  $\{h_1, \phi_0, w_2, k_2, v_2\}$  which we can manipulate although we set  $\phi_0 = 0$  (along with  $N_c = 1$ ) without loss of generality. Given a value of  $h_1$  we want to study the effects of the SUSY-breaking deformations for the remaining three  $\{w_2, k_2, v_2\}$ . We find that for a general deformation of these IR parameters the resulting solution does not exhibit the expected UV behaviour. Initially we find that the general behaviour of solutions in this parameter space has

$$b \sim \pm e^{2\rho} \quad \text{and} \quad e^{2g} \sim e^{2h} \sim e^{2k} \sim e^{8\rho/3} \tag{D.1}$$

going into the UV. The  $e^{8\rho/3}$  behaviour appears to be suppressed by a very small numerical factor relative to the expected  $e^{4\rho/3}$  term, and in fact is not visible in plots of  $g$ ,  $h$  and  $k$  themselves. It is apparent, however, if we examine quantities of the form

$$e^{2k} - e^{2k_{\text{SUSY}}} \sim e^{8\rho/3}, \tag{D.2}$$

in which the  $e^{4\rho/3}$  behaviour (almost) cancels.

Given a value for one of the three non-SUSY deformations, we believe it is possible to obtain the desired UV behaviour (C.1)–(C.6), i.e.

$$e^{-2\rho/3} \quad \text{and} \quad e^{2g} \sim e^{2h} \sim e^{2k} \sim e^{4\rho/3}, \tag{D.3}$$

with the correct choice of the remaining two. In practice it seems easier to vary three but keep one very close to its starting value.

Having obtained a numerical solution with the correct UV behaviour, we look to determine the corresponding values of the expansion coefficients in the UV, i.e.

$$\{c_+, c_-, \Phi_\infty, Q_o, \rho_o, H_{11}, W_{20}, \Phi_{30}, V_{40}\}. \tag{D.4}$$

We define the mismatch function

$$m = \sum_i \left[ f_i^{\text{Numerical}}(\rho_{\text{match}}) - f_i^{\text{Expansion}}(\rho_{\text{match}}) \right]^2, \tag{D.5}$$

with  $f_i \in \{g, h, k, \Phi, a, b, g', h', k', \Phi', a', b'\}$ . We then minimise  $m$  to match our numerical solution and a UV expansion using `NMinimize` (with 60-digit `WorkingPrecision`) at a large  $\rho$  value,  $\rho_{\text{match}}$ .

With this setup and given the SUSY IR, `NMinimize` recovers the SUSY values for the UV parameters with an acceptable accuracy, even allowing all nine parameters to vary. The only restrictions we apply to the parameter space are  $c_+ \geq 0$  and  $\Phi_\infty \geq \phi_0 = 0$ .

We now present a non-SUSY solution found using the above methods for one set of values of the IR parameters. It has the expected behaviour for all functions at least up to  $\rho \sim 11$  (where the corresponding SUSY solution fails) and possibly as far as  $\rho \sim 17$ . We

first choose  $h_1 = 5$  (and have set  $\phi_0 = 0$  as mentioned above). The corresponding SUSY solution has

$$w_2 = \frac{8}{3h_1} - 2 = -\frac{22}{15}, \quad k_2 = \frac{2}{5h_1}(h_1^2 - 4) = \frac{42}{25}, \quad v_2 = -\frac{2}{3}.$$

This results in an `NMinimize` output (with  $\rho_{\text{match}} = 6$ ) of

$$\begin{aligned} c_+ &\approx 1.6, & c_- &\approx 2.0 \times 10^3, & \Phi_\infty &\approx 0.076, \\ Q_o &\approx -1.0, & \rho_o &\approx -6.8 \times 10^{-11}, & W_{20} &\approx 6.9 \times 10^{-14}, \\ V_{40} &\approx 2.7 \times 10^{-9}, & H_{11} &\approx 0.50, & \Phi_{30} &\approx 0.38. \end{aligned}$$

The associated mismatch value is  $m \lesssim 10^{-29}$ . We take this as a good value for the mismatch as we know that the SUSY solution does indeed exist, and these values are in good agreement with (3.5).

