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Towards a realistic F-theory GUT

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Abstract: We consider semi-local F-theory GUTs arising from a single E_8 point of local enhancement, leading to simple GUT groups based on E_6, SO(10) and SU(5) with SU(3), SU(4) and SU(5) spectral covers, respectively. Assuming the minimal $\mathcal{Z}_2$ monodromy, we determine the homology classes and the associated spectra after flux breaking for each case. Using these results we construct an E_6 based model that demonstrates, for the first time, that it is possible to construct a phenomenologically viable model which leads to the MSSM at low energies. The exotics that result from flux breaking all get a large mass when singlet fields acquire vacuum expectation values driven by D- and F-flatness. Due to the underlying GUT symmetry and the U(1)s descending from E_8, bare baryon- and lepton-number violating terms are forbidden up to and including dimension 5. As a result nucleon decay is naturally suppressed below present bounds. The μ-term is generated by non-perturbative U(1) breaking effects. After including the effect of flux and instanton corrections acceptable quark and charged lepton masses and mixing angles can be obtained. Neutrinos get a mass from the see-saw mechanism through their coupling to singlet neutrinos that acquire large Majorana mass as a result of the monodromy.

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Contents

1 Introduction

Almost forty years after their inception, Grand Unified Theories (GUTs) [1] remain a tantalising combination of successes and challenges that still provides our best glimpse into the possible unity of all particle forces. By combining GUTs with supersymmetry (SUSY) the possibility of gauge coupling unification at a higher scale remains an attractive possibility, while the hierarchy of scales is stabilised by the non-renormalisation theorem of SUSY, and gauge mediated proton decay is suppressed below current limits. However new dimension-5 operators can occur in SUSY models which may destabilise the proton, and R-parity violating operators (baryon- and lepton-number violating) must be very carefully controlled to avoid phenomenological disasters. In addition the SUSY Higgs/Higgsino mass parameter μ must be forbidden at leading order (the GUT or Planck scale), but then subsequently must be generated at the TeV scale. Moreover SUSY GUTs do not explain why there are three chiral families of quarks and leptons in complete SU(5) representations, with one pair of Higgs doublets, H_u and H_d, in incomplete SU(5) representations (i.e. without their colour triplet SU(5) partners). Nor do they shed much light on the origin of the pattern of Yukawa couplings, although $b - \tau$ unification predicted by SU(5) remains viable.

Recently there has been considerable activity [2–7] in the reformulation of GUTs as 8D theories arising from F-theory versions of string theory [8] (for reviews and related work see e.g. [9–12]). The reason for the renewed interest is that F-theory provides new opportunities for addressing some of the above outstanding issues facing GUTs, such as GUT breaking and Higgs doublet-triplet splitting by flux [4, 5]. The original formulation of such theories on a del Pezzo surface [3] allows for gravity to be decoupled, $M_{\rm GUT} \ll M_{Planck}$, simplifying the analysis of possible effective GUT models. In this the GUT gauge group lives on seven branes wrapping the del Pezzo surface, while quarks and leptons and Higgs live on restricted (complex) matter curves constituting the intersections of this surface with other seven branes. Yukawa interactions occur at points on the surface at the intersection of three matter curves.

Using this structure there has been remarkable progress on model building in F-theory over the last two or three years [13]–[65]. A considerable amount of this work deals with the reconciliation of F-theory models with the low energy Standard Model and the related phenomenology. These include papers related to fermion mass structure and the computation of Yukawa couplings in the context of F-theory and del Pezzo singularities [13, 16, 19, 27–29, 33, 44, 47–49, 58]. In particular, some interesting mechanisms were suggested to generate Yukawa hierarchy either with the use of fluxes [13, 44] and the notion of T-branes [57] or with the implementation of the Froggatt-Nielsen mechanism [27–29, 33, 58]. More specifically, in [44] it is argued that when three-form fluxes are turned on in F-theory compactifications, rank-one fermion mass matrices are modified, leading to masses for lighter generations and CKM mixing. Ibanez et al. [49] have recently shown that flux and instanton effects can generate a realistic hierarchy of fermion masses. In the F-theory context, such non-perturbative contributions were computed in [50], although the magnitude of such corrections remains somewhat unclear.

Larger GUT groups than SU(5) have also been considered, such as the F-theory E_6 model of ref [52] where non-Abelian fluxes are introduced to break the symmetry. Flipped SU(5) [13, 22, 28, 53, 55] has also been considered, including an attempt using an SU(4) spectral cover [54]. Some examples of SO(10) F-theory models were also considered in [13, 34, 35].

Many (or all) of these models predict exotic states below the unification scale, and the renormalization group (RG) analysis of gauge coupling unification including the effect of such states and flux effects has been discussed in a series of papers [38]–[43]. Other phenomenological issues such as neutrinos from KK-modes [46], proton decay [37] and the origin of CP violation [56] have also been discussed. The possibility of obtaining the Standard Model directly from F-theory [45] has also been considered.

Following this work some generic challenges have been identified that result from the highly constrained nature of the constructions, in particular the constraints related to the compatibility of unification (due to the appearance of exotics), the suppression of proton decay (due to R-parity violating operators and dimension-5 operators) the suppression of the μ term and the generation of realistic Yukawa couplings. These occur when flux is used to break the GUT group and generate doublet-triplet splitting. To date no fully realistic model has been constructed using just the symmetries descending from the underlying unified gauge group [27, 58, 63] and this provides additional motivation for the present paper.

In this paper we classify semi-local F-theory GUTs arising from a single E_8 point of local enhancement, leading to simple GUT groups based on E_6, SO(10) and SU(5) on the del Pezzo surface with SU(3), SU(4) and SU(5) spectral covers, respectively. In the semi-local approach to F-theory, it is normally assumed that there exists a single point of E_8 enhancement in the internal geometry [24], from which all the interactions come. We study the matter that descends from the adjoint of E_8 for the following breaking patterns:

$$E_8 \supset E_6 \times \mathrm{SU}(3)_\perp \,,$$
$$E_8 \supset \mathrm{SO}(10) \times \mathrm{SU}(4)_\perp \,,$$
$$E_8 \supset \mathrm{SU}(5) \times \mathrm{SU}(5)_\perp \,.$$

Assuming the minimal case of a $\mathcal{Z}_2$ monodromy, we discuss the flux breaking and homology classes of the spectrum for each case, and provide a dictionary relating the representations of the different GUT groups that can lead to new physical insights into model building. We assume that all breaking of the GUT gauge group to the Standard Model occurs when fluxes associated with the U(1)s in the perpendicular groups are turned on. To determine the chiral spectrum we need to know how the fluxes restrict on the various matter curves. There two kinds of flux that we need to consider. Firstly, we have the fluxes associated with the U(1)s remaining after the imposition of a $\mathcal{Z}_2$ monodromy and the perpendicular gauge group has been broken. These fluxes determine the chirality of the complete GUT representations. Secondly, we have the hypercharge flux inside the GUT group, which breaks the remaining gauge symmetry down to that of the Standard Model. To determine the effect of the flux it is convenient to work in the spectral cover approach. Using this we determine the spectrum after flux breaking. As an example of an application of our results we consider the construction of a viable low-energy-model in which the U(1) symmetries and flux effects answer all the challenges posed above. We start with the identification of R-parity in an E_6 GUT. After flux breaking the model has some undesirable features but it proves possible to eliminate these by relaxing the E_6 constraints on the spectrum. However the dangerous $R-$parity violating operators are still forbidden. In addition the dimension 5 nucleon decay operators, allowed by R-parity, are also forbidden due to the U(1) global symmetries of the model.

Due to the flux breaking the spectrum has additional vector-like states beyond those of the minimal supersymmetric extension of the Standard Model (MSSM). We show that these exotic states get a large mass, close to the compactification scale, if certain SM (and SU(5)) singlet fields acquire vacuum expectation values (VEV). We identify the necessary singlet fields and show that these VEVs are needed for D- and F-flatness of the scalar potential, the VEVs being driven close to the compactification scale. Moreover we show that these VEVs do not re-introduce terms that can give rapid proton decay.

Finally we show that the model may have a realistic structure for the quark and charged lepton masses in which the light generation masses and mixings are driven by flux and instanton effects. The neutrinos can get mass from the (type I) "see-saw" through the coupling of the doublet neutrinos to singlet neutrinos that acquire Majorana mass due to the monodromy.

The layout of the remainder of the paper is as follows. In section 2 we discuss a dictionary connecting the $SU(3)_\perp \times E_6$ and $SU(5)_\perp \times SU(5)$ representations and their $U(1)_\perp$ charges. This proves to be useful when constructing viable models. In section 3, using the spectral cover approach, we determine the homology of the gauge non-singlet and gauge singlet fields for the three breaking patterns given above. We discuss how the homology gives constraints on the spectrum after flux breaking. In section 4 we discuss the D-and F-flatness conditions that apply for the case only the SU(5) singlets acquire VEVs. In section 5 we discuss the construction of a realistic model that, after a definite set of singlet VEVs are switched on, has just the MSSM spectrum and renormalisable couplings. Using the results of section 4 we show that the F-and D-flatness conditions do indeed induce the needed VEVs and we determine their relative magnitude. We show that, due to the U(1) symmetries and the underlying GUT structure, the model avoids dangerous baryon- and lepton-number terms up to and including the dangerous dimension 5 operators. We also discuss how a μ term of the required order is generated. Finally we consider the structure of the quark, charged lepton and neutrino masses and mixings and show that they can be realistic.

2 Group theory dictionary between E_6 and SU(5)

In this paper we are concerned with the sequence of rank preserving symmetry breakings, induced by flux breaking. Starting from the highest allowed symmetry in elliptic fibration, that is the E_8 exceptional group, there exists a variety of breaking patterns to obtain the Standard Model gauge symmetry. A complete classification of these possibilities from

JHEP04(2012)094

the F-theory perspective has been given in the appendix of ref [5]. Here, we shall be interested in the general embeddings discussed in the Introduction, where the adjoint of E_8 decomposes in each case as

$$\begin{aligned} E_8 &\supset E_6 \times \mathrm{SU}(3)_\perp\,, && (2.1)\\ 248 &\to (78,1) + (27,3) + (\overline{27},\overline{3}) + (1,8)\,, && (2.2)\\ E_8 &\supset \mathrm{SO}(10) \times \mathrm{SU}(4)_\perp\,, && (2.3)\\ 248 &\to (1,15) + (45,1) + (10,6) + (16,4) + (\overline{16},\overline{4})\,, && (2.4)\\ E_8 &\supset \mathrm{SU}(5) \times \mathrm{SU}(5)_\perp\,, && (2.5)\\ 248 &\to (24,1) + (1,24) + (10,5) + (\overline{5},10) + (\overline{5},\overline{10}) + (5,\overline{10}). && (2.6) \end{aligned}$$

The last one in particular has been extensively studied by many authors including [5, 21, 26–28]. In this case, the $\mathrm{SU}(5)_{\mathrm{GUT}}$ is the maximal subgroup $\mathrm{SU}(5) \in E_8$ while the corresponding matter content transforms non-trivially under the Cartan subalgebra of $\mathrm{SU}(5)_\perp$ with weight vectors $t_{1,\ldots,5}$ satisfying

$$t_1 + t_2 + t_3 + t_4 + t_5 = 0. \tag{2.7}$$

In principle, the superpotential can be maximally constrained by four U(1)'s according to the breaking pattern

$$E_8 \supset \mathrm{SU}(5) \times \mathrm{SU}(5)_\perp \to \mathrm{SU}(5) \times \mathrm{U}(1)_\perp^4\,. \tag{2.8}$$

The 5 representation of $\mathrm{SU}(5)_\perp$ may be expressed in the conventional basis of the five weight vectors t_i in which the 4 Cartan generators corresponding to $\mathrm{U}(1)_\perp^4$ are expressed as:

$$\begin{aligned} H_1 &= \frac{1}{2}\mathrm{diag}(1,-1,0,0,0), & H_2 &= \frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,-2,0,0),\\ H_3 &= \frac{1}{2\sqrt{6}}\mathrm{diag}(1,1,1,-3,0), & H_4 &= \frac{1}{2\sqrt{10}}\mathrm{diag}(1,1,1,1,-4). \end{aligned} \tag{2.9}$$

In general, however, there is an action on t_i's of a non-trivial monodromy group which is a subgroup of the Weyl group $W(\mathrm{SU}(5)_\perp) = S_5$. Such subgroups are the alternating groups $\mathcal{A}_n$, the dihedral groups $\mathcal{D}_n$ and cyclic groups $\mathcal{Z}_n$, $n \leq 5$ and the Klein four-group $\mathcal{Z}_2 \times \mathcal{Z}_2$. Throughout this paper we shall assume the minimal $\mathcal{Z}_2$ monodromy, $t_1 \leftrightarrow t_2$.

