

Baryon asymmetric Universe from spontaneous CP violation

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ABSTRACT: Spontaneous CP violation, such as the Nelson-Barr (NB) mechanism, is an attractive scenario for addressing the strong CP problem while realizing the observed phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. However, not only the CKM phase but also the baryon asymmetric Universe requires sources of CP violation. In this study, we show that a supersymmetric NB mechanism can naturally accommodate the Affleck-Dine (AD) baryogenesis within a CP-invariant Lagrangian. The model provides flat directions associated with new heavy quarks. Focusing on one of the directions, we find that the correct baryon asymmetry is obtained with a sufficiently low reheating temperature which does not cause the gravitino problem. Some parameter space is consistent with the gravitino dark matter. We assess radiative corrections to the strong CP phase induced by gauge-mediated supersymmetry breaking and CP-violating heavy fields and show that the strong CP problem is solved in a viable parameter space where the visible sector supersymmetric particles must be lighter than $\mathcal{O}(100)$ TeV. Even in the case that they are heavier than the TeV scale, our model predicts the neutron electric dipole moment within the reach of the near future experiments. Our model addresses the electroweak naturalness problem, strong CP problem, baryon asymmetric Universe, and dark matter. Then, the model may give a new compelling paradigm of physics beyond the Standard Model.

KEYWORDS: Baryo- and Leptogenesis, Supersymmetry, Cosmology of Theories BSM, Early Universe Particle Physics

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1 Introduction

The electroweak naturalness and the strong CP problem are two major issues of fine-tuning in the Standard Model (SM). The electroweak scale is unstable under quantum effects and significant fine-tuning is required to set it to a much smaller scale than the UV energy scales, such as the Planck scale. An elegant solution to this problem is provided by supersymmetry (SUSY): quadratically divergent contributions to the Higgs mass-squared parameter are canceled by those of superpartners. However, if SUSY is realized in nature, it must be spontaneously broken. SUSY breaking introduces numerous new parameters into the theory. Arbitrary choices of these parameters lead to dangerous CP and flavor violating processes, and the superpartner mass must be heavier than about 1000 TeV (see, e.g., ref. [1]), which reintroduces a need for fine-tuning. This issue can be addressed using gauge-mediated SUSY breaking (see refs. [2, 3] for reviews). Superpartner masses in the visible sector are generated via flavor-independent SM gauge interactions. Gauge mediation predicts a light gravitino as the lightest supersymmetric particle (LSP). To avoid overproduction of this light gravitino, the reheating temperature of the Universe must be sufficiently low [4–7].

The strong CP problem is the question of why a large CP violation has never been observed in QCD [8–10]. The absence of observation of the neutron electric dipole moment (EDM) leads to a significant constraint on the effective θ -angle, such as $\bar{\theta} \lesssim 10^{-10}$ [11, 12]. The most popular solution to the strong CP problem is the Peccei-Quinn (PQ)

mechanism [10], where the dynamical axion [13, 14] associated with the spontaneous breaking of the $U(1)_{PQ}$ global symmetry sets $\bar{\theta}$ to zero at the potential minimum. However, it has been discussed that the quantum gravity effects explicitly break any global symmetry. This introduces a fine-tuning of the theory [15–26].¹ Moreover, depending on the scenarios of the $U(1)_{PQ}$ symmetry breaking, there are challenges in the field of cosmology, including the domain wall problem [59] and isocurvature problem [60–65]. Although the effects on cosmological history by the supersymmetric partners, axino and saxion, are highly model dependent (see, e.g., refs. [66–68]), they could still cause a cosmological problem. An alternative approach to the strong CP problem is to assume that the theory respects CP symmetry, but breaks it spontaneously to realize the observed Cabibbo-Kobayashi-Maskawa (CKM) phase.² The Nelson-Barr (NB) mechanism [72–74] introduces a vector-like heavy quark, whose realization does not regenerate a dangerous $\bar{\theta}$. As the mechanism also requires new scalar fields to break the CP, which causes another naturalness problem, a supersymmetric extension of the model is a natural possibility [75]. However, CP and flavor violations in SUSY breaking parameters are still significantly constrained to avoid the regeneration of a sizable $\bar{\theta}$. The constraints are stronger than those of flavor-changing neutral currents and are independent of superpartner mass scales. Therefore, gauge-mediated SUSY breaking is necessary in this framework [76, 77].

Considering spontaneous CP violation, it should be noted that not only the CKM phase but also the baryon asymmetric Universe requires sources of CP violation. Therefore, it is natural to consider how the baryon asymmetric Universe is realized if CP symmetry, whose violation is one of the necessary conditions to generate the baryon asymmetry, is fundamentally preserved in the Lagrangian. In addition, baryogenesis presents several challenges in the minimal supersymmetric Standard Model (MSSM). Although thermal leptogenesis [78] is one of the most common scenarios for generating baryon asymmetry, this requires a high reheating temperature, which leads to the overproduction of gravitinos in gauge mediation. The Affleck-Dine (AD) mechanism [79–81] is an alternative method for baryogenesis that is compatible with a low reheating temperature in a supersymmetric theory. The mechanism uses a flat direction, called an AD field, that carries a nonzero baryon or lepton number. The AD field develops a large expectation value during inflation, after which a coherent oscillation is produced, and a sizable baryon or lepton charge density is stored. Finally, the condensate of the AD field decays, converting the charge density to a baryon asymmetry via the electroweak anomaly. However, in the MSSM with gauge-mediated SUSY breaking, there are challenges to this option too. The AD mechanism is preceded [82–86] by the formation of non-topological solitons, called Q-balls [87–90]. The Q-balls may or may not dominate the Universe and decay into quarks (baryons), gravitinos, and next-to-lightest SUSY particles. In this scenario, the correct relic abundance of dark matter (DM) and the baryon asymmetry are obtained only in a limited parameter space [86, 91–94].

¹There are several models that allow for the quality of the $U(1)_{PQ}$ symmetry by accidental symmetry from discrete gauge symmetries [27–32], abelian gauge symmetries [33–37], and non-abelian gauge symmetries [24, 38–46]. In refs. [47–57], models with extra dimensions were also considered. Furthermore, a model based on superconformal dynamics was considered in ref. [58].

²The idea of spontaneous CP violation has been used in SUSY to suppress contributions to EDMs [69–71].

Problems	Solutions	Challenges	NB with gauge mediation?
Hierarchy	SUSY	Gauge mediation is favored	✓
Strong CP	PQ mechanism	$\left\{ \begin{array}{l} \text{Quality problem} \\ \text{Cosmological issues} \end{array} \right.$	
	SCPV (NB)	Gauge mediation is necessary	✓
BAU	Thermal LG	Gravitino problem in SUSY	No CP-violating source
	ADBG	Maybe too many Q-balls in the MSSM	✓ (The present work)

Table 1. Summary of possible solutions for the problems in the SM and their challenges in 3+1 dimensions (NB = the NB model; SCPV = spontaneous CP violation; BAU = baryon asymmetric Universe; LG = leptogenesis; ADBG = Affleck-Dine baryogenesis).

In this study, we discuss a viable possibility for generating baryon asymmetry in the framework of spontaneous CP violation based on the supersymmetric NB mechanism.³ The source of CP violation is the spontaneous CP violations that are driven by an AD field during inflation, which are different from the source of spontaneous CP violation at present. Since the AD field is a complex scalar field, its large vacuum expectation value (VEV) violates the CP as well as the baryon (and/or lepton) symmetry. The AD mechanism, therefore, works with the CP-conserving Lagrangian. As mentioned earlier, the supersymmetric NB model contains a new vector-like heavy quark, and the flat directions associated with these quarks can be used to generate baryon asymmetry via the AD mechanism. Since the heavy quark has an explicit supersymmetric mass term, there is no danger of long-lived Q-ball formation. Even if Q-balls are formed, they immediately decay into lighter SUSY particles. Also, the reheating temperature can be low enough to avoid the gravitino overproduction problem. In parameter regions that are consistent with the correct baryon asymmetry, the gravitino LSP can give the correct DM abundance. The model then simultaneously addresses the electroweak naturalness problem, strong CP problem, baryon asymmetry, and cosmological DM. The logic leading to the current scenario is summarized in table 1, which shows that this is a natural direction for solving fine-tuning problems as well as cosmological issues. Interestingly, we find that our model predicts the neutron EDM within the reach of the near future experiments.

To confirm that the supersymmetric NB model gives a viable solution to the strong CP problem, it is important to assess radiative corrections to $\bar{\theta}$. Such radiative corrections have been investigated in NB models without SUSY [96] and with SUSY in gravity mediation [75, 97]. Refs. [76, 77] initiated to study radiative corrections to $\bar{\theta}$ in the supersymmetric NB model with gauge-mediated SUSY breaking while a detailed analysis has not been done so far. We then provide a further study of this issue. When the messenger scale of gauge mediation is lower than the scale of spontaneous CP violation, the radiative corrections can

³Ref. [77] assumes thermal leptogenesis in the supersymmetric NB model, which may encounter the gravitino overproduction problem. In ref. [95], the authors commented on the possibility of realizing resonant leptogenesis in a non-SUSY model of spontaneous CP violation, wherein a certain amount of fine-tuning is required to generate a sufficient amount of baryon (or $B - L$) asymmetry.

be analyzed by using an effective field theory (EFT) consisting of the MSSM and messenger fields. In this paper, we explicitly construct such an EFT by integrating out new fields in the NB model and estimate the radiative corrections. We find that the corrections to $\bar{\theta}$ are suppressed by the ratio of the messenger scale to the scale of SCPV. Our result smoothly connects with the case that the messenger scale is higher than the SCPV scale.

The rest of the paper is organized as follows. In section 2, we review the supersymmetric NB mechanism to obtain a solution to the strong CP problem and analyze the observed CKM phase. Section 3 includes a detailed discussion of radiative corrections to $\bar{\theta}$ induced by gauge-mediated SUSY breaking and CP-violating heavy fields. Section 4 discusses the AD mechanism for baryogenesis in the model. Focusing on one of the flat directions associated with new heavy quarks, we find that the correct baryon asymmetry is obtained with a sufficiently low reheating temperature to avoid the gravitino problem. In section 5, we determine the parameter space in which the strong CP problem, baryon asymmetric Universe, and DM are explained. Section 6 is devoted to conclusions and discussions.

2 SUSY Nelson-Barr model

We introduce a vector-like pair of heavy down-type quark chiral superfields D and \bar{D} , and the SM singlet chiral superfields η_α ($\alpha = 1, \dots, n_\eta$), whose scalar components develop complex VEVs and break CP symmetry spontaneously. To accommodate a physical CP-breaking phase, $n_\eta \geq 2$ is required. In this paper, we do not specify the η sector and just assume they have VEVs. The representations of these fields under the SM gauge symmetry $SU(3)_C \times SU(2)_W \times U(1)_Y$ are summarized as follows:

$$D : \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3} \right), \quad \bar{D} : \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3} \right), \quad \eta_\alpha : (\mathbf{1}, \mathbf{1}, 0). \quad (2.1)$$

We assume the model respects a discrete Z_N symmetry,

$$D \rightarrow e^{2\pi i/N} D, \quad \bar{D} \rightarrow e^{-2\pi i/N} \bar{D}, \quad \eta_\alpha \rightarrow e^{-2\pi i/N} \eta_\alpha. \quad (2.2)$$

Then, the superpotential of the model is given by

$$W_{\text{NB}} = y_{\alpha i}^D \eta_\alpha D \bar{d}_i + M_D D \bar{D} + W_{\text{MSSM}}, \quad (2.3)$$

where we sum over repeated indices, \bar{d}_i ($i = 1, 2, 3$) denote three generations of MSSM right-handed down-type quark supermultiplets, $y_{\alpha i}^D$ are Yukawa couplings, M_D is a mass parameter, and W_{MSSM} denotes the superpotential of the MSSM, which includes

$$W_{\text{MSSM}} \supset -y_{ij}^u \bar{u}_i Q_j H_u + y_{ij}^d \bar{d}_i Q_j H_d + \mu H_u H_d. \quad (2.4)$$

In this case, Q_j , \bar{u}_i , and $H_{u,d}$ are the MSSM left-handed quark, the right-handed up-type quark, and the up and down-type Higgs supermultiplets, respectively. As leptons do not play a role in this section, we have omitted the interactions with leptons. Other renormalizable operators such as $Q \bar{D} H_d$ and $\eta_\alpha D \bar{D}$ are forbidden by Z_N symmetry. Given that CP symmetry is assumed to be exact in the NB model, all parameters in the superpotential interactions (2.3) and (2.4) are real.

