

Dark matter particle spectroscopy at the LHC: generalizing M_{T2} to asymmetric event topologies

Partha Konar,^a Kyoungchul Kong,^b Konstantin T. Matchev^a and Myeonghun Park^a

^aPhysics Department, University of Florida,
Gainesville, FL 32611, U.S.A.

^bTheoretical Physics Department, SLAC,
Menlo Park, CA 94025, U.S.A.

E-mail: konar@phys.ufl.edu, kckong@slac.stanford.edu,
matchev@phys.ufl.edu, ishaed@phys.ufl.edu

ABSTRACT: We consider SUSY-like missing energy events at hadron colliders and critically examine the common assumption that the missing energy is the result of two *identical* missing particles. In order to experimentally test this hypothesis, we generalize the subsystem M_{T2} variable to the case of asymmetric event topologies, where the two SUSY decay chains terminate in different “children” particles. In this more general approach, the endpoint $M_{T2(max)}$ of the M_{T2} distribution now gives the mass $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ of the parent particles as a function of *two* input children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. We propose two methods for an independent determination of the *individual* children masses $M_c^{(a)}$ and $M_c^{(b)}$. First, in the presence of upstream transverse momentum P_{UTM} the corresponding function $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM})$ is independent of P_{UTM} at precisely the right values of the children masses. Second, the previously discussed M_{T2} “kink” is now generalized to a “ridge” on the 2-dimensional surface $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$. As we show in several examples, quite often there is a special point along that ridge which marks the true values of the children masses. Our results allow collider experiments to probe a multi-component dark matter sector directly and without any theoretical prejudice.

KEYWORDS: Supersymmetry Phenomenology

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1 Introduction

A general expectation in high energy physics today is that physics beyond the standard model (BSM) should emerge at the TeV scale in order to stabilize the hierarchy between the Planck and electroweak scales. Further evidence in support of this belief is provided by the dark matter problem of astro-particle physics, which can be quite naturally solved by postulating the existence of a new, weakly-interacting dark matter particle with a mass in the TeV range. Such dark matter particles are naturally present in the most popular BSM scenarios such as supersymmetry [1], extra dimensions [2–4], little Higgs theory [5, 6] etc. They will be produced in the upcoming high-energy collisions at the Large Hadron Collider (LHC) at CERN, which offers an exciting opportunity to study dark matter in a high-energy lab. Since the dark matter particles are weakly interacting, they do not leave any deposits inside the detector and can only manifest themselves in the form of missing energy. Recently, there has been a lot of theoretical effort directed at testing the dark matter hypothesis at the LHC [7–16] and the future International Linear Collider (ILC) [7, 9–11, 17–20]. Unfortunately, most of these studies have been performed in some very model-dependent as well as very complex setup.¹ In the literature, a typical collider study of dark matter most often starts with the assumption of a specific model with a dark matter candidate (usually supersymmetry with its myriad of parameters) and then investigates the model’s predictions for the expected rates at the LHC in one or several missing energy channels. Rarely, if ever, has the question been posed in reverse: what does the observation of a missing energy signal at the LHC tell us about the dark matter particle and its properties in a generic and model-independent way [21].

1.1 Probing the dark matter sector at colliders

Naturally, the most pertinent question after the discovery of any BSM missing energy signal at the LHC is simply whether the new signal is indeed due to the production of new massive particles, or whether it is just an enhancement in the production of SM neutrinos [21]. In principle, there are two handles that can be used in addressing this question. In order to prove dark matter production, one can measure the *mass* of the missing particle and show that it is different (heavier) from the SM neutrino masses. Alternatively, one can try to measure the *spin* of the missing particle and show that it is different from 1/2 (the spin of the neutrino). While there is a large body of recent work on spin measurements in missing energy events [22–48], once again very few of those methods are model-independent² [45, 47]. Furthermore, in all considered examples in the literature the spin measurement appears to be very difficult. Therefore, in this paper we shall concentrate on the question of measuring the mass(es) of the particles responsible for the missing energy. In doing so, we are motivated by two reasons. First, previous experience indicates that the mass question will be answered long before any spin measurements, and second, many of the spin determination methods require prior knowledge of the mass spectrum anyway.

¹Some notable exceptions are the studies in refs. [7, 8].

²In the sense that the method does not rely on an experimentally unverified hypothesis about the chirality of the fermion couplings and the size of the mixing angles of the new particles.

The difficulty in measuring the mass of the dark matter particle at a hadron collider like the Tevatron or the LHC is widely appreciated and has generated a lot of recent activity [49–103]. The main problem can be understood as follows. In a typical BSM dark matter scenario, the cosmological longevity of the dark matter particle is ensured by some new symmetry³ under which the SM particles are singlets. At the same time, there are additional particles in the spectrum which are charged under the new symmetry. If the lightest one among those is electrically and color neutral, it is a potential dark matter candidate, whose lifetime is protected by the new symmetry. With any such setup, it is clear that *single* production of dark matter particles at colliders is forbidden by the symmetry. Therefore, each event has *at least two* missing particles, whose energies and momenta are unknown. As a rule, it is typically impossible to fully reconstruct the kinematics of such events and observe the mass of the missing particle directly as an invariant mass peak.⁴ Consequently, one has to resort to various indirect methods of extracting the mass of the dark matter particle.

Unfortunately, most⁵ existing studies in the literature have explicitly or implicitly made the following two assumptions:

- *Single dark matter component.* A common assumption throughout the collider phenomenology literature is that colliders are probing only one dark matter species at a time, i.e. that the missing energy signal at colliders is due to the production of one and only one type of dark matter particles. Of course, there is no astrophysical evidence that the dark matter is made up of a single particle species: it may very well be that the dark matter world has a rich structure, just like ours [104]. Consequently, if there exist several types of dark matter particles, each contributing some fraction to the total relic density, a priori there is no reason why they cannot *all* be produced in high energy collisions. Theoretical models with multiple dark matter candidates have also been proposed [105–113].
- *Identical missing particles in each event.* A separate assumption, common to most previous studies, is that the two missing particles in each event are *identical*. This assumption could in principle be violated as well, even if the single dark matter component hypothesis is true. The point is that one of the missing particles in the event may *not* be a dark matter particle, but simply some heavier cousin which decays invisibly. An invisibly decaying heavy neutralino ($\tilde{\chi}_i^0 \rightarrow \nu\bar{\nu}\tilde{\chi}_1^0$ with $i > 1$) and an invisibly decaying sneutrino ($\tilde{\nu} \rightarrow \nu\tilde{\chi}_1^0$) are two such examples from supersymmetry. As far as the event kinematics is concerned, the mass of the heavier cousin *is* a relevant parameter and approximating it with the mass of the dark matter particle will simply give nonsensical results. Another relevant example is provided by models in which the SUSY cascade may terminate in any one of several light neutral particles [114].

³Some popular examples are: R -parity in supersymmetry, KK parity in Universal Extra Dimensions, T -parity in Little Higgs models, Z -parity in warped extra dimensions, U -parity in extended gauge theories, etc.

⁴For studies attempting full event reconstruction in long cascade chains, see refs. [63, 75, 89, 93, 102].

⁵To our knowledge, the only exception is the discussion in ref. [97] and the previous unpublished work in [78, 96].

Given our utter ignorance about the structure of the dark matter sector, in this paper we set out to develop the necessary formalism for carrying out missing energy studies at hadron colliders in a very general and model-independent way, without relying on any assumptions about the nature of the missing particles. In particular, we shall *not* assume that the two missing particles in each event are the same. We shall also allow for the simultaneous production of *several* dark matter species, or alternatively, for the production of a dark matter candidate in association with a heavier, invisibly decaying particle. Under these very general circumstances, we shall try to develop a method for measuring the individual masses of *all* relevant particles - the various missing particles which are responsible for the missing energy, as well as their parents which were originally produced in the event.

1.2 Generalizing M_{T2} to asymmetric event topologies

In general, by now there is a wide variety of techniques available for mass measurements in SUSY-like missing energy events. Such events are characterized by the pair production of two new particles, each of which undergoes a sequence of cascade decays ending up in a particle which is invisible in the detector. Each technique has its own advantages and disadvantages.⁶ For our purposes, we chose to revamp the method of the Cambridge M_{T2} variable [50] and adapt it to the more general case of an asymmetric event topology shown in figure 1. Consider the inclusive production of two identical⁷ parents of mass M_p as shown in figure 1. The parent particles may be accompanied by any number of “upstream” objects, such as jets from initial state radiation [66, 67, 80], or visible decay products of even heavier (grandparent) particles [86]. The exact origin and nature of the upstream objects will be of no particular importance to us, and the only information about them that we shall use will be their total transverse momentum \vec{P}_{UTM} . In turn, each parent particle initiates a decay chain (shown in red) which produces a certain number $n^{(\lambda)}$ of Standard Model (SM) particles (shown in gray) and an intermediate “child” particle of mass $M_c^{(\lambda)}$. Throughout this paper we shall use the index λ to classify various objects as belonging to the upper ($\lambda = a$) or lower ($\lambda = b$) branch in figure 1. The child particle may or may not be a dark matter candidate: in general, it may decay further as shown by the dashed lines in figure 1. We shall apply the “subsystem” M_{T2} concept [77, 86] to the subsystem within the blue rectangular frame. The SM particles from each branch within the subsystem form a composite particle of known⁸ transverse momentum $\vec{p}_T^{(\lambda)}$ and invariant mass $m_{(\lambda)}$. Since the children masses $M_c^{(a)}$ and $M_c^{(b)}$ are a priori unknown, the subsystem M_{T2} will be defined in terms of two “test” masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. In figure 1, $\vec{q}_T^{(\lambda)}$ are the trial transverse momenta of the two children. The individual momenta $\vec{q}_T^{(\lambda)}$ are also a priori unknown, but they are constrained by transverse momentum conservation:

$$\vec{q}_T^{(a)} + \vec{q}_T^{(b)} \equiv \vec{Q}_{\text{tot}} = -(\vec{p}_T^{(a)} + \vec{p}_T^{(b)} + \vec{P}_{\text{UTM}}). \quad (1.1)$$

⁶For a comparative review of the three main techniques, see [86].

⁷In principle, the assumption of identical parents can also be relaxed, by a suitable generalization of the M_{T2} variable, in which the mass ratio of the two parents is treated as an additional input parameter [97].

⁸We assume that there are no neutrinos among the SM decay products in each branch.

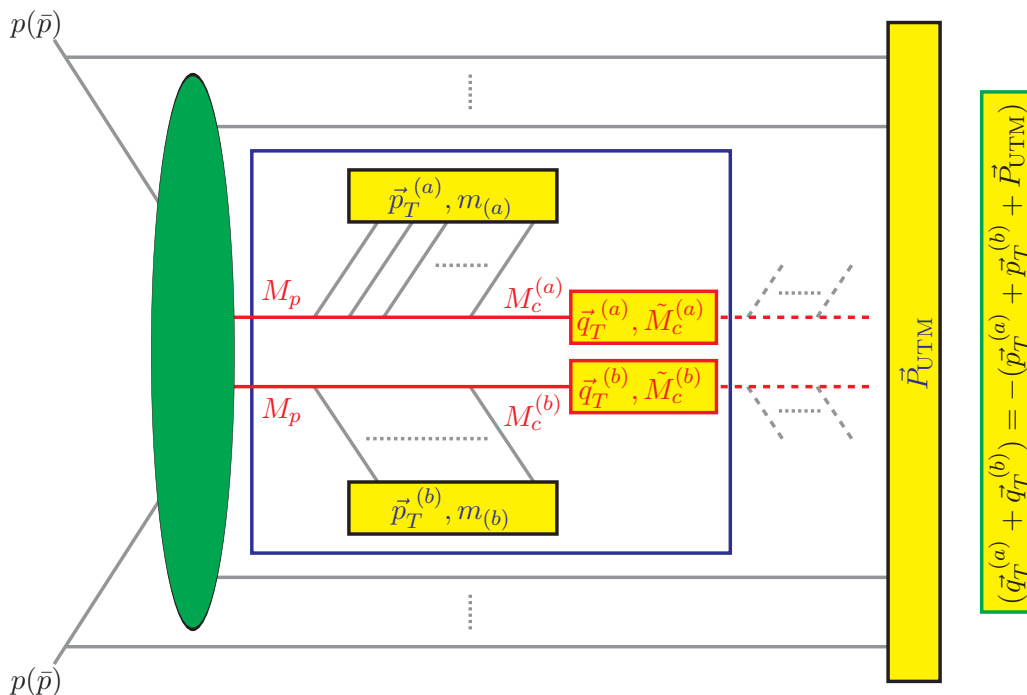


Figure 1. The generic event topology under consideration in this paper. We consider the inclusive pair-production of two “parent” particles with identical masses M_p . The parents may be accompanied by “upstream” objects, e.g. jets from initial state radiation, visible decay products of even heavier particles, etc. The transverse momentum of all upstream objects is measured and denoted by \vec{P}_{UTM} . In turn, each parent particle initiates a decay chain (shown in red) which produces a certain number $n^{(\lambda)}$ of SM particles (shown in gray) and an intermediate “child” particle of mass $M_c^{(\lambda)}$, where $\lambda = a$ ($\lambda = b$) for the branch above (below). In general, the child particle does not have to be the dark matter candidate, and may decay further as shown by the dashed lines. The M_{T2} variable is defined for the subsystem inside the blue box and is defined in terms of two arbitrary children “test” masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. The $n^{(\lambda)}$ SM particles from each branch form a composite particle of transverse momentum $\vec{p}_T^{(\lambda)}$ and invariant mass $m_{(\lambda)}$, correspondingly. The trial transverse momenta $\vec{q}_T^{(\lambda)}$ of the children obey the transverse momentum conservation relation shown inside the green box. In general, the number $n^{(\lambda)}$, as well as the type of SM decay products in each branch do not have to be the same.

Given this very general setup, in section 3 we shall consider a generalization⁹ of the usual M_{T2} variable which can apply to the asymmetric event topology of figure 1. There will be two different aspects of the asymmetry:

- First and foremost, we shall avoid the common assumption that the two children have the same mass. This will be important for two reasons. On the one hand, it will allow us to study events in which there are indeed two different types of missing particles. We shall give several such examples in the subsequent sections. More importantly, the endpoint of the asymmetric M_{T2} variable will allow us to measure

⁹The possibility of applying the M_{T2} variable to an event topology with different children was previously mentioned in refs. [96, 97].

the two children masses *separately*. Therefore, even when the events contain identical missing particles, as is usually assumed throughout the literature, one would be able to establish this fact experimentally from the data, instead of relying on an ad hoc theoretical assumption.

- As can be seen from figure 1, in general, the number as well as the types of SM decay products in each branch may be different as well. Once we allow for the children to be different, and given the fact that we start from identical parents, the two branches of the subsystem will naturally involve different sets of SM particles.

In what follows, when referring to the more general M_{T2} variable defined in section 3, we shall interchangeably use the terms “asymmetric” or “generalized” M_{T2} . In contrast, we shall use the term “symmetric” when referring to the more conventional M_{T2} definition with identical children.

