

## Dark radiation and decaying matter

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M.C. Gonzalez-Garcia,<sup>a,b</sup> V. Niro<sup>c</sup> and Jordi Salvado<sup>d</sup>

<sup>a</sup>*Institució Catalana de Recerca i Estudis Avançats (ICREA),  
Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos,  
Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain*

<sup>b</sup>*C.N. Yang Institute for Theoretical Physics, State University of New York at Stony Brook,  
Stony Brook, NY 11794-3840, U.S.A.*

<sup>c</sup>*Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos,  
Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain*

<sup>d</sup>*Wisconsin IceCube Particle Astrophysics Center (WIPAC) and Department of Physics,  
University of Wisconsin, Madison, WI 53706, U.S.A.*

*E-mail:* [concha@insti.physics.sunysb.edu](mailto:concha@insti.physics.sunysb.edu), [niro@ecm.ub.es](mailto:niro@ecm.ub.es),  
[jordi.salvado@icecube.wisc.edu](mailto:jordi.salvado@icecube.wisc.edu)

**ABSTRACT:** Recent cosmological measurements favour additional relativistic energy density beyond the one provided by the three active neutrinos and photons of the Standard Model (SM). This is often referred to as “dark radiation”, suggesting the need of new light states in the theory beyond those of the SM. In this paper, we study and numerically explore the alternative possibility that this increase comes from the decay of some new form of heavy matter into the SM neutrinos. We study the constraints on the decaying matter density and its lifetime, using data from the Wilkinson Microwave Anisotropy Probe, the South Pole Telescope, measurements of the Hubble constant at present time, the results from high-redshift Type-I supernovae and the information on the Baryon Acoustic Oscillation scale. We, moreover, include in our analysis the information on the presence of additional contributions to the expansion rate of the Universe at the time of Big Bang Nucleosynthesis. We compare the results obtained in this decaying matter scenario with those obtained with the standard analysis in terms of a constant  $N_{\text{eff}}$ .

**KEYWORDS:** Cosmology of Theories beyond the SM, Neutrino Physics

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**1 Introduction**

In recent years an enormous progress has been made in the field of cosmology. Data on the Cosmic Microwave Background (CMB) has brought cosmology into a precision science and have revealed a Universe made by roughly 23% of Dark Matter (DM) and 72% of Dark Energy (DE). For a review on the physics of the CMB we refer to [1]. Furthermore, at present, recent analysis of cosmological data suggest a trend towards the existence of “dark radiation” (see for example [2–8]). Dark radiation alters the time of matter-radiation equality with a corresponding impact on the observed CMB anisotropies as well as an affect on the Large Scale Structure (LSS) distributions. The amount of dark radiation is usually parametrized using the parameter  $N_{\text{eff}}$ , that indicates the “effective number” of neutrino-like relativistic degrees of freedom. The value associated with the standard case of three active neutrino flavours was calculated in detail in ref. [9] and was found to be  $N_{\text{eff}}^{\text{SM}} = 3.046$ .

The Wilkinson Microwave Anisotropy Probe (WMAP) collaboration found  $N_{\text{eff}} = 4.34_{-0.88}^{+0.86}$  based on their 7-year data release and additional LSS data [2] at  $1\sigma$  in a  $\Lambda$ CDM cosmology. More recent measurements of the CMB anisotropy on smaller scales by the Atacama Cosmology Telescope (ACT) [10] and South Pole Telescope (SPT) [11] experiments seem to also favour a value of  $N_{\text{eff}}$  higher than predicted in SM. In a  $\Lambda$ CDM cosmology the current constraints on  $N_{\text{eff}}$  at 68% C.L at CMB time read [12]:

$$N_{\text{eff}}^{\text{CMB}} = 4.34_{-0.88}^{+0.86} \quad \text{WMAP7+BAO+}H_0, \quad (1.1)$$

$$N_{\text{eff}}^{\text{CMB}} = 3.86 \pm 0.42 \quad \text{WMAP7+SPT+BAO+}H_0, \quad (1.2)$$

$$N_{\text{eff}}^{\text{CMB}} = 3.89 \pm 0.41 \quad \text{WMAP7+ACT+SPT+BAO+}H_0, \quad (1.3)$$

where  $H_0$  refers to the constraint  $H_0 = 74.2 \pm 3.6 \text{ km s}^{-1}$  found by the Hubble Space Telescope [13]. This evidence of dark radiation is robust under consideration of more generalized cosmologies. For example in ref. [3],  $N_{\text{eff}} = 4.35_{-0.54}^{+1.4}$  was found in a global

analysis including the data from cosmic microwave background experiments (in particular from WMAP-7), the Hubble constant  $H_0$  measurement [13], the high-redshift Type-I supernovae [14] and the LSS results from the Sloan Digital Sky Survey (SDSS) data release 7 (DR7) halo power spectrum [15]. The value refers to generalized cosmologies which depart from  $\Lambda$ CDM models by allowing not only the presence of dark radiation but also dark energy with equation of state with  $\omega \neq -1$ , neutrino masses, and non-vanishing curvature.

Independent information on the amount of radiation at earlier times is provided by its effect on the expansion rate of the Universe at the time of Nucleosynthesis. Faster expansion would lead to an earlier freeze-out of the neutron to proton ratio and would lead to a higher  ${}^4\text{He}$  abundance generated during Big Bang Nucleosynthesis (BBN) [16]. Consequently the value of  $N_{\text{eff}}$  at the time of BBN can be constrained using primordial Nucleosynthesis yields of deuterium and helium. Old data and analysis reported a value of  $N_{\text{eff}}$  at BBN consistent with the standard model prediction:  $N_{\text{eff}}^{\text{BBN}} = 2.4 \pm 0.4$  at 68% C.L. [17]. However recent results indicate, as well, that the relatively high  ${}^4\text{He}$  abundance can be interpreted in terms of additional radiation during the BBN epoch [18–20] (see [21] for a recent review). In particular in ref. [20] it was found

$$N_{\text{eff}}^{\text{BBN}} = 3.83^{+0.22}_{-0.61} \tag{1.4}$$

at 68% C.L. which suggests that  $\Delta N_{\text{eff}}^{\text{BBN}} < 1$  at 95% C.L. at BBN time. This bound is reasonably independent of measurements on the baryon density from CMB anisotropy data and of the neutron lifetime input.

The number of active neutrinos is constrained by measurements of the decay width of the Z boson [22] to be  $2.984 \pm 0.008$ . For this reason, several authors invoked the presence of a sterile neutrino to explain the above data [4, 23–26]. Furthermore, the possibility of one or two additional eV scale mass sterile neutrinos, brought to equilibrium with the SM ones via their mixing, could also account for some of the anomalies observed in short-baseline (SBL) neutrino experiments, that favour one or two sterile neutrinos [27–32].

Light sterile neutrinos account for dark radiation in the same amount at BBN and CMB times, *i.e.* in these scenarios  $N_{\text{eff}}^{\text{BBN}} = N_{\text{eff}}^{\text{CMB}}$ . Alternatively and in the light of the old BBN value for  $N_{\text{eff}}$  [17], other models were proposed to explain an increasing on  $N_{\text{eff}}$  at CMB time, while having  $N_{\text{eff}}$  compatible with three at BBN time. In ref. [33], for example, the authors suggested that the increase in radiation could be explained by the decay of non-relativistic matter into relativistic states — see also [34–36] for a general study on dark radiation production from particle decays — while, in ref. [37], it was proposed that non-thermal DM production, such as late-time decays of a long-lived state into DM and neutrinos and photons, could mimic an additional neutrino species. Recently, in ref. [38], the authors discussed the possibility that a new particle of mass  $\lesssim 10$  MeV, that remains in thermal equilibrium with neutrinos until it becomes non-relativistic, leads to a value of  $N_{\text{eff}}$  that is greater than three.

