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6D effective action of heterotic compactification on K3 with nontrivial gauge bundles

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Abstract: We compute the six-dimensional effective action of the heterotic string compactified on $K3$ for the standard embedding and for a class of backgrounds with line bundles and appropriate Yang-Mills fluxes. We compute the couplings of the charged scalars and the bundle moduli as functions of the geometrical $K3$ moduli from a Kaluza-Klein analysis. We derive the D-term potential and show that in the flux backgrounds $U(1)$ vector multiplets become massive by a Stückelberg mechanism.

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1 Introduction

Heterotic model building is one of the possibilities to connect string theory with particle phenomenology. The requirement of a light chiral spectrum in four space-time dimensions (4D) together with stability arguments suggests to consider string backgrounds with $\mathcal{N}=1$ supersymmetry in 4D. This in turn singles out Calabi-Yau threefolds [1], appropriate $\mathbb{Z}_n$ orbifolds [2] or more generally two-dimensional $(0,2)$ superconformal field theories [3, 4] as backgrounds.

The revival of Grand Unified Theories (GUTs) in recent years resulted in renewed attempts to embed these field theories also in the heterotic string. In particular field theoretic models where a GUT-group is only unbroken in a higher-dimensional space-time background seem attractive due the simplicity of the Higgs-sector [5–8]. This led to the study of anisotropic orbifold compactification with an intermediate 5D or 6D effective theory [9–12].

One of the problems of orbifold compactifications is the vast number of massless moduli fields. However, it is well known that some of them gain mass when one considers the theory away from the orbifold point, i.e. in blown-up orbifolds or more generally in smooth Calabi-Yau backgrounds. The relation between orbifold and smooth Calabi-Yau compactifications is addressed in [13–20]. In this paper we focus instead on the 6D intermediate theory and derive the effective action for smooth $K3$ compactifications from a Kaluza-Klein reduction. The resulting 6D effective theory has the minimal amount of eight supercharges corresponding to $\mathcal{N} = 2$ in 4D. The scalar fields appear in tensor- and hypermultiplets but not in vector multiplets. In perturbative heterotic compactifications there is exactly one tensor multiplet containing the dilaton while all other scalars are members of hypermultiplets. In this case supersymmetry constrains the action to depend on a gauge coupling function given by the dilaton, a quaternionic-Kähler metric of the hypermultiplet scalars and a D-term potential [21–23].

A consistent heterotic string background has to satisfy the Bianchi identity which in turn requires a nontrivial gauge bundle on $K3$. As a consequence the resulting light scalar Kaluza-Klein (KK) spectrum consists of the moduli of $K3$, the moduli of the gauge bundle and a set of matter fields charged under the unbroken gauge group. For these fields we systematically compute their couplings in the effective action, extending the analysis in [24–30]. However, since the effective action sensitively depends on the choice of the gauge bundle we cannot give a model-independent answer. Instead we focus on two prominent subclasses of gauge bundles embedded in $E_8 \times E_8$: we discuss the well known standard embedding of the gauge bundle into the tangent bundle in section 3 and backgrounds with $U(1)$ line bundles in section 4.

In the derivation of the 6D effective action we focus on the bundle moduli and the matter fields and compute their couplings as a function of the $K3$ moduli. While low energy supersymmetry restricts the compactification manifold to be Calabi-Yau, it also restricts the gauge bundle to be a solution of the hermitean Yang-Mills equations (HYM) [1]. These solutions are generally constructed from a stability condition using algebraic geometry [31–36]. However, on $K3$ the HYM equations take a simple form, stating that the background field strength is anti-selfdual (ASD) [37, 38]. Its massless deformations determine the light 6D particle spectrum and lead to ASD-preserving bundle moduli which deform the holomorphic bundle structure and charged matter fields which deform the structure group embedding.

This paper is organized as follows. In section 2 we set the stage for the later analysis and briefly recall the multiplets and effective action of 6D minimal supergravity (in section 2.1), and some basic facts about $K3$ (in section 2.2). In section 3 we then turn to the standard embedding and derive the effective action. We determine the couplings of the matter fields and the bundle moduli as a function of the $K3$ moduli. Unfortunately for the bundle moduli these couplings can only be given in terms of moduli-dependent integrals on $K3$ but they are not explicitly evaluated. As a consequence we cannot show in general that the final metric is quaternionic-Kähler as required by supersymmetry [21]. However, in an appropriate orbifold limit we show that the couplings of the matter fields in the untwisted sector are quaternionic-Kähler and agree with the results of [39]. We further compute the scalar potential and show that it consistently descends from a D-term.

In section 4 we consider backgrounds with line bundles [13, 24, 30, 36, 40–44]. In this case the Bianchi identity is satisfied by Abelian Yang-Mills fluxes on internal $K3$ two-cycles. The fluxes are characterized by their group theoretical embedding inside the Cartan subalgebra of $E_8 \times E_8$ and the localization inside the second cohomology lattice of $K3$. Using a vanishing theorem we show that the resulting effective action is consistent with 6D supergravity in that the scalar potential descends from a D-term. We determine the couplings of the matter fields in terms of $K3$-moduli dependent integrals. The Abelian factors of the gauge bundle are also part of the unbroken gauge group and the fluxes affect the effective action in two ways. First of all, the scalars descending from the heterotic B-field get affinely gauged under the Abelian factors. Due to the Stückelberg mechanism this is equivalent to the Abelian gauge bosons becoming massive. Second of all, in the scalar potential the (selfdual components of the) fluxes appear as Fayet-Iliopoulos D-terms, leading to a stabilization of s subset of the $K3$ moduli. Together, for every independent gauge flux a vector multiplet and a hypermultiplet gain a non-zero mass, consistent with the 6D anomaly constraint.

In appendix A we describe in detail the local deformation theory of gauge connections, which is essential for the Kaluza-Klein reduction in the main text. In particular we establish the connection of massless internal deformations and Dolbeault cohomology which, to our knowledge, is not discussed in detail in the literature. Finally, appendix B provides further details about the metric in the untwisted sector of the previously considered orbifold limit.

2 Preliminaries

In this paper we consider Kaluza-Klein reductions of the heterotic string in space-time backgrounds of the form

$$M^{1,5} \times K3\,, \tag{2.1}$$

where $M^{1,5}$ is the six-dimensional Minkowski space-time with Lorentzian signature and $K3$ is the unique compact four-dimensional Calabi-Yau manifold.

The starting point of the analysis is the ten-dimensional heterotic supergravity characterized by the bosonic Lagrangian[1]

$$\mathcal{L} = \frac{1}{2} e^{-2\Phi} \left(R * 1 + 4 d\Phi \wedge * d\Phi - \frac{1}{3} H \wedge * H + \alpha' (\mathrm{tr}\ F \wedge * F - \mathrm{tr}\ \tilde{R} \wedge * \tilde{R}) \right) \,. \tag{2.2}$$

Φ is the ten-dimensional dilaton, F is the Yang-Mills field strength in the adjoint representation of $E_8 \times E_8$ and H is the field strength of the Kalb-Ramond field B defined as

$$H = dB + \alpha' (\omega^L - \omega^{YM})\,, \tag{2.3}$$

[1]Throughout this paper we use the space-time metric signature $(-,+,+,+,\ldots)$ and antihermitean generators for the gauge group.

where ω^L, ω^{YM} are the gravitational and Yang-Mills Chern-Simons 3-forms, respectively. As a consequence H satisfies the Bianchi identity

$$dH = \alpha'(\text{tr } R \wedge R - \text{tr } F \wedge F)\,, \tag{2.4}$$

where R is the Riemann curvature 2-form.[2] Finally, the last term in (2.2) is the Gauss-Bonnet form [45]

$$\text{tr}\tilde{R} \wedge *\tilde{R} := R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2\,. \tag{2.5}$$

The Bianchi identity (2.4) requires a nontrivial gauge bundle over $K3$. As a consequence the original $E_8 \times E_8$ gauge group breaks to G according to

$$E_8 \times E_8 \longrightarrow G \times \langle H \rangle\,. \tag{2.6}$$

Here $\langle H \rangle$ is the structure group of the nontrivial bundle and G is the unbroken maximal commutant.

Before compactification, i.e. in flat ten-dimensional Minkowski space-time $M^{1,9}$, the theory has 16 supercharges corresponding to an $\mathcal{N} = 1$ supergravity in $D = 10$. In a background of the form (2.1) half of the supersymmetries are broken due to the properties of $K3$. Unbroken supersymmetry also constrains the gauge bundle to satisfy the hermitean Yang-Mills equations [1]

$$\mathcal{F} \in H^{1,1}(K3, \mathfrak{h}), \qquad \mathcal{F} \wedge J = 0\,, \tag{2.7}$$

where $H^{1,1}(K3, \mathfrak{h})$ denotes the $(1,1)$ Dolbeault cohomology group with values in the adjoint bundle $\mathfrak{h}$ of H and J is the Kähler-form of $K3$.[3] On $K3$ the hermitean Yang-Mills equations are equivalent to the anti-selfduality condition [37, 38]

$$\mathcal{F} \in \Lambda^2_-(K3, \mathfrak{h})\,, \tag{2.8}$$

where $\Lambda^2_-(K3, \mathfrak{h})$ denotes the -1 eigenspace of the Hodge-$\star$ operator acting on 2-forms. The resulting low energy effective theory is an $\mathcal{N} = 1$ supergravity in $D = 6$, which we shall briefly review.

2.1 $\mathcal{N} = 1$ supergravity in $D = 6$

The supercharges of the $6D, \mathcal{N} = 1$ supergravity form a doublet of two Weyl spinors with the same chirality, satisfying a symplectic Majorana condition. They are rotated into each other under the R symmetry group $\text{Sp}(1)_R \cong \text{SU}(2)_R$. The massless supermultiplets are [46]

$$\begin{aligned} \text{gravity multiplet}: &\quad \{g_{\mu\nu}, \psi^-_\mu, B^+_{\mu\nu}\}\,, \\ \text{tensor multiplet}: &\quad \{B^-_{\mu\nu}, \lambda^+, \phi\}\,, \\ \text{vector multiplet}: &\quad \{V_\mu, \lambda^-\}\,, \\ \text{hypermultiplet}: &\quad \{\chi^+, 4q\}\,, \end{aligned} \tag{2.9}$$

[2] The trace $\text{tr}R \wedge R$ is evaluated in the vector representation **10** of $\text{SO}(1,9)$ and $\text{tr}F \wedge F := \frac{1}{30}\text{Tr}F \wedge F$ is $\frac{1}{30}$ of the trace in the adjoint representation of $E_8 \times E_8$.

[3] Note that a solution of (2.7) also solves the full Yang-Mills equations.

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where $g_{\mu\nu}$ is the graviton of the six-dimensional space-time, ψ^-_μ the negative chirality gravitino and $B^+_{\mu\nu}$ is an antisymmetric tensor with selfdual field strength. The tensor multiplet contains a tensor $B^-_{\mu\nu}$ with anti-selfdual field strength, the dilatino λ^- and the 6D dilaton ϕ. The vector multiplet contains a gauge boson V_μ and the gaugino λ^+. Finally the hypermultiplet features the hyperino χ^+ together with four real scalars q. Note that all scalars, except the dilaton, are in hypermultiplets. The massless spectrum is intrinsically chiral, since the fermions of each supermultiplet have definite chirality.

The doublet structure of the 6D supercharges has further consequences for possible gauge representations. Especially, the four scalars in a hypermultiplet form a complex doublet of the R-symmetry group.[4] A hypermultiplet in a complex representation $\mathbf{R}$ cannot be CPT-selfconjugate, so hypermultiplets always occur in vector-like representations $\mathbf{R}\oplus\overline{\mathbf{R}}$ in the spectrum. The four scalars correspondingly group into two complex scalars in $\mathbf{R}$ and $\overline{\mathbf{R}}$, respectively.

The absence of local anomalies does not constrain the gauge group as in 10D, but rather the massless spectrum to obey [47, 48]

$$29 n_T + n_H - n_V = 273\,, \tag{2.10}$$

where n_T denotes the number of tensor multiplets, n_H the number of hypermultiplets and n_V the number of vector multiplets. This condition is automatically satisfied in any $K3$ compactifications with supersymmetric bundle (2.7). In this paper we only consider perturbative $K3$-compactifications where $n_T = 1$, such that $n_H - n_V = 244$ holds.

For gauge groups of the form

$$G = \prod_\alpha G_\alpha \times \prod_m U(1)_m\,, \tag{2.11}$$

where G_α denotes any simple factor and $U(1)_m$ any Abelian factor, the bosonic Lagrangian is given by [22, 23]

$$\begin{aligned}
\mathcal{L}_6 = {} & \frac{1}{4} R * 1 - \frac{1}{2} e^{-2\phi} H \wedge * H + \frac{1}{4} d\phi \wedge * d\phi \\
& + \frac{1}{2}(c_\alpha e^{-\phi} + \tilde{c}_\alpha e^{\phi}) \mathrm{tr} F^{\mathfrak{g}_\alpha} \wedge * F^{\mathfrak{g}_\alpha} - \tilde{c}_\alpha B \wedge \mathrm{tr} F^{\mathfrak{g}_\alpha} \wedge F^{\mathfrak{g}_\alpha} \\
& + \frac{1}{2}(c_{mn}\, e^{-\phi} + \tilde{c}_{mn}\, e^{\phi}) F^m \wedge * F^n - \tilde{c}_{mn}\, B \wedge F^m \wedge F^n \\
& - \frac{1}{2} g_{uv}(q) \mathcal{D} q^u \wedge * \mathcal{D} q^v - V * 1\,,
\end{aligned} \tag{2.12}$$

where the non-Abelian Yang-Mills field strengths are labeled as $F^{\mathfrak{g}_\alpha}$ and the Abelian field strengths as F^m. Due to supersymmetry, the gauge kinetic functions only depend on the 6D dilaton ϕ, with numerical factors $c_\alpha, \tilde{c}_\alpha, c_{mn}, \tilde{c}_{mn}$.[5] For the Abelian factors kinetic mixing, parametrized by the off-diagonal part of $c_{mn}, \tilde{c}_{mn}$ is possible [50]. B is the sum of

[4] A half-hypermultiplet, which is the smallest CPT self-conjugate multiplet, can only exist, if it is in a pseudoreal gauge representation. If it is a gauge singlet, the two real scalars are both their own CPT-conjugate but cannot build a $\mathrm{SU}(2)_R$-doublet [4].

[5] It was shown recently that these numerical factors are constrained to take values in a selfdual lattice [49].

B^+ and B^-, and it is coupled to the vector multiplets via Chern-Simons forms appearing in its field strength $H = dB + \omega^L - c_\alpha \omega^{YM}_{\mathfrak{g}_\alpha} - c_{mn}\omega^{YM}_{mn}$, where ω^L and $\omega^{YM}_{\mathfrak{g}_\alpha}$ are standard Chern-Simons forms while the "mixed" Abelian Chern-Simons form is given by

$$\omega^{YM}_{mn} = dV^m \wedge V^n \ . \tag{2.13}$$

The real hypermultiplet scalars $q^u, u = 1, \dots, 4n_H$ constitute a quaternionic Kähler target manifold $\mathcal{M}$ with metric $g_{uv}(q)$ which only depends on the hyperscalars [21]. The gauge group can be any isometry group of $\mathcal{M}$, with Killing vectors K^{ua} appearing in the gauge covariant derivatives:

$$\mathcal{D}q^u = dq^u - V^a K^{ua}(q) \,, \tag{2.14}$$

where a denotes the adjoint index of the gauge group.

Finally, there only exists a D-term potential given by

$$V = -\frac{1}{4}\sum_a \frac{(D^a)^A{}_B (D^a)^B{}_A}{c_\alpha e^{-\phi} + \tilde{c}_\alpha e^{\phi}} - \frac{1}{4}\sum_{m,n} \frac{(D^m)^A{}_B (D^n)^B{}_A}{c_{mn} e^{-\phi} + \tilde{c}_{mn} e^{\phi}} \,, \tag{2.15}$$

where

$$(D^{a,m})^A{}_B = \Gamma^A_{uB} K^{ua,m} \,, \qquad A, B = 1, 2 \,, \tag{2.16}$$

with Γ^A_{uB} being a composite $\mathfrak{su}(2)_R$-valued connection on $\mathcal{M}$ [22, 23]. Our main interest in the following will be to derive the 6D couplings, i.e. the hyperscalar metric $g_{uv}(q)$ and the explicit form of the D-term.

2.2 $K3$ compactification

Before we proceed let us collect a few facts about the (unique) Calabi-Yau two-fold $K3$ (for a review see [52]). It has a reduced holonomy group $\mathrm{SU}(2)_{\mathrm{hol}}$, so its frame bundle splits as $\mathrm{SO}(4) \to \mathrm{SU}(2)_R \times \mathrm{SU}(2)_{\mathrm{hol}}$ into an $\mathrm{SU}(2)_R$ bundle which is flat over $K3$ and the nontrivial $\mathrm{SU}(2)_{\mathrm{hol}}$ bundle. A covariantly constant spinor on $K3$ transforms as a doublet under $\mathrm{SU}(2)_R$, so this generates the R-symmetry in 6D. Moreover, the $K3$ surface is hyper-Kähler and its curvature 2-form is anti-selfdual [37]. Its Hodge numbers are

$$\begin{matrix} & & h^{0,0} & & \\ & h^{1,0} & & h^{0,1} & \\ h^{2,0} & & h^{1,1} & & h^{0,2} \\ & h^{2,1} & & h^{1,2} & \\ & & h^{2,2} & & \end{matrix} \;=\; \begin{matrix} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{matrix} \ . \tag{2.17}$$

The nontrivial part is the second cohomology group $H^2(K3, \mathbb{R})$. It is a vector space of signature (3,19) with respect to the scalar product

$$\langle v, w \rangle = \int v \wedge w \,, \qquad v, w \in H^2(K3, \mathbb{R}) \ . \tag{2.18}$$

In a basis of 2-forms $\eta_I \in H^2(K3, \mathbb{R})$ the scalar product is given by the matrix[6]

$$\rho_{IJ} = \int \eta_I \wedge \eta_J \,, \qquad I, J = 1, \dots, 22 \ . \tag{2.19}$$

[6]For integral 2-forms this is the intersection matrix of the Poincaré dual 2-cycles [52].

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A Riemannian metric on $K3$ is defined by a positive definite three-dimensional subspace $\Sigma := H^2_+(K3,\mathbb{R}) \subset H^2(K3,\mathbb{R})$ and the overall volume $\mathcal{V}$. Then we have the orthogonal splitting $H^2(K3) = H^2_+(K3) \oplus H^2_-(K3)$ and the two subspaces are eigenspaces of the Hodge $\star$-operator. The corresponding elements are called selfdual and anti-selfdual, respectively.

Locally the moduli space of Ricci-flat metrics takes the form [53]

$$\mathcal{M}_{\rm K3} = \frac{O(3,19)}{O(3)\times O(19)} \times \mathbb{R}^+ \,, \tag{2.20}$$

which has dimension 58. A complex structure is defined by the choice of an orthonormal dreibein $\{J_s\}_{s=1,2,3} \in H^2_+(K3,\mathbb{R})$ such that

$$J = \sqrt{2\mathcal{V}} J_3 \,, \quad \Omega = J_1 + iJ_2 \tag{2.21}$$

are the Kähler form and the holomorphic 2-form, respectively. They are normalized as

$$\int J\wedge J = 2\mathcal{V} \,, \qquad \int \Omega\wedge\overline{\Omega} = 2 \,, \qquad \|\Omega\|^2 = \frac{1}{2}\Omega_{\alpha\beta}\overline{\Omega}^{\alpha\beta} = \frac{2}{\mathcal{V}} \,. \tag{2.22}$$

The metric moduli combine with the 22 scalars b^I, arising from zero modes of the Kalb-Ramond field on $K3$ to form 20 hypermultiplets in 6D. Including the b^I the geometrical moduli space given in (2.20) locally turns into the quaternionic-Kähler manifold [54]

$$\mathcal{M} = \frac{O(4,20)}{O(4)\times O(20)} \,. \tag{2.23}$$

3 Standard embedding on $K3$

In the previous section we recalled that heterotic theories have to satisfy the Bianchi identity (2.4). For compactifications on $K3$ the integrated version yields

$$\frac{1}{2}\int_{\rm K3} {\rm tr}(F\wedge F) = \frac{1}{2}\int_{\rm K3} {\rm tr}(R\wedge R) = \chi(K3) = 24 \,, \tag{3.1}$$

where $\chi(K3)$ is the Euler characteristic of $K3$. In order to preserve 6D Poincaré invariance all background fields have to be tangent to $K3$. Then (3.1) implies that the second Chern characters of the tangent- and Yang-Mills bundle must coincide. In the following we denote the Kaluza-Klein expansion around these backgrounds as

$$A = \mathcal{A} + a \,, \qquad F = \mathcal{F} + f \,, \qquad f = d_{\mathcal{A}} a + \frac{1}{2}[a,a] \,. \tag{3.2}$$

(We denote background fields by calligraphic symbols such as $\mathcal{A},\mathcal{F},\mathcal{H},\mathcal{R}$.) Since (3.1) is a topological equation, continuous fluctuations cannot contribute to (3.1).