It is of interest to consider the possibility of a sequence of flux breaking, which may be associated with different scales. Here we consider the sequence

$$\begin{aligned} E_8 &\to E_6 \times \mathrm{U}(1)_\perp^2 && (2.10)\\ &\to \mathrm{SO}(10) \times \mathrm{U}(1)_\psi \times \mathrm{U}(1)_\perp^2 && (2.11)\\ &\to \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi \times \mathrm{U}(1)_\perp^2. && (2.12) \end{aligned}$$

E_6	SO(10)	SU(5)	Weight vector
$27_{t_1'}$	16	$\overline{5}_3$	t_1+t_5
$27_{t_1'}$	16	10_M	t_1
$27_{t_1'}$	16	θ_{15}	t_1-t_5
$27_{t_1'}$	10	5_1	$-t_1-t_3$
$27_{t_1'}$	10	$\overline{5}_2$	t_1+t_4
$27_{t_1'}$	1	θ_{14}	t_1-t_4
$27_{t_3'}$	16	$\overline{5}_5$	t_3+t_5
$27_{t_3'}$	16	10_2	t_3
$27_{t_3'}$	16	θ_{35}	t_3-t_5
$27_{t_3'}$	10	5_{H_u}	$-2t_1$
$27_{t_3'}$	10	$\overline{5}_4$	t_3+t_4
$27_{t_3'}$	1	θ_{34}	t_3-t_4

Table 1. Complete 27s of E_6 and their SO(10) and SU(5) decompositions. For the SU(5) states we use the notation of ref [27] where indices in $5_i, 10_j$ representations are associated to the corresponding matter curves $\Sigma_{5_i}, \Sigma_{10_j}$.

which for the E_6 representations gives

$$\begin{aligned}
78 \to\ & [24_{(0,0)} + 10_{(4,0)} + \overline{10}_{(-4,0)} + 1_{(0,0)}]_{45} \\
& +[10_{(-1,-3)} + \overline{5}_{(3,-3)} + 1_{(-5,-3)}]_{16} \\
& +[\overline{10}_{(1,3)} + 5_{(-3,3)} + 1_{(5,3)}]_{\overline{16}} \\
& +[1_{(0,0)}]_1 \\
27 \to\ & [10_{(-1,1)} + \overline{5}_{(3,1)} + 1_{(-5,1)}]_{16} \\
& +[5_{(2,-2)} + \overline{5}_{(-2,-2)}]_{\overline{10}} \\
& +[1_{(0,4)}]_1\,, \qquad (2.13)
\end{aligned}$$

where the subscripts refer to the $U(1)_\chi, U(1)_\psi$ charges and SO(10) representation respectively. It is convenient to choose a basis for the weight vectors such that the charge generators have the form

$$\begin{aligned}
Q_\chi &\propto \text{diag}[-1,-1,-1,-1,4] \\
Q_\psi &\propto \text{diag}[1,1,1,-3,0] \\
Q_\perp &\propto \text{diag}[1,1,-2,0,0]\,. \qquad (2.14)
\end{aligned}$$

where $Q_\perp$ is the charge of the $U(1)_\perp$ in the breaking pattern of eq. (2.10) that remains after imposing the $t_1 \leftrightarrow t_2$ monodromy. This is in fact the same as the conventional basis for the $SU(5)_\perp$ generators in eq. (2.9), and the normalisation of the generators is given by identifying,

$$H_1 = H_1', \quad H_2 = Q_\perp, \quad H_3 = Q_\psi, \quad H_4 = -Q_\chi. \qquad (2.15)$$

This almost trivial equivalence shows that the $\mathrm{SU}(5)_{\mathrm{GUT}}$ states in eq. (2.6) have well defined E_6 charges Q_χ and Q_ψ. For example SU(5) singlets will in general carry Q_χ and Q_ψ charges which originate from E_6 and which may be unbroken. The equivalence will provide insights into both anomaly cancellation and the origin of R-parity for example, in terms of the underlying E_6 structure, in the explicit models discussed later. Throughout this paper we shall assume the minimal $\mathcal{Z}_2$ monodromy, $t_1 \leftrightarrow t_2$ [29] which trivially corresponds to the minimal $\mathcal{Z}_2$ monodromy, $t'_1 \leftrightarrow t'_2$. It is clear from eq. (2.9) that this corresponds to $H_1 = H'_1$ being broken leaving only three independent Cartan symmetries $\{H_2, H_3, H_4\}$ or equivalently $\{Q_\perp, Q_\psi, Q_\chi\}$.

In this basis the weight vectors t'_1, t'_2, t'_3 $(t'_1 + t'_2 + t'_3 = 0)$ of $\mathrm{SU}(3)_\perp$ are related to the $\mathrm{SU}(5)_\perp$ weight vectors by $t'_i = t_i + (t_4 + t_5)/3, \;\; i = 1, 2, 3$. As an example of the use of this dictionary that will play an important role when building a realistic theory we can now connect the two independent representations $27_{t'_1}$ and $27_{t'_3}$ that appear in the E_6 breaking pattern of eq. (2.10) to the SU(5) representations of eq. (2.12). These are shown in table 1 with SU(5) states given in the notation of [27].

3 Flux breaking and matter content in F-theory GUTs

In this section we determine the light matter content that results if the underlying E_8 is broken to some subgroup by a Higgs bundle on the del-Pezzo surface S [21]. We are interested in the cases that the unbroken gauge group is E_6, SO(10) or SU(5). One reason to study these patterns of breaking is because subsequent breaking to the Standard Model may proceed via the normal Higgs mechanism with fields acquiring VEVs along flat directions. In this knowledge of the multiplet content before such breaking is crucial. A second reason to study these patterns is because it can suggest promising phenomenological models based on a high degree of unification even though they are subsequently further broken by flux to just the Standard Model. We will present a viable model in the next section.

We proceed by studying the spectral cover of the transverse groups for the three cases of interest $E_6 \times \mathrm{SU}(3)_\perp$, $\mathrm{SO}(10) \times \mathrm{SU}(4)_\perp$ and $\mathrm{SU}(5) \times \mathrm{SU}(5)_\perp$. This will allow us to determine the homology of the matter fields and hence the effect of flux breaking. In dealing with singlets, we note that for a given surface S with associated singularity G_S, there are singlet fields residing on curves that extend away from S and can be affected by $\mathrm{U}(1)_\perp$ fluxes not supported by S. There are also singlet fields emerging from the decomposition of GUT representations after the breaking of the covering group G_S by the flux mechanism. The latter singlets localise on certain line bundles of the corresponding surface S and as a consequence they are affected by the fluxes breaking G_S. In this case the homologies of the corresponding matter curves can be determined and, as shown in this paper, certain properties including chirality and multiplicities can be expressed in terms of a few integers parameterising the associated U(1) fluxes.

3.1 $\mathrm{SU}(3)_\perp$ spectral cover

E_6 models are quite attractive and have been extensively studied in compactifications on Calabi-Yau manifolds, in the context of the heterotic superstring with underlying $E_8 \times$

E_8 symmetry (see [66–68] and references therein). Furthermore, recent phenomenological investigations based on string motivated versions with E_6 gauge symmetry have inspired the exceptional supersymmetric standard model [69]. This is distinguished from the minimal one by the appearance of an additional Z' boson and extra matter content at the TeV scale. Interestingly, although these new ingredients are also potentially present in the F-theory E_6-analogue, they are subject to constraints from flux restrictions on matter curves and the topological properties of the compact manifold, and in the model considered later the Z' boson has a GUT scale mass and the extra matter also has a similarly large mass.

In the context of F-theory in which the GUT group on the brane is E_6, we need to look at the breaking

$$E_8 \to E_6 \times \mathrm{SU}(3)_\perp \,. \tag{3.1}$$

We can determine what matter curves arise by decomposing the adjoint of E_8 as follows

$$248 \to (78,1) + (27,3) + (\overline{27},\overline{3}) + (1,8)\,. \tag{3.2}$$

The E_6 content consists of three 27s (and $\overline{27}s$) plus eight singlet matter curves. In terms of the weight vectors t_i, i=1,2,3, of $\mathrm{SU}(3)_\perp$ the equations of these curves are

$$\Sigma_{27} : t_i = 0\,, \tag{3.3}$$

$$\Sigma_1 : \pm(t_i - t_j) = 0 \quad i \neq j\,. \tag{3.4}$$

The SU(3) spectral cover is found by determining two sections U, V of the projective bundle $\mathcal{P}(\mathcal{O} \oplus K_S)$ over the compact surface S. Let c_1 be the 1^{st} Chern class of the *tangent* bundle to S_{GUT} and $c_1(NS) = -t$ that of the *normal* bundle. Note that the homological classes are $[U] = -t$ and $[V] = c_1 - t$. The SU(3) spectral cover is

$$\mathcal{C}^3 : \; b_0 U^3 + b_2 U V^2 + b_3 V^3 = 0\,. \tag{3.5}$$

Associated to this is the polynomial

$$P_3 = \sum_{k=0}^{3} b_k s^{3-k} \; = \; b_3 + b_2 s + b_1 s^2 + b_0 s^3\,, \tag{3.6}$$

where we have introduced the affine parameter $s = \frac{U}{V}$.

We define for convenience $\eta = 6c_1 - t$ and, as usual, we demand that the coefficients b_k are sections of

$$b_k : \; [b_k] = \eta - k\, c_1\,, \tag{3.7}$$

where k spans the integers $k = 1, 2, 3, 4, 5$. The roots of the spectral cover equation

$$0 = b_3 + b_2 s + b_0 s^3 \propto \prod_{i=1}^{3}(s + t'_i)$$

are identified as the $\mathrm{SU}(3)_\perp$ weight vectors t'_i. In the above the coefficient b_1 is taken to be zero since it corresponds to the sum of the roots which, for SU(n), is always zero, $\sum_i t'_i = 0$.

JHEP04(2012)094

3.1.1 27 and $\overline{27}$ fields

The coefficient b_3 is equal to the product of the roots, i.e. $b_3 = t'_1 t'_2 t'_3$ and the Σ_{27} curves where the corresponding matter multiplets are localized are determined by its three zeros

$$\Sigma_{27_i}, \quad b_3 = \prod_{i=1}^{3} t'_i = 0 \to t'_i = 0, \quad i = 1, 2, 3\,. \tag{3.8}$$

To obtain different curves for 27's we need to split the spectral cover. (If the polynomial is not factorized, there is only one matter curve). There are two possible ways to split a third degree polynomial: either to a binomial-monomial $(2-1)$ or to three monomials $(1-1-1)$. Since we need to impose a monodromy action, we choose this to be $\mathcal{Z}_2$ and therefore we get a $(2-1)$ split. The $\mathcal{Z}_2$ monodromy corresponds to the following split of the spectral cover equation

$$\begin{aligned} 0 = \Pi_3(s) &= (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s) \\ &= a_1 a_4 + (a_2 a_4 + a_1 a_5)s + (a_2 a_5 + a_3 a_4)s^2 + a_3 a_5 s^3\,, \end{aligned} \tag{3.9}$$

with $s = U/V$ and a_i coefficients, constituting sections of line bundles each of them being of specific Chern class to be determined.

The first bracket contains the polynomial factor that corresponds to the $\mathcal{Z}_2$ monodromy $t'_1 \leftrightarrow t'_2$, so that the corresponding two Σ_{27} curves lift to a common one in the spectral cover. The Σ_{27} curves are found setting $s = 0$ in the polynomial

$$b_3 \equiv \Pi_3(0) = a_1 a_4 = 0 \to a_1 = 0,\, a_4 = 0\,.$$

Thus, after the monodromy action, we obtain two matter curves. When building a realistic theory it is necessary to assign the three families of quarks and leptons and the Higgs to these curves. As there are more than one way to do this, the optimal choice will be dictated by phenomenology.

To determine the distribution of families and Higgs on the two matter curves we need to know how the flux restricts on the available curves. To do this we first determine their homology classes $[a_k]$ corresponding to the sections a_k, $k = 1, 4$. This can be done comparing the coefficients of eqs. (3.6), (3.9). We get

$$\begin{aligned} b_0 &= a_3 a_5\,, \\ b_1 &= a_2 a_5 + a_3 a_4 = 0\,, \\ b_2 &= a_2 a_4 + a_1 a_5\,, \\ b_3 &= a_1 a_4\,. \end{aligned}$$

The homology classes $[b_k]$ of the sections b_k are given in eq. (3.7), while those of a_i can be determined by the system of linear equations in one to one correspondence with the above relations. This linear system consists of four equations with five unknowns $[a_i]$, therefore we can solve the system in terms of one arbitrary parameter. Let a_5 be of some unspecified homology class $[a_5] = \chi$. For the remaining a_i, we find that they are sections of

$$[a_1] = \eta - 2c_1 - \chi, [a_2] = \eta - \chi - c_1,\ [a_3] = \eta - \chi,\ [a_4] = \chi - c_1,\ [a_5] = \chi\,. \tag{3.10}$$

Matter	Section	Homology
$27_{t_{1,2}}$	a_1	$\eta - 2c_1 - \chi$
27_{t_3}	a_4	$\chi - c_1$

Table 2. The three columns show the quantum numbers of matter curves under $E_6 \times \mathrm{U}(1)_{t_i}$, the section and the homology class.

For the two curves we obtain the results of table 2. For the homology classes of the two curves $\mathcal{C}^3 = \mathcal{C}_{t_{1,2}}\mathcal{C}_{t_3}$ from eq. (3.5) we get

$$\mathcal{C}_{t_{1,2}} = a_1 V^2 + a_2 UV + a_2 U^2 \,, \tag{3.11}$$

$$\mathcal{C}_{t_3} = a_4 V + a_5 U \,, \tag{3.12}$$

so that their homology classes are given by

$$[\mathcal{C}_{t_{1,2}}] = \eta - \chi - 2\,t, \quad [\mathcal{C}_{t_3}] = \chi - t \,.$$

Using the data of table 2, we can turn on a $\mathcal{F}_{\mathrm{U}(1)}$ flux on the external U(1) and find the restriction on the curves of 27's:

$$n_{t_1} = \mathcal{F}_{\mathrm{U}(1)} \cdot (\eta - \chi - 2c_1) \;;\; n_{t_3} = \mathcal{F}_{\mathrm{U}(1)} \cdot (\chi - c_1) \,. \tag{3.13}$$

These determine the chiral content of states arising from the decomposition of 27's along the matter curves. We have also seen that χ is some unspecified homology class (associated to a_5) and it can be chosen at will. For acceptable choices it can be seen from table 2 the two curves cannot be of the same homology class. Since the two curves belong to different homology classes, in general flux restricts differently on them. The two conditions can be combined as follows

$$n_{t_3} + n_{t_1} = \mathcal{F}_{\mathrm{U}(1)} \cdot (\eta - 3c_1) = \mathcal{F}_{\mathrm{U}(1)} \cdot (3c_1 - t) \,. \tag{3.14}$$

From eq. (3.14) we deduce that if $\mathcal{F}_{\mathrm{U}(1)} \cdot (3c_1 - t) = 0$, then $n_{t_3} = -n_{t_1}$ i.e., we get opposite flux restrictions on 27_{t_1} and 27_{t_3}. Notice that the choice $\mathcal{F}_{\mathrm{U}(1)} \cdot c_1 \neq 0$ implies that the corresponding gauge boson becomes massive. This is not a problem however, for the extra U(1)'s that do not participate in the hypercharge definition.[1]

3.1.2 E_6 singlets

Singlet fields are important for the construction of the low energy effective field theory model. Some of them may develop VEVs that can be used to create mass terms for the fermion generations and make massive other potentially dangerous fields mediating proton decay. In certain models, those carrying charges under the weights t_i' undergoing a monodromy action can play the role of the right handed neutrino [46]. The E_6 singlets θ_{ij}

[1] For a recent work on the U(1) symmetries in F-theory see [65].

lie in the $t'_i - t'_j$ directions of the corresponding Cartan subalgebra, and because of their central role in phenomenology, it would be useful to determine their homology classes. If the worldvolume theory on S has gauge group E_6, these singlets θ_{12}, θ_{13} and θ_{23} are localised on curves which do not lie within the surface S, and as such, spectral cover analysis can no longer be used to determine their properties.