The traditional M_{T2} approach assumes that the children have a common test mass $\tilde{M}_c \equiv \tilde{M}_c^{(a)} = \tilde{M}_c^{(b)}$ and then proceeds to find one functional relation between the true child mass M_c and the true parent mass M_p as follows [50]. Construct several M_{T2} distributions for different input values of the test children mass \tilde{M}_c and then read off their upper kinematic endpoints $M_{T2(max)}(\tilde{M}_c)$. These endpoint measurements are then interpreted as an output parent mass \tilde{M}_p , which is a function of the input test mass \tilde{M}_c :

$$\tilde{M}_p(\tilde{M}_c) \equiv M_{T2(max)}(\tilde{M}_c). \tag{1.2}$$

The importance of this functional relation is that it is automatically satisfied for the *true* values M_p and M_c of the parent and child masses:

$$M_p = M_{T2(max)}(M_c). \tag{1.3}$$

In other words, if we could somehow guess the correct value M_c of the child mass, the function (1.2) will provide the correct value M_p of the parent mass. However, since the true child mass M_c is a priori unknown, the individual masses M_p and M_c still remain undetermined and must be extracted by some other means.

At this point, it may seem that by considering the asymmetric M_{T2} variable with non-identical children particles, we have regressed to some extent. Indeed, we are introducing an additional degree of freedom in eq. (1.2), which now reads

$$\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \equiv M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}). \tag{1.4}$$

The standard M_{T2} endpoint method will still allow us to find the parent mass \tilde{M}_p , but now it is a function of *two* input parameters $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ which are completely unknown. Of course, if one knew the correct values of the two children masses $M_c^{(a)}$ and $M_c^{(b)}$ entering eq. (1.4), the true parent mass M_p will be given in a manner analogous to eq. (1.3):

$$M_p = M_{T2(max)}(M_c^{(a)}, M_c^{(b)}). \tag{1.5}$$

Our main result in this paper is that in spite of the apparent remaining arbitrariness in eq. (1.4), one can nevertheless uniquely determine *all three* masses M_p , $M_c^{(a)}$ and $M_c^{(b)}$,

just by studying the behavior of the measured function $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$. More importantly, this determination can actually be done in two different ways! Our first method is simply a generalization of the observation made in refs. [65–68, 86] that under certain circumstances (varying $m_{(\lambda)}$ or nonvanishing upstream momentum P_{UTM}), the function (1.2) develops a “kink” precisely at the correct value M_c of the child mass:

$$\left(\frac{\partial \tilde{M}_p(\tilde{M}_c)}{\partial \tilde{M}_c}\right)_{\tilde{M}_c+\epsilon} - \left(\frac{\partial \tilde{M}_p(\tilde{M}_c)}{\partial \tilde{M}_c}\right)_{\tilde{M}_c-\epsilon} \begin{cases} = 0, & \text{if } \tilde{M}_c \neq M_c, \\ \neq 0, & \text{if } \tilde{M}_c = M_c. \end{cases} \quad (1.6)$$

In other words, the function (1.2) is continuous, but not differentiable at the point $\tilde{M}_c = M_c$. In the asymmetric M_{T2} case, we find that the function (1.4) is similarly non-differentiable at a *set* of points $\{(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})\}$, so that the kink of eq. (1.6) is generalized to a “ridge” on the 2-dimensional hypersurface defined by (1.4) in the three-dimensional parameter space of $\{\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, \tilde{M}_p\}$.¹⁰ Interestingly enough, the ridge often (albeit not always) exhibits a special point which marks the exact location of the true values $(M_c^{(a)}, M_c^{(b)})$.

Our second method for determining the two children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ is even more general and is applicable under any circumstances. The main starting point is that just like the endpoint of the symmetric M_{T2} , the endpoint of the asymmetric M_{T2} also depends on the value of the upstream transverse momentum P_{UTM} , so that eq. (1.4) is more properly written as

$$\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}}) = M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}}). \quad (1.7)$$

Now we can explore the P_{UTM} dependence in (1.7) and note that it is absent for precisely the right values of $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$:

$$\left.\frac{\partial M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}})}{\partial P_{\text{UTM}}}\right|_{\tilde{M}_c^{(a)}=M_c^{(a)}, \tilde{M}_c^{(b)}=M_c^{(b)}} = 0. \quad (1.8)$$

While this property has been known, it was rarely used in the case of the symmetric M_{T2} , since it offers redundant information: once the correct child mass M_c is found through the M_{T2} kink (1.6), the parent mass M_p is given by (1.2) and there are no remaining unknowns, thus there is no need to further investigate the P_{UTM} dependence. In the case of the asymmetric M_{T2} , however, we start with one additional unknown parameter, which cannot always be determined from the “ridge” information alone. Therefore, in order to pin down the complete spectrum, we are forced to make use of (1.8). The nice feature of the P_{UTM} method is that it always allows us to determine *both* children masses $M_c^{(a)}$ and $M_c^{(b)}$, without relying on the “ridge” information at all. In this sense, our two methods are complementary and each can be used to cross-check the results obtained by the other.

The paper is organized as follows.¹¹ In section 2 we begin with a review of the conventional symmetric M_{T2} variable and its properties. Then in section 3 we introduce

¹⁰Ref. [97] studied the orthogonal scenario of different parents ($M_p^{(a)} \neq M_p^{(b)}$) and identical children ($M_c^{(a)} = M_c^{(b)}$) and found a similar non-differentiable feature, called a “crease”, on the corresponding two-dimensional hypersurface within the three-dimensional parameter space $\{\tilde{M}_c, \tilde{M}_p^{(a)}, \tilde{M}_p^{(b)}\}$.

¹¹Readers who are unfamiliar with the M_{T2} concept may benefit from consulting refs. [54, 68, 86, 97] first.

the asymmetric M_{T2} variable and highlight its properties which are relevant for our mass measurements. We also discuss some experimental subtleties in the construction of the asymmetric M_{T2} distribution, which are not present in the case of the symmetric M_{T2} . Sections 4, 5.1 and 5.2 present some simple examples of asymmetric event topologies. Finally, section 6 summarizes our main results and outlines some possible directions for future work. Appendix A revisits the examples of section 4 in the case of $P_{UTM} \rightarrow \infty$, which can be handled by purely analytical means [97].

2 The conventional symmetric M_{T2}

2.1 Definition

We begin our discussion by revisiting the conventional definition of the symmetric M_{T2} variable with identical daughters, following the general notation introduced in figure 1. Let us consider the inclusive production of two parent particles with common mass M_p . Each parent initiates a decay chain producing a certain number $n^{(\lambda)}$ of SM particles. In this section we assume that the two chains terminate in children particles of the same mass: $M_c^{(a)} = M_c^{(b)} = M_c$. (From section 3 on we shall remove this assumption.) In most applications of M_{T2} in the literature, the children particles are identified with the very last particles in the decay chains, i.e. the dark matter candidates. However, the symmetric M_{T2} can also be usefully applied to a subsystem of the original event topology, where the children are some other pair of (identical) particles appearing further up the decay chain [77, 86]. The M_{T2} variable is defined in terms of the measured invariant mass $m_{(\lambda)}$ and transverse momentum $\vec{p}_T^{(\lambda)}$ of the visible particles on each side (see figure 1). With the assumption of identical children, the transverse mass of each parent is

$$M_T^{(\lambda)}(\vec{p}_T^{(\lambda)}; \vec{q}_T^{(\lambda)}; m_{(\lambda)}; \tilde{M}_c) = \sqrt{m_{(\lambda)}^2 + \tilde{M}_c^2 + 2(e^{(\lambda)}\tilde{e}^{(\lambda)} - \vec{p}_T^{(\lambda)} \cdot \vec{q}_T^{(\lambda)})}, \quad (2.1)$$

where \tilde{M}_c is the common test mass for the children, which is an input to the M_{T2} calculation, while $\vec{q}_T^{(\lambda)}$ is the unknown transverse momentum of the child particle in the λ -th chain. In eq. (2.1) we have also introduced shorthand notation for the transverse energy of the composite particle made from the visible SM particles in the λ -th chain

$$e^{(\lambda)} = \sqrt{m_{(\lambda)}^2 + \vec{p}_T^{(\lambda)} \cdot \vec{p}_T^{(\lambda)}} \quad (2.2)$$

and for the transverse energy of the corresponding child particle in the λ -th chain

$$\tilde{e}^{(\lambda)} = \sqrt{\tilde{M}_c^2 + \vec{q}_T^{(\lambda)} \cdot \vec{q}_T^{(\lambda)}}. \quad (2.3)$$

Then the event-by-event symmetric M_{T2} variable is defined through a minimization procedure over all possible partitions of the two children momenta $\vec{q}_T^{(\lambda)}$ [50]

$$M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c, P_{UTM}) = \min_{\vec{q}_T^{(a)} + \vec{q}_T^{(b)} = \vec{Q}_{tot}} \left[\max \left\{ M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)}; m_{(a)}; \tilde{M}_c), M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)}; m_{(b)}; \tilde{M}_c) \right\} \right], \quad (2.4)$$

consistent with the momentum conservation constraint (1.1) in the transverse plane.

2.2 Computation

The standard definition (2.4) of the M_{T2} variable is sufficient to compute the value of M_{T2} numerically, given a set of input values for its arguments. The right-hand side of eq. (2.4) represents a simple minimization problem in two variables, which can be easily handled by a computer. In fact, there are publicly available computer codes for computing M_{T2} [115, 116]. The public codes have even been optimized for speed [85] and give results consistent with each other (as well as with our own code).¹² Nevertheless, it is useful to have an analytical formula for calculating the event-by-event M_{T2} for several reasons. First, an analytical formula is extremely valuable when it comes to understanding the properties and behavior of complex mathematical functions like (2.4). Second, computing M_{T2} from a formula will be faster than any numerical scanning algorithm. The computing speed becomes an issue especially when one considers variations of M_{T2} like M_{T2gen} , where in addition one needs to scan over all possible partitions of the visible objects into two decay chains [64]. Therefore, in this paper we shall pay special attention to the availability of analytical formulas and we shall quote such formulas whenever they are available.

In the symmetric case with identical children, an analytical formula for the event-by-event M_{T2} exists only in the special cases of $Q_{tot} = 0$ [54] or $P_{UTM} = 0$ [64]. We review the latter result here for completeness. (In the next section we shall present its generalization for the asymmetric case of different children.) The symmetric M_{T2} is known to have two types of solutions: “balanced” and “unbalanced” [54, 64]. The balanced solution is achieved when the minimization procedure in eq. (2.4) selects a momentum configuration for $\vec{q}_T^{(\lambda)}$ in which the transverse masses of the two parents are the same: $M_T^{(a)} = M_T^{(b)}$. In that case, typically neither $M_T^{(a)}$ nor $M_T^{(b)}$ is at its global (unconstrained) minimum. In what follows, we shall use a superscript B to refer to such balanced-type solutions. The formula for the balanced solution M_{T2}^B of the symmetric M_{T2} variable is given by [64, 68]

$$\left[M_{T2}^B(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c) \right]^2 = \tilde{M}_c^2 + A_T + \sqrt{\left(1 + \frac{4\tilde{M}_c^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} \right) \left(A_T^2 - m_{(a)}^2 m_{(b)}^2 \right)}, \tag{2.5}$$

where A_T is a convenient shorthand notation introduced in [68]

$$A_T = e^{(a)} e^{(b)} + \vec{p}_T^{(a)} \cdot \vec{p}_T^{(b)} \tag{2.6}$$

and $e^{(\lambda)}$ was already defined in eq. (2.2).

On the other hand, unbalanced solutions arise when one of the two parent transverse masses ($M_T^{(a)}$ or $M_T^{(b)}$, as the case may be) is at its global (unconstrained) minimum. Denoting the two unbalanced solutions with a superscript $U\lambda$, we have [54]

$$M_{T2}^{Ua}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c) = m_{(a)} + \tilde{M}_c, \tag{2.7}$$

$$M_{T2}^{Ub}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c) = m_{(b)} + \tilde{M}_c. \tag{2.8}$$

¹²Unfortunately, the assumption of identical children is hardwired in the public codes and they cannot be used to calculate the asymmetric M_{T2} introduced below in section 3 without additional hacking. We shall return to this point in section 3.

Given the three possible options for M_{T2} , eqs. (2.5), (2.7) and (2.8), it remains to specify which one actually takes place for a given set of values for $\vec{p}_T^{(a)}$, $\vec{p}_T^{(b)}$, $m_{(a)}$, $m_{(b)}$, \tilde{M}_c and $P_{\text{UTM}} = 0$ in the event.¹³ The balanced solution (2.5) applies when the following two conditions are simultaneously satisfied:

$$M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)} = -\vec{q}_T^{(a)} + \vec{Q}_{\text{tot}}; m_{(b)}; \tilde{M}_c) \geq M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)} = \vec{q}_T^{(a)}; m_{(a)}; \tilde{M}_c) = m_{(a)} + \tilde{M}_c, \quad (2.9)$$

$$M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)} = -\vec{q}_T^{(b)} + \vec{Q}_{\text{tot}}; m_{(a)}; \tilde{M}_c) \geq M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)} = \vec{q}_T^{(b)}; m_{(b)}; \tilde{M}_c) = m_{(b)} + \tilde{M}_c, \quad (2.10)$$

where

$$\vec{q}_T^{(\lambda)} = \frac{\tilde{M}_c}{m_{(\lambda)}} \vec{p}_T^{(\lambda)}, \quad (\lambda = a, b), \quad (2.11)$$

gives the global (unconstrained) minimum of the corresponding parent transverse mass $M_T^{(\lambda)}$. The unbalanced solution M_{T2}^{Ua} applies when the condition (2.9) is false and condition (2.10) is true, while the unbalanced solution M_{T2}^{Ub} applies when the condition (2.9) is true and condition (2.10) is false. It is easy to see that conditions (2.9) and (2.10) cannot be simultaneously violated, so these three cases exhaust all possibilities.

2.3 Properties

Given its definition (2.4), one can readily form and study the differential M_{T2} distribution. Although its shape in general does carry some information about the underlying process, it has become customary to focus on the upper endpoint $M_{T2(\text{max})}$, which is simply the maximum value of M_{T2} found in the event sample:

$$M_{T2(\text{max})}(\tilde{M}_c, P_{\text{UTM}}) = \max_{\text{all events}} \left[M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c) \right]. \quad (2.12)$$

Notice that in the process of maximizing over all events, the dependence on $\vec{p}_T^{(a)}$, $\vec{p}_T^{(b)}$, $m_{(a)}$ and $m_{(b)}$ disappears, and $M_{T2(\text{max})}$ depends only on two input parameters: \tilde{M}_c and P_{UTM} , the latter entering through \vec{Q}_{tot} in the momentum conservation constraint (1.1). The measured function (2.12) is the starting point of any M_{T2} -based mass determination analysis. We shall now review its three basic properties which make it suitable for such studies [101].

2.3.1 Property I: knowledge of M_p as a function of M_c

This property was already identified in the original papers and served as the main motivation for introducing the M_{T2} variable in the first place [50, 54]. Mathematically it can be expressed as

$$\tilde{M}_p(\tilde{M}_c, P_{\text{UTM}}) \equiv M_{T2(\text{max})}(\tilde{M}_c, P_{\text{UTM}}). \quad (2.13)$$

This is the same as eq. (1.2), but now we have been careful to include the explicit dependence on P_{UTM} , which will be important in our subsequent discussion. As indicated in eq. (2.13), the function $\tilde{M}_p(\tilde{M}_c, P_{\text{UTM}})$ can be experimentally measured from the M_{T2}

¹³Recall that (2.5) only applies for $P_{\text{UTM}} = 0$.