In this paper, we explore the possibility of generating the dark radiation without the need of additional light states besides the ones of the SM. In particular, we consider a scenario in which dark radiation consists of SM neutrinos which have been populated in an extra amount by the decay of some heavy form of thermally produced matter as described

in section 2. One important constraint in our study is the requirement that the decay products, which constitute the dark radiation, are SM neutrinos. This is so because before neutrino decoupling any neutrinos produced by the decay would be brought to equilibrium by SM interactions and, in this way, the additional contribution from dark radiation to the expansion would be erased.

We perform a global analysis of the relevant cosmological observables to determine the allowed range for the neutrino decaying matter density and its lifetime in this framework. Our results, presented in section 3.2, show that this scenario can explain the tendency of the data to favour more radiation at the BBN and CMB times without the need of adding one or more sterile neutrinos nor any other new relativistic states to the three active ones of the SM. In particular we find that a lifetime of order  $\tau_{\text{dec}} \sim 10^3$  s is favoured, implying a decay that happens during BBN time. We summarize our conclusions in section 4.

## 2 Decaying matter

Generically when discussing the possibility of decay matter as the origin of dark radiation one can think of two possibilities: in the first scenario the decaying matter is the main component of the DM itself, while in the second case the particle that decays is not the relic particle present nowadays in the Universe, but it is another particle species.

The possibility of decaying DM (DDM) has been exhaustively explored in the literature. CMB constraints on decaying warm DM have been derived in [39], specifically in the context of a Majoron DM. Cosmological constraints on the DM decay rate into neutrinos have been obtained in ref. [40], using only Type Ia supernova data (SNIa) and the first CMB peak, see also [41–44] for more references. More recently, in ref. [45], the authors extended the analysis using the full CMB anisotropy spectrum, large scale structure (LSS), Lyman- $\alpha$  data and weak lensing observations: the authors found a DM lifetime of the order  $\Gamma_{\text{dec}}^{-1} \gtrsim 100$  Gyr. Cosmological data, thus, tightly constrain the DM decay rate into neutrinos. This happens because DDM alters the time of matter-radiation equality, and the early integrated Sachs-Wolfe effect, with an increase in the first CMB peak. DDM also changes the late integrated Sachs-Wolfe effect, with a direct consequence on the CMB anisotropy spectrum at small multipoles, see [39] for a detailed explanation. Bounds on the DM lifetime can be obtained independently also from neutrino telescopes data. The Super-Kamiokande bounds on the DM flux from the Galactic Center, from the Earth and the Sun are given in [46] and, using these data, the constrain on the DM lifetime has been calculated in refs. [47–49].

Consequently since the DM decay rate into neutrinos is constrained to be much longer than the age of the Universe, the increase in the value of  $N_{\text{eff}}$  that we expect in the DDM scenario is, in general, tiny.<sup>1</sup>

Alternatively one can consider scenarios — denoted in the following as *decM* — in which the decaying state is different than the one dominantly present nowadays in the Universe as dark matter, such as those in ref. [33–37]. In what follows, we will consider

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<sup>1</sup>Considering a DM lifetime that depends on time, as suggested in ref. [50], it is possible to explain the excess in the radiation within the DDM scenario.

that the *decM* is thermally produced and dominantly decays into neutrinos. We, moreover, restrict our study to the case of a non-relativistic *decM* at BBN time: its mass is greater than roughly 10 MeV (see also discussion about eq. (3.13)).

### 2.1 Boltzmann equations for decaying matter

In the synchronous gauge, the line element is defined as:

$$ds^2 = a^2(\tau) \{-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\}, \tag{2.1}$$

where  $\tau$  is the conformal time, while  $t$  is the cosmological time ( $dt = a(\tau) d\tau$ ), and  $a = R/R_0$  is the scale factor. The metric perturbation  $h_{ij}$  can be expanded in Fourier space as [51]

$$h_{ij}(\vec{x}, \tau) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \left\{ \hat{k}_i \hat{k}_j h(\vec{k}, \tau) + \left( \hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij} \right) 6\eta(\vec{k}, \tau) \right\}, \tag{2.2}$$

with  $\vec{k} = k\hat{k}$ . We will use the notation defined above when writing the equations for the perturbations, i.e. the fields  $h(\vec{k}, \tau)$  and  $\eta(\vec{k}, \tau)$ .

In the *decM* scenario, the set of Boltzmann equations describing the background evolution of the different components of the energy density of the Universe is complemented with two equations describing the evolution of the *decM* and dark radiation density as [52, 53]:

$$\dot{\rho}_{\text{dec}} = -3aH\rho_{\text{dec}} - a\Gamma_{\text{dec}}\rho_{\text{dec}}, \tag{2.3}$$

$$\dot{\rho}_{\text{dr}} = -4aH\rho_{\text{dr}} + a\Gamma_{\text{dec}}\rho_{\text{dec}}, \tag{2.4}$$

where the over-dot denotes the derivative respect to conformal time  $\tau$ . The subscript “*dr*” indicates the dark radiation coming from the *decM*, while “*dec*” is the density of the *decM*.  $\Gamma_{\text{dec}}$  is the *decM* decay width.

Correspondingly we derive the equations for the density fluctuations [51] generalized for the case of *decM* into SM neutrinos by adapting the equations for of DDM in [54–56] to the specifics of our case, in which the decaying products are SM neutrinos. We find:

$$\dot{\delta}_{DM} = -\frac{1}{2}\dot{h}, \tag{2.5}$$

$$\dot{\delta}_R = -\frac{2}{3}\dot{h} - \frac{4}{3}\theta_R + a\Gamma_{\text{dec}}\frac{\rho_{\text{dec}}}{\rho_R}(\delta_{DM} - \delta_R), \tag{2.6}$$

$$\dot{\theta}_R = k^2\left(\frac{1}{4}\delta_R - \sigma_R\right) - a\Gamma_{\text{dec}}\frac{\rho_{\text{dec}}}{\rho_R}\theta_R, \tag{2.7}$$

$$\dot{\sigma}_R = \frac{1}{2}\left(\frac{8}{15}\theta_R - \frac{3}{5}kF_3 + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta}\right) - a\Gamma_{\text{dec}}\frac{\rho_{\text{dec}}}{\rho_R}\sigma_R, \tag{2.8}$$

$$\dot{F}_l = \frac{k}{2l+1}[lF_{l-1} - (l+1)F_{l+1}] - a\Gamma_{\text{dec}}\frac{\rho_{\text{dec}}}{\rho_R}F_l, \tag{2.9}$$

where we have defined  $\rho_R \equiv \rho_\nu + \rho_{\text{dr}}$ ,  $\rho_{DM} \equiv \rho_c + \rho_{\text{dec}}$ , with  $\rho_\nu$  the standard neutrino contribution, i.e. non coming from matter decay, and  $\rho_c$  the density of standard cold dark matter. In eq. (2.9)  $l \geq 3$  and  $F_2 = 2\sigma_R$  and we have used the conventions in refs. [45, 51]. Note that the density perturbation equation for the DM does not depend on the value of  $\Gamma_{\text{dec}}$  since both the background DM density and overdensity decay at the same rate [45].

We have implemented the previous equations for the background and the perturbations in the CLASS (Cosmic Linear Anisotropy Solving System) code [57]. We considered flat cosmology and an equation of state for the cosmological constant parameter given by  $w = -1$ .

