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Three-dimensional Maxwellian extended Bargmann supergravity

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Abstract: We present a novel three-dimensional non-relativistic Chern-Simons supergravity theory invariant under a Maxwellian extended Bargmann superalgebra. We first study the non-relativistic limits of the minimal and the $\mathcal{N}=2$ Maxwell superalgebras. We show that a well-defined Maxwellian extended Bargmann supergravity requires to construct by hand a supersymmetric extension of the Maxwellian extended Bargmann algebra by introducing additional fermionic and bosonic generators. The new non-relativistic supergravity action presented here contains the extended Bargmann supergravity as a sub-case.

Keywords: Supergravity Models, Classical Theories of Gravity, Extended Supersymmetry, Gauge Symmetry

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1 Introduction

Three-dimensional non-relativistic (NR) and ultra-relativistic (UR) versions of supergravity theory have only been explored recently in [1–7]. Although several generalizations and applications of supergravity have been developed by diverse authors these last four decades, its NR construction remains challenging and has only been approached in three spacetime dimensions. In particular, the formulation of a well-defined NR supergravity action has required the introduction of additional fermionic generators. As in NR bosonic cases, the addition of new generators allows to construct a non-degenerate invariant bilinear form which assures the proper construction of a Chern-Simons (CS) action.

The NR theories have received a renewed interest since they play an important role to approach condensed matter systems [8–15] and NR effective field theories [16–19]. It seems then natural to extend NR gravity theories [20–36] to the presence of supersymmetry. In particular, NR supergravity models can be seen as a starting point to approach supersymmetric field theories on curved backgrounds by means of localization [37, 38].

On the other hand, the Maxwell algebra has received a growing interest these last decades. Such symmetry has been first introduced to describe Minkowski space in the presence of a constant electromagnetic field background [39–41]. In the gravity context, the Maxwell algebra and its generalizations have been useful to recover standard General Relativity from CS and Born-Infeld gravity theories [42–46]. More recently, a Maxwell CS formulation in three spacetime dimensions has been explored in [47]. Its solution [48, 49], generalization to higher spin [50], and asymptotic symmetry [49, 51] have been subsequently studied by diverse authors. Further application of the Maxwell algebra can be

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found in [52–58]. At the supersymmetric level, the minimal Maxwell superalgebra appears to describe a constant Abelian supersymmetric gauge field background in a four-dimensional superspace [59]. Generalizations of the Maxwell superalgebras have then been explored with diverse applications [60–69]. More recently, a three-dimensional CS supergravity theory invariant under the Maxwell superalgebra and its $\mathcal{N}$-extended versions have been explored in [70–73].

The NR version of the Maxwell CS gravity theory has only been presented recently [74] (see also [75], where the related algebra has been recovered through Lie algebra expansion). Interestingly, the relativistic theory required the presence of three U(1) gauge fields in order to establish a well-defined NR limit and to avoid degeneracy. In the presence of supersymmetry, the NR version of the Maxwell CS supergravity was unknown till now. In this work, we explore the NR limit of the Maxwell superalgebra for $\mathcal{N}=1$ and $\mathcal{N}=2$. In particular, we show that a well-defined NR Maxwellian CS supergravity action requires to introduce by hand additional fermionic and bosonic generators. Our model is not only a novel NR supergravity theory without cosmological constant but contains the extended Bargmann supergravity as a sub-case.

The paper is organized as follows: in section 2, we briefly review the Maxwellian extended Bargmann gravity introduced in [74]. Sections 3 and 4 contain our main results. In section 3, we introduce the NR limits of the minimal and the $\mathcal{N}=2$ Maxwell superalgebras. In section 4, we present the Maxwellian extendad Bargmann superalgebra and the NR CS supergravity action. Section 5 is devoted to discussion and possible future developments. Some large formulas are collected in the appendix.

2 Maxwellian extended Bargmann gravity

In this section, we briefly review the Maxwellian extended Bargmann algebra introduced in [74] and the associated CS gravity theory developed in the same paper in three (2+1) dimensions. In [74] the authors proved that an alternative way to circumvent the degeneracy of the bilinear form in the [Maxwell] $\oplus$ $u(1)\oplus u(1)$ system analyzed in the same paper, is to add one more $u(1)$ gauge field.

The non-vanishing commutation relations of the Maxwell algebra are given by

$$\begin{aligned}
[J_A, J_B] &= \epsilon_{ABC} J^C \,, \\
[J_A, P_B] &= \epsilon_{ABC} P^C \,, \\
[J_A, Z_B] &= \epsilon_{ABC} Z^C \,, \\
[P_A, P_B] &= \epsilon_{ABC} Z^C \,,
\end{aligned} \tag{2.1}$$

where J_A are the spacetime rotations, P_A the spacetime translations, and Z_A are new generators characterized and introduced in [39, 40] ($A=0,1,2$ and $\eta^{AB}=\text{diag}(-,+,+)$). A gauge-invariant CS gravity action in three (meaning 2+1, here as well as in the sequel) dimensions based on the above written Maxwell algebra has been constructed in [47–49, 74]. The CS action is constructed using the connection one-form $A=A^A T_A$ taking values in the Maxwell algebra generated by $\{J_A, P_A, Z_A\}$, that is

$$A = E^B P_B + W^B J_B + K^B Z_B \,, \tag{2.2}$$

where E^B, W^B, and K^B are one-form fields.

The CS form constructed with the invariant bilinear form defines an action for the relativistic gauge theory for the symmetry under consideration as

$$I_{\text{CS}} = \int \left\langle A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right\rangle = \int \left\langle A \wedge dA + \frac{1}{3} A \wedge [A, A] \right\rangle . \tag{2.3}$$

In the specific case we are now reviewing, when the Maxwell algebra is supplemented with the three additional U(1) generators (Y_1, Y_2, and Y_3), the connection one-form involved in the construction reads

$$A = E^B P_B + W^B J_B + K^B Z_B + M Y_1 + S Y_2 + T Y_3 \,, \tag{2.4}$$

where M, S, and T are the additional bosonic gauge fields. Also the bilinear form acquires further non-zero entries due to the presence of the new generators (see [74] for details). In particular, a non-degenerate bilinear form can be obtained from the aforesaid relativistic bilinear form, allowing for a well-defined and finite NR CS action.

Specifically, in [74], the contraction leading to the NR generators is defined through the identifications

$$\begin{aligned}
P_0 &= \frac{\tilde{H}}{2\xi} + \xi \tilde{M} \,, & P_a &= \tilde{P}_a \,, & Y_1 &= \frac{\tilde{H}}{2\xi} - \xi \tilde{M} \,, \\
J_0 &= \frac{\tilde{J}}{2} + \xi^2 \tilde{S} \,, & J_a &= \xi \tilde{G}_a \,, & Y_2 &= \frac{\tilde{J}}{2} - \xi^2 \tilde{S} \,, \\
Z_0 &= \frac{\tilde{Z}}{2\xi^2} + \tilde{T} \,, & Z_a &= \frac{\tilde{Z}_a}{\xi} \,, & Y_3 &= \frac{\tilde{Z}}{2\xi^2} - \tilde{T} \,,
\end{aligned} \tag{2.5}$$

and by subsequently taking $\xi \to \infty$. Let us note that the index $A = 0, 1, 2$ has previously been decomposed as $A \to \{0, a\}$, with $a = 1, 2$. Furthermore, Y_1, Y_2, and Y_3 are the three U(1) generators introduced at the relativistic level.

In terms of the NR generators and fields, the gauge connection one-form of [74], $\tilde{A} = A^A \tilde{T}_A$, is given by

$$\tilde{A} = \tau \tilde{H} + e^a \tilde{P}_a + \omega \tilde{J} + \omega^a \tilde{G}_a + k \tilde{Z} + k^a \tilde{Z}_a + m \tilde{M} + s \tilde{S} + t \tilde{T} \,. \tag{2.6}$$

The NR version of the Maxwell algebra presented in [74] was called by the authors Maxwellian Exotic Bargmann (MEB) algebra, and its non-trivial commutations relations read

$$\begin{aligned}
\left[\tilde{G}_a, \tilde{P}_b\right] &= -\epsilon_{ab} \tilde{M} \,, & \left[\tilde{G}_a, \tilde{Z}_b\right] &= -\epsilon_{ab} \tilde{T} \,, \\
\left[\tilde{H}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{P}_b \,, & \left[\tilde{J}, \tilde{Z}_a\right] &= \epsilon_{ab} \tilde{Z}_b \,, \\
\left[\tilde{J}, \tilde{P}_a\right] &= \epsilon_{ab} \tilde{P}_b \,, & \left[\tilde{H}, \tilde{P}_a\right] &= \epsilon_{ab} \tilde{Z}_b \,, \\
\left[\tilde{J}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{G}_b \,, & \left[\tilde{P}_a, \tilde{P}_b\right] &= -\epsilon_{ab} \tilde{T} \,, \\
\left[\tilde{G}_a, \tilde{G}_b\right] &= -\epsilon_{ab} \tilde{S} \,, & \left[\tilde{Z}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{Z}_b \,.
\end{aligned} \tag{2.7}$$

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Such NR algebra admits the following non-vanishing components of the invariant tensor

$$\begin{aligned}
\left\langle \tilde{G}_a \tilde{G}_b \right\rangle &= \tilde{\alpha}_0 \delta_{ab} \,, \\
\left\langle \tilde{G}_a \tilde{P}_b \right\rangle &= \tilde{\alpha}_1 \delta_{ab} \,, \\
\left\langle \tilde{G}_a \tilde{Z}_b \right\rangle &= \tilde{\alpha}_2 \delta_{ab} \;=\; \left\langle \tilde{P}_a \tilde{P}_b \right\rangle \,, \\
\left\langle \tilde{J} \tilde{S} \right\rangle &= -\tilde{\alpha}_0 \,, \\
\left\langle \tilde{J} \tilde{M} \right\rangle &= -\tilde{\alpha}_1 \;=\; \left\langle \tilde{H} \tilde{S} \right\rangle \,, \\
\left\langle \tilde{J} \tilde{T} \right\rangle &= -\tilde{\alpha}_2 \;=\; \left\langle \tilde{H} \tilde{M} \right\rangle \,.
\end{aligned} \tag{2.8}$$