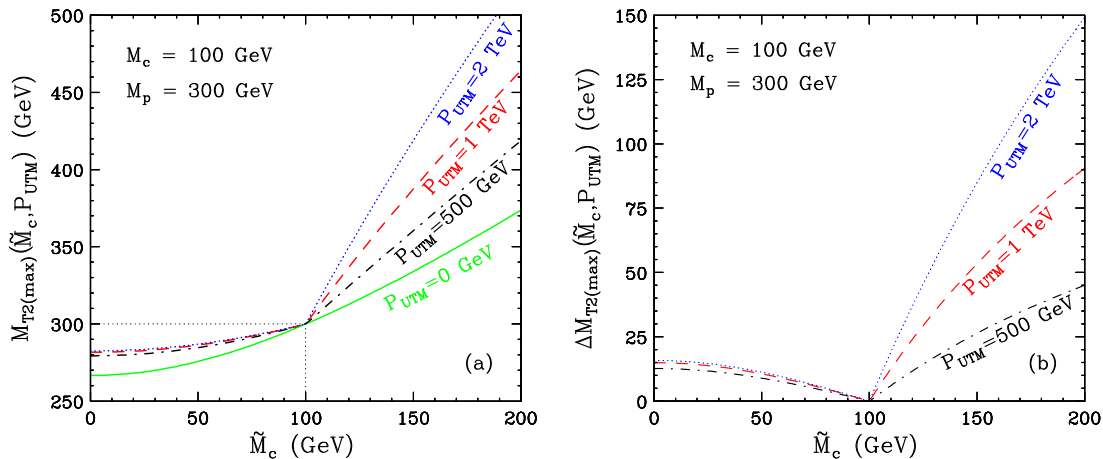


Figure 2. Plots of (a) the M_{T2} endpoint $M_{T2(max)}(\tilde{M}_c, P_{UTM})$ defined in eq. (2.12), and (b) the function $\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM})$ defined in (2.16) as a function of the test child mass \tilde{M}_c , for several fixed values of P_{UTM} : $P_{UTM} = 0$ GeV (solid, green), $P_{UTM} = 500$ GeV (dot-dashed, black), $P_{UTM} = 1$ TeV (dashed, red), and $P_{UTM} = 2$ TeV (dotted, blue). The process under consideration is pair production of sleptons of mass $M_p = 300$ GeV, which decay directly to the lightest neutralino $\tilde{\chi}_1^0$ of mass $M_c = 100$ GeV.

endpoint (2.12). The crucial point now is that the relation (2.13) is satisfied by the true values M_p and M_c of the parent and child mass, correspondingly:

$$M_p = M_{T2(max)}(M_c, P_{UTM}). \tag{2.14}$$

Notice that eq. (2.14) holds for *any* value of P_{UTM} , so in practical applications of this method one could choose the most populated P_{UTM} bin to reduce the statistical error. On the other hand, since a priori we do not know the true mass M_c of the missing particle, eq. (2.14) gives only one relation between the masses of the mother and the child. This is illustrated in figure 2(a), where we consider the simple example of direct slepton pair production,¹⁴ where each slepton ($\tilde{\ell}$) decays to the lightest neutralino ($\tilde{\chi}_1^0$) by emitting a single lepton ℓ : $\tilde{\ell} \rightarrow \ell + \tilde{\chi}_1^0$. Here the slepton is the parent and the neutralino is the child. Their masses were chosen to be $M_p = 300$ GeV and $M_c = 100$ GeV, correspondingly, as indicated with the black dotted lines in figure 2(a). In this example, the upstream transverse momentum P_{UTM} is provided by jets from initial state radiation. In figure 2(a) we plot the function (2.13) versus \tilde{M}_c , for several fixed values of P_{UTM} . The green solid line represents the case of no upstream momentum $P_{UTM} = 0$. In agreement with eq. (2.14), this line passes through the point (M_c, M_p) corresponding to the true values of the mass parameters. Notice that the property (2.14) continues to hold for other values of P_{UTM} . Figure 2(a) shows three more cases: $P_{UTM} = 500$ GeV (dotdashed black line), $P_{UTM} = 1$ TeV (dashed red line) and $P_{UTM} = 2$ TeV (dotted blue line). All those curves still pass through the point (M_c, M_p) with the correct values of the masses, illustrating the robustness of the property (2.14) with respect to variations in P_{UTM} .

¹⁴The corresponding event topology is shown in figure 3(a) below with $M_c^{(a)} = M_c^{(b)} = M_c$.

2.3.2 Property II: Kink in $M_{T2(max)}$ at the true M_c

The second important property of the M_{T2} variable was identified rather recently [65–68, 86]. Interestingly, the M_{T2} endpoint $M_{T2(max)}$, when considered as a function of the unknown input test mass \tilde{M}_c , often develops a kink (1.6) at precisely the correct value $\tilde{M}_c = M_c$ of the child mass. The appearance of the kink is a rather general phenomenon and occurs under various circumstances. It was originally noticed in event topologies with composite visible particles, whose invariant mass $m_{(\lambda)}$ is a variable parameter [65, 68]. Later it was realised that a kink also occurs in the presence of non-zero upstream momentum P_{UTM} [66, 67, 86], as in the example of figure 2(a), where P_{UTM} arises due to initial state radiation. As can be seen in the figure, the kink is absent for $P_{UTM} = 0$, but as soon as there is some non-vanishing P_{UTM} , the kink becomes readily apparent. As expected, the kink location (marked by the vertical dotted line) is at the true child mass ($M_c = 100$ GeV), where the corresponding value of $M_{T2(max)}$ (marked by the horizontal dotted line) is at the true parent mass ($M_p = 300$ GeV). Figure 2(a) also demonstrates that with the increase in P_{UTM} , the kink becomes more pronounced, thus the most favorable situations for the observation of the kink are cases with large P_{UTM} , e.g. when the upstream momentum is due to the decays of heavier (grandparent) particles [86].

In section 3.3 we shall see how the kink feature (1.6) of the symmetric M_{T2} endpoint $\tilde{M}_p(\tilde{M}_c)$ defined by eq. (1.2) is generalized to a “ridge” feature on the asymmetric M_{T2} endpoint $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ defined in (1.4).

2.3.3 Property III: P_{UTM} invariance of $M_{T2(max)}$ at the true M_c

This property is the one which has been least emphasized in the literature. Notice that the M_{T2} endpoint function (2.13) in general depends on the value of P_{UTM} . However, the first property (2.14) implies that the P_{UTM} dependence disappears at the correct value M_c of the child mass:

$$\left. \frac{\partial M_{T2(max)}(\tilde{M}_c, P_{UTM})}{\partial P_{UTM}} \right|_{\tilde{M}_c=M_c} = 0. \quad (2.15)$$

In order to quantify this feature, let us define the function

$$\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM}) \equiv M_{T2(max)}(\tilde{M}_c, P_{UTM}) - M_{T2(max)}(\tilde{M}_c, 0), \quad (2.16)$$

which measures the shift of the M_{T2} endpoint due to variations in P_{UTM} . The function $\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM})$ can be measured experimentally: the first term on the right-hand side of (2.16) is simply the M_{T2} endpoint observed in a subsample of events with a given (preferably the most common) value of P_{UTM} , while the second term on the right-hand side of (2.16) contains the endpoint $M_{T2\perp}^{(max)}$ of the 1-dimensional $M_{T2\perp}$ variable introduced in [101]:

$$M_{T2(max)}(\tilde{M}_c, 0) = M_{T2\perp}^{(max)}(\tilde{M}_c). \quad (2.17)$$

Given the definition (2.16), the third property (2.15) can be rewritten as

$$\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM}) \geq 0, \quad (2.18)$$

where the equality holds only for $\tilde{M}_c = M_c$:

$$\Delta M_{T2(max)}(M_c, P_{UTM}) = 0, \quad \forall P_{UTM}. \quad (2.19)$$

Eqs. (2.18) and (2.19) provide an alternative way to determine the true child mass M_c : simply find the value of \tilde{M}_c which minimizes the function $\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM})$. This procedure is illustrated in figure 2(b), where we revisit the slepton pair production example of figure 2(a) and plot the function $\Delta M_{T2(max)}(\tilde{M}_c, P_{UTM})$ defined in (2.16) versus the test mass \tilde{M}_c , for the same set of (fixed) values of P_{UTM} . Clearly, the zero of the function (2.16) occurs at the true child mass $\tilde{M}_c = M_c = 100$ GeV, in agreement with eq. (2.19). In our studies of the asymmetric M_{T2} case in the next sections, we shall find that the third property (2.19) is extremely important, since it will *always* allow us the complete determination of the mass spectrum, including *both* children masses $M_c^{(a)}$ and $M_c^{(b)}$.

3 The generalized asymmetric M_{T2}

After this short review of the basic properties of the conventional symmetric M_{T2} variable (2.4), we now turn our attention to the less trivial case of $\tilde{M}_c^{(a)} \neq \tilde{M}_c^{(b)}$. Following the logic of section 2, in section 3.1 we first introduce the asymmetric M_{T2} variable and then in sections 3.2 and 3.3 we discuss its computation and mathematical properties, correspondingly.

3.1 Definition

The generalization of the usual definition (2.4) to the asymmetric case of $\tilde{M}_c^{(a)} \neq \tilde{M}_c^{(b)}$ is straightforward [97]. We continue to follow the conventions and notation of figure 1, but now we simply avoid the assumption that the children masses are equal, and we let each one be an independent input parameter $\tilde{M}_c^{(\lambda)}$. Without loss of generality, in what follows we assume $M_c^{(b)} \geq M_c^{(a)}$. The transverse mass of each parent (2.1) is now a function of the corresponding child mass $\tilde{M}_c^{(\lambda)}$:

$$M_T^{(\lambda)}(\vec{p}_T^{(\lambda)}; \vec{q}_T^{(\lambda)}; m_{(\lambda)}; \tilde{M}_c^{(\lambda)}) = \sqrt{m_{(\lambda)}^2 + \left(\tilde{M}_c^{(\lambda)}\right)^2 + 2\left(e^{(\lambda)}\tilde{e}^{(\lambda)} - \vec{p}_T^{(\lambda)} \cdot \vec{q}_T^{(\lambda)}\right)}, \quad (3.1)$$

where the transverse energy $e^{(\lambda)}$ of the composite SM particle on the λ -th side of the event was already defined in (2.2), while the transverse energy $\tilde{e}^{(\lambda)}$ of the child is now generalized from (2.3) to

$$\tilde{e}^{(\lambda)} = \sqrt{\left(\tilde{M}_c^{(\lambda)}\right)^2 + \vec{q}_T^{(\lambda)} \cdot \vec{q}_T^{(\lambda)}}. \quad (3.2)$$

The event-by-event asymmetric M_{T2} variable is defined in analogy to (2.4) and is given by [97]

$$M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM}) = \min_{\vec{q}_T^{(a)} + \vec{q}_T^{(b)} = \vec{Q}_{tot}} \left[\max \left\{ M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)}; m_{(a)}; \tilde{M}_c^{(a)}), M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)}; m_{(b)}; \tilde{M}_c^{(b)}) \right\} \right], \quad (3.3)$$

which is now a function of two input test children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. In the special case of $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} \equiv \tilde{M}_c$, the asymmetric M_{T2} variable defined in (3.3) reduces to the conventional symmetric M_{T2} variable (2.4).

3.2 Computation

In this subsection we generalize the discussion in section 2.2 and present an analytical formula for computing the event-by-event asymmetric M_{T2} variable (3.3). Just like the formula (2.5) for the symmetric case, our formula will hold only in the special case of $P_{UTM} = 0$. As before, the asymmetric M_{T2} variable has two types of solutions — balanced and unbalanced. The balanced solution occurs when the following two conditions are simultaneously satisfied (compare to the analogous conditions (2.9) and (2.10) for the symmetric case)

$$M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)} = -\vec{q}_T^{(a)} + \vec{Q}_{\text{tot}}; m_{(b)}; \tilde{M}_c^{(b)}) \geq M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)} = \vec{q}_T^{(b)}; m_{(a)}; \tilde{M}_c^{(a)}) = m_{(a)} + \tilde{M}_c^{(a)}, \quad (3.4)$$

$$M_T^{(a)}(\vec{p}_T^{(a)}; \vec{q}_T^{(a)} = -\vec{q}_T^{(b)} + \vec{Q}_{\text{tot}}; m_{(a)}; \tilde{M}_c^{(a)}) \geq M_T^{(b)}(\vec{p}_T^{(b)}; \vec{q}_T^{(b)} = \vec{q}_T^{(a)}; m_{(b)}; \tilde{M}_c^{(b)}) = m_{(b)} + \tilde{M}_c^{(b)}, \quad (3.5)$$

where, in analogy to (2.11),

$$\vec{q}_T^{(\lambda)} = \frac{\tilde{M}_c^{(\lambda)}}{m_{(\lambda)}} \vec{p}_T^{(\lambda)}, \quad (\lambda = a, b), \quad (3.6)$$

is the test child momentum at the global unconstrained minimum of $M_T^{(\lambda)}$. The balanced solution for M_{T2} is now given by

$$\begin{aligned} \left[M_{T2}^B(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \right]^2 &= \tilde{M}_+^2 + A_T + \left(\frac{m_{(b)}^2 - m_{(a)}^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} \right) \tilde{M}_-^2 \\ &\pm \sqrt{1 + \frac{4\tilde{M}_+^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} + \left(\frac{2\tilde{M}_-^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} \right)^2} \times \sqrt{A_T^2 - m_{(a)}^2 m_{(b)}^2}, \end{aligned} \quad (3.7)$$

where A_T was defined in (2.6). For convenience, in (3.7) we have introduced two alternative mass parameters

$$\tilde{M}_+^2 \equiv \frac{1}{2} \left\{ (\tilde{M}_c^{(b)})^2 + (\tilde{M}_c^{(a)})^2 \right\}, \quad (3.8)$$

$$\tilde{M}_-^2 \equiv \frac{1}{2} \left\{ (\tilde{M}_c^{(b)})^2 - (\tilde{M}_c^{(a)})^2 \right\}, \quad (3.9)$$

in place of the original trial masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. The new parameters \tilde{M}_+ and \tilde{M}_- are simply a different parametrization of the two degrees of freedom corresponding to the unknown child masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ entering the definition of the asymmetric M_{T2} . The parameters \tilde{M}_+ and \tilde{M}_- allow us to write formula (3.7) in a more compact form. More importantly, they also allow to make easy contact with the known results from section 2 by taking the symmetric limit $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} \equiv \tilde{M}_c$ as

$$\tilde{M}_+ \rightarrow \tilde{M}_c, \quad \tilde{M}_- \rightarrow 0. \quad (3.10)$$

It is easy to see that in the symmetric limit (3.10) our balanced solution (3.7) for the asymmetric M_{T2} reduces to the known result (2.5) for the symmetric M_{T2} .