It should also be noted that the discrete group $\mathcal{Z}_2$ which identifies $t_1 \leftrightarrow t_2$ leads also to the identification of the singlet fields $\theta_{12} \leftrightarrow \theta_{21}$. This will also lead to geometric identification of the corresponding matter curves in the covering theory. Therefore these singlets carry no U(1)-charges and are treated as moduli of the spectral cover and differently from the θ_{13} singlet fields, in accordance with previous studies [21, 24].

3.2 SU(4)$_\perp$ spectral cover

SO(10) GUT is one of the most promising Unified Theories and the smallest one incorporating the right-handed neutrino into the same multiplet with the remaining fundamental particles (quarks and leptons). For the case that the GUT group on the brane is SO(10) we need to consider the breaking

$$E_8 \to \mathrm{SO}(10) \times \mathrm{SU}(4)_\perp\,. \tag{3.15}$$

We can determine which matter curves arise from the decomposition of the adjoint of E_8:

$$248 \to (1,15) + (45,1) + (10,6) + (16,4) + (\overline{16},\overline{4})\,. \tag{3.16}$$

Thus there are four 16 (and $\overline{16}$) matter curves, six 10 matter curves, and fifteen singlets. The equations for these curves in terms of the weight vectors t_i, i=1,2,3,4, of SU(4)$_\perp$ are

$$\Sigma_{16} : t_i = 0\,, \tag{3.17}$$

$$\Sigma_{10} : (-t_i - t_j) = 0, \quad i \neq j\,, \tag{3.18}$$

$$\Sigma_1 : \pm(t_i - t_j) = 0 \quad i \neq j\,, \tag{3.19}$$

where $\sum_i t_i = 0$. In order to determine how fluxes restrict on these matter curves, taking into account the effects of monodromy, the spectral cover approach is used. The SU(4) spectral cover is a hypersurface given by the constraint

$$\mathcal{C}^4 : \;\; b_0 U^4 + b_1 V U^3 + b_2 V^2 U^2 + b_3 V^3 U + b_4 V^4 = 0\,. \tag{3.20}$$

We can set an affine parameter $s = U/V$ which eq. (3.20) is a polynomial in, and whose 5 roots are the t_i. This s can be equated with the value of the Higgs field that breaks the E_8 gauge theory. In terms of s, we have:

$$\begin{aligned} \mathcal{C}^4 &= b_0 s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4 = 0\,, & (3.21)\\ &= b_0(s+t_1)(s+t_2)(s+t_3)(s+t_4) = 0\,, & (3.22) \end{aligned}$$

where the second line reflects the fact that the t_i are the roots of the polynomial. This polynomial describes the 16 matter curves, which are given by setting s to zero in the above equations, leading to $b_4 = 0$. Equations for the b's in terms of the t's can be found by comparing powers of s in eqs. (3.21) and (3.22). This leads to the following equations, once t_4 has been eliminated by using the fact that the sum of the t_i is zero:

$$b_1 = -b_0(t_1 + t_2 + t_3 + t_4) = 0\,, \tag{3.23}$$
$$b_2 = b_0(t_1^2 + t_2^2 + t_3^2 + t_1 t_2 + t_2 t_3 + t_1 t_3)\,, \tag{3.24}$$
$$b_3 = b_0(t_1 + t_2)(t_2 + t_3)(t_1 + t_3)\,, \tag{3.25}$$
$$b_4 = -b_0 t_1 t_2 t_3 (t_1 + t_2 + t_3)\,. \tag{3.26}$$

It can be seen that the equation $b_4 = 0$ does indeed reproduce eq. (3.17) for the 16 matter curves in terms of the t_i.

3.2.1 $\mathcal{Z}_2$ monodromy

Imposing a $\mathcal{Z}_2$ monodromy implies the splitting of eq. (3.21) as follows

$$\mathcal{C}^4 = (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)\,. \tag{3.27}$$

The first bracket is quadratic in s reflecting the fact that we have chosen a $\mathcal{Z}_2$ monodromy, which in the weight language corresponds to an identification of two weights $t_1 \leftrightarrow t_2$. We can now match powers of s in eqs. (3.21) and (3.27) to get equations for the b_i in terms of the a_i.

$$b_0 = a_3 a_5 a_7\,, \tag{3.28}$$
$$b_1 = a_2 a_5 a_7 + a_3 a_5 a_6 + a_3 a_4 a_7\,, \tag{3.29}$$
$$b_2 = a_1 a_5 a_7 + a_2 a_4 a_7 + a_2 a_5 a_6 + a_3 a_4 a_6\,, \tag{3.30}$$
$$b_3 = a_1 a_4 a_7 + a_1 a_5 a_6 + a_2 a_4 a_6\,, \tag{3.31}$$
$$b_4 = a_1 a_4 a_6\,. \tag{3.32}$$

Solving for $b_1 = 0$ gives[2]

$$a_2 = -\gamma(a_5 a_6 + a_4 a_7)\,, \tag{3.33}$$
$$a_3 = \gamma a_5 a_7\,, \tag{3.34}$$

where γ is unspecified. Now we can demand that the homology classes of the b_n are

$$[b_n] = \eta - n c_1\,, \tag{3.35}$$

where, as before, $\eta = 6c_1 - t$, c_1 is the first Chern class of the tangent bundle to S_{GUT} and $-t$ is the first Chern class of the normal bundle. We can now determine the homology classes of the a_i coefficients by using eqs. (3.28)–(3.32), setting the homology class of a

[2] It is understood that some solutions of the $b_1 = 0$ constraint might lead to additional degeneracies. However, for each case in the paper, we pick up the solution which leads to acceptable factorization, avoiding non-Kodaira singularities. We are also aware that subtleties could in principle appear on split spectral covers. However, we mainly concentrate on general phenomenological issues of F-GUT model building, and it is not our intention to address all these issues in the present paper.

JHEP04(2012)094

Coefficient	Homology
a_1	$\eta - 2c_1 - \tilde{\chi}$
a_2	$\eta - c_1 - \tilde{\chi}$
a_3	$\eta - \tilde{\chi}$
a_4	$-c_1 + \chi_5$
a_5	χ_5
a_6	$-c_1 + \chi_7$
a_7	χ_7

Table 3. Homology classes of the a_i coefficients.

given b_n equal to the homology class of each product of a_is on the left hand side of the appropriate equation. This leads to

$$\eta = [a_3] + [a_5] + [a_7]\,, \tag{3.36}$$
$$\eta - c_1 = [a_2] + [a_5] + [a_7]\,, \tag{3.37}$$
$$\eta - 2c_1 = [a_1] + [a_5] + [a_7]\,, \tag{3.38}$$
$$\eta - 3c_1 = [a_1] + [a_4] + [a_7]\,, \tag{3.39}$$
$$\eta - 4c_1 = [a_1] + [a_4] + [a_6]\,. \tag{3.40}$$

As such, we have 5 equations in 7 unknowns, and so we can solve the equations in terms of two free parameters, which we can set as

$$[a_5] = \chi_5\,, \tag{3.41}$$
$$[a_7] = \chi_7\,, \tag{3.42}$$
$$\tilde{\chi} = \chi_5 + \chi_7\,. \tag{3.43}$$

Solving the system of equations gives the homology classes of the remaining a_i

$$[a_1] = \eta - 2c_1 - \tilde{\chi}\,, \tag{3.44}$$
$$[a_2] = \eta - c_1 - \tilde{\chi}\,, \tag{3.45}$$
$$[a_3] = \eta - \tilde{\chi}\,, \tag{3.46}$$
$$[a_4] = -c_1 + \chi_5\,, \tag{3.47}$$
$$[a_6] = -c_1 + \chi_7\,. \tag{3.48}$$

We now have determined the homology classes of all the a_i coefficients (which are summarised in table 3), and can use them in order to find the homology classes of the matter curves.

JHEP04(2012)094

3.2.2 Homology of the 16 matter curves

As discussed after eq. (3.22), the 16 matter curves are given by $b_4 = 0$. From eq. (3.32), this means that the equations of the 16s are

$$a_1 = 0, \quad a_4 = 0, \quad a_6 = 0 \tag{3.49}$$

and so the homology classes are

$$[16_1] = \eta - 2c_1 - \tilde{\chi}\,, \tag{3.50}$$
$$[16_2] = -c_1 + \chi_5\,, \tag{3.51}$$
$$[16_3] = -c_1 + \chi_7\,. \tag{3.52}$$

3.2.3 Homology of the 10 matter curves

Just as the correct polynomial to describe the 16 matter curves was the spectral cover polynomial, the polynomial for the 10s is given by

$$\begin{aligned} P_{10} &= b_0^2 \prod_{i<j} (s + t_i + t_j) \\ &= b_0^2 (s - t_1 - t_2)(s + t_1 + t_2)(s - t_1 - t_3)(s + t_1 + t_3)(s - t_2 - t_3)(s + t_2 + t_3) \quad (3.53) \\ &= s^6 + c_1 s^5 + c_2 s^4 + c_3 s^3 + c_4 s^2 + c_5 s + c_6\,, \quad (3.54) \end{aligned}$$

where in eq. (3.53), t_4 has been eliminated by using $\sum_i t_i = 0$. Comparing coefficients of s between eqs. (3.53) and (3.54) the following equations for the c_i in terms of the t_i are obtained

$$c_1 = 0 \tag{3.55}$$
$$c_2 = -2(t_1^2 + t_2^2 + t_3^2 + t_1 t_2 + t_1 t_3 + t_2 t_3) b_0^2 \tag{3.56}$$
$$c_3 = 0 \tag{3.57}$$
$$\begin{aligned} c_4 = [&t_1^4 + 2t_1^3(t_2 + t_3) + (t_2^2 + t_2 t_3 + t_3^2)^2 + t_1^2(3t_2^2 + 8t_2 t_3 + 3t_3^2) \\ &+ 2t_1(t_2^3 + 4t_2^2 t_3 + 4t_2 t_3^2 + t_3^3)] b_0^2 \end{aligned} \tag{3.58}$$
$$c_5 = 0 \tag{3.59}$$
$$c_6 = -(t_1 + t_2)^2 (t_1 + t_3)^2 (t_2 + t_3)^2 b_0^2\,. \tag{3.60}$$

We can now use eqs. (3.23)–(3.26) to write the c_i coefficients in terms of the b_i. The results are

$$c_2 = -2b_0 b_2\,, \tag{3.61}$$
$$c_4 = b_2^2 - 4b_4 b_0\,, \tag{3.62}$$
$$c_6 = -b_3^2\,. \tag{3.63}$$

Substituting into eq. (3.54) gives

$$P_{10} = s^6 - 2b_0 b_2 s^4 + (b_2^2 - 4b_4 b_0) s^2 - b_3^2\,. \tag{3.64}$$

Matter	Equation	Homology	$\mathrm{U}(1)_X$
$16_{t_{1,2}}$	a_1	$\eta - 2c_1 - \tilde{\chi}$	$M - P$
16_{t_3}	a_4	$-c_1 + \chi_5$	P_5
16_{t_4}	a_6	$-c_1 + \chi_7$	P_7
$10_{(t_1+t_3)}$	$a_1 - \gamma a_4 a_6$	$\eta - 2c_1 - \tilde{\chi}$	$M - P$
$10_{(t_1+t_2)}$	$a_5 a_6 + a_4 a_7$	$-c_1 + \tilde{\chi}$	P
$10_{(t_1+t_4)}$	$a_1 - \gamma a_4 a_6$	$\eta - 2c_1 - \tilde{\chi}$	$M - P$
$10_{(t_3+t_4)}$	$a_5 a_6 + a_4 a_7$	$-c_1 + \tilde{\chi}$	P

Table 4. 16 and 10 matter curves and their equations and homology classes.

As in the case of the 16 polynomial, the 10 matter curves are found by setting s to zero in this equation, giving $b_3^2 = 0$. In order to know the equations and homology classes for the 10 matter curves when the monodromy is imposed, we must express this equation in terms of the a_i coefficients. From eq. (3.31), we know b_3 in terms of the a_i. Substituting eq. (3.33) in for a_2 leads to

$$b_3 = (a_5 a_6 + a_4 a_7)(a_1 - \gamma a_4 a_6)\,. \tag{3.65}$$

As such, the 10 matter curves are defined by the equation

$$(a_5 a_6 + a_4 a_7)(a_1 - \gamma a_4 a_6)(a_5 a_6 + a_4 a_7)(a_1 - \gamma a_4 a_6) = 0\,. \tag{3.66}$$

We therefore have four 10 matter curves, two of which have homology class $[a_1] = \eta - 2c_1 - \tilde{\chi}$, and two of which have homology class $[a_5 a_6] = [a_5] + [a_6] = -c_1 + \tilde{\chi}$. The information about the homology classes of all the 16 and 10 matter curves is summarised in table 4. For convenience, the following notation is introduced

$$M = \mathcal{F}_1 \cdot (\eta - 3c_1)\,, \tag{3.67}$$
$$P = \mathcal{F}_1 \cdot (\chi - c_1)\,, \tag{3.68}$$
$$P_n = \mathcal{F}_1 \cdot (\chi_n - c_1)\,, \tag{3.69}$$
$$C = \mathcal{F}_1 \cdot (-c_1)\,. \tag{3.70}$$

3.2.4 Homology of the SO(10) singlets

We have already pointed out that singlet fields can play a decisive role in building the low energy effective model. If the worldvolume theory on S is seen to have gauge group SO(10), then the same argument about singlets applies as before. The SO(10) singlets will reside on curves which extend away from S, forbidding us from computing the homology classes in the local prescription. If we look at a model where the worldvolume group on S is E_6 however, only the singlets θ_{12} and θ_{13} do not live on S. The other SO(10) singlets then have the homologies of the 27 curves which they originate from in the E_6 formalism.