### 3 Numerical analysis

#### 3.1 Cosmological inputs

In our analysis we include the results from the 7-year data of WMAP [2] on the temperature and polarization anisotropies, using the likelihood function as provided by the collaboration. The ACT [58] and SPT [11] experiments have probed higher multipole moments than WMAP. We implement the SPT data only in our analysis, since they give a bound in  $N_{\text{eff}}$  of the same order as the one obtained considering both the ACT and the SPT data, see eqs. (1.2), (1.3). We build the corresponding likelihood functions from the data, covariance matrix and window functions, introducing other three parameters in the analysis: the Sunyaev-Zel'dovich (SZ) amplitude, the amplitude of Poisson distributed point sources and the amplitude of clustered point sources. For these parameters, we used gaussian priors as given in refs. [11, 59] and the templates of ref. [11]. We fixed the foreground terms to be positive. In the following, with the notation ‘‘CMB’’ we will always refer to the combination of WMAP7 and SPT data.

We introduce a Hubble parameter prior, based on the latest Hubble Space Telescope value [60]:  $H_0 = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . This measurement of  $H_0$  is obtained from the magnitude-redshift relation of 240 low- $z$  Type Ia supernovae at  $z < 0.1$ .

We also include the luminosity measurements of high- $z$  SNIa as given in ref. [14]. This compilation, the ‘‘Constitution’’ set, consists of 397 supernovae and it is an extension of the previous sample, the ‘‘Union’’ set [61].

Finally we use the measurement of BAO scale obtained from the Two-Degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey Data Release 7 (SDSS DR7) [62]. We use as input data the two distance ratios  $d_z$  at  $z = 0.2$  and  $z = 0.35$ , with  $d_z \equiv r_s(z_d)/D_V(z)$ , where  $r_s(z_d)$  is the comoving sound horizon at the baryon drag epoch and  $D_V(z) = [(1+z)^2 D_A^2 cz/H(z)]^{1/3}$ , with  $D_A$  the angular diameter distance and  $H(z)$  the Hubble parameter. We build the corresponding likelihood function using the covariance matrix as given in ref. [62]. Since in the fitting procedure of [62], the value of  $d_z$  is obtained by first assuming some fiducial cosmology ( $h = 0.72$ ,  $\Omega_b h^2 = 0.0223$ ,  $\Omega_m = 0.25$ ), we rescale the predictions for the BAO scale as explained in ref. [63]. We do not introduce the information on the full power-spectrum of the SDSS DR7 survey [15], but we consider only the BAO measurement, since we will neglect the neutrino mass in our analysis.<sup>2</sup>

The analysis method we adopt is based on a Markov Chain Monte Carlo (MCMC) generator which employs the Metropolis-Hasting algorithm, see [3, 64] for more details.

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<sup>2</sup>As long as the population of neutrinos produced in the decay of the massive particle of mass  $M_{\text{dec}}$  are energetic enough to still be relativistic at the time of structure formation they will not affect the matter power spectrum. A conservative bound on the range of validity of this assumption can be obtained by demanding  $m_\nu < E_\nu^{\text{dec}}(\text{today}) \sim (T_0^\nu/T_{\text{dec}}^\nu) E_\nu^{\text{dec}}(\tau_{\text{dec}}) \sim 0.3\text{--}0.5 (T_0^\nu/T_{\text{dec}}^\nu) M_{\text{dec}} \lesssim (0.6\text{--}1) \times 10^{-9} M_{\text{dec}}$  for decay into two or three neutrinos and for the favoured decay times that we will find  $T_0^\nu/T_{\text{dec}}^\nu \lesssim 2 \times 10^{-9}$ .

For convenience for the case of *decM* in the MC generator we use the parameters in the table 1 for which we assume a flat prior. The parameters  $n_s, \tau, A_S$  and  $\Gamma_{\text{dec}}/\text{Gyr}^{-1}$  are respectively the scalar spectral index, the optical depth at reionization, the amplitude of scalar power spectrum at  $k = 0.05 \text{ Mpc}^{-1}$  and the decay rate into neutrinos.

The first four parameters in table 1 are related to the initial cosmological constant, baryon, DM and *decM* density as:

$$\rho_{\Lambda}^i = \tilde{\Omega}_{\Lambda} \cdot \tilde{H}_0^2 \tag{3.1}$$

$$\rho_b^i = \tilde{\Omega}_b \cdot \tilde{H}_0^2 \cdot a^{-3} \tag{3.2}$$

$$\rho_c^i = \tilde{\Omega}_c \cdot \tilde{H}_0^2 \cdot a^{-3} \tag{3.3}$$

$$\rho_{\text{dec}}^i = \tilde{\Omega}_{\text{dec}} \cdot \tilde{H}_0^2 \cdot a^{-3} \tag{3.4}$$

where  $\tilde{H}_0^2$  has a fixed value. The densities are all in units of  $[3c^2/8\pi G]$ , with  $c$  the speed of light and  $G$  the Newton's constant. Note that we have decided to use  $\tilde{\Omega}_{\Lambda}$  as MC parameter rather than  $\tilde{H}_0^2$  and that the parameter  $\tilde{H}_0^2$  is not equivalent to the Hubble parameter today. The ‘‘physical’’ Hubble parameter is calculated at each redshift using the Friedmann equation:

$$H(z) = \sqrt{\sum_i \rho_i(z)}, \tag{3.5}$$

with  $i$  that runs over all the components of the Universe. After solving the Boltzmann equations, we obtain the present values ( $z = 0$ ) of the Hubble constant  $H_0$ , of the baryon density  $\Omega_b$  and of the cold dark matter density  $\Omega_c$ . More specifically, the latter parameter is defined as  $\Omega_c = (\rho_c(z = 0) + \rho_{\text{dec}}(z = 0))/\rho_{\text{crit}}$ , but we will see that our results favour a negligible contribution of  $\rho_{\text{dec}}$  to the dark matter density at present time. When presenting the results of our analysis, we will use these derived parameters (which we refer to as ‘‘analysis parameters’’) to better compare with the  $\Lambda\text{CDM}+N_{\text{eff}}$  case.

At any time (or equivalently redshift  $z$ ) the densities  $\rho_{\text{dec}}(z)$  and  $\rho_{dr}(z)$  can be translated in a time dependent effective number of neutrinos depending on how this one is related with the observations. At BBN  $N_{\text{eff}}$  is determined by its contribution to the expansion rate of the Universe which is what affects the primordial abundances. Any form of energy density, relativistic or non-relativistic enters in this observable. Consequently, at BBN

$$\Delta N_{\text{eff}}^{\text{BBN}} = \frac{\rho_{\text{dec}}(z_{\text{BBN}}) + \rho_{dr}(z_{\text{BBN}})}{\left(\frac{7}{8}\right) \left(\frac{4}{11}\right)^{4/3} \rho_{\gamma}(z_{\text{BBN}})} \equiv N_{\text{eff}}^{\text{BBN}} - N_{\text{eff}}^{\text{SM}}, \tag{3.6}$$

with  $N_{\text{eff}}^{\text{SM}} = 3.046$ . To account for the constraints from BBN we impose in the analysis a prior on the value of  $N_{\text{eff}}^{\text{BBN}}$ , eq. (1.4) defined for definiteness, at the time at which BBN ends,  $z_{\text{BBN}} = T_{\text{BBN}}/T_{\gamma}^0 - 1$  with  $T_{\text{BBN}} = 0.01 \text{ MeV}$  ( $T_{\gamma}^0 = 2.726 \text{ K}$ ).

In CMB observables, the information on  $N_{\text{eff}}$  arises mostly from its contribution to the determination of the matter-radiation equality epoch. Thus, when translating our results in terms of  $N_{\text{eff}}$  at CMB time, we will define

$$\Delta N_{\text{eff}}^{\text{CMB}} = \frac{\rho_{dr}(z_{\text{CMB}})}{\left(\frac{7}{8}\right) \left(\frac{4}{11}\right)^{4/3} \rho_{\gamma}(z_{\text{CMB}})}, \tag{3.7}$$

with  $z_{\text{CMB}} = 1100$ .

