Soft hair of dynamical black hole and Hawking radiation

Chong-Sun Chu[a,b] **and Yoji Koyama**[a]

[a]*Physics Division, National Center for Theoretical Sciences, National Tsing-Hua University,*
Hsinchu, 30013, Taiwan

[b]*Department of Physics, National Tsing-Hua University,*
Hsinchu 30013, Taiwan

E-mail: cschu@phys.nthu.edu.tw, koyama811@cts.nthu.edu.tw

ABSTRACT: Soft hair of black hole has been proposed recently to play an important role in the resolution of the black hole information paradox. Recent work has emphasized that the soft modes cannot affect the black hole S-matrix due to Weinberg soft theorems. However as soft hair is generated by supertranslation of geometry which involves an angular dependent shift of time, it must have non-trivial quantum effects. We consider supertranslation of the Vaidya black hole and construct a non-spherical symmetric dynamical spacetime with soft hair. We show that this spacetime admits a trapping horizon and is a dynamical black hole. We find that Hawking radiation is emitted from the trapping horizon of the dynamical black hole. The Hawking radiation has a spectrum which depends on the soft hair of the black hole and this is consistent with the factorization property of the black hole S-matrix.

Contents

1 Introduction

Black hole model in general relativity identifies deep insufficiency in our understanding of gravity. As explained by Bekenstein [1] and Hawking [2, 3], black holes obey the first law of thermodynamics [4], with a temperature that arises from the quantum process of Hawking radiation. From the no-hair theorem [5], one would expect that the Hawking radiation to be completely independent of the state of matters entering the black hole. Therefore if the matters were in a pure quantum state, and since the Hawking radiation is described by a mixed thermal state, the black hole evaporation process would be non-unitary which is in contradiction with quantum mechanics. See, for example, [6, 7] for a review of the black hole information problem.

Recently, Strominger initiated a study of the infrared structure of gravity and its connection with the asymptotic BMS symmetry [8–13]. Furthermore, Hawking, Perry and Strominger [14, 15] and Strominger [16] advocated a new approach to the black hole information paradox based on a new kind of black hole soft hair.

BMS symmetry [17, 18] is the symmetry of asymptotically flat spacetime generated by supertranslation, an angular dependent shift of the time coordinate in the asymptotic region. With the BMS symmetry, asymptotically flat spacetimes are characterized by, in addition to the standard ADM charges of mass, electric charge and angular momentum [19], also an infinite number of BMS charges of supertranslation and superrotation charges [20, 21]. The existence of an infinite number of new charges for asymptotically flat spacetime is a very interesting observation. When applied to black hole in Minkowski

spacetime, this means that supertranslated black hole carries an infinite amount of soft hair characterized by the BMS charges. The existence of soft hair is not a contradiction to the standard no-hair theorem of black hole since the supertranslated black hole is diffeomorphic to the untranslated one, and so they carry the same ADM charges. Nevertheless soft hair modifies the definition of time at null infinity and it acts non-trivially on the classical phase space. It is physical and its effects can be observed classically with the gravitational memory effect [22–25].

Soft hair can be generated via physical process. In the case of Schwarzschild black hole, [15] showed that one could grow soft hair on it by throwing in a shockwave of radiation. This relation of the soft hair with the collapsed matter suggests that soft hair could encode information about the black hole creation and evaporation process. It was further argued [14] that, if viewed as a scattering amplitude in the quantum theory, the process of the black hole formation and evaporation should be constrained by the infinite symmetries of BMS and hence the existence of an infinite amount of soft hair could help to resolve the information paradox. The problem of how the black hole S-matrix could be constrained by the soft modes has been further analyzed by a number of authors recently. As emphasized by [26–29], the scattering of the hard modes (like the black hole process) is factorized from the scattering of the soft modes. The decoupling can be seen in the dressed state approach. The decoupling has also been argued to be a generic feature of quantum gravity, at least perturbatively [30].

Although the S-matrix turns out not to be a good observable for studying the evaporation process of black hole, it does not exclude soft hair from having physical effects on the black hole physics. Quantum mechanically, the change of time is translated to a change of the quantum vacuum, so soft hair is expected to leave quantum effects on the black hole physics. The main motivation of our work is to find out how and what aspects of the quantum physics of black hole is affected by its soft hair.

In this regard, we consider the effects of soft hair on the spectrum of the Hawking radiation. For the soft-hairy Schwarzschild black hole, we find that the Hawking radiation is insensitive to the soft hair. In this process, soft hair is implanted on the eternal Schwarzschild black hole with an energy flux of shock wave. However this is not entirely consistent as Hawking radiation carries away energy and should backreact on the metric. One should allow for the time dependence of the metric and consider a dynamical black hole. We therefore consider the more realistic process of black hole evaporation due to Hawking radiation of this dynamical black hole and investigate how the soft hair would affect the Hawking radiation. The black hole evaporation process can be modeled with a supertranslated Vaidya spacetime. We employ the tunneling method to compute the Hawking radiation and find that it has a dependence on the soft hair configuration. Our result is consistent with the factorization property of the S-matrix: the final state of black hole evaporation consists of Hawking radiation plus a sea of soft modes. If one use the dressed states as a basis for the soft modes, then the Hawking radiation would have a spectrum which is independent of the soft hair configuration. On the other hand, if one use the undressed states as in our computation, then the Hawking radiation would develop a dependence on the soft hair.

The organization of the paper is as follow. In section 2, we introduce the Vaidya spacetime and its supertranslation. We show that this spacetime can be obtained from the collapse of a certain energy momentum tensor with soft charges. In section 3, we discuss some properties of the supertranslated Vaidya black hole. We briefly review the definition of trapping horizon for dynamical black holes and show that the supertranslated Vaidya spacetime is a dynamical black hole with a trapping horizon. Next we define the horizon surface gravity associated with the trapping horizon of the supertranslated Vaidya black hole by extending the definition of the horizon surface gravity of spherically symmetric black holes using Kodama vector. In section 4, we apply the Hamilton-Jacobi tunneling method [31], which has been used as frequently as the null geodesic method developed by Parikh and Wilczek [32], to compute the Hawking radiation from the supertranslated Vaidya black hole. The Hawking radiation spectrum takes the standard form and the dependence on soft hair is encoded in the temperature of the radiation. We discuss the results with respect to dressing of soft modes in the final state of observation. We conclude the paper with some further discussion.

2 Supertranslation of Vaidya spacetime

2.1 Supertranslated Vaidya spacetime

Let us start with a brief review on supertranslation for an asymptotically flat metric in four dimensions. At the infinity, depending on the physical situation one wants to describe, one may impose different falloff conditions on the metric. In general one wants to choose the falloff conditions such that interesting solutions such as gravitational radiations are included, but unphysical solutions (e.g. those with infinite energy) are ruled out. The choice of falloff conditions of Bondi, van der Burg, Metzner and Sachs (BMS) [17, 18, 44, 45] considers metric with the asymptotic expansion near the past null infinity $\mathcal{I}^-$ [15],

$$
\begin{aligned}
ds^2 = &-dv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B \\
&+ \frac{2m}{r}dv^2 + rC_{AB}d\Theta^A d\Theta^B + \frac{1}{4}\gamma_{AB}C_{CD}C^{CD}d\Theta^A d\Theta^B \\
&- D^B C_{AB}dvd\Theta^A - \frac{1}{r}\left(\frac{4}{3}N_A - \frac{4v}{3}\partial_A m - \frac{1}{8}\partial_A(C_{BD}C^{BD})\right)dvd\Theta^A \\
&- \frac{1}{16r^2}C_{AB}C^{AB}dvdr,
\end{aligned}
\tag{2.1}
$$

where γ_{AB} is the metric on the unit two sphere and $\Theta^A = (z, \bar{z})$ are the angular coordinates. In (2.1) the Bondi mass aspect m, the traceless tensor C_{AB} and the angular momentum aspect N_A depend on (v, Θ^A). BMS supertranslation is the diffeomorphism which preserves the Bondi gauge and the asymptotic falloff conditions. It is generated by the vector field:

$$
\zeta_f = f\partial_v - \frac{1}{2}D^2 f\partial_r + \frac{1}{r}D^A f\partial_A,
\tag{2.2}
$$

where D_A is the covariant derivative with respect to γ_{AB}. Supertranslations are characterized by an arbitrary function of the angular variables, $f = f(\Theta)$.