3.3 $\mathrm{SU}(5)_\perp$ spectral cover

Our final investigation in the present work concerns the $\mathrm{SU}(5)_{\mathrm{GUT}}$. Considering again the maximal symmetry E_8, the spectral cover encoding the relevant information (bundle structure etc.) is associated to the commutant of the GUT group, which is $\mathrm{SU}(5)_\perp$. Hence, in this case the breaking pattern is

$$E_8 \to \mathrm{SU}(5) \times \mathrm{SU}(5)_\perp \,. \tag{3.71}$$

This case has been extensively studied and the homology of the gauge non-singlets determined. The associated adjoint representation decomposition is

$$248 \to (24,1) + (1,24) + (10,5) + (\overline{5},10) + (\overline{5},\overline{10}) + (5,\overline{10})\,. \tag{3.72}$$

Although this case has been analysed by many authors in the recent F-theory model building literature, a detailed examination of the breaking mechanism of the higher intermediate symmetries and possible implications is still lacking. In the following we attempt to implement the constraints obtained from the previous symmetry breaking stages into the $\mathrm{SU}(5)_{\mathrm{GUT}}$ model.

To start with, we recall that the global model is assumed in the context of elliptically fibered Calabi-Yau compact complex fourfold over a three-fold base. Using the Tate's algorithm [70, 71], the $\mathrm{SU}(\mathbf{5})$ singularity can be described by the following form of Weierstrass' equation [2]

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y\,.$$

We determine the corresponding spectral cover by defining homogeneous coordinates

$$z \to U,\ x \to V^2,\ y \to V^3$$

so that the spectral cover equation becomes

$$0 = b_0 U^5 + b_2 V^2 U^3 + b_3 V^3 U^2 + b_4 V^4 U + b_5 V^5\,.$$

We can see this equation as a fifth degree polynomial in terms of the affine parameter $s = U/V$:

$$P_5 = \sum_{k=0}^{5} b_k s^{5-k} \ = \ b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_1 s^4 + b_0 s^5\,,$$

where we have divided by V^5, so that each term in the last equation becomes section of $c_1 - t$. The roots of the spectral cover equation.

$$0 = b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 \propto \prod_{i=1}^{5}(s + t_i)$$

are identified as the $\mathrm{SU}(5)$ weights t_i.

In the above the coefficient b_1 is taken to be zero since it corresponds to the sum of the roots which for $\mathrm{SU}(n)$ is always zero, $\sum t_i = 0$. Also, it can be seen that the coefficient

b_5 is equal to the product of the roots, i.e. $b_5 = t_1t_2t_3t_4t_5$ and the Σ_{10} curves where the corresponding matter multiplets are localized are determined by the five zeros

$$\Sigma_{10_i}, \quad b_5 = \prod_{i=1}^{5} t_i = 0 \to t_i = 0, \quad i = 1, 2, 3, 4, 5\,. \tag{3.73}$$

Following [29], we impose the $\mathcal{Z}_2$ monodromy corresponding to the following splitting of the spectral cover equation

$$0 = (a_1 + a_2 s + a_3 s^2)(a_4 + a_5 s)(a_6 + a_7 s)(a_8 + a_9 s)\,, \tag{3.74}$$

with $s = U/V$ and a_i undetermined coefficients, constituting sections of line bundles each of them being of specific Chern class to be determined. The first bracket contains the polynomial factor which corresponds to the $\mathcal{Z}_2$ monodromy, while the remaining monomials leave three U(1)'s intact. Expanding, we may determine the homology class for each of the coefficients a_i by comparison with the b_k's. Thus,

$$\begin{aligned}
b_0 &= a_3a_5a_7a_9\,,\\
b_1 &= a_3a_5a_7a_8 + a_3a_4a_9a_7 + a_2a_5a_7a_9 + a_3a_5a_6a_9\,,\\
b_2 &= a_3a_5a_6a_8 + a_2a_5a_8a_7 + a_2a_5a_9a_6 + a_1a_5a_9a_7 + a_3a_4a_7a_8 + a_3a_4a_6a_9 + a_2a_4a_7a_9\,,\\
b_3 &= a_3a_4a_8a_6 + a_2a_5a_8a_6 + a_2a_4a_8a_7 + a_1a_7a_8a_5 + a_2a_4a_6a_9 + a_1a_5a_6a_9 + a_1a_4a_7a_9\,,\\
b_4 &= a_2a_4a_8a_6 + a_1a_5a_8a_6 + a_1a_4a_8a_7 + a_1a_4a_6a_9\,,\\
b_5 &= a_1a_4a_6a_8\,.
\end{aligned}$$

We first solve the constraint $b_1 = 0$. We make the Ansatz:

$$a_2 = -c(a_5a_7a_8 + a_4a_9a_7 + a_5a_6a_9),\;\; a_3 = ca_5a_7a_9$$

Substituting into b_n's we get

$$\begin{aligned}
b_0 &= c\, a_5^2a_7^2a_9^2\,,\\
b_2 &= a_1a_5a_7a_9 - \left(a_5^2a_7^2a_8^2 + a_5a_7\,(a_5a_6 + a_4a_7)\,a_9a_8 + \left(a_5^2a_6^2 + a_4a_5a_7a_6 + a_4^2a_7^2\right)a_9^2\right)c\,,\\
b_3 &= a_1\,(a_5a_7a_8 + a_5a_6a_9 + a_4a_7a_9) - (a_5a_6 + a_4a_7)\,(a_5a_8 + a_4a_9)\,(a_7a_8 + a_6a_9)\,c\,,\\
b_4 &= a_1\,(a_5a_6a_8 + a_4a_7a_8 + a_4a_6a_9) - a_4a_6a_8\,(a_5a_7a_8 + a_5a_6a_9 + a_4a_7a_9)\,c\,,\\
b_5 &= a_1a_4a_6a_8\,.
\end{aligned}$$

Next, we observe that we have to determine the homology classes of nine unknowns $a_1, \dots a_9$ in terms of the b_k-classes, which we demand to be $\eta - kc_1$. Three classes are left unspecified which we choose them to be $[a_l] = \chi_l, l = 5, 7, 9$. The rest are computed easily, and the results are $[a_1] = \eta - 2c_1 - \chi$, $[a_2] = \eta - c_1 - \chi$, $[a_3] = \eta - \chi$, $[a_4] = -c_1 + \chi_5$, $[a_5] = \chi_5$, $[a_6] = -c_1 + \chi_7$, $[a_7] = \chi_7$, $[a_8] = -c_1 + \chi_9$, $[a_9] = \chi_9$.

The Σ_{10} curves are found setting $s = 0$ in the polynomial

$$b_5 \equiv \Pi_5(0) = a_1a_4a_5a_6 = 0 \to a_1 = 0, a_4 = 0, a_5 = 0, a_6 = 0\,. \tag{3.75}$$

Thus, after the monodromy action, we obtain four curves (one less) to arrange the appropriate pieces of the three (3) families.

The Σ_5 curves are treated similarly in [29] so we do not present the details here.

Matter	Charge	Equation	Homology	N_Y	$M_{\mathrm{U}(1)}$
5_{H_u}	$-2t_1$	$a_8a_5a_7+a_6a_5a_9+a_4a_7a_9$	$-c_1+\tilde{\chi}$	$\tilde{N}$	M_{H_u}
5_1	$-t_1-t_3$	$a_1-ca_4a_8a_7-ca_4a_6a_9$	$\eta-2c_1-\tilde{\chi}$	$-\tilde{N}$	M_{5_1}
5_2	$-t_1-t_4$	$a_1-ca_6a_8a_5-ca_4a_6a_9$	$\eta-2c_1-\tilde{\chi}$	$-\tilde{N}$	M_{5_2}
5_3	$-t_1-t_5$	$a_1-ca_6a_8a_5-ca_4a_8a_7$	$\eta-2c_1-\tilde{\chi}$	$-\tilde{N}$	M_{5_3}
5_4	$-t_3-t_4$	$a_6a_5+a_4a_7$	$-c_1+\chi_5+\chi_7$	N_5+N_7	M_{5_4}
5_5	$-t_3-t_5$	$a_8a_5+a_4a_9$	$-c_1+\chi_5+\chi_9$	N_5+N_9	M_{5_5}
5_6	$-t_4-t_5$	$a_8a_7+a_6a_9$	$-c_1+\chi_7+\chi_9$	N_7+N_9	M_{5_6}
10_M	t_1	a_1	$\eta-2c_1-\tilde{\chi}$	$-\tilde{N}$	$-(M_{5_1}+M_{5_2}+M_{5_3})$
10_2	t_3	a_4	$-c_1+\chi_5$	N_5	M_{10_2}
10_3	t_4	a_6	$-c_1+\chi_7$	N_7	M_{10_3}
10_4	t_5	a_8	$-c_1+\chi_9$	N_9	M_{10_4}
θ_{13}	t_1-t_3	-	-	0	M_{13}
θ_{14}	t_1-t_4	-	-	0	M_{14}
θ_{15}	t_1-t_5	-	-	0	M_{15}
θ_{34}	t_3-t_4	-	-	0	M_{34}
θ_{35}	t_3-t_5	-	-	0	M_{35}
θ_{45}	t_4-t_5	-	-	0	M_{45}

Table 5. Table showing curves and flux restrictions with $\mathcal{Z}_2$ monodromy $t_1 \leftrightarrow t_2$. $\tilde{N} = N_5+N_7+N_9$. The homologies of the singlet fields θ_{ij} are also shown. Due to monodromy, θ_{12} and θ_{21} do not couple to fluxes so they are not included.

3.4 Singlets in the $\mathrm{SU}(5)_\perp$ spectral cover

The way in which the singlets fit into the E_6 and SO(10) pictures can be found by working out the $\mathrm{U}(1)_\chi$ and $\mathrm{U}(1)_\psi$ charges using the generators in eq. (2.14) and matching the charges to the singlets in the decomposition in eq. (2.13). Putting this information together leads to table 6. When the GUT group on S is taken to be SU(5), we again can't compute the homologies of the SU(5) singlets in the spectral cover formalism as they are localised on curves which don't lie within S.

In the subsequent model building, if the GUT group on S is taken to be E_6, we cannot know the properties of the singlets θ_{12} and θ_{13} using the spectral cover approach for the reasons discussed previously. If the GUT group is taken to be SO(10) or SU(5), the situation

Singlet	Q_χ	Q_ψ	Representations
θ_{12}	0	0	SO(10) singlet in 78
θ_{13}	0	0	$45 \subset 78$
θ_{14}	0	4	SO(10) singlet in $27_{t_{1,2}}$
θ_{15}	-5	1	$16_{t_{1,2}} \subset 27_{t_{1,2}}$
θ_{34}	0	4	SO(10) singlet in 27_{t_3}
θ_{35}	-5	-1	$16_{t_3} \subset 27_{t_3}$
θ_{45}	-5	-3	$16_{t_4} \subset 78$

Table 6. Table showing the E_6 charges and origin of some of the singlets in table 5.

is clearly worse, as then there are more GUT singlets for which we can't compute homology classes. As such, we can never have a complete knowledge of the singlet properties in a local framework. This means that in model building, we will simply make assumptions about the singlet spectrum (which in turn would amount to making assumptions about the global completion of the model).

4 Singlet VEVs and D- and F-flatness conditions

The homology constraints just discussed can be used to construct models capable of accommodating the Standard Model — an example of this is given in section 5. To obtain a viable model it is usually necessary to remove additional Standard Model "vectorlike" states by generating mass for them through their coupling to E_6 singlets which acquire VEVs. Any such VEVs should be consistent with F and D flatness conditions and we turn now to a discussion of this. Since, in this paper, we have assumed all GUT breaking is driven by flux no GUT non-singlet fields acquire VEVs until the electroweak scale and so these VEVs can be ignored when determining high scale VEVs.

In general the superpotential for the massless singlet fields is given by

$$W = \mu_{ijk}\theta_{ij}\theta_{jk}\theta_{ki}\,. \tag{4.1}$$

The F-flatness conditions are given by

$$\frac{\partial W}{\partial \theta_{ij}} = \mu_{ijk}\theta_{jk}\theta_{ki} = 0\,. \tag{4.2}$$

The D-flatness condition for $U_A(1)$ is

$$\sum_j Q^A_{ij}(|\langle\theta_{ij}\rangle|^2 - |\langle\theta_{ji}\rangle|^2) = -\frac{\mathrm{Tr}Q^A}{192\pi^2} g_s^2 M_S^2\,, \tag{4.3}$$

where the right-hand side (rhs) is the anomalous contribution, Q^A_j are the singlet charges and the trace $\mathrm{Tr}Q^A$ is over all singlet and non-singlet states. The D-flatness conditions must be checked for each of the $U_A(1)$s.

4.1 E_6 case

In this case after the monodromy action there is only a single U(1) and, in the t'_i basis the charge is given by diag$[1, 1, -2]$. As both the 27s and the θ_{ij} are charged under the U(1), we must know the number of each after the monodromy action and the flux breaking mechanism in order to compute the trace. The contribution of the 27_{t_i} to TrQ^A is

$$27(q_1 n_1 + q_3 n_3) = 27(n_1 - 2n_3)q_1 \tag{4.4}$$

and the contribution of the θ_{ij} is

$$1 \times [(q_1 - q_2)n_{12} + (q_1 - q_3)n_{13}] = 3n_{13}q_1 \,. \tag{4.5}$$

The multiplicities are given in terms of the flux restrictions as the flux dotted with the homology class, and so we have

$$n_1 + n_3 = \mathcal{F} \cdot (\eta - 3c_1)\,, \tag{4.6}$$

$$n_{12} = n_{13} = \mathcal{F} \cdot (\eta - 2c_1)\,. \tag{4.7}$$

Assuming that only the pair θ_{13}, θ_{31} get VEVs, the flatness condition is

$$q_3(|\langle\theta_{13}\rangle|^2 - |\langle\theta_{31}\rangle|^2) + \frac{9(n_1 - 2n_3) + n_{13}}{64\pi^2} q_1 g_s^2 M_S^2 = 0 \tag{4.8}$$

and as we have $q_3 = -2q_1$

$$|\langle\theta_{13}\rangle|^2 - |\langle\theta_{31}\rangle|^2 = \frac{9(n_1 - 2n_3) + n_{13}}{128\pi^2} g_s^2 M_S^2 \,. \tag{4.9}$$

In order to relate the multiplicities to each other, we define for convenience $\omega = \mathcal{F}_{\mathrm{U}(1)} \cdot \eta$, $p = \mathcal{F}_{\mathrm{U}(1)} \cdot c_1$ and $x = \mathcal{F}_{\mathrm{U}(1)} \cdot \chi$. As such, in this notation, we have

$$n_1 = \omega - 2p - x\,, \tag{4.10}$$

$$= n_{13} - x\,, \tag{4.11}$$

$$n_3 = x - p\,. \tag{4.12}$$

As chirality requires $n_1 > 0$ and $n_3 < 0$, the term $9(n_1 - 2n_3)$ is always positive. If we take the case $n_1 = 4$ and $n_3 = -1$ (i.e. the minimal case of three 27's accommodating the three families and a pair $27_H + \overline{27}_{\bar{H}}$), we have $n_{13} = 3 + p$, and

$$|\langle\theta_{13}\rangle|^2 - |\langle\theta_{31}\rangle|^2 = \frac{54 + n_{13}}{128\pi^2} g_s^2 M_S^2 \,. \tag{4.13}$$

This condition is consistent with $\langle\theta_{13}\rangle \neq 0$ and $\langle\theta_{31}\rangle = 0$ for any $n_{13} > 0$, but not with the case $\langle\theta_{13}\rangle = 0$, $\langle\theta_{31}\rangle \neq 0$ as this would require $n_{13} < -54$.