To obtain a non-SUSY deformation, we follow the procedure described and modify the three IR parameters  $\{k_2, v_2, w_2\}$  away from their SUSY values so as to manually scan the parameter space, until we gain a solution with the correct UV behaviour. We find that a suitable choice of deformations is<sup>11</sup>

$$\Delta k_2 \approx -2.471 \times 10^{-5}, \quad \Delta v_2 \approx 2.574 \times 10^{-5}, \quad \Delta w_2 \approx 1.029 \times 10^{-4}.$$

The minimization routine (again at  $\rho_{\text{match}} = 6$ ) then finds that the UV parameters are modified from their SUSY values according to

$$\begin{aligned} \Delta c_+ &\approx -6.6 \times 10^{-6}, & \Delta c_- &\approx 1.6, & \Delta \Phi_\infty &\approx -4.0 \times 10^{-7}, \\ \Delta Q_o &\approx -1.5 \times 10^{-4}, & \Delta \rho_o &\approx -7.1 \times 10^{-5}, & \Delta W_{20} &\approx 5.2 \times 10^{-5}, \\ \Delta V_{40} &\approx 5.6 \times 10^{-4}, & \Delta H_{11} &\approx 9.1 \times 10^{-5}, & \Delta \Phi_{30} &\approx -5.0 \times 10^{-5}, \end{aligned}$$

again with a mismatch value of  $m \lesssim 10^{-29}$ . However, we are unsure of the precision of these values — they appear to be slightly sensitive to the value of  $\rho_{\text{match}}$ , and so should be interpreted with caution. We present plots of the functions (figure 1) in the main text.

## E Free energy

Consider the Euclidean action  $\mathcal{I}$  for the wrapped D5 background of section 3. The free energy is  $F = \mathcal{I}/\beta$ , where  $\beta$  is the period of the compactified time direction.

$$\begin{aligned} \mathcal{I} &= \mathcal{I}_{\text{grav}} + \mathcal{I}_{\text{surf}} \\ &= -\frac{1}{16\pi} \int_{\mathcal{M}} d^{10}x \sqrt{\mathbf{g}} R + \frac{1}{32\pi} \int_{\mathcal{M}} (d\Phi \wedge \star d\Phi + e^\Phi F_3 \wedge \star F_3) \\ &\quad - \frac{1}{8\pi} \oint_{\Sigma_r} {}^9 K d\Sigma_r. \end{aligned} \tag{E.1}$$

<sup>11</sup>The exact values used were  $\Delta k_2 = -24\,705\,875 \times 10^{-12}$ ,  $\Delta v_2 = 25\,744\,091\,286\,331\,971\,640\,358 \times 10^{-27}$  and  $\Delta w_2 = 1\,029\,383\,373\,181\,636\,875 \times 10^{-22}$ .

$\mathcal{M}$  is a ten dimensional volume enclosed by a nine dimensional boundary  $\Sigma_r$ . The boundary  $\Sigma_r$  is taken to be surface at constant radial direction  $r$ .  ${}^9K$  is the extrinsic curvature of the boundary,

$${}^9K = \nabla_\mu n^\mu = \frac{1}{\sqrt{\mathbf{g}}} \partial_\mu (\sqrt{\mathbf{g}} n^\mu) = \frac{1}{4} e^{-\Phi/4} e^{-k} [9\Phi' + 8(g' + h') + 4k'] \quad (\text{E.2})$$

where  $n^\mu$  is the boundary outward normal vector,  $n^\mu = \sqrt{g^{rr}} \delta_r^\mu$ . Using the equations of motion  $\mathcal{I}_{\text{grav}}$  reduces to a volume integral of a total derivative,