We are interested in dynamical black holes in asymptotically flat spacetime. An example is the Vaidya metric. There are two kinds of Vaidya spacetime, one written in terms of the retarded (outgoing) null coordinates and one in terms of the advanced (ingoing) null coordinates. The Vaidya metric in the advanced Bondi coordinates (v, r, Θ^A) is given by

$$ds^2 = \overline{g}_{\mu\nu}dx^\mu dx^\nu = -Vdv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B, \qquad V \equiv 1 - \frac{2M(v)}{r}. \qquad (2.3)$$

The Bondi mass aspect $M = M(v)$ is a function of the advanced time v. The ingoing Vaidya metric satisfies the Einstein equation with the energy flux (for Newton constant $G_N = 1$)

$$\bar{T}_{vv} = \frac{M'(v)}{4\pi r^2}, \qquad M' := \partial_v M(v). \qquad (2.4)$$

Null energy condition implies that $M'(v) \geq 0$ which corresponds to ingoing energy flux being absorbed by the black hole. This geometry naturally describes the formation of a black hole by the collapse of matter. However with the sign reversed, $M' < 0$, the metric can also be taken as a model describing the evaporation of black hole by an outgoing energy flux. In this paper, we will determine the Hawking radiation from the Vaidya black hole with soft hair using the tunneling method. We note that in order to utilize the tunneling method, one must employ a metric which is smooth across the location where tunneling occurs, i.e. the horizon. This is suitable for the ingoing Vaidya metric which covers the both the interior and exterior of the black hole; but not for the outgoing Vaidya metric since it covers the exterior region of a black hole and the interior region of a white hole. Therefore we will employ the ingoing Vaidya metric with $M' < 0$ as a model to discuss the evaporation of dynamical black hole due to Hawking radiation.[1]

The supertranslated Vaidya metric $g_{\mu\nu}$ is obtained by acting the vector field ζ_f of (2.2) on the Vaidya metric (2.3),

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + \mathcal{L}_{\zeta_f}\overline{g}_{\mu\nu}. \qquad (2.5)$$

The result is

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(V - \frac{2fM'}{r} - \frac{MD^2f}{r^2}\right)dv^2 + 2dvdr - D_A(2Vf + D^2f)dvd\Theta^A$$
$$+ (r^2\gamma_{AB} + 2rD_AD_Bf - r\gamma_{AB}D^2f)d\Theta^A d\Theta^B. \qquad (2.6)$$

Note that the metric (2.6) can be extended to finite distance and is a solution to the linearized Einstein equations for all r. In this paper, we restrict ourselves to linearized theory in the metric perturbation, or equivalently in f, which means that the back reaction caused by the energy momentum tensor of the gravitational waves and the quantization of gravitational fluctuations will not be considered in our analysis. From the form of (2.6), it is clear that the supertranslated Vaidya spacetime is non-spherical. This may also be seen physically from the fact that the metric (2.6) can be obtained, as we will show in the next subsection, by throwing in a non-spherically symmetric energy momentum flux to the Vaidya

---

[1]Most part of our analysis actually holds true for any sign of M'. For example, with $M' > 0$, the results, (4.20), (4.21) give the influence of the soft hair on the Hawking radiation during the stage the black hole was formed. However the identification (4.22) of the energy loss from Hawking radiation with the change of mass of the black hole holds only in the case of mass loss $M' < 0$.

spacetime. We also remark that unlike the static case of the supertranslated Schwarzschild spacetime, the supertranslated Vaidya spacetime does not have an event horizon. In fact for a dynamical black hole, the concept of event horizon has to be generalized.

In the next section, we will review the definition of dynamical black holes and show that the supertranslated Vaidya spacetime admits a trapping horizon and describes a non-spherical dynamical black hole. As far as we are aware of, (2.6) is the first example of a non-spherical dynamical black hole. Before that, let us first demonstrate that the supertranslated hair configuration (2.6) can be obtained by throwing in a shockwave-like energy momentum flux to the Vaidya black hole.

2.2 Implantation of supertranslation hair

The spacetime (2.6) describes a black hole with supertranslation hair implanted on it. Physically, the configuration of soft hair can be implanted by throwing in an energy flux of a particular form. Consider an ansatz for the perturbed energy momentum tensor,

$$\hat{T}_{vv} = \frac{1}{4\pi r^2}\left(\hat{\mu}(v,\Theta) + \hat{T}(\Theta)\delta(v-v_0)\right) + \frac{1}{4\pi r^3}\left(\hat{T}^{(1)}(\Theta)\delta(v-v_0) + \hat{t}^{(1)}(\Theta)\theta(v-v_0)\right),$$

$$\hat{T}_{vA} = \frac{1}{4\pi r^2}\left(\hat{T}_A(\Theta)\delta(v-v_0) + \hat{t}_A(\Theta)\theta(v-v_0)\right), \tag{2.7}$$

where $\theta(v-v_0)$ is step function. The covariant conservation $\bar{\nabla}^\mu\hat{T}_{\mu\nu} = 0$ imposes that

$$D^A\hat{T}_A(\Theta) = \hat{T}^{(1)}(\Theta), \quad D^A\hat{t}_A(\Theta) = \hat{t}^{(1)}(\Theta), \tag{2.8}$$

where $\bar{\nabla}_\mu$ stands for the covariant derivative with respect to the background Vaidya metric $\bar{g}_{\mu\nu}$. In addition, we consider energy momentum tensor satisfying the relation

$$(D^2 + 2)\hat{T}^{(1)}(\Theta) = -6M(v)\hat{T}(\Theta). \tag{2.9}$$

Now for general asymptotically flat metric (2.1) in the Bondi gauge, the Einstein equations at $\mathcal{I}^-$ give the following constraints at order $\mathcal{O}(r^{-2})$:

$$\partial_v m = \frac{1}{4}D^A D^B \partial_v C_{AB} + 4\pi \lim_{r\to\infty}(r^2 T_{vv}), \tag{2.10}$$

$$\partial_v N_A = vD_A\partial_v m + \frac{1}{4}D^B(D_B D^C C_{AC} - D_A D^C C_{BC}) - 8\pi\lim_{r\to\infty}(r^2 T_{vA}), \tag{2.11}$$

where we neglected quadratic perturbation terms off from flat space. Our claim is that one can reproduce the supertranslated Vaidya metric with an appropriate choice of the energy momentum tensor

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu}. \tag{2.12}$$

To start, the constraint (2.10) can be solved by taking

$$\frac{1}{4}D^A D^B \partial_v C_{AB} = -\hat{T}(\Theta)\delta(v-v_0) \tag{2.13}$$

and

$$\hat{\mu}(v,\Theta) = \partial_v\left(\theta(v-v_0)(\hat{C}M' + \mu)\right), \tag{2.14}$$

where μ is a constant and $\hat{C} = \hat{C}(\Theta)$ is an arbitrary function of angles. Then

$$m = M + \theta(v - v_0)(\hat{C}M' + \mu). \tag{2.15}$$

As for the equation (2.13), let us consider

$$\hat{T}(\Theta) = -\frac{1}{4}D^2(D^2 + 2)\hat{C}, \tag{2.16}$$

which implies, from (2.8) and (2.9), that

$$\hat{T}^{(1)}(\Theta) = \frac{3M}{2}D^2\hat{C}, \qquad \hat{T}_A(\Theta) = \frac{3M}{2}D_A\hat{C}. \tag{2.17}$$

Then (2.13) can be solved and gives

$$C_{AB} = \theta(v - v_0)(2D_AD_B\hat{C} - \gamma_{AB}D^2\hat{C}). \tag{2.18}$$

In turn from (2.11), we obtain

$$N_A = \theta(v - v_0)(vM' - 3M)D_A\hat{C}, \tag{2.19}$$

and

$$\hat{t}_A(\Theta) = M'D_A\hat{C}, \qquad \hat{t}^{(1)}(\Theta) = M'D^2\hat{C}, \tag{2.20}$$

where $\hat{t}^{(1)}$ is obtained by solving (2.8).