4.2 SO(10) case

Analogous to the E_6 case, the D-flatness condition for the anomalous U(1)s is given by eq. (4.3). In this case there are two $U_A(1)$s with charges that can be taken as $Q^1 =$ diag$[1, 1, 1, -3]$ and $Q^2 =$ diag$[1, 1, -2, 0]$. For example, for the case of Q_1, using table 4, the trace is given by

$$\mathrm{Tr}Q^1_j = 16(n^{16}_1 + n^{16}_3 - 3n^{16}_4) + 10(2n^{10}_{13} + 2n^{10}_{12} - 2n^{10}_{14} - 2n^{10}_{34}) + 4n^1_{14} + 4n^1_{34} \,. \tag{4.14}$$

4.3 SU(5) case

In this case there are three $U_A(1)$s with charges given in eq. (2.14). In the next section we discuss F- and D-flatness in detail for a realistic model.

5 Model building: a realistic model based on E_6

There are several important ingredients to building a phenomenologically realistic low energy theory. The first is the need to control the baryon- and lepton-number violating terms in the Lagrangian that generate rapid nucleon decay. In addition to the dimension 3 and 4 terms (forbidden by R-parity in the MSSM) it is necessary to forbid the dimension 5 nucleon decay terms too. Although the latter are suppressed by an inverse mass factor, this mass must be some 10^7 times the Planck mass, unacceptably large in string theory.

A second necessary ingredient is the control of the "μ term". The Higgs doublet supermultiplet mass term in the superpotential, $\mu H_u H_d$, is allowed by the Standard Model Gauge symmetry but, for a viable theory, its coefficient, μ, must be of order the SUSY breaking scale in the visible sector. To control this requires additional symmetry. At the same time the Higgs colour triplets that are expected as partners of the Higgs doublets in a unified theory must be very heavy — the "doublet-triplet splitting" problem.

Finally the quark and lepton masses and mixings must be consistent. In particular it is necessary to explain why the quark masses and mixing angles have a hierarchical structure while the leptons must have large mixing angles and a relatively small mass hierarchy to explain the observed neutrino oscillation phenomena.

There has been a significant effort to build F-theory based models that use its U(1) symmetries to obtain these ingredients but, to date, no fully satisfactory model has been obtained; indeed it has been speculated that it is not possible. Here, using the results obtained above, we construct an explicit example to demonstrate how the U(1) symmetries alone are sufficient to build a viable theory.

5.1 The E_6 inspired model

The first, most important, step in model building is to find a matter and Higgs multiplet assignment that can eliminate rapid nucleon decay. In this we find that starting from an underlying unified group is very helpful and we consider the case of E_6. After imposing a $\mathcal{Z}_2$ monodromy there are just two multiplets, $27_{t'_{1,3}}$. The $SU(5)\times SU(5)_\perp$ properties of these multiplets are given in table 1. The only E_6 allowed trilinear term in the superpotential is $27_{t'_1}27_{t'_1}27_{t'_3}$. As a result, if we assign the quark and lepton supermultiplets to 27_{t_1} and the Higgs supermultiplets to 27_{t_3}, there will be no dimension 3 or dimension 4 baryon- or lepton-number violating terms.

Anomaly cancellation leads to constraints between the number of SU(5) 10 and 5 dimensional representations [29, 51]. These conditions are automatically satisfied for multiplets descending from complete E_6 multiplets. In particular for the E_6 27 dimensional

E_6	SO(10)	SU(5)	Weight vector	N_Y	$M_{\mathrm{U}(1)}$	SM particle content
$27_{t_1'}$	16	$\overline{5}_3$	t_1+t_5	$\tilde{N}$	$-M_{5_3}$	$-M_{5_3}d^c+(-M_{5_3}+\tilde{N})L$
$27_{t_1'}$	16	10_M	t_1	$-\tilde{N}$	$-M_{5_3}$	$-M_{5_3}Q+(-M_{5_3}+\tilde{N})u^c+(-M_{5_3}-\tilde{N})e^c$
$27_{t_1'}$	16	θ_{15}	t_1-t_5	0	$-M_{5_3}$	$-M_{5_3}\nu^c$
$27_{t_1'}$	10	5_1	$-t_1-t_3$	$-\tilde{N}$	$-M_{5_3}$	$-M_{5_3}D+(-M_{5_3}-\tilde{N})H_u$
$27_{t_1'}$	10	$\overline{5}_2$	t_1+t_4	$\tilde{N}$	$-M_{5_3}$	$-M_{5_3}\overline{D}+(-M_{5_3}+\tilde{N})H_d$
$27_{t_1'}$	1	θ_{14}	t_1-t_4	0	$-M_{5_3}$	$-M_{5_3}S$
$27_{t_3'}$	16	$\overline{5}_5$	t_3+t_5	$-\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}d^c+(M_{5_{H_u}}-\tilde{N})L$
$27_{t_3'}$	16	10_2	t_3	$\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}Q+(M_{5_{H_u}}-\tilde{N})u^c+(M_{5_{H_u}}+\tilde{N})e^c$
$27_{t_3'}$	16	θ_{35}	t_3-t_5	0	$M_{5_{H_u}}$	$M_{5_{H_u}}\nu^c$
$27_{t_3'}$	10	5_{H_u}	$-2t_1$	$\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}D+(M_{5_{H_u}}+\tilde{N})H_u$
$27_{t_3'}$	10	$\overline{5}_4$	t_3+t_4	$-\tilde{N}$	$M_{5_{H_u}}$	$M_{5_{H_u}}\overline{D}+(M_{5_{H_u}}-\tilde{N})H_d$
$27_{t_3'}$	1	θ_{34}	t_3-t_4	0	$M_{5_{H_u}}$	$M_{5_{H_u}}S$

Table 7. Complete 27s of E_6 and their SO(10) and SU(5) decompositions. The indices of the SU(5) non-trivial states $10, 5$ refer to the labeling of the corresponding matter curve (we use the notation of [29]). We impose the extra conditions on the integers N_Y and $M_{\mathrm{U}(1)}$ from the requirement of having complete 27s of E_6 and no 78 matter. The SU(5) matter states decompose into SM states as $\overline{5}\to d^c, L$ and $10\to Q,u^c,e^c$ with right-handed neutrinos $1\to\nu^c$, while SU(5) Higgs states decompose as $5\to D,H_u$ and $\overline{5}\to\overline{D},H_d$, where $D,\overline{D}$ are exotic colour triplets and antitriplets. We identify RH neutrinos as $\nu^c=\theta_{15,35}$ and extra singlets from the 27 as $S=\theta_{14,34}$.

representations we have, in the notation of [29]

$$M_{10_M}=M_{5_1}=-M_{5_2}=-M_{5_3}, \tag{5.1}$$

$$M_{10_2}=-M_{5_4}=-M_{5_5}=M_{5_{H_u}}. \tag{5.2}$$

Furthermore, in the absence of matter in the 78 dimensional representation we have

$$M_{10_3}=M_{10_4}=M_{5_6}=N_8=N_9=0, \tag{5.3}$$

which implies:

$$N_7=\tilde{N}. \tag{5.4}$$

The resulting states arising from complete 27s are shown in table 7 where we have allowed also for the breaking of SU(5) through hypercharge flux. The SM particle content is also shown in table 7 in the usual notation where a generation of quarks and leptons is Q,u^c,d^c,L,e^c. The Higgs doublets H_u,H_d are accompanied by exotic colour triplets and anti-triplets $D,\overline{D}$. The 27s also contain the CP conjugates of the right-handed neutrinos ν^c and extra singlets S.

The only undetermined parameters in table 7 are the three integers M_{5_3}, $M_{5_{H_u}}$ and $\tilde{N}$. To maintain the E_6 based suppression of the baryon- and lepton-number violating terms

JHEP04(2012)094

we require that the Higgs should come from $27_{t'_3}$ and the matter from $27_{t'_1}$ and that any states transforming as $H_{u,d}$ in $27_{t'_1}$ be heavy.

We first choose $M_{5_3} = -3$ to get three families of quarks and leptons in $27_{t'_1}$. To get a single pair of Higgs doublets in $27_{t'_3}$ without colour triplet partners we next choose $M_{5_{H_u}} = 0$ and $\tilde{N} = 1$. According to table 7 this gives the following SM spectrum, grouped according to SO(10) origin:

$$\begin{aligned}
&\left[\overline{5}_3 \to 3d^c + 4L,\ 10_M \to 3Q + 4u^c + 2e^c,\ \theta_{15} \to 3\nu^c\right]_{16},\\
&\left[5_1 \to 3D + 2H_u,\ \overline{5}_2 \to 3\overline{D} + 4H_d\right]_{10},\\
&[\theta_{14} \to 3S]_1,\\
&\left[\overline{5}_5 \to \overline{L},\ 10_2 \to \overline{u^c} + e^c\right]_{16},\\
&\left[5_{H_u} \to H_u,\ \overline{5}_4 \to \overline{H_d}\right]_{10}.
\end{aligned} \tag{5.5}$$

Note that the matter content is just that contained in 3 complete 27s of E_6: $3[Q, u^c, d^c, L, e^c, \nu^c]_{16}$, $3[H_u, D, H_d, \overline{D}]_{10}$, $3[S]_1$ plus some extra vector pairs $L + \overline{L}, e^c + \overline{e^c}, u^c + \overline{u^c}, H_d + \overline{H_d}$ that may be expected to get a large mass if some of the singlet states acquire large VEVs.

It may be seen that the U(1) flux breaking has resulted in one of the lepton super-multiplets, e^c, being assigned to $27_{t'_3}$ in conflict with our original strategy of assigning all matter states to $27_{t'_1}$. However this does not lead to the dimension 4 R-parity violating superpotential term LLe^c because one of the e^c comes from the 16 of SO(10) and there is no 16^3 coupling allowed by SO(10). In this case it is a combination of the original R-parity and the underlying GUT symmetry that eliminates dangerous baryon- and lepton-number violating terms. In fact the combination is more effective than R-parity alone for it also forbids the dangerous dimension 5 terms.

More troublesome is the fact that H_d now comes from $27_{t'_1}$ so that down quark masses are forbidden in tree level. However there is an allowed coupling of $H_d L e^c$ for the e^c belonging to $27_{t'_3}$. This discrepancy between down quark and charged lepton masses looks unacceptable even if the remaining masses are generated in higher order through coupling to singlet fields that acquire large VEVs. To avoid this we look at a slightly modified structure choosing

$$\begin{aligned}
M_{10_M} &= -M_{5_3} = 4,\\
M_{5_1} &= -M_{5_2} = 3,\\
M_{10_2} &= -M_{5_5} = -1,\\
M_{5_4} &= M_{H_u} = 0,\\
M_{\theta_{15}} &= 2,\\
\tilde{N} &= 1.
\end{aligned} \tag{5.6}$$

This leads to the spectrum given in table 8 where now both the down quarks and leptons originate in $27_{t'_1}$, avoiding the troublesome difference in their mass matrices just discussed.

The difference in the spectrum compared to the previous case is in the vectorlike sector with additional pairs of $L+\overline{L}, Q+\overline{Q}, u^c+\overline{u^c}, d^c+\overline{d^c}$ and $H_d+\overline{H_d}$ and no $e^c+\overline{e^c}$. Provided

E_6	SO(10)	SU(5)	Weight vector	N_Y	$M_{\mathrm{U}(1)}$	SM particle content	Low energy spectrum
$27_{t_1'}$	16	$\overline{5}_3$	$t_1 + t_5$	1	4	$4d^c + 5L$	$3d^c + 3L$
$27_{t_1'}$	16	10_M	t_1	-1	4	$4Q + 5u^c + 3e^c$	$3Q + 3u^c + 3e^c$
$27_{t_1'}$	16	θ_{15}	$t_1 - t_5$	0	3	$3\nu^c$	-
$27_{t_1'}$	10	5_1	$-t_1 - t_3$	-1	3	$3D + 2H_u$	-
$27_{t_1'}$	10	$\overline{5}_2$	$t_1 + t_4$	1	3	$3\overline{D} + 4H_d$	H_d
$27_{t_3'}$	16	$\overline{5}_5$	$t_3 + t_5$	-1	-1	$\overline{d^c} + 2\overline{L}$	-
$27_{t_3'}$	16	10_2	t_3	1	-1	$\overline{Q} + 2\overline{u^c}$	-
$27_{t_3'}$	16	θ_{35}	$t_3 - t_5$	0	0	$-$	-
$27_{t_3'}$	10	5_{H_u}	$-2t_1$	1	0	H_u	H_u
$27_{t_3'}$	10	$\overline{5}_4$	$t_3 + t_4$	-1	0	$\overline{H_d}$	-
$27_{t_3'}$	1	θ_{34}	$t_3 - t_4$	0	1	θ_{34}	-
-	1	θ_{31}	$t_3 - t_1$	0	4	θ_{31}	-
-	1	θ_{53}	$t_5 - t_3$	0	1	θ_{53}	-
-	1	θ_{14}	$t_1 - t_4$	0	3	θ_{14}	-
-	1	θ_{45}	$t_4 - t_5$	0	2	θ_{45}	-

Table 8. Complete 27s of E_6 and their SO(10) and SU(5) decompositions. We use the notation of ref [29] for the indices of the SU(5) states and impose the extra conditions on the integers N_Y and $M_{\mathrm{U}(1)}$ from the requirement of having complete 27s of E_6 and no 78 matter. The SU(5) matter states decompose into SM states as $\overline{5} \to d^c, L$ and $10 \to Q, u^c, e^c$ with right-handed neutrinos $1 \to \nu^c$, while SU(5) Higgs states decompose as $5 \to D, H_u$ and $\overline{5} \to \overline{D}, H_d$, where $D, \overline{D}$ are exotic colour triplets and antitriplets. We identify RH neutrinos as $\nu^c = \theta_{15}$. The extra singlets are needed for giving mass to neutrinos and exotics and to ensure F- and D-flatness.

the vectorlike states are heavy the absence of the dimension 3 and 4 R-parity violating operators is now guaranteed by the underlying U(1) symmetries.[3] As we shall see the underlying GUT symmetry still also eliminates the dimension 5 terms that would cause nucleon decay.