This bilinear form is non-degenerate if $\tilde{\alpha}_2 \neq 0$. The MEB curvature two-forms are given by

$$\begin{aligned}
R\left(\omega\right) &= d\omega \,, \\
R^a\left(\omega^b\right) &= d\omega^a + \epsilon^{ac}\omega\omega_c \,, \\
R\left(\tau\right) &= d\tau \,, \\
R^a\left(e^b\right) &= de^a + \epsilon^{ac}\omega e_c + \epsilon^{ac}\tau\omega_c \,, \\
R\left(k\right) &= dk \,, \\
R^a\left(k^b\right) &= dk^a + \epsilon^{ac}\omega k_c + \epsilon^{ac}\tau e_c + \epsilon^{ac} k\omega_c \,, \\
R\left(m\right) &= dm + \epsilon^{ac} e_a \omega_c \,, \\
R\left(s\right) &= ds + \frac{1}{2}\epsilon^{ac}\omega_a\omega_c \,, \\
R\left(t\right) &= dt + \epsilon^{ac}\omega_a k_c + \frac{1}{2}\epsilon^{ac} e_a e_c \,.
\end{aligned} \tag{2.9}$$

The NR three-dimensional CS action obtained in [74] reads, up to boundary terms, as follows:

$$\begin{aligned}
I_{\mathrm{MEB}} = \int \Big\{ & \tilde{\alpha}_0 \Big[\omega_a R^a(\omega^b) - 2sR\left(\omega\right) \Big] + \tilde{\alpha}_1 \Big[2e_a R^a(\omega^b) - 2mR(\omega) - 2\tau R(s) \Big] \\
& + \tilde{\alpha}_2 \Big[e_a R^a\left(e^b\right) + k_a R^a\left(\omega^b\right) + \omega_a R^a\left(k^b\right) - 2sR\left(k\right) - 2mR\left(\tau\right) \\
& - 2tR\left(\omega\right) \Big] \Big\} \,.
\end{aligned} \tag{2.10}$$

As was noticed in [74], the NR CS action has three independent sectors proportional to three arbitrary constants, $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, and $\tilde{\alpha}_2$. The first term corresponds to the so-called exotic NR gravity. The second term is the CS action for the extended Bargmann algebra [76–81], while the last term reproduces the CS action for a new NR Maxwell algebra. Let us note that, since the bilinear form does not result to acquire degeneracy in the contraction process, the equations of motion from the NR action (2.10) are given by the vanishing of all the curvatures (2.9).

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3 On the supersymmetric extension of the Maxwellian extended Bargmann algebra

In this section, we explore the supersymmetric extension of the NR Maxwell algebra by applying a NR limit to the $\mathcal{N}=1$ and $\mathcal{N}=2$ Maxwell superalgebra. Interestingly, we show that, in order to have a well-defined NR superalgebra, we have to consider the NR limit of a centrally extended $\mathcal{N}=2$ Maxwell superalgebra endowed with a $\mathfrak{so}(2)$ generator. Indeed, a true supersymmetric extension of the MEB algebra in which the anti-commutator of two fermionic charges gives a time and a space translation requires, as in the Bargmann case, at least $\mathcal{N}=2$ supersymmetry. However, as we shall see in the next section, it is necessary to introduce by hand additional fermionic and bosonic generators in order to obtain a MEB superalgebra which allows the proper construction of a NR CS supergravity.

In three spacetime dimensions, the minimal Maxwell superalgebra is spanned by the set of generators $\{J_A, P_A, Z_A, Q_\alpha, \Sigma_\alpha\}$ [72], where, in particular, Q_α are the supersymmetry generators. Besides, such supersymmetric extension of the Maxwell algebra is characterized by the presence of an additional Majorana fermionic generator Σ_α whose presence assures the Jacobi identity (P_a, Q_α, Q_β). The introduction of a second spinorial charge is not new and have previously been considered in superstring theory [82] and $D=11$ supergravity [83–85]. The (anti-)commutation relations of the minimal Maxwell superalgebra are given by

$$\begin{aligned}
[J_A, J_B] &= \epsilon_{ABC} J^C \,, \\
[J_A, P_B] &= \epsilon_{ABC} P^C \,, \\
[J_A, Z_B] &= \epsilon_{ABC} Z^C \,, \\
[P_A, P_B] &= \epsilon_{ABC} Z^C \,, \\
[J_A, Q_\alpha] &= -\frac{1}{2} \left(\gamma_A\right)_\alpha^{\ \beta} Q_\beta \,, \\
[J_A, \Sigma_\alpha] &= -\frac{1}{2} \left(\gamma_A\right)_\alpha^{\ \beta} \Sigma_\beta \,, \\
[P_A, Q_\alpha] &= -\frac{1}{2} \left(\gamma_A\right)_\alpha^{\ \beta} \Sigma_\beta \,, \\
\{Q_\alpha, Q_\beta\} &= -\left(\gamma^A C\right)_{\alpha\beta} P_A \,, \\
\{Q_\alpha, \Sigma_\beta\} &= -\left(\gamma^A C\right)_{\alpha\beta} Z_A \,,
\end{aligned} \tag{3.1}$$

where $\alpha, \beta = 1, 2$ are spinorial indices, C is the charge conjugation matrix, and γ^A are the Dirac matrices in three spacetime dimensions.

As was discussed in [74], it is necessary to include three additional $U(1)$ generators given by Y_1, Y_2, and Y_3 in order to get the bosonic MEB algebra as a NR limit. At the supersymmetric level, a NR contraction can be applied by considering the rescaling of the bosonic generators as in (2.5) and the following rescaling, with a dimensionless parameter ξ, of the Majorana fermionic generators Q_α and Σ_α:

$$Q_\alpha = \sqrt{\xi}\tilde{Q}_\alpha^- \,, \qquad \Sigma_\alpha = \frac{1}{\sqrt{\xi}}\tilde{\Sigma}_\alpha^- \,. \tag{3.2}$$

A particular supersymmetric extension of the MEB algebra is obtained from the NR contraction $\xi \to \infty$ of (3.1):

$$\begin{aligned}
\left[\tilde{G}_a, \tilde{P}_b\right] &= -\epsilon_{ab}\tilde{M}\,, & \left[\tilde{G}_a, \tilde{Z}_b\right] &= -\epsilon_{ab}\tilde{T}\,,\\
\left[\tilde{H}, \tilde{G}_a\right] &= \epsilon_{ab}\tilde{P}_b\,, & \left[\tilde{J}, \tilde{Z}_a\right] &= \epsilon_{ab}\tilde{Z}_b\,,\\
\left[\tilde{J}, \tilde{P}_a\right] &= \epsilon_{ab}\tilde{P}_b\,, & \left[\tilde{H}, \tilde{P}_a\right] &= \epsilon_{ab}\tilde{Z}_b\,,\\
\left[\tilde{J}, \tilde{G}_a\right] &= \epsilon_{ab}\tilde{G}_b\,, & \left[\tilde{P}_a, \tilde{P}_b\right] &= -\epsilon_{ab}\tilde{T}\,,\\
\left[\tilde{G}_a, \tilde{G}_b\right] &= -\epsilon_{ab}\tilde{S}\,, & \left[\tilde{Z}, \tilde{G}_a\right] &= \epsilon_{ab}\tilde{Z}_b\,,\\
\left[\tilde{J}, \tilde{Q}^-_\alpha\right] &= -\frac{1}{2}\left(\gamma_0\right)_\alpha^{\ \beta}\tilde{Q}^-_\beta\,, & \left[\tilde{J}, \tilde{\Sigma}^-_\alpha\right] &= -\frac{1}{2}\left(\gamma_0\right)_\alpha^{\ \beta}\tilde{\Sigma}^-_\beta\,,\\
\left[\tilde{H}, \tilde{Q}^-_\alpha\right] &= -\frac{1}{2}\left(\gamma_0\right)_\alpha^{\ \beta}\tilde{\Sigma}^-_\beta\,, & &\\
\left\{\tilde{Q}^-_\alpha, \tilde{Q}^-_\beta\right\} &= -\left(\gamma^0 C\right)_{\alpha\beta}\tilde{M}\,, & \left\{\tilde{Q}^-_\alpha, \tilde{\Sigma}^-_\beta\right\} &= -\left(\gamma^0 C\right)_{\alpha\beta}\tilde{T}\,.
\end{aligned} \tag{3.3}$$

Although the (anti-)commutation relations (3.3) are well-defined and satisfy the Jacobi identities, we cannot say that the $\mathcal{N}=1$ MEB superalgebra obtained here is a true supersymmetry algebra. Indeed, the anti-commutator of two supercharges leads to a central charge transformation instead of a time and space translation. This is analogous to the $\mathcal{N}=1$ Bargmann superalgebra case [1].

One way to circumvent such difficulty is to apply the NR contraction to a $\mathcal{N}=2$ relativistic Maxwell superalgebra. The $\mathcal{N}=2$ supersymmetric extension of the Maxwell algebra has been explored by diverse authors [62, 64, 71]. Here, we shall focus on the $\mathcal{N}=2$ centrally extended Maxwell superalgebra endowed with a $\mathfrak{so}\left(2\right)$ internal symmetry generator introduced in [73]. Such $\mathcal{N}=2$ Maxwell superalgebra is spanned by the set of generators $\left\{J_A, P_A, Z_A, \mathcal{B}, \mathcal{Z}, Q^i_\alpha, \Sigma^i_\alpha\right\}$, which satisfy the following non-vanishing (anti-)commutation relations:

$$\begin{aligned}
\left[J_A, J_B\right] &= \epsilon_{ABC}J^C\,, & \left[J_A, P_B\right] &= \epsilon_{ABC}P^C\,,\\
\left[J_A, Z_B\right] &= \epsilon_{ABC}Z^C\,, & \left[P_A, P_B\right] &= \epsilon_{ABC}Z^C\,,\\
\left[J_A, Q^i_\alpha\right] &= -\frac{1}{2}\left(\gamma_A\right)_\alpha^{\ \beta}Q^i_\beta\,, & \left[J_A, \Sigma^i_\alpha\right] &= -\frac{1}{2}\left(\gamma_A\right)_\alpha^{\ \beta}\Sigma^i_\beta\,,\\
\left[P_A, Q^i_\alpha\right] &= -\frac{1}{2}\left(\gamma_A\right)_\alpha^{\ \beta}\Sigma^i_\beta\,, & \left[Q^i_\alpha, \mathcal{B}\right] &= \frac{1}{2}\epsilon^{ij}\Sigma^j_\alpha\,,\\
\left\{Q^i_\alpha, Q^j_\beta\right\} &= -\delta^{ij}\left(\gamma^A C\right)_{\alpha\beta}P_A - C_{\alpha\beta}\epsilon^{ij}\mathcal{B}\,, & &\\
\left\{Q^i_\alpha, \Sigma^j_\beta\right\} &= -\delta^{ij}\left(\gamma^A C\right)_{\alpha\beta}Z_A - C_{\alpha\beta}\epsilon^{ij}\mathcal{Z}\,, & &
\end{aligned} \tag{3.4}$$

where $i=1,2$ is the number of supercharges. Let us note that the presence of a $\mathfrak{so}\left(2\right)$ internal symmetry generator is crucial in order to admit a non-degenerate invariant inner product [73]. Then, following [86], let us consider the following definitions of the fermionic generators

$$
\begin{aligned}
Q_\alpha^{\pm} &= \frac{1}{\sqrt{2}}\left(Q_\alpha^1 \pm \epsilon_{\alpha\beta} Q_\beta^2\right) , \\
\Sigma_\alpha^{\pm} &= \frac{1}{\sqrt{2}}\left(\Sigma_\alpha^1 \pm \epsilon_{\alpha\beta} \Sigma_\beta^2\right) .
\end{aligned} \tag{3.5}
$$

A dimensionless parameter ξ can be introduced by considering the rescaling of the generators and central extension,

$$
\begin{aligned}
&J_0 = \tilde{J} , && J_a = \xi \tilde{G}_a , && \\
&P_0 = \frac{\tilde{H}}{2\xi} + \xi \tilde{M} , && P_a = \tilde{P}_a , && \mathcal{B} = \frac{\tilde{H}}{2\xi} - \xi \tilde{M} , \\
&Z_0 = \frac{\tilde{Z}}{2\xi^2} + \tilde{T} , && Z_a = \frac{\tilde{Z}_a}{\xi} , && \mathcal{Z} = \frac{\tilde{Z}}{2\xi^2} - \tilde{T} , \\
&Q_\alpha^- = \sqrt{\xi} \tilde{Q}_\alpha^- , && Q_\alpha^+ = \frac{1}{\sqrt{\xi}} \tilde{Q}_\alpha^+ , && \\
&\Sigma_\alpha^- = \frac{1}{\sqrt{\xi}} \tilde{\Sigma}_\alpha^- , && \Sigma_\alpha^+ = \frac{1}{\xi^{3/2}} \tilde{\Sigma}_\alpha^+ . &&
\end{aligned} \tag{3.6}
$$

Then, after taking the limit $\xi \to \infty$, a particular $\mathcal{N} = 2$ Maxwellian Bargmann superalgebra is obtained; its (anti-)commutation relations are given by the purely bosonic commutators

$$
\begin{aligned}
\left[\tilde{G}_a, \tilde{P}_b\right] &= -\epsilon_{ab} \tilde{M} , & \left[\tilde{G}_a, \tilde{Z}_b\right] &= -\epsilon_{ab} \tilde{T} , \\
\left[\tilde{H}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{P}_b , & \left[\tilde{J}, \tilde{Z}_a\right] &= \epsilon_{ab} \tilde{Z}_b , \\
\left[\tilde{J}, \tilde{P}_a\right] &= \epsilon_{ab} \tilde{P}_b , & \left[\tilde{H}, \tilde{P}_a\right] &= \epsilon_{ab} \tilde{Z}_b , \\
\left[\tilde{J}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{G}_b , & \left[\tilde{P}_a, \tilde{P}_b\right] &= -\epsilon_{ab} \tilde{T} , \\
\left[\tilde{Z}, \tilde{G}_a\right] &= \epsilon_{ab} \tilde{Z}_b , & &
\end{aligned} \tag{3.7}
$$

along with

$$
\begin{aligned}
\left[\tilde{J}, \tilde{Q}_\alpha^{\pm}\right] &= -\frac{1}{2} \left(\gamma_0\right)_\alpha^{\ \beta} \tilde{Q}_\beta^{\pm} , & \left[\tilde{J}, \tilde{\Sigma}_\alpha^{\pm}\right] &= -\frac{1}{2} \left(\gamma_0\right)_\alpha^{\ \beta} \tilde{\Sigma}_\beta^{\pm} , \\
\left[\tilde{H}, \tilde{Q}_\alpha^-\right] &= -\left(\gamma_0\right)_\alpha^{\ \beta} \tilde{\Sigma}_\beta^- , & \left[\tilde{P}_a, \tilde{Q}_\alpha^+\right] &= -\frac{1}{2} \left(\gamma_a\right)_\alpha^{\ \beta} \tilde{\Sigma}_\beta^- , \\
\left[\tilde{G}_a, \tilde{Q}_\alpha^+\right] &= -\frac{1}{2} \left(\gamma_a\right)_\alpha^{\ \beta} \tilde{Q}_\beta^- , & \left[\tilde{G}_a, \tilde{\Sigma}_\alpha^+\right] &= -\frac{1}{2} \left(\gamma_a\right)_\alpha^{\ \beta} \tilde{\Sigma}_\beta^- , \\
\left\{\tilde{Q}_\alpha^-, \tilde{Q}_\beta^-\right\} &= -2 \left(\gamma^0 C\right)_{\alpha\beta} \tilde{M} , & \left\{\tilde{Q}_\alpha^+, \tilde{Q}_\beta^-\right\} &= -\left(\gamma^a C\right)_{\alpha\beta} \tilde{P}_a , \\
\left\{\tilde{Q}_\alpha^+, \tilde{Q}_\beta^+\right\} &= -\left(\gamma^0 C\right)_{\alpha\beta} \tilde{H} , & \left\{\tilde{Q}_\alpha^-, \tilde{\Sigma}_\beta^-\right\} &= -2 \left(\gamma^0 C\right)_{\alpha\beta} \tilde{T} , \\
\left\{\tilde{Q}_\alpha^{\pm}, \tilde{\Sigma}_\beta^{\mp}\right\} &= -\left(\gamma^a C\right)_{\alpha\beta} \tilde{Z}_a , & \left\{\tilde{Q}_\alpha^+, \tilde{\Sigma}_\beta^+\right\} &= -\left(\gamma^0 C\right)_{\alpha\beta} \tilde{Z} .
\end{aligned} \tag{3.8}
$$

Notice that, unlike the $\mathcal{N} = 1$ superalgebra, the $\mathcal{N} = 2$ Maxwellian Bargmann superalgebra obtained here can be seen as a true supersymmetry algebra. In particular, let us

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note the presence of the non-vanishing commutator between the $\tilde{G}_a$ generator and supersymmetry generator. Nevertheless, this superalgebra does not contain the MEB algebra as a subalgebra. Indeed, the bosonic subalgebra (3.7) can be seen as a non-relativistic version of a [Maxwell]$\oplus u(1) \oplus u(1)$ algebra.

Moreover, although the $\mathcal{N}=2$ NR Maxwell superalgebra (3.7)–(3.8) has the desired features of a true superalgebra, it is not a good candidate to construct a three-dimensional CS supergravity action. Indeed, in order to have a NR supergravity action based on a supersymmetric extension of the MEB algebra, we need a well-defined invariant tensor, which requires to introduce by hand additional fermionic generators. The explicit Maxwellian extended Bargmann superalgebra allowing to construct a NR supergravity action is presented in the next section.

4 Maxwellian extended Bargmann supergravity

Here, we present the explicit form of the Maxwellian extended Bargmann superalgebra allowing to construct a NR supergravity action. Consequently, we develop the aforementioned NR supergravity action by exploiting the CS construction in three dimensions.

4.1 Maxwellian extended Bargmann superalgebra

As we have discussed in the previous section, the $\mathcal{N}=2$ NR Maxwell superalgebra given by (3.7)–(3.8) does not allow for the proper construction of a NR CS supergravity action although its relativistic analogue is well-defined. In order to have a proper NR CS supergravity action based on a supersymmetric extension of the MEB algebra, one requires to find a NR superalgebra which not only contains the MEB algebra as a subalgebra but also admits a non-degenerate invariant supertrace. Indeed, when studying the NR limit of a theory, one has that the symplectic form of the NR model might become degenerate, making some fields not determined by the field equations, thus reducing the number of dynamical fields. In the case of a CS formulation in three dimensions, the non-degeneracy of the bilinear invariant trace of gauge generators implies the non-degeneracy of the symplectic form, which would ensure dynamically indeterminate fields in the NR theory. In particular, the non-degeneracy of the bilinear form is related to the physical requirement that the CS action involves a kinematical term for each field and the equation of motions imply that all curvatures vanish.