$$\begin{aligned} \mathcal{I}_{\text{grav}} &= \frac{1}{32\pi} \int_{\mathcal{M}} d^{10}x \sqrt{\mathbf{g}} \nabla_\mu \nabla^\mu \Phi = \frac{1}{32\pi} \int_{\mathcal{M}} d^{10}x \partial_\mu (\sqrt{\mathbf{g}} \mathbf{g}^{\mu\nu} \partial_\nu \Phi) \\ &= \text{vol}_8 \beta \frac{1}{32\pi} \lim_{r \rightarrow \infty} \left( \frac{1}{8} e^{2(\Phi+g+h)} \Phi' \right). \end{aligned} \quad (\text{E.3})$$

Explicitly, the surface term is

$$\mathcal{I}_{\text{surf}} = -\text{vol}_8 \beta \frac{1}{8\pi} \lim_{r \rightarrow \infty} \left\{ \frac{1}{32} e^{2(\Phi+g+h)} [9\Phi' + 8(g' + h') + 4k'] \right\}. \quad (\text{E.4})$$

Thus,

$$\begin{aligned} \mathcal{I} &= \mathcal{I}_{\text{grav}} + \mathcal{I}_{\text{surf}} \\ &= -\frac{\text{vol}_8 \beta}{256\pi} \lim_{r \rightarrow \infty} \left\{ e^{2(\Phi+g+h)} [8(\Phi' + g' + h') + 4k'] \right\}. \end{aligned} \quad (\text{E.5})$$

Equation (E.5) gives the value of the on-shell action in terms of the asymptotic fields at infinity. It typically contains divergences and has to be regularized. One way of doing this is to subtract the action of a reference background. In our case the natural choice is to subtract a supersymmetric background. We also require that both backgrounds induce the same metric at the boundary,  $\Sigma_r$ ,

$$\begin{aligned} e^{\frac{\Phi_{ns}}{2}} e^{2g_{ns}} &= e^{\frac{\Phi_{su}}{2}} e^{2g_{su}}, & e^{\frac{\Phi_{ns}}{2}} e^{2h_{ns}} &= e^{\frac{\Phi_{su}}{2}} e^{2h_{su}}, \\ e^{\frac{\Phi_{ns}}{2}} e^{2k_{ns}} &= e^{\frac{\Phi_{su}}{2}} e^{2k_{su}}, & e^{\frac{\Phi_{ns}}{2}} &= e^{\frac{\Phi_{su}}{2}} \end{aligned} \quad (\text{E.6})$$

and that the matter fields coincide at the boundary. In order to achieve the matching of the induced metrics and matter fields at the boundary we have to choose particular values for the integration constants of the supersymmetric background that we use as a regulator. We can then evaluate the free energy,

$$\begin{aligned} F &= \frac{1}{\beta} (\mathcal{I}^{ns} - \mathcal{I}^{su}) \\ &= -\frac{\text{vol}_8}{256\pi} \lim_{r_c \rightarrow \infty} \left\{ e^{2\Phi_{ns}+2g_{ns}+2h_{ns}} (8\Phi'_{ns} + 8g'_{ns} + 8h'_{ns} + 4k'_{ns}) \right. \\ &\quad \left. - e^{2\Phi_s+2g_s+2h_s} (8\Phi'_s + 8g'_s + 8h'_s + 4k'_s) \right\}. \end{aligned} \quad (\text{E.7})$$

Using the UV expansion (3.1), to first order in  $W_{20}$ ,

$$F = E = \frac{\text{vol}_8}{24\pi} c_+^2 e^{2\rho_0+2\Phi_\infty} W_{20}. \quad (\text{E.8})$$

which agrees with the ADM calculation. A similar evaluation of the free energy can be carried out for the backgrounds after the rotation. Due to the presence of  $F_5$  and the Chern-Simons term the calculation is more involved and the equality of the energy before and after rotation cannot be expressed as simply as (5.12). Nevertheless, after plugging in the appropriate UV expansions we get, to first order in  $W_{20}$ ,  $F_{\text{before}} \sim F_{\text{after}} \sim c_+^2 e^{2\rho_0+2\Phi_\infty} W_{20}$  as expected.