5.2 Doublet-triplet splitting and vector-like masses

There remains the doublet-triplet problem of giving large mass to the D and $\overline{D}$ fields and the problem of giving large mass to the vectorlike pairs of fields. Since the D and $\overline{D}$ fields also come in vectorlike pairs these problems are related and are solved by generating mass for vectorlike fields through their coupling to SM singlet fields that acquire large VEVs. For the case the vectorlike pairs have components in both the $27_{t_1'}$ and $27_{t_3'}$ multiplets the extra vector pairs are removed by introducing θ_{31}, an E_6 singlet, with couplings:

$$\theta_{31} 27_{t_1'} \overline{27_{t_3'}} = \theta_{31} Q\overline{Q} + \theta_{31}(2u^c)(2\overline{u^c}) + \theta_{31} d^c \overline{d^c} + \theta_{31}(2L)(2\overline{L}) + \theta_{31} H_d \overline{H_d}. \qquad (5.7)$$

[3]Note that these operators do not involve H_d and so the fact that H_d originates in $27_{t_1'}$ does not cause problems.

If θ_{31} gets a large VEV these vector states get large masses as required. We shall discuss how the D-terms associated with the anomalous $U_A(1)$s can require a VEV for this field close to the Planck scale.

To remove the remaining exotics we introduce θ_{34} which has the couplings:

$$\theta_{34}5_1\overline{5_2} = \theta_{34}[3D+2H_u][3\overline{D}+3H_d] = \theta_{34}[3(D\overline{D})] + \theta_{34}[2(H_uH_d)]. \tag{5.8}$$

If it too acquires a large VEV it generates large mass to the three copies of $D+\overline{D}$ (solving the doublet-triplet splitting problem) and two families of Higgs H_u, H_d, leaving just the MSSM spectrum as shown in the last column of table 8.

5.3 Singlet VEVS

In the model under consideration we assume the SUSY breaking soft masses are such that only the SM singlet fields acquire very large VEVs. To determine them we consider the F- and D-flatness conditions. Taking account of the $\mathcal{Z}_2$ monodromy, $t_1 \leftrightarrow t_2$ the D-flatness conditions are of the form given in eq. (4.3) where there are three $U_A(1)$s with charges given in eq. (2.14). We wish to show that the D-flatness conditions are satisfied by the massless fields θ_{31}, θ_{34}, θ_{53} needed to give mass to exotics and, as discussed below, to generate viable neutrino masses. Using the spectrum given in table 8 we compute $\mathrm{Tr}Q^A$ for the three $U_A(1)$s. In a general basis, $Q = \mathrm{diag}[t_1,t_2,t_3,t_4,t_5]$, eq. (4.3) can be written

$$(t_5-t_3)|\theta_{53}|^2 + (t_3-t_4)|\theta_{34}|^2 + (t_3-t_1)|\theta_{31}|^2 = -X\mathrm{Tr}Q^A\,. \tag{5.9}$$

The trace is taken over all states, and is given by

$$\mathrm{Tr}Q^A = 5\sum n_{ij}(t_i+t_j) + 10\sum n_k t_k + \sum m_{ij}(t_i-t_j)\,. \tag{5.10}$$

For our model, this trace is computed to be

$$\mathrm{Tr}Q^A = 61t_1 - 26t_3 + 14t_4 + 11t_5\,. \tag{5.11}$$

Applying this to the three $U_A(1)$s using the generators given in eq. (2.14) leads to

$$\begin{aligned} 5|\theta_{53}|^2 &= 5\,X \;\; (Q_\chi) \\ -|\theta_{53}|^2 + 4|\theta_{34}|^2 &= 7\,X \;\; (Q_\psi) \\ 2|\theta_{53}|^2 - 2|\theta_{34}|^2 - 3|\theta_{31}|^2 &= -113\,X \;\; (Q_\perp)\,, \end{aligned} \tag{5.12}$$

where $X = \frac{g_s^2 M_S^2}{192\pi^2}$. These equations are solved by

$$\begin{aligned} |\theta_{53}|^2 &= X\,, \\ |\theta_{34}|^2 &= 2\,X\,, \\ |\theta_{31}|^2 &= 37\,X\,. \end{aligned} \tag{5.13}$$

It remains to demonstrate F-flatness. The only allowed superpotential terms that can give a non-zero F-term involves the fields with VEVs plus at most a single additional light field. The only problematic terms have the form $\lambda_{ij}\theta_{53}\theta_{31}^i\theta_{15}^j$ where $i=1,2,3,4$ and

$j = 1, 2, 3$. The F-terms of θ_{15}^j are potentially non-zero but minimisation of the singlet potential will make $\lambda_{i1}\langle\theta_{31}^i\rangle = 0$ and $\lambda_{i2}\langle\theta_{31}^i\rangle = 0$. This means three independent θ_{31}^i fields have zero VEVs but the fourth one can have a VEV as it decouples from θ_{15}^j. It is this combination that enters in eqs. (5.12) and (5.13).

To complete the singlet discussion we note that most of the singlets acquire mass through the singlet VEVs. In particular the coupling $\theta_{14}\theta_{45}\theta_{53}\theta_{31}/M$ is allowed by the symmetries and generated a high scale mass, $\langle\theta_{53}\theta_{31}\rangle/M$ for the θ_{14} and θ_{45} fields. The reason these vector-like sets of singlet fields were included in the spectrum was to ensure the D-flat conditions could be satisfied — they play no role in the low energy phenomenology. Since we have three θ_{14} fields and two θ_{45} fields in the massless spectrum below the string scale there will be one field θ_{14} left over.

5.4 Gauge coupling unification

As we have seen, the result of flux splitting is to add SM non-singlet states in vector-like representations. Although these all acquire a large mass they still affect gauge coupling running. If they come in complete SU(5) representations they do not affect the relative gauge coupling running at the one loop order. However this is not the case as may be seen from table 8 where there are incomplete multiplets. Therefore we would not expect the gauge couplings to meet at some unification scale $M_{\rm GUT}$. On the other hand, as discussed in appendix A, in F-theory it has been observed [38] that $\mathrm{U}(1)_Y$ flux mechanism splits the gauge couplings at the unification scale. Taking into account flux threshold effects, at the GUT scale the gauge couplings $\alpha_i = g_i^2/4\pi$ are found to satisfy the relation,

$$\frac{1}{\alpha_Y(M_{\rm GUT})} = \frac{5}{3}\frac{1}{\alpha_1(M_{\rm GUT})} = \frac{1}{\alpha_2(M_{\rm GUT})} + \frac{2}{3}\frac{1}{\alpha_3(M_{\rm GUT})}, \tag{5.14}$$

Although this is not sufficient to yield a prediction for the low energy gauge couplings, we find that the spectrum of exotics in the considered model tends to compensate for the flux splitting at $M_{\rm GUT}$, so that the low energy gauge couplings can remain close to the same values as predicted in conventional GUT models, independently of the exotic mass scale. We shall also find that this exotic mass scale independence remains true even when two different exotic mass thresholds are taken into account. The compensation is not exact however as noted in [6] and in [63] for the case of models with one or two $\mathrm{U}(1)_\perp$s, and we shall see that this also applies in the model with three $\mathrm{U}(1)_\perp$s. Moreover, the situation here is more involved because the exotic states are not expected to be degenerate, so this requires a dedicated analysis for this particular model.

In our model we have the following vector pairs of exotics, which get large masses when θ_{31} gets a VEV according to eq. (5.7): $(Q+\overline{Q})$, $2(u^c+\overline{u}^c)$, $(d+\overline{d}^c)$, $2(L+\overline{L})$, $(H_d+\overline{H}_d)$. Below some scale $M_X < M_{\rm GUT}$ these exotics decouple. We also have $3(D+\overline{D})$, $2(H_u, H_d)$ exotics which get masses when θ_{34} gets a VEV according to eq. (5.8). Below a scale $M_{X'} < M_X$ these exotics decouple and only the MSSM spectrum remains massless for scales $\mu < M_{X'}$. As discussed above, the VEV for θ_{31} is expected to be much larger than that of θ_{34}. Consequently the exotic states getting mass from different VEVs will have significantly different masses. From eqs. (5.7) and (5.8) we see that a vectorlike pair of D, $\bar{D}$ quarks in an incomplete SU(5) multiplet are much lighter than the net $2L$, $2\bar{L}$ incomplete multiplets. We find that this effect goes in the direction of cancelling the flux contribution. Since the effect depends sensitively on the scale at which gauge coupling running ceases, we now investigate the implications of these scales using a one-loop renormalisation group analysis.

Assuming that extra matter decouples at mass scales M_X and $M_{X'}$ ($M_Z < M_{X'} < M_X < M_{\rm GUT}$) we can express the GUT scale as follows (see appendix A for details)

$$M_{\rm GUT} = e^{\frac{2\pi}{\beta\mathcal{A}}\rho}\, M_Z^{\rho} M_{X'}^{\gamma-\rho} M_X^{1-\gamma} \tag{5.15}$$
$$\rho = \frac{\beta}{\beta_x}$$
$$\gamma = \frac{\beta_{x'}}{\beta_x} M_Z\,,$$

where $\mathcal{A}$ is a function of the experimentally known low energy values of the SM gauge coupling constants

$$\frac{1}{\mathcal{A}} = \frac{5}{3}\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} - \frac{2}{3}\frac{1}{\alpha_3(M_Z)}\,, \tag{5.16}$$

where $\beta, \beta_{x'}, \beta_x$ are the beta-function combinations in the regions $M_Z < \mu < M_{X'}$, $M_{X'} < \mu < M_X$ and $M_X < \mu < M_{\rm GUT}$ respectively

$$\beta_x = b_Y^x - b_2^x - \frac{2}{3}b_3^x\,, \tag{5.17}$$
$$\beta_{x'} = b_Y^{x'} - b_2^{x'} - \frac{2}{3}b_3^{x'}\,, \tag{5.18}$$
$$\beta = b_Y - b_2 - \frac{2}{3}b_3. \tag{5.19}$$

Imposing the well known condition $c_1(\mathcal{L})^2 = -2$ which eliminates the exotic states $(3,2)_{-5/6} + (\bar{3},2)_{5/6}$ originating from the SU(5)-adjoint decomposition in the bulk, we note that all other types of these extra states descend from Σ_{10}, Σ_5 matter curves. Let us denote with

$$n_Q, n_{u^c}, n_{d^c}, n_L, n_{e^c}, n_h$$

the multiplicities of all types of possible extra states, in a self explanatory notation. We find

$$\beta_{x'} - \beta = -2\, n_Q + n_{u^c} + n_{e^c}$$

and a similar equation for the exotics which are in the massless spectrum above the M_X scale. We observe that the above difference depends on the number of additional quark doublets, u-type right handed quarks and electrons n_Q, n_{u^c}, n_{e^c} and is independent of the number of other types of additional states.

In the model under consideration, we analyse this information for the spectrum with two decoupling scales in appendix A, where we find the remarkable result that the GUT scale becomes independent of the decoupling scales M_X, $M_{X'}$,

$$M_{\rm GUT} = e^{\frac{2\pi}{\beta\mathcal{A}}}\, M_Z = e^{\frac{\pi}{6\mathcal{A}}}\, M_Z \approx 2\times 10^{16}\,{\rm GeV} \tag{5.20}$$

that is, it is identical with the MSSM GUT scale. Although one cannot predict the low energy QCD coupling constant in F-theory, in appendix A we obtain an approximate lower bound $\alpha_3 \geq 0.11$, which is consistent with experiment.

5.5 Baryon- and lepton-number violating terms

As discussed above the R-parity violating superpotential couplings $u^c d^c d^c$, $Q d^c L$, $L e^c L$, $\kappa L H_u$ are not allowed because of the underlying U(1) symmetries which play the role of R-parity. Dimension 5 terms in the Lagrangian, corresponding to the superpotential terms $QQQL$ and $u^c u^c d^c e^c$, which would be allowed by usual R-parity, are forbidden by the U(1) symmetries that originate in the underlying E_6.

Of course one must be careful that spontaneous symmetry breaking terms coming from SM singlet field VEVs do not allow these dangerous operators to appear. Allowing for arbitrary singlet fields to acquire VEVs the dangerous the baryon- and lepton-number violating operators arise through the terms $\theta_{15} L H_u$, $(\theta_{31}\theta_{45}+\theta_{41}\theta_{35})10_M\overline{5_3}^2$ and $\theta_{31}\theta_{41}10_M^3\overline{5_3}$. Thus, provided θ_{15}, θ_{41} and θ_{45} do *not* acquire VEVs these dangerous terms will not arise.

However this is not sufficient to ensure the absence of baryon and lepton number violating terms because, even in the absence of these VEVs, tree level graphs can generate the dangerous operators at higher order in the singlet fields. The dangerous graph is shown in figure 1 and is driven by colour triplet exchange coming from the couplings

$$\begin{aligned}
10_M\, 10_M\, 5_{H_u} &\to QQD_h + \ldots \\
5_{H_u}\bar{5}_{\bar{H}_u} &\to M_D D_h \bar{D}_h + \ldots \\
\theta_{34} 5_1 \bar{5}_2 &\to \langle\theta_{34}\rangle\, D_h' \bar{D}_h{}''' + \ldots = \langle\theta_{34}\rangle\, D\bar{D} + \ldots.
\end{aligned}$$

As may be seen from table 8 only the states D_h' and $\bar{D}_h'''$ appear in the spectrum with mass generated by the singlet VEV $\langle\theta_{34}\rangle$ which from eq. (5.13) is predicted to be somewhat below the GUT scale. Since the choice of fluxes in table 8 eliminates light colour triplet states D_h in the low energy spectrum, arising from 5_{H_u}, there is no reason to expect any KK modes with the quantum numbers of D_h below the string scale since there is no ground state with the colour triplet quantum numbers of D_h below the string scale. Similarly the choice of fluxes in table 8 eliminates light colour triplet states $D_h{}''$ in the low energy spectrum, arising from 5_4, so there is no reason to expect any KK modes with the quantum numbers of $D_h{}''$ below the string scale.