Here we construct by hand a supersymmetric extension of the MEB algebra by introducing six Majorana fermionic generators $\tilde{Q}^{+}_{\alpha}$, $\tilde{Q}^{-}_{\alpha}$, $\tilde{\Sigma}^{+}_{\alpha}$, $\tilde{\Sigma}^{-}_{\alpha}$, $\tilde{R}_{\alpha}$, and $\tilde{W}_{\alpha}$. Let us note that the presence of the $\tilde{R}_{\alpha}$ and $\tilde{W}_{\alpha}$ generators is similar to what happens in the extended Bargmann superalgebra presented in [3] and in the extended Newtonian superalgebra of [4], in which a $\tilde{R}_{\alpha}$ generator is considered. Furthermore, we introduce six extra bosonic generators Y_1, Y_2, U_1, U_2, B_1, and B_2. Both B_1 and B_2 are central, while the others act non-trivially on the spinor generators, similarly to the extra bosonic generators introduced in the extended Newton-Hooke supergravity [5]. The proposed supersymmetric extension of the MEB algebra is generated by the set of bosonic and fermionic generators

$$\{\tilde{J},\tilde{G}_a,\tilde{S},\tilde{H},\tilde{P}_a,\tilde{M},\tilde{Z},\tilde{Z}_a,\tilde{T},\tilde{Y}_1,\tilde{Y}_2,\tilde{U}_1,\tilde{U}_2,\tilde{B}_1,\tilde{B}_2,\tilde{Q}^{+}_{\alpha},\tilde{Q}^{-}_{\alpha},\tilde{R}_{\alpha},\tilde{\Sigma}^{+}_{\alpha},\tilde{\Sigma}^{-}_{\alpha},\tilde{W}_{\alpha}\}. \tag{4.1}$$

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Such generators satisfy the MEB algebra (2.7) along with the following non-vanishing (anti-)commutation relations:

$$
\begin{aligned}
&\left[\tilde{J},\tilde{Q}^{\pm}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{Q}^{\pm}_{\beta}\,, &&\left[\tilde{J},\tilde{\Sigma}^{\pm}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{\Sigma}^{\pm}_{\beta}\,, &&\left[\tilde{M},\tilde{Q}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,,\\
&\left[\tilde{H},\tilde{Q}^{\pm}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{\Sigma}^{\pm}_{\alpha}\,, &&\left[\tilde{P}_a,\tilde{Q}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{\Sigma}^{-}_{\beta}\,, &&\left[\tilde{Y}_1,\tilde{Q}^{\pm}_{\alpha}\right]=\pm\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{Q}^{\pm}_{\beta}\,,\\
&\left[\tilde{G}_a,\tilde{Q}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{Q}^{-}_{\beta}\,, &&\left[\tilde{G}_a,\tilde{Q}^{-}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{R}_{\beta}\,, &&\left[\tilde{Y}_1,\tilde{\Sigma}^{\pm}_{\alpha}\right]=\pm\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{\Sigma}^{\pm}_{\beta}\,,\\
&\left[\tilde{G}_a,\tilde{\Sigma}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{\Sigma}^{-}_{\beta}\,, &&\left[\tilde{G}_a,\tilde{\Sigma}^{-}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,, &&\left[\tilde{U}_1,\tilde{Q}^{\pm}_{\alpha}\right]=\pm\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{\Sigma}^{\pm}_{\beta}\,,\\
&\left[P_a,\tilde{Q}^{-}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_a\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,, &&\left[\tilde{J},\tilde{R}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{R}_{\beta}\,, &&\left[\tilde{Y}_2,\tilde{Q}^{+}_{\alpha}\right]=\left[\tilde{Y}_1,\tilde{R}_{\alpha}\right]=\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{R}_{\beta}\,,\\
&\left[\tilde{J},\tilde{W}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,, &&\left[\tilde{H},\tilde{R}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,, &&\left[\tilde{Y}_2,\tilde{\Sigma}^{+}_{\alpha}\right]=\left[\tilde{Y}_1,\tilde{W}_{\alpha}\right]=\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{W}_{\beta}\,,\\
&\left[\tilde{S},\tilde{Q}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{R}_{\beta}\,, &&\left[\tilde{S},\tilde{\Sigma}^{+}_{\alpha}\right]=-\frac{1}{2}\left(\gamma_0\right)_{\alpha}^{\ \beta}\tilde{W}_{\beta}\,, &&\left[\tilde{U}_2,\tilde{Q}^{+}_{\alpha}\right]=\left[\tilde{U}_1,\tilde{R}_{\alpha}\right]=\frac{1}{2}\left(\gamma_0\right)_{\alpha\beta}\tilde{W}_{\beta}\,,
\end{aligned}
$$

$$
\begin{aligned}
&\left\{\tilde{Q}^{-}_{\alpha},\tilde{Q}^{-}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{M}+\left(\gamma^0C\right)_{\alpha\beta}\tilde{U}_2\,, &&\left\{\tilde{Q}^{+}_{\alpha},\tilde{\Sigma}^{+}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{Z}-\left(\gamma^0C\right)_{\alpha\beta}\tilde{B}_1\,,\\
&\left\{\tilde{Q}^{+}_{\alpha},\tilde{Q}^{+}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{H}-\left(\gamma^0C\right)_{\alpha\beta}\tilde{U}_1\,, &&\left\{\tilde{Q}^{-}_{\alpha},\tilde{\Sigma}^{-}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{T}+\left(\gamma^0C\right)_{\alpha\beta}\tilde{B}_2\,,\\
&\left\{\tilde{Q}^{+}_{\alpha},\tilde{R}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{M}-\left(\gamma^0C\right)_{\alpha\beta}\tilde{U}_2\,, &&\left\{\tilde{Q}^{+}_{\alpha},\tilde{W}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{T}-\left(\gamma^0C\right)_{\alpha\beta}\tilde{B}_2\,,\\
&\left\{\tilde{\Sigma}^{+}_{\alpha},\tilde{R}_{\beta}\right\}=-\left(\gamma^0C\right)_{\alpha\beta}\tilde{T}-\left(\gamma^0C\right)_{\alpha\beta}\tilde{B}_2\,, &&\left\{\tilde{Q}^{\pm}_{\alpha},\tilde{\Sigma}^{\mp}_{\beta}\right\}=-\left(\gamma^aC\right)_{\alpha\beta}\tilde{Z}_a\,,\\
&\left\{\tilde{Q}^{+}_{\alpha},\tilde{Q}^{-}_{\beta}\right\}=-\left(\gamma^aC\right)_{\alpha\beta}\tilde{P}_a\,.
\end{aligned}
\tag{4.2}
$$

The superalgebra given by (2.7) and (4.2) will be denoted as the Maxwellian extended Bargmann superalgebra. One can note that the $\tilde{S}$ generator is no longer a central charge in this supersymmetric extension of the MEB algebra but acts non-trivially on the spinor generators $\tilde{Q}^{+}_{\alpha}$ and $\tilde{\Sigma}^{+}_{\alpha}$. It is important to emphasize that the MEB superalgebra obtained here has not been obtained through a NR limit of a relativistic superalgebra. Furthermore, the supersymmetric extension of the MEB algebra allowing a well-defined CS supergravity action could not be unique. Then, it would be interesting to study further supersymmetric extensions of the MEB algebra and the possibility of obtaining them by applying a NR limit to a relativistic theory.

4.2 Non-relativistic Chern-Simons supergravity action

Let us construct a NR CS supergravity action based on the MEB superalgebra previously introduced.

The non-vanishing components of the invariant tensor for the MEB superalgebra are given by (2.8) along with

$$
\begin{aligned}
\left\langle \tilde{Z}\tilde{S} \right\rangle &= -\tilde{\alpha}_2 \,, \\
\left\langle \tilde{Y}_1\tilde{Y}_2 \right\rangle &= \tilde{\alpha}_0 \,, \\
\left\langle \tilde{Y}_1\tilde{U}_2 \right\rangle &= \tilde{\alpha}_1 \; = \left\langle \tilde{U}_1\tilde{Y}_2 \right\rangle \,, \\
\left\langle \tilde{Y}_1\tilde{B}_2 \right\rangle &= \tilde{\alpha}_2 \; = \left\langle \tilde{U}_1\tilde{U}_2 \right\rangle \; = \left\langle \tilde{B}_1\tilde{Y}_2 \right\rangle \,, \\
\left\langle \tilde{Q}^-_\alpha\tilde{Q}^-_\beta \right\rangle &= 2\tilde{\alpha}_1 C_{\alpha\beta} \; = \left\langle \tilde{Q}^+_\alpha\tilde{R}_\beta \right\rangle \,, \\
\left\langle \tilde{Q}^-_\alpha\tilde{\Sigma}^-_\beta \right\rangle &= 2\tilde{\alpha}_2 C_{\alpha\beta} \; = \left\langle \tilde{\Sigma}^+_\alpha\tilde{R}_\beta \right\rangle \; = \left\langle \tilde{Q}^+_\alpha\tilde{W}_\beta \right\rangle \,,
\end{aligned}
\tag{4.3}
$$

where $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, and $\tilde{\alpha}_2$ are arbitrary constants. The bilinear form associated with the MEB superalgebra is non-degenerate for $\tilde{\alpha}_2 \neq 0$, analogously to the purely bosonic case [74]. On the other hand, the gauge connection one-form $\tilde{A}$ for the MEB superalgebra reads[1]

$$
\begin{aligned}
\tilde{A} = {} & \omega\tilde{J} + \omega^a\tilde{G}_a + \tau\tilde{H} + e\tilde{P}_a + k\tilde{Z} + k^a\tilde{Z}_a + m\tilde{M} + s\tilde{S} + t\tilde{T} + y_1\tilde{Y}_1 + y_2\tilde{Y}_2 + b1\tilde{B}_1 \\
& + b_2\tilde{B}_2 + u_1\tilde{U}_1 + u_2\tilde{U}_2 + \psi^+\tilde{Q}^+ + \psi^-\tilde{Q}^- + \xi^+\tilde{\Sigma}^+ + \xi^-\tilde{\Sigma}^- + \rho\tilde{R} + \chi\tilde{W} \,.
\end{aligned}
\tag{4.4}
$$

The corresponding curvature two-form $\tilde{F} = d\tilde{A} + \tilde{A}\wedge\tilde{A} = d\tilde{A} + \frac{1}{2}\left[\tilde{A},\tilde{A}\right]$ in terms of the generators is given by

$$
\begin{aligned}
\tilde{F} = {} & R(\omega)\,\tilde{J} + R^a\left(\omega^b\right)\tilde{G}_a + F(\tau)\,\tilde{H} + F^a\left(e^b\right)\tilde{P}_a + F(k)\,\tilde{Z} + F^a\left(k^b\right)\tilde{Z}_a + F(m)\,\tilde{M} \\
& + R(s)\,\tilde{S} + F(t)\,\tilde{T} + F(y_1)\,\tilde{Y}_1 + F(y_2)\,\tilde{Y}_2 + F(b_1)\,\tilde{B}_1 + F(b_2)\,\tilde{B}_2 + F(u_1)\,\tilde{U}_1 \\
& + F(u_2)\,\tilde{U}_2 + \nabla\psi^+\tilde{Q}^+ + \nabla\psi^-\tilde{Q}^- + \nabla\xi^+\tilde{\Sigma}^+ + \nabla\xi^-\tilde{\Sigma}^- + \nabla\rho\tilde{R} + \nabla\chi\tilde{W} \,.
\end{aligned}
\tag{4.5}
$$