An interesting feature of the asymmetric balanced solution is the appearance of a \pm sign on the second line of (3.7). In principle, this sign ambiguity is present in the symmetric case as well, but there the minus sign always turns out to be unphysical and the sign issue does not arise [64]. However, in the asymmetric case, both signs can be physical sometimes and one must make the proper sign choice in eq. (3.7) as follows. For the given set of test masses $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$, calculate the transverse center-of-mass energy

$$\begin{aligned} \sqrt{\hat{s}_T^\pm} = e^{(a)} + e^{(b)} + \frac{2(e^{(b)} - e^{(a)})\tilde{M}_-^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} \pm \frac{(e^{(b)} + e^{(a)})A_T - (e^{(b)}m_{(a)}^2 + e^{(a)}m_{(b)}^2)}{\sqrt{A_T^2 - m_{(a)}^2 m_{(b)}^2}} \\ \times \sqrt{1 + \frac{4\tilde{M}_+^2}{2A_T - m_{(a)}^2 - m_{(b)}^2} + \left(\frac{2\tilde{M}_-^2}{2A_T - m_{(a)}^2 - m_{(b)}^2}\right)^2}, \end{aligned} \quad (3.11)$$

corresponding to each sign choice in eq. (3.7), and compare the result to the minimum allowed value of $\sqrt{\hat{s}_T}$

$$\sqrt{\hat{s}_{T(min)}} = e^{(a)} + e^{(b)} + \sqrt{Q_{tot}^2 + (\tilde{M}_c^{(a)} + \tilde{M}_c^{(b)})^2}. \quad (3.12)$$

The minus sign in eq. (3.7) takes precedence and applies whenever it is physical, i.e. whenever $\sqrt{\hat{s}_T^-} > \sqrt{\hat{s}_{T(min)}}$. In the remaining cases when $\sqrt{\hat{s}_T^-} < \sqrt{\hat{s}_{T(min)}}$ and the minus sign is unphysical, the plus sign in eq. (3.7) applies.

If one of the conditions (3.4), (3.5) is not satisfied, the asymmetric M_{T2} is given by an unbalanced solution, in analogy to (2.7) and (2.8):

$$M_{T2}^{Ua}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = m_{(a)} + \tilde{M}_c^{(a)}, \quad (3.13)$$

$$M_{T2}^{Ub}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = m_{(b)} + \tilde{M}_c^{(b)}. \quad (3.14)$$

The unbalanced solution M_{T2}^{Ua} of eq. (3.13) applies when the condition (3.4) is false and condition (3.5) is true, while the unbalanced solution M_{T2}^{Ub} of eq. (3.14) applies when the condition (3.4) is true and condition (3.5) is false.

Eqs. (3.7), (3.13) and (3.14) represent one of our main results. They generalize the analytical results of refs. [64, 68] and allow the direct computation of the asymmetric M_{T2} variable without the need for scanning and numerical minimizations. This is an important benefit, since the existing public codes for M_{T2} [115, 116] only apply in the symmetric case $M_c^{(a)} = M_c^{(b)}$.

3.3 Properties

All three properties of the symmetric M_{T2} discussed in section 2.3 readily generalize to the asymmetric case.

3.3.1 Property I: knowledge of M_p as a function of $M_c^{(a)}$ and $M_c^{(b)}$

In the asymmetric case, the endpoint $M_{T2(max)}$ of the M_{T2} distribution still gives the mass of the parent, only this time it is a function of two input test masses for the children:

$$\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM}) = M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM}). \quad (3.15)$$

The important property is that this relation is satisfied by the true values of the children and parent masses:

$$M_p = M_{T2(max)}(M_c^{(a)}, M_c^{(b)}, P_{UTM}). \quad (3.16)$$

Thus the true parent mass M_p will be known once we determine the two children masses $M_c^{(a)}$ and $M_c^{(b)}$.

3.3.2 Property II: ridge in $M_{T2(max)}$ through the true $M_c^{(a)}$ and $M_c^{(b)}$

In the symmetric M_{T2} case, the endpoint function (2.13) is not continuously differentiable and has a “kink” at the true child mass $\tilde{M}_c = M_c$. In the asymmetric M_{T2} case, the endpoint function (3.15) is similarly non-differentiable at a set of points

$$\left\{ \left(\tilde{M}_c^{(a)}(\theta), \tilde{M}_c^{(b)}(\theta) \right) \right\} \quad (3.17)$$

parametrized by a single¹⁵ continuous parameter θ . The gradient of the endpoint function (3.15) suffers a discontinuity as we cross the curve defined by (3.17). Since (3.15) represents a hypersurface in the three-dimensional parameter space of $\{\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, \tilde{M}_p\}$, the gradient discontinuity will appear as a “ridge” (sometimes also referred to as a “crease” [97]) on our three-dimensional plots below. The important property of the ridge is that it passes through the correct values for the children masses, even when they are different:

$$M_c^{(a)} = \tilde{M}_c^{(a)}(\theta_0), \quad (3.18)$$

$$M_c^{(b)} = \tilde{M}_c^{(b)}(\theta_0), \quad (3.19)$$

for some θ_0 . Thus the ridge information provides a relation among the two children masses and leaves us with just a single unknown degree of freedom — the parameter θ in eq. (3.17).

Interestingly, the shape of the ridge provides a quick test whether the two missing particles are identical or not.¹⁶ If the shape of the ridge in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane is symmetric with respect to the interchange $\tilde{M}_c^{(a)} \leftrightarrow \tilde{M}_c^{(b)}$, i.e. under a mirror reflection with respect to the 45° line $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)}$, then the two missing particles are the same. Conversely, when the shape of the ridge is *not* symmetric under $\tilde{M}_c^{(a)} \leftrightarrow \tilde{M}_c^{(b)}$, the missing particles are in general expected to have different masses.

¹⁵In general, the set of points (3.17) may consist of several lines, each individually parameterized (see e.g. figures 12, 14, 18(b) and 19(b) below).

¹⁶To be more precise, the ridge shape tests whether the two missing particles have the same mass or not.

3.3.3 Property III: P_{UTM} invariance of $M_{T2(\text{max})}$ at the true $M_c^{(a)}$ and $M_c^{(b)}$

The third M_{T2} property, which was discussed in section 2.3.3, is readily generalized to the asymmetric case as well. Note that eq. (3.16) implies that the P_{UTM} dependence of the asymmetric M_{T2} endpoint (3.15) disappears at the true values of the children masses:

$$\left. \frac{\partial M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}})}{\partial P_{\text{UTM}}} \right|_{\tilde{M}_c^{(a)}=M_c^{(a)}, \tilde{M}_c^{(b)}=M_c^{(b)}} = 0. \quad (3.20)$$

This equation is the asymmetric analogue of eq. (2.15). Proceeding as in section 2.3.3, let us define the function

$$\Delta M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}}) \equiv M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}}) - M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, 0), \quad (3.21)$$

which quantifies the shift of the asymmetric M_{T2} endpoint (3.15) in the presence of P_{UTM} . By definition,

$$\Delta M_{T2(\text{max})}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}}) \geq 0, \quad (3.22)$$

with equality being achieved only for the correct values of the children masses:

$$\Delta M_{T2(\text{max})}(M_c^{(a)}, M_c^{(b)}, P_{\text{UTM}}) = 0, \quad \forall P_{\text{UTM}}. \quad (3.23)$$

The last equation reveals the power of the P_{UTM} invariance method. Unlike the kink method discussed in section 3.3.2, which was only able to find a relation between the two children masses $M_c^{(a)}$ and $M_c^{(b)}$, the P_{UTM} invariance implied by eq. (3.23) allows us to determine *each* individual children mass, without any theoretical assumptions, and even in the case when the two children masses happen to be different ($M_c^{(a)} \neq M_c^{(b)}$).

3.4 Examples

In the next two sections we shall illustrate the three properties discussed so far in section 3.3 with some concrete examples. Instead of the most general event topology depicted figure 1, here we limit ourselves to the three simple examples shown in figure 3.

The simplest possible case is when $n^{(\lambda)} = 1$, i.e. when each cascade decay contains a single SM particle, as in figure 3(a). In this example, $m_{(\lambda)}$ is constant. For simplicity, we shall take $m_{(\lambda)} \approx 0$, which is the case for a lepton or a light flavor jet. If the SM particle is a Z -boson or a top quark, its mass cannot be neglected, and one must keep the proper value of $m_{(\lambda)}$. This, however, is only a technical detail, which does not affect our main conclusions below. In spite of its simplicity, the topology of figure 3(a) is actually the most challenging case, due to the limited number of available measurements [86]. In order to be able to determine all individual masses in that case, one must consider events with upstream momentum \vec{P}_{UTM} , as illustrated in figure 3(a). This is not a particularly restrictive assumption, since there is always a certain amount of P_{UTM} in the event (at the very least, from initial state radiation). In section 4 the topology of figure 3(a) will be extensively studied - first for the asymmetric case of $M_c^{(a)} \neq M_c^{(b)}$ in section 4.1, and then for the symmetric case of $M_c^{(a)} = M_c^{(b)}$ in section 4.2.

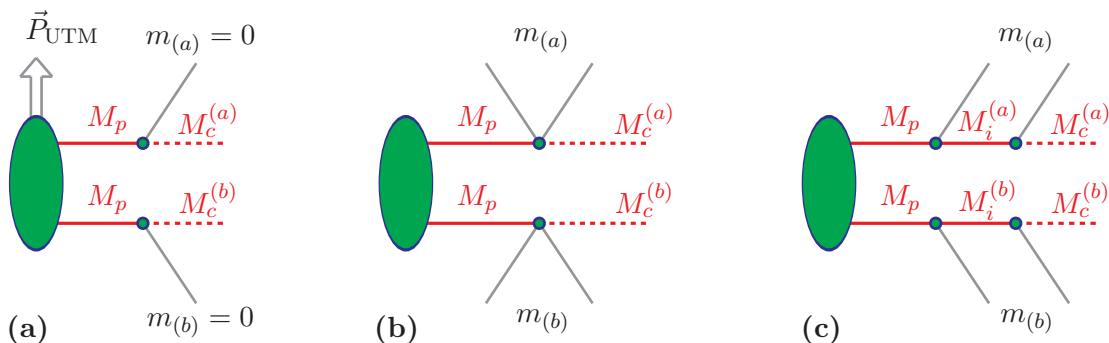


Figure 3. The three different event-topologies under consideration in this paper. In each case, two parents with mass M_p are produced onshell and decay into two daughters of (generally different) masses $M_c^{(a)}$ and $M_c^{(b)}$. Case (a), which is the subject of section 4, has a single massless visible SM particle in each leg and some arbitrary upstream transverse momentum \vec{P}_{UTM} . In the remaining two cases (b) and (c), which are discussed in section 5, there are two massless visible particles in each leg, which form a composite visible particle with varying invariant mass $m_{(\lambda)}$. The intermediate particle of mass $M_i^{(\lambda)}$ is (b) heavy and off-shell ($M_i^{(\lambda)} > M_p$), or (c) on-shell ($M_p > M_i^{(\lambda)} > M_c^{(\lambda)}$). For simplicity, we do not consider any upstream momentum in cases (b) and (c).

Another simple situation arises when there are *two* massless visible SM particles in each leg, as illustrated in figures 3(b) and 3(c). In either case, the invariant mass $m_{(\lambda)}$ is not constant any more, but varies within a certain range $m_{(\lambda)}^{\min} \leq m_{(\lambda)} \leq m_{(\lambda)}^{\max}$, where $m_{(\lambda)}^{\min} = 0$, while the value of $m_{(\lambda)}^{\max}$ depends on the mass $M_i^{(\lambda)}$ of the corresponding intermediate particle. In figure 3(b) we assume $M_i^{(\lambda)} > M_p$, so that the intermediate particle is off-shell and

$$m_{(\lambda)}^{\max} = M_p - M_c^{(\lambda)}. \quad (3.24)$$

The “off-shell” case of figure 3(b) will be discussed in section 5.1.

In contrast, in figure 3(c) we take $M_p > M_i^{(\lambda)} > M_c^{(\lambda)}$, in which case the intermediate particle is on-shell and the range for $m_{(\lambda)}$ is now limited from above by

$$m_{(\lambda)}^{\max} = M_p \sqrt{\left[1 - \left(\frac{M_i^{(\lambda)}}{M_p}\right)^2\right] \left[1 - \left(\frac{M_c^{(\lambda)}}{M_i^{(\lambda)}}\right)^2\right]}. \quad (3.25)$$

We shall discuss the “on-shell” case of figure 3(c) in section 5.2.

In the event topologies of figures 3(b) and 3(c), the mass $m_{(\lambda)}$ is varying and the ridge of eq. (3.17) will appear even if there were no upstream transverse momentum in the event. Therefore, in our discussion of figures 3(b) and 3(c) in section 5 below we shall assume $P_{\text{UTM}} = 0$ for simplicity. The presence of non-zero P_{UTM} will only additionally enhance the ridge feature.

3.5 Combinatorial issues

Before going on to the actual examples in the next two sections, we need to discuss one minor complication, which is unique to the asymmetric M_{T2} variable and was not present

in the case of the symmetric M_{T2} variable. The question is, how does one associate the visible decay products observed in the detector with a particular decay chain $\lambda = a$ or $\lambda = b$. This is the usual combinatorics problem, which now has two different aspects:

- The first issue is also present in the symmetric case, where one has to decide how to partition the SM particles observed in the detector into two disjoint sets, one for each cascade. In the traditional approach, where the children particles are assumed to be identical, the two sets are indistinguishable and it does not matter which one is first and which one is second. This particular aspect of the combinatorial problem will also be present in the asymmetric case.
- In the asymmetric case, however, there is an additional aspect to the combinatorial problem: now the two cascades are distinguishable (by the masses of the child particles), so even if we correctly divide the visible objects into the proper subsets, we still do not know which subset goes together with $M_c^{(a)}$ and thus gets a label $\lambda = a$, and which goes together with $M_c^{(b)}$ and gets labelled by $\lambda = b$. This leads to an additional combinatorial factor of 2 which is absent in the symmetric case with identical children.

The severity of these two combinatorial problems depends on the event topology, as well as the type of signature objects. For example, there are cases where the first combinatorial problem is easily resolved, or even absent altogether. Consider the event topology of figure 3(a) with a lepton as the SM particle on each side. In this case, the partition is unique, and the upstream objects are jets, which can be easily identified [98]. Now consider the event topologies of figures 3(b) and 3(c), with two opposite sign, same flavor leptons on each side. Such events result from inclusive pair production of heavier neutralinos in supersymmetry. By selecting events with different lepton flavors: $e^+e^-\mu^+\mu^-$, we can overcome the first combinatorial problem above and uniquely associate the e^+e^- pair with one cascade and the $\mu^+\mu^-$ pair with the other. However, the second combinatorial problem remains, as we still have to decide which of the two lepton pairs to associate with $\lambda = a$ and which to associate with $\lambda = b$. Recall that the labels $\lambda = a$ and $\lambda = b$ are already attached to the child particles, which are distinguishable in the asymmetric case. In this paper we use the convention that $\lambda = a$ is attached to the lighter child particle:

$$\tilde{M}_c^{(a)} \leq \tilde{M}_c^{(b)}, \tag{3.26}$$

which also ensures that the \tilde{M}_- parameter defined in (3.9) is real.