The standard embedding is defined as the solution of (3.1) with the integrands identified, i.e. $\mathcal{F}\equiv\mathcal{R}$ and $\mathcal{H}\equiv 0$ in (2.4) [1]. In this case the nontrivial gauge bundle is an SU(2)-bundle embedded inside one E_8, which is identified with the SU(2) structure-bundle associated with the holomorphic tangent bundle $\mathcal{T}_{\rm K3}$. The standard embedding breaks one E_8 to the maximal commutant E_7, i.e.

$$E_8 \times E_8 \longrightarrow E_8 \times E_7 \times \langle \mathrm{SU}(2) \rangle \,, \tag{3.3}$$

where $\langle H \rangle$ denotes the broken group factor. For the standard embedding the hermitean Yang-Mills equations (2.7) take the form

$$\mathcal{F} \in H^{1,1}(\mathrm{End}\ \mathcal{T}_{\rm K3}) \,, \qquad \mathcal{F} \wedge J = 0 \,, \tag{3.4}$$

where End $\mathcal{T}_{\rm K3}$ is the bundle of linear transition functions on $\mathcal{T}_{\rm K3}$, i.e. locally $\mathfrak{su}(2)$ valued matrix functions. Note that $\mathcal{F}$ is automatically anti-selfdual since the $K3$-curvature is.

3.1 Reduction of the Yang-Mills sector

All bosonic charged matter multiplets arise from zero modes of the 10D vector fields A of the broken E_8. The massless fields are determined by deformations of the background gauge connection $A = \mathcal{A} + a$. Group theoretically a transforms in the **10**-dimensional representation of the Lorentz group SO$(9,1)$ and in the **248**-dimensional adjoint representation of E_8. Decomposing the **248** under $E_8 \to E_7 \times \mathrm{SU}(2)$ we have

$$\mathbf{248} \to (\mathbf{133}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{56}, \mathbf{2}) \,, \tag{3.5}$$

while decomposing the **10** under $\mathrm{SO}(9,1) \to \mathrm{SO}(5,1) \times \mathrm{SO}(4)$ yields

$$\mathbf{10} \to (\mathbf{6}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{4}) \ . \tag{3.6}$$

In terms of the gauge potential we denote the latter split by $a = a_1 + a_{\bar{1}}$ where a_1 denotes a one-form on $M^{1,5}$ while $a_{\bar{1}}$ is an 'internal' one-form on $K3$.

The non-linearity of the free 10D Yang-Mills equation complicates the determination of the massless modes in the Kaluza-Klein procedure. In appendix A we perform the Kaluza-Klein reduction in detail and show that generically the scalar zero modes are in the cohomology $H^{0,1}(K3, E)$, where E is a bundle associated with the right entries in the decomposition (3.5).[7] The result is

$$a_1 = V^{\mathbf{133}} \,, \qquad a_{\bar{1}} = C_j^{\mathbf{56}} \omega_j^{\mathbf{2}} + \overline{C}_j^{\overline{\mathbf{56}}} \overline{\omega}_j^{\overline{\mathbf{2}}} + \xi_k \alpha_k^{\mathbf{3}} + \bar{\xi}_k \overline{\alpha}_k^{\mathbf{3}} \,, \tag{3.7}$$

where $V^{\mathbf{133}}$ is the 6D gauge potential of the unbroken E_7. The $C_j^{\mathbf{56}}$ are complex charged scalars and ξ_k are complex singlet scalars, called bundle moduli. The latter are deformations that preserve the ASD condition of the background. Their multiplicities are given by the cohomology groups of their corresponding zero modes

$$\begin{aligned} \omega_j^{\mathbf{2}} &= (\omega_j)_{\bar{\alpha}}^{\beta} dz^{\bar{\alpha}} \in H^{0,1}(\mathcal{T}_{\rm K3}), \qquad && j = 1, \ldots, 20 \,, \\ \alpha_k^{\mathbf{3}} &= (\alpha_k)_{\bar{\alpha}}^{s} dz^{\bar{\alpha}} \in H^{0,1}(\mathrm{End}\ \mathcal{T}_{\rm K3}), \qquad && k = 1, \ldots, 90 \ . \end{aligned} \tag{3.8}$$

[7] This result is usually derived counting zero modes of the Dirac operator and then using supersymmetry. In appendix A we rederive this result directly from the deformation of the gauge connection.

The $\omega_j^{\mathbf{2}}$ and $\alpha_k^{\mathbf{3}}$ are one-forms which take values in the vector bundles $E_{\mathbf{2}} \cong \mathcal{T}_{\mathrm{K3}}$ and $E_{\mathbf{3}} \cong \mathfrak{su}(2) \subset \mathrm{End}\,\mathcal{T}_{\mathrm{K3}}$, respectively. This is denoted by the indices $\beta = 1, 2$ and $s = 1, 2, 3$ in (3.8). Note that the $\mathbf{3} = \mathfrak{su}(2)$ is a real representation while $\mathbf{56}$ and $\mathbf{2}$ are both pseudoreal. Therefore the 20 complex scalars $C_j^{\mathbf{56}}$ align in 20 half-hypermultiplets, or equivalently 10 hypermultiplets. The 90 complex bundle moduli align in 45 hypermultiplets and 20 additional hypermultiplets arise from the 58 geometrical moduli combined with the 22 Kalb-Ramond axions. The second E_8 remains unbroken and yields a 6D pure Yang-Mills hidden sector with one vector multiplet in the $\mathbf{248}$. The constraint for anomaly freedom (2.10) is fulfilled as follows:

$$n_V = 133 + 248 = 381\,, \quad n_H = 10 \cdot 56 + 45 + 20 = 625\,. \tag{3.9}$$

From (3.7) we derive the Kaluza-Klein expansion of the Yang-Mills field strength,

$$f = f_2^{(\mathbf{133},\mathbf{1})} + f_{1,\bar{1}}^{(\mathbf{1},\mathbf{3})} + f_{1,\bar{1}}^{(\mathbf{56},\mathbf{2})} + f_{\bar{2}}^{(\mathbf{1},\mathbf{3})} + f_{\bar{2}}^{(\mathbf{133},\mathbf{1})}\,. \tag{3.10}$$

Here and in the following, we write $f_{R,\bar{S}}$ for an $(R+S)$-form with R external (6D space-time) and S internal ($K3$) indices. The different terms are orthogonal with respect to the scalar product $\langle F, G\rangle = \mathrm{tr} F \wedge *G$. The first term in (3.10) is the 6D field strength of the unbroken E_7

$$f_2^{(\mathbf{133},\mathbf{1})} = dV^{\mathbf{133}} + \frac{1}{2}[V^{\mathbf{133}}, V^{\mathbf{133}}]\,. \tag{3.11}$$

The next two terms in (3.10) are given by

$$\begin{aligned} f_{1,\bar{1}}^{(\mathbf{1},\mathbf{3})} &= d\xi_k \wedge \alpha_k^s + d\overline{\xi}_k \wedge \overline{\alpha}_k^s\,, \\ f_{1,\bar{1}}^{(\mathbf{56},\mathbf{2})} &= \mathcal{D}C_j^x \wedge \omega_j^\beta + \mathcal{D}\overline{C}_j^{\bar{x}} \wedge \overline{\omega}_j^{\bar{\beta}}\,, \end{aligned} \tag{3.12}$$

where we label the $\mathbf{56}$ by the index $x = 1, \ldots, 56$. In this notation the E_7-covariant derivative reads $\mathcal{D}C_i^x = dC_i^x + V^a (\tau^a)_y{}^x C_i^y$ with τ^a being the E_7 generator. Finally, let us derive the last two terms $f_{\bar{2}}$ in (3.10). Using the zero-mode property $d_{\mathcal{A}} a_{\bar{1}} = 0$ (derived in appendix A) we obtain

$$f_{\bar{2}} = \left[\overline{\xi}_k \overline{\alpha}_k^{\mathbf{3}},\ \xi_k \alpha_k^{\mathbf{3}}\right] + \frac{1}{2}\left[C_j^{\mathbf{56}}\omega_j^{\mathbf{2}} + \overline{C}_j^{\overline{\mathbf{56}}}\overline{\omega}_j^{\bar{\mathbf{2}}},\ C_j^{\mathbf{56}}\omega_j^{\mathbf{2}} + \overline{C}_j^{\overline{\mathbf{56}}}\overline{\omega}_j^{\bar{\mathbf{2}}}\right]. \tag{3.13}$$

The first commutator transforms in the $(\mathbf{1}, \mathbf{3})$ representation. Furthermore we show in appendix A that it preserves the hermitean Yang-Mills equations (3.4) and therefore can be viewed as a flat deformation of the background field strength $\delta\mathcal{F}$. The second commutator results in two representations

$$(\mathbf{56}, \mathbf{2}) \otimes_A (\mathbf{56}, \mathbf{2}) = (\mathbf{1}_A, \mathbf{3}_S) \oplus (\mathbf{133}_S, \mathbf{1}_A)\,, \tag{3.14}$$

which in terms of generators amounts to

$$\begin{aligned} [T_{x\alpha}, T_{y\beta}] &= \varepsilon_{xy}\sigma^s_{\alpha\beta} T_s + \tau^a_{xy}\varepsilon_{\alpha\beta} T_a\,, \\ [T_{x\alpha}, \overline{T}_{\bar{y}\bar{\beta}}] &= \varepsilon_{x\bar{y}}\sigma^s_{\alpha\bar{\beta}} T_s + \tau^a_{x\bar{y}} h_{\alpha\bar{\beta}} T_a\,. \end{aligned} \tag{3.15}$$

ε_{xy} and $\varepsilon_{\alpha\beta}$ are the invariant antisymmetric tensors of E_7 and SU(2) respectively. $\sigma^s_{\alpha\beta}$ are the Pauli matrices and τ^a_{xy} the E_7-generators in the **56**-representation. Since we have the complex conjugated fields in (3.7) we also need the second commutator with different invariant tensors: For the $\tau^a_{x\bar{y}}$ to be again antihermitean, $\overline{\tau^a_{x\bar{y}}} = -\tau^a_{y\bar{x}}$, the tensor h must satisfy

$$\overline{h_{\alpha\bar{\beta}}} = h_{\beta\bar{\alpha}} \ . \tag{3.16}$$

However, since the commutator in (3.13) is a product of global 1-forms, the result is a global 2-form on $K3$. Therefore the invariant tensors $\sigma^s_{\alpha\beta}$, $\varepsilon_{\alpha\beta}$ and $h_{\alpha\beta}$ must be extended to global tensors on $K3$. In fact, (3.16) is the property of a Kähler metric and $\varepsilon_{\alpha\beta}$ is a local expression of the holomorphic 2-form. Hence, we set

$$\begin{aligned} h_{\alpha\bar{\beta}} &\longrightarrow \frac{1}{\sqrt{2\mathcal{V}}} g_{\alpha\bar{\beta}} \, , \\ \varepsilon_{\alpha\beta} &\longrightarrow \Omega_{\alpha\beta} \, , \\ \sigma^s_{\alpha\beta} &\longrightarrow \sigma^s_{\alpha\beta} \in \Lambda^2(\mathrm{End}\ \mathcal{T}_{\mathrm{K3}}) \ . \end{aligned} \tag{3.17}$$

Since the **56** is pseudoreal we will omit the bar on the indices $\bar{x}, \bar{y}$ in the following. With this we get

$$f_{\bar{2}} = \delta\mathcal{F} + f^{(\mathbf{1},\mathbf{3})}_{\bar{2}} + f^{(\mathbf{133},\mathbf{1})}_{\bar{2}} \, , \tag{3.18}$$

where

$$f^{(\mathbf{1},\mathbf{3})}_{\bar{2}} = \begin{pmatrix} \overline{C}^x_i \\ C^x_i \end{pmatrix}^T \begin{pmatrix} \sigma^s_{\bar{\alpha}\beta} \overline{\omega}^{\bar{\alpha}}_i \wedge \omega^{\beta}_j & \sigma^s_{\bar{\alpha}\bar{\beta}}\ \overline{\omega}^{\bar{\alpha}}_i \wedge \overline{\omega}^{\bar{\beta}}_j \\ \sigma^s_{\alpha\beta}\ \omega^{\alpha}_i \wedge \omega^{\beta}_j & \sigma^s_{\alpha\bar{\beta}}\ \omega^{\alpha}_i \wedge \overline{\omega}^{\bar{\beta}}_j \end{pmatrix} \varepsilon_{xy} \begin{pmatrix} C^y_j \\ \overline{C}^y_j \end{pmatrix} , \tag{3.19}$$

$$f^{(\mathbf{133},\mathbf{1})}_{\bar{2}} = \begin{pmatrix} \overline{C}^x_i \\ C^x_i \end{pmatrix}^T \begin{pmatrix} \frac{1}{\sqrt{2\mathcal{V}}} g_{\bar{\alpha}\beta}\ \overline{\omega}^{\bar{\alpha}}_i \wedge \omega^{\beta}_j & \overline{\Omega}_{\bar{\alpha}\bar{\beta}}\ \overline{\omega}^{\bar{\alpha}}_i \wedge \overline{\omega}^{\bar{\beta}}_j \\ \Omega_{\alpha\beta}\ \omega^{\alpha}_i \wedge \omega^{\beta}_j & \frac{1}{\sqrt{2\mathcal{V}}} g_{\alpha\bar{\beta}}\ \omega^{\alpha}_i \wedge \overline{\omega}^{\bar{\beta}}_j \end{pmatrix} (\tau^a)_{xy} \begin{pmatrix} C^y_j \\ \overline{C}^y_j \end{pmatrix} . \tag{3.20}$$

We included the factor $\frac{1}{\sqrt{2\mathcal{V}}}$ in (3.17) such that all matrix elements of the final expression are independent of the $K3$-volume.

3.2 Reduction of the Kalb-Ramond sector

We now turn to the reduction of the $H \wedge *H$-term in the 10D Lagrangian (2.2) where $H = dB + \alpha'(\omega^L - \omega^{YM})$ is a gauge invariant and thus globally defined 3-form. In the KK-reduction H splits into two pieces

$$H \longrightarrow H_3 \ + \ H_{1,\bar{2}} \, , \tag{3.21}$$

where H_3 is the standard 6D Kalb-Ramond term with all indices in the space-time direction. This term reduces straightforwardly yielding the second term in (2.12). For $H_{1,\bar{2}}$ on the other hand we need to perform the KK-reduction with more care.

Let us start by considering the Yang-Mills Chern-Simons form which in 10D is defined by $\omega^{YM} = \mathrm{tr}(F \wedge A) - \frac{1}{3}\mathrm{tr}(A \wedge A \wedge A)$. For the $\omega_{1,\bar{2}}$ component we then have

$$\omega^{YM}_{1,\bar{2}} = \mathrm{tr}(f_{1,\bar{1}} \wedge A_{\bar{1}}) + \mathrm{tr}(F_{\bar{2}} \wedge a_1) - \mathrm{tr}(A_{\bar{1}} \wedge A_{\bar{1}} \wedge a_1) \ . \tag{3.22}$$

Inserting the Kaluza-Klein expansions (3.7) and (3.10), including the background fields $A_{\bar 1} = \mathcal{A} + a_{\bar 1}$, $F_{\bar 2} = \mathcal{F} + \delta\mathcal{F} + f_{\bar 2}$, the nonvanishing terms are

$$\omega^{YM}_{1,\bar 2} = \mathrm{tr}\Big(f^{(\mathbf{56},\mathbf{2})}_{1,\bar 1} \wedge a^{(\mathbf{56},\mathbf{2})}_{\bar 1}\Big) + \mathrm{tr}\Big(f^{(\mathbf{1},\mathbf{3})}_{1,\bar 1} \wedge (\mathcal{A} + a^{(\mathbf{1},\mathbf{3})}_{\bar 1})\Big) \ . \tag{3.23}$$

Similar to the commutators (3.14), the traces of antihermitean generators yield invariant tensors that are extended to global tensors on $K3$

$$\begin{aligned} -\mathrm{tr}(T_s T_t) &= \delta_{st} \longrightarrow h_{st} \ , \\ -\mathrm{tr}(T_{x\alpha} T_{y\beta}) &= \varepsilon_{xy}\varepsilon_{\alpha\beta} \longrightarrow \varepsilon_{xy}\Omega_{\alpha\beta} \ , \\ -\mathrm{tr}(T_{x\alpha} \overline{T}_{y\bar\beta}) &= \delta_{xy} h_{\alpha\bar\beta} \longrightarrow \frac{1}{\sqrt{2\mathcal{V}}}\delta_{xy} g_{\alpha\bar\beta} \ . \end{aligned} \tag{3.24}$$

Here h_{st} is a hermitean metric on the adjoint End $\mathcal{T}_{\mathrm{K3}}$-bundle. Inserting (3.24) and (3.12) into (3.23) we arrive at

$$\begin{aligned} \omega^{YM}_{1,\bar 2} = &- \begin{pmatrix} \mathcal{D}\overline{C}^x_i \\ \mathcal{D}C^x_i \end{pmatrix}^T \begin{pmatrix} \frac{1}{\sqrt{2\mathcal{V}}}\delta_{xy} g_{\bar\alpha\beta}\overline{\omega}^{\bar\alpha}_i \wedge \omega^\beta_j & \varepsilon_{xy}\overline{\Omega}_{\bar\alpha\bar\beta}\overline{\omega}^{\bar\alpha}_i \wedge \overline{\omega}^{\bar\beta}_j \\ \varepsilon_{xy}\Omega_{\alpha\beta}\omega^\alpha_i \wedge \omega^\beta_j & \frac{1}{\sqrt{2\mathcal{V}}}\delta_{xy} g_{\alpha\bar\beta}\omega^\alpha_i \wedge \overline{\omega}^{\bar\beta}_j \end{pmatrix} \begin{pmatrix} C^y_j \\ \overline{C}^y_j \end{pmatrix} \\ &- \begin{pmatrix} d\bar\xi_k \\ d\xi_k \end{pmatrix}^T \begin{pmatrix} h_{st}\overline{\alpha}^s_k \wedge \alpha^t_l & h_{st}\overline{\alpha}^s_k \wedge \overline{\alpha}^t_l \\ h_{st}\alpha^s_k \wedge \alpha^t_l & h_{st}\alpha^s_k \wedge \overline{\alpha}^t_l \end{pmatrix} \begin{pmatrix} \xi_l \\ \bar\xi_l \end{pmatrix} - d\xi_k \left(h_{st}\alpha^s_k \wedge \mathcal{A}^t\right) - d\bar\xi_k \left(h_{st}\overline{\alpha}^s_k \wedge \mathcal{A}^t\right) . \end{aligned} \tag{3.25}$$