The energy momentum tensor $\hat{T}_{\mu\nu}$ is shockwave-like, with a delta function and a step function component. And the corresponding spacetime $g_{\mu\nu}$ can be written as a perturbation over the background Vaidya metric of mass $M(v)$:

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu} \tag{2.21}$$

with the perturbations:

$$h_{vv} = \theta(v - v_0)\left(\frac{2}{r}(\mu + \hat{C}M') + \frac{1}{r^2}MD^2\hat{C}\right),$$
$$h_{vA} = -\theta(v - v_0)D_A\left(V\hat{C} + \frac{1}{2}D^2\hat{C}\right),$$
$$h_{AB} = r\theta(v - v_0)\left(2D_AD_B\hat{C} - \gamma_{AB}D^2\hat{C}\right). \tag{2.22}$$

Like in the static case [15], the perturbation can be written as

$$h_{\mu\nu} = \theta(v - v_0)\left(\mathcal{L}_{f=\hat{C}}\overline{g}_{\mu\nu} + \frac{2\mu}{r}\delta_\mu^v\delta_\nu^v\right). \tag{2.23}$$

Therefore our shockwave-like energy momentum flux creates a supertranslation of the Vaidya metric with the supertranslation parameter $f = \hat{C}$. Also it shifts the mass parameter by a constant amount in case μ is nonzero. Physically it means that during the process of the formation of the dynamical Vaidya black hole, we can also implant a configuration of soft hair on it with the aid of the energy momentum flux $\hat{T}_{\mu\nu}$. Note that the

metric (2.21) actually satisfies the linearized Einstein equations for all r and thus describes linear perturbations not only at around $\mathcal{I}^-$ but also in the interior of spacetime.

The fact that the metric (2.21) holds for all r, especially in the neighborhood of the black hole trapping horizon, has important physical consequences. Below we will extend the consideration of Hawking radiation for dynamical black hole and show that Hawking radiation are created at the dynamical horizon of the soft hair implanted Vaidya black hole. Naturally the Hawking radiation spectrum can be expected to have dependence on the soft charges (superrotation charges) of the black hole. We will show that this is indeed the case.

We end this section by noting that while in this paper we focus on BMS supertranslations as asymptotic symmetries defined at null infinity and its effect on the physics at the black hole horizon, there is a different sort of supertranslations (and superrotations) as asymptotic symmetries defined at black hole horizons, also known as horizon supertranslations, which has been studied for example in [14, 33–40]. Recent work on gravitational memory and quantum mechanical effects associated with the horizon supertranslations on Rindler horizon can be found in [41–43].

3 Properties of supertranslated Vaidya black hole

In this section we investigate some properties of the supertranslated Vaidya black hole which will be helpful for interpreting and understanding of the results of Hawking radiation from the supertranslated Vaidya black hole obtained in the section 4.

3.1 Trapping horizon

The event horizon of a spacetime is a global concept and requires a knowledge of the entire causal structure of spacetime. This is physically impossible unless the spacetime is stationary and nothing changes. In this case one can invoke the theorem of Hawking which says that the event horizon of a stationary asymptotically flat spacetime is a Killing horizon. In addition, using the null Killing vector, one can associate with the Killing horizon a surface gravity which plays the role of temperature in the black hole thermodynamics and Hawking radiation effect.

However the situation becomes much more complicated in the general dynamical case where Killing horizon generally does not exist. Much efforts have been spent on looking for appropriate local definitions of horizon for dynamical black hole spacetime [46–49] (see also [50–54] for reviews). These local horizons are typically defined in terms of trapped surfaces, which are space-like 2-surfaces for which the expansion of the outgoing null rays normal to the surface vanishes. Let us review the precise definition here. Consider a bundle of null geodesics with tangent vector l_-^μ and satisfies $g_{\mu\nu}l_-^\mu l_-^\nu = 0$, $l_-^\mu \nabla_\mu l_-^\nu = 0$. Let us pick another null vector field l_+^μ such that $g_{\mu\nu}l_+^\mu l_+^\nu = 0$ and a relative normalization $l_-{}^\mu l_{+\mu} = -1$. Then the metric in the two-space S orthogonal to l_+ and l_- can be written as

$$q_{\mu\nu} = g_{\mu\nu} + l_{-\mu}l_{+\nu} + l_{-\nu}l_{+\mu}. \tag{3.1}$$

By construction, $l_\pm$ is orthogonal to S: $q_{\mu\nu}l_\pm^\nu = 0$. In general given a null flow characterized by the tangent vector l_μ, one can characterize the flow with a shear, rotation and an expansion part. Of particular interests to general relativity is the expansion scalar which is defined by the divergence of the flow:

$$\theta := q_{\mu\nu}\nabla^\mu l^\nu. \qquad (3.2)$$

Physically, θ measures the expansion rate of the infinitesimally nearby surrounding radial null geodesics [55]: the bundle of null geodesics is expanding if $\theta > 0$ (gravity is not so strong) and contracting if $\theta < 0$. Now a space-like closed and orientable two surface in four-dimensions has two independent normal directions, corresponding to the ingoing and outgoing null rays. It is thus natural to take l_-^μ and l_+^μ to be the tangent vectors of the bundles of ingoing and outgoing null geodesics, and use their behaviour to characterize the gravitational field surrounding S. A normal surface would have $\theta_- < 0$ and $\theta_+ > 0$. A *trapped surface* is one with $\theta_- < 0$ and $\theta_+ < 0$, i.e. the outgoing null rays are contracting at S instead of expanding. The surface is *marginally trapped* if $\theta_- < 0$ and $\theta_+ = 0$, i.e. the outgoing null rays momentarily stop expanding. Trapped surfaces are interesting since under certain physically reasonable assumptions, they lead to the presence of singularities. The cosmic censorship hypothesis then suggests that there must be an event horizon, with the trapped surface located inside of certain black hole horizon. This is a highly non-trivial problem in the dynamical case since when embedded in a dynamical spacetime, the horizon is not expected to be a null hypersurface although it should still exhibit infinite redshift. Various definitions of black hole horizon have been proposed and considered. Among them of particular importance is the future outer trapping horizon (FOTH) introduced by Hayward [46], where later refinements and generalizations are based on. In this paper, we find that the superstranslated Vaidya spacetime is a dynamical black hole with a future outer trapping horizon of Hayward.

A future outer (marginally) trapping horizon is a smooth three-dimensional submanifold of spacetime which is foliated by closed space-like surfaces S_t, $t \in \mathbf{R}$, with null normals $l_\pm$ constructed by ingoing and outgoing rays such that:

$$\theta_+ = 0 \quad \text{(marginally trapped)}, \quad \theta_- < 0 \quad \text{(future type)}, \quad l_-^\mu \partial_\mu \theta_+ < 0 \quad \text{(outer type)}. \qquad (3.3)$$

The first condition in (3.3) specifies the location of marginally trapped surfaces where nearby surrounding radial outgoing null geodesics are parallel. The second condition says that the trapping horizon is of future type, i.e. a black hole rather than a white hole. The third condition says a motion of S_t along l_- makes it trapped, hence it is outer rather than inner kind.