We can put this discussion in more formal terms as follows. The correct association of the visible particles with the corresponding children will yield

$$M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}), \tag{3.27}$$

while the other, wrong association will give simply

$$M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(b)}, \tilde{M}_c^{(a)}). \tag{3.28}$$

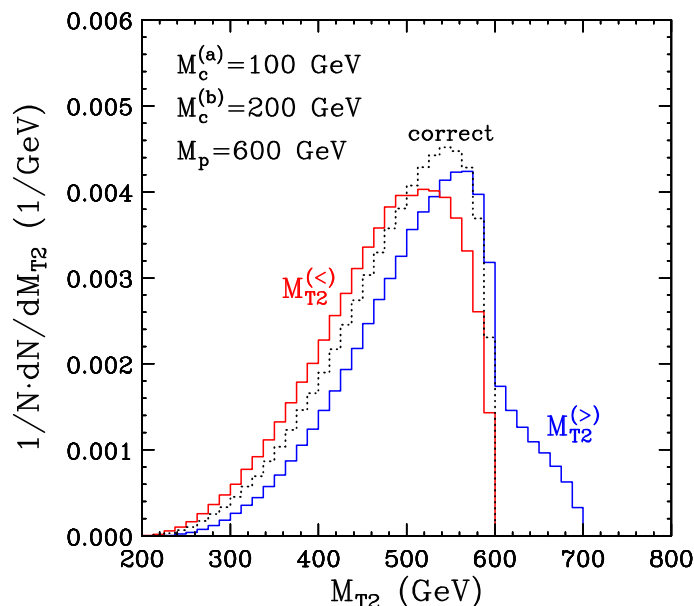


Figure 4. Unit-normalized M_{T2} distributions for the event topology of figure 3(b). The mass spectrum is chosen as $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV and $M_p = 600$ GeV. The test children masses are taken to be the true masses: $\tilde{M}_c^{(a)} = M_c^{(a)}$ and $\tilde{M}_c^{(b)} = M_c^{(b)}$. The dotted black distribution is the true M_{T2} distribution, ignoring the combinatorial problem. The red histogram shows the distribution of the $M_{T2}^{(<)}$ variable defined in (3.29) while the blue histogram shows the distribution of the $M_{T2}^{(>)}$ variable defined in (3.31).

Both of these two M_{T2} values can be computed from the data, but a priori we do not know which one corresponds to the correct association. The solution to this problem is however already known [64, 86]: one can conservatively use the smaller of the two

$$M_{T2}^{(<)} \equiv \min \{ M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}), M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(b)}, \tilde{M}_c^{(a)}) \} \quad (3.29)$$

in order to preserve the location of the upper M_{T2} endpoint. This is illustrated in figure 4, where we show results for the event topology of figure 3(b) with a mass spectrum as follows: $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV and $M_p = 600$ GeV. The test children masses are taken to be the true masses: $\tilde{M}_c^{(a)} = M_c^{(a)}$ and $\tilde{M}_c^{(b)} = M_c^{(b)}$. The dotted black distribution is the unit-normalized true M_{T2} distribution, where one ignores the combinatorial problem and uses the Monte Carlo information to make the correct association. The red histogram shows the unit-normalized distribution of the $M_{T2}^{(<)}$ variable defined in (3.29). We see that the definition (3.29) preserves the corresponding endpoint:

$$M_{T2(max)}^{(<)} = M_{T2(max)}. \quad (3.30)$$

Of course, we can also consider the alternative combination

$$M_{T2}^{(>)} \equiv \max \{ M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}), M_{T2}(\vec{p}_T^{(a)}, \vec{p}_T^{(b)}; m_{(a)}, m_{(b)}; \tilde{M}_c^{(b)}, \tilde{M}_c^{(a)}) \}, \quad (3.31)$$

whose unit-normalized distribution is shown in figure 4 with the blue histogram. One can see that some of the wrong combination entries in the $M_{T2}^{(>)}$ histogram violate the original

Spectrum	Case	$M_c^{(a)}$	$M_c^{(b)}$	M_p
I	Different children	250	500	600
II	Identical children	100	100	300

Table 1. Mass spectra for the two examples studied in sections 4.1 and 4.2. All masses are given in GeV.

endpoint $M_{T2(max)}$, yet there is still a well defined $M_{T2}^{(>)}$ endpoint

$$M_{T2(max)}^{(>)} \geq M_{T2(max)}. \tag{3.32}$$

Strictly speaking, in our analysis in the next sections, we only need to study the $M_{T2}^{(<)}$ endpoint (3.30), which contains the relevant information about the physical M_{T2} endpoint. At the same time, with our convention (3.26) for the children masses, we only need to concentrate on the upper half $\tilde{M}_c^{(b)} \geq \tilde{M}_c^{(a)}$ of the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane. However, for completeness we shall also present results for the $M_{T2}^{(>)}$ endpoint (3.32), and we shall use the lower $(\tilde{M}_c^{(b)} < \tilde{M}_c^{(a)})$ half of the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane to show those. Thus the M_{T2} endpoint shown in our plots below should be interpreted as follows

$$M_{T2(max)} = \begin{cases} M_{T2(max)}^{(<)}, & \text{if } \tilde{M}_c^{(a)} \leq \tilde{M}_c^{(b)}, \\ M_{T2(max)}^{(>)}, & \text{if } \tilde{M}_c^{(a)} > \tilde{M}_c^{(b)}. \end{cases} \tag{3.33}$$

4 The simplest event topology: one SM particle on each side

In this section, we consider the simplest topology with a single visible particle on each side of the event. We already introduced this example in section 3.4, along with its event topology in figure 3(a). In section 4.1 below we first discuss an asymmetric case with different children. Later in section 4.2 we consider a symmetric situation with identical children masses. The mass spectra for these two study points are listed in table 1.

4.1 Asymmetric case

Before we present our numerical results, it will be useful to derive an analytical expression for the asymmetric M_{T2} endpoint (3.15) in terms of the corresponding physical spectrum of table 1 and the two test children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. Our result will generalize the corresponding formula derived in [68] for the symmetric case of $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} \equiv \tilde{M}_c$ and no upstream momentum ($P_{UTM} = 0$). For the event topology of figure 3(a) the M_{T2} endpoint is always obtained from the balanced solution and is given by [68]

$$M_{T2(max)}(\tilde{M}_c, P_{UTM} = 0) = \mu_{\text{npc}} + \sqrt{\mu_{\text{npc}}^2 + \tilde{M}_c^2}. \tag{4.1}$$

Here we made use of the convenient shorthand notation introduced in [86] for the relevant combination of physical masses

$$\mu_{\text{npc}} \equiv \frac{M_n}{2} \left\{ 1 - \left(\frac{M_c}{M_p} \right)^2 \right\}. \tag{4.2}$$

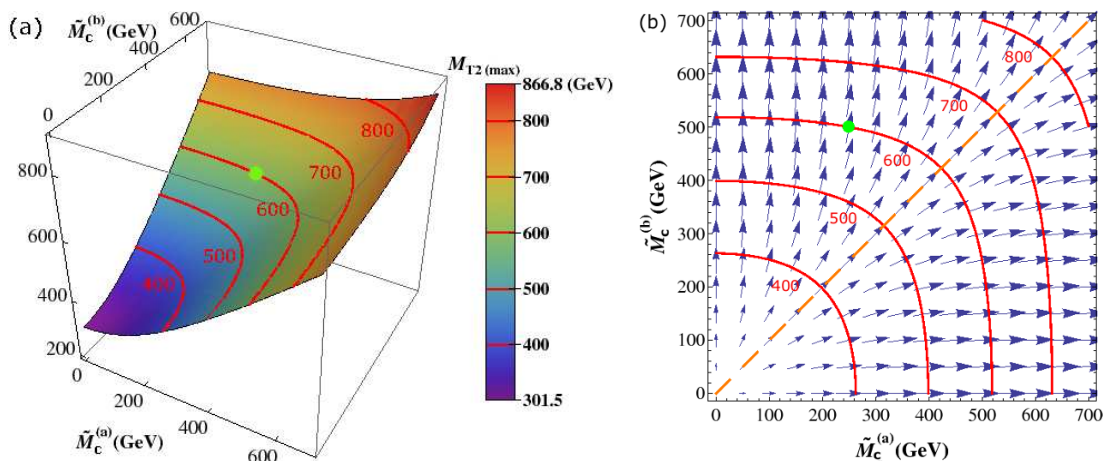


Figure 5. $M_{T2(max)}$ as a function of the two test children masses, $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$, for the event topology of figure 3(a) with no upstream momentum ($P_{UTM} = 0$), and the asymmetric mass spectrum I from table 1: $(M_c^{(a)}, M_c^{(b)}, M_p) = (250, 500, 600)$ GeV. We show (a) a three dimensional view and (b) contour plot projection on the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane (red contour lines). The green dot marks the true values of the children masses. Panel (b) also shows a gradient plot, where longer (shorter) arrows imply steeper (gentler) slope. A kink structure is absent in this case. The symmetric endpoint $M_{T2(max)}(\tilde{M}_c)$ of eq. (4.1) can be obtained by going along the diagonal orange line $\tilde{M}_c^{(b)} = \tilde{M}_c^{(a)}$.

The μ parameter defined in (4.2) is simply the transverse momentum of the (massless) visible particle in those events which give the maximum value of M_{T2} [98]. Squaring (4.1), we can equivalently rewrite it as

$$M_{T2(max)}^2(\tilde{M}_c, P_{UTM} = 0) = 2\mu_{ppc}^2 + \tilde{M}_c^2 + \sqrt{4\mu_{ppc}^2(\mu_{ppc}^2 + \tilde{M}_c^2)}. \quad (4.3)$$

Now let us derive the analogous expressions for the asymmetric case $M_c^{(a)} \neq M_c^{(b)}$. Just like the symmetric case, the asymmetric endpoint $M_{T2(max)}$ also comes from a balanced solution and is given by

$$M_{T2(max)}^2(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM} = 0) = 2\bar{\mu}_{ppc}^2 + \tilde{M}_+^2 + \sqrt{4\bar{\mu}_{ppc}^2(\bar{\mu}_{ppc}^2 + \tilde{M}_+^2) + \tilde{M}_-^4}, \quad (4.4)$$

where the parameters \tilde{M}_+^2 and \tilde{M}_-^2 were already defined in (3.8) and (3.9), while $\bar{\mu}_{ppc}$ is now the geometric average of the corresponding individual μ_{ppc} parameters

$$\bar{\mu}_{ppc}^2 \equiv \mu_{ppc_a} \mu_{ppc_b} \equiv \frac{(M_p^2 - (M_c^{(a)})^2)(M_p^2 - (M_c^{(b)})^2)}{4M_p^2}. \quad (4.5)$$

It is easy to check that in the symmetric limit

$$\tilde{M}_c^{(b)} \rightarrow \tilde{M}_c^{(a)} \implies \bar{\mu}_{ppc} \rightarrow \mu_{ppc}, \quad \tilde{M}_+ \rightarrow \tilde{M}_c, \quad \tilde{M}_- \rightarrow 0, \quad (4.6)$$

eq. (4.4) reduces to its symmetric counterpart (4.3), as it should.

We are now ready to present our numerical results for the event topology of figure 3(a). We first take the asymmetric mass spectrum I from table 1 and consider the case with no upstream momentum, when formula (4.4) applies. Figure 5 shows the corresponding M_{T2} endpoint as a function of the two test children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$. In panel (a) we present a three dimensional view, while in panel (b) we show a contour plot projection on the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane (red contour lines). On either panel, the green dot marks the true values of the children masses, $M_c^{(a)}$ and $M_c^{(b)}$. Panel (b) also shows a gradient plot, where longer (shorter) arrows imply steeper (gentler) slope. The symmetric endpoint $M_{T2(max)}(\tilde{M}_c, P_{UTM} = 0)$ of eq. (4.1) can be obtained by going along the diagonal orange line $\tilde{M}_c^{(b)} = \tilde{M}_c^{(a)}$ in figure 5(b). We remind the reader that the endpoint $M_{T2(max)}$ plotted in figure 5 should be interpreted as in eq. (3.33).

Figure 5 illustrates the first basic property of the asymmetric M_{T2} variable, which was discussed in section 3.3.1. The M_{T2} endpoint allows us to find one relation between the two children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ and the parent mass $\tilde{M}_p = M_{T2(max)}$, and in order to do so, we do *not* have to assume equality of the children masses, as is always done in the literature. The crucial advantage of our approach, in which we allow the two children masses to be arbitrary, is its generality and model-independence. It allows us to extract the basic information contained in the M_{T2} endpoint, without muddling it up with additional theoretical (and unproven) assumptions.

Unfortunately, to go any further and determine each individual mass, we must make use of the additional properties discussed in sections 3.3.2 and 3.3.3. In the case of the simplest event topology of figure 3(a) considered here, they both require the presence of some upstream momentum [67, 86]. As a proof of concept, we now reconsider the same type of events, but with a fixed upstream momentum of $P_{UTM} = 1$ TeV. (The upstream momentum may be due to initial state radiation, or decays of heavier particles upstream.) The corresponding results are shown in figure 6.

Figure 6 demonstrates the second basic property of the asymmetric M_{T2} variable discussed in section 3.3.2. Unlike the result shown in figure 5(a), which was perfectly smooth, this time the $M_{T2(max)}$ function in figure 6(a) shows a ridge, corresponding to the slope discontinuity marked with the black solid line in figure 6(b). The most important feature of the ridge is the fact that it passes through the green dot marking the true values of the children masses. Notice that applying the traditional symmetric M_{T2} approach in this case will give a completely wrong result. If we were to assume equal children masses from the very beginning, we will be constrained to the diagonal orange line in figure 6(b). The M_{T2} endpoint will then still exhibit a kink, but the kink will be in the wrong location. In the example shown in figure 6(b), we will underestimate the parent mass, while for the child mass we will find a value which is somewhere in between the two true masses $M_c^{(a)}$ and $M_c^{(b)}$.