After the scalar component of η_α develops a VEV with a non-vanishing CP-violating phase, we have obtained the following 4×4 down-type quark mass matrix:

$$\mathcal{L}_{\text{mass}} = (\bar{d} \ \bar{D})\mathcal{M}\begin{pmatrix} d \\ D \end{pmatrix} + \text{h.c.}, \quad \mathcal{M} = \begin{pmatrix} m_d & B \\ 0 & M_D \end{pmatrix}, \quad (2.5)$$

where $(B)_i \equiv y_{\alpha i}^D \langle \eta_\alpha \rangle$ is a 3×1 matrix containing a nonzero CP phase. The MSSM down-type quark mass $m_d \equiv y^d \langle H_d \rangle$ is assumed to be much smaller than M_D . Based on the structure of \mathcal{M} , we can find that $\arg(\det \mathcal{M}) = 0$, and thus, the strong CP phase $\bar{\theta}$ does not appear at the tree level. Let us now decompose \bar{d}, \bar{D}, d, D into light (\hat{d}, \hat{d}) and heavy (\hat{D}, \hat{D}) states, which are defined as

$$(\bar{d} \ \bar{D}) = (\hat{d} \ \hat{D})U_R^\dagger, \quad \begin{pmatrix} d \\ D \end{pmatrix} = U_L \begin{pmatrix} \hat{d} \\ \hat{D} \end{pmatrix}, \quad (2.6)$$

with the 4×4 unitary matrices $U_R = (u_i, u_4)$, $U_L = (v_i, v_4)$, to diagonalize $\mathcal{M}\mathcal{M}^\dagger$ and $\mathcal{M}^\dagger\mathcal{M}$ (neglecting the small mass m_d), respectively, where u_i, v_i are the eigenvectors of the light SM quarks, and u_4, v_4 are those of the heavy quark. The eigenvectors of the heavy quarks are explicitly given by

$$u_4 = \frac{1}{M_{\text{CP}}} \begin{pmatrix} B \\ M_D \end{pmatrix}, \quad v_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (2.7)$$

where $M_{\text{CP}}^2 \equiv M_D^2 + B^\dagger B$ is the physical mass squared of the heavy quark. In the following discussion, we take the flavor basis $(v_i)_j = \delta_{ij}$, where the mass matrix of the light states \hat{d}, \hat{d} is given by $(m_{\hat{d}})_{ij} = (u_i^*)_k (m_d)_{kj}$. Considering the up-type quark for which mass matrix is diagonal, the CKM matrix K is defined by the matrix that diagonalizes $m_d^\dagger m_{\hat{d}}$. Hence, we find that $\bar{m}_d^2 = K^\dagger m_d^T u_i u_i^\dagger m_d K$, where $\bar{m}_d \equiv \text{diag}(m_d, m_s, m_b)$ is the diagonal SM down-type quark mass matrix. Using $U_R U_R^\dagger = \mathbf{1}_{4 \times 4}$, we can obtain that

$$u_i u_i^\dagger + u_4 u_4^\dagger = \mathbf{1}_{4 \times 4}. \quad (2.8)$$

This equation and eq. (2.7) lead to the following relation [96, 98–100]:

$$K \bar{m}_d^2 K^\dagger \simeq m_d^T m_d - \frac{m_d^T B B^\dagger m_d}{M_{\text{CP}}^2}. \quad (2.9)$$

Thus, it can be found that a CP-violating phase vanishes for $M_D^2 \gg B^\dagger B$. Then, we assume $M_D^2 \sim B^\dagger B$ in the following discussion.

The SM contribution to $\bar{\theta}$ originating from the CKM phase is much smaller than 10^{-10} [101–103] because of the significant suppression by the Jarlskog invariant and the GIM mechanism. To address the strong CP problem, we must also make sure that radiative corrections and Planck-suppressed operators do not generate a large strong CP phase. We will discuss radiative corrections to $\bar{\theta}$ induced by gauge-mediated SUSY breaking and

CP-violating heavy fields in the next section. Here, our focus is on the effect of the Planck-suppressed operators. For concreteness, we define the effective strong CP phase that is invariant by field rotations as:

$$\bar{\theta} \equiv \theta - \arg \det(y^u y^d) - 3 \arg(m_{\tilde{g}}), \quad (2.10)$$

where $m_{\tilde{g}}$ is the gluino mass. Corrections to the phase of the quark and gluino masses must be suppressed by a factor of 10^{-10} to solve the strong CP problem. The lowest order Planck-suppressed operators in the superpotential are given by

$$W_{\text{CP}} = \frac{c_{1,\alpha}}{M_{\text{Pl}}^{N-1}} \eta_\alpha^N H_u H_d + \frac{c_{2,\alpha}}{M_{\text{Pl}}^{N-1}} \eta_\alpha^N D \bar{D} + \frac{c_{3,\alpha}}{M_{\text{Pl}}^{N-1}} \eta_\alpha^{N-1} Q \bar{D} H_d, \quad (2.11)$$

where $c_{1,\alpha}, c_{2,\alpha}, c_{3,\alpha}$ are $\mathcal{O}(1)$ dimensionless constants, and M_{Pl} ($\simeq 2.4 \times 10^{18}$ GeV) is the reduced Planck scale. The first term generates a CP-violating μ -term after η_α develops a complex VEV. This CP-violating μ -term leads to a correction of the phase of the quark masses through a loop of the Higgsino with the insertion of the μ and A -terms. It has been shown that $\bar{\theta} < 10^{-10}$ requires $\arg(\mu) < 10^{-8} \tan \beta$ [104], which turns out to be

$$\langle \eta_\alpha \rangle \lesssim 10^{-8/N} \left(\frac{\text{Re}(\mu) \tan \beta}{M_{\text{Pl}}} \right)^{1/N} M_{\text{Pl}}, \quad (2.12)$$

for $\text{Re}(\mu) \gg \text{Im}(\mu)$. The second and third terms in eqs. (2.11) may lead to a CP-violating phase of the order: $(\langle \eta_\alpha \rangle / M_{\text{Pl}})^{N-1}$. This phase must be smaller than 10^{-10} , which requires $\langle \eta_\alpha \rangle < 10^{-10/(N-1)} M_{\text{Pl}}$. There can also be Planck-suppressed Kähler potential terms such as

$$K_{\text{CP}} \supset \frac{c_{\alpha\beta,ij}}{M_{\text{Pl}}^2} \eta_\alpha^\dagger \eta_\beta \bar{d}_i^\dagger \bar{d}_j + \text{h.c.}, \quad (2.13)$$

with $\mathcal{O}(1)$ dimensionless coefficients $c_{\alpha\beta,ij}$. This term generates non-canonical CP-violating kinetic terms for the \bar{d}_i fields. However, it has been noted in refs. [104, 105] that, owing to the hermiticity of the wavefunction renormalization factors, the kinetic terms are canonically normalized without an additional contribution to the strong CP phase. Hence, the Kähler potential terms, such as those in eq. (2.13) do not lead to an additional constraint on the model parameters.

To set the cosmological constant to zero, we require $\langle W \rangle \sim m_{3/2} M_{\text{Pl}}^2$ where $m_{3/2}$ is the gravitino mass. This term generally has an $\mathcal{O}(1)$ complex phase which generates a gluino mass phase via the anomaly mediation at the one-loop order [106, 107]. Thus, the following constraint can be found [76]:

$$\frac{\alpha_s}{4\pi} \frac{m_{3/2}}{m_{\tilde{g}}} < 10^{-10}. \quad (2.14)$$

In the present work, we focus on gauge-mediated SUSY breaking, wherein $m_{3/2}$ is much smaller than the gluino mass $m_{\tilde{g}}$. Hence, this constraint can be fulfilled by having a sufficiently low value for the messenger mass scale.

3 Radiative corrections

We now discuss radiative corrections to $\bar{\theta}$ induced by the soft SUSY breaking parameters and CP-violating heavy fields. Radiative corrections to $\bar{\theta}$ in NB models have been investigated without SUSY [96] and with SUSY in gravity mediation [75, 97]. In ref. [76], it was argued that radiative corrections to $\bar{\theta}$ are quite problematic in those cases, whereas they are controlled in gauge-mediated SUSY breaking, as we will see below.

In the exact supersymmetric theory, the superpotential is not renormalized because of the non-renormalization theorem [108]. In addition, $\bar{\theta}$ is not renormalized by the wavefunction renormalization as discussed in the previous section.⁴ However, $\bar{\theta}$ is not protected against SUSY breaking. Hence, radiative corrections to $\bar{\theta}$ depend on the SUSY breaking scale. The mass scale at which SUSY breaking effects are mediated to the visible sector is denoted by M_* . It M_* is given by a messenger mass scale in gauge mediation and by the Planck scale in gravity mediation. In the case of $M_{\text{CP}} > M_*$, we can analyze a correction to the strong CP phase $\delta\bar{\theta}$ by using an EFT with the MSSM fields and messengers. Since there is no SUSY breaking source at the scale of M_{CP} , this EFT is a supersymmetric theory with higher dimensional operators suppressed by $1/M_{\text{CP}}$. The radiative correction to $\bar{\theta}$ in the EFT should involve both SUSY breaking and one of the higher dimensional operators, and we can see a suppression factor $1/M_{\text{CP}}^2$ in $\delta\bar{\theta}$. Although this analysis is valid only if $M_* < M_{\text{CP}}$, there exists a contribution to $\delta\bar{\theta}$ in the case of $M_* > M_{\text{CP}}$ which is smoothly connected at $M_* \sim M_{\text{CP}}$. In this section, we mostly focus on the EFT analysis on $\delta\bar{\theta}$ for the case with $M_* < M_{\text{CP}}$ and then discuss the case with $M_* > M_{\text{CP}}$ briefly.

After the scalar components of η_α develop CP-violating VEVs, the superpotential (2.3) is rewritten in the basis of $(\hat{d}, \hat{d}, \hat{D}, \hat{D})$ as

$$W_{\text{NB}} = y_{ij}^{\hat{d}} \hat{d}_i Q_j H_d + y_i^{Q\hat{D}} \hat{D} Q_i H_d + y_{\alpha i}^{\hat{D}} \delta\eta_\alpha \hat{D} \hat{d}_i + y_\alpha^{\hat{D}\hat{D}} \delta\eta_\alpha \hat{D} \hat{D} + M_{\text{CP}} \hat{D} \hat{D}, \quad (3.1)$$

where we have used eqs. (2.6), (2.7) and $\delta\eta_\alpha \equiv \eta_\alpha - \langle \eta_\alpha \rangle$ parameterizes the fluctuations around the CP-breaking vacuum. The coupling constants $y_{ij}^{\hat{d}}, y_i^{Q\hat{D}}, y_{\alpha i}^{\hat{D}}, y_\alpha^{\hat{D}\hat{D}}$ in the superpotential are defined by

$$y_{ij}^{\hat{d}} \equiv (u_i^*)_k y_{kj}^d, \quad y_i^{Q\hat{D}} \equiv (u_4^*)_k y_{ki}^d, \quad y_{\alpha i}^{\hat{D}} \equiv (u_i^*)_k y_{\alpha k}^D, \quad y_\alpha^{\hat{D}\hat{D}} \equiv (u_4^*)_k y_{\alpha k}^D. \quad (3.2)$$

Let us now construct an effective theory below M_{CP} by integrating the heavy fields. The F -term conditions, $\partial W_{\text{NB}}/\partial \hat{D} = \partial W_{\text{NB}}/\partial \hat{D} = 0$, and $\delta\eta_\alpha = 0$ lead to

$$\hat{D} = 0, \quad \hat{D} = -\frac{y_i^{Q\hat{D}}}{M_{\text{CP}}} Q_i H_d. \quad (3.3)$$

Then, the following effective Kähler potential is obtained at the tree level,

$$\Delta K_{\text{tree}} = \frac{(y_j^{Q\hat{D}})^* y_i^{Q\hat{D}}}{M_{\text{CP}}^2} (Q_j H_d)^\dagger Q_i H_d. \quad (3.4)$$

⁴Even if we include threshold corrections to $\bar{\theta}$ that is generated by integrating heavy fields, this conclusion does not change provided that the theory is exactly supersymmetric. See ref. [104] for detailed discussions.