Now let us apply these concepts to the superstranslated Vaidya spacetime. Let us consider radial ingoing and outgoing null geodesics. The corresponding tangent vectors are

$$l_-^\mu = (0, -1, 0, 0), \qquad l_+^\mu = \left(1, \frac{1}{2}\left(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\right), 0, 0\right). \qquad (3.4)$$

Using (2.6), (3.1) and (3.4), we get

$$\theta_- = -\frac{2}{r}, \qquad \theta_+ = \frac{1}{r}\left(V - \frac{2fM'}{r} - \frac{MD^2f}{r^2}\right),$$ (3.5)

where we have discarded terms of $\mathcal{O}(f^2)$. For the supertranslated Vaidya black hole, the first condition in (3.3) yields[2]

$$r = r_h = 2M + 2fM' + \frac{1}{2}D^2f.$$ (3.6)

In our calculation, f is taken as a small perturbation and the f dependent terms give correction to the Schwarzschild radius. For consistency, we will consider the case where the corrections are small such that the first term dominates, $M \gg |M'f|, |D^2f|$, and so $r_h > 0$. The second condition in (3.3) is trivially satisfied. The third condition yields

$$-\frac{1}{2r_hM}\left(1 - f\frac{M'}{M}\right) < 0,$$ (3.7)

which is satisfied for the same assumption as above. The condition (3.7) is actually equivalent to a positive horizon surface gravity which we will discuss in the next section. Therefore, a trapping horizon foliated by marginally trapped surfaces (3.6) is of future and outer type and is a FOTH of the supertranslated Vadiya black hole. A FOTH is in general not null but it still has infinite red shift. In fact, substituting (3.6) into (2.6) and fixing angular coordinates $d\Theta^A = 0$ gives

$$ds^2 = 4\left(M' + fM'' + \frac{1}{4}M'D^2f\right)dv^2.$$ (3.8)

Hence the FOTH of supertranslated Vaidya black hole at a fixed angular coordinates is time-like if $M' < -fM'' - \frac{1}{4}M'D^2f$ for which the null energy condition will be violated and space-like if $M' > -fM'' - \frac{1}{4}M'D^2f$. For the stage of evaporation the black hole mass will be decreasing $M' < 0$ due to ingoing negative energy flux and the FOTH would be time-like.

3.2 Surface gravity

Next we discuss the horizon surface gravity associated with the trapping horizon of the supertranslated Vaidya black hole.

For stationary black hole, surface gravity is defined in terms of the null Killing vector of the Killing horizon. In a time-dependent spacetime, there is generally no asymptotically time-like Killing vector to define a preferred time coordinate. As a result, quantities such as four-acceleration and surface gravity cannot be defined unambiguously. Instead, depending on the local definition of horizons that is adopted, one may define a notion of surface gravity

---

[2]Actually $\theta_+ = 0$ has two solutions. One is given by (3.6) and corresponds to a marginally trapped surface of outer type as discussed above. The other solution $r = -\frac{1}{2}D^2f$ corresponds to a marginally trapped surface of inner type with $l^\mu\partial_\mu\theta_+ > 0$, whose foliation gives a future inner trapping horizon if $r = -\frac{1}{2}D^2f > 0$.

correspondingly, see e.g. [54, 56, 57]. For the FOTH of Hayward, there is a quite explicit and satisfactory definition of surface gravity for the spherically symmetric case. In fact, for any spherically symmetric metric, one can show the existence of a unique vector field K^μ, called the Kodama vector, which satisfies [58]

$$\nabla^\nu(G_{\mu\nu}K^\mu) = 0, \qquad \nabla_\mu K^\mu = 0. \tag{3.9}$$

Here $G_{\mu\nu}$ is the Einstein tensor. Kodama vector gives a preferred time direction for dynamical spacetimes in spherical symmetry. In an asymptotically flat spacetime, with an appropriate normalization, the Kodama vector coincides with the time translation Killing vector at spatial infinity. Hayward has shown that the following relation holds for Kodama vector [59],

$$K^\mu\nabla_{[\nu}K_{\mu]} = -\kappa K_\nu \tag{3.10}$$

where the equality holds when it is evaluated on a trapping horizon. The coefficient κ defines the horizon surface gravity of the FOTH of Hayward. Kodama vector coincides with the time translation Killing vector field for stationary solutions, and the defining equation for surface gravity (3.10) reduces to that of Killing surface gravity in static limit by virtue of Killing equation and thus can be used to define surface gravity κ for dynamical black holes with spherical symmetry. For non-spherical dynamical black holes, it is not known in general whether the Kodama vector exists. However if it exists, we can use (3.10) to define the surface gravity.

For a spherically symmetric Vaidya spacetime, the Kodama vector is $\bar{K}^\mu = \delta^\mu_v$ and the horizon surface gravity of Vaidya black hole defined by (3.10) is $\bar{\kappa} = \frac{1}{4M(v)}$. We would like to extend the definition of surface gravity to the supertranslated Vaidya black hole case using the Kodama vector. For the supertranslated Vaidya metric $g_{\mu\nu}$ (2.6), although it is not spherically symmetric, we find that

$$K^\mu = \delta^\mu_v, \qquad K_\mu = g_{\mu\nu}K^\nu, \tag{3.11}$$

solves the equation (3.9) up to $\mathcal{O}(f^2)$ term. Evaluated on the FOTH, the surface gravity in (3.10) is found to be

$$\kappa = \frac{1}{4M}\left(1 - f\frac{M'}{M}\right), \tag{3.12}$$

at the linear order in f. Note that the Kodama-like vector (3.11) coincides with the time-translation Killing vector for the supertranslated Schwarzschild in static limit. In the supertranslated case, the horizon surface gravity of the supertranslated Vaidya black hole has a dependence on the supertranslation hair and is not a constant on the FOTH. Physically this means that the black hole does not correspond to a system in thermal equilibrium. In general one cannot define a specific temperature for a nonequilibrium system. Nevertheless one may still define a temperature locally provided that the system is in equilibrium locally. We propose to interpret κ as a local measure of the temperature of the superstranslated Vaidya black hole. As we mentioned above, the outer trapping horizon corresponds to the positive surface gravity κ given by (3.12). The inner trapping horizon would correspond to $\kappa < 0$ and the degenerate (extremal) one corresponds to $\kappa = 0$. It is interesting to think about what these thermodynamical notions mean in terms of spacetime physics.

4 Hawking radiation from supertranslated Vaidya black hole

4.1 Hamilton-Jacobi method

Now let us consider Hawking radiation from the supertranslated Vaidya black hole. As argued by Parikh and Wilczek [32], the Hawking radiation can be computed as a tunneling process based on null geodesic motion of particle in the black hole geometry. That this is possible without using the full-fledged quantum field theory is because of the huge red shift factor at the horizon of the black hole. As a result, Hawking radiation observed at asymptotic infinity arises from emitted wave with vanishing wavelength near the horizon, and therefore, as far as the tunneling process is of concern, a point particle approximation near the horizon is good. We refer the readers to [57] for a review on the tunneling methods and Hawking radiation. In addition to the original method of null geodesic, the Hamilton-Jacobi method has also been developed to compute the Hawking radiation of black hole. It has been shown for stationary black hole spacetimes that the two methods provide the same result for the semi-classical emission rate at the leading order of the energy of the emitted particles [60]. While the null geodesic method is based on the self-gravitation (backreaction) of emitted particles and the energy conservation of the whole spacetime, the Hamilton-Jacobi method would be simple to compute the imaginary part of the particle action without reference to the self-gravitation of emitted particles. Also covariance of the method is well understood in the Hamilton-Jacobi equation. Moreover, the Hamilton-Jacobi method can be applied to either stationary or dynamical black holes [61–63] so that it is suitable for our purpose. We will therefore adopt the Hamilton-Jacobi method to compute the Hawking radiation spectrum for the supertranslated Vaidya black hole.

Consider now, for example, a minimally coupled massless scalar particles, $\Box\phi(x) = 0$. We look for a solution to this equation by a WKB ansatz $\phi = A(x)e^{iS/\hbar} + \mathcal{O}(\hbar)$. At the leading non-trivial order of $\hbar$, the Klein-Gordon (KG) equation reduces to the Hamilton-Jacobi equation

$$g^{\mu\nu}\partial_\mu S\partial_\nu S = 0. \tag{4.1}$$

Generally there are two solutions to this equation and they correspond to the two different solutions of the KG equation. Physically, they describe motion of a particle getting out (outgoing solution) and falling in to the black hole (ingoing solution). The motion of particle is given by a null geodesic in the black hole background. One can then reconstruct the particle action by a line integral

$$S = \int_P dx^\mu \partial_\mu S, \tag{4.2}$$

where P is the null geodesic. S is real in general, but can become complex if the trajectory is a classically forbidden one. Since particles can fall into a black hole along a classically permitted path, we expect S to possibly become imaginary only for the outgoing solution. In the case of static black holes, this occurs for the outgoing solution along a trajectory crossing the event horizon which we call a tunneling path, and the imaginary contribution arises from the residue of the pole in $\partial_\mu S$ at the location of horizon. To leading order in

$\hbar$, the semiclassical emission rate Γ_{em} is given by the WKB formula

$$\Gamma_{\text{em}} \propto \exp\left(-2\,\text{Im}S_{\text{out}}\right), \tag{4.3}$$

where S_{out} denotes the particle action for the outgoing solution and we have set $\hbar = 1$. For static black hole spacetimes, it is easy to show that the semiclassical emission rate satisfies the detailed balance relation [31, 64] $\Gamma_{\text{em}} = e^{-\beta\omega}\Gamma_{\text{ab}}$ where Γ_{ab} is the semiclassical absorption rate and ω is the energy of the emitted particles. This allows one to have a thermal interpretation of the result with β^{-1} being the (Hawking) temperature of the system.