## F Calculation of $B_2$

In the SUSY case, we have

$$B_2 = \kappa \frac{e^{\frac{3}{2}\Phi}}{\hat{h}^{1/2}} \left[ e^{\rho^3} - \cos \alpha (e^{\theta\varphi} + e^{12}) - \sin \alpha (e^{\theta^2} + e^{\varphi^1}) \right], \quad (\text{F.1})$$

with

$$\cos \alpha = \frac{\cosh(2\rho) - a}{\sinh(2\rho)}, \quad \sin \alpha = -\frac{2e^{h-g}}{\sinh(2\rho)}. \quad (\text{F.2})$$

This is not valid in the general non-SUSY case. We obtain the same  $H_3$  as in the SUSY case (2.12), but the relationship to (F.1) requires the BPS equations, as does the consistency of the definitions (F.2).

Instead, we must determine  $B_2$  by requiring that  $dB_2 = H_3$ . We assume that  $B_2$  has the same general structure as (F.1),

$$B_2 = b_1(\rho)e^{\rho^3} + b_2(\rho)e^{\theta\varphi} + b_3(\rho)e^{12} + b_4(\rho)e^{\theta^2} + b_5(\rho)e^{\varphi^1}, \quad (\text{F.3})$$

which results in

$$\begin{aligned} dB_2 = & \frac{e^{-h-k-\frac{\Phi}{4}} (ab_3e^g + 2b_4e^h)}{\hat{h}^{1/4}} e^{1\theta 3} + \frac{e^{-h-k-\frac{\Phi}{4}} (ab_3e^g + 2b_5e^h)}{\hat{h}^{1/4}} e^{\varphi 23} \\ & - \frac{(b_4 - b_5) e^{-h-\frac{\Phi}{4}} \cot \theta}{\hat{h}^{1/4}} e^{\theta\varphi 1} \\ & + \frac{e^{-2g-k-\frac{\Phi}{4}}}{2\hat{h}^{5/4}} \left( e^{2g} \left\{ \hat{h} [b_3 (4g' + \Phi') + 2b'_3] + b_3 \hat{h}' \right\} + 4b_1 \hat{h} e^{2k} \right) e^{\rho 12} \\ & + \frac{e^{-h-k-\frac{\Phi}{4}}}{2\hat{h}^{5/4}} \left( b_3 e^g \hat{h} a' - 2ab_1 e^{-g} \hat{h} e^{2k} \right. \\ & \quad \left. + e^h \left\{ \hat{h} [b_4 (2g' + 2h' + \Phi') + 2b'_4] + b_4 \hat{h}' \right\} \right) e^{\rho \theta 2} \\ & + \frac{e^{-h-k-\frac{\Phi}{4}}}{2\hat{h}^{5/4}} \left( b_3 e^g \hat{h} a' - 2ab_1 e^{-g} \hat{h} e^{2k} \right. \\ & \quad \left. + e^h \left\{ \hat{h} [b_5 (2g' + 2h' + \Phi') + 2b'_5] + b_5 \hat{h}' \right\} \right) e^{\rho \varphi 1} \\ & + \frac{e^{-h-k-\frac{\Phi}{4}}}{2\hat{h}^{5/4}} \left( -(b_4 + b_5) e^g \hat{h} a' - (a^2 - 1) b_1 \hat{h} e^{2k-h} \right. \\ & \quad \left. + e^h \left\{ \hat{h} [b_2 (4h' + \Phi') + 2b'_2] + b_2 \hat{h}' \right\} \right) e^{\rho \theta \varphi}. \end{aligned} \quad (\text{F.4})$$

Comparing with (2.12), we see that the  $e^{\theta\varphi^1}$  component of  $H_3$  is zero, from which we immediately obtain that  $b_4 = b_5$ . The  $e^{\rho\theta^2}$  and  $e^{\rho\varphi^1}$  components of (F.4) are then identical, as are the  $e^{1\theta^3}$  and  $e^{\varphi^{23}}$  components. This is also the case in  $H_3$ , so we are left with four independent equations.