One-loop corrections to the effective Kähler potential are expressed as [109]

$$\Delta K_{1\text{-loop}} = -\text{Tr} \left[\frac{M^\dagger M}{32\pi^2} \log \left(\frac{M^\dagger M}{\Lambda^2} \right) \right], \quad (M)_{ab} \equiv \frac{\partial^2 W_{\text{NB}}}{\partial \Phi_a \partial \Phi_b}, \quad (3.5)$$

where $\Lambda \sim M_{\text{CP}}$ is a renormalization scale and $\Phi_a \equiv (\hat{D}, \hat{\bar{D}}, \delta\eta_\alpha)$. Explicitly, the mass matrix M is given by

$$M = \begin{pmatrix} 0 & M_{\text{CP}} & y^{\hat{D}} \hat{\bar{d}} \\ M_{\text{CP}} & 0 & 0 \\ y^{\hat{D}} \hat{\bar{d}} & 0 & M_\eta \end{pmatrix}, \quad (3.6)$$

where M_η is the $n_\eta \times n_\eta$ mass matrix of the η_α field. Assuming $M_{\text{CP}} \sim M_\eta$, we can expand the one-loop effective Kähler potential with respect to M_{CP}^{-1} as given below.

$$\Delta K_{1\text{-loop}} \sim -\frac{1}{32\pi^2} (y_{\alpha i}^{\hat{D}} \hat{\bar{d}}_i)^\dagger y_{\alpha j}^{\hat{D}} \hat{\bar{d}}_j - \frac{1}{32\pi^2 M_{\text{CP}}^2} (y_{\alpha i}^{\hat{D}} \hat{\bar{d}}_i)^\dagger y_{\alpha k}^{\hat{D}} \hat{\bar{d}}_k (y_{\beta j}^{\hat{D}} \hat{\bar{d}}_j)^\dagger y_{\beta l}^{\hat{D}} \hat{\bar{d}}_l + \mathcal{O} \left(\frac{1}{M_{\text{CP}}^4} \right). \quad (3.7)$$

In this equation, the first term gives the non-canonically normalized kinetic term for $\hat{\bar{d}}$. As discussed in the previous section, the wavefunction renormalization does not reintroduce $\bar{\theta}$.

The relevant soft SUSY breaking terms generated by gauge mediation are

$$-\mathcal{L}_{\text{soft}} \supset \tilde{Q}_i^\dagger (m_{\tilde{Q}}^2)_{ij} \tilde{Q}_j + \tilde{u}_i (m_{\tilde{u}}^2)_{ij} \tilde{u}_j^\dagger + \hat{\bar{d}}_i (m_{\hat{\bar{d}}}^2)_{ij} \hat{\bar{d}}_j^\dagger + \left(\frac{1}{2} m_{\tilde{g}} \tilde{g} \tilde{g} - A_{ij}^u H_u \tilde{u}_i \tilde{Q}_j + A_{ij}^{\hat{d}} H_{\hat{d}} \hat{\bar{d}}_i \tilde{Q}_j + \text{h.c.} \right), \quad (3.8)$$

where \tilde{X} denotes the superpartner of a SM field X . In gauge mediation, the gluino mass is generated in the one-loop order: $m_{\tilde{g}} = \alpha_s F / (4\pi M_*)$, where \sqrt{F} is the SUSY breaking scale. The soft scalar mass-squared parameters and A -parameters are generated at the two-loop order: $m_{\text{soft}}^2 \sim \alpha^2 F^2 / (16\pi^2 M_*^2)$ and $A_{ij}^{u,\hat{d}} \sim y_{ij}^{u,\hat{d}} \alpha^2 F / (16\pi^2 M_*)$ with $\alpha \equiv g^2 / 4\pi$ (g denotes a SM gauge coupling). A remarkable feature of gauge-mediated SUSY breaking is that $m_{\tilde{Q}}^2, m_{\tilde{u}}^2, m_{\hat{\bar{d}}}^2$ are flavor-universal and $A^u, A^{\hat{d}}$ are proportional to the corresponding Yukawa couplings. However, the effective Kähler potential obtained from eqs. (3.4), (3.5), leads to corrections to those soft SUSY breaking parameters based on the diagrams shown in figure 1. These corrections are roughly evaluated as

$$\begin{aligned} (\delta m_{\hat{\bar{d}}}^2)_{ij} &\sim \left(\frac{1}{16\pi^2} \right)^2 (y_{\alpha j}^{\hat{D}} y_{\beta k}^{\hat{D}})^* y_{\alpha k}^{\hat{D}} y_{\beta i}^{\hat{D}} \left(\frac{M_*}{M_{\text{CP}}} \right)^2 m_{\text{soft}}^2, \\ (\delta m_{\tilde{Q}}^2)_{ij} &\sim \frac{1}{16\pi^2} (y_i^{Q\hat{D}})^* y_j^{Q\hat{D}} \left(\frac{M_*}{M_{\text{CP}}} \right)^2 m_{\text{soft}}^2, \\ (\delta A^{\hat{d}})_{ij} &\sim \frac{1}{16\pi^2} (y_k^{Q\hat{D}})^* y_j^{Q\hat{D}} \left(\frac{M_*}{M_{\text{CP}}} \right)^2 A_{ik}^{\hat{d}}. \end{aligned} \quad (3.9)$$

Therefore, now the soft mass-squared parameters are not flavor-universal and the A -parameters are not precisely proportional to the corresponding Yukawa couplings in the presence of the heavy fields.

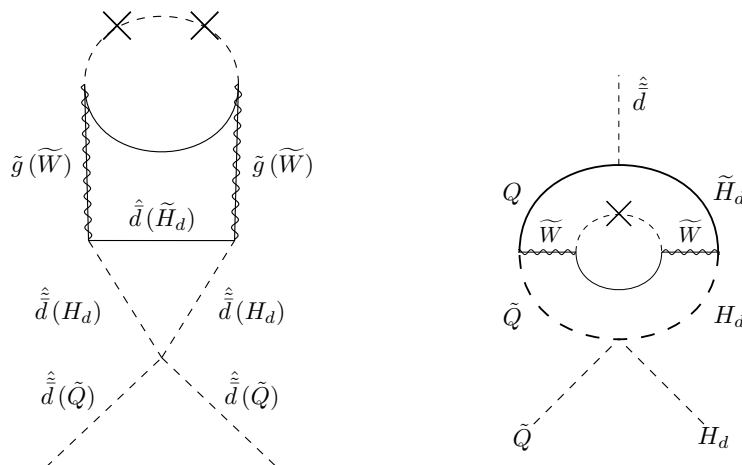


Figure 1. The soft mass-squared parameters $m_{\hat{d}, \tilde{Q}}^2$ (left) and the A -parameter $A^{\hat{d}}$ (right) induced by the effective Kähler potential obtained in eqs. (3.4), (3.5). The cross denotes the insertion of the F -term of a SUSY breaking field.

The presence of soft SUSY breaking parameters (3.9) induces complex phases to the SM quark and gluino masses, potentially leading to a large correction to $\bar{\theta}$. Such a correction can be described as [75]

$$\delta\bar{\theta} = -\text{Im Tr} \left(m_{\hat{d}}^{-1} \delta m_{\hat{d}} + m_u^{-1} \delta m_u \right) - 3 \text{Im} \left(m_{\tilde{g}}^{-1} \delta m_{\tilde{g}} \right), \tag{3.10}$$

where $\delta m_{\hat{d}, u, \tilde{g}}$ denotes mass corrections of the \hat{d}, u, \tilde{g} fields induced by the soft SUSY breaking parameters (3.9). To calculate the mass corrections, we treat the flavor non-universal parts of the soft mass-squared parameters, $\delta m_{\hat{d}, \tilde{Q}}^2$, as perturbations. The Feynman diagram of the lowest non-vanishing contribution to $\delta m_{\tilde{g}}$ with the CP-violating phase is shown in the left panel of figure 2. This contribution can be evaluated as [104]⁵

$$\begin{aligned} \delta\theta_{\tilde{g}}^m &\sim \frac{\alpha_s}{4\pi} \frac{v_d^4}{m_{\tilde{g}} m_{\text{soft}}^8} \text{Im Tr} \left[y^{\hat{d}} (y^{\hat{d}})^\dagger y^{\hat{d}} \delta m_{\tilde{Q}}^2 (\tilde{A}^{\hat{d}})^\dagger \delta m_{\hat{d}}^2 \right] \\ &\sim \frac{\alpha_s}{4\pi} \frac{v_d^3 v_u}{m_{\tilde{g}} m_{\text{soft}}^7} \text{Im Tr} \left[y^{\hat{d}} (y^{\hat{d}})^\dagger y^{\hat{d}} \delta m_{\tilde{Q}}^2 (y^{\hat{d}})^\dagger \delta m_{\hat{d}}^2 \right], \end{aligned} \tag{3.11}$$

where $v_{u(d)} \equiv \langle H_{u(d)} \rangle$ are Higgs VEVs and $\tilde{A}^{\hat{d}} \equiv A^{\hat{d}} + y^{\hat{d}} \mu v_u / v_d$. In the second line, we use $\tilde{A}^{\hat{d}} \sim y^{\hat{d}} \mu v_u / v_d \sim y^{\hat{d}} m_{\text{soft}} v_u / v_d > A^{\hat{d}}$, and the dependence of m_{soft} is estimated using dimensional analysis. The Feynman diagram of the lowest non-vanishing contribution to $\delta m_{\hat{d}}$ with the CP-violating phase is shown in the right panel of figure 2. This contribution can be expressed as

$$\begin{aligned} \delta\theta_{\hat{d}}^m &\sim \frac{\alpha_s}{4\pi} \frac{1}{m_{\text{soft}}^5} \text{Im Tr} \left[(y^{\hat{d}})^{-1} \delta m_{\hat{d}}^2 \tilde{A}^{\hat{d}} \delta m_{\tilde{Q}}^2 \right] \\ &\sim \frac{1}{16\pi^2} \frac{\alpha_s}{4\pi} \frac{1}{m_{\text{soft}}^2} \frac{v_u}{v_d} \text{Im} \left[(y^{\hat{d}})^{-1} \delta m_{\hat{d}}^2 y^{\hat{d}} (y^u)^\dagger y^u \right]. \end{aligned} \tag{3.12}$$

⁵The contribution to $\delta\theta_{\tilde{g}}^m$ from a single insertion of $m_{\hat{d}}$ vanishes because of the hermiticity of the matrices $\delta m_{\hat{d}}^2$ and $\delta m_{\tilde{Q}}^2$. Therefore, the non-vanishing leading contribution arises from the three insertions of $m_{\hat{d}}$.