MC Parameters	symbols
Cosmological constant density	$\tilde{\Omega}_\Lambda$
Baryon density	$\tilde{\Omega}_b$
Dark Matter density	$\tilde{\Omega}_c$
Decaying Matter density	$\tilde{\Omega}_{\text{dec}}$
Scalar spectral index	$n_s$
Optical depth at reionization	$\tau$
Amplitude of scalar power spectrum at $k = 0.05 \text{ Mpc}^{-1}$	$A_S$
Decay rate	$\Gamma_{\text{dec}}/\text{Gyr}^{-1}$

**Table 1.** Parameters used in the MC generation in the *decM* scenario.

Furthermore, to implement thermal equilibrium condition on the dark radiation before neutrino decoupling, we impose

$$\frac{\rho_{dr}(z_{\nu\text{-decoup}}) + \rho_\nu(z_{\nu\text{-decoup}})}{\left(\frac{7}{8}\right) \left(\frac{4}{11}\right)^{4/3} \rho_\gamma(z_{\nu\text{-decoup}})} = 3.046, \quad (3.8)$$

with  $T_{\nu\text{-dec}} \geq 1 \text{ MeV}$ .

Using the above data and their theoretical predictions — obtained from the modified Boltzmann equations presented in the previous section — we construct the combined likelihood function and the corresponding probability distribution function from which we obtain the one-dimensional and two-dimensional probability distributions, as described in ref. [3].

For the sake of comparison, we also perform the corresponding analysis in the framework of a  $\Lambda\text{CDM}$  model with a constant  $\Delta N_{\text{eff}}$  (but without imposing the condition eq. (3.8)). In this case, the parameters used in the MC, for which flat priors are used, are the present Hubble constant  $H_0$ , baryon density  $\Omega_b$ , cold dark matter density  $\Omega_c$  and  $\Delta N_{\text{eff}}$ , together with  $n_s, \tau$ , and  $A_S$ .

### 3.2 Results

The results of our cosmological fits are summarized in table 2 and in figures 1 and 2. In table 2, we report the best-fit values as well as the  $1\sigma$  and 95% C.L. range for the analysis parameters. We present the results for our model, the *decM* case, as well as for the standard  $\Lambda\text{CDM}$  model with the addition of a fix  $N_{\text{eff}}$  and with the implementation of the constraint on  $N_{\text{eff}}^{\text{BBN}}$  in both cases. We denote these two analysis as *decM*+BBN and  $\Lambda\text{CDM}+N_{\text{eff}}+\text{BBN}$ . As seen from the table, in the *decM*+BBN the set of data considered is better described with an amount of decaying matter with lifetime short enough to allow for a good fraction of it to have decayed into neutrinos at BBN time.

In figure 1, we present the one-dimensional probabilities for the two analysis. For a better understanding, we report both the value of  $\Delta N_{\text{eff}}$  at BBN and CMB time (which for  $\Lambda\text{CDM}+N_{\text{eff}}+\text{BBN}$  are identical by definition). Since the BBN data we are using favours



Parameter	decM+BBN			$\Lambda$ CDM+ $N_{\text{eff}}$ +BBN		
	best	1 $\sigma$	95%	best	1 $\sigma$	95%
$H_0$ [km/s/Mpc]	72.6	+1.5 -1.4	+3.0 -2.8	73.0	+1.5 -1.5	+2.8 -3.0
$\Omega_b h^2 \times 100$	2.254	+0.034 -0.037	+0.069 -0.068	2.258	+0.032 -0.037	+0.065 -0.070
$\Omega_c h^2$	0.125	+0.005 -0.005	+0.012 -0.010	0.127	+0.006 -0.006	+0.011 -0.012
$\log(\rho_{\text{dec}}/\rho_\gamma)$ at $t = 10^{-4}$ s	-4.61	+0.61 -0.73	+0.92 -1.72	—	—	—
$n_s$	0.973	+0.009 -0.009	+0.018 -0.018	0.975	+0.010 -0.010	+0.019 -0.019
$\tau$	0.084	+0.013 -0.015	+0.026 -0.026	0.083	+0.013 -0.013	+0.027 -0.024
$A_s \times 10^9$	2.452	+0.082 -0.083	+0.164 -0.157	2.449	+0.075 -0.083	+0.155 -0.159
$\log(\tau_{\text{dec}}/\text{s})$	2.9	+1.7 -1.0	+3.7 -1.5	—	—	—
$\Delta N_{\text{eff}}^{\text{CMB}}$	0.50	+0.30 -0.19	+0.58 -0.42	0.70	+0.25 -0.30	+0.44 -0.59
$\Delta N_{\text{eff}}^{\text{BBN}}$	—	—	$\leq 0.90$	0.70	+0.25 -0.30	+0.44 -0.59

**Table 2.** Best-fit values, 68% C.L. and 95% C.L. errors for the analysis parameters.

values of  $\Delta N_{\text{eff}} < 1$  at 95% C.L., adding the BBN information slightly changes the best-fit value for  $\Delta N_{\text{eff}}$  in the  $\Lambda$ CDM+ $N_{\text{eff}}$ +BBN analysis respect to the analysis of  $\Lambda$ CDM+ $N_{\text{eff}}$  (without the BBN information) and reduces the 1 $\sigma$  range from  $\Delta N_{\text{eff}} \simeq 0.71_{-0.38}^{+0.42}$  (without BBN) to  $\Delta N_{\text{eff}} \simeq 0.70_{-0.30}^{+0.25}$  (with BBN).

We find that the preferred lifetime is  $\tau_{\text{dec}} \sim 10^3$  sec which means decaying slightly before or during BBN. This can be understood because at any time  $t$  the contribution of the decaying matter and the dark radiation produced in its decay to the energy density of the universe, parametrized as  $\Delta N_{\text{eff}}^{\text{dec}}$  and  $\Delta N_{\text{eff}}^{\text{dr}}$  respectively, can be approximated by integrating eqs. (2.3) and (2.4):