In (A.56) and (A.57) we show that the zero modes of the first two terms can be written in terms of harmonic 2-forms and thus are globally defined. The last two terms on the other hand contain the gauge connection $\mathcal{A}$ explicitly and therefore are gauge-variant and globally not well defined. However, they are a total derivative in 6D and thus can be absorbed into $dB_{1,\bar 2}$ by redefining $B_{\bar 2}$. This has the additional benefit that after the redefinition $B_{1,\bar 2}$ is also gauge invariant which follows from the fact that H and the first two terms of $\omega^{YM}_{1,\bar 2}$ in (3.25) are gauge invariant. Therefore the internal redefined $B_{\bar 2}$-field can be expanded globally as

$$B_{\bar 2} = b^I \eta_I \ . \tag{3.26}$$

Finally let us note that the Lorentz Chern-Simons form $\omega^L_{1,\bar 2}$ also is a total space-time derivative in 6D and can similarly be absorbed into a redefinition of $B_{\bar 2}$. Thus altogether we have

$$\begin{aligned} H_{1,\bar 2} &= dB_{\bar 2} \ + \ \alpha'\omega^{CS}_{1,\bar 2} \\ &= db^I \wedge \eta_I - \alpha' \begin{pmatrix} \mathcal{D}\overline{C}^x_i \\ \mathcal{D}C^x_i \end{pmatrix}^T \begin{pmatrix} \frac{1}{\sqrt{2\mathcal{V}}}\delta_{xy} g_{\bar\alpha\beta}\overline{\omega}^{\bar\alpha}_i \wedge \omega^\beta_j & \varepsilon_{xy}\overline{\Omega}_{\bar\alpha\bar\beta}\overline{\omega}^{\bar\alpha}_i \wedge \overline{\omega}^{\bar\beta}_j \\ \varepsilon_{xy}\Omega_{\alpha\beta}\omega^\alpha_i \wedge \omega^\beta_j & \frac{1}{\sqrt{2\mathcal{V}}}\delta_{xy} g_{\alpha\bar\beta}\omega^\alpha_i \wedge \overline{\omega}^{\bar\beta}_j \end{pmatrix} \begin{pmatrix} C^y_j \\ \overline{C}^y_j \end{pmatrix} \\ &\quad - \alpha' \begin{pmatrix} d\xi_k \\ d\bar\xi_k \end{pmatrix}^T \begin{pmatrix} h_{st}\alpha^s_k \wedge \overline{\alpha}^t_l & h_{st}\alpha^s_k \wedge \alpha^t_l \\ h_{st}\overline{\alpha}^s_k \wedge \overline{\alpha}^t_l & h_{st}\overline{\alpha}^s_k \wedge \alpha^t_l \end{pmatrix} \begin{pmatrix} \bar\xi_l \\ \xi_l \end{pmatrix} \ . \end{aligned} \tag{3.27}$$

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3.3 6D effective action

Using the results from the previous sections we now derive the 6D effective action, first focusing on the kinetic terms. The effective action of the gravity-dilaton sector has been determined in ref. [55] and we include their result in the following. In the Einstein frame the 6D dilaton ϕ has to be defined as

$$\phi = \Phi - \frac{1}{2}\ln\mathcal{V}\,, \tag{3.28}$$

where Φ is the 10D dilaton and $\mathcal{V}$ is the $K3$ volume. The Einstein-frame metric is given by $g_{\mu\nu} = e^{-\phi}g^{(10)}_{\mu\nu}$. From this redefinition one gets a factor of $\mathcal{V}^{-1}$ in front of all terms in the Lagrangian with nontrivial $K3$ integral. Altogether we get

$$\begin{aligned}\mathcal{L}_6 = \frac{1}{2}R*1 \;-\; \frac{1}{6}e^{-2\phi}H\wedge *H \;+\; \frac{\alpha'}{2}e^{-\phi}\mathrm{tr}F^{\mathbf{133}}\wedge *F^{\mathbf{133}} + \frac{9}{2}d\phi\wedge *d\phi \\ -\;\frac{\alpha'}{\mathcal{V}}\mathcal{G}_{kl}d\bar{\xi}_k\wedge *d\xi_l \;+\; \frac{1}{4}h_{IJ}dt^I_s\wedge *dt^J_s \;-\; \frac{1}{8\mathcal{V}^2}d\mathcal{V}\wedge *d\mathcal{V} \\ -\;\alpha' G_{ij}\delta_{xy}\mathcal{D}\overline{C}^x_i\wedge *\mathcal{D}C^y_j \;-\; \frac{1}{6\mathcal{V}}g_{IJ}\mathcal{D}_c b^I\wedge *\mathcal{D}_c b^J - V*1\,,\end{aligned} \tag{3.29}$$

where the t^I_s are the $K3$ moduli which, together with the volume, span the moduli space (2.20) with the metric denoted by h_{IJ}.[8] The charged scalars are gauged under the unbroken E_7 via the covariant derivative

$$\mathcal{D}C^x_i = dC^x_i + V^a(\tau^a)_y{}^x C^y_i\;. \tag{3.30}$$

For the b-scalars we have

$$\begin{aligned}\mathcal{D}_c b^I = db^I - \alpha'\delta_{xy}M^I_{ij}\overline{C}^x_i\overleftrightarrow{\mathcal{D}}C^y_j - \alpha'\varepsilon_{xy}(N^I_{ij}C^x_i\mathcal{D}C^y_j + c.c.) \\ -\,\alpha'\mathcal{M}^I_{kl}\bar{\xi}_k\overleftrightarrow{d}\xi_l - \alpha'(\mathcal{N}^I_{kl}\xi_k d\xi_l + c.c.)\;.\end{aligned} \tag{3.31}$$

Here $\bar{\xi}_k\overleftrightarrow{d}\xi_l := \bar{\xi}_k d\xi_l - \xi_l d\bar{\xi}_k$ is the skew-symmetric derivative and we use the same definition for the E_7-covariant derivative $\overleftrightarrow{\mathcal{D}}$. The scalar couplings in (3.29) depend on the $K3$ moduli and are given by

$$\mathcal{G}_{kl} = \int h_{st}\overline{\alpha}^s_k\wedge\star\alpha^t_l\,,\qquad G_{ij} = \frac{\gamma_i\gamma_j}{2\sqrt{2\mathcal{V}}}g_{ij}\,,\qquad g_{IJ} = \int\eta_I\wedge\star\eta_J\;. \tag{3.32}$$

The coupling G_{ij} of the charged scalars (no summation over i,j implied) is proportional to b-scalar coupling g_{IJ}, restricted to $H^{1,1}(K3,\mathbb{R})$. Moreover, it contains the moduli dependent functions

$$\gamma_i = \frac{\mathcal{V}^{\frac{1}{4}}}{(\int J\wedge\eta_i)^{\frac{1}{2}}}\;. \tag{3.33}$$

We find that these are necessary in the charged zero mode isomorphy (A.36), in order to match with the orbifold limit known from [39]. $\mathcal{G}_{kl}$ is the metric on the space of ASD connections. All couplings are derived in more detail in the appendices A.2 and A.3.

[8] For an explicit expression of h_{IJ} see, for example, [55]. Note that from the classical 10D supergravity we cannot deduce the 6D Green-Schwarz term and the full dilaton couplings of (2.12).

The coupling functions appearing in (3.31) read

$$N^I_{ij} = \int \Omega_{\alpha\beta}\omega^\alpha_i \wedge \omega^\beta_j \wedge \eta^I = \frac{1}{2}\gamma_i\gamma_j\rho_{ij}\rho^{IJ}(\langle J_1, \eta_J\rangle - i\langle J_2, \eta_J\rangle)\,,$$
$$M^I_{ij} = \frac{1}{\sqrt{2\mathcal{V}}}\int g_{\bar\alpha\beta}\overline{\omega}^{\bar\alpha}_i \wedge \omega^\beta_j \wedge \eta^I = \frac{i}{2}\gamma_i\gamma_j\rho^{IJ}\Big(\rho_{ij}\langle J_3, \eta_J\rangle - \langle J_3, \eta_i\rangle\rho_{jJ} - \langle J_3, \eta_j\rangle\rho_{iJ}\Big)\,, \tag{3.34}$$

(no summation over i, j implied) and are derived in (A.53) and (A.56). Here $\langle\cdot,\cdot\rangle$ is the scalar product on $H^2(K3,\mathbb{R})$ and ρ_{ij} is the $K3$ intersection matrix restricted to $H^{1,1}(K3,\mathbb{R})$. Since the definition of $H^{1,1}(K3,\mathbb{R})$ depends on a choice of the complex structure ρ_{ij} depends on the $K3$ moduli. For the couplings of the ξ_k in (3.31) we find

$$\mathcal{M}^I_{kl} = \rho^{IJ}\int h_{st}\overline{\alpha}^s_k \wedge \alpha^t_l \wedge \eta_J = \rho^{IJ}\rho_{iJ}c^i_{kl}\,,$$
$$\mathcal{N}^I_{kl} = \rho^{IJ}\int h_{st}\alpha^s_k \wedge \alpha^t_l \wedge \eta_J = e_{kl}(\langle \eta_J, J_1\rangle - i\langle\eta_J, J_2\rangle)\,, \tag{3.35}$$

where we defined c^i_{kl}, e_{kl} as the (antisymmetric) "intersection" matrices

$$h_{st}\overline{\alpha}^s_k \wedge \alpha^t_l = c^i_{kl}\eta_i\,, \qquad h_{st}\alpha^s_k \wedge \alpha^t_l = e_{kl}\overline{\Omega}\,. \tag{3.36}$$

The scalar target manifold is a fibration of the bundle moduli ξ and the charged scalars C over the $K3$ moduli space $\mathcal{M}$ given in (2.23). Supersymmetry imposes that this scalar manifold is quaternionic-Kähler which, however, we did not verify explicitly. In appendix B we show that our results are consistent with the orbifold limit $T^4/\mathbb{Z}_3$ (with standard embedding). The scalars of the truncated spectrum corresponding to the untwisted sector span the quaternionic-Kähler (and simultaneously Kähler) manifold

$$\frac{\mathrm{SU}(2,2+56)}{U(1)\times \mathrm{SU}(2)\times \mathrm{SU}(2+56)}\,. \tag{3.37}$$

We now turn to the scalar potential which consists of all terms descending from (2.2) with space-time indices tangent to $K3$. Since $K3$ is Ricci-flat the Gauss-Bonnet term (2.5) reduces to the square of the curvature 2-form. Moreover, since the curvature is anti-selfdual for all metric deformations, the term gives a constant topological contribution equal to the Euler number of $K3$

$$-\frac{1}{2}\int_{\mathrm{K3}} \mathrm{tr}(\tilde{\mathcal{R}}\wedge\star\tilde{\mathcal{R}}) = \frac{1}{2}\int_{\mathrm{K3}}\mathrm{tr}(\mathcal{R}\wedge\mathcal{R}) = 24\,. \tag{3.38}$$

Together with the contribution from the Yang-Mills field strength we obtain

$$V = -\frac{\alpha'}{2\mathcal{V}}e^\phi\Big(\int \mathrm{tr}F_{\bar 2}\wedge\star F_{\bar 2} + 48\Big)\,. \tag{3.39}$$

Dividing into background and fluctuations $F_{\bar 2} = \mathcal{F} + f_{\bar 2}$ we arrive at

$$V = -\frac{\alpha'}{\mathcal{V}}e^\phi\Big(-\frac{1}{2}\int \mathrm{tr}(\mathcal{F}\wedge\mathcal{F}) + 24 + \frac{1}{2}\int \mathrm{tr}(f_{\bar 2}\wedge\star f_{\bar 2})\Big)\,. \tag{3.40}$$

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The first two terms vanish due to the tadpole condition (3.1) while the third can be decomposed into selfdual and anti-selfdual parts. The tadpole condition additionally constrains

$$0 = \int \mathrm{tr}(f_{\bar{2}} \wedge f_{\bar{2}}) = \int \mathrm{tr}(f_{\bar{2}+} \wedge f_{\bar{2}+}) + \int \mathrm{tr}(f_{\bar{2}-} \wedge f_{\bar{2}-}) \,, \tag{3.41}$$

since continuous fluctuations cannot change a topological invariant. Therefore we can express the potential entirely in terms of the selfdual part $f_{\bar{2}+}$ to obtain

$$\begin{aligned} V &= -\frac{\alpha'}{2\mathcal{V}} e^{\phi} \int \mathrm{tr}(f_{\bar{2}} \wedge \star f_{\bar{2}}) \\ &= -\frac{\alpha'}{2\mathcal{V}} e^{\phi} \int \mathrm{tr}\left(f_{\bar{2}+} \wedge f_{\bar{2}+} - f_{\bar{2}-} \wedge f_{\bar{2}-}\right) \\ &= -\frac{\alpha'}{\mathcal{V}} e^{\phi} \int \mathrm{tr}(f_{\bar{2}+} \wedge f_{\bar{2}+}) \,. \end{aligned} \tag{3.42}$$

This is positive definite since for antihermitean generators the trace gives a negative Killing form.

One thus has to compute the selfdual components $f_{\bar{2}+}^{(\mathbf{1},\mathbf{3})}, f_{\bar{2}+}^{(\mathbf{133},\mathbf{1})}$ of the terms given in (3.19) and (3.20). In appendix A.3 we show that $f_{\bar{2}+}^{(\mathbf{1},\mathbf{3})}$ vanishes, due to the nontriviality of the adjoint End $\mathcal{T}_{\mathrm{K3}}$-bundle. This is crucial for consistency with 6D supergravity, since D-terms necessarily are valued in the adjoint of the unbroken gauge group. On the other hand, the selfdual part of (3.20) reads

$$f_{\bar{2}+}^{(\mathbf{133},\mathbf{1})} \equiv f_{\bar{2}+}^{a} = \begin{pmatrix} \overline{C}_i^x \\ C_i^x \end{pmatrix}^T \begin{pmatrix} -i\sqrt{2\mathcal{V}} G_{ij} J_3 & \frac{1}{2}\tilde{\rho}_{ij}\Omega \\ \frac{1}{2}\tilde{\rho}_{ij}\overline{\Omega} & i\sqrt{2\mathcal{V}} G_{ij} J_3 \end{pmatrix} (\tau^a)_{xy} \begin{pmatrix} C_j^y \\ \overline{C}_j^y \end{pmatrix} \tag{3.43}$$

(see (A.71)). Here $\tilde{\rho}_{ij} = \gamma_i \gamma_j \rho_{ij}$ is the rescaled $K3$-intersection matrix (2.19), restricted to $H^{1,1}(K3, \mathbb{R})$ and G_{ij} is the same coupling as in (3.32). The D-term is identified by expanding

$$f_{\bar{2}+}^{a} = \left(\int f_{\bar{2}+}^{a} \wedge J_s \right) J_s \,, \tag{3.44}$$

Inserting this into (3.42) we arrive at

$$\begin{aligned} V &= -\frac{\alpha'}{\mathcal{V}} e^{\phi} \int J_s \wedge J_t \left(\int f_{\bar{2}+}^{a} \wedge J_s \right) \left(\int f_{\bar{2}+}^{a} \wedge J_t \right) \\ &= -\frac{\alpha'}{2\mathcal{V}} e^{\phi} \mathrm{tr}(\sigma^{(s)} \sigma^{(t)}) \left(\int f_{\bar{2}+}^{a} \wedge J_s \right) \left(\int f_{\bar{2}+}^{a} \wedge J_t \right) \,. \end{aligned} \tag{3.45}$$

Comparing with the generic scalar potential (2.15) yields

$$(D^a)^A{}_B = \frac{1}{\sqrt{2\mathcal{V}}} (\sigma^{(s)})^A{}_B \int f_{\bar{2}+}^{a} \wedge J_s \,. \tag{3.46}$$

Hence, the standard embedding on $K3$ leads to a quartic D-term potential for the charged scalars in consistency with the generic 6D supergravity. If there exist D-flat directions the moduli space of vacua has a Higgs branch, where the gauge group is broken further.

Finally, we identify the $\mathfrak{su}(2)_R$-valued connection 1-form Γ on the charged scalar field space, defined in (2.16). Separating the Killing vectors $K_i^{xa} = (\tau^a)_y{}^x C_i^y$ in the D-term (3.46) yields

$$(\Gamma_j^x)^A{}_B = \begin{pmatrix} iG_{ij}(-\overline{C}_i^x, C_i^x) & \frac{1}{\sqrt{2\mathcal{V}}}\tilde{\rho}_{ij}(0, \overline{C}_i^x) \\ \frac{1}{\sqrt{2\mathcal{V}}}\tilde{\rho}_{ij}(C_i^x, 0) & iG_{ij}(\overline{C}_i^x, -C_i^x) \end{pmatrix}^A{}_B . \tag{3.47}$$

The corresponding curvature tensor is nonvanishing.

3.4 Deviation from the standard embedding

Before we continue let us briefly discuss the scalar potential for deviations from the standard embedding. A first generalization is to drop the condition $\mathcal{F} = \mathcal{R}$ but keep the anti-selfduality of $\mathcal{F}$. This is automatically satisfied for any instanton configuration. In this class the scalar potential for the $K3$ moduli is trivially zero. The second generalization is to consider an arbitrary Yang-Mills bundle. Under metric deformations the curvature 2-form of $K3$ stays anti-selfdual, but the Yang-Mills curvature generically loses this property. In this case the selfdual part contributes an additional term to the scalar potential given by

$$\begin{aligned} V_6 &\sim -\frac{1}{2}\int \mathrm{tr}(\mathcal{F}\wedge\star\mathcal{F}) + \frac{1}{2}\int \mathrm{tr}(\mathcal{R}\wedge\mathcal{R}) \\ &= -\frac{1}{2}\int \mathrm{tr}(\mathcal{F}\wedge\star\mathcal{F}) - \frac{1}{2}\int \mathrm{tr}(\mathcal{F}\wedge\mathcal{F}) \\ &= -\int \mathrm{tr}(\mathcal{F}_+\wedge\star\mathcal{F}_+) \, . \end{aligned} \tag{3.48}$$

This term is positive definite, because the Killing form is negative on antihermitean generators. There are two ways how the system can go back to the minimum of the potential. Either $\mathcal{F}$ is dynamically driven to a new ASD ground state or the $K3$ metric deforms in such a way that $\mathcal{F}$ becomes anti-selfdual again. It follows that for a fixed $\mathcal{F}$ only metric deformations which preserve the ASD condition are true moduli, while the others generate a potential like (3.48). In the next section we will consider Yang-Mills fluxes which are rigid backgrounds, fixed by a quantization condition. In particular they cannot deform dynamically to different ASD ground states. This will stabilize some of the $K3$ metric moduli.

4 Line bundles on $K3$

In this section we look for solutions of the tadpole condition different from the standard embedding, i.e. backgrounds which only satisfy the integrated equation (2.4) in terms of characteristic classes. Strictly speaking, this is not possible with $\mathcal{H} \equiv 0$ in this background. One has to include torsion into the internal geometry and the proper back reaction is given by the Strominger equations. For six internal dimensions one loses the Calabi-Yau property or even more structure, but for $K3$ the torsion can be completely absorbed in a conformal factor of the metric [56].

In the following we consider $K3$ compactifications with line bundles, where the tadpole condition is solved by assigning $\mathcal{F}$ to be the curvature of one (or several) principal $U(1)$

bundle(s) [13, 24, 29, 30, 36, 40–44]. For one line bundle L inside one E_8 factor we then have

$$E_8 \longrightarrow G \times \langle U(1) \rangle \,, \tag{4.1}$$

which implies the following decomposition of the adjoint representation

$$\mathbf{248} \longrightarrow (\mathfrak{g}, \mathbf{1}_0) \oplus (\mathbf{1}, \mathbf{1}_0) \bigoplus_i \left((\mathbf{R}_i, \mathbf{1}_{q_i}) \oplus (\overline{\mathbf{R}}_i, \mathbf{1}_{-q_i}) \right) \,, \tag{4.2}$$

where $\mathfrak{g}$ is the adjoint representation of G while the second term includes $\mathbf{1}_0$ as the adjoint representation of $U(1)$. The $\mathbf{R}_i$ are model dependent representations of G and $\mathbf{1}_{q_i}$ are representations of $U(1)$ with charge q_i. The right entries define associated vector bundles $E_{\mathbf{1}_q}$ which are tensor products of the line bundle L with charge q:

$$E_{\mathbf{1}_q} = L^q = L \otimes \ldots \otimes L \,. \tag{4.3}$$

Negative charges correspond to the dual bundle, $L^{-1} = L^*$, and $L^0 = \mathcal{O}$ is the trivial bundle.