In particular, the bosonic curvature two-forms are given by

$$
\begin{aligned}
F(y_1) &= dy_1 \,, \\
F(y_2) &= dy_2 \,, \\
F(b_1) &= db_1 + \bar{\psi}^+\gamma^0\xi^+ \,, \\
F(b_2) &= db_2 - \bar{\psi}^-\gamma^0\xi^- + \bar{\psi}^+\gamma^0\chi + \bar{\xi}^+\gamma^0\rho \,, \\
F(u_1) &= du_1 + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ \,, \\
F(u_2) &= du_2 - \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- + \bar{\psi}^+\gamma^0\rho \,.
\end{aligned}
\tag{4.6}
$$

[1]Here and in the sequel, we omit the spinor index α, in order to lighten the notation.

together with

$$
\begin{aligned}
F(\omega) &= R(\omega) \,, \\
F^a\left(\omega^b\right) &= R^a\left(\omega^b\right) \,, \\
F(\tau) &= R(\tau) + \frac{1}{2}\bar{\psi}^+\gamma^0\psi^+ \,, \\
F^a\left(e^b\right) &= R^a\left(e^b\right) + \bar{\psi}^+\gamma^a\psi^- \,, \\
F(k) &= R(k) + \bar{\psi}^+\gamma^0\xi^+ \,, \\
F^a\left(k^b\right) &= R^a\left(k^b\right) + \bar{\psi}^+\gamma^a\xi^- + \bar{\psi}^-\gamma^a\xi^+ \,, \\
F(m) &= R(m) + \frac{1}{2}\bar{\psi}^-\gamma^0\psi^- + \bar{\psi}^+\gamma^0\rho \,, \\
F(s) &= R(s) \,, \\
F(t) &= R(t) + \bar{\psi}^-\gamma^0\xi^- + \bar{\psi}^+\gamma^0\chi + \bar{\xi}^+\gamma^0\rho \,,
\end{aligned}
\tag{4.7}
$$

where $R(\omega)$, $R^a\left(\omega^b\right)$, $R(\tau)$, $R^a\left(e^b\right)$, $R(k)$, $R^a\left(k^b\right)$, $R(m)$, $R(s)$, and $R(t)$ correspond to the MEB curvatures already defined in (2.9). On the other hand, the covariant derivatives of the spinor 1-form fields read

$$
\begin{aligned}
\nabla\psi^+ &= d\psi^+ + \frac{1}{2}\omega\gamma_0\psi^+ - \frac{1}{2}y_1\gamma_0\psi^+ \,, \\
\nabla\psi^- &= d\psi^- + \frac{1}{2}\omega\gamma_0\psi^- + \frac{1}{2}\omega^a\gamma_a\psi^+ + \frac{1}{2}y_1\gamma_0\psi^- \,, \\
\nabla\xi^+ &= d\xi^+ + \frac{1}{2}\omega\gamma_0\xi^+ + \frac{1}{2}\tau\gamma_0\psi^+ - \frac{1}{2}y_1\gamma_0\xi^+ - \frac{1}{2}u_1\gamma_0\psi^+ \,, \\
\nabla\xi^- &= d\xi^- + \frac{1}{2}\omega\gamma_0\xi^- + \frac{1}{2}\tau\gamma_0\psi^- + \frac{1}{2}e^a\gamma_a\psi^+ + \frac{1}{2}\omega^a\gamma_a\xi^+ \\
&\quad + \frac{1}{2}y_1\gamma_0\xi^- + \frac{1}{2}u_1\gamma_0\psi^- \,, \\
\nabla\rho &= d\rho + \frac{1}{2}\omega\gamma_0\rho + \frac{1}{2}\omega^a\gamma_a\psi^- + \frac{1}{2}s\gamma_0\psi^+ - \frac{1}{2}y_2\gamma_0\psi^+ - \frac{1}{2}y_1\gamma_0\rho \,, \\
\nabla\chi &= d\chi + \frac{1}{2}\omega\gamma_0\chi + \frac{1}{2}\omega^a\gamma_a\xi^- + \frac{1}{2}e^a\gamma_a\psi^- + \frac{1}{2}\tau\gamma_0\rho + \frac{1}{2}s\gamma_0\xi^+ + \frac{1}{2}m\gamma_0\psi^+ \\
&\quad - \frac{1}{2}y_2\gamma_0\xi^+ - \frac{1}{2}y_1\gamma_0\chi - \frac{1}{2}u_2\gamma_0\psi^+ - \frac{1}{2}u_1\gamma_0\rho \,.
\end{aligned}
\tag{4.8}
$$

Let us note that the gauge fields related to the additional bosonic generators $\{Y_1, Y_2, U_1, U_2\}$ appear explicitly on the spinorial curvature. One can see that b_1 and b_2 do not give any contribution to the fermionic curvatures since they are related to central charges.

A CS supergravity action based on the Maxwellian extended Bargmann superalgebra can be constructed by combining the non-zero invariant tensors (2.8) and (4.3) with the gauge connection one-form $\tilde{A}$ (4.4), and it reads, up to boundary terms, as follows:

$$
\begin{aligned}
I = \int \Big\{ & \tilde{\alpha}_0 \Big[\omega_a R^a(\omega^b) - 2sR(\omega) + 2y_1 dy_2 \Big] + \tilde{\alpha}_1 \Big[2e_a R^a(\omega^b) - 2mR(\omega) - 2\tau R(s) + 2y_1 du_2 \\
& + 2u_1 dy_2 + 2\bar{\psi}^+ \nabla \rho + 2\bar{\rho} \nabla \psi^+ + 2\bar{\psi}^- \nabla \psi^- \Big] + \tilde{\alpha}_2 \Big[e_a R^a \left(e^b \right) + k_a R^a \left(\omega^b \right) + \omega_a R^a \left(k^b \right) \\
& - 2sR(k) - 2mR(\tau) - 2tR(\omega) + 2y_1 db_2 + 2u_1 du_2 + 2y_2 db_1 + 2\bar{\psi}^- \nabla \xi^- + 2\bar{\xi}^- \nabla \psi^- \\
& + 2\bar{\psi}^+ \nabla \chi + 2\bar{\chi} \nabla \psi^+ + 2\bar{\xi}^+ \nabla \rho + 2\bar{\rho} \nabla \xi^+ \Big] \Big\} .
\end{aligned}
\tag{4.9}
$$

The CS action (4.9) obtained here describes the so-called Maxwellian extended Bargmann supergravity theory. Let us note that the NR CS supergravity action (4.9) contains three independent sectors proportional to $\tilde{\alpha}_0$, $\tilde{\alpha}_1$, and $\tilde{\alpha}_2$. In particular, the term proportional to $\tilde{\alpha}_0$ corresponds to a NR exotic Lagrangian, while the extended Bargmann supergravity introduced in [3] appears in the $\tilde{\alpha}_1$ sector, endowed with some additional terms related to the presence of the bosonic 1-form fields y_1, y_2, u_1, and u_2. The term proportional to $\tilde{\alpha}_2$ can be seen as the CS Lagrangian for a new NR Maxwell superalgebra. In addition, one can see that the bosonic part of the CS action (4.9) corresponds to the MEB gravity action presented in [74], supplemented with the bosonic 1-form fields y_1, y_2, b_1, b_2, u_1, and u_2.

Note that, for $\alpha_2 \neq 0$, the field equations from the NR CS supergravity action (4.9) reduce to the vanishing of the curvature two-forms (4.7), (4.6), and (4.8) associated with the MEB superalgebra. These curvatures transform covariantly with respect to the supersymmetry transformation laws given in (A.1).

The three-dimensional Maxwellian extended Bargmann supergravity theory obtained here corresponds to an alternative NR supergravity theory which contains the extended Bargmann supergravity [3] (supplemented with some additional bosonic 1-form fields) as a sub-case and which is distinct from the Newton-Cartan supergravity introduced in [1, 2]. It is important to emphasize that, although our result generalizes the extended Bargmann supergravity [3], the new NR supergravity obtained here do not contain a cosmological constant. It would be then interesting to study the introduction of a cosmological constant in our model.

5 Discussion

In this work, we have studied the NR limit of the relativistic Maxwell superalgebra. A well-defined NR superalgebra with the desired features has been obtained by contracting the $\mathcal{N} = 2$ Maxwell superalgebra introduced in [73]. Nevertheless, the construction of a proper NR supergravity action based on a NR version of the Maxwell superalgebra has required to introduce by hand new fermionic and bosonic generators. The new structure has been called as the Maxwellian extended Bargmann superalgebra and corresponds to a supersymmetric extension of the MEB algebra presented in [74]. In particular the MEB superalgebra admits a non-degenerate invariant bilinear form allowing to construct a proper NR CS supergravity action. Interestingly, the MEB CS supergravity theory presented here contains the extended Bargmann supergravity as a sub-case.

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The NR supergravity action constructed in this work, see (4.9), could serve as a starting point for diverse studies. In particular, our result is the first step to construct a new family of NR supergravity beyond the extended Bargmann supergravity. Furthermore, the MEB supergravity could be useful in the construction of a three-dimensional Horava-Lifshitz supergravity. Indeed, as was noticed in [3, 33], the extended Bargmann gravity can be seen as a particular kinetic term of the Horava-Lifshitz gravity. In particular, it would be intriguing to explore the effects arising from the presence of the additional gauge field appearing in the Maxwell version of the extended Bargmann (super)algebra.

Moreover, the NR supergravity action (4.9) could have applications in the context of holography, as it already happens for NR gravity at the purely bosonic level [11, 13], as well as in effective field theory, where gravitational fields are used as geometrical background response functions [16, 98]. Also at this level, it would be interesting to examine the role played by the additional field involved in Maxwell version of the extended Bargmann (super)algebra. As a further remark, let us highlight that additional gauge fields appearing in (super)Maxwell algebras have already been proven to be of particular relevance in the study of the supersymmetry invariance of flat (relativistic) supergravity on a manifold with non-trivial boundary [68], where it has also been conjectured that the presence of the new gauge fields in the boundary would allow to regularize the supergravity action in the holographic renormalization language.