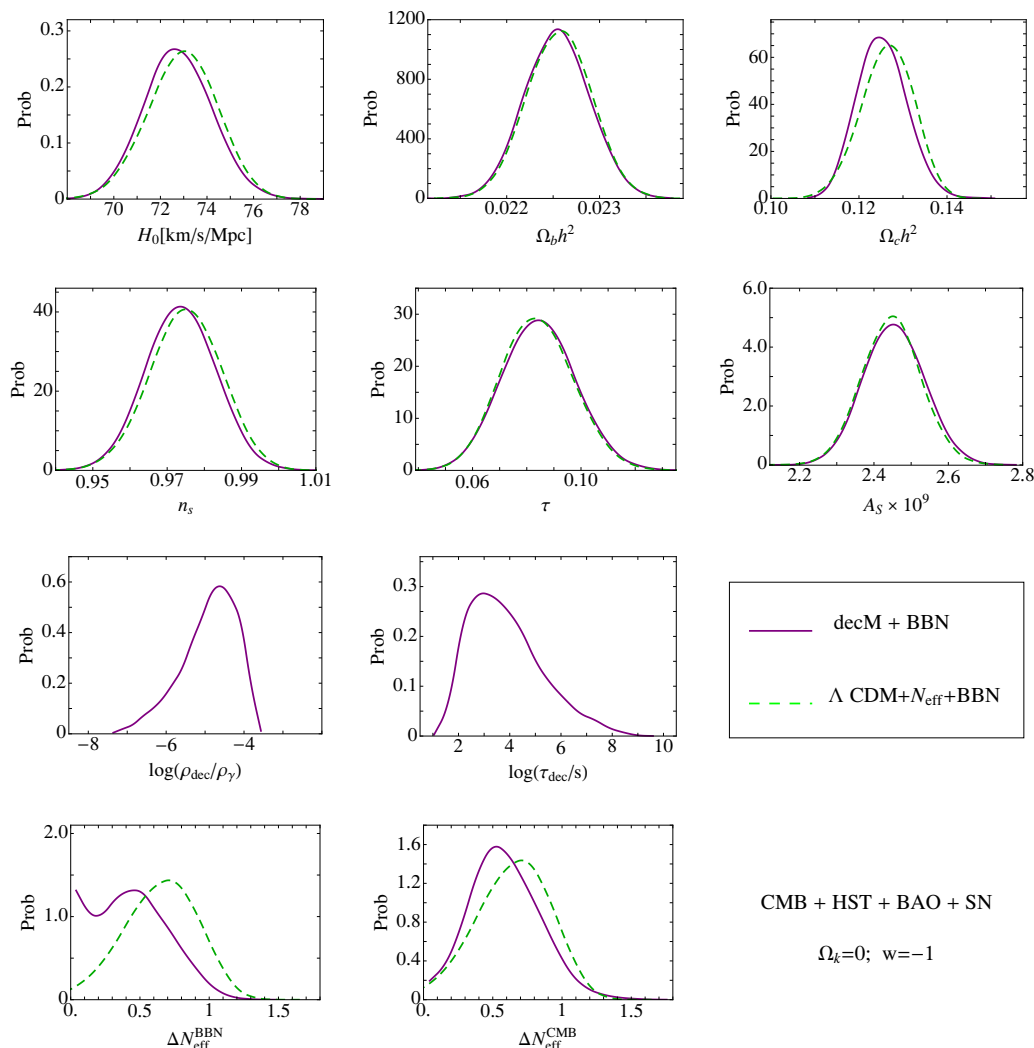
$$\Delta N_{\text{eff}}^{\text{dec}}(t) = \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{\frac{4}{3}} \left(\frac{t}{t_0}\right)^{\frac{1}{2}} \frac{\rho_{\text{dec}}(t_0)}{\rho_\gamma(t_0)} e^{-\frac{t}{\tau_{\text{dec}}}}, \quad (3.9)$$

$$\Delta N_{\text{eff}}^{\text{dr}}(t) = \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{\frac{4}{3}} \left(\frac{\tau_{\text{dec}}}{t_0}\right)^{\frac{1}{2}} \frac{\rho_{\text{dec}}(t_0)}{\rho_\gamma(t_0)} \left( \frac{\sqrt{\pi}}{2} \text{Erf} \left[ \sqrt{\frac{t}{\tau_{\text{dec}}}} \right] - \sqrt{\frac{t}{\tau_{\text{dec}}}} e^{-\frac{t}{\tau_{\text{dec}}}} \right) \quad (3.10)$$

$$\xrightarrow{t \gg \tau_{\text{dec}}} \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{\frac{4}{3}} \left(\frac{\tau_{\text{dec}}}{t_0}\right)^{\frac{1}{2}} \frac{\sqrt{\pi}}{2} \frac{\rho_{\text{dec}}(t_0)}{\rho_\gamma(t_0)}, \quad (3.11)$$

where we have assumed that the decay occurs in a radiation dominated era,  $a \propto \sqrt{t}$  and we have chosen for normalization  $t_0 = 10^{-4}$ s (up to the factor  $\frac{\sqrt{\pi}}{2}$  eq. (3.11) can be also obtained with the assumption of fast decay at  $t = \tau_{\text{dec}}$  [33]).

At CMB time  $\Delta N_{\text{eff}}^{\text{CMB}}$  is well approximated by eq. (3.11) and the analysis constraints the product of  $\rho_{\text{dec}}(t_0) \times \sqrt{\tau_{\text{dec}}}$  so the density of decaying matter and its lifetime become



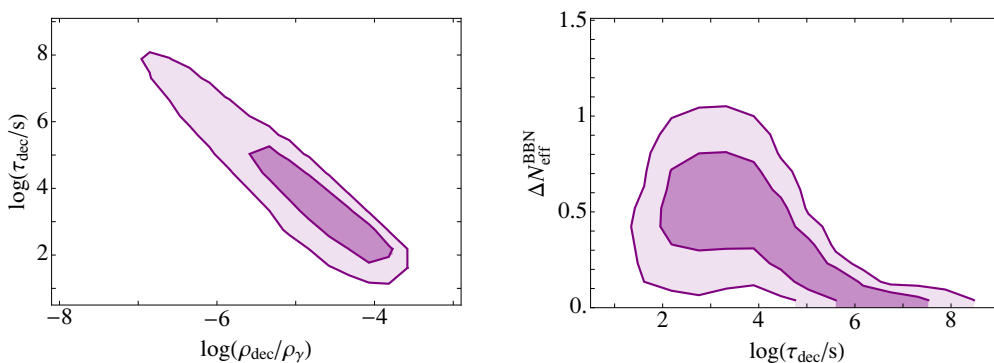
**Figure 1.** Marginalized one-dimensional probability distributions for the analysis parameters. We present the results for the case of *decM*+BBN with solid lines. The dashed line shows the results for  $\Lambda$ CDM+ $N_{\text{eff}}$ +BBN.

strongly anticorrelated (see also figure 2). At BBN time

$$\begin{aligned} \Delta N_{\text{eff}}^{\text{BBN}} &= \Delta N_{\text{eff}}^{\text{dec}}(t_{\text{BBN}}) + \Delta N_{\text{eff}}^{\text{dr}}(t_{\text{BBN}}) \\ &= \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{4/3} \left(\frac{\tau_{\text{dec}}}{t_0}\right)^{1/2} \frac{\rho_{\text{dec}}(t_0)}{\rho_{\gamma}(t_0)} \frac{\sqrt{\pi}}{2} \text{Erf} \left[ \sqrt{\frac{t_{\text{BBN}}}{\tau_{\text{dec}}}} \right]. \end{aligned} \quad (3.12)$$

The present analysis favours  $N_{\text{eff}}^{\text{BBN}}$  not very different from  $\Delta N_{\text{eff}}^{\text{CMB}}$  and eq. (3.12) and eq. (3.11) give comparable numerical results for  $\tau_{\text{dec}} \sim t_{\text{BBN}}$ .

In our calculations we have assumed that the decay product neutrinos are decoupled from the plasma. However if sufficiently energetic they could re-thermalize with the plasma and transfer energy to the other particles present. The minimum energy for this to happen can be estimated [65] — see also ref. [66] for a specific model example — for a radiation



**Figure 2.** Two-dimensional probability distribution at the 68% and 95% C.L. for the *decM* parameters  $\{\log(\rho_{\text{dec}}/\rho_\gamma), \log(\tau_{\text{dec}}/s)\}$ , and  $\{\log(\tau_{\text{dec}}/s), \Delta N_{\text{eff}}^{\text{BBN}}\}$ .

dominated universe to be

$$E_\nu^{\text{re-therm}} = \left(\frac{8\pi}{3}\rho\right)^{1/2} (G_F^2 m_{pl} T^4)^{-1} \simeq 0.42 g_\star \left(\frac{t}{s}\right) \text{ MeV}, \quad (3.13)$$

where  $\rho$  is the total mass-energy density,  $G_F$  is the Fermi constant,  $m_{pl}$  is the Planck mass and  $g_\star$  is the number of effective degrees of freedom at decay time. From our numerical analysis, we find that *decM* with lifetimes above  $10^2$  s is favoured. Assuming instantaneous decay at  $t = \tau_{\text{dec}}$ ,  $E_\nu^{\text{re-therm}} \simeq 140$  MeV. Thus, as long as the decaying particle is lighter than roughly 300–450 MeV (for decay in two or three neutrinos respectively), the decaying product neutrinos can be considered decoupled from the plasma.