If string states with the quantum numbers of D_h, D_h'' exist they are expected to have string scale masses, of $O(M_S)$. In this case the diagram of figure 1 gives the proton decay operator $QQQL$ with coefficient $1/\Lambda_{\rm eff}$ given by

$$\frac{1}{\Lambda_{\rm eff}} = \lambda^5 \left(\frac{\langle\theta_{31}\rangle}{M_S}\right)^2 \frac{1}{\langle\theta_{34}\rangle} \tag{5.21}$$

In (5.21), λ^5 represents the the product of the five Yukawa couplings in the relevant diagram and according to ref [33] it is expected to be

$$\lambda^5 = \lambda_{10\cdot 10\cdot 5}\lambda_{10\cdot\bar{5}\cdot\bar{5}}\lambda^3_{5\cdot\bar{5}\cdot 1} \approx 10^{-3}.$$

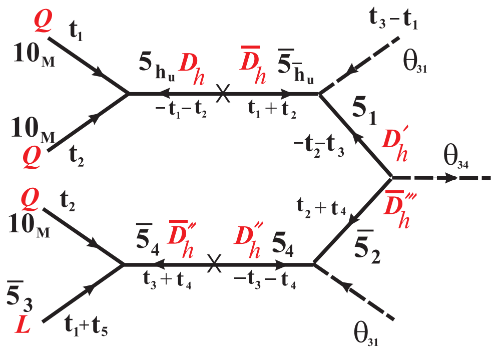

Figure 1. The proton decay diagram generating dim. 5 operator $QQQL$.

We can further determine the mass ratios taking into account the solution eq. (5.13) to flatness conditions to estimate the effective scale

$$\Lambda_{\rm eff} \approx 10^3 \left(\frac{M_S}{\langle\theta_{31}\rangle}\right)^2 \frac{\langle\theta_{34}\rangle}{M_S} M_S \approx \frac{8\sqrt{6}\,\pi}{37 g_s} \times 10^3\, M_S \gtrsim 10^3\, M_S.$$

This, multiplied by the appropriate loop-factor due to higgsino/gaugino dressing and other theoretical factors [73–77], should be compared to experimental bounds on nucleon decay. This bound, relevant to the case that the operator $QQQL$ involves quarks from the two lighter generations only, requires $\Lambda_{\rm eff}^{light} > (10^8 - 10^9) M_S$. Given the large discrepancy between $\Lambda_{\rm eff}^{light}$ and $\Lambda_{\rm eff}$ it is clearly important to determine whether, in the absence of flux, this light quark operator is generated by the diagram of figure 1.

Consider first trilinear couplings involving light fields only. They are given by an integral over the coordinates about the point of intersection, z_i, of the surface on which the matter curves reside. For the case there are N multiple fields associated with a matter curve the orthogonal wave functions may be chosen proportional to powers of the coordinates, $(z_i)^j$, $j = 1, \ldots, N$. On integration only the coupling involving the fields with $j = 0$ are non-zero, corresponding to a $U(1)_i$ invariance, $z_i \to z_i e^{i\alpha_i}$. For the case the three families live on the same matter curve this means the mass matrices are rank 1 in the absence of flux. Switching on the flux gives a rank 3 mass matrix and generates the mixing between the generations.

Now consider the case that there are vertices involving both light and heavy fields. In this case, because the heavy field wave function can involve powers of $\bar{z}_i$ [49], there can be couplings involving light states with $j \neq 0$. However, as the $U(1)_i$ invariance is intact, higher order operators with only external light fields are generated only if all the external fields have $j = 0$. In the absence of flux and assuming that the θ fields that acquire large vevs are also light fields this means that the operator generated by Fig 1 does not involve the light quarks. Thus its contribution to nucleon decay vanishes in the absence of flux and hence is significantly suppressed. To estimate this suppression we use the fact that the same flux effects generate the masses and mixings of the light quarks. Using these mixing angles we can convert the heavy quark operator to one involving light quarks. For the least suppressed case involving two down quarks and an up quark, this gives $\Lambda_{\rm eff}^{light} \approx \sqrt{\frac{m_t}{m_u}\frac{m_b}{m_d}}\Lambda_{\rm eff} \approx 10^9 M_S$, consistent with the experimental bound. A similar result applies to the operator involving right handed quarks.

JHEP04(2012)094

5.6 The μ term

From table 8 it is clear that the U(1) symmetries discussed above forbid a μ term. We expect these local symmetries to be anomalous and the associated gauge bosons to become massive due to the Stueckelberg mechanism, leaving three global U(1) symmetries which act as selection rules in determining the allowed Yukawa couplings [37]. However these global symmetries are only approximate and are spontaneously broken by the VEVs in eq. (5.13). The U(1) symmetries are also explicitly broken by non-perturbative effects [78] with breaking characterised by the Kähler moduli, τ_i, components of the complex fields T_i, whose complex components provide the longitudinal components of the U(1) gauge bosons. These non-perturbative effects will generate an explicit $\mu H_u H_d$ term with the $\mu = O(M_s e^{-t/M_s})$ where t is the VEV of the appropriate combination of τ_i moduli, and M_s represents the string scale. Due to the exponential dependence on t this term can be of the electroweak scale as required. Of course it is important that such breaking effects do not re-introduce nucleon decay terms at an unacceptable level. Provided the breaking of all the U(1) symmetries are of the same order this will be the case because each of the dangerous operators will be suppressed by the factor $e^{-t/M} = O(\mu/M_s)$. Thus the nucleon decay amplitude due to these terms will be suppressed by two powers of the string scale and be negligible.

5.7 Quark and charged lepton masses

Up to SM singlets the surviving low energy spectrum is that of the MSSM given by:

$$\begin{aligned}
&\left[\overline{5}_3 \to 3d^c + 3L,\ 10_M \to 3Q + 3u^c + 3e^c\right],\\
&\left[\overline{5}_2 \to H_d\right]_{10_{t_1}},\\
&\left[5_{H_u} \to H_u\right]_{10_{t_3}}.
\end{aligned} \tag{5.22}$$

The allowed low energy couplings in the superpotential originate from:

$$\begin{aligned}
27_{t_1}27_{t_1}27_{t_3} &\to 16_{t_1}16_{t_1}10_{t_3}\\
&\to 10_M 10_M 5_{H_u} + \overline{5}_3\theta_{15}5_{H_u} + \overline{5}_3 10_2 \overline{5}_2\\
&\to (3Q)(3u^c)H_u + (3L)(3\nu^c)H_u.
\end{aligned} \tag{5.23}$$

A 3×3 up-type and Dirac neutrino mass matrix is allowed at dimension three. In the absence of flux these matrices are rank one. However, as recently shown by Aparicio, Font, Ibanez and Marchesano [49], nonperturbative flux effects can generate an acceptable pattern for the light up quarks.

The down quark and charged leptons acquire mass through the non-renormalisable Yukawa couplings:

$$\begin{aligned}
\theta_{31}27_{t_1}27_{t_1}27_{t_1}/M &\to \theta_{31}16_{t_1}16_{t_1}10_{t_1}/M\\
&\to \theta_{31}\overline{5}_3 10_M \overline{5}_2/M\\
&\to (\theta_{31}(3d^c)(3Q) + \theta_{31}(3L)(3e^c))H_d/M.
\end{aligned} \tag{5.24}$$

JHEP04(2012)094

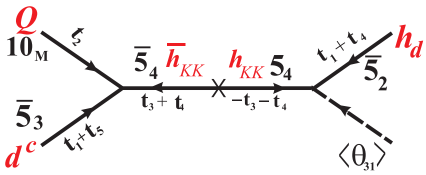

Figure 2. Tree-level diagram contributing to the bottom mass.

Note that, from table 8, the relevant graph 2 is generated by the exchange of a massive vectorlike pair that is given a mass by $\langle\theta_{31}\rangle$. We already saw that θ_{31} must have a large VEV to give mass to exotics so this term can lead to down quark and charged lepton Yukawa couplings that are only mildly suppressed relative to the up quark couplings $(\langle\theta_{31}\rangle/M \geq m_b/m_t)$. This suppression provides an origin for the relative magnitude of the top quark to the bottom quark. Although the mass matrices for the down quarks and charged leptons coming from eq. (5.24) are rank one, non-perturbative flux effects will generate the remaining terms and can lead to an acceptable mass structure [49].

5.8 Neutrino masses

As discussed in [46] models with a monodromy have states with Majorana mass. In this case, due to the monodromy, θ_{12} and θ_{21} are identified so, in the covering theory, the superpotential term $M_M\theta_{12}\theta_{21}$, that is allowed by all the symmetries, is a Majorana mass in the quotient theory. In what follows we shall use the notation $\Theta_{51} \equiv \theta_{53}\theta_{31}/M$. The RH neutrinos, θ_{15}^i couple to θ_{12}^j via $\lambda_{ij}\Theta_{51}\theta_{12}^i\theta_{15}^j$ where we have allowed for several states on the θ_{12} matter curve. In the absence of flux the mass mixing matrix $\lambda_{ij}\langle\Theta_{51}\rangle\theta_{12}^i\theta_{15}^j$ has rank one and we work in a basis in which only λ_{11} is non-zero. With flux the remaining θ_{15} fields mix but this can be considerably suppressed. In the case of the up quark mass matrix such flux effects can explain th! e up quark mass hierarchy $m_c/m_t = 10^{-2}$, $m_u/m_t = 10^{-4}$ so we may expect similar hierarchies in the mass mixing parameters λ_{ij}.

Since Θ_{51} has a VEV the fields θ_{15}^j acquire a Majorana mass, M_{15}, through the see-saw mechanism giving

$$M_{15} = \frac{\lambda^2\langle\Theta_{51}^\dagger\rangle^2 M_M}{(|\lambda\langle\Theta_{51}\rangle|^2 + |M_M|^2)}. \tag{5.25}$$

where we have suppressed the family indices for clarity. In the absence of flux only θ_{15}^1 acquires a mass. With flux the remaining fields also get a Majorana mass but this can be significantly suppressed due to the expectation of smaller λs. Since the light neutrinos couple to θ_{15} via the term $\lambda' L H_u \theta_{15}$[4] they, in turn, acquire Majorana masses given by

$$M_\nu = \frac{\lambda'^2\langle H_u\rangle^2 M_{15}}{(|\kappa\langle\Theta_{53}\rangle|^2 + |M_{15}|^2)} \tag{5.26}$$

where the states θ_{15} also get mass from the coupling $\kappa\theta_{15}\theta_{31}\theta_{53}$.

[4] λ' is a matrix of couplings. In the absence of flux it is also rank 1.

What is the range of neutrino masses to be expected? From eqs. (5.25) and (5.26) we have

$$M_\nu < \frac{\lambda'^2 \langle H_u\rangle^2}{\lambda\langle\Theta_{51}\rangle} \tag{5.27}$$

For $\lambda,\ \lambda' = O(1)$, to get a mass of $O(10^{-1})eV$ requires $\langle\Theta_{51}\rangle \sim 10^{14}\,\mathrm{GeV}$ which is too low given that (c.f. eq. (5.13)) $\langle\Theta_{51}\rangle/M \sim \theta_{31}/M \geq m_b/m_t$ and we expect $M \sim M_S$.

However it is clear the result is very sensitive to the couplings λ and λ' (and κ). Given the approximate rank 1 form of the matrix of couplings λ, from eq. (5.25), two of the singlet states θ_{15} have suppressed mixing to θ_{12}, which we characterise by $\tilde{\lambda}$, and consequently smaller Majorana mass. The resulting spectrum of doublet neutrino masses is one light one satisfying the bound of eq. (5.26) due to the exchange of the heavy θ_{15} state plus two heavier states with mass satisfying the bound

$$M_\nu < \frac{\lambda'^2 \langle H_u\rangle^2}{\tilde{\lambda}\langle\Theta_{51}\rangle} \tag{5.28}$$

due to the exchange of the lighter θ_{15} states. Note that in this equation we have kept the leading $\lambda' = O(1)$ coupling because we expect the light θ_{15} mass eigenstates states to contain a significant component of the state that has the leading λ' coupling. As discussed above $\tilde{\lambda}$ can readily be of $O(10^{-2})$ or smaller. As a result, if this bound is saturated, two neutrino masses in the $10^{-1}eV$ range can readily be generated for $\langle\Theta_{51}\rangle/M_S \sim \theta_{31}/M \geq m_b/m_t$. However the saturation can be spoilt by the term involving $\kappa\langle\Theta_{53}\rangle$ in eq. (5.26) so a determination of the precise result depends on the relative alignment of the leading contributions to $\lambda,\ \lambda'$ and κ. This in turn depends on the relative proximity of the relevant intersections of the matter curves involved in the three couplings.

5.9 Relation to previous work

In [58] a general analysis was presented of the possible R-symmetries coming from the $\mathrm{U}(1)_\perp$ factors in the local analysis of F-theory. Two possibilities were identified but it was shown that it was not possible to realise them in the semi-local picture. The model presented above corresponds to the Matter Parity Case 1 of [58] and we have shown that it is consistent with the semi-local picture. The explanation of the apparent conflict is straightforward. In [58], seeking to generate viable fermion mass matrices without flux effects, the analysis considered only the case that the matter coming from the 10 dimensional representation of SU(5) should come from two matter curves, 10_M and 10_{t_4}. As a result, in order to suppress the dimension 5 nucleon decay operators, a VEV for the field θ_{31} was forbidden and hence, c.f. the discussion above, no down-type mass terms could be generated and the Matter Parity Case 1 was ruled out. However in the case of interest here all three generations are assigned to 10_M. As a result a VEV for θ_{31} is allowed without generating dimension 5 nucleon decay operators. Hence a down-type mass matrix proportional to $\langle\theta_{31}\rangle$ is possible and, allowing for flux effects, the resulting mass matrix can be of rank 3.