Applying the deformation theory of gauge connections to this setup (for more details see appendix A.4) yields the multiplicities of the corresponding massless fields. Specifically one finds

$$h^{0,1}(L^q) = -2 - q^2 ch_2(L) \,, \tag{4.4}$$

where $ch_2(L) = -\frac{1}{2}\int \mathrm{tr}\mathcal{F} \wedge \mathcal{F}$ is the second Chern-character. Moreover, no bundle moduli exist, as End L^q is the trivial line bundle with $H^{0,1}(\mathrm{End}\ L^q) = 0$. Since the only nonvanishing Chern class is $c_1(L) = i\,\mathrm{tr}\mathcal{F} \in H^{1,1}(K3, \mathbb{Z})$, nontrivial line bundles are equivalent to integral, Abelian Yang-Mills fluxes.[9] Therefore, to specify a line bundle, one chooses a vector X in the Cartan subalgebra $E_8 \times E_8$ and an integral linear combination of the 2-cycles of $K3$.[10] X determines the group theoretical embedding and the unbroken gauge group while the 2-cycles determine the location of the flux

$$i\mathcal{F} = X \otimes m^I \eta_I \,, \qquad I = 1, \ldots, 22 \,, \tag{4.5}$$

with η_I being an integral basis of $H^2(K3, \mathbb{Z})$. The flux satisfies the quantization condition

$$i \int_{\Gamma^I} \mathrm{tr}\mathcal{F} = -\|X\|\ m^I \quad \in \mathbb{Z} \,, \tag{4.6}$$

for all integral 2-cycles $\Gamma_I \in H_2(K3, \mathbb{Z})$. Here $\|X\|$ is the Euclidean norm in the Cartan subalgebra of E_8. For a given $K3$-metric a supersymmetry preserving background must in addition satisfy the ASD condition $\mathcal{F} \in H^{1,1}_-(K3, \mathbb{Z})$, which is a restriction on the $K3$ metric as we already said in section 3.4.

[9] There exist no Abelian local instantons on $K3$ because in 4D these are characterized by the winding number of the mapping $S^3 \mapsto U(1)$, however $\pi_3(U(1)) = 0$.

[10] The specific choice of 2-cycles can be motivated by making contact with heterotic orbifold models which arise as singular limits of $K3$ with shrinking 2-cycles [13, 14].

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We can extend the construction to several line bundles, each with field strength

$$i\mathcal{F}^n = X^n \otimes m^{In}\eta_I \ . \tag{4.7}$$

Since $E_8 \times E_8$ has rank 16, there are at most 16 independent line bundles available. For the tadpole condition we must have

$$24 = \frac{1}{2}\int \mathrm{tr}(\mathcal{F}\wedge\mathcal{F}) \quad = -\frac{1}{2}(X^n \cdot X^m)\; m^{In}m^{Jm}\rho_{IJ} \ . \tag{4.8}$$

Here $\cdot$ is the Euclidean scalar product in the Cartan subalgebra and ρ_{IJ} is the 2-cycle intersection matrix (2.19) of $K3$.

4.1 Reduction of the Yang-Mills sector

Using the results from appendix A.4, the Kaluza-Klein expansion of the gauge potential reads

$$a_1 = V^{\mathfrak{g}} + V^{\mathbf{1}} \,, \qquad a_{\bar{1}} \;=\; \sum_i \, (C^{\mathbf{R}_i}_{k_i}\omega^{q_i}_{k_i} + \overline{C}^{\overline{\mathbf{R}}_i}_{k_i}\overline{\omega}^{-q_i}_{k_i}) + (\overline{D}^{\overline{\mathbf{R}}_i}_{k_i}\varpi^{-q_i}_{k_i} + D^{\mathbf{R}_i}_{k_i}\varpi^{q_i}_{k_i}) \ . \tag{4.9}$$

Here $V^{\mathfrak{g}}$ is the 6D gauge potential in the adjoint representation of G. For one line bundle, we have additionally the Abelian gauge potential $V^{\mathbf{1}}$. For $q_i \neq 0$ the representations in (4.2) are complex and always occur pairwise, with corresponding charged scalars C_{k_i} and $\overline{D}_{k_i}$, respectively. Their four real degrees of freedom align in one hypermultiplet in the representation $\mathbf{R}_i \oplus \overline{\mathbf{R}}_i$. The zero modes belong to

$$\begin{aligned} \omega^{q_i}_{k_i} &\in H^{0,1}(L^{q_i}) \ , \qquad & \overline{\omega}^{-q_i}_{k_i} &\in H^{1,0}(L^{-q_i}) \,, \\ \varpi^{q_i}_{k_i} &\in H^{1,0}(L^{q_i}) \ , \qquad & \overline{\varpi}^{-q_i}_{k_i} &\in H^{0,1}(L^{-q_i}) \,, \end{aligned} \tag{4.10}$$

with multiplicities $k_i = 1, \ldots, h^{0,1}(L^{q_i})$. For notational simplicity we define doublets of the charged scalars as

$$\Phi^{\mathbf{R}_i}_{k_i} := (C^{\mathbf{R}_i}_{k_i}, D^{\mathbf{R}_i}_{k_i}) \ . \tag{4.11}$$

From (4.9) we derive the Kaluza-Klein expansion of the field strength

$$f = f^{\mathbf{1}}_2 + f^{\mathfrak{g}}_2 + \sum_i (f^{\mathbf{R}_i}_{1,\bar{1}} + \overline{f}^{\overline{\mathbf{R}}_i}_{1,\bar{1}}) + f_{\bar{2}} \ . \tag{4.12}$$

Here $f^{\mathbf{1}}_2 = dV^{\mathbf{1}}$ and $f^{\mathfrak{g}}_2 = dV^{\mathfrak{g}} + \frac{1}{2}[V^{\mathfrak{g}}, V^{\mathfrak{g}}]$ are the 6D field strengths. The terms with one external and one internal tangent index give rise to gauge covariant derivatives of the charged scalars,

$$f^{\mathbf{R}_i}_{1,\bar{1}} = \mathcal{D}\Phi^{\mathbf{R}_i}_{k_i} \wedge \omega^{q_i}_{k_i}, \qquad \mathcal{D}\Phi^{\mathbf{R}_i} = d\Phi^{\mathbf{R}_i} - q_i V^{\mathbf{1}}\Phi^{\mathbf{R}_i} - V^a(\tau_a\Phi)^{\mathbf{R}_i} \ . \tag{4.13}$$

Using the zero mode property $d_{\mathcal{A}}\omega^{q_i}_{k_i} = d_{\mathcal{A}}\varpi^{q_i}_{k_i} = 0$, the internal fluctuation is given by the commutator

$$f_{\bar{2}} = \frac{1}{2}\sum_{i,j}\left[a^{(\mathbf{R}_i,\mathbf{1}_{q_i})}_{\bar{1}}, a^{(\mathbf{R}_j,\mathbf{1}_{q_j})}_{\bar{1}}\right] \ . \tag{4.14}$$

Depending on the surviving gauge group G, several representations can arise in (4.14). For $i = j$ the commutator generates the adjoint representations of the unbroken gauge group $G \times U(1)$

$$(\mathbf{R}_i, \mathbf{1}_{q_i}) \otimes (\overline{\mathbf{R}}_i, \mathbf{1}_{-q_i}) = (\mathfrak{g}, \mathbf{1}_0) \oplus (\mathbf{1}, \mathbf{1}_0) \oplus \dots \ . \tag{4.15}$$

It results in field strength fluctuations of the form

$$f_{\bar{2}}^{\mathfrak{g}} = \sum_i \begin{pmatrix} \overline{C}^{\overline{\mathbf{R}}_i} \\ \overline{D}^{\overline{\mathbf{R}}_i} \end{pmatrix}^T \begin{pmatrix} \overline{\omega}^{-q_i} \wedge \omega^{q_i} & \overline{\omega}^{-q_i} \wedge \varpi^{q_i} \\ \overline{\varpi}^{-q_i} \wedge \omega^{q_i} & \overline{\varpi}^{-q_i} \wedge \varpi^{q_i} \end{pmatrix} (\tau^a) \begin{pmatrix} C^{\mathbf{R}_i} \\ D^{\mathbf{R}_i} \end{pmatrix} , \tag{4.16}$$

$$f_{\bar{2}}^{\mathbf{1}} = \sum_i q_i \begin{pmatrix} \overline{C}^{\overline{\mathbf{R}}_i} \\ \overline{D}^{\overline{\mathbf{R}}_i} \end{pmatrix}^T \begin{pmatrix} \overline{\omega}^{-q_i} \wedge \omega^{q_i} & \overline{\omega}^{-q_i} \wedge \varpi^{q_i} \\ \overline{\varpi}^{-q_i} \wedge \omega^{q_i} & \overline{\varpi}^{-q_i} \wedge \varpi^{q_i} \end{pmatrix} (\mathbb{1}) \begin{pmatrix} C^{\mathbf{R}_i} \\ D^{\mathbf{R}_i} \end{pmatrix} , \tag{4.17}$$

where we suppressed the multiplicity indices. τ^a are the $\mathfrak{g}$-generators in the appropriate representation $\mathbf{R}_i$. The products of zero modes belong to $H^2(L^{q_i} \otimes L^{-q_i}) = H^2(K3, \mathbb{R})$. Other representations can occur if the adjoint decomposition allows for other tensor products. Let us illustrate this with an explicit example: There exists a Cartan generator for the line bundle [13, 24] that breaks

$$\begin{aligned} E_8 &\longrightarrow \mathrm{SO}(14) \times U(1) : \\ \mathbf{248} &\longrightarrow \mathbf{91}_0 \oplus \mathbf{1}_0 \oplus (\mathbf{64}_1 \oplus \overline{\mathbf{64}}_{-1}) \oplus (\mathbf{14}_2 \oplus \overline{\mathbf{14}}_{-2}), \end{aligned} \tag{4.18}$$

where $\mathbf{64}$ is the Weyl-spinor of SO(14). Then the commutator (4.14) realizes the tensor products

$$\begin{aligned} \mathbf{64}_1 \otimes \mathbf{64}_1 &= \mathbf{14}_2 \oplus ..., \\ \overline{\mathbf{64}}_{-1} \otimes \overline{\mathbf{64}}_{-1} &= \overline{\mathbf{14}}_{-2} \oplus ..., \\ \mathbf{64}_1 \otimes \overline{\mathbf{14}}_{-2} &= \overline{\mathbf{64}}_{-1} \oplus ..., \\ \overline{\mathbf{64}}_{-1} \otimes \mathbf{14}_2 &= \mathbf{64}_1 \oplus ... \ . \end{aligned} \tag{4.19}$$

The first two tensor products generate a field strength fluctuation of the form

$$\begin{aligned} f_{\bar{2}}^{\mathbf{14}_2 \oplus \mathbf{14}_{-2}} = & \begin{pmatrix} C^u \\ D^u \end{pmatrix}^T \begin{pmatrix} \omega^1 \wedge \omega^1 & \omega^1 \wedge \varpi^1 \\ \varpi^1 \wedge \omega^1 & \varpi^1 \wedge \varpi^1 \end{pmatrix} (\sigma^x)_{uv} \begin{pmatrix} C^v \\ D^v \end{pmatrix} \\ & + \begin{pmatrix} \overline{C}_u \\ \overline{D}_u \end{pmatrix}^T \begin{pmatrix} \overline{\omega}^{-1} \wedge \overline{\omega}^{-1} & \overline{\omega}^{-1} \wedge \overline{\varpi}^{-1} \\ \overline{\varpi}^{-1} \wedge \overline{\omega}^{-1} & \overline{\varpi}^{-1} \wedge \overline{\varpi}^{-1} \end{pmatrix} (\sigma_x)^{uv} \begin{pmatrix} \overline{C}_v \\ \overline{D}^v \end{pmatrix} , \end{aligned} \tag{4.20}$$

where we again suppressed the multiplicity indices. The products of zero modes belong to $H^{1,1}(L^2 \oplus L^{-2})$. The latter two tensor products in (4.19) yield an analogous term $f_{\bar{2}}^{\mathbf{64}_1 \oplus \overline{\mathbf{64}}_{-1}}$. Together we have for this example

$$f_{\bar{2}} \ = \ f_{\bar{2}}^{\mathbf{91}_0} \ + \ f_{\bar{2}}^{\mathbf{1}_0} \ + \ f_{\bar{2}}^{\mathbf{14}_2 \oplus \mathbf{14}_{-2}} \ + \ f_{\bar{2}}^{\mathbf{64}_1 \oplus \overline{\mathbf{64}}_{-1}} \ . \tag{4.21}$$

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4.2 Reduction of the Kalb-Ramond sector

The reduction is essentially the same as in section 3.2, so we only present the new features. The coupling between the b-scalars and the charged scalars again arises from the $\omega^{YM}_{1,\bar{2}}$ component of the Chern-Simons 3-form. But due to the Abelian character of the flux the nonvanishing terms are

$$\omega^{YM}_{1,\bar{2}} = \sum_i \mathrm{tr}\Big(f^{\mathbf{R}_i}_{1,\bar{1}} \wedge a^{(\mathbf{R}_i,\mathbf{1}_{q_i})}_{\bar{1}}\Big) + \mathrm{tr}\Big(\mathcal{F}^{\mathbf{1}} \wedge a^{\mathbf{1}}_{\bar{1}}\Big) \,. \tag{4.22}$$

Compared to (3.23) we see that the second term in (4.22) vanishes in the standard embedding (as well as for any non-Abelian gauge bundle) since in that case there cannot be a 6D vector in the same representation as the background field strength $\mathcal{F}$. The first term in (4.22) generates the skew-symmetric $\bar{\Phi}\overleftrightarrow{\mathcal{D}}\Phi$ couplings and the second term affinely gauges the b-scalars under the unbroken $U(1)$. Using the expansion $\mathcal{F} = -iXm^I\eta_I$ we get

$$dB_{1,\bar{2}} + \alpha'\omega^{YM}_{1,\bar{2}} = \big(db^I - \alpha' V^{\mathbf{1}}\|X\|^2 m^I\big)\,\eta_I + \alpha'\sum_i \mathrm{tr}\Big(\overline{\Phi}^{\mathbf{R}_i}\overleftrightarrow{\mathcal{D}}\,\Phi^{\mathbf{R}_i}\Big) \,, \tag{4.23}$$

where

$$\begin{aligned}\overline{\Phi}^{\mathbf{R}_i}\overleftrightarrow{\mathcal{D}}\,\Phi^{\mathbf{R}_i} = {} & \frac{1}{2}\begin{pmatrix}\overline{C}^{\mathbf{R}_i}_{k_i}\\ \overline{D}^{\mathbf{R}_i}_{k_i}\end{pmatrix}\begin{pmatrix}\overline{\omega}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\omega}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\\ \overline{\varpi}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\varpi}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\end{pmatrix}\begin{pmatrix}\mathcal{D}C^{\mathbf{R}_i}_{l_i}\\ \mathcal{D}D^{\mathbf{R}_i}_{l_i}\end{pmatrix}\\ & -\frac{1}{2}\begin{pmatrix}\mathcal{D}\overline{C}^{\mathbf{R}_i}_{k_i}\\ \mathcal{D}\overline{D}^{\mathbf{R}_i}_{k_i}\end{pmatrix}\begin{pmatrix}\overline{\omega}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\omega}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\\ \overline{\varpi}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\varpi}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\end{pmatrix}\begin{pmatrix}C^{\mathbf{R}_i}_{l_i}\\ D^{\mathbf{R}_i}_{l_i}\end{pmatrix} \,.\end{aligned} \tag{4.24}$$

4.3 6D effective action

Let us now turn to the effective action combining the previous results

$$\begin{aligned}\mathcal{L}_6 = {} & \frac{1}{2}R*1 \;-\; \frac{1}{6}e^{-2\phi}H\wedge *H \;+\; \frac{\alpha'}{2}e^{-\phi}\mathrm{tr}F^{\mathfrak{g}}\wedge *F^{\mathfrak{g}} \;-\; \frac{\alpha'}{2}e^{-\phi}\|X\|^2F^{\mathbf{1}}\wedge *F^{\mathbf{1}}\\ & + \frac{9}{2}d\phi\wedge *d\phi \;+\; \frac{1}{4}h_{IJ}dt^I_s\wedge *dt^J_s \;-\; \frac{1}{8\mathcal{V}^2}d\mathcal{V}\wedge *d\mathcal{V}\\ & - \alpha'\sum_i G^{\mathbf{R}_i}_{k_il_i}\mathrm{tr}\big(\mathcal{D}\overline{\Phi}^{\mathbf{R}_i}_{k_i}\wedge *\mathcal{D}\Phi^{\mathbf{R}_i}_{l_i}\big) \;-\; \frac{1}{6\mathcal{V}}\,g_{IJ}\mathcal{D}b^I\wedge *\mathcal{D}b^J - V*1 \,.\end{aligned} \tag{4.25}$$

$F^{\mathfrak{g}}$ is the Yang-Mills field strength of the semi-simple part of the unbroken gauge group and $F^{\mathbf{1}}$ is the field strength of the unbroken $U(1)$ corresponding to the line bundle. The derivatives of the scalars read

$$\begin{aligned}\mathcal{D}\Phi^{\mathbf{R}_i} &= d\Phi^{\mathbf{R}_i} - q_iV^{\mathbf{1}}\Phi^{\mathbf{R}_i} - V^a(\tau_a\Phi)^{\mathbf{R}_i}\,,\\ \mathcal{D}b^I &= db^I - \alpha' V^{\mathbf{1}}\|X\|^2m^I + \alpha'\rho^{IJ}\mathrm{tr}\Big(\overline{\Phi}^{\mathbf{R}_i}_{k_i}(N^{q_i}_{Jk_il_i})\overleftrightarrow{\mathcal{D}}\,\Phi^{\mathbf{R}_i}_{l_i}\Big) \,.\end{aligned} \tag{4.26}$$

We see that the scalars $\Phi^{\mathbf{R}_i}$ are linearly gauged under the entire unbroken gauge group. The b-scalars are affinely gauged under the unbroken $U(1)$ due to the flux of the line bundle, with charges given by the flux vector m^I. The 2×2 coupling matrix $(N^{q_i}_{Jk_il_i})$ is given by

$$(N^{q_i}_{Jk_il_i}) = \int \eta_J \wedge \begin{pmatrix}\overline{\omega}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\omega}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\\ \overline{\varpi}^{-q_i}_{k_i}\wedge\omega^{q_i}_{l_i} & \overline{\varpi}^{-q_i}_{k_i}\wedge\varpi^{q_i}_{l_i}\end{pmatrix} \,. \tag{4.27}$$

The scalar metrics read

$$
\begin{aligned}
g_{IJ} &= \int \eta_I \wedge \star \eta_J \,, \\
(G^{\mathbf{R}_i}_{k_i l_i}) &= \mathcal{V}^{-1} \int \begin{pmatrix} \omega^{q_i}_{k_i} \wedge \star \overline{\omega}^{-q_i}_{l_i} & 0 \\ 0 & \varpi^{q_i}_{k_i} \wedge \star \overline{\varpi}^{-q_i}_{l_i} \end{pmatrix} \,,
\end{aligned}
\tag{4.28}
$$

so the latter is diagonal in the $C^{\mathbf{R}_i}$ and $D^{\mathbf{R}_i}$ fields.