From the one-dimensional probability plots, it can also be inferred that the probability distributions for the parameter  $\Delta N_{\text{eff}}$  in the *decM*+BBN and in the model  $\Lambda\text{CDM}+N_{\text{eff}}+\text{BBN}$  are not equivalent. Besides the expected differences due to the fact that in the second scenario  $N_{\text{eff}}$  is constant in time while in the first one it is not, there is a more subtle difference that is associated with the prior distribution assumed for the parameters in the two analysis. In the *decM* model,  $N_{\text{eff}}$  is a derived parameter and it arises from a combination of two model parameters  $\tilde{\Omega}_{\text{dec}}$  and  $\Gamma_{\text{dec}}$  as given in eq. (3.11) and eq. (3.12). Let us introduce an auxiliary variable  $Z$ , function of  $\tilde{\Omega}_{\text{dec}}$  and  $\Gamma_{\text{dec}}$ , which takes values along the  $N_{\text{eff}}=\text{cnt}$  direction. Using that the probability has to be invariant under reparametrizations:  $\mathcal{P}(\tilde{\Omega}_{\text{dec}}, \Gamma_{\text{dec}})d\tilde{\Omega}_{\text{dec}}d\Gamma_{\text{dec}} = \tilde{\mathcal{P}}(N_{\text{eff}}, Z)dN_{\text{eff}}dZ$ . The flat prior for  $\tilde{\Omega}_{\text{dec}}$  and  $\Gamma_{\text{dec}}$  that we are using in the analysis of the *decM* scenario means that  $\mathcal{P}(\tilde{\Omega}_{\text{dec}}, \Gamma_{\text{dec}})=\text{cnt}$ . Consequently, from eqs. (3.9)–(3.11), we find that the jacobian for the change of variables leads to  $\tilde{\mathcal{P}}(N_{\text{eff}}, Z) \propto 1/N_{\text{eff}}$ . In other words, the effective prior in  $N_{\text{eff}}$ , that corresponds to having a flat prior in  $\{\tilde{\Omega}_{\text{dec}}, \Gamma_{\text{dec}}\}$ , pushes  $N_{\text{eff}}$  to lower values. Furthermore, since  $H_0$ ,  $\Omega_b h^2$ ,  $\Omega_c h^2$  and  $n_s$  are correlated with the value of  $\Delta N_{\text{eff}}$ , this difference leads to the shift in the best-fit values for the parameters  $H_0$ ,  $\Omega_b h^2$ ,  $\Omega_c h^2$  and  $n_s$  in the two models seen in table 2 and in figure 1.

This also explains that the one-dimensional probability distribution for the parameter  $\Delta N_{\text{eff}}^{\text{BBN}}$  presents two peaks: one around  $\Delta N_{\text{eff}}^{\text{BBN}} \simeq 0$  and one around  $\Delta N_{\text{eff}}^{\text{BBN}} \simeq 0.5$ . The first one is a consequence of the  $1/N_{\text{eff}}$  “initial” probability distribution in combination

with the condition of vanishing dark radiation before neutrino decoupling, eq. (3.8). The second is induced by the condition of having an amount of dark radiation at BBN, eq. (1.4). Note, however, that the region below the first peak is much smaller than the region below the second peak. Indeed, the integral of  $\int_{x_{\min}}^{+\infty} dx p_{1-\dim}(\Delta N_{\text{eff}}^{\text{BBN}}) \sim 0.68$ , with  $x_{\min} \equiv 0.28$ . This means that even if the height of the first peak is higher than the second one, a value of  $\Delta N_{\text{eff}}^{\text{BBN}} \gtrsim 0.28$  is favoured at roughly 68% C.L.. This is, in turn, consistent with the best-fit value of the lifetime  $\tau_{\text{dec}}$ , that is around BBN time (see right panel in figure 2).

In the right panel of figure 2 we present the two-dimensional 68% and 95% C.L. credibility regions for the parameters  $\log(\tau_{\text{dec}}/\text{s})$  and  $\Delta N_{\text{eff}}^{\text{BBN}}$  for the *decM*+BBN analysis. In the figure, it is clearly visible that the regions are formed by two connected “islands” around the two favoured values of  $\Delta N_{\text{eff}}^{\text{BBN}}$ . Also as expected the two parameters are anticorrelated: a shorter lifetime is associated with a value of  $\Delta N_{\text{eff}}^{\text{BBN}}$  bigger than zero, while a lifetime bigger than  $10^4$  will increase the value of  $N_{\text{eff}}^{\text{CMB}}$  but not the value of  $N_{\text{eff}}^{\text{BBN}}$ .

The left panel of figure 2 shows the two-dimensional 68% and 95% C.L. credibility regions for the parameters  $\{\log(\rho_{\text{dec}}/\rho_{\gamma})$  and  $\log(\tau_{\text{dec}}/\text{s})\}$ . Here we also see the expected anticorrelation between the total amount of decaying matter and its lifetime required to produce a certain amount of dark radiation (see eq. (3.11)). The figure clearly shows how the best fit values of the analysis correspond to a decay during the BBN time while the contribution of the favoured range of  $\rho_{\text{dec}}$  implies that its contribution to the dark matter at present times is totally negligible.

## 4 Conclusions

In this paper, we have explored the possibility that dark radiation is not due to new relativistic particles but it is formed by SM neutrinos whose density is increased by the decay of some heavy thermally produced particle. With this aim we have performed a global analysis of all the relevant cosmological data from the WMAP and SPT collaborations, measurements of the Hubble constant at present time, the results from high-redshift Type-I supernovae and the BAO scale. We have also included the information on additional radiation at the time of BBN as inferred from the relatively high  ${}^4\text{He}$  abundance determination. We have concluded that the inclusion of the BBN information favours a decay that happens during BBN time with favoured lifetime  $\tau_{\text{dec}}$  between  $10^2$  and  $10^4$  seconds and leads to similar values of  $N_{\text{eff}}$  at BBN and CMB times. We have discussed the difference in the analysis between this scenario and the standard  $\Lambda\text{CDM}+N_{\text{eff}}$  associated with the different physical assumptions on the initial probability distribution of the model parameters.

The discussion of a specific theoretical model in which this scenario is implemented is beyond the scope of this paper. Different possibilities have been suggested in the literature in the context of a *decM* that decays into radiation and that can be applied with some extents also to our case of decay into neutrinos. We refer to refs. [33, 35, 67, 68] for different examples.

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## References

- [1] W. Hu and S. Dodelson, *Cosmic microwave background anisotropies*, *Ann. Rev. Astron. Astrophys.* **40** (2002) 171 [[astro-ph/0110414](#)] [[INSPIRE](#)].
- [2] WMAP collaboration, E. Komatsu et al., *Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation*, *Astrophys. J. Suppl.* **192** (2011) 18 [[arXiv:1001.4538](#)] [[INSPIRE](#)].
- [3] M. Gonzalez-Garcia, M. Maltoni and J. Salvado, *Robust cosmological bounds on neutrinos and their combination with oscillation results*, *JHEP* **08** (2010) 117 [[arXiv:1006.3795](#)] [[INSPIRE](#)].
- [4] J. Hamann, S. Hannestad, G.G. Raffelt, I. Tamborra and Y.Y. Wong, *Cosmology seeking friendship with sterile neutrinos*, *Phys. Rev. Lett.* **105** (2010) 181301 [[arXiv:1006.5276](#)] [[INSPIRE](#)].
- [5] K.M. Nollett and G.P. Holder, *An analysis of constraints on relativistic species from primordial nucleosynthesis and the cosmic microwave background*, [arXiv:1112.2683](#) [[INSPIRE](#)].
- [6] A.X. Gonzalez-Morales, R. Poltis, B.D. Sherwin and L. Verde, *Are priors responsible for cosmology favoring additional neutrino species?*, [arXiv:1106.5052](#) [[INSPIRE](#)].
- [7] S. Joudaki, *Constraints on neutrino mass and light degrees of freedom in extended cosmological parameter spaces*, [arXiv:1202.0005](#) [[INSPIRE](#)].
- [8] M. Archidiacono, E. Giusarma, A. Melchiorri and O. Mena, *Dark radiation in extended cosmological scenarios*, *Phys. Rev. D* **86** (2012) 043509 [[arXiv:1206.0109](#)] [[INSPIRE](#)].
- [9] G. Mangano et al., *Relic neutrino decoupling including flavor oscillations*, *Nucl. Phys. B* **729** (2005) 221 [[hep-ph/0506164](#)] [[INSPIRE](#)].
- [10] S. Das et al., *The Atacama Cosmology Telescope: a measurement of the cosmic microwave background power spectrum at 148 and 218 GHz from the 2008 southern survey*, *Astrophys. J.* **729** (2011) 62 [[arXiv:1009.0847](#)] [[INSPIRE](#)].
- [11] R. Keisler et al., *A measurement of the damping tail of the cosmic microwave background power spectrum with the South Pole Telescope*, *Astrophys. J.* **743** (2011) 28 [[arXiv:1105.3182](#)] [[INSPIRE](#)].
- [12] K. Abazajian et al., *Light sterile neutrinos: a white paper*, [arXiv:1204.5379](#) [[INSPIRE](#)].
- [13] A.G. Riess et al., *A redetermination of the Hubble constant with the Hubble Space Telescope from a differential distance ladder*, *Astrophys. J.* **699** (2009) 539 [[arXiv:0905.0695](#)] [[INSPIRE](#)].