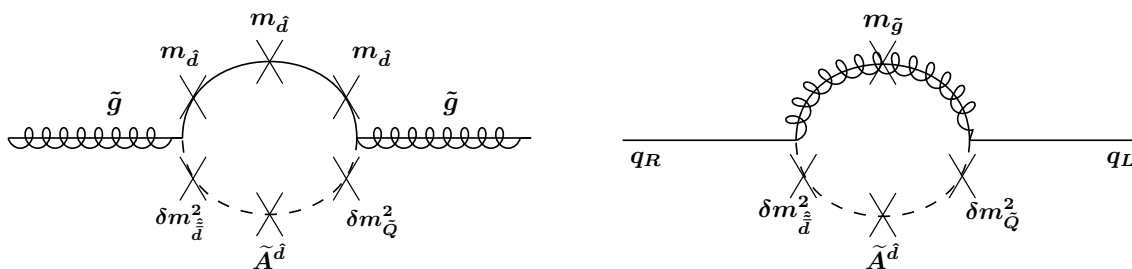


Figure 2. Radiative corrections to the phase of the gluino mass (the left panel) and quark mass (the right panel) induced by flavor violating SUSY-breaking parameters, $\delta m_{\hat{d}}^2$ and $\delta m_{\hat{Q}}^2$.

In the second relation, we have used $\tilde{A}^{\hat{d}} \sim y^{\hat{d}} m_{\text{soft}} v_u / v_d$ and $\delta m_{\hat{Q}}^2 \sim (y^u)^\dagger y^u m_{\text{soft}}^2 / (16\pi^2)$, which are dominated by the radiative correction of the top Yukawa coupling. Note that $\delta\theta_d^m$ vanishes for $\delta m_{\hat{d}}^2 \sim y^{\hat{d}} (y^{\hat{d}})^\dagger m_{\text{soft}}^2 / 16\pi^2$ because of the hermiticity. Therefore, the constraint from $\delta m_{\hat{Q}}^2$ in eq. (3.9) is subdominant.

In general, when A -parameters are not precisely proportional to the corresponding Yukawa coupling, additional radiative corrections to $\bar{\theta}$ proportional to $\text{Im Tr} \left[(y^{\hat{d}})^\dagger A^{\hat{d}} \right]$ and $\text{Im Tr} \left[(y^{\hat{d}})^{-1} A^{\hat{d}} \right]$ are generated [104]. However, these contributions vanish for the A -parameter given by eq. (3.9). We have also confirmed that radiative corrections to phases of the up-type SM quark masses are not induced by the soft SUSY breaking parameters given by eq. (3.9). Therefore, there is no additional constraint from the up-type sector.

The radiative corrections to the phase of the gluino are subdominant compared to those of the SM quarks because they are suppressed by powers of $v_{u,d}/m_{\text{soft}}$. Then, the stringent constraint originates from $\delta\theta_d^m$. By substituting the expression in eq. (3.9) into eq. (3.12), the bound is given by

$$\left(y_{\alpha i}^{\hat{D}} y_{\beta k}^{\hat{D}} \right)^* y_{\alpha k}^{\hat{D}} y_{\beta j}^{\hat{D}} \tan \beta \frac{M_*^2}{M_{\text{CP}}^2} \lesssim 10^{-3}. \quad (3.13)$$

In this calculation, we have used $\tan \beta \equiv v_u / v_d \gg 1$ and the condition $\delta\theta_d^m < 10^{-10}$.

Finally, let us comment on the case with $M_* > M_{\text{CP}}$. Although the EFT analysis presented above cannot be applied, we have checked that the similar Feynman diagram gives the most important contribution to $\delta\bar{\theta}$. The constraint can be obtained by replacing M_{CP}^2 in eq. (3.13) to M_*^2 . To summarize, in the case of either $M_* < M_{\text{CP}}$ or $M_* > M_{\text{CP}}$, $\delta\bar{\theta} \lesssim 10^{-10}$ is satisfied if

$$\left(y_{\alpha i}^{\hat{D}} y_{\beta k}^{\hat{D}} \right)^* y_{\alpha k}^{\hat{D}} y_{\beta j}^{\hat{D}} \tan \beta \times \min \left[\frac{M_*^2}{M_{\text{CP}}^2}, 1 \right] \lesssim 10^{-3}. \quad (3.14)$$

This constraint will be used in section 5 to determine the parameter space where the strong CP problem, baryon asymmetric Universe, and DM are explained.

	$B - L$
$D\bar{D}$	0
$D\bar{d}$	0
$\bar{u}\bar{d}\bar{D}$	-1
$Q\bar{D}L$	-1
$\bar{u}\bar{e}D$	-1
QQD	1
$Q\bar{u}Q\bar{D}$	0
$\bar{u}\bar{u}\bar{D}\bar{e}$	0
$\bar{d}\bar{d}\bar{D}LL$	-3

Table 2. Some flat directions associated with D and/or \bar{D} and their $B - L$ charges. The Dirac mass term is neglected to define the flat directions.

4 Affleck-Dine baryogenesis via a heavy quark flat direction

Let us consider baryogenesis in the SUSY NB model. We will verify that the AD baryogenesis works without introducing further new fields nor CP-violating operators in the model.

The scalar potential in a supersymmetric model becomes zero if all of the F -terms and D -terms are zero. In some cases, there are non-trivial scalar VEV solutions with $F_i = D_a = 0$ which can be parameterized by continuous parameters. The scalar potential is flat in such directions, which are called flat directions.⁶ It has been known that flat directions are characterized by gauge invariant combinations of chiral superfields [110–112]. A part of flat directions in our model are then listed in table 2, where we neglect the Dirac mass term and only write the ones associated with D and/or \bar{D} . The flat directions in the MSSM can be found in ref. [113]. The AD baryogenesis is the mechanism that generates $B - L$ asymmetry via the dynamics of a flat direction with a nonzero $B - L$ charge, called an AD field [79–81]. The resulting $B - L$ asymmetry is converted to the baryon asymmetry via the sphaleron effect before the electroweak phase transition [78, 114].

In this study, we assume that the CP symmetry is not restored by the dynamics of the AD field, to avoid the domain wall problem of spontaneous CP violation. In particular, we should not use flat directions including \bar{d} and/or D . This is because the CP violating field η obtains a large effective mass via the Yukawa term if the flat direction including \bar{d} and/or D obtains a large VEV, and then the CP symmetry is restored in the early Universe. For concreteness, we identify the $Q\bar{D}L$ flat direction as the AD field to attain the AD mechanism. This flat direction contains a heavy squark and hence provides a unique scenario for the NB model. Note that there is no Yukawa term that directly connects Q and \bar{D} . This is in contrast to the $Q\bar{d}L$ flat direction in the MSSM, which has a renormalizable F -term potential from the SM Yukawa interaction.

⁶In the context of the AD baryogenesis, even if a direction has a Dirac mass from F -terms, it is conventional to call it a flat direction.

4.1 The potential of the AD field

First, we describe the potential of the AD field during and after inflation. We consider the scenario of non-instantaneous reheating after inflation, where inflation is followed by an inflaton-oscillation dominated era (or so-called early matter-dominated era) and then by the radiation dominated era. We denote the Hubble parameter during inflation as H_{inf} and the reheating temperature as T_{RH} . It is assumed that H_{inf} is larger than the mass of the AD field.

If the AD field has a very flat potential, it may have a large VEV in the early Universe. Therefore, higher-dimensional operators are important in the stabilization of its VEV at a high-energy scale. We may write a non-renormalizable superpotential for the AD field [81],

$$W \supset \frac{\lambda}{(N!)^3 M_{\text{Pl}}^{3(N-1)}} (Q\bar{D}L)^N, \tag{4.1}$$

where the flavor indices have been omitted for notational simplicity. The superpotential of the AD field ϕ is obtained by identifying $\phi^3 \approx Q\bar{D}L$. This superpotential leads to the corresponding A -term via the gravity-mediated SUSY breaking effect. Although this A -term can produce baryon asymmetry, it is strongly suppressed by a small gravitino mass in our model. We thus instead consider the case where the above non-renormalizable superpotential is absent owing to an implicit approximate (discrete) R-symmetry. For example, we can use the Z_{4R} R-symmetry to forbid the above superpotential for the case of $N = 4$. In this case, non-renormalizable Kähler potentials (rather than superpotentials) are important for the dynamics of the AD field. Here, we only consider the minimal terms that are important for our discussion [81, 115, 116]:

$$K \supset K_H + K_A, \tag{4.2}$$

$$K_H = -\frac{c_H}{M_{\text{Pl}}^2} I^\dagger I \phi^\dagger \phi + \frac{c_K}{M_{\text{Pl}}^6} I^\dagger I (Q^\dagger \bar{D}^\dagger L^\dagger) (Q\bar{D}L), \tag{4.3}$$

$$K_A = -\frac{c_A}{(N!)^3 M_{\text{Pl}}^{3N}} I^\dagger I (Q\bar{D}L)^N + \text{h.c.} \tag{4.4}$$

where $c_{H,K,A}$ are $\mathcal{O}(1)$ real constants. In the first term, $\phi^\dagger \phi$ can be understood as a collective notation for $Q^\dagger Q$, $\bar{D}^\dagger \bar{D}$, and $L^\dagger L$. The field whose F -term dominates the energy density of the Universe during inflation and inflaton-oscillation dominated era is collectively denoted as I , such that $|F_I|^2 \sim 3H^2(t)M_{\text{Pl}}^2$, where $H(t)$ is the Hubble parameter.⁷ We include the factorial prefactors in the denominator of the non-renormalizable Kähler potential K_A such that the cutoff scale is of the order of M_{Pl} by canceling the combinatorial factor. Only the smallest dimensional $U(1)_{B-L}$ -breaking term, such as K_A , is important. Note that the Kähler potential violates $B - L$ symmetry but maintains CP symmetry. Thus, it is consistent with the NB mechanism.

⁷Here, we have assumed that inflation is driven by an F -term potential for concreteness. Even if inflation is driven by a D -term potential [117, 118], our result is not significantly changed [119–122].

With the non-renormalizable Kähler potentials, the potential of the AD field $\phi^3 \approx Q\bar{D}L$ during inflation and the inflaton-oscillation dominated era is given by:

$$V(\phi) = m_\phi^2 |\phi|^2 - c_H H^2(t) |\phi|^2 + \frac{c_K H^2(t)}{M_{\text{Pl}}^4} |\phi|^6 - \left(\frac{c_A H^2(t)}{(N!)^3 M_{\text{Pl}}^{3N-2}} \phi^{3N} + \text{c.c.} \right). \quad (4.5)$$

Here, some $\mathcal{O}(1)$ factors are absorbed into $c_{H,K,A}$.⁸ To realize the AD mechanism, we assume $c_H > 0$ and $c_K > 0$. Hereafter, we assume $c_A > 0$ for simplicity although a negative c_A is also allowed in our scenario.⁹ Note again that there is no CP violating term in the potential. In the first term, m_ϕ is the mass of the AD field that arises after turning on the Yukawa interaction and the Dirac mass term for the heavy quark. Since D and \bar{D} have the Dirac mass term, the mass of the AD field is given by $m_\phi \sim M_D$. This is a bare mass term in the superpotential so that it is not suppressed even for an energy scale larger than the messenger mass. This is in contrast to other MSSM flat directions, for which the soft mass terms are suppressed at an energy scale above the messenger scale [126].