We now turn to the scalar potential. By the same argument as given in (3.42) for the standard embedding, only the selfdual parts of the field strength fluctuations (4.21) contribute to the potential. It is shown in appendix A.4 that the selfdual parts vanish for all terms which are not in the adjoint representation of the unbroken gauge group

$$
f^{\mathbf{R}_i \oplus \overline{\mathbf{R}}_i}_{2+} = 0 \,. \tag{4.29}
$$

On the other hand the selfdual parts of (4.16) and (4.17) take the form

$$
f^{\mathfrak{g}}_{2+} = \sum_i \overline{\Phi}^{\overline{\mathbf{R}}_i}_{k_i} (U_{k_i l_i}) (\tau^a \Phi)^{\mathbf{R}_i}_{l_i} \,, \qquad f^{\mathbf{1}}_{2+} = \sum_i q_i \overline{\Phi}^{\overline{\mathbf{R}}_i}_{k_i} (U_{k_i l_i}) \Phi^{\mathbf{R}_i}_{l_i} \,, \tag{4.30}
$$

where

$$
U_{k_i l_i} = \begin{pmatrix} \frac{i}{2} G^C_{k_i l_i} J & \frac{1}{2} c_{k_i l_i} \Omega \\ \frac{1}{2} \overline{c}_{k_i l_i} \overline{\Omega} & \frac{i}{2} G^D_{k_i l_i} J \end{pmatrix} \,. \tag{4.31}
$$

Note that a is used for the adjoint $\mathfrak{g}$ index and that the matrix U depends on the representation $\mathbf{R}_i$. As in the standard embedding we find on the diagonal the scalar metrics $G^C_{k_i l_i}$ and $G^D_{k_i l_i}$, which are the two matrix elements of (4.28). In the off-diagonal elements we find a generalized "intersection matrix"

$$
c_{k_i l_i} = \int \overline{\omega}^{-q_i}_{k_i} \wedge \varpi^{q_i}_{l_i} \wedge \overline{\Omega} \,. \tag{4.32}
$$

Identifying the Killing vectors

$$
K^a_{k_i} = (\tau^a \Phi_{k_i})^{\mathbf{R}_i} \,, \qquad K^{\mathbf{1}}_{k_i} = q_i \Phi^{\mathbf{R}_i}_{k_i} \,, \qquad K^{I\mathbf{1}} = \|X\|^2 m^I \,, \tag{4.33}
$$

we see that the terms (4.30), (4.31) generate D-terms in the 6D potential. The third Killing vector corresponds to the gauge flux, whose selfdual component appears as a Fayet-Iliopoulos term in the Abelian D-term

$$
V = -\frac{\alpha'}{\mathcal{V}} e^{\phi} \int \mathrm{tr} \left((\mathcal{F}_+ + f^{\mathbf{1}}_{2+}) \wedge \star (\mathcal{F}_+ + f^{\mathbf{1}}_{2+}) \right) - \frac{\alpha'}{\mathcal{V}} e^{\phi} \int \mathrm{tr} \left(f^{\mathfrak{g}}_{2+} \wedge \star f^{\mathfrak{g}}_{2+} \right) \,. \tag{4.34}
$$

Similar to the analysis in (3.45) and (3.46), the individual D-terms can be extracted from (4.34) by the ($K3$ metric dependent) expansion

$$
\begin{aligned}
(D^a)^A{}_B &= \frac{1}{\sqrt{2}} \int f^a_{2+} \wedge J_s \otimes (\sigma^{(s)})^A{}_B \,, \\
(D^{\mathbf{1}})^A{}_B &= \frac{1}{\sqrt{2}} \int (\mathcal{F}_+ + f^{\mathbf{1}}_{2+}) \wedge J_s \otimes (\sigma^{(s)})^A{}_B \,.
\end{aligned}
\tag{4.35}
$$

The generalization of the above results to several line bundles is straightforward. The b-scalars are then gauged under all Abelian factors $U(1)_m$ with charges proportional to the flux vectors m^{In}. For line bundles which are not orthogonal, $X^n \cdot X^m \neq 0$, kinetic mixing of the different $F^{\mathbf{1}_m}$ field strengths occurs

$$\mathcal{L}_6^{kin} \sim -\frac{\alpha'}{2} e^{-\phi} \sum_{m,n} (X^m \cdot X^n) F^{\mathbf{1}_m} \wedge *F^{\mathbf{1}_n} \; . \tag{4.36}$$

Also the 6D H-field may contain mixed Abelian Chern-Simons couplings (see (2.13)). The scalar potential takes the form

$$V = \frac{\alpha'}{\mathcal{V}} e^{\phi} (X^n \cdot X^m) \int (\mathcal{F}_+^n + f_{2+}^{\mathbf{1}_n}) \wedge \star (\mathcal{F}_+^m + f_{2+}^{\mathbf{1}_m}) - \frac{\alpha'}{\mathcal{V}} e^{\phi} \int \mathrm{tr}\left(f_{2+}^{\mathfrak{g}} \wedge \star f_{2+}^{\mathfrak{g}} \right) \; . \tag{4.37}$$

Here $f_{2+}^{\mathbf{1}_n}$ is the direct generalization of (4.17) containing all charged matter fields charged under $U(1)_n$. The explicit form of the scalar potential reads

$$\begin{aligned} V = \frac{\alpha' e^{\phi}}{\mathcal{V}} (X^n \cdot X^m) \int &\left(\mathcal{F}_+^n + \sum_i q_i^n \overline{\Phi}_{k_i}^{\overline{\mathbf{R}}_i} (U_{k_i l_i}) \Phi_{l_i}^{\mathbf{R}_i} \right) \wedge \star \left(\mathcal{F}_+^m + \sum_i q_i^m \overline{\Phi}_{k_i}^{\overline{\mathbf{R}}_i} (U_{k_i l_i}) \Phi_{l_i}^{\mathbf{R}_i} \right) \\ &+ \frac{\alpha' e^{\phi}}{\mathcal{V}} \sum_a \int \left(\sum_i \overline{\Phi}_{k_i}^{\overline{\mathbf{R}}_i} (U_{k_i l_i}) (\tau^a \Phi)_{l_i}^{\mathbf{R}_i} \right) \wedge \star \left(\sum_i \overline{\Phi}_{k_i}^{\overline{\mathbf{R}}_i} (U_{k_i l_i}) (\tau^a \Phi)_{l_i}^{\mathbf{R}_i} \right) , \end{aligned} \tag{4.38}$$

where q_i^n is the charge of the field $\Phi^{\mathbf{R}_i}$ under the group $U(1)_n$.

Recalling the general argument in section 3.4, the rigid fluxes of the line bundle background stabilize some of the $K3$ moduli. The Fayet-Iliopoulos term $\mathcal{F}_+$ in (4.34) is generated by those $K3$ metric deformations that violate the ASD condition of the Yang-Mills background. Hence, their mass is lifted to a nonzero value. Since we have an Abelian gauge flux in the case of line bundles, i.e. $\mathcal{F} \in H^2(K3, \mathbb{Z})$, we get an intuitive picture of the moduli stabilization in terms of the 3-plane $\Sigma \in H^2(K3, \mathbb{R})$, introduced in section 2.2. The ASD condition (3.4) can be written as

$$\mathcal{F} \perp \Sigma \, , \tag{4.39}$$

where orthogonality is defined with respect to the intersection matrix ρ. Hence, massless deformations of the $K3$ metric are given by all motions of Σ, preserving (4.39). For N line bundles the massless metric deformations are constrained to the subspace orthogonal to the flux vectors $\{m^1, \ldots, m^N\}$. If all N flux vector are linearly independent, the remaining moduli space is described by the Grassmannian manifold

$$\tilde{\mathcal{M}}_{\mathrm{K3}} = \frac{O(3, 19-N)}{O(3) \times O(19-N)} \times \mathbb{R}^+ , \tag{4.40}$$

so there are $3N$ moduli stabilized and $\dim \tilde{\mathcal{M}}_{\mathrm{K3}} = 58 - 3N$. For $E_8 \times E_8$ we have $N_{\max} = 16$, which stabilizes all but 10 moduli and leaves $U(1)^{16}$ unbroken. For a GUT group to survive in 6D a larger number of moduli has to stay unfixed.

Finally, let us mention that there exists also a moduli space for the charged scalars which consists of all D-flat directions $C_{k_i}^{\mathbf{R}}, D_{k_i}^{\mathbf{R}} \neq 0$, satisfying $D^a = D^{\mathbf{1}} = 0$. The corresponding Higgs branch has a smaller gauge group with less massless hypermultiplets [57].

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4.4 Stückelberg mechanism and massive U(1)s

We close this paper by analyzing the effect of the affinely gauged scalars b^I (cf. (4.26)). Let us first focus on one line bundle for simplicity. In this case the $U(1)$ gauge symmetry acts according to

$$V^{\mathbf{1}} \longrightarrow V^{\mathbf{1}} + d\chi \,, \qquad b^I \longrightarrow b^I + \alpha' m^I \chi \,. \tag{4.41}$$

This implies that one combination of b^I can be gauged to zero with $V^{\mathbf{1}}$ becoming massive which is known as the Stückelberg mechanism.[11] The mass term (in the Einstein frame) is found from (4.26) to be

$$\frac{\alpha'^2}{6\mathcal{V}}\|X\|^2 V^{\mathbf{1}} \wedge *V^{\mathbf{1}} \int \mathrm{tr}(\mathcal{F} \wedge \star\mathcal{F}) = -\frac{\alpha'^2}{6\mathcal{V}}\|X\|^4 V^{\mathbf{1}} \wedge *V^{\mathbf{1}} \rho_{IJ} m^I m^J \,, \tag{4.42}$$

where we used the ASD condition $\star\mathcal{F} = -\mathcal{F}$. To identify the physical mass we need to absorb a factor $\sqrt{\alpha'}\|V\|$ into $V^{\mathbf{1}}$ in order to get a canonical kinetic term as can be seen from (3.29). Using the tadpole condition (4.8) the physical mass reads

$$m = 4\sqrt{\frac{\alpha'}{\mathcal{V}}} \,. \tag{4.43}$$

Note that the physical mass only depends on the $K3$ volume.

If there are N line bundles with flux parameters $m^{In} = (m^{I1}, \ldots, m^{IN})$, the b^I are coupled to all of them and generically all "fluxed" $U(1)$'s become massive. However, if some flux vectors are linearly dependent, $\dim \mathrm{span}\{m^1, \ldots, m^N\} = K < N$, the rank of the mass matrix is reduced and there remain $N-K$ massless $U(1)$'s in the spectrum. Let us show which combination of b_I-scalars is eaten by which combination of $U(1)$'s. In an integral basis of $H^2(K3,\mathbb{Z})$ we define $q^{In} = \|V^n\| m^{In} \in \mathbb{Z}$ and look for the orthogonalization

$$\mathcal{L}_6 \sim g_{IJ}(db^I - q^{In}V_n^{\mathbf{1}})^2 = \tilde{g}_{IJ}(d\tilde{b}^I - \lambda^{In}\tilde{V}_n^{\mathbf{1}})^2 \,. \tag{4.44}$$

For K linear independent flux vectors the $22 \times N$ matrix q^{In} has rank K and hence can be be brought to the following form (e.g. $N=3, K=2$)

$$q^{In} \mapsto O^I{}_J q^{Jm} U^n{}_m = \lambda^{In} = \begin{pmatrix} \lambda^1 & 0 & 0 & \ldots \\ 0 & \lambda_2 & 0 & \ldots \\ 0 & 0 & 0 & \ldots \end{pmatrix} \,, \tag{4.45}$$

where $O \in O(22)$ and $U \in O(N)$. This determines the preferred basis

$$\tilde{V}_n^{\mathbf{1}} = U_n{}^p V_p^{\mathbf{1}} \,, \qquad \tilde{b}^I = O^I{}_J b^J \,, \tag{4.46}$$

in which the first K $\tilde{b}$ scalars are the Goldstone bosons of the first K gauge potentials. More precisely, one goes to a basis of $H^2(K3,\mathbb{Z})$ where the flux hyperplane $\mathrm{span}(m^1, \ldots, m^n)$ is spanned by the first K harmonic 2-forms $\tilde{\eta}_1, \ldots \tilde{\eta}_K$. The special form of λ^{In} however does

[11] In 6D this effect is independent of possible Abelian anomalies [30].

not tell us if this basis is orthogonal with respect to the intersection matrix (2.19). Since we have $\star\mathcal{F}^n = -\mathcal{F}^n$ for each gauge flux, the mass terms read

$$\frac{\alpha'}{6\mathcal{V}} V_n^{\mathbf{1}} \wedge * V_m^{\mathbf{1}} \int \mathcal{F}^n \wedge \star\mathcal{F}^m = -\frac{\alpha'}{6\mathcal{V}} \tilde{V}_n^{\mathbf{1}} \wedge * \tilde{V}_m^{\mathbf{1}} \tilde{\rho}_{IJ} \lambda^{In} \lambda^{Jm} \,, \tag{4.47}$$

where $\tilde{\rho}_{IJ} = O_I{}^K O_J{}^L \rho_{KL}$. In general $\tilde{\rho}_{IJ}$ will not be diagonal and hence the mass term will not be diagonal in n, m. Therefore, the mass eigenbasis is generically different from the "Goldstone eigenbasis". Note that again the mass matrix only depends on the volume modulus and that the trace of the (squared) mass matrix is fixed by the tadpole condition

$$\mathrm{tr}(M^2) = \sum_n \left(-\frac{1}{3} \frac{\alpha'}{\mathcal{V}} \tilde{\rho}_{IJ} \lambda^{In} \lambda^{Jn} \right) = 16 \, \frac{\alpha'}{\mathcal{V}} \,. \tag{4.48}$$

5 Conclusion

In this paper we derived the six-dimensional low energy effective action of the heterotic string compactified on $K3$. Consistency requires a nontrivial gauge bundle on $K3$ and for concreteness we chose to consider first the standard embedding and second a flux background with $U(1)$ line bundles. In both cases we performed a Kaluza-Klein reduction starting from the ten-dimensional action. Specifically we focused on the gauge sector where charged and neutral scalars (bundle moduli) arise as massless deformations of the internal gauge bundle. We carefully performed a KK-reduction and computed the sigma-model metric and the scalar potential of the six-dimensional action as a functions of the geometrical $K3$ moduli and the axionic scalars arising from the NS B-field. For the scalar potential we showed the consistency with the generic 6D, $\mathcal{N} = 1$ supergravity in that it arises solely from a D-term. The sigma-model metric is constrained to be a quaternionic-Kähler metric which, however, we could only show in an appropriate orbifold limit. The proof that the full metric computed in this paper is indeed quaternionic-Kähler is left for a future project.

The line bundle backgrounds are realized by Abelian Yang-Mills fluxes on $K3$. They affect the 6D theories in that the scalars arising from the B-field become affinely gauged under the unbroken $U(1)$'s. This in turn gives a mass to the $U(1)$ gauge fields via a Stückelberg mechanism. For several line bundles which are linearly dependent in $H^2(K3, \mathbb{Z})$, massless $U(1)$ gauge fields remain in the 6D theory. At the same time the fluxes stabilize those $K3$ moduli which violate the anti-selfduality of the Yang-Mills field strength. In the effective potential this is realized as a Fayet-Iliopoulos term proportional to the flux vector. Together, one line bundle eliminates four scalars (one B scalar and three $K3$ moduli) from the effective theory, which are absorbed into a massive vector multiplet.

Recently [58] derived the 6D effective action of F-theory compactified on a Calabi-Yau three-fold X. When X is a $K3$ fibration, this background is dual to the heterotic theory compactified on $K3$ studied in this paper. It would be interesting to compare the two effective actions. On the F-theory side one may use our results to get information on the couplings of the charged matter (in [58] the action was derived on a generic point in the Coulomb branch, where these fields are massive, but eventually one has to go away

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from this branch in the F-theory limit). On the heterotic side one may use the results of [58] to understand the couplings of non-perturbative tensors (that in F-theory appear at perturbative level).

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A Details of the Kaluza-Klein reduction

A.1 Deformations of gauge connections

In this appendix we give a detailed derivation of the Kaluza-Klein reduction of the gauge potential, from which all bosonic matter fields descend. The low energy spectrum is determined by the gauge background consisting of a nontrivial holomorphic H-bundle over $K3$ and a flat G-bundle over $M^{1,5}$

$$E_8 \times E_8 \longrightarrow G \times \langle H \rangle \,, \tag{A.1}$$

where G is the maximal commutant of H. The H-bundle satisfies the Bianchi identity (3.1) and its nonzero field strength $\mathcal{F}$ satisfies the hermitean Yang-Mills equations (HYM)

$$\mathcal{F} \in H^{1,1}(K3, \mathfrak{h}), \qquad \mathcal{F} \wedge J = 0 \,. \tag{A.2}$$

Here we write $\mathfrak{h}$ for the adjoint H-bundle. (A.2) is equivalent to the anti-selfduality (ASD) of the field strength, $\star\mathcal{F} = -\mathcal{F}$ [37, 38]. We denote the background connection, valued in $\mathfrak{h}$, as $\mathcal{A}$ and its deformations give rise to massless 6D fields.[12] These deformations are grouped into multiplets according to the decomposition

$$\mathbf{496} \rightarrow \bigoplus_i \, (\mathbf{R}_i, \mathbf{S}_i) \oplus (\mathfrak{g}, \mathbf{1}) \oplus (\mathbf{1}, \mathfrak{h}) \,, \tag{A.3}$$

where $\mathfrak{g}$ and $\mathfrak{h}$ denote the adjoint representations of G and H, respectively. The $\mathbf{1}$ is the trivial representation and $(\mathbf{R}_i, \mathbf{S}_i)$ are group specific representations. It is known from supersymmetry that massless 6D hypermultiplets in representations $\mathbf{R}_i$ occur with multiplicities given by the chiral index [24]

$$\chi(E_{\mathbf{S}_i}) = h^{0,0}(K3, E_{\mathbf{S}_i}) - h^{0,1}(K3, E_{\mathbf{S}_i}) + h^{0,2}(K3, E_{\mathbf{S}_i}) \,, \tag{A.4}$$

where $E_{\mathbf{S}_i}$ denotes the vector bundle associated with $\mathbf{S}_i$.[13] In fact, $h^{0,0}(K3, E)$ and $h^{0,2}(K3, E)$ vanish for a HYM background. This can be seen as follows: $H^{0,0}(K3, E)$ is

[12] Since we insist on six-dimensional Lorentz invariance we do not include the possibility of a background value for the 6D gauge field.

[13] χ is called chiral index due to the equivalent definition $\chi(E) = n_E^+ - n_E^-$, where $n_E^\pm$ count the chiral zero modes of the Dirac operator. On $K3$ one has $\chi(E) = \chi(E^*)$, so complex conjugate representations always occur with equal multiplicities. Due to the definite chiralities in the vector- and hypermultiplets, $\chi(E)$ counts the difference of them.

the space of global sections of E, which are closed with respect to the covariant Dolbeault operator $\bar{\partial}_{\mathcal{A}}$ on $K3$. But for sections of a HYM-bundle we have the identity[14]

$$d^*_{\mathcal{A}} d_{\mathcal{A}} = 2\bar{\partial}^*_{\mathcal{A}}\bar{\partial}_{\mathcal{A}}\,, \tag{A.5}$$

where $d_{\mathcal{A}} = \partial_{\mathcal{A}} + \bar{\partial}_{\mathcal{A}}$. Therefore any such section is also covariantly constant. When E is nontrivial and irreducible, no constant sections exist. The vanishing of $H^{0,2}(K3, E)$ then follows by Serre duality [60].

For the Kaluza-Klein reduction of the bosonic action it is not enough to know this multiplicity. One has to know which internal differential equation the zero modes satisfy. Therefore we analyze the deformations of the gauge connection without referring to supersymmetry. Starting from the 10D Yang-Mills Lagrangian

$$\mathcal{L}^{YM} \sim \langle F, F\rangle = \mathrm{tr}(F \wedge *F)\,, \tag{A.6}$$

we parametrize the deformations by $A = \mathcal{A} + a$ with $a \in \Lambda^1(\mathfrak{e}_8)$. For simplicity we assume that the background H-bundle is inside one E_8 and consider only deformations inside this E_8. We restrict a to be compatible with the metric on the adjoint E_8 bundle.[15] The field strength deforms as

$$F = \mathcal{F} + f\,, \qquad f = d_{\mathcal{A}} a + \frac{1}{2}[a, a]\ . \tag{A.7}$$

As in the main text we decompose $a = a_1 + a_{\bar{1}}$ into 1-forms on $M^{1,5}$ and on $K3$. They deform the flat G- and the curved H-connection, respectively. Their 6D effective mass terms are given by

$$\mathcal{L}_6^{\mathrm{mass}}[a_1] \sim \int_{\mathrm{K3}} \mathrm{tr}(d_{\mathcal{A}} a_1 \wedge \star d_{\mathcal{A}} a_1)\,, \tag{A.8}$$

$$\mathcal{L}_6^{\mathrm{mass}}[a_{\bar{1}}] \sim \int_{\mathrm{K3}} \mathrm{tr}(d_{\mathcal{A}} a_{\bar{1}} \wedge \star d_{\mathcal{A}} a_{\bar{1}}) + \int_{\mathrm{K3}} \mathrm{tr}(a_{\bar{1}} \wedge \star[\mathcal{F}, a_{\bar{1}}])\ . \tag{A.9}$$

From (A.8) it follows that massless 6D vectors V_i arise from deformations with $d_{\mathcal{A}} a_1 = 0$. Therefore the Kaluza-Klein expansion reads

$$a_1 = V \cdot \psi\,, \qquad d_{\mathcal{A}}\psi = 0\,, \tag{A.10}$$

with internal covariantly constant functions (sections) ψ. Since there exist no globally constant sections on nontrivial vector bundles, massless 6D vectors can only occur from the term $(\mathfrak{g}, \mathbf{1})$ in (A.3). From the identity (A.5) (on sections) it follows that $\ker(d_{\mathcal{A}}) = \ker(\bar{\partial}_{\mathcal{A}})$. Hence the multiplicity is given by Dolbeault cohomology

$$h^{0,0}(K3, E_{\mathbf{1}}) = h^{0,0}(K3) = 1\ . \tag{A.11}$$

[14] A proof can be found, for example, in appendix E of [59].