Let us consider the semiclassical emission rate for supertranslated Schwarzschild black hole. In the advanced Bondi coordinates, the metric reads [15]

$$ds^2 = -\left(V - \frac{MD^2f}{r^2}\right)dv^2 + 2dvdr - dvd\Theta^A D_A(2Vf + D^2f)$$
$$+ \left(r^2\gamma_{AB} + 2rD_A D_B f - r\gamma_{AB}D^2 f\right)d\Theta^A d\Theta^B, \tag{4.4}$$

where $V = 1 - \frac{2M}{r}$. This coordinate system has a nice feature for tunneling computation since it is non-singular at the event horizon $r = 2M + \frac{1}{2}D^2 f$. As shown by [15], the metric can be extended to the interior and is still a solution of the Einstein equations. We can see that this coordinate system covers both interior and exterior regions of a black hole and hence it is an appropriate coordinate system to compute the semiclassical emission rate for black holes. On the other hand, retarded Bondi coordinates would be suitable for the computation of the semiclassical absorption rate for white holes. We consider radial null geodesics in $(v - r)$ plane for which the Hamilton-Jacobi equation is

$$\left(V - \frac{MD^2f}{r^2}\right)(\partial_r S)^2 - 2\omega\partial_r S = 0, \tag{4.5}$$

where

$$\omega = -\xi^\mu p_\mu = -\xi^\mu \partial_\mu S = -\partial_v S. \tag{4.6}$$

ξ is the time-translation Killing vector field $\xi = \partial_v$ and p_μ is the 4-momentum of particle. ω is the energy of particle and is a constant in motion. The solutions to (4.5) are

$$\partial_r S_{\text{out}} = \frac{2\omega}{V - \frac{MD^2f}{r^2}}, \qquad \partial_r S_{\text{in}} = 0, \tag{4.7}$$

where S_{out} corresponds to the outgoing solution ($\partial_r S > 0$) for $r > 2M$, and S_{in} to the ingoing solution. Using (4.2), the corresponding action is

$$S_{\text{out}} = -\int \omega dv + \int \frac{2\omega dr}{V - \frac{MD^2f}{r^2}}, \qquad S_{\text{in}} = -\int \omega dv. \tag{4.8}$$

As mentioned above, the ingoing action does not have an imaginary part since absorption is complete for black holes in the classical limit [65]. The integrand for S_{out} has poles at, up to order f terms, $r = r_h := 2M + \frac{1}{2}D^2 f$ which is the location of the (shifted) event horizon, and at $r = r_f := -\frac{1}{2}D^2 f$.
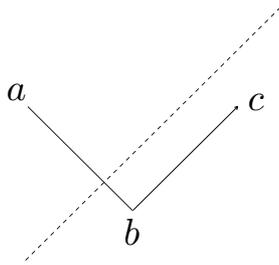
Figure 1. A trajectory from inside to outside of the black hole. The dashed line represents the event horizon.

To obtain S_{out}, we need to perform the integration for along a trajectory of motion. As we consider s-wave motion, there are two types of path:

$$dv = 0, \qquad\qquad\qquad \text{(type-I)}$$
$$\frac{dr}{dv} = -\frac{1}{2}\left(V - \frac{MD^2 f}{r^2}\right), \qquad \text{(type-II)} \tag{4.9}$$

In figure 1, we show a trajectory $\overrightarrow{abc}$ which contains a path $\overrightarrow{ab}$ moving backward in time and crossing the horizon. This part is described by a type-I path. After tunneling out from the horizon, the emitted particle escapes to infinity on a type-II path and would be observed as Hawking radiation.[3] It is easy to see that S_{out} vanishes on a type-II path. For a type-I path, we have

$$S_{\text{out}} = \int_{\text{I}} \frac{2\omega r^2 dr}{(r - r_h)(r - r_f)}. \tag{4.10}$$

As our path crosses the horizon $r = r_h$, the integral is divergent. Adopting a Feynman's $i\epsilon$ prescription to deform the integral,[4] the imaginary part of S_{out} is

$$\text{Im}\, S_{\text{out}} = \text{Im} \int_{a \to b} \frac{2\omega r^2 dr}{(r - r_h - i\epsilon)(r - r_f)} = 4\pi\omega M, \tag{4.11}$$

dropping terms of order f^2 or higher. The semiclassical emission rate is thus obtained as

$$\Gamma_{\text{em}} \propto \exp(-2\text{Im}S_{\text{out}}) = \exp(-8\pi\omega M). \tag{4.12}$$

---

[3]This tunneling process can also be interpreted in a different way. A pair of virtual particles is created near b. One with negative energy falls into the black hole and the other with positive energy escapes to infinity [64].

[4]Here $i\epsilon$ is introduced in such a way that the positive energy exponential function decays and the relation $\Gamma_{\text{em}} = e^{-8\pi\omega M}\Gamma_{\text{ab}}$ for black hole semiclassical emission rate is obtained [31]. If $i\epsilon$ is introduced with the opposite sign, one obtains a result which would be regarded as that for the time reversed one $\Gamma_{\text{ab}} = e^{-8\pi\omega M}\Gamma_{\text{em}}$ for white hole semiclassical absorption rate. In [57], it is discussed that the $i\epsilon$ prescription is consistent with the analytic continuation when spacetime allows the Kruskal extension as in [64]. $i\epsilon$ prescription should correspond to boundary conditions on the wave function and choice of vacuum while in the tunneling method with the WKB approximation identification of the positive frequency solutions is usually implicit in the coordination and in making ansatz for the solution to the Hamilton-Jacobi equation [66].

Identifying Γ_{em} with the Boltzmann factor, $\exp(-\omega/T)$, we can read off the Hawking temperature as $T = \frac{\kappa_s}{2\pi}$ with $\kappa_s = \frac{1}{4M}$ the surface gravity of Schwarzschild black hole. Note that in this computation, only the infinitesimal region across the trapping horizon contributes to the result. Note also that the Hawking radiation in this case is independent of the soft hair features of the black hole.

4.2 Supertranslated Vaidya black hole

In the above, we find that the Hawking radiation is insensitive to the features of soft hair of a stationary Schwarzschild black hole. Next, we would like to consider the process of black hole evaporation and compute the Hawking radiation of this dynamical black hole; and investigate how the soft hair would affect the Hawking radiation. The black hole evolution process can be modeled with a supertranslated Vaidya spacetime with a time-dependent mass $M(v)$ where v is the advanced null coordinate. The decrease of the black hole mass due to the ingoing negative energy flux can be interpreted in the tunneling method as one having the negative energy particle of a virtual particle pair created near the FOTH falls into the black hole. Here we effectively incorporate this back reaction effect of ingoing Hawking radiation by the diminishing behaviour of the black hole mass $M(v)$ of the background geometry with $M' < 0$ during the evaporation process.