4.2 The dynamics

Let us now discuss the dynamics of the AD field. For this purpose, we decompose the AD field into radial and phase directions, such as $\phi = \varphi e^{i\theta} / \sqrt{2}$. The equation of motion of the AD field is then given by

$$\ddot{\varphi} + 3H\dot{\varphi} - \dot{\theta}^2 \varphi + \frac{\partial V}{\partial \varphi} = 0, \quad (4.6)$$

$$\ddot{\theta} + 3H\dot{\theta} + 2\frac{\dot{\varphi}}{\varphi}\dot{\theta} + \frac{1}{\varphi^2} \frac{\partial V}{\partial \theta} = 0. \quad (4.7)$$

We consider the case where the Hubble parameter during inflation $H(t) \simeq H_{\text{inf}}$ is larger than M_D . Specifically, we assume that:

$$\sqrt{c_H} H_{\text{inf}} \gtrsim m_\phi \sim M_D. \quad (4.8)$$

Then, we can neglect m_ϕ in eq. (4.5) and the AD field remains at a large VEV during inflation:

$$\langle \varphi \rangle \simeq \left(\frac{4c_H}{3c_K} \right)^{1/4} M_{\text{Pl}}. \quad (4.9)$$

The condition (4.8) is necessary to obtain the large AD field VEV in eq. (4.9). One might worry about the domain wall problem from the restoration of CP symmetry because of a large H_{inf} , however, this problem can be avoided by introducing a negative Hubble induced mass for the η field. Note that the effect of the last term in eq. (4.5) on $\langle \varphi \rangle$ is negligible because of the suppression by $1/(N!)^3$. Regarding the phase direction, the solution to eq. (4.7) is given by

$$\theta \simeq \theta_{\text{min}} + (\theta_{\text{ini}} - \theta_{\text{min}}) \exp \left[- \left(\frac{r_\theta}{3} \right) H_{\text{inf}} t \right], \quad (4.10)$$

⁸A factor of 3 for $H^2|\phi|^2$ originates from the supergravity potential, even if K_H is absent. It is absorbed into c_H in eq. (4.5) for notational simplicity.

⁹If the VEV of ϕ is as large as the Planck scale, as we will verify shortly, the thermal effect on the AD field is negligible [81, 123–125].

for $r_\theta \ll 1$ and $|\theta_{\text{ini}} - \theta_{\text{min}}| \lesssim 1/3N$, where θ_{ini} is the initial phase and $\theta_{\text{min}} (= (2\pi/3N)i, i = 0, 1, 2, \dots, 3N - 1)$ represents the potential minima of the phase direction. We have also defined

$$r_\theta \equiv \frac{\partial^2 V}{\varphi^2 \partial \theta^2} \frac{1}{H_{\text{inf}}^2} \Big|_{\phi=\langle \phi \rangle} \quad (4.11)$$

$$\sim \left(\frac{(3N)^2 c_A}{(N!)^3} \right) \left(\frac{c_H}{3c_K} \right)^{(3N-2)/4}. \quad (4.12)$$

We consider the case in which the phase direction θ does not reach the potential minimum during inflation. Here, the inflation should last longer than $t \sim H_{\text{inf}}^{-1} N_e$, where N_e (~ 50) is the e-folding number of inflation. This implies that for the phase direction not to reach the potential minimum during inflation, r_θ should be much smaller than $3/N_e$ ($\lesssim 1/20$). It can be verified that r_θ is much smaller than $\mathcal{O}(0.01)$ for $N \geq 3$, depending on $c_{H,K,A}$ ($= \mathcal{O}(1)$). For example, $r_\theta \simeq 3 \times 10^{-3}$ for $N = 3$ with $5c_H/c_K = c_A = 1$ whereas $r_\theta \simeq 7 \times 10^{-4}$ for $N = 4$ with $c_H = c_A = c_K = 1$. Hence, the phase of the AD field is almost massless during inflation and stays at a certain phase for a sufficiently long e-folding number. We denote the phase of the AD field at the end of inflation as $\theta_{\text{ini}} (\in (-\pi, \pi))$. In general, θ_{ini} is nonzero and is expected to be $\mathcal{O}(1)$ without fine-tuning. Therefore, CP symmetry is spontaneously broken by the AD field during inflation. This is a source of CP violation that is required to generate the $B - L$ asymmetry.

After inflation, the Hubble parameter decreases and eventually becomes comparable to $m_\phi/\sqrt{c_H}$. Then, the AD field starts to oscillate around $\varphi = 0$. We denote the Hubble parameter and the time at the onset of the oscillation as $H_{\text{osc}} (\simeq m_\phi/\sqrt{c_H})$ and $t_{\text{osc}} (= 2/3H_{\text{osc}})$, respectively. The amplitude of the oscillation φ_{osc} is again given by eq. (4.9). Simultaneously, the phase direction is kicked by the last term in eq. (4.5), and the AD field starts to rotate in the phase space. Let us see how $B - L$ number asymmetry is generated from the time evolution of the AD field. The Noether current of $B - L$ charge is $j_{B-L,\mu} = q_{B-L}(i\phi\partial_\mu\phi^\dagger - i\phi^\dagger\partial_\mu\phi) = q_{B-L}\varphi^2\partial_\mu\theta$, where q_{B-L} ($= -1$) is the $B - L$ charge of the AD field. We obtain $B - L$ charge density as

$$n_{B-L} = q_{B-L}\varphi^2\dot{\theta}. \quad (4.13)$$

We can see, provided that the AD field carries the nonzero $B - L$ charge, its rotation generates $B - L$ asymmetry. By using the equations of motion (4.6), (4.7), we obtain the time evolution of n_{B-L} defined in eq. (4.13) as

$$\frac{1}{a(t)^3} \frac{\partial}{\partial t} (a(t)^3 n_{B-L}) = -q_{B-L} \frac{\partial V}{\partial \theta}. \quad (4.14)$$

We can see that the explicit $U(1)_{B-L}$ breaking term $\partial V/\partial \theta$ works as a source term for $B - L$ asymmetry.

The numerical results of time evolution of $B - L$ charge density of the AD field in a comoving volume are shown in figure 3, where we choose $N = 3$ with $c_K = 5$, $N = 4$ with $c_K = 1$, and $N = 5$ with $c_K = 1/3$. We take $3N\theta_{\text{ini}} = 1$ for each N . Other $\mathcal{O}(1)$ parameters

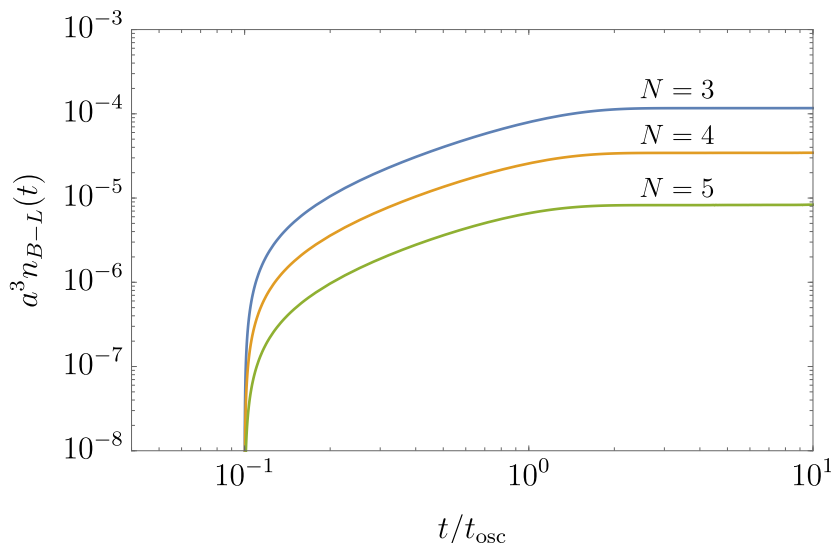


Figure 3. Time evolution of $B - L$ charge density of the AD field in a comoving volume. We choose $N = 3$ with $c_K = 5$ (blue curve), $N = 4$ with $c_K = 1$ (yellow curve), and $N = 5$ with $c_K = 1/3$ (green curve). We take $3N\theta_{\text{ini}} = c_A = c_H = -q_{B-L} = 1$ for each case.

are set to unity. It can be seen that the $B - L$ asymmetry is generated around $t = t_{\text{osc}}$ and its comoving density is conserved for $t \gg t_{\text{osc}}$. We can also obtain the analytic formula by approximating the time integral in eq. (4.14) as $\int dt \varphi^{3N} \sin(3N\theta) \sim t_{\text{osc}} \varphi_{\text{osc}}^{3N} \sin(3N\theta_{\text{ini}})$,

$$n_{B-L}(t) = q_{B-L} \epsilon m_\phi \varphi_{\text{osc}}^2 \frac{a(t_{\text{osc}})^3}{a(t)^3}. \quad (4.15)$$

Here, the ellipticity ϵ is given by

$$\epsilon = -c_\epsilon \sin(3N\theta_{\text{ini}}) \left(\frac{4Nc_A}{(N!)^3 \sqrt{c_H}} \right) \left(\frac{c_H}{3c_K} \right)^{(3N-2)/4}, \quad (4.16)$$

where $H_{\text{osc}} = 2/(3t_{\text{osc}}) = m_\phi/\sqrt{c_H}$, and c_ϵ is the $\mathcal{O}(1)$ numerical factor. This is approximately proportional to the initial phase and ratio in the potential curvature between the radial and phase directions. Using our numerical results, we can verify that eqs. (4.15) and (4.16) with $c_\epsilon = 0.5-1$ gives the correct value for the parameter in the region of interest (namely, for $N, \theta_{\text{ini}}, c_{H,K,A} = \mathcal{O}(1)$).

The amplitude of ϕ decreases owing to the cosmic expansion, and the non-renormalizable $B - L$ breaking operators become ineffective after the onset of the oscillation. It can be observed from eq. (4.14) that the $B - L$ number-to-entropy ratio is conserved for $t \gg t_{\text{osc}}$ and is given by

$$\begin{aligned} \frac{n_{B-L}}{s} &= \frac{n_{B-L}}{4\rho/3T} \Big|_{T=T_{\text{RH}}} = \frac{3T_{\text{RH}}}{4} \frac{n_{B-L}}{3H^2 M_{\text{Pl}}^2} \Big|_{t=t_{\text{osc}}} \\ &= q_{B-L} \epsilon c_H \frac{T_{\text{RH}}}{4m_\phi} \frac{\varphi_{\text{osc}}^2}{M_{\text{Pl}}^2} \\ &= -\sin(3N\theta_{\text{ini}}) q_{B-L} \frac{T_{\text{RH}}}{m_\phi} \left(\frac{2Nc_A c_\epsilon \sqrt{c_H}}{(N!)^3} \right) \left(\frac{c_H}{3c_K} \right)^{3N/4}. \end{aligned} \quad (4.17)$$

We have used eqs. (4.9), (4.15), (4.16). Here, T_{RH} is the reheating temperature, that is, the temperature at the beginning of the radiation-dominated era. We do not assume instantaneous reheating, but the case in which the inflaton-oscillation dominated era is followed by the radiation-dominated era is considered. For example, the $B - L$ number-to-entropy ratio is given by

$$\frac{n_{B-L}}{s} \simeq 3 \times 10^{-10} \sin(3N\theta_{\text{ini}}) \left(\frac{T_{\text{RH}}}{10^3 \text{ GeV}} \right) \left(\frac{m_\phi}{10^8 \text{ GeV}} \right)^{-1}, \quad (4.18)$$

for $N = 3$ with $c_A = c_H = c_K/5 = 1$ (in which case, $c_\epsilon \simeq 1.1$), and

$$\frac{n_{B-L}}{s} \simeq 2 \times 10^{-10} \sin(3N\theta_{\text{ini}}) \left(\frac{T_{\text{RH}}}{10^3 \text{ GeV}} \right) \left(\frac{m_\phi}{10^8 \text{ GeV}} \right)^{-1}, \quad (4.19)$$

for $N = 4$ with $c_A = c_H = c_K = 1$ (in which case $c_\epsilon \simeq 0.97$). Note that $m_\phi \sim M_D$, which is much larger than the visible sector superpartner mass scale.

4.3 Dissipation into thermal plasma

After the $B - L$ asymmetry is generated by the dynamics of the AD field, it should be dissipated into thermal plasma. If the dissipation is completed before the electroweak phase transition, the $B - L$ asymmetry is converted into a combination of baryon and lepton asymmetries via the electroweak sphaleron. We will verify that this is indeed the case in our model.