On the other hand, concerning, in particular, the purely bosonic restriction of our model, one could explore the phenomenological implications possibly related to Dark Matter at the non-relativistic level. Another aspect of the MEB gravity which deserves some investigation is the study of the asymptotic symmetries. As it was shown in [49], the asymptotic symmetry of the three-dimensional CS theory invariant under the Maxwell algebra is given by an extension and deformation of the BMS_3 algebra denoted as $\mathfrak{Max}_3$ [99]. One could then expect that, since the MEB gravity is the NR version of the Maxwell CS gravity, the corresponding asymptotic symmetry of the MEB gravity would be given by a NR version of the $\mathfrak{Max}_3$ algebra. Interestingly, the presence of the additional gauge field Z_A modifies not only the asymptotic sector but also the vacuum of the theory. It would be then interesting to study the physical implications of the additional bosonic gauge field appearing in our model. One could go even further and extend this study to supersymmetric level. However, this would require first to establish the relativistic version of our NR model and then study its asymptotic structure by considering suitable boundary conditions.

It would also be interesting to develop higher-dimensional extensions of our NR supergravity theory, since we argue that, due to the fact that the introduction of (at least) one second spinorial charge not only is required to construct the minimal supersymmetric extension of the Maxwell algebra but has also been considered in superstring [82] and in $D = 11$ supergravity [83–85], they could be relevant in the study and analysis of the NR limit of higher-dimensional supergravity models, and, more specifically, of $D = 11$ supergravity.

A future development could also consist in exploring the possibility to obtain the MEB superalgebra introduced here through a NR limit or contraction process from a relativistic superalgebra. An alternative limit which could be used to recover the MEB superalgebra

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is the vanishing cosmological constant limit. In particular, it would be worth exploring the possibility to accommodate a cosmological constant to the MEB supergravity presented here. One could conjecture that a supersymmetric extension of the recent enlarged extended Bargmann gravity, introduced in [87], reproduces the present MEB supergravity in a flat limit [work in progress].

On the other hand, it would be interesting to apply the Lie algebra expansion method [88–91] to obtain the MEB superalgebra. One could follow the procedure used in [92–94] and consider the expansion of a relativistic Maxwell superalgebra. Alternatively, one might also extend the results obtained in [87, 95] in which NR algebras appear as semigroup expansions of the so-called Nappi-Witten algebra.

Another aspect that deserves further investigation regards the development of a Maxwellian version of the extended Newtonian gravity [96] and its supersymmetric extension [4]. In particular, the result obtained here could correspond to a subcase of a Maxwellian generalization of the extended Newtonian supergravity. A cosmological constant has recently been accommodated in the extended Newtonian gravity action by including new generators to the Newton-Hooke algebra [97]. One could expect to obtain a Maxwellian Newtonian algebra by generalizing the MEB one in a similar way to [4, 97]. It would be then compelling to explore possible matter couplings.

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A Supersymmetry transformation laws

The curvatures (4.7), (4.6), and (4.8) associated with the MEB superalgebra transform covariantly with respect to the following supersymmetry transformation laws:

$$
\begin{aligned}
\delta\omega &= 0\,,\\
\delta\omega^a &= 0\,,\\
\delta\tau &= -\bar{\epsilon}^{+}\gamma^{0}\psi^{+}\,,\\
\delta e^a &= -\bar{\epsilon}^{+}\gamma^{a}\psi^{-} - \bar{\epsilon}^{-}\gamma^{a}\psi^{+}\,,\\
\delta k &= -\bar{\epsilon}^{+}\gamma^{0}\xi^{+} - \bar{\varphi}^{+}\gamma^{0}\psi^{+}\,,\\
\delta k^a &= -\bar{\epsilon}^{\pm}\gamma^{a}\xi^{\mp} - \bar{\varphi}^{\pm}\gamma^{a}\psi^{\mp}\,,\\
\delta m &= -\bar{\epsilon}^{-}\gamma^{0}\psi^{-} - \bar{\epsilon}^{+}\gamma^{0}\rho - \bar{\eta}\gamma^{0}\psi^{+}\,,\\
\delta s &= 0\,,\\
\delta t &= -\bar{\epsilon}^{-}\gamma^{0}\xi^{-} - \bar{\varphi}^{-}\gamma^{0}\psi^{-} - \bar{\epsilon}^{+}\gamma^{0}\chi - \bar{\zeta}\gamma^{0}\psi^{+} - \bar{\varphi}^{+}\gamma^{0}\rho - \bar{\eta}\gamma^{0}\xi^{+}\,,\\
\delta y_1 &= 0\,,\\
\delta y_2 &= 0\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta b_1 &= -\bar{\epsilon}^+\gamma^0\xi^+ - \bar{\varphi}^+\gamma^0\psi^+ \,, \\
\delta b_2 &= \bar{\epsilon}^-\gamma^0\xi^- + \bar{\varphi}^-\gamma^0\psi^- - \bar{\epsilon}^+\gamma^0\chi - \bar{\zeta}\gamma^0\psi^+ - \bar{\varphi}^+\gamma^0\rho - \bar{\eta}\gamma^0\xi^+ \,, \\
\delta u_1 &= -\bar{\epsilon}^+\gamma^0\psi^+ \,, \\
\delta u_2 &= \bar{\epsilon}^-\gamma^0\psi^- - \bar{\epsilon}^+\gamma^0\rho - \bar{\eta}\gamma^0\psi^+ \,, \\
\delta\epsilon^+ &= d\epsilon^+ + \frac{1}{2}\omega\gamma_0\epsilon^+ - \frac{1}{2}y_1\gamma_0\epsilon^+ \,, \\
\delta\epsilon^- &= d\epsilon^- + \frac{1}{2}\omega\gamma_0\epsilon^- + \frac{1}{2}\omega^a\gamma_a\epsilon^+ + \frac{1}{2}y_1\gamma_0\epsilon^- \,, \\
\delta\varphi^+ &= d\varphi^+ + \frac{1}{2}\omega\gamma_0\varphi^+ + \frac{1}{2}\tau\gamma_0\epsilon^+ - \frac{1}{2}y_1\gamma_0\varphi^+ - \frac{1}{2}u_1\gamma_0\epsilon^+ \,, \\
\delta\varphi^- &= d\varphi^- + \frac{1}{2}\omega\gamma_0\varphi^- + \frac{1}{2}\tau\gamma_0\epsilon^- + \frac{1}{2}e^a\gamma_a\epsilon^+ + \frac{1}{2}\omega^a\gamma_a\varphi^+ + \frac{1}{2}y_1\gamma_0\varphi^- + \frac{1}{2}u_1\gamma_0\epsilon^- \,, \\
\delta\eta &= d\eta + \frac{1}{2}\omega\gamma_0\eta + \frac{1}{2}\omega^a\gamma_a\epsilon^- + \frac{1}{2}s\gamma_0\epsilon^+ - \frac{1}{2}y_2\gamma_0\epsilon^+ - \frac{1}{2}y_1\gamma_0\eta \,, \\
\delta\zeta &= d\zeta + \frac{1}{2}\omega\gamma_0\zeta + \frac{1}{2}\omega^a\gamma_a\varphi^- + \frac{1}{2}e^a\gamma_a\epsilon^- + \frac{1}{2}\tau\gamma_0\eta + \frac{1}{2}s\gamma_0\varphi^+ + \frac{1}{2}m\gamma_0\epsilon^+ \\
&\quad -\frac{1}{2}y_2\gamma_0\varphi^+ - \frac{1}{2}y_1\gamma_0\zeta - \frac{1}{2}u_2\gamma_0\epsilon^+ - \frac{1}{2}u_1\gamma_0\eta \,,
\end{aligned} \tag{A.1}
$$

where $\epsilon^\pm$, $\varphi^\pm$, η, and ζ are the fermionic gauge parameters related to the Majorana fermionic generators $\tilde{Q}^\pm$, $\tilde{\Sigma}^\pm$, $\tilde{R}$, and $\tilde{W}$, respectively (in order to lighten the notation, we have omitted the spinor index α).

References

[1] R. Andringa, E.A. Bergshoeff, J. Rosseel and E. Sezgin, *3D Newton-Cartan supergravity*, *Class. Quant. Grav.* **30** (2013) 205005 [arXiv:1305.6737] [INSPIRE].

[2] E. Bergshoeff, J. Rosseel and T. Zojer, *Newton-Cartan supergravity with torsion and Schrödinger supergravity*, *JHEP* **11** (2015) 180 [arXiv:1509.04527] [INSPIRE].

[3] E.A. Bergshoeff and J. Rosseel, *Three-Dimensional Extended Bargmann Supergravity*, *Phys. Rev. Lett.* **116** (2016) 251601 [arXiv:1604.08042] [INSPIRE].

[4] N. Ozdemir, M. Ozkan, O. Tunca and U. Zorba, *Three-Dimensional Extended Newtonian (Super)Gravity*, *JHEP* **05** (2019) 130 [arXiv:1903.09377] [INSPIRE].

[5] N. Ozdemir, M. Ozkan and U. Zorba, *Three-dimensional extended Lifshitz, Schrödinger and Newton-Hooke supergravity*, *JHEP* **11** (2019) 052 [arXiv:1909.10745] [INSPIRE].

[6] L. Ravera, *AdS Carroll Chern-Simons supergravity in* $2+1$ *dimensions and its flat limit*, *Phys. Lett.* **B 795** (2019) 331 [arXiv:1905.00766] [INSPIRE].

[7] F. Ali and L. Ravera, *$\mathcal{N}$-extended Chern-Simons Carrollian supergravities in* $2+1$ *spacetime dimensions*, *JHEP* **02** (2020) 128 [arXiv:1912.04172] [INSPIRE].

[8] D.T. Son, *Toward an AdS/cold atoms correspondence: A Geometric realization of the Schrödinger symmetry*, *Phys. Rev.* **D 78** (2008) 046003 [arXiv:0804.3972] [INSPIRE].

[9] K. Balasubramanian and J. McGreevy, *Gravity duals for non-relativistic CFTs*, Phys. Rev. Lett. **101** (2008) 061601 [arXiv:0804.4053] [INSPIRE].

[10] S. Kachru, X. Liu and M. Mulligan, *Gravity duals of Lifshitz-like fixed points*, Phys. Rev. **D 78** (2008) 106005 [arXiv:0808.1725] [INSPIRE].

[11] A. Bagchi and R. Gopakumar, *Galilean Conformal Algebras and AdS/CFT*, JHEP **07** (2009) 037 [arXiv:0902.1385] [INSPIRE].