For the supertranslated Vaidya metric, the Hamilton-Jacobi method can be applied in a similar way to the previous example of Schwarzschild black hole. However there are important differences. We start with the Hamilton-Jacobi equation (4.1) with the metric given by (2.6).

$$2\partial_r S \partial_v S + \left(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2} \right) (\partial_r S)^2$$
$$+ \frac{1}{r^2} D^A (2Vf + D^2 f) \partial_r S \partial_A S + \frac{1}{r^4} \left(r^2 \gamma^{AB} - 2r D^A D^B f + r \gamma^{AB} D^2 f \right) \partial_A S \partial_B S = 0. \quad (4.13)$$

Here we define the invariant energy of particle ω in favor of the Kodama-like vector which generalizes the particle energy in the static case.

$$\omega \equiv -K^\mu p_\mu = -K^\mu \partial_\mu S = -\partial_v S. \quad (4.14)$$

(4.13) is rather complicated to solve in general. The simplest possibility is to consider radial null geodesics with $\Theta^A = $ const. along the geodesics. In this case, the Hamilton-Jacobi equation has simple solutions

$$\partial_r S_{\text{out}} = 2\omega \left(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2} \right)^{-1}, \qquad \partial_r S_{\text{in}} = 0. \quad (4.15)$$

And the particle action is reconstructed as

$$S_{\text{out}} = -\int \omega dv + \int \frac{2\omega r^2 dr}{Vr^2 - 2fM'r - MD^2 f}, \qquad S_{\text{in}} = -\int \omega dv. \quad (4.16)$$

As before, S_{in} has no imaginary contribution since the energy is real. As for S_{out}, the line integral contribution (4.16) to S_{out} is real for most part of the path except possibly
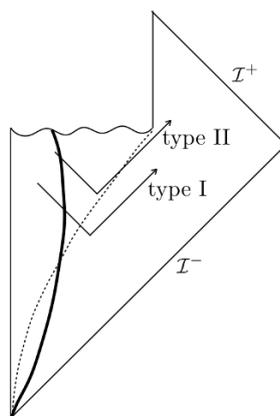
Figure 2. Typical Penrose diagram of the supertranslated Vaidya black hole at a fixed value of Θ^A. The dashed line represents the FOTH $r = r_h$, the wavy line represents the singularity $r = 0$ and the thick curve represents a collapsing matter.

at around the location where the integrand diverges, which occurs at the location of the trapping horizon. However since now the trapping horizon is dynamical, it is no longer null. This leads to two types of tunneling paths, depending on how the path crosses the trapping horizon. See figure 2. On the segment of the geodesic which crosses the trapping horizon, they are described by the type-I and type-II paths respectively:

$$v = \text{const.}, \qquad\qquad (\text{type-I}), \qquad\qquad (4.17)$$

$$\frac{dr}{dv} = \frac{1}{2}\left(V - \frac{2fM'}{r} - \frac{MD^2f}{r^2}\right), \qquad (\text{type-II}). \qquad (4.18)$$

The two types of tunneling paths correspond to whether the virtual pair is created inside or outside of the trapping horizon. The crossing with type-I path corresponds to a pair forming outside and describes a backward radial null ray which comes out from the future singularity at $r = 0$, crosses the trapping horizon and escapes to infinity. The crossing with type-II path corresponds to a pair forming inside and describes a backward null ray which comes out from the future singularity, and reaches some interior point in the trapped region, and then crosses the trapping horizon and eventually escapes to infinity. The main difference between the two types of tunneling paths is that type-I tunneling path crosses the FOTH along a classically forbidden trajectory backward in time, while type-II tunneling path crosses the FOTH along a classically allowed trajectory forward in time. Note that the crossing with type-II path is absent for a static black hole.

Let us now compute $\text{Im}S_{\text{out}}$. It is clear that S_{out} vanishes and no tunneling occurs for type-II paths. For type-I paths, we have (4.10) with r_h given by (3.6) and it is clear that the integrand has a pole on the FOTH $r = r_h$. The imaginary contribution due to the pole is evaluated by an $i\epsilon$ prescription as before and yields

$$\text{Im}S_{\text{out}} = \frac{2\pi\omega r_h^2}{r_h - r_f} = \frac{\pi\omega}{\kappa}, \qquad\qquad (4.19)$$

– 15 –

where ω is evaluated on the FOTH for a fixed v and Θ^A. In the last equality of (4.19), we have used (3.12) to write the result in terms of the horizon surface gravity κ,

$$\kappa = \frac{r_h - r_f}{r_h^2} = \frac{1}{4M}\left(1 - f\frac{M'}{M}\right).$$
(4.20)

The semiclassical emission rate Γ_{em} is thus formally written in the same form as that in the static case.

$$\Gamma_{\text{em}} \propto e^{-2\text{Im}S_{\text{out}}} = \exp\left(-\frac{2\pi\omega}{\kappa}\right).$$
(4.21)

We find that the supertranslation $f(\Theta)$ dependence comes into the spectrum of Hawking radiation through the surface gravity κ of the FOTH.

Note that our result (4.21) of the emission rate is consistent with the factorization property of the black hole S-matrix [26–29]. The non-trivial dependence on the soft hair in (4.21) is due to the fact that we are performing an observation of the undressed hard Hawking radiation quanta. If one is to dress the hard modes with soft gravitons, then the soft hair dependence would disappear. This is similar to the discussion of gravitational memory where one may also remove the memory with dressing [29]. It should be clear that the factorization of soft modes and hard modes in the S-matrix does not imply that the soft hair has no physical implication at all. The effects of soft hair on gravitational memory and Hawking radiation spectrum is physical and is observable.

So far we have left M' free and determined the Hawking radiation in terms of it. In a consistent model of evaporation, the mass loss [3, 67] is caused by the Hawking radiation. The leading contribution to the mass loss of the supertranslated Vaidya black hole can be estimated from the Hawking radiation spectrum as:

$$-M' = \frac{1}{2\pi^2}\int\left(\int_0^\infty \frac{\omega^3}{e^{\beta\omega}-1}d\omega\right) r_h^2\sqrt{\gamma}d^2\Theta = \frac{\pi^2}{30}\int\frac{r_h^2}{\beta^4}\sqrt{\gamma}d^2\Theta$$
$$= \left(1 - \frac{a_{00}}{\sqrt{\pi}}\frac{M'}{M}\right)P_0,$$
(4.22)

where $\beta := 2\pi/\kappa$ and $P_0 := (7680\pi M^2)^{-1}$ is the standard power loss due to Hawking radiation in the leading order approximation of a constant M. Note that non-trivial angle-dependent supertranslations corresponding to $l \geq 1$ do not contribute to the mass loss in the case of supertranslated Vadiya black hole with $M = M(v)$. Solving (4.22) for M' we get

$$M' = -P_0\left(1 - \frac{a_{00}P_0}{\sqrt{\pi}M}\right)^{-1}.$$
(4.23)

With this value of M', we obtain a consistent model of dynamical black hole whose evaporation is driven by the power loss due to Hawking radiation.

Let us discuss the entropy of the supertranslated Vaidya black hole. A proposal for dynamical black hole entropy is presented by Iyer and Wald [68]. For Einstein gravity with matter minimally coupled to gravity, the entropy of dynamical black hole is given by the

same formula $S_{\text{dyn}} = A_h/4$ as for stationary black holes. We find

$$S_{\text{dyn}} = \frac{1}{4}\int r_h^2 \sqrt{\gamma} d^2\Theta$$
$$= 4\pi M^2 + 4\sqrt{\pi}a_{00}MM'. \tag{4.24}$$

The entropy (4.24) can be expressed as $S_{\text{dyn}} = 4\pi M^2(v + a_{00}/\sqrt{4\pi})$ and it is consistent with the fact that only the zero mode part of the shift of time can appear in the mass function.

When the black hole emits a massless Hawking radiation, the black hole mass changes by amount of ω, $M \to M - \omega$ due to the energy conservation, and the entropy changes accordingly $S_{\text{dyn}} \to S_{\text{dyn}} + \Delta S$ with

$$\Delta S = -\omega(8\pi M + 4\sqrt{\pi}a_{00}M') + \mathcal{O}(\omega^2). \tag{4.25}$$

This can be related to the semiclassical emission rate (4.21) as

$$\Delta S = \int \frac{d\Omega}{4\pi}\ln\Gamma_{\text{em}}. \tag{4.26}$$

This generalizes the standard relation for spherical black holes [32, 57]. Furthermore, one may define a differential entropy change:

$$\frac{d\Delta S}{d\Omega} = \ln\Gamma_{\text{em}} \tag{4.27}$$

and interpret the relation (4.26) as saying that the Hawking radiation carries away from the black hole different amount of entropy (4.27) at different angles. As Hawking radiation originates locally at the surface of the trapping horizon, it is consistent that a local change of entropy of the black hole occurs.