Equating the  $(e^{1\theta^3} + e^{\varphi^{23}})$  components results in

$$b_4 = -\frac{1}{2}e^{g-h}ab_3 - \frac{\kappa N_c e^{\frac{3\Phi}{2}-g-h}b'}{4\hat{h}^{1/2}}, \quad (\text{F.5})$$

and the  $e^{\rho^{12}}$  component gives

$$b_1 = \frac{e^{2g-2k}}{4\hat{h}} \left[ 2b_3\Phi' - 3\hat{h}b_3\Phi' - 4\hat{h}b_3g' - 2\hat{h}b_3' \right. \\ \left. + \kappa N_c e^{\frac{3\Phi}{2}-2h}\hat{h}^{\frac{1}{2}}(a^2 - 2ab + 1) \right]. \quad (\text{F.6})$$

This leaves  $b_2$  and  $b_3$  to be determined. Substituting these results into (F.4), we find that the  $(e^{\rho\theta^2} + e^{\rho\varphi^1})$  component of  $H_3 = dB_2$  reduces to the equation of motion (B.9) for  $b$ . The only remaining equation is then the  $e^{\rho\theta\varphi}$  component. This is a first order differential equation in  $b_2$  and  $b_3$ ,

$$0 = 8\hat{h}e^{2g+4h}b_2' + 2(a^2 - 1)\hat{h}e^{4g+2h}b_3' + e^{2(g+h)}\hat{h}' \left[ (a^2 - 1)e^{2g}b_3 + 4e^{2h}b_2 \right] \\ + \hat{h}e^{2(g+h)} \left[ 4ae^{2g}a'b_3 + (a^2 - 1)e^{2g}(4g' + \Phi')b_3 + 4b_2e^{2h}(4h' + \Phi') \right] \\ - \kappa N_c \sqrt{\hat{h}}e^{3\Phi/2} \left[ -2a'b'e^{2(g+h)} + (a^4 - 1)e^{4g} \right. \\ \left. - 2(a^2 - 1)abe^{4g} + 2abe^{4g} - 16e^{4h} \right]. \quad (\text{F.7})$$

Solving for  $b_2$  we obtain

$$b_2 = \frac{e^{-2h-\Phi/2}}{\sqrt{\hat{h}}} \int^\rho d\rho' \left( \frac{e^{-2g-2h+\frac{\Phi}{2}}}{8\sqrt{\hat{h}}} \left\{ -(a^2 - 1)e^{4g+2h}\hat{h}'b_3 \right. \right. \\ \left. \left. - \hat{h}e^{4g+2h} \left[ 4aa' + a^2(4g' + \Phi') - 4g' - \Phi' \right] b_3 \right. \right. \\ \left. \left. + \kappa N_c \sqrt{\hat{h}}e^{3\Phi/2} \left[ (a^4 - 1)e^{4g} - 2(a^2 - 1)abe^{4g} - 2a'b'e^{2(g+h)} - 16e^{4h} \right] \right\} \right. \\ \left. - \frac{1}{4}(a^2 - 1)\sqrt{\hat{h}}e^{2g+\frac{\Phi}{2}}b_3' \right), \quad (\text{F.8})$$

which does not appear to be very useful. Instead, we can use the fact that we want  $Q_{\text{Page, D3}} = 0$  (see eq. (6.12)). We therefore impose that the  $e^{\theta\varphi^{123}}$  component of  $F_5 - B_2 \wedge F_3$  vanishes. The resulting equation is algebraic in  $b_2$  and  $b_3$ , and results in

$$b_2 = \frac{e^{-2h}}{4\hat{h}^{1/2}} \left\{ e^{2g}\hat{h}^{\frac{1}{2}}(1 - a^2)b_3 - \frac{\kappa}{N_c}e^{\frac{3\Phi}{2}} \left[ N_c^2(a - b)b' + 4e^{2(g+h)}\Phi' \right] \right\}. \quad (\text{F.9})$$