[15] This amounts to the condition that the deformed connection $A = \mathcal{A} + a$ satisfies $d(h(\psi_1, \psi_2)) = h(d_A\psi_1, \psi_2) + h(\psi_1, d_A\psi_2)$, where h is the adjoint metric, i.e. locally the Killing form of the Lie algebra, and ψ_1, ψ_2 are sections of the adjoint bundle.

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The mass operator for 6D scalars is identified from (A.9) as

$$\Delta_{YM} a_{\bar{1}} := d^*_{\mathcal{A}} d_{\mathcal{A}} a_{\bar{1}} + \star[\mathcal{F}_{\mathcal{A}}, a_{\bar{1}}] \ . \tag{A.12}$$

Since this is not a proper Laplacian, the connection to Dolbeault cohomology is obscure at first sight. We now show that 1-form zero modes of Δ_{YM} are in one-to-one correspondence with zero modes of $\Delta_{\bar{\partial}_{\mathcal{A}}} := \bar{\partial}^*_{\mathcal{A}} \bar{\partial}_{\mathcal{A}} + \bar{\partial}_{\mathcal{A}} \bar{\partial}^*_{\mathcal{A}}$. Using the Kähler identities $\bar{\partial}^*_{\mathcal{A}} = i[\partial_A, J\cdot]$ and $\partial^*_{\mathcal{A}} = -i[\bar{\partial}_A, J\cdot]$ [61], we find the following operator identity on 1-forms

$$d^*_{\mathcal{A}} d_{\mathcal{A}} a_{\bar{1}} = 2\Delta_{\bar{\partial}_{\mathcal{A}}} a_{\bar{1}} - d_{\mathcal{A}} d^*_{\mathcal{A}} a_{\bar{1}} + iJ \cdot [\mathcal{F}, a_{\bar{1}}] \ . \tag{A.13}$$

Here $J\cdot$ is the contraction with the Kähler form. (There is an equivalent identity with $\Delta_{\partial_{\mathcal{A}}}$ instead of $\Delta_{\bar{\partial}_{\mathcal{A}}}$.) We prove (A.13) at the end of this section. The second term on the r.h.s. vanishes in the Lorenz gauge $d^*_{\mathcal{A}} a_{\bar{1}} = 0$. Moreover, on a complex Kähler surface with a HYM-bundle (i.e. anti-selfdual field strength) one can show that

$$\star\, [\mathcal{F}, a_{\bar{1}}] = -iJ \cdot [\mathcal{F}, a_{\bar{1}}] \ . \tag{A.14}$$

Inserting (A.13) and (A.14) into the mass operator (A.12), we are left with the (gauge fixed) identity on 1-forms

$$\Delta_{YM} = 2\Delta_{\bar{\partial}_{\mathcal{A}}} = 2\Delta_{\partial_{\mathcal{A}}} \ . \tag{A.15}$$

Since on holomorphic bundles the Dolbeault operator satisfies $\bar{\partial}^2_{\mathcal{A}} = 0$, the harmonic 1-forms of $\Delta_{\bar{\partial}_{\mathcal{A}}}$ are unique representatives of $H^{0,1}(K3, E)$. From (A.15) it also follows that the massless modes are zero modes of $d_{\mathcal{A}}$. This is obvious from (A.12) as a sufficient condition, but here we have shown that it is also necessary. Another way of seeing this is the following: Whereas the first term in (A.12) is a positive, symmetric operator, the second is in fact antisymmetric with respect to the YM-scalar product on $K3$

$$\langle a_{\bar{1}}, \star[\mathcal{F}, a_{\bar{1}}]\rangle = -\langle \star[\mathcal{F}, a_{\bar{1}}], a_{\bar{1}}\rangle \ . \tag{A.16}$$

Hence, the two terms correspond to real and imaginary part of the squared mass eigenvalues and have to vanish separately. Hence, we derived the supersymmetric result from pure bosonic Yang-Mills deformation theory.

Returning to the different terms in (A.3), no 6D scalars in the adjoint representation $\mathfrak{g}$ can occur, because $H^{0,1}(K3, E_{\mathbf{1}}) = H^{0,1}(K3) = 0$. Generically one gets scalars from representations $(\mathbf{R}, \mathbf{S})$ with some multiplicity $h^{0,1}(K3, E_{\mathbf{S}})$. Here two cases can arise: First, if $(\mathbf{R}, \mathbf{S})$ is a real representation and $\mathbf{R}$ is pseudoreal and we are left with $\mathbf{R}$-half-hypermultiplets in 6D. To have complex fields in 6D one decomposes the deformation as $a_{\bar{1}} = a^{0,1} + a^{1,0}$, using a complex structure on $K3$. Since a is restricted to preserve the hermitean structure of the $\mathfrak{e}_8$ bundle, the two terms satisfy [61]

$$(a^{1,0})^\dagger = -a^{0,1} \ . \tag{A.17}$$

Hence, the Kaluza-Klein expansion reads

$$a_{\bar{1}} = C^{\mathbf{R}}_k \omega_k + \overline{C}^{\overline{\mathbf{R}}}_k \overline{\omega}_k \ . \tag{A.18}$$

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Second, if there are complex representations occuring in conjugated pairs, $(\mathbf{R},\mathbf{S})\oplus(\overline{\mathbf{R}},\overline{\mathbf{S}})$, two sets of independent 6D scalars arise

$$a_{\bar{1}} = C_k^{\mathbf{R}}\omega_k + \overline{C}_k^{\overline{\mathbf{R}}}\overline{\omega}_k + D_k^{\mathbf{R}}\varpi_k + \overline{D}_k^{\overline{\mathbf{R}}}\overline{\varpi}_k \; . \tag{A.19}$$

The zero modes of both cases are given by

$$\begin{aligned} \omega_k &\in H^{0,1}(K3,E_{\mathbf{S}})\,, & \overline{\omega}_k &\in H^{1,0}(K3,E_{\overline{\mathbf{S}}})\,, \\ \varpi_k &\in H^{1,0}(K3,E_{\mathbf{S}})\,, & \overline{\varpi}_k &\in H^{0,1}(K3,E_{\overline{\mathbf{S}}}) \; . \end{aligned} \tag{A.20}$$

Here $E_{\overline{\mathbf{S}}} = (E_{\mathbf{S}})^*$ is the dual vector bundle. On $K3$ all multiplicities are the same due to Serre duality

$$\overline{H^{0,1}(K3,E_{\mathbf{S}})} \cong H^{0,1}(K3,E_{\overline{\mathbf{S}}}) \tag{A.21}$$

and can be computed via the chiral index (A.4).[16] Thus, in 6D one has hypermultiplets with scalar components $\Phi_k^{\mathbf{R}\oplus\overline{\mathbf{R}}} = (C_k^{\mathbf{R}}, \overline{D}_k^{\overline{\mathbf{R}}})$.

Let us now show that the 6D singlet scalars coming from the term $(\mathbf{1},\mathfrak{h})$ in (A.3) are special in that they are not only massless but exact flat directions of the potential. They are termed bundle moduli. Applying the previous analysis it follows that there exist massless deformations with multiplicity $h^{0,1}(K3,\mathfrak{h})$. In fact, any such deformation preserves (A.2) and hence the ASD condition of the background $\mathcal{F}$. It is known that the moduli space of ASD connections modulo gauge transformations is equivalent to the moduli space of holomorphic structures (see for example [38]). A holomorphic structure is defined by a Dolbeault operator satisfying $\bar{\partial}_{\mathcal{A}}^2 = \mathcal{F}^{0,2} = 0$. A deformation $A = \mathcal{A} + a$, with $a \in \Lambda^1(K3,\mathfrak{h})$ defines another holomorphic structure if $\mathcal{F}_A^{0,2} = 0$, i.e.

$$\bar{\partial}_{\mathcal{A}} a^{0,1} + \frac{1}{2}[a^{0,1},a^{0,1}] = 0 \; . \tag{A.22}$$

Infinitesimally this yields $a^{0,1} \in \ker(\bar{\partial}_{\mathcal{A}})$. However $a \in \ker(\bar{\partial}_{\mathcal{A}})$ contains directions which lead to gauge-equivalent holomorphic structures which have to be modded out. Their Dolbeault operators are related by conjugation in H

$$\bar{\partial}_{\mathcal{A}}^h = h^{-1}\bar{\partial}_{\mathcal{A}} h \approx \bar{\partial}_{\mathcal{A}} + \bar{\partial}_{\mathcal{A}}\delta h\,, \tag{A.23}$$

where $h \in \Lambda^0(K3,H)$ and $h \approx \mathbf{1} + \delta h$, $\delta h \in \Lambda^0(K3,\mathfrak{h})$. Modding out the term $\bar{\partial}_{\mathcal{A}}\delta h \in \mathrm{Im}(\bar{\partial}_{\mathcal{A}})$, infinitesimal deformations of the holomorphic structure are given by $a^{0,1} \in H^{0,1}(K3,\mathfrak{h})$, in agreement with the result from the mass operator. But since the effective scalar potential from the background takes the form

$$V_6 \sim -\int \mathrm{tr}(\mathcal{F}_+ \wedge \star\mathcal{F}_+)\,, \tag{A.24}$$

(see (3.48)) all deformations preserving the ASD condition are moduli, i.e. flat directions of the scalar potential. Finally, the Kaluza-Klein expansion of the $(\mathbf{1},\mathfrak{h})$-scalars reads

$$a_{\bar{1}} = \xi_k\alpha_k + \overline{\xi}_k\overline{\alpha}_k\,, \qquad \alpha_k \in H^{0,1}(K3,\mathfrak{h}) \; . \tag{A.25}$$

[16] On a Calabi Yau 3-fold the $C^{\mathbf{R}}$ and $\overline{D}^{\overline{\mathbf{R}}}$ occur with different multiplicities, yielding the 4D chiral spectrum.

The complex 6D scalars ξ_k are called bundle moduli. In the following sections the above results are applied to the standard embedding and the line bundle background.

We finally give a proof of the formula (A.13) for $a \in \Lambda^1(K3, E)$:

$$d_{\mathcal{A}}^* d_{\mathcal{A}} a = (\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} + \partial_{\mathcal{A}}^* \partial_{\mathcal{A}}) a + (\bar{\partial}_{\mathcal{A}}^* \partial_{\mathcal{A}} + \partial_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}}) a \tag{A.26}$$

The first term can be written as

$$\begin{aligned}(\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} + \partial_{\mathcal{A}}^* \partial_{\mathcal{A}}) a &= i([\partial_{\mathcal{A}}, J\cdot]\bar{\partial}_{\mathcal{A}} - [\bar{\partial}_{\mathcal{A}}, J\cdot]\partial_{\mathcal{A}}) a \\ &= i(\partial_{\mathcal{A}} J \cdot \bar{\partial}_{\mathcal{A}} - \bar{\partial}_{\mathcal{A}} J \cdot \partial_{\mathcal{A}}) a - iJ \cdot (\partial_{\mathcal{A}} \bar{\partial}_{\mathcal{A}} a - \bar{\partial}_{\mathcal{A}} \partial_{\mathcal{A}} a) \\ &= (\partial_{\mathcal{A}} \partial_{\mathcal{A}}^* + \bar{\partial}_{\mathcal{A}} \bar{\partial}_{\mathcal{A}}^*) a + iJ \cdot [\mathcal{F}, a] - 2iJ \cdot (\partial_{\mathcal{A}} \bar{\partial}_{\mathcal{A}} a) \ .\end{aligned} \tag{A.27}$$

Here we used the Kähler identities $\partial_{\mathcal{A}}^* = -i[\bar{\partial}_{\mathcal{A}}, J\cdot]$, $\bar{\partial}_{\mathcal{A}}^* = i[\partial_{\mathcal{A}}, J\cdot]$, $J \cdot \partial_{\mathcal{A}} a = [J\cdot, \partial_{\mathcal{A}}] a$ since $J \cdot a = 0$, and we identified $\mathcal{F} = \partial_{\mathcal{A}} \bar{\partial}_{\mathcal{A}} + \bar{\partial}_{\mathcal{A}} \partial_{\mathcal{A}}$. We now write the last term in (A.27) as

$$\begin{aligned}2iJ \cdot (\partial_{\mathcal{A}} \bar{\partial}_{\mathcal{A}} a) &= 2i([J\cdot, \partial_{\mathcal{A}}] + \partial_{\mathcal{A}} J\cdot) \bar{\partial}_{\mathcal{A}} a \\ &= -2\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} a + 2i \partial_{\mathcal{A}} [J\cdot, \bar{\partial}_{\mathcal{A}}] a \\ &= -2\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} a + 2\partial_{\mathcal{A}} \partial_{\mathcal{A}}^* a \ .\end{aligned} \tag{A.28}$$

With this we get

$$(\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} + \partial_{\mathcal{A}}^* \partial_{\mathcal{A}}) a = (\bar{\partial}_{\mathcal{A}} \bar{\partial}_{\mathcal{A}}^* - \partial_{\mathcal{A}} \partial_{\mathcal{A}}^*) a + iJ \cdot [\mathcal{F}, a] + 2\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} a \ . \tag{A.29}$$

Now we consider the second term in (A.26)

$$(\bar{\partial}_{\mathcal{A}}^* \partial_{\mathcal{A}} + \partial_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}}) a = (\partial_{\mathcal{A}} \bar{\partial}_{\mathcal{A}}^* + \bar{\partial}_{\mathcal{A}} \partial_{\mathcal{A}}^*) a = d_{\mathcal{A}} d_{\mathcal{A}}^* a - (\partial_{\mathcal{A}} \partial_{\mathcal{A}}^* + \bar{\partial}_{\mathcal{A}} \bar{\partial}_{\mathcal{A}}^*) a \,, \tag{A.30}$$

where we used $\{\partial_{\mathcal{A}}, \bar{\partial}_{\mathcal{A}}^*\} = 0$ (which follows from the Kähler identities). Together we end up with the claimed result (A.13)

$$d_{\mathcal{A}}^* d_{\mathcal{A}} a = -d_{\mathcal{A}} d_{\mathcal{A}}^* a + 2(\bar{\partial}_{\mathcal{A}}^* \bar{\partial}_{\mathcal{A}} + \bar{\partial}_{\mathcal{A}} \bar{\partial}_{\mathcal{A}}^*) a + iJ \cdot [\mathcal{F}, a] \ . \tag{A.31}$$

A.2 Zero modes in the standard embedding

For the standard embedding the nontrivial SU(2) bundle is inside one E_8 factor, yielding the breaking

$$E_8 \longrightarrow E_7 \times \langle \mathrm{SU}(2) \rangle \ . \tag{A.32}$$

Focusing on this E_8 factor we have the decomposition

$$\mathbf{248} \to (\mathbf{56}, \mathbf{2}) \oplus (\mathbf{133}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \ . \tag{A.33}$$

The vector bundles E corresponding to the right entries are identified as $E_{\mathbf{2}} = \mathcal{T}_{\mathrm{K3}}$, which is the holomorphic tangent bundle, $E_{\mathbf{3}} = \mathfrak{su}(2) = \mathrm{End}\ \mathcal{T}_{\mathrm{K3}}$, which is the adjoint bundle and $E_{\mathbf{1}} = \mathcal{O}$, which is the trivial bundle over $K3$. Since $(\mathbf{56}, \mathbf{2})$ is a real representation, its massless Kaluza-Klein components are given by

$$a_{\bar{1}}^{(\mathbf{56},\mathbf{2})} = C_j^{\mathbf{56}} \omega_j + \overline{C}_j^{\overline{\mathbf{56}}} \overline{\omega}_j \,, \qquad j = 1, \ldots, 20 \ . \tag{A.34}$$

Here the zero modes are

$$\omega_j \in H^{0,1}(\mathcal{T}_{\rm K3}) \cong H^{1,1}(K3)\,,$$
$$\overline{\omega}_j \in H^{1,0}(\overline{\mathcal{T}_{\rm K3}}) \cong H^{1,1}(K3)\;. \tag{A.35}$$

From the Hodge diamond (2.17) we see that the multiplicity is 20. We realize the isomorphy to $H^{1,1}(K3)$ with the holomorphic 2-form Ω and a particular prefactor, i.e. in components (no summation over j implied)[17]

$$(\omega_j)_{\bar{\alpha}}{}^{\beta} = \frac{\gamma_j}{||\Omega||^2}\overline{\Omega}^{\alpha\beta}(\eta_j)_{\alpha\bar{\alpha}}\,,$$
$$(\overline{\omega}_j)_{\alpha}{}^{\bar{\beta}} = \frac{\gamma_j}{||\Omega||^2}\Omega^{\bar{\alpha}\bar{\beta}}(\eta_j)_{\alpha\bar{\alpha}}\,, \tag{A.36}$$

where η_j are the harmonic $(1,1)$ forms on $K3$ and γ_j is the real function

$$\gamma_j = \frac{\mathcal{V}^{\frac{1}{4}}}{\left(\int J\wedge\eta_j\right)^{\frac{1}{2}}}\;. \tag{A.37}$$

This function is motivated by matching with the orbifold limit of the standard embedding which we discuss in appendix B. In fact, the zero modes of the charged scalars depend on the complex structure of $K3$ by the very definition of $\mathcal{T}_{\rm K3}$. For a fixed complex structure the prefactor γ_j depends on the remaining Kähler moduli in such a way that the full zero mode is independent of them.

The term $(\mathbf{133},\mathbf{1})$ gives rise to one 6D vector $A^{\mathbf{133}}$, as stated in (A.11). The term $(\mathbf{1},\mathbf{3})$ corresponds to the bundle moduli as specified in (A.25)

$$a_{\bar{1}}^{(\mathbf{1},\mathbf{3})} = \xi_k\alpha_k + \overline{\xi}_k\overline{\alpha}_k\,,\qquad \alpha_k \in H^{0,1}(\text{End}\;\mathcal{T}_{\rm K3})\;. \tag{A.38}$$

The multiplicity cannot be related to the Hodge numbers but can be computed with the chiral index (A.4). Here $h^{0,0}(\text{End}\;\mathcal{T}_{\rm K3}) = 0$, since a covariantly constant section $g \in \Gamma(K3, \text{End}\;\mathcal{T}_{\rm K3})$ must take values in the centralizer of the holonomy group, which is empty for $\text{hol}(K3) = \mathfrak{su}(2)$ [37]. Thus, one obtains

$$\chi(\text{End}\;\mathcal{T}_{\rm K3}) = -h^{0,1}(\text{End}\;\mathcal{T}_{\rm K3})\;. \tag{A.39}$$

χ can be computed via the Hirzebruch-Riemann-Roch theorem[18] which states

$$\chi(E_{\mathbf{S}}) = \int_{\rm K3}\text{Td}(K3)\wedge ch(E_{\mathbf{S}}) = 2\text{rk}(E_{\mathbf{S}}) + ch_2(E_{\mathbf{S}})\,, \tag{A.40}$$

where $\text{Td}(K3)$ is the Todd-class of $K3$, $\text{rk}(E)$ is the rank of the vector bundle and $ch_2(E_{\mathbf{S}}) = -\frac{1}{2}\int \text{tr}_{\mathbf{S}}\mathcal{F}\wedge\mathcal{F}$ is the second Chern-character. Using $\text{rk}(\text{End}\;\mathcal{T}_{\rm K3}) = 3$ we get

$$h^{0,1}(\text{End}\;\mathcal{T}_{\rm K3}) = -6 + \frac{1}{2}\int \text{tr}_{\mathbf{3}}(\mathcal{F}\wedge\mathcal{F}) = -6 + \frac{4}{2}\int \text{tr}_{\mathbf{2}}(\mathcal{F}\wedge\mathcal{F}) = -6 + 4\cdot 24 = 90\,, \tag{A.41}$$

[17]There exists an alternative isomorphism, $\omega_{\bar{\alpha}}^{\beta} \propto g^{\beta\bar{\gamma}}(t_{(\bar{\alpha}\bar{\gamma})} + t_{[\bar{\alpha}\bar{\gamma}]}) = g^{\beta\bar{\gamma}}(\overline{\Omega}^{\delta}{}_{(\bar{\alpha}}\omega_{\bar{\gamma})\delta} + \overline{\Omega}_{\bar{\alpha}\bar{\gamma}})$, which maps $H^{0,1}(\mathcal{T}_{\rm K3})$ to the anti-holomorphic 2-form $\overline{\Omega}$ plus all $(1,1)$-forms except the Kähler form. We always use the simpler one (A.36).