5 Conclusion and discussion

Supertranslation of black hole adds soft hair to it. These soft hairs are physical and can be observed through the classical memory effect. In this paper, we show that soft hairs also have non-trivial effects on the quantum physics of black hole. In particular we computed the Hawking radiation for a dynamical black hole modeled by the Vaidya spacetime with soft hair. We find that tunneling occurs at the trapping horizon and the semiclassical emission rate of Hawking radiation is characterized by the horizon surface gravity κ defined by the Kodama-like vector of the supertranslated Vaidya black hole. The Hawking radiation spectrum has a dependence on the soft hair distribution over the black hole. Our results make it clear that soft hair of black hole is physical and it has clear observable effects on the Hawking radiation spectrum. Of course this depends on what is being observed. If one wishes, one may also choose to make detection of the dressed Hawking quanta, then the observed spectrum would become the canonical one without any sight of the soft hair.

Recently Strominger [16] has emphasized the presence of soft modes in the final state of the black hole evaporation and proposed that the final state of the black hole evaporation process to be a pure state of the form

$$|\Psi\rangle = \sum_\alpha c_\alpha |H_\alpha\rangle|S_\alpha\rangle, \tag{5.1}$$

where $|H_\alpha\rangle$ describes the state of radiation in the thermal ensemble of Hawking radiation and $|S_\alpha\rangle$ describes the cloud of soft modes that is required by charge conservation. In other words, the purity of the quantum state is restored by the soft graviton modes. However if only the hard modes of the Hawking radiation are to be observed, the resulting reduced density matrix $\rho_r = \mathrm{tr}_{\mathrm{soft}}|\Psi\rangle\langle\Psi| = \rho_{\mathrm{thermal}}$ gives the thermal density matrix of Hawking radiation. Our result is consistent with this. The loss of coherence when "environmental variables" are traced out is not special and can be expected generically. Indeed, similar decoherence effect of hard particles as a result of the tracing out of unobservable soft modes has been demonstrated for certain kind of events in QED [69, 70]. It is important to understand deeper how (5.1) is related to the BMS symmetry and the infrared structure of gravity.

Acknowledgments

References

[1] J.D. Bekenstein, *Black holes and entropy*, *Phys. Rev.* **D 7** (1973) 2333 [ɪɴSPIRE].

[2] S.W. Hawking, *Black hole explosions*, *Nature* **248** (1974) 30 [ɪɴSPIRE].

[3] S.W. Hawking, *Particle Creation by Black Holes*, *Commun. Math. Phys.* **43** (1975) 199 [*Erratum ibid.* **46** (1976) 206] [ɪɴSPIRE].

[4] R.M. Wald, *Black hole entropy is the Noether charge*, *Phys. Rev.* **D 48** (1993) R3427 [gr-qc/9307038] [ɪɴSPIRE].

[5] P.T. Chrusciel, J. Lopes Costa and M. Heusler, *Stationary Black Holes: Uniqueness and Beyond*, *Living Rev. Rel.* **15** (2012) 7 [arXiv:1205.6112] [ɪɴSPIRE].

[6] S.D. Mathur, *The Information paradox: A Pedagogical introduction*, *Class. Quant. Grav.* **26** (2009) 224001 [arXiv:0909.1038] [ɪɴSPIRE].

[7] D. Marolf, *The Black Hole information problem: past, present and future*, *Rept. Prog. Phys.* **80** (2017) 092001 [arXiv:1703.02143] [ɪɴSPIRE].

[8] A. Strominger, *On BMS Invariance of Gravitational Scattering*, *JHEP* **07** (2014) 152 [arXiv:1312.2229] [INSPIRE].

[9] T. He, V. Lysov, P. Mitra and A. Strominger, *BMS supertranslations and Weinberg's soft graviton theorem*, *JHEP* **05** (2015) 151 [arXiv:1401.7026] [INSPIRE].

[10] F. Cachazo and A. Strominger, *Evidence for a New Soft Graviton Theorem*, arXiv:1404.4091 [INSPIRE].

[11] D. Kapec, V. Lysov, S. Pasterski and A. Strominger, *Semiclassical Virasoro symmetry of the quantum gravity S-matrix*, *JHEP* **08** (2014) 058 [arXiv:1406.3312] [INSPIRE].

[12] A. Strominger and A. Zhiboedov, *Gravitational Memory, BMS Supertranslations and Soft Theorems*, *JHEP* **01** (2016) 086 [arXiv:1411.5745] [INSPIRE].

[13] A. Strominger, *Lectures on the Infrared Structure of Gravity and Gauge Theory*, arXiv:1703.05448 [INSPIRE].

[14] S.W. Hawking, M.J. Perry and A. Strominger, *Soft Hair on Black Holes*, *Phys. Rev. Lett.* **116** (2016) 231301 [arXiv:1601.00921] [INSPIRE].

[15] S.W. Hawking, M.J. Perry and A. Strominger, *Superrotation Charge and Supertranslation Hair on Black Holes*, *JHEP* **05** (2017) 161 [arXiv:1611.09175] [INSPIRE].

[16] A. Strominger, *Black Hole Information Revisited*, arXiv:1706.07143 [INSPIRE].

[17] H. Bondi, M.G.J. van der Burg and A.W.K. Metzner, *Gravitational waves in general relativity. 7. Waves from axisymmetric isolated systems*, *Proc. Roy. Soc. Lond.* **A 269** (1962) 21 [INSPIRE].

[18] R.K. Sachs, *Gravitational waves in general relativity. 8. Waves in asymptotically flat space-times*, *Proc. Roy. Soc. Lond.* **A 270** (1962) 103 [INSPIRE].

[19] R.L. Arnowitt, S. Deser and C.W. Misner, *The Dynamics of general relativity*, *Gen. Rel. Grav.* **40** (2008) 1997 [gr-qc/0405109] [INSPIRE].

[20] R.M. Wald and A. Zoupas, *A General definition of 'conserved quantities' in general relativity and other theories of gravity*, *Phys. Rev.* **D 61** (2000) 084027 [gr-qc/9911095] [INSPIRE].

[21] G. Barnich and C. Troessaert, *BMS charge algebra*, *JHEP* **12** (2011) 105 [arXiv:1106.0213] [INSPIRE].

[22] Ya.B. Zel'dovich and A.G. Polnarev, *Radiation of gravitational waves by a cluster of superdense stars*, Sov. Astron. **18** (1974) 17.

[23] V.B. Braginsky and L.P. Grishchuk, *Kinematic Resonance and Memory Effect in Free Mass Gravitational Antennas*, Sov. Phys. JETP **62** (1985) 427 [INSPIRE].

[24] V.B. Braginsky and K.S. Thorne, *Gravitational-wave bursts with memory and experimental prospects*, *Nature* **327** (1987) 123.

[25] L. Blanchet and T. Damour, *Hereditary effects in gravitational radiation*, *Phys. Rev.* **D 46** (1992) 4304 [INSPIRE].

[26] M. Mirbabayi and M. Porrati, *Dressed Hard States and Black Hole Soft Hair*, *Phys. Rev. Lett.* **117** (2016) 211301 [arXiv:1607.03120] [INSPIRE].

[27] B. Gabai and A. Sever, *Large gauge symmetries and asymptotic states in QED*, *JHEP* **12** (2016) 095 [arXiv:1607.08599] [INSPIRE].

[28] C. Gomez and M. Panchenko, *Asymptotic dynamics, large gauge transformations and infrared symmetries*, `arXiv:1608.05630` [InSPIRE].

[29] R. Bousso and M. Porrati, *Soft Hair as a Soft Wig*, *Class. Quant. Grav.* **34** (2017) 204001 [`arXiv:1706.00436`] [InSPIRE].

[30] W. Donnelly and S.B. Giddings, *How is quantum information localized in gravity?*, *Phys. Rev.* **D 96** (2017) 086013 [`arXiv:1706.03104`] [InSPIRE].

[31] K. Srinivasan and T. Padmanabhan, *Particle production and complex path analysis*, *Phys. Rev.* **D 60** (1999) 024007 [`gr-qc/9812028`] [InSPIRE].