Together with the above results for  $b_{1,4,5}$  this completes (6.2).

It remains to check that this  $b_2$  is also compatible with the requirement that  $dB_2 = H_3$ . Substituting into (F.7) we find that  $b_3$  cancels, giving

$$\begin{aligned}
 0 = & 4e^{4(g+h)} \left\{ 2\hat{h} \left[ 2g'\Phi' + 2h'\Phi' + \Phi'' + 2(\Phi')^2 \right] - 2g'\hat{h}' - 2h'\hat{h}' - \hat{h}'' \right\} \\
 & + N_c^2 \left[ a^4 e^{4g} - 2a^3 b e^{4g} + 2(a-b)b'' e^{2(g+h)} + 4(a-b)b' e^{2(g+h)} \Phi' \right. \\
 & \left. + 2abe^{4g} - 2(b')^2 e^{2(g+h)} - e^{4g} - 16e^{4h} \right].
 \end{aligned}
 \tag{F.10}$$

This is solved by the equations of motion (B.7), (B.9) for  $\Phi$  and  $b$ .

To determine the effect of the undetermined function  $b_3$ , we can look at the difference  $\Delta B_2 = B_2 - (B_2)_{b_3=0}$ , which we find to be of the form

$$\begin{aligned}
 \Delta B_2 = & F_1(\rho) \sin \theta \, d\theta \wedge d\varphi + F_2(\rho) \sin \tilde{\theta} \, d\tilde{\theta} \wedge d\tilde{\varphi} + F_3(\rho) \cos \theta \, d\rho \wedge d\varphi \\
 & + F_4(\rho) \cos \tilde{\theta} \, d\rho \wedge d\tilde{\varphi} + F_5(\rho) \, d\rho \wedge d\psi,
 \end{aligned}
 \tag{F.11}$$

where the  $F_i$  depend on  $g, \Phi, \hat{h}, b_3$  and their derivatives. If we set this equal to

$$d \left[ \beta_1(\rho) \cos \theta \, d\varphi + \beta_2(\rho) \cos \tilde{\theta} \, d\tilde{\varphi} + \beta_3(\rho) \, d\psi \right]
 \tag{F.12}$$

we can solve for the  $\beta_i$ , giving

$$\begin{aligned}
 \Delta B_2 = & -\frac{1}{4} d \left[ e^{2g+\Phi/2} \sqrt{\hat{h}} b_3 (\cos \theta \, d\varphi + \cos \tilde{\theta} \, d\tilde{\varphi} + d\psi) \right] \\
 = & -\frac{1}{2} d \left( e^{2g-k+\Phi/4} \hat{h}^{1/4} b_3 e^3 \right).
 \end{aligned}
 \tag{F.13}$$

## G Seiberg-like duality

In section 6.1.6 we discuss how the operation known as Seiberg duality in the KS cascade acts for our non-SUSY solution. In order to do so, we find it instructive to compare to two different cases: the KS case and the baryonic branch case. These are summarized here.