Since the amplitude of ϕ is as large as the Planck scale, the energy density of its oscillation is comparable to that of the inflaton oscillation. If the reheating is completed before the dissipation of ϕ , its energy density dominates the Universe. In this case, T_{RH} in eq. (4.17) must be replaced by the decay temperature of the AD field. Since the decay rate of the AD field is quite large, its decay temperature cannot be as low as the scale of interest. Thus, it is necessary to check whether the dissipation temperature is higher than the reheating temperature.

If the AD field has a very large field value, it cannot decay into particles that directly couple to the AD field at the tree level. Instead, the AD field can dissipate into light particles via loop diagrams. The rate of dissipation into gauge fields via loop diagrams is estimated as (see, for example, refs. [127, 128])

$$\Gamma_{\text{diss}}(t) \sim 10^{-2} \alpha^2 y^2 \frac{m_\phi^3}{(y\phi)^2}, \quad (4.20)$$

where α and y generically represent the fine-structure constants for SM gauge interactions and Yukawa interactions for the AD field, respectively. The dependence of the Yukawa coupling is cancelled between the denominator and numerator. Here, we implicitly assume $m_\phi > \alpha T$ and $y\phi \gg m_\phi$, which is satisfied during the regime of interest. We compare the dissipation rate with the Hubble expansion rate and obtain

$$\frac{\Gamma_{\text{diss}}(t)}{H(t)} \sim 10^{-2} \alpha^2 \left(\frac{m_\phi}{M_{\text{Pl}}} \right)^2 \left(\frac{M_{\text{Pl}}}{\phi_{\text{osc}}} \right)^2 \left(\frac{m_\phi}{H(t)} \right)^3, \quad (4.21)$$

where we have considered that the amplitude of ϕ decreases as $\propto H(t)$. The ratio becomes larger than $\mathcal{O}(1)$ by the completion of the reheating for a typical parameter space of interest. This means that the AD field dissipates into the thermal plasma well before the completion of the reheating, at which $H(T_{\text{RH}}) \sim 10^{-12}$ GeV for $T_{\text{RH}} \sim 1$ TeV. Therefore, we conclude that the AD field does not dominate the Universe, and thus, the calculation of the baryon asymmetry can be justified.

In the preceding discussions, we have assumed that the AD field does not form Q-balls [87], which are localized condensation with a large number of $B - L$ charges. If the potential of the AD field is less than quadratic, spacial perturbations of the AD field increase and eventually Q-balls form after the onset of the oscillation [82–86, 89, 129]. In our case, the mass of the AD field m_ϕ has logarithmic quantum corrections via the renormalization group running and its potential can be written as

$$V(\phi) \simeq m_\phi^2 \left[1 + K \log\left(\frac{|\phi|}{M_{\text{Pl}}}\right) \right] |\phi|^2, \tag{4.22}$$

where K is a small parameter determined by the quantum corrections. Here, we neglect the higher-dimensional operators because they are not important for $|\phi| \ll M_{\text{Pl}}$. If K is negative, the potential is shallower than the quadratic and Q-ball forms after the onset of the oscillation. The sign of K depends on the value of the quantum corrections from the Yukawa interactions compared to those from the gauge interactions. If the Yukawa coupling is sufficiently small, then K is negative. Depending on the value of the Yukawa coupling, the Q-balls may or may not form in our scenario.

Q-balls are localized condensations with a high number density, and their decay rate into fermions is strongly suppressed because the flux of fermions is saturated by the Pauli exclusion principle on the surface of the Q-balls [130]. In the standard AD baryogenesis, Q-balls are therefore long-lived (or stable), and the thermal history and the calculation of baryon asymmetry may drastically change. In our case, however, the AD field consists of the heavy squark, which can decay into light bosons (the MSSM squarks and Higgs), without being suppressed by the Pauli exclusion principle. Rather, their decay rate is enhanced exponentially by the Bose enhancement effect [131, 132] similarly to the preheating process [133–137]. This means that even if Q-balls are formed, they quickly dissipate into light particles. Therefore, we conclude that the formation of the Q-balls does not affect the calculations in our case.

The AD field dissipates before the electroweak phase transition, and the $B - L$ asymmetry is converted to a combination of baryon and lepton asymmetries via the electroweak sphaleron. This is the same even if the reheating temperature is lower than the electroweak scale because the electroweak sphaleron is efficient within the ambient plasma even before the reheating is completed (see, for example, ref. [138]). In any case, the resulting baryon-to-entropy ratio is given by [139]

$$Y_B = \frac{8}{23} \frac{n_{B-L}}{s}. \tag{4.23}$$

The observed value is 9×10^{-11} [140]. Thus, using eq. (4.17), we can realize the baryon asymmetric Universe in the SUSY NB model.

4.4 Quantum fluctuations

The phase of the AD field is almost massless, so that it acquires quantum fluctuations during inflation,

$$|\delta\theta_{\text{ini}}| \simeq \frac{H_{\text{inf}}}{2\pi\langle\varphi\rangle}. \quad (4.24)$$

The produced baryon asymmetry depends on θ_{ini} (see eq. (4.17)), and hence the fluctuations result in isocurvature fluctuations of baryon asymmetry [120–122, 141, 142]:

$$S_{b\gamma} \equiv \frac{\delta Y_B}{Y_B} \simeq 3N \cot(3N\theta_{\text{ini}}) \delta\theta_{\text{ini}}. \quad (4.25)$$

Since the observed density perturbations are predominantly adiabatic, the isocurvature fluctuations are constrained by the Planck results, such as $S_{b\gamma} \lesssim 5.0 \times 10^{-5}$. Together with eqs. (4.9) and (4.24), we obtain

$$H_{\text{inf}} \lesssim 8.1 \times 10^{14} \text{ GeV} \left(\frac{\tan(3N\theta_{\text{ini}})}{3N} \right) \left(\frac{c_H}{c_K} \right)^{1/4}. \quad (4.26)$$

This bound will be improved by a factor of a few by future observations of the cosmic microwave background (CMB), such as LiteBIRD [143, 144] and CORE [145].

The energy scale of inflation is constrained by the measurement of the tensor modes for CMB temperature anisotropies. The upper bound on the tensor-to-scalar ratio is approximately given by $r \simeq 0.036$ [146], which implies that

$$H_{\text{inf}} \simeq 4.6 \times 10^{13} \text{ GeV} \left(\frac{r}{0.036} \right)^{1/2}. \quad (4.27)$$

The future CMB missions, such as LiteBIRD [143, 144], CORE [145], and CMB-S4 [147], will measure r with uncertainty of $\mathcal{O}(10^{-3})$. Combining eq. (4.25) with eq. (4.27), we predict a consistency relation between the isocurvature fluctuations and the tensor-to-scalar ratio such as

$$\left(\frac{S_{b\gamma}}{5 \times 10^{-5}} \right) = \left(\frac{\theta_{\text{ini}}}{0.06} \right)^{-1} \left(\frac{\tan(3N\theta_{\text{ini}})}{3N\theta_{\text{ini}}} \right)^{-1} \left(\frac{c_H}{c_K} \right)^{-1/4} \left(\frac{r}{0.036} \right)^{1/2}. \quad (4.28)$$

If H_{inf} is close to the upper bounds, both isocurvature fluctuations and the tensor-to-scalar ratio would be observed in future.

5 Summary plots and other constraints

We now combine the results and constraints obtained from the previous sections to determine the parameter space in which the strong CP problem, baryon asymmetric Universe, and DM are explained. However, it is still needed to take account of some constraints in cosmological scenarios, such as the domain-wall formation and the gravitino overproduction problem. We first explain these issues and then show summary plots.

5.1 The domain-wall problem

Let us consider the constraint on the maximum temperature of the Universe after inflation. If CP symmetry is spontaneously broken after inflation, domain walls are formed. The domain walls soon dominate the Universe and lead to a highly inhomogeneous Universe. To avoid this problem, we consider a scenario in which CP symmetry is spontaneously broken before inflation and is never restored after inflation. For this, the maximal temperature after inflation, T_{\max} , must be lower than the mass of η , which is expected to be at the order of the VEV of η . The maximal temperature is estimated as

$$T_{\max} \sim \left(\frac{\Gamma_I}{H_{\text{inf}}} \rho_{\text{inf}} \right)^{1/4} \sim \left(T_{\text{RH}}^2 H_{\text{inf}} M_{\text{Pl}} \right)^{1/4}, \quad (5.1)$$

where $\rho_{\text{inf}} (\simeq 3H_{\text{inf}}^2 M_{\text{Pl}}^2)$ and $\Gamma_I (\sim T_{\text{RH}}^2/M_{\text{Pl}})$ are the energy density during inflation and the inflaton decay rate, respectively. We require

$$T_{\max} \lesssim \langle \eta \rangle \sim \frac{M_{\text{CP}}}{y^D}. \quad (5.2)$$

Note that eq. (5.1) overestimates the maximal temperature because it usually requires a finite time for inflaton-decay products to be thermalized [138]. Therefore, the preceding condition should be regarded as a conservative upper bound on the reheating temperature.

5.2 The gravitino problem and Lyman- α constraint

Gravitino overproduction results in a stringent constraint on the reheating temperature. As we have discussed in section 2, the gravitino mass must be smaller than $\mathcal{O}(10\text{--}100)$ keV to solve the strong CP problem in the NB model. In this case, if the temperature of thermal plasma is higher than the mass scale of the MSSM SUSY particles m_{soft} , the stable gravitino is in thermal equilibrium via the decay and inverse decay of the MSSM SUSY particles. If the gravitino mass $m_{3/2}$ is larger than $\mathcal{O}(1)$ keV, and if $T_{\text{RH}} \gg m_{\text{soft}}$, the gravitino abundance is larger than the DM abundance. This is known as the gravitino problem. One may consider a lighter gravitino, which is favored for the NB model, as explained in the previous section. In this case, the reheating temperature can be arbitrarily large. However, the free-streaming constraint by the Lyman- α forest data excludes the possibility that such a light gravitino is DM.

Instead, one can consider a scenario in which the reheating temperature is lower than the mass of the lightest MSSM SUSY particle. In this case, the gravitino is diluted by the reheating process even though it is produced during the inflaton-oscillation dominated era. Specifically, the gravitino abundance is given by

$$\begin{aligned} \frac{\rho_{3/2}}{s} &\simeq \frac{3T_{\text{RH}}}{4} \left(\frac{\rho_{3/2} \rho_{\text{rad}}}{\rho_{\text{rad}} \rho_{\text{tot}}} \right)_{T=T_{\text{dec},3/2}} \\ &\simeq m_{3/2} \frac{45\zeta(3)}{2\pi^4 g_*} \left(\frac{T_{\text{RH}}}{T_{\text{dec},3/2}} \right)^5, \end{aligned} \quad (5.3)$$

where $\rho_{\text{rad}}/\rho_{\text{tot}} = (2/3)(T_{\text{RH}}/T_{\text{dec},3/2})^4$ in the second line. Here, $T_{\text{dec},3/2} (\sim m_{\text{soft}})$ denotes the decoupling temperature of the gravitino, and g_* is the number of the effective relativistic

degrees of freedom at $T = T_{\text{dec},3/2}$. We can choose T_{RH} to explain the DM abundance by the gravitino, such as

$$T_{\text{RH}} \simeq 0.27 T_{\text{dec},3/2} \left(\frac{m_{3/2}}{100 \text{ keV}} \right)^{1/5}, \quad (5.4)$$

where we have used $\rho_{3/2}/s \simeq 0.4 \text{ eV}$ for the observed DM density and $g_* \simeq 106.75$. The reheating temperature must be lower than m_{soft} , but it should not be too small because the suppression factor in the gravitino abundance has a power of five. In addition, the free-streaming velocity of the gravitino must be suppressed. The Lyman- α constraint reads [148, 149] (see also ref. [150])

$$m_{3/2} \gtrsim 5.3 \text{ keV}. \quad (5.5)$$

Our results (4.18) and (4.19) show that a sufficient baryon asymmetry can be successfully produced, while satisfying the constraints on T_{RH} and $m_{3/2}$. We have found that the Dirac mass of the heavy quark should be 10^9 GeV or smaller, depending on the T_{RH} and $\mathcal{O}(1)$ constants. This is consistent with the constraints discussed in section 3 to suppress quantum corrections to the strong CP phase.