Using the ridge information, we now know an additional relation among the children masses, which allows us to express all three masses in terms of a single unknown parameter θ , as illustrated in figure 7(a). Let us choose to parametrize the ridge by the polar angle

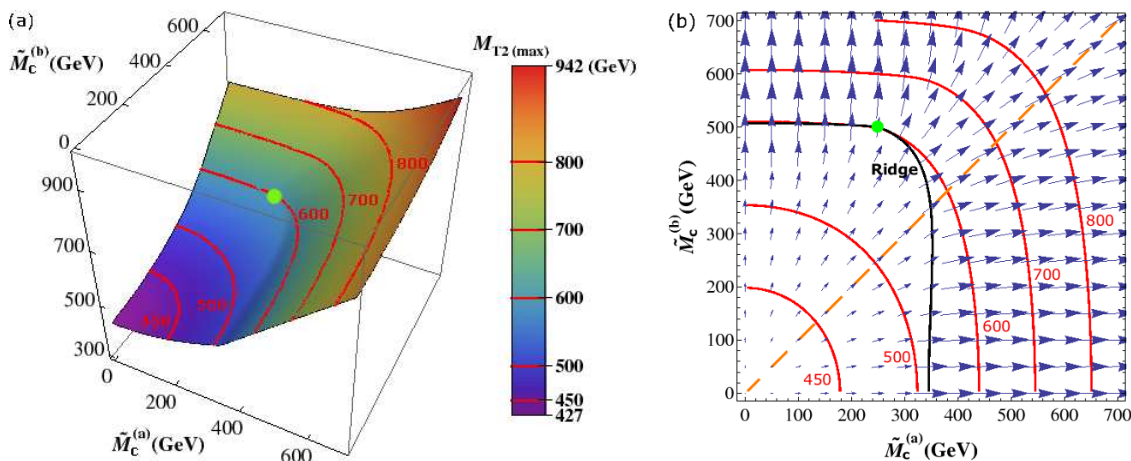


Figure 6. The same as in figure 5 but with fixed upstream momentum of $P_{UTM} = 1$ TeV. The ridge structure (shown as the black solid line) is revealed by the sudden increase in the slope (gradient) in panel (b). Notice that the ridge goes through the true values of the children masses marked by the green dot.

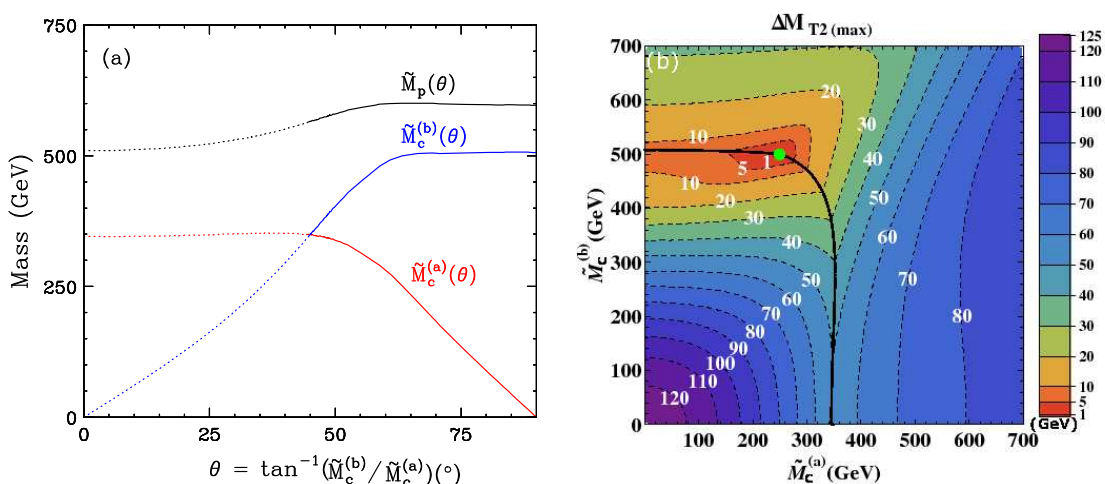


Figure 7. (a) Particle masses obtained along the $M_{T2(max)}$ ridge seen in figure 6. The ridge is parametrized by the angle θ defined in (4.7). The two children masses $\tilde{M}_c^{(a)}(\theta)$ (in red) and $\tilde{M}_c^{(b)}(\theta)$ (in blue) as well as the parent mass \tilde{M}_p (in black) are then plotted as a function of θ . In our convention (3.26) only values of $\theta \geq 45^\circ$ are physical, and the corresponding masses are shown with solid lines. Dotted lines show the extrapolation for $\theta < 45^\circ$. (b) Contour plot of the quantity $\Delta M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM} = 1 \text{ TeV})$ defined in eq. (3.21), in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane. This plot is obtained simply by taking the difference between figure 6(a) and figure 5(a). The solid black curve indicates the location of the $M_{T2(max)}$ ridge. Only the point corresponding to the true children masses (the green dot) satisfies the P_{UTM} invariance condition $\Delta M_{T2(max)} = 0$ from eq. (3.23).

in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane:

$$\theta = \tan^{-1} \left(\frac{\tilde{M}_c^{(b)}}{\tilde{M}_c^{(a)}} \right). \quad (4.7)$$

Using the ridge information from figure 6, we can then find all three masses as a function of θ . The result is shown in figure 7(a). The mass $\tilde{M}_c^{(a)}$ of the lighter child is plotted in red, the mass $\tilde{M}_c^{(b)}$ of the heavier child is plotted in blue, while the parent mass \tilde{M}_p is plotted in black. With our convention (3.26) for the children masses, only values of $\theta \geq 45^\circ$ are physical, and the corresponding masses are shown with solid lines. The dotted lines in figure 7(a) show the extrapolation into the unphysical region $\theta < 45^\circ$.

Figure 7(a) has some important and far reaching implications. For example, one may now start asking the question: Are there really any *massive* invisible particles in those events, or is the missing energy simply due to neutrino production [21]? The ridge results shown in figure 7(a) begin to provide the answer to that quite fundamental question. According to figure 7(a), for *any* value of the (still unknown) parameter θ , the two children particles cannot be simultaneously massless. This means that the missing energy cannot be simply due to neutrinos, i.e. there is *at least* one new, massive invisible particle produced in the missing energy events. At this point, we cannot be certain that this is a dark matter particle, but establishing the production of a WIMP candidate at a collider is by itself a tremendously important result. Notice that while we cannot be sure about the masses of the children, the parent mass M_p is determined with a very good precision from figure 7(a): the function $\tilde{M}_p(\theta)$ is almost flat and rather insensitive to the particular value of θ .¹⁷

Once we have proved that some kind of WIMP production is going on, the next immediate question is: how many such WIMP particles are present in the data — one or two? Unfortunately, the ridge analysis of figure 7(a) alone cannot provide the answer to this question, since the value of θ is still undetermined. If $\theta = 90^\circ$, one of the missing particles is massless, which is consistent with a SM neutrino. Therefore, if θ were indeed 90° , the most plausible explanation of this scenario would be that only one of the missing particles is a genuine WIMP, while the other is a SM neutrino. On the other hand, almost any other value of $\theta < 90^\circ$ would guarantee that there are *two* WIMP candidates in each event. In that case, the next immediate question is: are they the same or are they different? Fortunately, our asymmetric approach will allow answering this question in a model-independent way. If θ is determined to be 45° , the two WIMP particles are the same, i.e. we are producing a single species of dark matter. On the other hand, if $45^\circ < \theta < 90^\circ$, then we can be certain that there are not one, but *two* different WIMP particles being produced.

We see that in order to completely understand the physics behind the missing energy signal, we must determine the value of θ , i.e. we must find the exact location of the true children masses along the ridge. One of our main results in this paper is that this can be done by using the third M_{T2} property discussed in section 3.3.3. The idea is illustrated in figure 7(b), where we show a contour plot in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane of the quantity $\Delta M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{UTM})$ defined in eq. (3.21), for a fixed $P_{UTM} = 1$ TeV. This plot is obtained simply by taking the difference between figure 6(a) and figure 5(a). (A more practical method for obtaining this information was proposed in [101].) Recall that the

¹⁷ Interestingly, for the example in figure 7(a), the maximum value of $\tilde{M}_p(\theta)$ happens to give the true parent mass M_p , but we have checked that this is a coincidence and does not hold in general for other examples which we have studied.

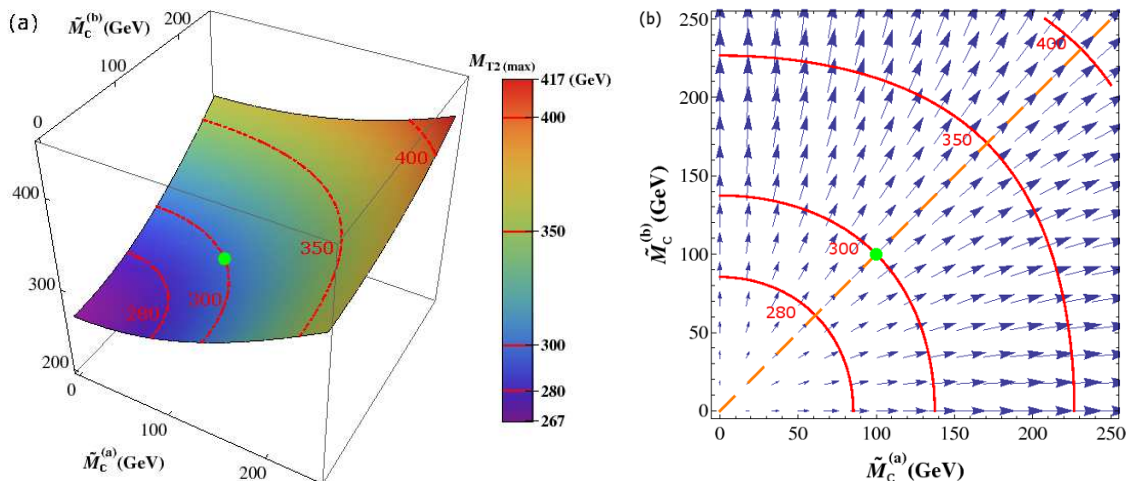


Figure 8. The same as figure 5, but for the symmetric mass spectrum II from table 1, i.e. $(M_c^{(a)}, M_c^{(b)}, M_p) = (100, 100, 300)$ GeV.

function $\Delta M_{T2(max)}$ was introduced in order to quantify the P_{UTM} invariance of the M_{T2} endpoint, and it is expected that $\Delta M_{T2(max)}$ vanishes at the correct values of the children masses (see eq. (3.23)). This expectation is confirmed in figure 7(b), where we find the minimum (zero) of the $\Delta M_{T2(max)}$ function exactly at the right spot (marked with the green dot) along the $M_{T2(max)}$ ridge. Thus the $\Delta M_{T2(max)}$ function in figure 7(b) completely pins down the spectrum, and in this case would reveal the presence of two *different* WIMP particles, with unequal masses $M_c^{(a)} \neq M_c^{(b)}$. Our analysis thus shows that colliders can not only produce a WIMP dark matter candidate and measure its mass, as discussed in the existing literature, but they can do a much more elaborate dark matter particle spectroscopy, as advertized in the title. In particular, they can probe the number and type of missing particles, including particles from subdominant dark matter species, which are otherwise unlikely to be discovered experimentally in the usual dark matter searches.

4.2 Symmetric case

While in our approach the two children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ are treated as independent inputs, this, of course, does not mean that the approach is only valid in cases when the children masses are different to begin with. The techniques discussed in the previous subsection remain applicable also in the more conventional case when the children are identical, i.e. when colliders produce a single dark matter component. In order to illustrate how our method works in that case, we shall now work out an example with equal children masses. We still consider the simplest event topology of figure 3(a), but with the symmetric mass spectrum II from table 1. We then repeat the analysis done in figures 5, 6, and 7 and show the corresponding results in figures 8, 9 and 10.

The conclusions from this exercise are very similar to what we found earlier in section 4.1 for the asymmetric case. The M_{T2} endpoint still provides one relation among the two children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ and the parent mass $\tilde{M}_p = M_{T2(max)}$. This relation is shown in figure 8 (figure 9) for the case without (with) upstream momentum P_{UTM} . As

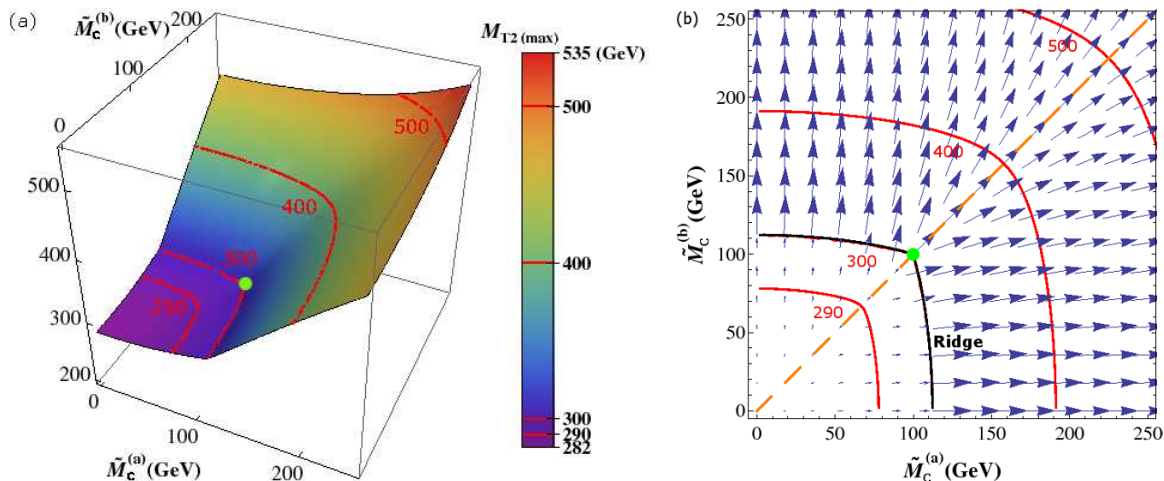


Figure 9. The same as figure 6 but for the symmetric mass spectrum II from table 1, i.e. $(M_c^{(a)}, M_c^{(b)}, M_p) = (100, 100, 300)$ GeV.

seen in figure 8, in the absence of any upstream P_{UTM} , the function $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ is smooth and reveals nothing about the children masses. However, the presence of upstream momentum significantly changes the picture and the function $\tilde{M}_p(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ again develops a ridge, which is clearly visible¹⁸ in both the three-dimensional view of figure 9(a), as well as the gradient plot in figure 9(b). The ridge information now further constrains the children masses to the black solid line in figure 9(b), leaving only one unknown degree of freedom. Parametrizing it with the polar angle θ as in (4.7), we obtain the spectrum as a function of θ , as shown in figure 10(a). Once again we find the fortuitous result that in spite of the remaining arbitrariness in the value of θ , the parent mass M_p is very well determined, since $\tilde{M}_p(\theta)$ is a very weakly varying function of θ . Furthermore, both figure 9(a) and figure 9(b) exhibit a high degree of symmetry under $\tilde{M}_c^{(a)} \leftrightarrow \tilde{M}_c^{(b)}$, which is a good hint that the children are in fact identical. This suspicion is confirmed in figure 10(b), where we find that the P_{UTM} dependence disappears at the symmetric point $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} = 100$ GeV, revealing the true masses of the two children.

In the two examples considered so far in sections 4.1 and 4.2, we used a fixed finite value of the upstream transverse momentum $P_{UTM} = 1$ TeV, which is probably rather extreme — in realistic models, one might expect typical values of P_{UTM} on the order of several hundred GeV. However, things begin to get much more interesting if one were to consider even *larger* values of P_{UTM} . On the one hand, the ridge feature becomes sharper and easier to observe [86]. More importantly, the ridge structure itself is modified, and

¹⁸We caution the reader that here we are presenting only a proof of concept. In the actual analysis the ridge may be rather difficult to see, for a variety of reasons - detector resolution, finite statistics, combinatorial and SM backgrounds, etc. Nevertheless, we expect that the ridge will be just as easily observable as the traditional kink in the symmetric M_{T2} endpoint. If the kink can be seen in the data, the ridge can be seen too, and there is no reason to make the assumption of equal children masses. Conversely, if the kink is too difficult to see, the ridge will remain hidden as well.