[12] A. Bagchi, R. Gopakumar, I. Mandal and A. Miwa, *GCA in 2d*, JHEP **08** (2010) 004 [arXiv:0912.1090] [INSPIRE].

[13] M.H. Christensen, J. Hartong, N.A. Obers and B. Rollier, *Torsional Newton-Cartan Geometry and Lifshitz Holography*, Phys. Rev. **D 89** (2014) 061901 [arXiv:1311.4794] [INSPIRE].

[14] M.H. Christensen, J. Hartong, N.A. Obers and B. Rollier, *Boundary Stress-Energy Tensor and Newton-Cartan Geometry in Lifshitz Holography*, JHEP **01** (2014) 057 [arXiv:1311.6471] [INSPIRE].

[15] M. Taylor, *Lifshitz holography*, Class. Quant. Grav. **33** (2016) 033001 [arXiv:1512.03554] [INSPIRE].

[16] D.T. Son, *Newton-Cartan Geometry and the Quantum Hall Effect*, arXiv:1306.0638 [INSPIRE].

[17] C. Hoyos and D.T. Son, *Hall Viscosity and Electromagnetic Response*, Phys. Rev. Lett. **108** (2012) 066805 [arXiv:1109.2651] [INSPIRE].

[18] M. Geracie, K. Prabhu and M.M. Roberts, *Curved non-relativistic spacetimes, Newtonian gravitation and massive matter*, J. Math. Phys. **56** (2015) 103505 [arXiv:1503.02682] [INSPIRE].

[19] A. Gromov, K. Jensen and A.G. Abanov, *Boundary effective action for quantum Hall states*, Phys. Rev. Lett. **116** (2016) 126802 [arXiv:1506.07171] [INSPIRE].

[20] C. Duval and H.P. Kunzle, *Minimal Gravitational Coupling in the Newtonian Theory and the Covariant Schrödinger Equation*, Gen. Rel. Grav. **16** (1984) 333 [INSPIRE].

[21] C. Duval, G. Burdet, H.P. Kunzle and M. Perrin, *Bargmann Structures and Newton-cartan Theory*, Phys. Rev. **D 31** (1985) 1841 [INSPIRE].

[22] C. Duval, G.W. Gibbons and P. Horvathy, *Celestial mechanics, conformal structures and gravitational waves*, Phys. Rev. **D 43** (1991) 3907 [hep-th/0512188] [INSPIRE].

[23] C. Duval, *On Galileian isometries*, Class. Quant. Grav. **10** (1993) 2217 [arXiv:0903.1641] [INSPIRE].

[24] R. De Pietri, L. Lusanna and M. Pauri, *Standard and generalized Newtonian gravities as 'gauge' theories of the extended Galilei group. I. The standard theory*, Class. Quant. Grav. **12** (1995) 219 [gr-qc/9405046] [INSPIRE].

[25] R. De Pietri, L. Lusanna and M. Pauri, *Standard and generalized Newtonian gravities as 'gauge' theories of the extended Galilei group. II. Dynamical three space theories*, Class. Quant. Grav. **12** (1995) 255 [gr-qc/9405047] [INSPIRE].

[26] P. Hořava, *Quantum Gravity at a Lifshitz Point*, Phys. Rev. **D 79** (2009) 084008 [arXiv:0901.3775] [INSPIRE].

[27] C. Duval and P.A. Horvathy, *Non-relativistic conformal symmetries and Newton-Cartan structures*, J. Phys. **A 42** (2009) 465206 [arXiv:0904.0531] [INSPIRE].

JHEP04(2020)051

[28] G. Papageorgiou and B.J. Schroers, *Galilean quantum gravity with cosmological constant and the extended q-Heisenberg algebra*, JHEP **11** (2010) 020 [arXiv:1008.0279] [INSPIRE].

[29] R. Andringa, E. Bergshoeff, S. Panda and M. de Roo, *Newtonian Gravity and the Bargmann Algebra*, Class. Quant. Grav. **28** (2011) 105011 [arXiv:1011.1145] [INSPIRE].

[30] R. Andringa, E. Bergshoeff, J. Gomis and M. de Roo, *'Stringy' Newton-Cartan Gravity*, Class. Quant. Grav. **29** (2012) 235020 [arXiv:1206.5176] [INSPIRE].

[31] R. Banerjee, A. Mitra and P. Mukherjee, *Localization of the Galilean symmetry and dynamical realization of Newton-Cartan geometry*, Class. Quant. Grav. **32** (2015) 045010 [arXiv:1407.3617] [INSPIRE].

[32] R. Banerjee and P. Mukherjee, *Torsional Newton-Cartan geometry from Galilean gauge theory*, Class. Quant. Grav. **33** (2016) 225013 [arXiv:1604.06893] [INSPIRE].

[33] J. Hartong, Y. Lei and N.A. Obers, *Nonrelativistic Chern-Simons theories and three-dimensional Hořava-Lifshitz gravity*, Phys. Rev. **D 94** (2016) 065027 [arXiv:1604.08054] [INSPIRE].

[34] E. Bergshoeff, A. Chatzistavrakidis, L. Romano and J. Rosseel, *Newton-Cartan Gravity and Torsion*, JHEP **10** (2017) 194 [arXiv:1708.05414] [INSPIRE].

[35] R. Banerjee and P. Mukherjee, *Galilean gauge theory from Poincaré gauge theory*, Phys. Rev. **D 98** (2018) 124021 [arXiv:1810.03902] [INSPIRE].

[36] D. Chernyavsky and D. Sorokin, *Three-dimensional (higher-spin) gravities with extended Schrödinger and l-conformal Galilean symmetries*, JHEP **07** (2019) 156 [arXiv:1905.13154] [INSPIRE].

[37] G. Festuccia and N. Seiberg, *Rigid Supersymmetric Theories in Curved Superspace*, JHEP **06** (2011) 114 [arXiv:1105.0689] [INSPIRE].

[38] V. Pestun, *Localization of gauge theory on a four-sphere and supersymmetric Wilson loops*, Commun. Math. Phys. **313** (2012) 71 [arXiv:0712.2824] [INSPIRE].

[39] R. Schrader, *The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields*, Fortsch. Phys. **20** (1972) 701 [INSPIRE].

[40] H. Bacry, P. Combe and J.L. Richard, *Group-theoretical analysis of elementary particles in an external electromagnetic field. 1. the relativistic particle in a constant and uniform field*, Nuovo Cim. **A 67** (1970) 267 [INSPIRE].

[41] J. Gomis and A. Kleinschmidt, *On free Lie algebras and particles in electro-magnetic fields*, JHEP **07** (2017) 085 [arXiv:1705.05854] [INSPIRE].

[42] J.D. Edelstein, M. Hassaine, R. Troncoso and J. Zanelli, *Lie-algebra expansions, Chern-Simons theories and the Einstein-Hilbert Lagrangian*, Phys. Lett. **B 640** (2006) 278 [hep-th/0605174] [INSPIRE].

[43] F. Izaurieta, E. Rodríguez, P. Minning, P. Salgado and A. Perez, *Standard General Relativity from Chern-Simons Gravity*, Phys. Lett. **B 678** (2009) 213 [arXiv:0905.2187] [INSPIRE].

[44] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Even-dimensional General Relativity from Born-Infeld gravity*, Phys. Lett. **B 725** (2013) 419 [arXiv:1309.0062] [INSPIRE].

[45] P.K. Concha, D.M. Penãfiel, E.K. Rodríguez and P. Salgado, *Chern-Simons and Born-Infeld gravity theories and Maxwell algebras type*, Eur. Phys. J. **C 74** (2014) 2741 [arXiv:1402.0023] [INSPIRE].

[46] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Generalized Poincaré algebras and Lovelock-Cartan gravity theory*, Phys. Lett. **B 742** (2015) 310 [arXiv:1405.7078] [INSPIRE].

[47] P. Salgado, R.J. Szabo and O. Valdivia, *Topological gravity and transgression holography*, Phys. Rev. **D 89** (2014) 084077 [arXiv:1401.3653] [INSPIRE].

[48] S. Hoseinzadeh and A. Rezaei-Aghdam, $(2+1)$*-dimensional gravity from Maxwell and semisimple extension of the Poincaré gauge symmetric models*, Phys. Rev. **D 90** (2014) 084008 [arXiv:1402.0320] [INSPIRE].

[49] P. Concha, N. Merino, O. Mišković, E. Rodríguez, P. Salgado-ReboLledó and O. Valdivia, *Asymptotic symmetries of three-dimensional Chern-Simons gravity for the Maxwell algebra*, JHEP **10** (2018) 079 [arXiv:1805.08834] [INSPIRE].

[50] R. Caroca, P. Concha, O. Fierro, E. Rodríguez and P. Salgado-ReboLledó, *Generalized Chern-Simons higher-spin gravity theories in three dimensions*, Nucl. Phys. **B 934** (2018) 240 [arXiv:1712.09975] [INSPIRE].

[51] R. Caroca, P. Concha, E. Rodríguez and P. Salgado-ReboLledó, *Generalizing the* $\mathfrak{bms}_3$ *and 2D-conformal algebras by expanding the Virasoro algebra*, Eur. Phys. J. **C 78** (2018) 262 [arXiv:1707.07209] [INSPIRE].

[52] J.A. de Azcarraga, K. Kamimura and J. Lukierski, *Generalized cosmological term from Maxwell symmetries*, Phys. Rev. **D 83** (2011) 124036 [arXiv:1012.4402] [INSPIRE].

[53] R. Durka, J. Kowalski-Glikman and M. Szczachor, *Gauged AdS-Maxwell algebra and gravity*, Mod. Phys. Lett. **A 26** (2011) 2689 [arXiv:1107.4728] [INSPIRE].

[54] J.A. de Azcarraga, K. Kamimura and J. Lukierski, *Maxwell symmetries and some applications*, Int. J. Mod. Phys. Conf. Ser. **23** (2013) 01160 [arXiv:1201.2850] [INSPIRE].

[55] O. Cebecioğlu and S. Kibaroğlu, *Maxwell-affine gauge theory of gravity*, Phys. Lett. **B 751** (2015) 131 [arXiv:1503.09003] [INSPIRE].