5.3 Summary plots

We now summarize our results and show some plots. The strong CP phase is given by the contributions from the Planck-suppressed operators in eq. (2.12), anomaly-mediated effect in eq. (2.14), and radiative corrections in eq. (3.14). They are summarized as

$$\bar{\theta} = \bar{\theta}_{\text{Planck}} + \bar{\theta}_{\text{anomaly}} + \bar{\theta}_{\text{radiative}}, \quad (5.6)$$

$$\bar{\theta}_{\text{Planck}} \sim 10^{-2} \left(\frac{\text{Re}(\mu) \tan \beta}{M_{\text{Pl}}} \right)^{-1} \left(\frac{\langle \eta_\alpha \rangle}{M_{\text{Pl}}} \right)^N, \quad (5.7)$$

$$\bar{\theta}_{\text{anomaly}} \sim \frac{\alpha_s}{4\pi} \frac{m_{3/2}}{m_{\tilde{g}}}, \quad (5.8)$$

$$\bar{\theta}_{\text{radiative}} \sim 10^{-7} (y_{\alpha i}^{\hat{D}} y_{\beta k}^{\hat{D}})^* y_{\alpha k}^{\hat{D}} y_{\beta j}^{\hat{D}} \tan \beta \times \min \left[\frac{M_*^2}{M_{\text{CP}}^2}, 1 \right]. \quad (5.9)$$

To show the results, we just sum up these contributions with equal signs without fine-tunings. The overall sign can be either positive or negative. To explain the observed baryon asymmetry, we require that Y_{B-L} is given by eq. (4.18) (for $N = 3$) or eq. (4.19) (for $N = 4$) and is approximately 3×10^{-10} . The Hubble parameter during inflation should satisfy eqs. (4.8) and (5.2). We can explain the observed DM abundance by the thermally produced gravitino as presented in eq. (5.4), for which case eq. (5.5) must be satisfied to avoid the Lyman- α constraint.

Let us summarize the parameters in our model. As explained in section 2, $M_D^2 \sim B^\dagger B$ should be satisfied to obtain the observed CKM matrix. Then, we take $M_{\text{CP}} = M_D = y^D \langle \eta_\alpha \rangle$ and $y^D \sim y^{\hat{D}} \sim y^{\hat{D}\hat{D}}$ for simplicity. The messenger scale M_* , which enters in eq. (3.14), is given by $M_* = (\alpha/4\pi) \langle F \rangle / m_{\text{soft}}$, where $\langle F \rangle = \sqrt{3} M_{\text{Pl}} m_{3/2}$ is the SUSY breaking scale and α represents the fine structure constants of the SM gauge couplings. The messenger scale and the SUSY breaking scale must satisfy the relation $\langle F \rangle < M_*^2$ for the scalar component

of a messenger chiral superfield to stay at the origin of the potential. We take $\alpha = 1/30$, $\mu = m_{\text{soft}}$, and $\tan\beta = 10$ as a benchmark point, although the dependence on $\tan\beta$ is very weak. In summary, we consider $m_{\text{soft}}, m_{3/2}, M_{\text{CP}}, y^D$, and N as free parameters in the model. Our prediction of $|\bar{\theta}|$ on the $(m_{3/2}, M_{\text{CP}})$ -plane for $y^D = 0.1$ and $m_{\text{soft}} = 10$ TeV with $N = 3$ (upper panel) and $N = 4$ (lower panel) is plotted in figure 4, assuming that the gravitino abundance is equal to the observed DM abundance as in eq. (5.4). The meshed green and brown colored regions are excluded by the Lyman- α constraint (5.5) and the formation of the domain wall (5.2), respectively. The light blue regions give $|\bar{\theta}| < 10^{-10}, 10^{-11}$. The light green shaded region denotes $Y_B = Y_B^{(\text{obs})}$, where $Y_B^{(\text{obs})}$ is the observed baryon-to-entropy ratio. Here, Y_B is given by eqs. (4.18) and (4.19), where $m_\phi = M_{\text{CP}}$ and $\sin(3N\theta_{\text{ini}}) \in (0.1, 1)$.

The strong CP phase $|\bar{\theta}|$ can be sufficiently small in the light blue region. The lower bound on M_{CP} originates from $\bar{\theta}_{\text{radiative}}$ to avoid a large radiative correction to $\bar{\theta}$ from the heavy vector-like quark. For a large $m_{3/2}$, the anomaly mediated contribution to the gluino soft mass, $\bar{\theta}_{\text{anomaly}}$, becomes large, and hence, there exists an upper bound for $m_{3/2}$. The upper bound on M_{CP} originates from $\bar{\theta}_{\text{Planck}}$ so that the Planck-suppressed CP-violating operator does not reintroduce $\bar{\theta} > 10^{-10}$. For $N = 2$, this bound is too strong, and there is no viable parameter region for our benchmark point. As the value of N is large, Z_N symmetry strongly forbids the Planck-suppressed operator, and hence, this constraint becomes relaxed for a large N , as shown in the figure. We can explain the non-observation of the strong CP phase, gravitino DM, and baryon asymmetry in the region where the light blue and green shaded regions are overlapped.

Figure 5 shows the prediction of $|\bar{\theta}|$ on the $(m_{\text{soft}}, M_{\text{CP}})$ -plane for $y^D = 0.1$ (upper panel) and $y^D = 0.01$ (lower panel) with $N = 3$. Here, T_{RH} and $m_{3/2}$ are taken such that the observed baryon asymmetry and DM energy density are accounted for by the AD mechanism and thermally produced gravitino, respectively. The meshed green region is excluded by the Lyman- α constraint. The TeV-scale SUSY is not compatible with both $|\bar{\theta}| < 10^{-10}$ and the Lyman- α constraint for the case of $y^D = 0.1$ but is compatible for a smaller y^D . The meshed brown region is excluded by the condition of $\langle F \rangle < M_*^2$. For a given m_{soft} , there is an upper bound on the allowed value of M_{CP} , which can be relaxed for a large N . The dashed line in the upper panel represents the lower bound on M_{CP} by the domain-wall formation for the case of $H_{\text{inf}} \simeq 4.6 \times 10^{13}$ GeV. This is the most conservative bound and is relaxed for a small (but large enough to realize the AD mechanism) H_{inf} . This constraint is evaded in the whole parameter space in the lower panel.

There is a lower bound on M_{CP} to explain the baryon asymmetry for a given m_{soft} . From eqs. (4.18) and (4.19), the reheating temperature is related to M_{CP} . Moreover, from eq. (5.4), the reheating temperature should be of the same order with m_{soft} to explain DM by the thermally produced gravitino. Requiring that m_{soft} is larger than the TeV scale, we obtain

$$M_{\text{CP}} \gtrsim 10^7 \text{ GeV}, \quad (5.10)$$

for $N = 3, 4$, where we omit $\mathcal{O}(1)$ factors.

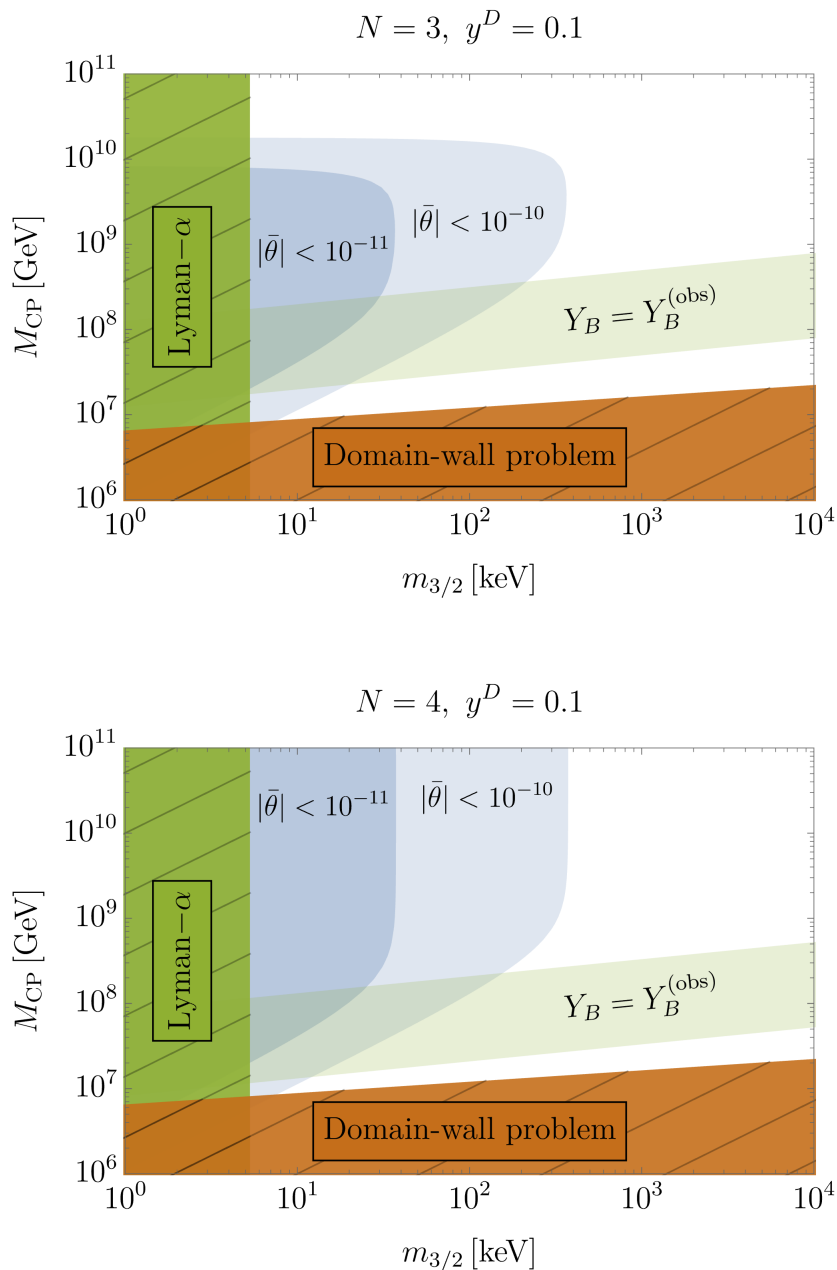


Figure 4. The prediction of $\bar{\theta}$ on $(m_{3/2}, M_{CP})$ -plane for $m_{\text{soft}} = 10 \text{ TeV}$, $y^D = 0.1$ with $N = 3$ (upper panel) and $N = 4$ (lower panel). We take the reheating temperature such that the gravitino abundance is equal to the observed DM abundance. The meshed green- and brown-colored regions are excluded because of the Lyman- α constraint and by the domain wall formation, respectively. The light blue shaded regions show $\bar{\theta} < 10^{-10}, 10^{-11}$, whereas the light green shaded region shows $Y_B = Y^{(\text{obs})}$.