[18]See for example chapter 5.1 of [61].

where in the last step we used the integrated tadpole condition (3.1)

$$\frac{1}{2}\int \mathrm{tr}_{\mathbf{2}}(\mathcal{F}\wedge\mathcal{F}) = \chi(K3) = 24 \; . \qquad \text{(A.42)}$$

Summarizing, the Kaluza-Klein expansion of the gauge potential reads

$$a_1 = V^{\mathbf{133}} \,, \qquad a_{\bar{1}} = C_j^{\mathbf{56}}\omega_j \oplus \overline{C}_j^{\overline{\mathbf{56}}}\overline{\omega}_j + \xi_k\alpha_k + \bar{\xi}_k\overline{\alpha}_k \,, \qquad \text{(A.43)}$$

with $j = 1, \ldots, 20$ and $k = 1, \ldots, 90$.

A.3 Coupling functions in the standard embedding

In this section we derive the coupling functions of the effective action. First we consider the kinetic terms in (3.29) and in particular the couplings of the charged scalars. Due to the correspondence of their zero-modes to harmonic $(1,1)$-forms (A.36) these functions exhibit a characteristic dependence on the $K3$ moduli.[19] To express this dependence in the following, let us review the parametrization of the $K3$ moduli space (2.20) from [55]. A Riemannian metric is given by a positive definite three-dimensional subspace $\Sigma := H^2_+(K3,\mathbb{R}) \subset H^2(K3,\mathbb{R})$, which is spanned by an orthonormal dreibein (J_1, J_2, J_3). The $K3$ moduli t_s^I are defined by the expansion

$$J_s = t_s^I \eta_I \,, \qquad I = 1, \ldots, 22 \; . \qquad \text{(A.44)}$$

They are constrained to be (positive) orthonormal

$$\rho_{IJ} t_s^I t_t^J = \delta_{st} \,, \qquad \text{(A.45)}$$

and subject to an equivalence relation which identifies equivalent metrics

$$t_s^I \sim \tilde{t}_s^I = R_s{}^t t_t^I \,, \qquad R \in \mathrm{SO}(3) \; . \qquad \text{(A.46)}$$

R rotates the dreibein inside Σ and corresponds to an S^2 of possible complex structures per metric.

In the following we want to relate the moduli space of the charged scalars to the moduli space of $K3$ metrics. Due to the very definition of $\mathcal{T}_{\mathrm{K3}}$ in the standard embedding, the charged scalar zero modes are defined with respect to a chosen complex structure. Hence, the discussion of their couplings implicitly requires the breaking of the Hyperkähler structure of $K3$. Defining the complex structure via the 2-form $\Omega = J_1 + iJ_2$, the harmonic $(1,1)$ forms in the charged scalars zero modes (A.36) are given the projection

$$\eta_I^{1,1} = (P^{1,1})_I{}^J \eta_J \,, \qquad (P^{1,1})_I{}^J = \delta_I^J - \sum_{s=1,2} \rho_{IK} t_s^K t_s^J \,, \qquad \text{(A.47)}$$

where ρ_{IJ} is the intersection form (2.19). They depend on the complex structure moduli t_1^I, t_2^I. In the following we fix the complex structure and discuss the dependence of the

[19] Recall that on $K3$ the embedding $H^{1,1}(K3,\mathbb{R}) \subset H^2(K3,\mathbb{R})$ is a moduli dependent subspace.

charged scalar couplings on the remaining Kähler moduli. As in (A.36) $\eta_j, j = 3, \ldots 22$ denotes a basis of $H^{1,1}(K3, \mathbb{R})$ with respect to the fixed complex structure.

Let us illustrate this by a first example. The KK reduction of (3.12) yields the kinetic term of the charged scalars in (3.29)

$$-\frac{\alpha'}{\sqrt{2}} \mathcal{D}\overline{C}_i^x \wedge *\mathcal{D}C_j^x \, \mathcal{V}^{-\frac{3}{2}} \int g_{\bar{\alpha}\beta} \overline{\omega}_i^{\bar{\alpha}} \wedge \star \omega_j^\beta \,, \tag{A.48}$$

where $g_{\bar{\alpha}\beta}$ is the Kähler metric on $K3$. We show now that the charged scalar metric G_{ij} is indeed related to the b-scalar metric g_{IJ} given in (3.32). Using the zero mode isomorphism (A.36) and the identities

$$\Omega^{\bar{\alpha}\bar{\beta}} = f(z)|g|^{-\frac{1}{2}} \varepsilon^{\bar{\alpha}\bar{\beta}} \,, \qquad |f|^2 = \|\Omega\|^2 \sqrt{g} \tag{A.49}$$

as well as the normalization $\|\Omega\|^2 = \frac{2}{\mathcal{V}}$ we obtain

$$\begin{aligned} G_{ij} &= \frac{1}{\sqrt{2}\mathcal{V}^{\frac{3}{2}}} \int g_{\bar{\alpha}\beta} \overline{\omega}_i^{\bar{\alpha}} \wedge \star \omega_j^\beta \\ &= \frac{\gamma_i \gamma_j}{\sqrt{2}\mathcal{V}^{\frac{3}{2}} \|\Omega\|^4} \int g_{\bar{\alpha}\beta} g^{\delta\bar{\delta}} |f|^2 \varepsilon^{\bar{\alpha}\bar{\gamma}} \varepsilon^{\beta\gamma} (\eta_i)_{\bar{\gamma}\delta} (\eta_j)_{\gamma\bar{\delta}} |g|^{-\frac{1}{2}} d^4x \\ &= \frac{\gamma_i \gamma_j}{\sqrt{2}\mathcal{V}^{\frac{3}{2}} \|\Omega\|^2} \int g^{\bar{\gamma}\gamma} g^{\delta\bar{\delta}} (\eta_i)_{\bar{\gamma}\delta} (\eta_j)_{\gamma\bar{\delta}} \sqrt{g} \; d^4x \\ &= \frac{\gamma_i \gamma_j}{2\sqrt{2\mathcal{V}}} \int \eta_i \wedge \star \eta_j \;. \end{aligned} \tag{A.50}$$

From the last line in (A.50) (no summation over i, j implied) one recognizes that this function is proportional to the projection of the b-scalar metric g_{IJ}

$$g_{ij} := \int \eta_i \wedge \star \eta_j = (P^{1,1})_i{}^I (P^{1,1})_j{}^J g_{IJ} \,, \qquad g_{IJ} = \int \eta_I \wedge \star \eta_J \;. \tag{A.51}$$

While $P^{1,1}$ depends on the fixed complex structure, g_{ij} also depends on the remaining Kähler moduli via the action of the Hodge $\star$ operator on $H^{1,1}(K3, \mathbb{R})$ [55]

$$\star\, \eta_i = \left(-\delta_i^j + 2\rho_{ik} t_3^k t_3^j \right) \eta_j \;. \tag{A.52}$$

For the coupling function N_{ij}^I in (3.34) which is obtained from a KK reduction of (3.27) we first use the same manipulations as above to get

$$N_{ij} \;=\; \Omega_{\alpha\beta} \, \omega_i^\alpha \wedge \omega_j^\beta \;=\; \frac{\gamma_i \gamma_j}{\|\Omega\|^2} \overline{\Omega} \cdot (\eta_i \wedge \eta_j) \;=\; \frac{\gamma_i \gamma_j}{\|\Omega\|^2} \rho_{ij} \overline{\Omega} \cdot \mathrm{vol} \;=\; \frac{\mathcal{V} \gamma_i \gamma_j}{2\sqrt{g}} \rho_{ij} \overline{\Omega} \,. \tag{A.53}$$

Here $\cdot$ denotes the contraction of forms and vol is the volume form, normalized to 1. In the second step we used $\eta_i \wedge \eta_j = \rho_{ij}\mathrm{vol}$ and in the third step we used $\overline{\Omega} \cdot \mathrm{vol} = g^{-\frac{1}{2}} \overline{\Omega}$. The coupling ρ_{ij} is defined as the projection

$$\rho_{ij} := \int \eta_i \wedge \eta_j = (P^{1,1})_i{}^I (P^{1,1})_j{}^J \rho_{IJ} \,, \tag{A.54}$$

where ρ_{IJ} is the moduli independent intersection matrix. Hence, the expansion into η_I has coefficients

$$N_{ij}^I = \rho^{IJ} \int N_{ij} \wedge \eta_J = \frac{1}{2}\gamma_i\gamma_j\rho_{ij}\rho^{IJ} \int \overline{\Omega} \wedge \eta_J = \frac{1}{2}\gamma_i\gamma_j\rho_{ij}\rho^{IJ}(\langle J_1, \eta_J\rangle - i\langle J_2, \eta_J\rangle)\,, \tag{A.55}$$

where $\langle\cdot,\cdot\rangle$ is the scalar product on $H^2(K3, \mathbb{R})$.

For the coupling function M_{ij}^I in (3.34) which also arise from (3.27) we proceed similarly to get

$$M_{ij} = \frac{1}{\sqrt{2\mathcal{V}}} g_{\alpha\bar\beta}\omega_i^\alpha \wedge \overline{\omega}_j^{\bar\beta} = \frac{\sqrt{\mathcal{V}}\gamma_i\gamma_j}{2\sqrt{2}} g^{\gamma\bar\delta}(\eta_i)_{\gamma\bar\alpha}(\eta_j)_{\beta\bar\delta} dz^\beta \wedge d\bar z^{\bar\alpha}\ . \tag{A.56}$$

Identifying the components of the Kähler form as $g_{\alpha\bar\beta} = -iJ_{\alpha\bar\beta}$ and $g_{\bar\alpha\beta} = iJ_{\bar\alpha\beta}$ we can express M_{ij} as the special contraction

$$\begin{aligned} M_{ij} &= -i\frac{\sqrt{\mathcal{V}}\gamma_i\gamma_j}{2\sqrt{2}}\Big(J\cdot(\eta_i\wedge\eta_j) - (J\cdot\eta_i)\ \eta_j - (J\cdot\eta_j)\ \eta_i\Big) \\ &= -i\frac{\sqrt{\mathcal{V}}\gamma_i\gamma_j}{2\sqrt{2}}\left(\rho_{ij}(J\cdot\mathrm{vol}) - \frac{\sqrt{2\mathcal{V}}}{\sqrt{g}}\langle J_3,\eta_i\rangle\eta_j - \frac{\sqrt{2\mathcal{V}}}{\sqrt{g}}\langle J_3,\eta_j\rangle\eta_i\right) \\ &= -i\frac{\mathcal{V}\gamma_i\gamma_j}{2\sqrt{g}}\Big(\rho_{ij}J_3 - \langle J_3,\eta_i\rangle\eta_j - \langle J_3,\eta_j\rangle\eta_i\Big)\ . \end{aligned} \tag{A.57}$$

Here we used the following identities

$$(J\cdot\eta_i)\mathrm{vol} = \frac{1}{\sqrt{g}} J\wedge\eta_i = \sqrt{\frac{2\mathcal{V}}{g}}\langle J_3,\eta_i\rangle\mathrm{vol}\,, \qquad J\cdot\mathrm{vol} = \frac{1}{\sqrt{g}}J\ . \tag{A.58}$$

Hence, the expansion into η_I has coefficients

$$M_{ij}^I = \rho^{IJ}\int M_{ij}\wedge\eta_J = i\frac{\gamma_i\gamma_j}{2}\rho^{IJ}\Big(\rho_{ij}\langle J_3,\eta_J\rangle - \langle J_3,\eta_i\rangle\rho_{jJ} - \langle J_3,\eta_j\rangle\rho_{iJ}\Big)\ . \tag{A.59}$$

Both couplings M and N depend on the $K3$ moduli but for a fixed complex structure we have the following simplification. In a basis (η_1, η_2, η_i) of $H^2(K3,\mathbb{R})$, where $\eta_{1,2}$ span the complex structure 2-plane, we have $\langle J_{1,2}, \eta_I\rangle = 0$ for $I = i$ and $\langle J_3, \eta_I\rangle = 0$ for $I = 1, 2$. This implies

$$\begin{aligned} N_{ij}^I &\neq 0 \quad \text{only for } I = 1, 2\,, \\ M_{ij}^I &\neq 0 \quad \text{only for } I = 3, \ldots, 22\ . \end{aligned} \tag{A.60}$$

In this basis the couplings (3.31) between the charged scalars and the b-scalars reduce to

$$\mathcal{D}_c b^I = \begin{pmatrix} db^{1,2} - \alpha'\varepsilon_{xy}(N_{kl}^{1,2}C_k^x\mathcal{D}C_l^y + c.c.) - \ldots \\ db^i - \alpha'\delta_{xy}M_{kl}^i\overline{C}_k^x\overleftrightarrow{\mathcal{D}}C_l^y - \ldots \end{pmatrix}, \tag{A.61}$$

where the dots stand for the $\bar\xi d\xi$ terms. Moreover, for the b-scalar combination $b^i\eta_i = t_3^i\eta_i$ proportional to the Kähler form of $K3$, the coupling function reduces to

$$M_{ij} = -i\frac{\gamma_i\gamma_j}{2}g_{ij}\,, \tag{A.62}$$

with g_{ij} known from (A.51). In appendix B we will use the second row in (A.61) to identify a quaternionic Kähler moduli subspace, containing complexified Kähler moduli and charged scalars.

Let us now turn to the scalar potential which contains quartic terms of the charged scalars. These arise from the squares of the expressions (3.19) and (3.20). The term in (3.20), which is in the adjoint representation of the surviving gauge group, gives rise to D-terms in 6D. The term in (3.19) is not allowed by 6D supergravity and we shall prove here that it vanishes due to properties of $K3$ and its bundles. First recall from (3.42) that only the selfdual components $\delta F_{\bar{2}+}$ contribute to the scalar potential which will be crucial to show the consistency with 6D supergravity. Recall (3.19)

$$f_{\bar{2}}^{(\mathbf{1},\mathbf{3})} = \begin{pmatrix} \overline{C}_i^x \\ C_i^x \end{pmatrix}^T \begin{pmatrix} \sigma^s_{\bar{\alpha}\beta} \overline{\omega}_i^{\bar{\alpha}} \wedge \omega_j^{\beta} & \sigma^s_{\bar{\alpha}\bar{\beta}}\, \overline{\omega}_i^{\bar{\alpha}} \wedge \overline{\omega}_j^{\bar{\beta}} \\ \sigma^s_{\alpha\beta}\, \omega_i^{\alpha} \wedge \omega_j^{\beta} & \sigma^s_{\alpha\bar{\beta}}\, \omega_i^{\alpha} \wedge \overline{\omega}_j^{\bar{\beta}} \end{pmatrix} \delta_{xy} \begin{pmatrix} C_j^y \\ \overline{C}_j^y \end{pmatrix}, \tag{A.63}$$

where all matrix elements are 2-forms in the group $H^2(\mathrm{End}\ \mathcal{T}_{\mathrm{K3}})$ as follows from the group representation $(\mathbf{1},\mathbf{3})$. We now use a local decomposition of $H^2(\mathrm{End}\ \mathcal{T}_{\mathrm{K3}})$ and show that its global extension does not exist. In fact any 2-form in $H^2(\mathrm{End}\ \mathcal{T}_{\mathrm{K3}})$ can be locally trivialized as

$$f^i \otimes \omega_i \ \in\ \Gamma(\mathrm{End}\ \mathcal{T}_{\mathrm{K3}}) \otimes \Lambda^2(K3)\,, \tag{A.64}$$

where $i = 1, \ldots, 6$. Since the zero modes in (A.63) are $d_{\mathcal{A}}$-closed also their products are $d_{\mathcal{A}}$-closed. This implies

$$0 = d_{\mathcal{A}}(f^i \otimes \omega_i) = (d_{\mathcal{A}} f^i) \wedge \omega_i + f^i (d\omega_i)\ . \tag{A.65}$$

For the scalar potential we restrict this equation to the selfdual 2-forms. Since there exists on $K3$ a basis of d-closed selfdual 2-forms, (A.65) reduces in this basis to

$$d_{\mathcal{A}} f^i = 0\ . \tag{A.66}$$

Hence, the f^i are covariantly constant sections of End $\mathcal{T}_{\mathrm{K3}}$, which have to extend to globally constant sections. However, since End $\mathcal{T}_{\mathrm{K3}}$ is an irreducible, nontrivial bundle, only the constant zero section exists. In other words, the deformation (A.63) preserves the ASD property of the background field strength and therefore does not contribute to the scalar potential.

Next we calculate the selfdual part of (3.20)

$$f_{\bar{2}}^{(\mathbf{133},\mathbf{1})} = \begin{pmatrix} \overline{C}_i^x \\ C_i^x \end{pmatrix}^T \begin{pmatrix} \frac{1}{\sqrt{2\mathcal{V}}} g_{\bar{\alpha}\beta}\, \overline{\omega}_i^{\bar{\alpha}} \wedge \omega_j^{\beta} & \overline{\Omega}_{\bar{\alpha}\bar{\beta}}\, \overline{\omega}_i^{\bar{\alpha}} \wedge \overline{\omega}_j^{\bar{\beta}} \\ \Omega_{\alpha\beta}\, \omega_i^{\alpha} \wedge \omega_j^{\beta} & \frac{1}{\sqrt{2\mathcal{V}}} g_{\alpha\bar{\beta}}\, \omega_i^{\alpha} \wedge \overline{\omega}_j^{\bar{\beta}} \end{pmatrix} (\tau^a)_{xy} \begin{pmatrix} C_j^y \\ \overline{C}_j^y \end{pmatrix}. \tag{A.67}$$

We recognize that the same coupling functions appear as in (A.53) and (A.56) so that we have

$$f_{\bar{2}}^{(\mathbf{133},\mathbf{1})} = \begin{pmatrix} \overline{C}_i^x \\ C_i^x \end{pmatrix}^T \begin{pmatrix} -M_{ij} & \overline{N}_{ij} \\ N_{ij} & M_{ij} \end{pmatrix} (\tau^a)_{xy} \begin{pmatrix} C_j^y \\ \overline{C}_j^y \end{pmatrix}. \tag{A.68}$$

The off-diagonal elements are already selfdual 2-forms given by (A.53), while the diagonal elements are generic $(1,1)$-forms. We get their selfdual part by projecting onto J_3

$$M_{ij+} = \left(\int M_{ij}\wedge J_3\right) J_3 = -i\frac{\gamma_i\gamma_j}{2}\Big(\rho_{ij}\langle J_3,J_3\rangle - 2\langle J_3,\eta_i\rangle\langle J_3,\eta_j\rangle\Big) J_3 = i\sqrt{2\mathcal{V}}G_{ij}J_3\ . \tag{A.69}$$

Here we identified the kinetic coupling G_{ij} using (A.52), (A.50) and

$$g_{ij} = \int \eta_i\wedge\star\eta_j = (-\delta^k_j + 2\rho_{jl}t^k_3 t^l_3)\rho_{ik} = -\rho_{ij}\langle J_3,J_3\rangle + 2\langle J_3,\eta_i\rangle\langle J_3,\eta_j\rangle\ . \tag{A.70}$$

Summarizing, we have

$$f^{(\mathbf{133},\mathbf{1})}_{\bar{2}+} = \begin{pmatrix}\overline{C}^x_i\\ C^x_i\end{pmatrix}^T \begin{pmatrix} -i\sqrt{2\mathcal{V}}G_{ij}J_3 & \frac{1}{2}\tilde\rho_{ij}\Omega\\ \frac{1}{2}\tilde\rho_{ij}\overline{\Omega} & i\sqrt{2\mathcal{V}}G_{ij}J_3\end{pmatrix}(\tau^a)_{xy}\begin{pmatrix}C^y_j\\ \overline{C}^y_j\end{pmatrix}\,, \tag{A.71}$$

where we $\tilde\rho_{ij} = \gamma_i\gamma_j\rho_{ij}$ denotes the rescaled intersection matrix on $H^{1,1}(K3,\mathbb{R})$.