- [14] M. Hicken et al., *CfA3: 185 type Ia supernova light curves from the CfA*, *Astrophys. J.* **700** (2009) 331 [[arXiv:0901.4787](#)] [[INSPIRE](#)].
- [15] B.A. Reid et al., *Cosmological constraints from the clustering of the Sloan Digital Sky Survey DR7 luminous red galaxies*, *Mon. Not. Roy. Astron. Soc.* **404** (2010) 60 [[arXiv:0907.1659](#)] [[INSPIRE](#)].
- [16] J.-M. Yang, D.N. Schramm, G. Steigman and R.T. Rood, *Constraints on cosmology and neutrino physics from Big Bang nucleosynthesis*, *Astrophys. J.* **227** (1979) 697 [[INSPIRE](#)].
- [17] V. Simha and G. Steigman, *Constraining the early-universe baryon density and expansion rate*, *JCAP* **06** (2008) 016 [[arXiv:0803.3465](#)] [[INSPIRE](#)].
- [18] Y. Izotov and T. Thuan, *The primordial abundance of  $^4\text{He}$ : evidence for non-standard Big Bang nucleosynthesis*, *Astrophys. J.* **710** (2010) L67 [[arXiv:1001.4440](#)] [[INSPIRE](#)].
- [19] E. Aver, K.A. Olive and E.D. Skillman, *A new approach to systematic uncertainties and self-consistency in Helium abundance determinations*, *JCAP* **05** (2010) 003 [[arXiv:1001.5218](#)] [[INSPIRE](#)].
- [20] G. Mangano and P.D. Serpico, *A robust upper limit on  $N_{\text{eff}}$  from BBN, circa 2011*, *Phys. Lett. B* **701** (2011) 296 [[arXiv:1103.1261](#)] [[INSPIRE](#)].
- [21] G. Steigman, *Neutrinos and Big Bang nucleosynthesis*, *Adv. High Energy Phys.* **2012** (2012) 268321 [[arXiv:1208.0032](#)] [[INSPIRE](#)].
- [22] PARTICLE DATA GROUP collaboration, K. Nakamura et al., *Review of particle physics*, *J. Phys. G* **37** (2010) 075021 [[INSPIRE](#)].
- [23] M.A. Acero and J. Lesgourgues, *Cosmological constraints on a light non-thermal sterile neutrino*, *Phys. Rev. D* **79** (2009) 045026 [[arXiv:0812.2249](#)] [[INSPIRE](#)].
- [24] E. Giusarma, M. Archidiacono, R. de Putter, A. Melchiorri and O. Mena, *Sterile neutrino models and nonminimal cosmologies*, *Phys. Rev. D* **85** (2012) 083522 [[arXiv:1112.4661](#)] [[INSPIRE](#)].
- [25] M. Archidiacono, E. Calabrese and A. Melchiorri, *The case for dark radiation*, *Phys. Rev. D* **84** (2011) 123008 [[arXiv:1109.2767](#)] [[INSPIRE](#)].
- [26] M. Archidiacono, N. Fornengo, C. Giunti and A. Melchiorri, *Testing  $3+1$  and  $3+2$  neutrino mass models with cosmology and short baseline experiments*, *Phys. Rev. D* **86** (2012) 065028 [[arXiv:1207.6515](#)] [[INSPIRE](#)].
- [27] J. Kopp, M. Maltoni and T. Schwetz, *Are there sterile neutrinos at the eV scale?*, *Phys. Rev. Lett.* **107** (2011) 091801 [[arXiv:1103.4570](#)] [[INSPIRE](#)].
- [28] C. Giunti and M. Laveder,  *$3+1$  and  $3+2$  sterile neutrino fits*, *Phys. Rev. D* **84** (2011) 073008 [[arXiv:1107.1452](#)] [[INSPIRE](#)].
- [29] C. Giunti and M. Laveder, *Status of  $3+1$  neutrino mixing*, *Phys. Rev. D* **84** (2011) 093006 [[arXiv:1109.4033](#)] [[INSPIRE](#)].
- [30] C. Giunti and M. Laveder, *Implications of  $3+1$  short-baseline neutrino oscillations*, *Phys. Lett. B* **706** (2011) 200 [[arXiv:1111.1069](#)] [[INSPIRE](#)].
- [31] G. Karagiorgi, M. Shaevitz and J. Conrad, *Confronting the short-baseline oscillation anomalies with a single sterile neutrino and non-standard matter effects*, [arXiv:1202.1024](#) [[INSPIRE](#)].