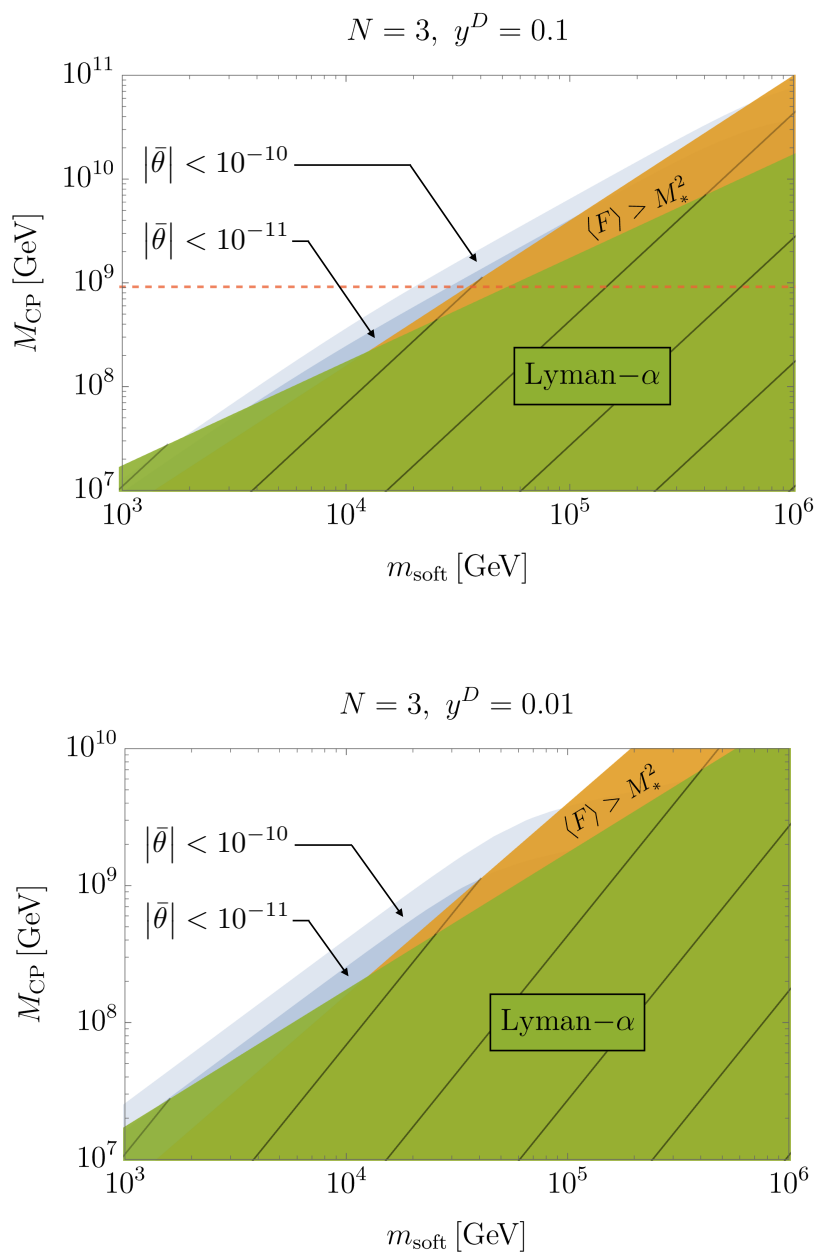


Figure 5. Same as figure 4 but with the observed baryon asymmetry on the $(m_{\text{soft}}, M_{\text{CP}})$ -plane for $y^D = 0.1$ (upper panel) and $y^D = 0.01$ (lower panel) with $N = 3$. The meshed green- and brown-colored regions are excluded because of the Lyman- α constraint and by the condition of $\langle F \rangle < M_*^2$, respectively. The red dashed line in the upper panel represents the most conservative lower bound on M_{CP} from the domain wall formation, which can be relaxed for a small value of H_{inf} .

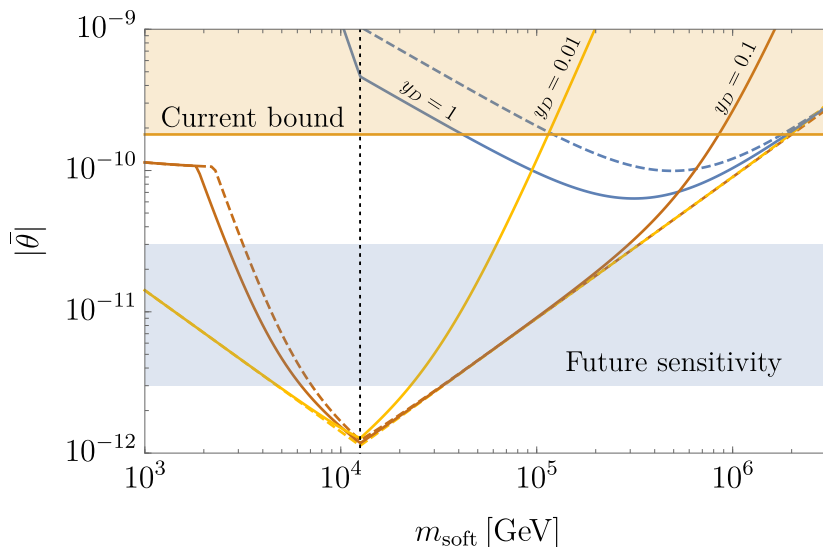


Figure 6. The lower bound on $|\bar{\theta}|$, which is consistent with the baryon asymmetry, gravitino DM, and Lyman- α constraint. We take $y^D = 0.01, 0.1$, and 1 for the yellow, brown and blue lines, respectively. The solid (dashed) lines represent the case of $N = 3$ ($N = 4$). The red-shaded region is excluded from the current constraint on $\bar{\theta}$ [151]. The blue shaded region represents the typical sensitivity of future experiments [152–158].

The condition of $\langle F \rangle < M_*^2$ leads to the upper bound on the SUSY scale,

$$m_{\text{soft}} < \frac{\alpha}{4\pi} \sqrt{\sqrt{3} M_{\text{Pl}} m_{3/2}}. \quad (5.11)$$

If we take the gravitino mass as the lower bound from the Lyman- α constraint, we obtain

$$m_{\text{soft}} < m_{\text{soft}}^{(\text{th})} \simeq 1.3 \times 10^4 \text{ GeV}. \quad (5.12)$$

It should be also noted that the gravitino mass has an upper bound from $|\bar{\theta}| \lesssim 10^{-10}$ because a larger gravitino mass gives a larger contribution from anomaly mediation for any y^D and N , as in eq. (5.8). This implies

$$m_{3/2} < \sqrt{3} M_{\text{Pl}} |\bar{\theta}_{\text{anomaly}}|^2 \lesssim 40 \text{ MeV}, \quad (5.13)$$

$$m_{\text{soft}} < \frac{\alpha}{4\pi} \sqrt{3} M_{\text{Pl}} |\bar{\theta}_{\text{anomaly}}| \lesssim 1.1 \times 10^6 \text{ GeV}, \quad (5.14)$$

where we have used $|\bar{\theta}_{\text{anomaly}}| \lesssim 10^{-10}$ in the last inequalities.

Eqs. (5.8) and (5.9) indicate that $\bar{\theta}$ is smaller for a smaller $m_{3/2}$. Thus, we can obtain the smallest $\bar{\theta}$ for the gravitino mass saturating the Lyman- α bound of eq. (5.5). In figure 6, we plot the smallest value of $\bar{\theta}$ as a function of m_{soft} , by taking the smallest allowed $m_{3/2}$ and determining M_{CP} and T_{RH} to explain the observed baryon asymmetry and DM. The value of $m_{3/2}$ is taken to be 5.3 keV for $m_{\text{soft}} < m_{\text{soft}}^{(\text{th})}$ and $(4\pi/\alpha)^2 m_{\text{soft}}^2 / (\sqrt{3} M_{\text{Pl}})$ for $m_{\text{soft}} > m_{\text{soft}}^{(\text{th})}$, where $m_{\text{soft}}^{(\text{th})}$ ($\simeq 1.3 \times 10^4 \text{ GeV}$) is represented by the vertical black dotted line in the figure. We take $y^D = 0.01, 0.1$, and 1 for the yellow, brown, and blue lines, respectively. The solid

(dashed) lines represent the cases of $N = 3$ ($N = 4$). From the figure, it is evident that there is an absolute lower bound on $|\bar{\theta}|$ ($\sim 10^{-12}$) even if we change y^D and N . This is due to the condition of $\langle F \rangle < M_*^2$ and the anomaly mediation contribution to $\bar{\theta}$ with a fixed $m_{3/2}$. The red shaded region is the current constraint on $|\bar{\theta}|$, where we have used $d_n = 10^{-3} \bar{\theta} e \text{ fm}$ (see refs. [159, 160] for the uncertainty of this relation) and the recent upper bound on the neutron EDM, $|d_n| < 1.8 \times 10^{-13} e \text{ fm}$ [151] (see also ref. [11], which yields $|d_n| < 2.9 \times 10^{-13} e \text{ fm}$). The blue shaded region represents a typical future prospect for the neutron EDM searches: $2\text{--}3 \times 10^{-15} e \text{ fm}$ by nEDM@SNS [152], $3 \times 10^{-14} e \text{ fm}$ by nEDM search at LANL [153], $\mathcal{O}(10^{-14}) e \text{ fm}$ by TUCAN [154], $\mathcal{O}(10^{-14}) e \text{ fm}$ by PanEDM [155], $\mathcal{O}(10^{-(14-15)}) e \text{ fm}$ by PNPI-ILL-PTI [156], $\mathcal{O}(10^{-(14-15)}) e \text{ fm}$ by n2EDM [157] (see also ref. [158] for beam EDM experiments). Thus, we can obtain information on relatively high scale SUSY, where the soft mass is of the order of 0.1–1 PeV, depending on y^D and N . This is an interesting “smoking-gun” signal of SUSY in our scenario. The prediction of observable $\bar{\theta}$ is unique for the case of spontaneously broken CP symmetry because it is negligibly small for the PQ mechanism.

6 Conclusions

In the present study, we have investigated the AD baryogenesis scenario in a NB model with gauge-mediated SUSY breaking. The model introduces a new massive vector-like quark and singlet chiral superfields, whose scalar components develop VEVs leading to spontaneous breaking of CP symmetry. The tree-level contribution to $\bar{\theta}$ is forbidden by the structure of the quark mass matrix. We have performed a detailed analysis of radiative corrections to $\bar{\theta}$ in the NB model with gauge mediation. We found that the model can be a plausible solution to the strong CP problem. The AD baryogenesis was realized by using a flat direction including a heavy vector-like quark, such that long-lived Q-balls do not form after the AD baryogenesis. We have shown that the observed baryon asymmetry can be produced in the CP invariant Lagrangian without causing any cosmological problems, such as the domain-wall problem and gravitino overproduction problem. The gravitino can be DM and the present DM density can be explained when the reheating temperature is lower than the soft mass scale of the SUSY particles.

The viable parameter region, leading to the successful production of baryon asymmetry and DM, was identified by several theoretical and observational constraints. First, the energy scale of spontaneous CP violation is bounded above and below to suppress corrections to the strong CP phase from Planck-suppressed operators and radiative corrections, respectively. Second, the anomaly-mediated contribution to the gluino mass may violate CP symmetry and should be also suppressed, which requires a low SUSY breaking scale. Third, the maximum temperature of the Universe must be lower than the energy scale of spontaneous CP violation to avoid restoration of CP symmetry. Otherwise, the domain wall forms after the spontaneous CP violation. Finally, the gravitino mass must be sufficiently heavy to evade the Lyman- α constraint on warm DM.

In the viable parameter space, the mass scale of the spontaneous CP-violating sector is larger than 10^7 GeV , the soft mass scale of the visible sector SUSY particles is smaller than

10^6 GeV, and the gravitino mass is between the Lyman- α bound ($\simeq 5.3$ keV) and the bound from the anomaly-mediation contribution in eq. (5.13) ($\simeq 40$ MeV). The strong CP phase originates from Planck-suppressed operators, anomaly mediation, and radiative corrections. We have found an interesting prediction of our scenario that the strong CP phase is larger than $\mathcal{O}(10^{-12})$ and is within the reach of future experiments, even if the soft mass scale of the visible sector SUSY particles is much larger than the TeV scale. The prediction is unique, particularly compared with the PQ mechanism, which predicts an extremely small neutron EDM which corresponds to $\bar{\theta} = \mathcal{O}(10^{-17})$ [161].

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