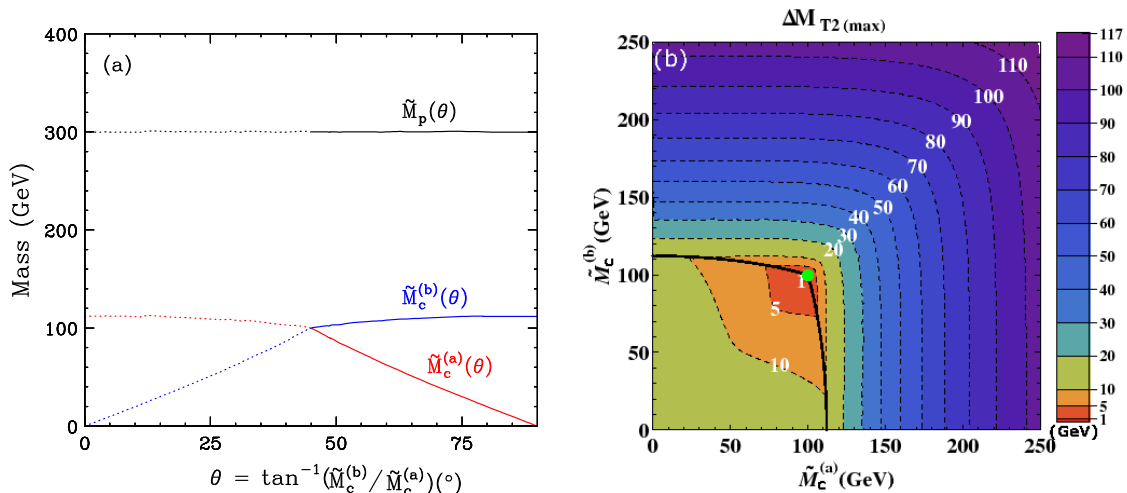


Figure 10. The same as in figure 7 but for the symmetric mass spectrum II from table 1, i.e. $(M_c^{(a)}, M_c^{(b)}, M_p) = (100, 100, 300)$ GeV. Notice that, in contrast to figure 7, the minimum of the $\Delta M_{T2(max)}$ function is now obtained at $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)}$, implying that the two missing particles are the same.

a second set of ridgelines appears¹⁹ at sufficiently large P_{UTM} . All ridgelines intersect precisely at the point marking the true values of the children masses, thus allowing the complete determination of the mass spectrum by the ridge method alone. This procedure was demonstrated explicitly in ref. [97], which investigated the extreme case of $P_{UTM} = \infty$ for a study point with different parents and identical children. The assumption of $P_{UTM} = \infty$ justified the use of a “decoupling argument”, in which the two branches $\lambda = a$ and $\lambda = b$ are treated independently, allowing the derivation of simple analytical expressions for the M_{T2} endpoint [97]. In appendix A we reproduce the analogous analytical results at $P_{UTM} \rightarrow \infty$ for the case of interest here (identical parents and different children) and study in detail the P_{UTM} dependence of the ridgelines. Unfortunately, we find that the values of P_{UTM} necessary to reveal the additional ridge structure, are too large to be of any interest experimentally. On the positive side, the P_{UTM} invariance method discussed in section 2.3.3 does not require such extremely large values of P_{UTM} and can in principle be tested in more realistic experimental conditions.

4.3 Mixed case

For simplicity, so far in our discussion we have been studying only one type of missing energy events at a time. In reality, the missing energy sample may contain several different types of events, and the corresponding M_{T2} measurements will first need to be disentangled from each other.

For concreteness, consider the inclusive pair production of some parent particle χ_p , which can decay either to a child particle χ_a of mass $M_c^{(a)}$, or a different child particle χ_b of mass $M_c^{(b)}$. Let the corresponding branching fractions be B_a and B_b , i.e. $B_a \equiv B(\chi_p \rightarrow \chi_a)$

¹⁹A keen observer may have already noticed a hint of those in figures 7(b) and 10(b).

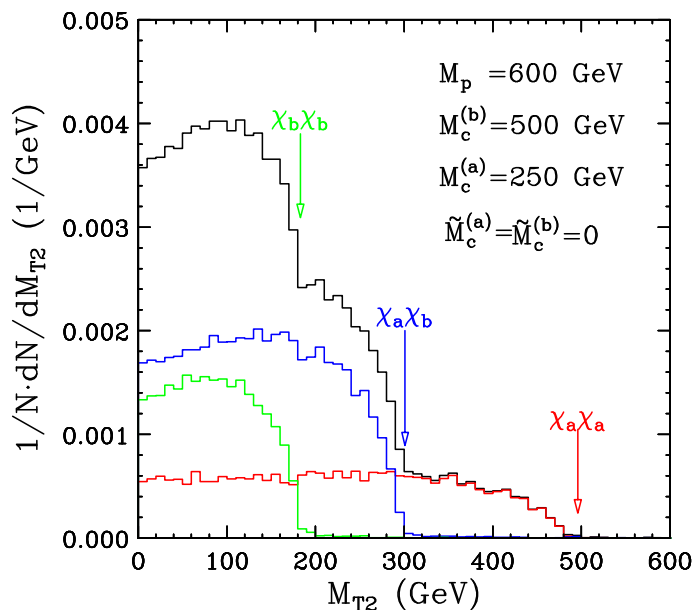


Figure 11. Unit-normalized, zero-bin subtracted M_{T2} distribution (black histogram) for the full mixed event sample, as well as the individual components $\chi_a\chi_a$ (red), $\chi_a\chi_b$ (blue) and $\chi_b\chi_b$ (green). We took zero test masses for the children $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} = 0$ and equal branching fraction for the parents $B_a = B_b = 50\%$. The mass spectrum is taken from the asymmetric study point I in table 1 with $M_c^{(a)} = 250$ GeV, $M_c^{(b)} = 500$ GeV and $M_p = 600$ GeV. The three arrows indicate the expected endpoints for each individual component in the sample.

and $B_b \equiv B(\chi_p \rightarrow \chi_b)$. Furthermore, let χ_b decay invisibly²⁰ to χ_a . Such a situation can be easily realized in supersymmetry, for example, with the parent being a squark, a slepton, or a gluino, the heavier child χ_b being a Wino-like neutralino $\tilde{\chi}_2^0$ and the lighter child χ_a being a Bino-like neutralino $\tilde{\chi}_1^0$. The heavier neutralino has a large invisible decay mode $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \nu \bar{\nu}$, if its mass happens to fall between the sneutrino mass and the left-handed slepton mass: $M_{\tilde{\nu}} < M_{\tilde{\chi}_2^0} < M_{\tilde{\ell}_L}$.

Let us start with a certain total number of events N_{pp} in which two parent particles χ_p have been produced. Then the missing energy sample will contain $N_{bb} = N_{pp} B_b^2$ symmetric events where the two children are χ_b and χ_b , $N_{aa} = N_{pp} B_a^2$ symmetric events where the two children are χ_a and χ_a , and $N_{ab} = 2N_{pp} B_a B_b$ asymmetric events where the two children are χ_a and χ_b . How can one analyze such a mixed event sample with a single M_{T2} variable?

The black histogram in figure 11 shows the unit-normalized M_{T2} distribution for the whole (mixed) event sample (for convenience, we do not show the zero bin [101]). For this plot, we used the asymmetric mass spectrum I from table 1: $M_c^{(a)} = 250$ GeV, $M_c^{(b)} = 500$ GeV and $M_p = 600$ GeV, and chose zero test masses for the children $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} = 0$. For definiteness, we fixed equal branching fractions $B_a = B_b = 50\%$, so that the relative normalization of the three individual samples is $N_{aa} : N_{bb} : N_{ab} = 1 : 1 : 2$. Figure 11 shows that the observable M_{T2} distribution is simply a superposition of the M_{T2} distributions of the three individual samples $\chi_a\chi_a$, $\chi_a\chi_b$ and $\chi_b\chi_b$, which are shown with the red, blue and

²⁰If χ_b decays visibly, then the respective types of events can in principle be sorted by their signature.

green histograms, correspondingly. Each individual sample exhibits its own M_{T2} endpoint, marked with a vertical arrow, which can also be seen in the combined M_{T2} distribution. Using eq. (4.4), the three endpoints are found to be

$$\chi_a\chi_a \rightarrow M_{T2(max)}^{(aa)}(0,0,0) = M_p \left[1 - \left(\frac{M_c^{(a)}}{M_p} \right)^2 \right] = 496 \text{ GeV}, \quad (4.8)$$

$$\chi_a\chi_b \rightarrow M_{T2(max)}^{(ab)}(0,0,0) = M_p \sqrt{\left[1 - \left(\frac{M_c^{(a)}}{M_p} \right)^2 \right] \left[1 - \left(\frac{M_c^{(b)}}{M_p} \right)^2 \right]} = 301 \text{ GeV}, \quad (4.9)$$

$$\chi_b\chi_b \rightarrow M_{T2(max)}^{(bb)}(0,0,0) = M_p \left[1 - \left(\frac{M_c^{(b)}}{M_p} \right)^2 \right] = 183 \text{ GeV}. \quad (4.10)$$

Now suppose that all three endpoints (4.8)–(4.10) are seen in the data. Their interpretation is far from obvious, and in fact, there will be different competing explanations. If one insists on the single missing particle hypothesis, there can be only one type of child particle, and the only way to get three different endpoints in figure 11 is to have production of three different pairs of parent particles, each of which decays in exactly the same way. Since the three parent masses are a priori unrelated, one does not expect any particular correlation among the three observed endpoints (4.8)–(4.10). Now consider an alternative explanation where we produce a single type of parents, but have two different children types. This situation also gives rise to three different event topologies, with three different M_{T2} endpoints, as we just discussed. However, now there is a predicted relation among the three M_{T2} endpoints, which follows simply from eqs. (4.8)–(4.10):

$$M_{T2(max)}^{(ab)}(0,0,0) = \sqrt{M_{T2(max)}^{(aa)}(0,0,0) M_{T2(max)}^{(bb)}(0,0,0)}. \quad (4.11)$$

If the parents are the same and the children are different, this relation must be satisfied. If the parents are different and the children are the same, a priori there is no reason why eq. (4.11) should hold, and if it does, it must be by pure coincidence. The prediction (4.11) therefore is a direct test of the number of children particles. Another test can be performed if we could estimate the individual event counts N_{aa} , N_{ab} and N_{bb} , although this appears rather difficult, due to the unknown shape of the M_{T2} distributions in figure 11. In the asymmetric example discussed here, we have another prediction, namely

$$N_{ab} = 2\sqrt{N_{aa}N_{bb}}, \quad (4.12)$$

which is another test of the different children hypothesis. Notice that eq. (4.12) holds regardless of the branching fractions B_a and B_b , although if one of them dominates, the two endpoints which require the other (rare) decay may be too difficult to observe.

Of course, the ultimate test of the single missing particle hypothesis is the behavior of the intermediate M_{T2} endpoint in figure 11 corresponding to the asymmetric events of

type $\chi_a\chi_b$. Applying either one of the two mass determination methods discussed earlier in figures 7 and 10, we should find that $M_{T2(max)}^{(ab)}$ is a result of asymmetric events, indicating the simultaneous presence of two *different* invisible particles in the data.

5 A more complex event topology: two SM particles on each side

In this section, we consider two more examples: the off-shell event topology of figure 3(b) is discussed in section 5.1, while the on-shell event topology of figure 3(c) is discussed in section 5.2. (For simplicity, we do not consider any P_{UTM} in this section.) Now there are two visible particles in each leg, which form a composite visible particle of varying mass $m_{(\lambda)}$. In general, by studying the invariant mass distribution of $m_{(\lambda)}$, one should be able to observe two different invariant mass endpoints, suggesting some type of an asymmetric scenario.

5.1 Off-shell intermediate particle

Here we concentrate on the example of figure 3(b). Since the intermediate particle is offshell, the maximum kinematically allowed value for $m_{(\lambda)}$ is given by eq. (3.24).

Recall that for the simple topology of figure 3(a) discussed in the previous section, the M_{T2} endpoint (4.4) always corresponded to a balanced solution. More precisely, the M_{T2} variable was maximized for a momentum configuration $\vec{p}_T^{(\lambda)}$ in which M_{T2} was given by the balanced solution (3.7). However, in this section we shall find that for the more complex topologies of figures 3(b) and 3(c), the M_{T2} endpoint may result from one of four different cases altogether: two different balanced solutions, which we shall label as B and B' , or the unbalanced solutions Ua and Ub discussed in section 3.2. Depending on the type of solution giving the endpoint $M_{T2(max)}$, the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ parameter plane divides into the three regions²¹ shown in figure 12. The green dot in figure 12 denotes the true children masses in this parameter space. Within each region, we show the relevant momentum configuration for the visible particles (red arrows) and the children particles (blue arrows) in each leg (a or b). The momenta are quoted in the “back-to-back boosted” (BB) frame [68], in which the two parents are at rest. The length of an arrow is indicative of the magnitude of the momentum. A blue dot implies that the corresponding daughter is at rest and therefore the two visible particles are emitted back-to-back. The two balanced solutions are denoted as B and B' , while the two unbalanced solutions are Ua and Ub . The black solid lines represent phase changes between different solution types and delineate the expected locations of the ridges in the $M_{T2(max)}$ function shown in figure 13 below. Perhaps the most striking feature of figure 12 is that the three (in fact, all four) regions come together precisely at the green dot marking the true values of the two children masses. The boundaries of the regions shown in figure 12 will manifest themselves as the locations of the ridges (i.e. gradient discontinuities) in the $M_{T2(max)}$ function. Therefore, we expect that by studying the ridge structure and finding its “triple” point, one will be able to completely determine the mass spectrum.

²¹The fourth case of the B' balanced solution happens to coincide with the two unbalanced solutions along the boundary between Ua and Ub .

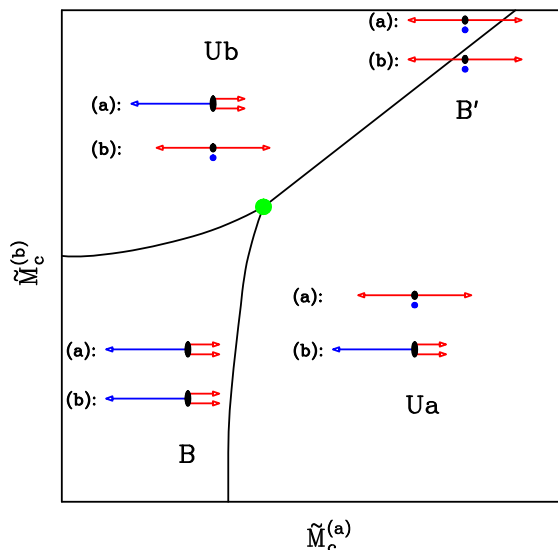


Figure 12. The four regions in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ parameter plane leading to the four different types of solutions for the M_{T2} endpoint, for the off-shell event topology of figure 3(b). The green dot marks the true location of the two children masses. Within each region, we indicate the relevant momentum configuration for the visible particles (red arrows) and the children particles (blue arrows) in each leg (a or b). The momenta are quoted in the “back-to-back boosted” (BB) frame [68], in which the two parents are at rest. A blue dot implies that the corresponding daughter is at rest and therefore the two visible particles are emitted back-to-back. The two balanced solutions are denoted as B and B' , while the two unbalanced solutions are Ua and Ub . The black solid lines represent phase changes between different solution types and delineate the expected locations of the ridges in the $M_{T2(max)}$ function shown in figure 13.