A.4 Zero modes in line bundle backgrounds

We now apply the results from appendix A.1 to deformations of a line bundle background. For one $U(1)$ principal bundle inside one E_8 factor we have the breaking

$$E_8 \longrightarrow G\times\langle U(1)\rangle\,, \tag{A.72}$$

and the adjoint decomposition

$$\mathbf{248}\longrightarrow \bigoplus_i\ \big((\mathbf{R}_i,\mathbf{1}_{q_i})\oplus(\overline{\mathbf{R}}_i,\mathbf{1}_{-q_i})\big)\oplus(\mathfrak{g},\mathbf{1}_0)\oplus(\mathbf{1},\mathbf{1}_0)\,, \tag{A.73}$$

which defines the associated vector bundles. Due to (A.11) we get again one 6D gauge potential $V^{\mathfrak{g}}$ in the adjoint of G. However, now the $\langle U(1)\rangle$ is part of the unbroken gauge group since it commutes with itself. Since here $\mathfrak{h} = \mathbf{1}_0$ corresponds to the trivial line bundle, there also exists a 6D Abelian gauge potential $V^{\mathbf{1}}$ in the same representation $(\mathbf{1},\mathbf{1}_0)$ as the background connection $\mathcal{A}$. There exist no bundle moduli, since End $L^q = \mathcal{O}$ is the trivial bundle and

$$H^{0,1}(\text{End } L^q) \cong H^{0,1}(K3,\mathbb{R}) = 0\ . \tag{A.74}$$

Finally, we get charged scalars in representations $\mathbf{R}_i$. Their multiplicity cannot be related to the Hodge numbers of $K3$, but we have

$$h^{0,1}(L^q) = -\chi(L^q)\,, \tag{A.75}$$

by the same argument as in (A.4). The chiral index of a line bundle over a four-dimensional manifold takes the simplified form (4.4) as we will show now. The total Chern-character $ch(L) = \operatorname{tr}\exp(\frac{i}{2\pi}\mathcal{F})$ factorizes for product bundles,

$$ch(L^q) = ch(L)\wedge\ldots\wedge ch(L)\,, \tag{A.76}$$

which implies

$$ch_2(L^q) = q\ ch_2(L) + \frac{1}{2}q(q-1)ch_1(L)^2\ . \tag{A.77}$$

For line bundles we have $ch_2(L) = \frac{1}{2}ch_1(L)^2$ such that

$$ch_2(L^q) = q^2 ch_2(L)\ . \tag{A.78}$$

Using $\mathrm{rk}(L^q) = \mathrm{rk}(L) = 1$, the chiral index reduces to

$$\chi(L^q) = 2\mathrm{rk}(L^q) + ch_2(L^q) = 2 + q^2 ch_2(L)\ . \tag{A.79}$$

Therefore (4.4) is verified.

The Kaluza-Klein expansion of the gauge potential is analogous to (A.19) and reads

$$a_1 = V^{\mathfrak{g}} + V^{\mathbf{1}}\,, \qquad a_{\bar{1}} = \sum_i \left(C^{\mathbf{R}_i}_{k_i}\omega^{q_i}_{k_i} + \overline{C}^{\overline{\mathbf{R}}_i}_{k_i}\overline{\omega}^{-q_i}_{k_i}\right) + \left(\overline{D}^{\overline{\mathbf{R}}_i}_{k_i}\overline{\varpi}^{-q_i}_{k_i} + D^{\mathbf{R}_i}_{k_i}\varpi^{q_i}_{k_i}\right)\,. \tag{A.80}$$

The zero modes belong to the Dolbeault cohomology groups

$$\begin{aligned} \omega^{q_i}_{k_i} \in H^{0,1}(L^{q_i}) \quad &, \quad \overline{\omega}^{-q_i}_{k_i} \in H^{1,0}(L^{-q_i})\,, \\ \varpi^{q_i}_{k_i} \in H^{1,0}(L^{q_i}) \quad &, \quad \overline{\varpi}^{-q_i}_{k_i} \in H^{0,1}(L^{-q_i})\,, \end{aligned} \tag{A.81}$$

with multiplicities $k_i = 1, \ldots, -\chi(L^{q_i})$.

The scalar potential of the charged scalars contains the selfdual parts of (4.16), (4.17) and (4.20), i.e.

$$f^{\mathfrak{g}}_{2+}\,, \quad f^{\mathbf{1}}_{2+}\,, \quad f^{\mathbf{R}_i\oplus\overline{\mathbf{R}}_i}_{2+}\ . \tag{A.82}$$

We show first that any term of the form $f^{\mathbf{R}_i\oplus\overline{\mathbf{R}}_i}_{2+}$ vanishes. The product of internal zero modes in (4.20) belong to $H^2(L^{q_i}\oplus L^{-q_i})$ and they are also closed under the gauge covariant derivative $d_{\mathcal{A}}$. Locally we can write these 2-forms as

$$s^i \otimes \alpha_i\,, \quad s^i \in \Gamma(L^{q_i} \oplus L^{-q_i})\,, \quad \alpha_i \in \Lambda^2(K3)\,, \tag{A.83}$$

where $i = 1, \ldots, 6$ is the number of locally independent 2-forms. Then we have

$$0 = d_{\mathcal{A}}(s^i \otimes \alpha_i) = (d_{\mathcal{A}} s^i) \wedge \alpha_i + s^i \otimes (d\alpha_i)\ . \tag{A.84}$$

If we restrict to the d-closed selfdual 2-forms, (A.84) reduces to

$$0 = (d_{\mathcal{A}} s^j) \wedge \alpha^+_j\ . \tag{A.85}$$

It follows that $f^{\mathbf{R}_i\oplus\overline{\mathbf{R}}_i}_{2+}$ is proportional to covariantly constant sections $s^j \in \Gamma(L^{q_i} \oplus L^{-q_i})$. However, since $L^{q_i}\oplus L^{-q_i}$ is nontrivial and irreducible, only the constant zero section exists. We conclude that all $f^{\mathbf{R}_i\oplus\overline{\mathbf{R}}_i}_{2+}$ vanish.

Next we derive the selfdual part of $f^{\mathfrak{g}}_{2+}$ and $f^{\mathbf{1}}_{2+}$. Considering the matrix of internal 2-forms in (4.16) and (4.17),

$$\begin{pmatrix} \overline{\omega}^{-q_i}_{k_i} \wedge \omega^{q_i}_{l_i} & \overline{\omega}^{-q_i}_{k_i} \wedge \varpi^{q_i}_{l_i} \\ \overline{\varpi}^{-q_i}_{k_i} \wedge \omega^{q_i}_{l_i} & \overline{\varpi}^{-q_i}_{k_i} \wedge \varpi^{q_i}_{l_i} \end{pmatrix}, \tag{A.86}$$

they take values in the trivial bundle, $H^2(K3, L^{q_i} \otimes L^{-q_i}) = H^2(K3)$. Hence, covariantly constant sections exist. Projecting to the selfdual components we get

$$\begin{aligned}
(\overline{\omega}_{k_i}^{-q_i} \wedge \omega_{l_i}^{q_i})_+ &= \frac{i}{2\mathcal{V}} \left(\int \overline{\omega}_{k_i}^{-q_i} \wedge \star\omega_{l_i}^{q_i} \right) J \,, \\
(\overline{\varpi}_{k_i}^{-q_i} \wedge \varpi_{l_i}^{q_i})_+ &= \frac{i}{2\mathcal{V}} \left(\int \overline{\varpi}_{k_i}^{-q_i} \wedge \star\varpi_{l_i}^{q_i} \right) J \,, \\
(\overline{\omega}_{k_i}^{-q_i} \wedge \varpi_{l_i}^{q_i})_+ &= \frac{1}{2} \left(\int (\overline{\omega}_{k_i}^{-q_i} \wedge \varpi_{l_i}^{q_i}) \wedge \overline{\Omega} \right) \Omega \,, \\
(\overline{\varpi}_{k_i}^{-q_i} \wedge \omega_{l_i}^{q_i})_+ &= \frac{1}{2} \left(\int (\overline{\varpi}_{k_i}^{-q_i} \wedge \omega_{l_i}^{q_i}) \wedge \Omega \right) \overline{\Omega} \,.
\end{aligned} \tag{A.87}$$

The diagonal elements are proportional to the scalar kinetic metric $g^C_{k_i l_i}$ and $g^D_{k_i l_i}$, that appeared in (4.28). The off-diagonal elements contain a generalized intersection matrix

$$c_{k_i l_i} = \int (\overline{\omega}_{k_i}^{-q_i} \wedge \varpi_{l_i}^{q_i}) \wedge \overline{\Omega} \,, \tag{A.88}$$

where the indices run over the multiplicity of the corresponding charged scalars.

B $T^4/\mathbb{Z}_3$ limit: hypermultiplet moduli space metric

In this appendix we focus on a specific orbifold corresponding to a heterotic compactification on a smooth $K3$ with standard embedding for the gauge bundle. In this case we are able to give an explicit form of the hypermultiplet field space for the untwisted moduli.

Specifically we consider the $E_8 \times E_8$ heterotic string compactified on the orbifold $T^4/\mathbb{Z}_3$ with gauge twist given by $\frac{1}{3}(1^2, 0^6)(0^8)$ [48]. In this case the unbroken gauge group is $E_7 \times U(1) \times E_8$. In the hypermultiplet sectors we have both untwisted and twisted states in the following representations:[20]

$$(\mathbf{56}, \mathbf{1})_1^{\text{untw}} \oplus (\mathbf{1}, \mathbf{1})_2^{\text{untw}} \oplus 2(\mathbf{1}, \mathbf{1})_0^{\text{untw}} \oplus 9(\mathbf{56}, \mathbf{1})_{\frac{1}{3}}^{\text{tw}} \oplus 45(\mathbf{1}, \mathbf{1})_{\frac{2}{3}}^{\text{tw}} \oplus 18(\mathbf{1}, \mathbf{1})_{\frac{4}{3}}^{\text{tw}} \,. \tag{B.1}$$

When we blow up the orbifold $T^4/\mathbb{Z}_3$ we get a smooth $K3$. After a field redefinition, the orbifold spectrum matches with the spectrum obtained by a smooth compactification with nontrivial gauge bundle [13]. In particular, the two $(\mathbf{1}, \mathbf{1})_0^{\text{untw}}$ are the two hypermultiplets containing the four geometric moduli and the four B-field moduli surviving the $\mathbb{Z}_3$ projection, the $(\mathbf{56}, \mathbf{1})_1^{\text{untw}}$ is a charged field, and the $(\mathbf{1}, \mathbf{1})_2^{\text{untw}}$ is eaten to give mass to the $U(1)$ gauge boson. Therefore the total orbifold spectrum matches the spectrum of the smooth compactification considered in section 3, i.e. 20 geometric, 45 bundle moduli and 10 charged hypermultiplets.

The metric on the hypermultiplet scalar field space in the untwisted sector, can be obtained by considering the 6D heterotic compactification on T^4 and performing a suitable

[20] The untwisted spectrum is obtained by taking the spectrum coming from compactification on T^4 and performing the $\mathbb{Z}_3$ projection. The twisted spectrum comes from strings localized around the orbifold singularities.

truncation [39, 62]. For the case at hand the truncation is

$$\frac{\mathrm{SO}(4,4+N)}{\mathrm{SO}(4)\times\mathrm{SO}(4+N)} \qquad \rightarrow \qquad \frac{\mathrm{SU}(2,2+n)}{U(1)\times\mathrm{SU}(2)\times\mathrm{SU}(2+n)}\,. \tag{B.2}$$

The latter space is simultaneously quaternionic-Kähler and Kähler, with a metric determined by the Kähler potential

$$K = -\log\det(T + T^\dagger - 2\Psi\Psi^\dagger)\,. \tag{B.3}$$

Ψ is a $2\times n$ complex matrix, which encodes the two complex scalars belonging to the n hypermultiplets in the untwisted charged spectrum (in our case $n = 56$.) T is a 2×2 complex matrix given by

$$(T_{ij}) = \begin{pmatrix} g_{1\bar 1} + iB_{1\bar 1} + \Psi_1\overline{\Psi}_1 & g_{12} + iB_{12} + \Psi_1\overline{\Psi}_2 \\ \overline{g_{12}} + i\overline{B_{12}} + \Psi_2\overline{\Psi}_1 & g_{2\bar 2} + iB_{2\bar 2} + \Psi_2\overline{\Psi}_2 \end{pmatrix}\,. \tag{B.4}$$

It contains the real $g_{1\bar 1}$, $g_{2\bar 2}$ and the complex g_{12} metric elements and the the corresponding components of the B-field. $\Psi_i\overline{\Psi}_j$ includes a summation over the n components. For simplicity let us fix the complex structure such that $g_{12} = 0$. In this limit, the Kähler potential (B.3) yields the kinetic terms

$$K_{T_{ij}\overline{T}_{kl}} dT_{ij} d\overline{T}_{kl} = \frac{1}{4g_{1\bar 1}^2} dT_{11} d\overline{T}_{11} + \frac{1}{4g_{2\bar 2}^2} dT_{22} d\overline{T}_{22} + \frac{1}{4g_{1\bar 1}g_{2\bar 2}}(dT_{12}d\overline{T}_{12} + dT_{21}d\overline{T}_{21})\,, \tag{B.5}$$

$$K_{\Psi_i\overline{\Psi}_j} d\Psi_i d\overline{\Psi}_j = \left(\frac{1}{g_{1\bar 1}} + \frac{\Psi_2\overline{\Psi}_2}{g_{1\bar 1}g_{2\bar 2}} + \frac{\Psi_1\overline{\Psi}_1}{g_{1\bar 1}^2}\right) d\Psi_1 d\overline{\Psi}_1 + \left(\frac{1}{g_{2\bar 2}} + \frac{\Psi_1\overline{\Psi}_1}{g_{1\bar 1}g_{2\bar 2}} + \frac{\Psi_2\overline{\Psi}_2}{g_{2\bar 2}^2}\right) d\Psi_2 d\overline{\Psi}_2\,, \tag{B.6}$$

$$K_{T_{ij}\overline{\Psi}_k} dT_{ij} d\overline{\Psi}_k = -\frac{\Psi_1}{2g_{1\bar 1}^2} dT_{11} d\overline{\Psi}_1 - \frac{\Psi_2}{2g_{2\bar 2}^2} dT_{22} d\overline{\Psi}_2 - \frac{\Psi_2}{2g_{1\bar 1}g_{2\bar 2}} dT_{12} d\overline{\Psi}_1 - \frac{\Psi_1}{2g_{1\bar 1}g_{2\bar 2}} dT_{21} d\overline{\Psi}_2\,. \tag{B.7}$$

Inserting (B.4) we get the kinetic terms in terms of the Kaluza-Klein modes [39, 62]. The leading term for the charged scalars reads

$$\sum_{i=1,2} \frac{1}{g_{i\bar i}} d\Psi_i d\overline{\Psi}_i\,. \tag{B.8}$$

The terms for the two complexified Kähler moduli read

$$\sum_{i=1,2} \frac{1}{4g_{i\bar i}^2} |dg_{i\bar i} + idB_{i\bar i} + \overline{\Psi}_i d\Psi_i - \Psi_i d\overline{\Psi}_i|^2\,. \tag{B.9}$$

The terms for the off-diagonal fields in T read

$$\frac{1}{4g_{1\bar 1}g_{2\bar 2}} \left(|idB_{12} + \Psi_1 d\overline{\Psi}_2 - \overline{\Psi}_2 d\Psi_1|^2 + |id\overline{B_{12}} + \Psi_2 d\overline{\Psi}_1 - \overline{\Psi}_1 d\Psi_2|^2\right)\,. \tag{B.10}$$

We now compare the above kinetic couplings with our results (3.29) coming from the smooth $K3$. To make contact with the ones just derived, we have to take the orbifold limit and identify the $K3$ moduli related to $g_{i\bar{i}}$. The $T^4/\mathbb{Z}_3$ limit of $K3$ corresponds to taking the 3-plane Σ orthogonal to 18 two-cycles with intersection matrix $A_2^{\oplus 9}$.[21] The orthogonal complement (where Σ lives) must contain the two complex 2-tori (that we call η_1, η_2) spanned by the coordinates z^i, plus two 2-cycles (called η_3, η_4) with positive self-intersection and that are not of type $(1,1)$. They have the following intersection matrix:

$$\begin{pmatrix} 0 & 3 & & \\ 3 & 0 & & \\ & & 2 & -1 \\ & & -1 & 2 \end{pmatrix} . \tag{B.11}$$

The chosen complex structure (i.e. $g_{12} = 0$) makes the metric hermitean, allowing us to identify the $g_{i\bar{i}}$ elements with the coefficient of the Kähler form J along the Poincaré dual of the two 2-tori. On the K3 side we need to take the two 2-tori of type $(1,1)$. This is done by making J be a linear combination of (the Poincaré dual of) η_1 and η_2 and Ω live in the positive definite subspace $\{\eta_3, \eta_4\}$. Also B will have components along η_1 and η_2:

$$J = t^1\,\eta_1 + t^2\,\eta_2\,, \qquad\qquad B = b^1\,\eta_1 + b^2\,\eta_2 + \dots\,, \tag{B.12}$$

and we have the identifications $g_{i\bar{i}} \leftrightarrow t^i$ and $B_{i\bar{i}} \leftrightarrow b^i$.

First, consider the coupling in front of (B.9). The smooth result reduces in the orbifold limit to

$$\frac{1}{\mathcal{V}} g_{IJ} = \frac{1}{\mathcal{V}} \int \eta_I \wedge \star\eta_J \;\longrightarrow\; \begin{pmatrix} \frac{1}{(t^1)^2} & \\ & \frac{1}{(t^2)^2} \end{pmatrix}, \tag{B.13}$$

which matches with (B.9) up to a numerical constant. For the leading charged scalar coupling we have

$$G_{ij} = \frac{\gamma_i\gamma_j}{2\sqrt{2\mathcal{V}}} \int \eta_i \wedge \star\eta_j \;\longrightarrow\; \frac{3}{2\sqrt{2}} \begin{pmatrix} \frac{1}{\langle J,\eta_1\rangle}\frac{t^2}{t^1} & \\ & \frac{1}{\langle J,\eta_2\rangle}\frac{t^1}{t^2} \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} \frac{1}{t^1} & \\ & \frac{1}{t^2} \end{pmatrix}, \tag{B.14}$$

which matches with (B.8). Here we see that for the orbifold match it is necessary to include the moduli dependent functions $\gamma_j = \mathcal{V}^{\frac{1}{4}}/\langle J, \eta_i\rangle^{\frac{1}{2}}$ in the isomorphy of zero modes (A.36). In fact, the moduli dependence of the skew-symmetric couplings M^I_{ij} drops out in the orbifold limit, as expected. The only nonvanishing components are

$$M^1_{11} = M^2_{22} = -\frac{i}{\sqrt{2}}\,. \tag{B.15}$$

This matches with (B.10).

[21] $T^4/\mathbb{Z}_3$ has nine A_2-singularities (i.e. locally $\mathbb{C}^2/\mathbb{Z}_3$). One ADE singularity of $K3$ is generated by shrinking a set of two-cycles with the intersection matrix given by (minus) the Cartan matrix of the corresponding ADE group.

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