[32] M.K. Parikh and F. Wilczek, *Hawking radiation as tunneling*, *Phys. Rev. Lett.* **85** (2000) 5042 [`hep-th/9907001`] [InSPIRE].

[33] J.-i. Koga, *Asymptotic symmetries on Killing horizons*, *Phys. Rev.* **D 64** (2001) 124012 [`gr-qc/0107096`] [InSPIRE].

[34] M. Hotta, K. Sasaki and T. Sasaki, *Diffeomorphism on horizon as an asymptotic isometry of Schwarzschild black hole*, *Class. Quant. Grav.* **18** (2001) 1823 [`gr-qc/0011043`] [InSPIRE].

[35] L. Donnay, G. Giribet, H.A. Gonzalez and M. Pino, *Supertranslations and Superrotations at the Black Hole Horizon*, *Phys. Rev. Lett.* **116** (2016) 091101 [`arXiv:1511.08687`] [InSPIRE].

[36] A. Averin, G. Dvali, C. Gomez and D. Lüst, *Gravitational Black Hole Hair from Event Horizon Supertranslations*, *JHEP* **06** (2016) 088 [`arXiv:1601.03725`] [InSPIRE].

[37] C. Eling and Y. Oz, *On the Membrane Paradigm and Spontaneous Breaking of Horizon BMS Symmetries*, *JHEP* **07** (2016) 065 [`arXiv:1605.00183`] [InSPIRE].

[38] L. Donnay, G. Giribet, H.A. González and M. Pino, *Extended Symmetries at the Black Hole Horizon*, *JHEP* **09** (2016) 100 [`arXiv:1607.05703`] [InSPIRE].

[39] R.-G. Cai, S.-M. Ruan and Y.-L. Zhang, *Horizon supertranslation and degenerate black hole solutions*, *JHEP* **09** (2016) 163 [`arXiv:1609.01056`] [InSPIRE].

[40] H. Afshar, D. Grumiller, W. Merbis, A. Perez, D. Tempo and R. Troncoso, *Soft hairy horizons in three spacetime dimensions*, *Phys. Rev.* **D 95** (2017) 106005 [`arXiv:1611.09783`] [InSPIRE].

[41] M. Hotta, J. Trevison and K. Yamaguchi, *Gravitational Memory Charges of Supertranslation and Superrotation on Rindler Horizons*, *Phys. Rev.* **D 94** (2016) 083001 [`arXiv:1606.02443`] [InSPIRE].

[42] S. Kolekar and J. Louko, *Gravitational memory for uniformly accelerated observers*, *Phys. Rev.* **D 96** (2017) 024054 [`arXiv:1703.10619`] [InSPIRE].

[43] S. Kolekar and J. Louko, *Quantum memory for Rindler supertranslations*, `arXiv:1709.07355` [InSPIRE].

[44] G. Barnich and C. Troessaert, *Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited*, *Phys. Rev. Lett.* **105** (2010) 111103 [`arXiv:0909.2617`] [InSPIRE].

[45] R. Sachs, *Asymptotic symmetries in gravitational theory*, *Phys. Rev.* **128** (1962) 2851 [InSPIRE].

[46] S.A. Hayward, *General laws of black hole dynamics*, *Phys. Rev.* **D 49** (1994) 6467 [InSPIRE].

[47] A. Ashtekar, C. Beetle and S. Fairhurst, *Isolated horizons: A generalization of black hole mechanics*, *Class. Quant. Grav.* **16** (1999) L1 [`gr-qc/9812065`] [InSPIRE].

[48] A. Ashtekar and B. Krishnan, *Dynamical horizons: Energy, angular momentum, fluxes and balance laws*, *Phys. Rev. Lett.* **89** (2002) 261101 [gr-qc/0207080] [inSPIRE].

[49] I. Booth and S. Fairhurst, *The First law for slowly evolving horizons*, *Phys. Rev. Lett.* **92** (2004) 011102 [gr-qc/0307087] [inSPIRE].

[50] A. Ashtekar and B. Krishnan, *Isolated and dynamical horizons and their applications*, *Living Rev. Rel.* **7** (2004) 10 [gr-qc/0407042] [inSPIRE].

[51] I. Booth, *Black hole boundaries*, *Can. J. Phys.* **83** (2005) 1073 [gr-qc/0508107] [inSPIRE].

[52] S.A. Hayward, *Dynamics of black holes*, arXiv:0810.0923 [inSPIRE].

[53] E. Gourgoulhon and J.L. Jaramillo, *New theoretical approaches to black holes*, *New Astron. Rev.* **51** (2008) 791 [arXiv:0803.2944] [inSPIRE].

[54] V. Faraoni, *Evolving black hole horizons in General Relativity and alternative gravity*, *Galaxies* **1** (2013) 114 [arXiv:1309.4915] [inSPIRE].

[55] R.M. Wald, *General Relativity*, University of Chicago Press (1984).

[56] A.B. Nielsen and J.H. Yoon, *Dynamical surface gravity*, *Class. Quant. Grav.* **25** (2008) 085010 [arXiv:0711.1445] [inSPIRE].

[57] L. Vanzo, G. Acquaviva and R. Di Criscienzo, *Tunnelling Methods and Hawking's radiation: achievements and prospects*, *Class. Quant. Grav.* **28** (2011) 183001 [arXiv:1106.4153] [inSPIRE].

[58] H. Kodama, *Conserved Energy Flux for the Spherically Symmetric System and the Back Reaction Problem in the Black Hole Evaporation*, *Prog. Theor. Phys.* **63** (1980) 1217 [inSPIRE].

[59] S.A. Hayward, *Unified first law of black hole dynamics and relativistic thermodynamics*, *Class. Quant. Grav.* **15** (1998) 3147 [gr-qc/9710089] [inSPIRE].

[60] R. Kerner and R.B. Mann, *Tunnelling, temperature and Taub-NUT black holes*, *Phys. Rev.* **D 73** (2006) 104010 [gr-qc/0603019] [inSPIRE].

[61] M. Visser, *Essential and inessential features of Hawking radiation*, *Int. J. Mod. Phys.* **D 12** (2003) 649 [hep-th/0106111] [inSPIRE].

[62] R. Di Criscienzo, M. Nadalini, L. Vanzo, S. Zerbini and G. Zoccatelli, *On the Hawking radiation as tunneling for a class of dynamical black holes*, *Phys. Lett.* **B 657** (2007) 107 [arXiv:0707.4425] [inSPIRE].

[63] S.A. Hayward, R. Di Criscienzo, L. Vanzo, M. Nadalini and S. Zerbini, *Local Hawking temperature for dynamical black holes*, *Class. Quant. Grav.* **26** (2009) 062001 [arXiv:0806.0014] [inSPIRE].

[64] J.B. Hartle and S.W. Hawking, *Path Integral Derivation of Black Hole Radiance*, *Phys. Rev.* **D 13** (1976) 2188 [inSPIRE].

[65] B. Chatterjee, A. Ghosh and P. Mitra, *Tunnelling from black holes in the Hamilton Jacobi approach*, *Phys. Lett.* **B 661** (2008) 307 [arXiv:0704.1746] [inSPIRE].

[66] S. Stotyn, K. Schleich and D. Witt, *Observer Dependent Horizon Temperatures: A Coordinate-Free Formulation of Hawking Radiation as Tunneling*, *Class. Quant. Grav.* **26** (2009) 065010 [arXiv:0809.5093] [inSPIRE].

[67] D.N. Page, *Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole*, *Phys. Rev.* **D 13** (1976) 198 [INSPIRE].

[68] V. Iyer and R.M. Wald, *Some properties of Noether charge and a proposal for dynamical black hole entropy*, *Phys. Rev.* **D 50** (1994) 846 [`gr-qc/9403028`] [INSPIRE].

[69] D. Carney, L. Chaurette, D. Neuenfeld and G.W. Semenoff, *Infrared quantum information*, *Phys. Rev. Lett.* **119** (2017) 180502 [`arXiv:1706.03782`] [INSPIRE].

[70] D. Carney, L. Chaurette, D. Neuenfeld and G.W. Semenoff, *Dressed infrared quantum information*, *Phys. Rev.* **D 97** (2018) 025007 [`arXiv:1710.02531`] [INSPIRE].