- [32] A. Donini, P. Hernández, J. Lopez-Pavon, M. Maltoni and T. Schwetz, *The minimal 3 + 2 neutrino model versus oscillation anomalies*, *JHEP* **07** (2012) 161 [[arXiv:1205.5230](#)] [[INSPIRE](#)].
- [33] W. Fischler and J. Meyers, *Dark radiation emerging after Big Bang nucleosynthesis?*, *Phys. Rev. D* **83** (2011) 063520 [[arXiv:1011.3501](#)] [[INSPIRE](#)].
- [34] J.L. Menestrina and R.J. Scherrer, *Dark radiation from particle decays during Big Bang nucleosynthesis*, *Phys. Rev. D* **85** (2012) 047301 [[arXiv:1111.0605](#)] [[INSPIRE](#)].
- [35] K. Ichikawa, M. Kawasaki, K. Nakayama, M. Senami and F. Takahashi, *Increasing effective number of neutrinos by decaying particles*, *JCAP* **05** (2007) 008 [[hep-ph/0703034](#)] [[INSPIRE](#)].
- [36] K. Choi, K.-Y. Choi and C.S. Shin, *Dark radiation and small-scale structure problems with decaying particles*, *Phys. Rev. D* **86** (2012) 083529 [[arXiv:1208.2496](#)] [[INSPIRE](#)].
- [37] D. Hooper, F.S. Queiroz and N.Y. Gnedin, *Non-thermal dark matter mimicking an additional neutrino species in the early universe*, *Phys. Rev. D* **85** (2012) 063513 [[arXiv:1111.6599](#)] [[INSPIRE](#)].
- [38] C. Boehm, M.J. Dolan and C. McCabe, *Increasing  $N_{\text{eff}}$  with particles in thermal equilibrium with neutrinos*, *JCAP* **12** (2012) 027 [[arXiv:1207.0497](#)] [[INSPIRE](#)].
- [39] M. Lattanzi and J. Valle, *Decaying warm dark matter and neutrino masses*, *Phys. Rev. Lett.* **99** (2007) 121301 [[arXiv:0705.2406](#)] [[INSPIRE](#)].
- [40] Y. Gong and X. Chen, *Cosmological constraints on invisible decay of dark matter*, *Phys. Rev. D* **77** (2008) 103511 [[arXiv:0802.2296](#)] [[INSPIRE](#)].
- [41] K. Ichikawa, T. Sekiguchi and T. Takahashi, *Probing the effective number of neutrino species with cosmic microwave background*, *Phys. Rev. D* **78** (2008) 083526 [[arXiv:0803.0889](#)] [[INSPIRE](#)].
- [42] K. Ichikawa, M. Kawasaki and F. Takahashi, *Constraint on the effective number of neutrino species from the WMAP and SDSS LRG power spectra*, *JCAP* **05** (2007) 007 [[astro-ph/0611784](#)] [[INSPIRE](#)].
- [43] S. Aoyama, K. Ichiki, D. Nitta and N. Sugiyama, *Formulation and constraints on decaying dark matter with finite mass daughter particles*, *JCAP* **09** (2011) 025 [[arXiv:1106.1984](#)] [[INSPIRE](#)].
- [44] M.-Y. Wang and A.R. Zentner, *Effects of unstable dark matter on large-scale structure and constraints from future surveys*, *Phys. Rev. D* **85** (2012) 043514 [[arXiv:1201.2426](#)] [[INSPIRE](#)].
- [45] S. De Lope Amigo, W. M.-Y. Cheung, Z. Huang and S.-P. Ng, *Cosmological constraints on decaying dark matter*, *JCAP* **06** (2009) 005 [[arXiv:0812.4016](#)] [[INSPIRE](#)].
- [46] SUPER-KAMIOKANDE collaboration, S. Desai et al., *Search for dark matter WIMPs using upward through-going muons in Super-Kamiokande*, *Phys. Rev. D* **70** (2004) 083523 [*Erratum ibid.* **D 70** (2004) 109901] [[hep-ex/0404025](#)] [[INSPIRE](#)].
- [47] S. Palomares-Ruiz, *Model-independent bound on the dark matter lifetime*, *Phys. Lett. B* **665** (2008) 50 [[arXiv:0712.1937](#)] [[INSPIRE](#)].
- [48] L. Covi, M. Grefe, A. Ibarra and D. Tran, *Neutrino signals from dark matter decay*, *JCAP* **04** (2010) 017 [[arXiv:0912.3521](#)] [[INSPIRE](#)].

- [49] A. Esmaili, A. Ibarra and O.L. Peres, *Probing the stability of superheavy dark matter particles with high-energy neutrinos*, *JCAP* **11** (2012) 034 [[arXiv:1205.5281](#)] [[INSPIRE](#)].
- [50] O.E. Bjaelde, S. Das and A. Moss, *Origin of  $\Delta N_{\text{eff}}$  as a result of an interaction between dark radiation and dark matter*, *JCAP* **10** (2012) 017 [[arXiv:1205.0553](#)] [[INSPIRE](#)].
- [51] C.-P. Ma and E. Bertschinger, *Cosmological perturbation theory in the synchronous and conformal Newtonian gauges*, *Astrophys. J.* **455** (1995) 7 [[astro-ph/9506072](#)] [[INSPIRE](#)].
- [52] R.J. Scherrer and M.S. Turner, *Decaying particles do not heat up the universe*, *Phys. Rev. D* **31** (1985) 681 [[INSPIRE](#)].
- [53] R.J. Scherrer and M.S. Turner, *Primordial nucleosynthesis with decaying particles. 1. Entropy producing decays. 2. Inert decays*, *Astrophys. J.* **331** (1988) 19 [[INSPIRE](#)].
- [54] M. Kaplinghat, R.E. Lopez, S. Dodelson and R.J. Scherrer, *Improved treatment of cosmic microwave background fluctuations induced by a late decaying massive neutrino*, *Phys. Rev. D* **60** (1999) 123508 [[astro-ph/9907388](#)] [[INSPIRE](#)].
- [55] K. Ichiki, M. Oguri and K. Takahashi, *WMAP constraints on decaying cold dark matter*, *Phys. Rev. Lett.* **93** (2004) 071302 [[astro-ph/0403164](#)] [[INSPIRE](#)].
- [56] M. Lattanzi, *Decaying majoron dark matter and neutrino masses*, *AIP Conf. Proc.* **966** (2007) 163 [[arXiv:0802.3155](#)] [[INSPIRE](#)].
- [57] D. Blas, J. Lesgourgues and T. Tram, *The Cosmic Linear Anisotropy Solving System (CLASS) II: approximation schemes*, *JCAP* **07** (2011) 034 [[arXiv:1104.2933](#)] [[INSPIRE](#)].
- [58] J. Dunkley et al., *The Atacama Cosmology Telescope: cosmological parameters from the 2008 power spectra*, *Astrophys. J.* **739** (2011) 52 [[arXiv:1009.0866](#)] [[INSPIRE](#)].
- [59] E. Shirokoff et al., *Improved constraints on cosmic microwave background secondary anisotropies from the complete 2008 South Pole Telescope data*, *Astrophys. J.* **736** (2011) 61 [[arXiv:1012.4788](#)] [[INSPIRE](#)].
- [60] A.G. Riess et al., *A 3% solution: determination of the Hubble constant with the Hubble Space Telescope and Wide Field Camera 3*, *Astrophys. J.* **730** (2011) 119 [Erratum *ibid.* **732** (2011) 129] [[arXiv:1103.2976](#)] [[INSPIRE](#)].
- [61] SUPERNOVA COSMOLOGY PROJECT collaboration, M. Kowalski et al., *Improved cosmological constraints from new, old and combined supernova datasets*, *Astrophys. J.* **686** (2008) 749 [[arXiv:0804.4142](#)] [[INSPIRE](#)].
- [62] SDSS collaboration, W.J. Percival et al., *Baryon acoustic oscillations in the Sloan Digital Sky Survey data release 7 galaxy sample*, *Mon. Not. Roy. Astron. Soc.* **401** (2010) 2148 [[arXiv:0907.1660](#)] [[INSPIRE](#)].
- [63] J. Hamann, S. Hannestad, J. Lesgourgues, C. Rampf and Y.Y. Wong, *Cosmological parameters from large scale structure — Geometric versus shape information*, *JCAP* **07** (2010) 022 [[arXiv:1003.3999](#)] [[INSPIRE](#)].
- [64] M. Gonzalez-Garcia, M. Maltoni and J. Salvado, *Direct determination of the solar neutrino fluxes from solar neutrino data*, *JHEP* **05** (2010) 072 [[arXiv:0910.4584](#)] [[INSPIRE](#)].
- [65] G.M. Fuller, C.T. Kishimoto and A. Kusenko, *Heavy sterile neutrinos, entropy and relativistic energy production and the relic neutrino background*, [arXiv:1110.6479](#) [[INSPIRE](#)].
- [66] M. Nemevšek, G. Senjanović and Y. Zhang, *Warm dark matter in low scale left-right theory*, *JCAP* **07** (2012) 006 [[arXiv:1205.0844](#)] [[INSPIRE](#)].



- [67] T. Higaki and F. Takahashi, *Dark radiation and dark matter in large volume compactifications*, *JHEP* **11** (2012) 125 [[arXiv:1208.3563](#)] [[INSPIRE](#)].
- [68] M. Cicoli, J.P. Conlon and F. Quevedo, *Dark radiation in LARGE volume models*, *Phys. Rev. D* **87** (2013) 043520 [[arXiv:1208.3562](#)] [[INSPIRE](#)].