### G.1 The KS case

We follow here the treatment in [33], specified in the case of no flavors ( $N_f = 0$ ). The NS potential  $B_2$  is given by,

$$B_2 = \frac{N_c}{2} [f g_1 \wedge g_2 + \tilde{k} g_3 \wedge g_4]
 \tag{G.1}$$

where the definition of  $g_1, \dots, g_4$  can be found in [33]. When specialized to the cycle

$$\Sigma_2 = [\theta = \tilde{\theta}, \varphi = 2\pi - \tilde{\varphi}, \psi = \psi_0]
 \tag{G.2}$$

we obtain that

$$B_2|_{\Sigma_2} = \frac{N_c}{2} [(f + \tilde{k}) + (\tilde{k} - f) \cos \psi_0] \sin \theta d\theta \wedge d\varphi
 \tag{G.3}$$

from which one finds

$$b_0 = \frac{1}{4\pi^2} \int_{\Sigma_2} B_2 = \frac{N_c}{\pi} \left[ f \sin^2 \left( \frac{\psi_0}{2} \right) + k \cos^2 \left( \frac{\psi_0}{2} \right) \right]
 \tag{G.4}$$

On the other hand, as computed in [33], we can see that the Maxwell charge of D3 branes is

$$Q_{\text{Max},D3} = \frac{N_c^2}{\pi} [f - (f - \tilde{k})F] \tag{G.5}$$

We see that under the change

$$f \rightarrow f - \frac{\pi}{N_c}, \quad \tilde{k} \rightarrow \tilde{k} - \frac{\pi}{N_c} \tag{G.6}$$

the D3-Maxwell charge changes by

$$Q_{\text{Max},D3} \rightarrow Q_{\text{Max},D3} - N_c, \quad b_0 \rightarrow b_0 - 1. \tag{G.7}$$

these transformations, are equivalent to changing the NS potential with a large gauge transformation

$$B_2 \rightarrow B_2 + \frac{\pi}{2} [g_1 \wedge g_2 + g_3 \wedge g_4] \tag{G.8}$$

which when evaluated on the cycle  $\Sigma_2$ , produces the changes in eq. (G.7). We move now to analyze the baryonic branch (SUSY) case.

## G.2 Baryonic branch case

In this case the NS potential is

$$B_2 = \frac{\kappa e^{3\Phi/2}}{\hat{h}^{1/2}} [e^{\rho^3} - \cos \alpha (e^{12} + e^{\theta\varphi}) - \sin \alpha (e^{\theta^2} + e^{\varphi^1})] \tag{G.9}$$

Evaluating this on the  $\Sigma_2$  we get

$$\begin{aligned} b_0 &= \frac{\kappa e^{2\Phi}}{\pi} [(\tilde{k} + f) + (\tilde{k} - f) \cos \psi_0], \\ \tilde{k} + f &= \frac{\kappa e^{2\Phi}}{N_c} \left[ \cos \alpha \left( \frac{e^{2g}}{4} (a^2 + 1) - e^{2h} \right) + \sin \alpha a e^{h+g} \right], \\ \tilde{k} - f &= \frac{\kappa e^{2\Phi}}{N_c} \left[ \cos \alpha \frac{e^{2g}}{2} a + \sin \alpha e^{h+g} \right], \end{aligned} \tag{G.10}$$

Using the explicit expressions, we have

$$\tilde{k} = -\frac{\kappa e^{2\Phi}}{4N_c} Q \coth(\rho), \quad f = -\frac{\kappa e^{2\Phi}}{4N_c} Q \tanh(\rho) \tag{G.11}$$

The Maxwell charge for D3 branes can be written as,

$$Q_{\text{Max},D3} = \frac{\kappa}{\pi} e^{2g+2h+2\Phi} \Phi' \tag{G.12}$$

and using the BPS equation for  $\Phi'$  we have

$$Q_{\text{Max},D3} = \frac{N_c^2}{\pi} [2f + (\tilde{k} - f)F], \tag{G.13}$$

where  $F = (1 - b)$ . So, once again, we obtain that under a large gauge transformation,

$$b_0 \rightarrow b_0 - 1, \quad Q_{\text{Max},D3} \rightarrow Q_{\text{Max},D3} - N_c. \tag{G.14}$$



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