In [63] a general discussion was presented of the difficulty in obtaining phenomenological viable F-theory models in the semi-local approach. The difficulty of reconciling the exotic spectrum necessitated by flux breaking with the μ-term, the suppression of nucleon decay operators and gauge unification was emphasised and studied in detail for the case of models with one or two $U(1)_\perp$s. The model constructed here has three $U(1)_\perp$s and demonstrates that the problems can be ameliorated but not eliminated. In particular we have shown that the suppression of the dangerous nucleon decay operators is maintained while generating a μ-term. However the constraints following from anomaly cancellation [29, 51] are still severe and lead to an extended exotic spectrum that affect gauge coupling running as discussed in section 5.4.

6 Conclusions

In the present work we have considered semi-local F-theory GUTs arising from a single E_8 point of local enhancement, involving simple GUT gauge groups based on E_6, SO(10) and SU(5) together with SU(3), SU(4) and SU(5) spectral covers, respectively. Assuming the minimal $\mathcal{Z}_2$ monodromy, we determined the homology classes of the spectrum for each case, and the implications for the resultant spectrum after flux breaking.

Using this, and aided by a dictionary relating the E_6, SO(10), SU(5) and singlet representations, we constructed a model that leads to the MSSM at low energies. We showed that D-and F-flatness constraints require VEVs for singlet fields, which spontaneously break the global U(1) symmetries, and which generate large masses for all the non-MSSM exotic fields. In the absence of flux, the quark and charged lepton mass matrices are of rank one. When flux and instanton corrections are included, light quark and lepton masses and mixings are generated that can be consistent with their observed values. In the absence of flux, the additional U(1) symmetries descending from E_8 ensure that dangerous baryon- and lepton-number violating terms are absent up to and including dimension 5, even taking into account the singlet VEVs which break the U(1) symmetries. Including the flux effects, dimension 5 terms involving light quarks are generated but at an acceptable level. As a result the nucleon is stable within present limits without requiring super-Planckian messenger masses. The μ term in the theory is also forbidden by the U(1) symmetries but can be generated at the SUSY breaking scale, again through non-perturbative effects which explicitly break the U(1) symmetries. Neutrino masses are generated via the see-saw mechanism, involving singlet neutrinos that acquire large Majorana masses allowed by the monodromy.

In conclusion, we have provided an example of a fully viable F-theory GUT, assuming flux breaking of all symmetries, satisfying the semi-local constraints, and employing only the additional (broken) U(1) symmetries descending from the E_8 point of local enhancement.

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A RGEs and extra matter

In this appendix we give a few details on the derivation of the GUT scale and constraints on other relevant quantities obtained from the renormalisation group analysis. It has been pointed out [38] that the $U(1)_Y$ flux mechanism used to break the SU(5) gauge symmetry down to the Standard Model one, splits the gauge couplings at the unification scale. The splitting at $M_{\rm GUT}$ is

$$\begin{aligned} \frac{1}{\alpha_3(M_G)} &= \frac{1}{\alpha_G} - y\,, \\ \frac{1}{\alpha_2(M_G)} &= \frac{1}{\alpha_G} - y + x\,, \\ \frac{1}{\alpha_1(M_G)} &= \frac{1}{\alpha_G} - y + \frac{3}{5}x\,. \end{aligned} \tag{A.1}$$

In the above we introduced the simplified notation $x = -\frac{1}{2}{\rm Re}S\int c_1^2(\mathcal{L}_Y)$ and $y = \frac{1}{2}{\rm Re}S\int c_1^2(\mathcal{L}_a)$ associated with a non-trivial line bundle $\mathcal{L}_a$ and $S = e^{-\phi} + i\,C_0$ the axion-dilaton field as discussed in [38]. Combining the above, the gauge couplings at $M_{\rm GUT}$ are found to satisfy the relation

$$\frac{1}{\alpha_Y(M_{\rm GUT})} = \frac{5}{3}\frac{1}{\alpha_1(M_{\rm GUT})} = \frac{1}{\alpha_2(M_{\rm GUT})} + \frac{2}{3}\frac{1}{\alpha_3(M_{\rm GUT})}\,. \tag{A.2}$$

To obtain the low energy couplings, we use renormalisation group analysis at one-loop level, taking into account threshold effects originating from possible existence of exotic states appear in the spectrum. In general, different exotics decouple at different scales.

In the considered model we have the following vector pairs of exotics, which get large masses when θ_{31} gets a VEV according to eq. (5.7): $(d+\overline{d}^c)$, $(Q+\overline{Q})$, $(H_d+\overline{H}_d)$, $2(L+\overline{L})$, $2(u^c+\overline{u}^c)$. Below some scale $M_X < M_{\rm GUT}$ these exotics decouple. We also have $3(D+\overline{D})$, $2(H_u, H_d)$ exotics which get masses when θ_{34} gets a VEV according to to eq. (5.8). Below a scale $M_{X'} < M_X$ these exotics decouple and only the MSSM spectrum remains massless for scales $\mu < M_{X'}$. The low energy values of the gauge couplings are then given by the evolution equations

$$\frac{1}{\alpha_a(M_Z)} = \frac{1}{\alpha_a(M_{\rm GUT})} + \frac{b_a^x}{2\pi}\ln\frac{M_{\rm GUT}}{M_X} + \frac{b_a^{x'}}{2\pi}\ln\frac{M_X}{M_{X'}} + \frac{b_a}{2\pi}\ln\frac{M_{X'}}{M_Z}\,, \tag{A.3}$$

where b_a^x is the beta-function above the scale M_X, $b_a^{x'}$ is the beta-function below M_X and b_a is the beta-function below $M_{X'}$. Combining the above equations, we find that the GUT scale is given by

$$M_{\rm GUT} = e^{\frac{2\pi}{\beta\mathcal{A}}\rho}\, M_Z^{\rho} M_{X'}^{\gamma-\rho} M_X^{1-\gamma}\,, \tag{A.4}$$

where $\mathcal{A}$ is a function of the experimentally known low energy values of the SM gauge coupling constants

$$\begin{aligned} \frac{1}{\mathcal{A}} &= \frac{5}{3}\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} - \frac{2}{3}\frac{1}{\alpha_3(M_Z)} \\ &= \frac{\cos(2\theta_W)}{\alpha_{em}} - \frac{2}{3}\frac{1}{\alpha_3(M_Z)}\,, \end{aligned} \tag{A.5}$$

where use has been made of the relations $\alpha_Y = \alpha_e/(1-\sin^2\theta_W)$ and $\alpha_2 = \alpha_e/\sin^2\theta_W$. We have also introduced the ratios ρ and γ

$$\rho = \frac{\beta}{\beta_x}\ \gamma = \frac{\beta_{x'}}{\beta_x}\,, \tag{A.6}$$

where $\beta, \beta_{x'}, \beta_x$ are the beta-function combinations in the regions $M_Z < \mu < M_{X'}$, $M_{X'} < \mu < M_X$ and $M_X < \mu < M_{\rm GUT}$ respectively

$$\beta_x = b_Y^x - b_2^x - \frac{2}{3}b_3^x\,, \tag{A.7}$$

$$\beta_{x'} = b_Y^{x'} - b_2^{x'} - \frac{2}{3}b_3^{x'}\,, \tag{A.8}$$

$$\beta = b_Y - b_2 - \frac{2}{3}b_3\,. \tag{A.9}$$

Recall now the beta-function coefficients ($b_1 = \frac{3}{5}\,b_Y$)

$$b_1 \;=\; -0 + 2n_G + \frac{3}{10}(n_h + n_L) + \frac{1}{5}n_{d^c} + \frac{1}{10}n_Q + \frac{4}{5}n_{u^c} + \frac{3}{5}\,n_{e^c}\,, \tag{A.10}$$

$$b_2 \;=\; -6 + 2n_G + \frac{1}{2}(n_h + n_L) + 0\,n_{d^c} + \frac{3}{2}\,n_Q + 0\,n_{u^c}\,, \tag{A.11}$$

$$b_3 \;=\; -9 + 2n_G + 0\,(n_h + n_L) + \frac{1}{2}\,n_{d^c} + n_Q + \frac{1}{2}\,n_{u^c}\,, \tag{A.12}$$

with $n_G = 3$ the number of families and $n_{h,L,\ldots}$ counting Higgses and extraneous matter.

Below $M_{X'}$ we have only the MSSM spectrum, thus $n_G = 3, n_h = 2$ and all extra matter contributions are zero, $n_i = 0$, thus

$$\{b_Y, b_2, b_3\} = \{11, 1, -3\} \to \beta = b_Y - b_2 - \frac{2}{3}b_3 = 12\,.$$

In our model we have additional matter of $3(D + \overline{D})$, $2(H_u, H_d)$ above the scale $M_{X'}$. Assuming n_Q, n_{d^c} and n_{u^c} extra $Q = (3, 2)$, $d^c = (\bar{3}, 1)$ and $u^c = (\bar{3}, 1)$ and n_h doublets while writing $\beta_{x'} = \beta + \delta\beta_{x'}$ we have

$$\delta\beta_{x'} = \beta^{x'} - \beta = -2\,n_Q + n_{u^c}\,.$$

In our model

$$n_Q = 0, n_{u^c} = 0, n_{d^c} = 6, n_L + n_h = 4, n_{e^c} = 0\,,$$

thus

$$\delta\beta_{x'} = 0 \to \beta_{x'} = \beta = 12,\ \frac{\rho}{\gamma} = 1\,.$$

Above the scale M_X, we have additional matter $(d+\overline{d}^c)$, $(Q+\overline{Q})$, $(H_d+\overline{H}_d)$, $2(L+\overline{L})$, $2(u^c + \overline{u}^c)$. As such

$$n_Q = 2, n_{u^c} = 4, n_{d^c} = 2, n_L + n_h = 6, n_{e^c} = 0$$

and so

$$\delta\beta_x = \beta^x - \beta^{x'} = 0 \to \beta_x = \beta = 12,\ \rho = \gamma = 1\,.$$

From (A.4) we see that $M_{\rm GUT}$ becomes independent of the M_X and $M_{X'}$ scales and in fact it is identified with the MSSM unification scale

$$M_U = M_{\rm GUT} \equiv e^{\frac{2\pi}{\beta\mathcal{A}}}\,M_Z \approx 2\times 10^{16}\text{GeV}\,. \tag{A.13}$$

A.1 A lower bound on the low energy QCD coupling constant

Recall now that the parameter x is given by $x = -\frac{1}{2}\,\mathrm{Re}S \int c_1(L_Y)^2$, with $S = e^{-\phi} + i\,C_0$ being the axion-dilaton. Notice that elimination of unwanted exotics $(3,2)_{-5/6} + (\bar{3},2)_{5/6}$ (arising from the SU(5) adjoint decomposition) impose the condition $\int c_1(L_Y)^2 = -2$. The low energy values of the gauge couplings are then given by the evolution equations eq. (A.3). These imply

$$\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_3}\right)_{M_Z} = x + \frac{b_2^x - b_3^x}{2\pi}\log\left(\frac{M_G}{M_X}\right) + \frac{b_2^{x'} - b_3^{x'}}{2\pi}\log\left(\frac{M_X}{M_{X'}}\right)\frac{b_2 - b_3}{2\pi}\log\left(\frac{M_{X'}}{M_Z}\right). \tag{A.14}$$

Let us investigate the implications of the parameter x which is found to play the decisive role in the gauge coupling splitting. We note first that a suitable twisting of $\mathcal{L}_a$ bundle by a trivial line bundle $\mathcal{L}_a \to \mathcal{L}_a \times \mathcal{R}_a$ implies the following change [38]

$$\int c_1(\mathcal{L}_Y)^2 \;\to\; \int c_1(\mathcal{L}_Y)^2 + 2c_1(\mathcal{L}_Y)\cdot c_1(\mathcal{L}_a)\,. \tag{A.15}$$

Since we have imposed the condition $\int c_1(\mathcal{L}_Y)^2 = -2$ and we assume that the manifold is a del Pezzo surface dP_n, we conclude that $\alpha_y = c_1(\mathcal{L}_Y)$ is a root of the corresponding Lie Algebra. If now the twisting is chosen so that $\alpha_a = c_1(\mathcal{L}_a)$ is also a root of the Lie Algebra, then $\alpha_Y \cdot \alpha_a = 1$. In this case $x = 0$ and the splitting effect vanishes since the remaining parameter y induces only a common shift to all gauge couplings, leading only to a redefinition $\alpha_G^{-1} \to \tilde{\alpha}_G^{-1} = \alpha_G^{-1} - y$. Thus, in this limiting case we get the standard gauge coupling unification scenario.

We proceed with the analysis for $x \neq 0$. In our case the required beta functions combinations are

$$b_2 - b_3 = 4\,, \tag{A.16}$$

$$b_2^{x'} - b_3^{x'} = 4\,, \tag{A.17}$$

$$b_2^x - b_3^x = 3\,. \tag{A.18}$$

Thus

$$x = \frac{1}{\alpha_2} - \frac{1}{\alpha_3} - \frac{2}{\pi}\ln\left(\frac{M_G}{M_Z}\right) - \frac{1}{2\pi}\ln\left(\frac{M_{X'}}{M_X}\right). \tag{A.19}$$

We have $\langle\theta_{31}\rangle = \sqrt{37X}$ and $\langle\theta_{34}\rangle = \sqrt{2X}$ from eq. (5.13), and so

$$\frac{M_{X'}}{M_X} = \sqrt{\frac{2}{37}}\,. \tag{A.20}$$

As such, we have

$$x = \frac{4}{3}\frac{1}{\alpha_2} - \frac{1}{3}\frac{1}{\alpha_Y} - \frac{7}{9}\frac{1}{\alpha_3} - \frac{1}{4\pi}\ln\left(\frac{2}{37}\right). \tag{A.21}$$

Since x is the dilaton field, $e^{-\phi}$ clearly we must have $x > 0$ which gives a lower bound in α_3

$$\alpha_3 \geq \frac{7}{9}\frac{1}{\frac{5\sin^2\theta_W - 1}{3\,\alpha_e} - \frac{1}{4\pi}\ln\left(\frac{2}{37}\right)} \approx 0.1130\,. \tag{A.22}$$

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