[56] S. Bansal and D. Sorokin, *Can Chern-Simons or Rarita-Schwinger be a Volkov-Akulov Goldstone?*, JHEP **07** (2018) 106 [arXiv:1806.05945] [INSPIRE].

[57] S. Kibaroğlu, M. Şenay and O. Cebecioğlu, $D=4$ *topological gravity from gauging the Maxwell-special-affine group*, Mod. Phys. Lett. **A 34** (2019) 1950016 [arXiv:1810.01635] [INSPIRE].

[58] P. Salgado-ReboLledó, *The Maxwell group in* $2+1$ *dimensions and its infinite-dimensional enhancements*, JHEP **10** (2019) 039 [arXiv:1905.09421] [INSPIRE].

[59] S. Bonanos, J. Gomis, K. Kamimura and J. Lukierski, *Maxwell Superalgebra and Superparticle in Constant Gauge Badkgrounds*, Phys. Rev. Lett. **104** (2010) 090401 [arXiv:0911.5072] [INSPIRE].

[60] S. Bonanos, J. Gomis, K. Kamimura and J. Lukierski, *Deformations of Maxwell Superalgebras and Their Applications*, J. Math. Phys. **51** (2010) 102301 [arXiv:1005.3714] [INSPIRE].

[61] J. Lukierski, *Generalized Wigner-Inönü contractions and Maxwell (super)algebras*, Proc. Steklov Inst. Math. **272** (2011) 183 [arXiv:1007.3405] [INSPIRE].

[62] J.A. de Azcarraga, J.M. Izquierdo, J. Lukierski and M. Woronowicz, *Generalizations of Maxwell (super)algebras by the expansion method*, Nucl. Phys. **B 869** (2013) 303 [arXiv:1210.1117] [INSPIRE].

JHEP04(2020)051

[63] J.A. de Azcarraga and J.M. Izquierdo, *Minimal $D = 4$ supergravity from the superMaxwell algebra*, Nucl. Phys. **B 885** (2014) 34 [arXiv:1403.4128] [INSPIRE].

[64] P.K. Concha and E.K. Rodríguez, *Maxwell Superalgebras and Abelian Semigroup Expansion*, Nucl. Phys. **B 886** (2014) 1128 [arXiv:1405.1334] [INSPIRE].

[65] P.K. Concha and E.K. Rodríguez, *$N = 1$ Supergravity and Maxwell superalgebras*, JHEP **09** (2014) 090 [arXiv:1407.4635] [INSPIRE].

[66] D.M. Peñafiel and L. Ravera, *On the Hidden Maxwell Superalgebra underlying $D = 4$ Supergravity*, Fortsch. Phys. **65** (2017) 1700005 [arXiv:1701.04234] [INSPIRE].

[67] L. Ravera, *Hidden role of Maxwell superalgebras in the free differential algebras of $D = 4$ and $D = 11$ supergravity*, Eur. Phys. J. **C 78** (2018) 211 [arXiv:1801.08860] [INSPIRE].

[68] P. Concha, L. Ravera and E. Rodríguez, *On the supersymmetry invariance of flat supergravity with boundary*, JHEP **01** (2019) 192 [arXiv:1809.07871] [INSPIRE].

[69] S. Kibaroğlu and O. Cebecioğlu, *$D = 4$ supergravity from the Maxwell-Weyl superalgebra*, Eur. Phys. J. **C 79** (2019) 898 [arXiv:1812.09861] [INSPIRE].

[70] P.K. Concha, O. Fierro, E.K. Rodríguez and P. Salgado, *Chern-Simons supergravity in $D = 3$ and Maxwell superalgebra*, Phys. Lett. **B 750** (2015) 117 [arXiv:1507.02335] [INSPIRE].

[71] P.K. Concha, O. Fierro and E.K. Rodríguez, *Inönü-Wigner contraction and $D = 2 + 1$ supergravity*, Eur. Phys. J. **C 77** (2017) 48 [arXiv:1611.05018] [INSPIRE].

[72] P. Concha, D.M. Peñafiel and E. Rodríguez, *On the Maxwell supergravity and flat limit in $2 + 1$ dimensions*, Phys. Lett. **B 785** (2018) 247 [arXiv:1807.00194] [INSPIRE].

[73] P. Concha, *$\mathcal{N}$-extended Maxwell supergravities as Chern-Simons theories in three spacetime dimensions*, Phys. Lett. **B 792** (2019) 290 [arXiv:1903.03081] [INSPIRE].

[74] L. Avilés, E. Frodden, J. Gomis, D. Hidalgo and J. Zanelli, *Non-Relativistic Maxwell Chern-Simons Gravity*, JHEP **05** (2018) 047 [arXiv:1802.08453] [INSPIRE].

[75] J. Gomis, A. Kleinschmidt, J. Palmkvist and P. Salgado-ReboLledó, *Newton-Hooke/Carrollian expansions of (A)dS and Chern-Simons gravity*, JHEP **02** (2020) 009 [arXiv:1912.07564] [INSPIRE].

[76] J.M. Lévy-Leblond, *Galilei Group and Galilean Invariance*, in *Group Theory and its Applications. Vol. II*, Academic Press, New York U.S.A. (1971), pg. 221.

[77] D.R. Grigore, *The Projective unitary irreducible representations of the Galilei group in $(1 + 2)$-dimensions*, J. Math. Phys. **37** (1996) 460 [hep-th/9312048] [INSPIRE].

[78] S.K. Bose, *The Galilean group in $(2 + 1)$ space-times and its central extension*, Commun. Math. Phys. **169** (1995) 385 [INSPIRE].

[79] C. Duval and P.A. Horvathy, *The 'Peierls substitution' and the exotic Galilei group*, Phys. Lett. **B 479** (2000) 284 [hep-th/0002233] [INSPIRE].

[80] R. Jackiw and V.P. Nair, *Anyon spin and the exotic central extension of the planar Galilei group*, Phys. Lett. **B 480** (2000) 237 [hep-th/0003130] [INSPIRE].

[81] P.A. Horvathy and M.S. Plyushchay, *Non-relativistic anyons, exotic Galilean symmetry and noncommutative plane*, JHEP **06** (2002) 033 [hep-th/0201228] [INSPIRE].

[82] M.B. Green, *Supertranslations, Superstrings and Chern-Simons Forms*, Phys. Lett. **B 223** (1989) 157 [INSPIRE].

JHEP04(2020)051

[83] R. D'Auria and P. Fré, *Geometric Supergravity in $d = 11$ and Its Hidden Supergroup*, Nucl. Phys. **B 201** (1982) 101 [*Erratum ibid.* **B 206** (1982) 496] [INSPIRE].

[84] L. Andrianopoli, R. D'Auria and L. Ravera, *Hidden Gauge Structure of Supersymmetric Free Differential Algebras*, JHEP **08** (2016) 095 [arXiv:1606.07328] [INSPIRE].

[85] L. Andrianopoli, R. D'Auria and L. Ravera, *More on the Hidden Symmetries of 11D Supergravity*, Phys. Lett. **B 772** (2017) 578 [arXiv:1705.06251] [INSPIRE].

[86] J. Lukierski, I. Prochnicka, P.C. Stichel and W.J. Zakrzewski, *Galilean exotic planar supersymmetries and nonrelativistic supersymmetric wave equations*, Phys. Lett. **B 639** (2006) 389 [hep-th/0602198] [INSPIRE].

[87] P. Concha and E. Rodríguez, *Non-Relativistic Gravity Theory based on an Enlargement of the Extended Bargmann Algebra*, JHEP **07** (2019) 085 [arXiv:1906.00086] [INSPIRE].

[88] M. Hatsuda and M. Sakaguchi, *Wess-Zumino term for the AdS superstring and generalized Inonu-Wigner contraction*, Prog. Theor. Phys. **109** (2003) 853 [hep-th/0106114] [INSPIRE].

[89] J.A. de Azcárraga, J.M. Izquierdo, M. Picón and O. Varela, *Generating Lie and gauge free differential (super)algebras by expanding Maurer-Cartan forms and Chern-Simons supergravity*, Nucl. Phys. **B 662** (2003) 185 [hep-th/0212347] [INSPIRE].

[90] J.A. de Azcárraga, J.M. Izquierdo, M. Picón and O. Varela, *Expansions of algebras and superalgebras and some applications*, Int. J. Theor. Phys. **46** (2007) 2738 [hep-th/0703017] [INSPIRE].

[91] F. Izaurieta, E. Rodríguez and P. Salgado, *Expanding Lie (super)algebras through Abelian semigroups*, J. Math. Phys. **47** (2006) 123512 [hep-th/0606215] [INSPIRE].

[92] E. Bergshoeff, J.M. Izquierdo, T. Ortín and L. Romano, *Lie Algebra Expansions and Actions for Non-Relativistic Gravity*, JHEP **08** (2019) 048 [arXiv:1904.08304] [INSPIRE].

[93] J.A. de Azcárraga, D. Gútiez and J.M. Izquierdo, *Extended $D = 3$ Bargmann supergravity from a Lie algebra expansion*, arXiv:1904.12786 [INSPIRE].

[94] L. Romano, *Non-Relativistic Four Dimensional p-Brane Supersymmetric Theories and Lie Algebra Expansion*, arXiv:1906.08220 [INSPIRE].

[95] D.M. Peñafiel and P. Salgado-ReboLledó, *Non-relativistic symmetries in three space-time dimensions and the Nappi-Witten algebra*, Phys. Lett. **B 798** (2019) 135005 [arXiv:1906.02161] [INSPIRE].

[96] D. Hansen, J. Hartong and N.A. Obers, *Action Principle for Newtonian Gravity*, Phys. Rev. Lett. **122** (2019) 061106 [arXiv:1807.04765] [INSPIRE].

[97] P. Concha, L. Ravera and E. Rodríguez, *Three-dimensional exotic Newtonian gravity with cosmological constant*, Phys. Lett. **B 804** (2020) 135392 [arXiv:1912.02836] [INSPIRE].

[98] M. Geracie, D.T. Son, C. Wu and S.-F. Wu, *Spacetime Symmetries of the Quantum Hall Effect*, Phys. Rev. **D 91** (2015) 045030 [arXiv:1407.1252] [INSPIRE].

[99] P. Concha and H.R. Safari, *On Stabilization of Maxwell-BMS Algebra*, arXiv:1909.12827 [INSPIRE].