We shall now give analytical formulas for the M_{T2} endpoint in each of the four regions of figure 12. We begin with the two balanced solutions B and B' , for which the event-by-event balanced solution for M_{T2} is given by eq. (3.7). In the parameter space region of figure 12 which is adjacent to the origin, we find the balanced configuration B , in which all visible particles have the same direction in the BB frame. As a result, we have

$$m_{(a)} = m_{(b)} = 0 \quad (5.1)$$

and

$$A_T = \frac{(M_p^2 - (M_c^{(a)})^2)(M_p^2 - (M_c^{(b)})^2)}{2M_p^2}. \quad (5.2)$$

Substituting eqs. (5.1) and (5.2) in the balanced M_{T2} solution (3.7), where we should take the plus sign, we obtain

$$\left[M_{T2(max)}^B(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \right]^2 = 2\bar{\mu}_{\text{ppc}}^2 + \tilde{M}_+^2 + \sqrt{4\bar{\mu}_{\text{ppc}}^2(\bar{\mu}_{\text{ppc}}^2 + \tilde{M}_+^2) + \tilde{M}_-^4}, \quad (5.3)$$

which we recognize as the balanced solution (4.4) found for the decay topology of figure 3(a).

Moving away from the origin in figure 12, we find a second balanced solution B' along the boundary of the unbalanced regions Ua and Ub . In this case the visible particles are

back-to-back, and their invariant mass is maximized:

$$m_{(\lambda)} = M_p - M_c^{(\lambda)}, \quad (5.4)$$

and correspondingly

$$A_T = \left(M_p - M_c^{(a)} \right) \left(M_p - M_c^{(b)} \right). \quad (5.5)$$

Substituting eqs. (5.4) and (5.5) in the balanced M_{T2} solution (3.7), we obtain the B' -type M_{T2} endpoint as

$$\left[M_{T2(max)}^{B'}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \right]^2 = \left(M_p - M_c^{(a)} \right) \left(M_p - M_c^{(b)} \right) + \tilde{M}_+^2 + \frac{2M_p - M_c^{(a)} - M_c^{(b)}}{M_c^{(b)} - M_c^{(a)}} \tilde{M}_-^2. \quad (5.6)$$

The corresponding formulas for the unbalanced cases Ua and Ub are obtained by taking the maximum value for the invariant mass of the visible particles in the corresponding decay chain:

$$m_{(a)} = m_{(a)}^{\max} = M_p - M_c^{(a)} \quad \text{for region (Ua)}, \quad (5.7)$$

$$m_{(b)} = m_{(b)}^{\max} = M_p - M_c^{(b)} \quad \text{for region (Ub)}. \quad (5.8)$$

The corresponding formula for $M_{T2(max)}$ is then given by

$$M_{T2(max)}^{Ua}(\tilde{M}_c^{(a)}) = M_p - M_c^{(a)} + \tilde{M}_c^{(a)}, \quad (5.9)$$

$$M_{T2(max)}^{Ub}(\tilde{M}_c^{(b)}) = M_p - M_c^{(b)} + \tilde{M}_c^{(b)}. \quad (5.10)$$

One can now use the analytical results (5.3), (5.6), (5.9) and (5.10) to understand the ridge structure shown in figure 12. For example, the boundary between the B and Ua regions is parametrically given by the condition

$$M_{T2(max)}^B(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = M_{T2(max)}^{Ua}(\tilde{M}_c^{(a)}), \quad (5.11)$$

while the boundary between the B and Ub regions is parametrically given by

$$M_{T2(max)}^B(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = M_{T2(max)}^{Ub}(\tilde{M}_c^{(b)}). \quad (5.12)$$

On the other hand, the boundary

$$M_{T2(max)}^{Ua}(\tilde{M}_c^{(a)}) = M_{T2(max)}^{Ub}(\tilde{M}_c^{(b)}) \quad (5.13)$$

between the two unbalanced regions Ua and Ub is quite interesting. The parametric equation (5.13) is nothing but a straight line in the $(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ plane:

$$\tilde{M}_c^{(b)} = M_c^{(b)} - M_c^{(a)} + \tilde{M}_c^{(a)}, \quad (5.14)$$

as seen in figure 12.

It is now easy to understand the triple point structure in figure 12. The triple point is obtained by the merging of all three boundaries (5.11), (5.12) and (5.13), i.e. when

$$M_{T2(max)}^B(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = M_{T2(max)}^{B'}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) = M_{T2(max)}^{Ua}(\tilde{M}_c^{(a)}) = M_{T2(max)}^{Ub}(\tilde{M}_c^{(b)}). \quad (5.15)$$

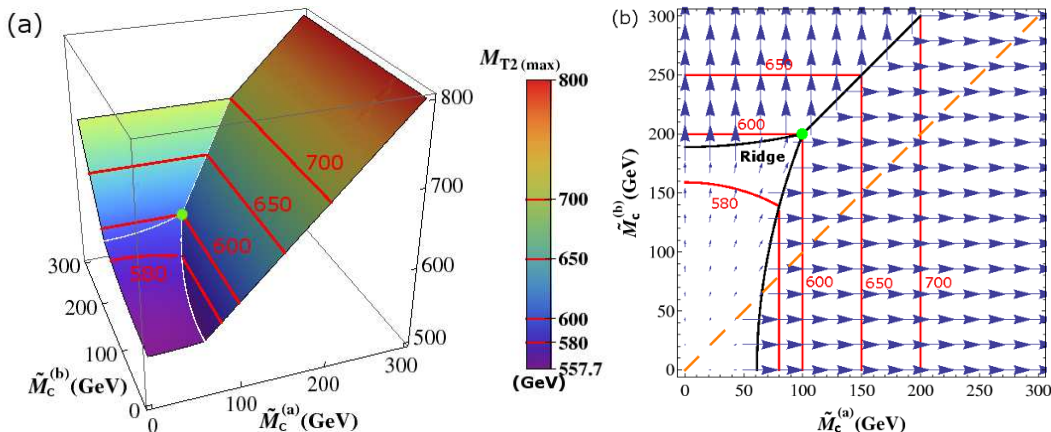


Figure 13. The same as figure 5, but for the off-shell event topology of figure 3(b). We use the mass spectrum from the example in figure 4: $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV and $M_p = 600$ GeV and for simplicity consider only events with $P_{UTM} = 0$.

It is easy to check that $\tilde{M}_c^{(a)} = M_c^{(a)}$ and $\tilde{M}_c^{(b)} = M_c^{(b)}$ identically satisfy these equations, thereby proving that the triple intersection of the boundaries seen in figure 12 indeed takes place at the true values of the children masses.

These results are confirmed in our numerical simulations. In figure 13 we present (a) a three dimensional view and (b) a gradient plot of the ridge structure found in events with the off-shell topology of figure 3(b). The mass spectrum for this study point was fixed as in figure 4, namely $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV and $M_p = 600$ GeV. Since the ridge structure for this topology does not require the presence of upstream momentum, for simplicity we consider only events with $P_{UTM} = 0$. The ridge pattern is clearly evident in figure 13(a), which shows a three-dimensional view of the M_{T2} endpoint function $M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$. It is even more apparent in figure 13(b), where one can see a sharp gradient change along the ridge lines: in regions Ua and Ub , the corresponding gradient vectors point in trivial directions (either horizontally or vertically), in accord with eqs. (5.9)–(5.10). On the other hand, the gradient in region B is very small, and the M_{T2} endpoint function is rather flat. The green dot marks the location of the true children masses ($M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV) and is indeed the intersection point of the three ridgelines. As expected, the corresponding $M_{T2(max)}$ at that point is the true parent particle mass $M_p = 600$ GeV.

At this point, it is interesting to ask the question, what would be the outcome of this exercise if one were to make the usual assumption of identical children, and apply the traditional symmetric M_{T2} to this situation. The answer can be deduced from figure 13(b), where the diagonal orange dotted line corresponds to the usual assumption of $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)}$. In that case, one still finds a kink, but at the wrong location: in figure 13(b) the intersection of the diagonal orange line and the solid black ridgeline occurs at $\tilde{M}_c^{(a)} = \tilde{M}_c^{(b)} = 65.3$ GeV and the corresponding parent mass is $\tilde{M}_p = 565.3$ GeV. Therefore, the traditional kink method can easily lead to a wrong mass measurement. Then the only way to know that there was something wrong with the measurement would be to study the effect of the upstream momentum and see that the observed kink is not invariant under P_{UTM} .

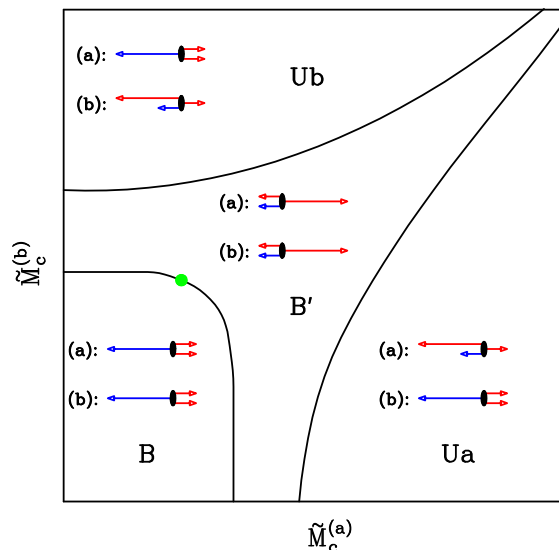


Figure 14. The same as figure 12 but for the onshell scenario illustrated in figure 3(c).

We should note that, depending on the actual mass spectrum, the two-dimensional ridge pattern seen in figures 12 and 13(b) may look very differently. For example, the balanced region B may or may not include the origin. One can show that if

$$M_p < \frac{M_c^{(b)}}{4M_c^{(a)}} \left(M_c^{(b)} + \sqrt{8(M_c^{(a)})^2 + (M_c^{(b)})^2} \right), \quad (5.16)$$

the boundary between B and Ua does not cross the $\tilde{M}_c^{(a)}$ axis. In this case the diagonal line in figure 13(b) does not cross any ridgelines and the traditional M_{T2} approach will not produce any kink structure, in contradiction with one’s expectations. This exercise teaches us that the failsafe approach to measuring the masses in missing energy events is to apply from the very beginning the asymmetric M_{T2} concept advertized in this paper.

5.2 On-shell intermediate particle

Our final example is the on-shell event topology illustrated in figure 3(c). Now there is an additional parameter which enters the game — the mass $M_i^{(\lambda)}$ of the intermediate particle in the λ -th decay chain. As a result, the allowed range of invariant masses for the visible particle pair on each side is limited from above by eq. (3.25).

In this case we find that the M_{T2} endpoint exhibits a similar phase structure as the one shown in figure 12. One particular pattern is illustrated in figure 14, which exhibits the same four regions B , B' , Ua and Ub seen in figure 12. The difference now is that region B' is considerably expanded, and as a result, region B does not have a common border with regions Ua and Ub any more. The triple point of figure 12 has now disappeared and the correct values of the children masses now lie somewhere on the border between regions B and B' , but their exact location along this ridgeline is at this point unknown.

Just like we did for the off-shell case in section 5.1, we shall now present analytical formulas for the M_{T2} endpoint in each region of figure 14. In the balanced region B , we find

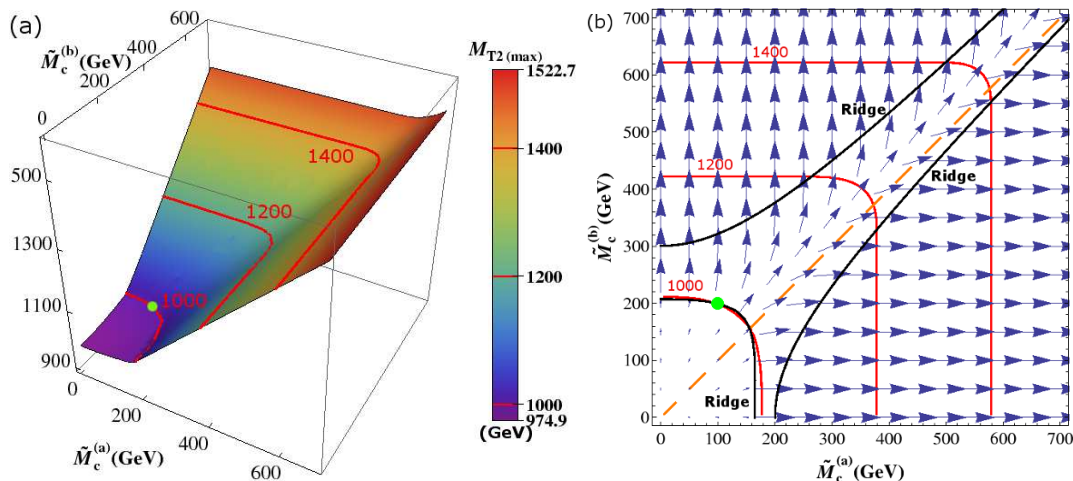


Figure 15. The same as in figure 13 but for the onshell scenario of figure 3(c), with a mass spectrum $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV, $M_i^{(a)} = M_i^{(b)} = 550$ GeV and $M_p = 1$ TeV.

the same results (5.1)–(5.3) as in the off-shell case considered in the previous section 5.1. The other balanced region B' is characterized by

$$m(\lambda) = m_{(\lambda)}^{\max}, \quad (5.17)$$

where $m_{(\lambda)}^{\max}$ is given by eq. (3.25), and

$$A_T = \frac{M_p^2}{4} \left[2 - \left(\frac{M_i^{(a)}}{M_p} \right)^2 - \left(\frac{M_c^{(a)}}{M_i^{(a)}} \right)^2 \right] \left[2 - \left(\frac{M_i^{(b)}}{M_p} \right)^2 - \left(\frac{M_c^{(b)}}{M_i^{(b)}} \right)^2 \right] + \frac{M_p^2}{4} \left| \left[\left(\frac{M_c^{(a)}}{M_i^{(a)}} \right)^2 - \left(\frac{M_i^{(a)}}{M_p} \right)^2 \right] \left[\left(\frac{M_c^{(b)}}{M_i^{(b)}} \right)^2 - \left(\frac{M_i^{(b)}}{M_p} \right)^2 \right] \right|. \quad (5.18)$$

The formula for the endpoint $M_{T2(max)}^{B'}$ in region B' is then simply obtained by substituting (5.17) and (5.18) into the balanced solution (3.7).

Finally, the M_{T2} endpoint in the unbalanced regions Ua and Ub is given by

$$M_{T2(max)}^{Ua}(\tilde{M}_c^{(a)}) = m_{(a)}^{\max} + \tilde{M}_c^{(a)}, \quad (5.19)$$

$$M_{T2(max)}^{Ub}(\tilde{M}_c^{(b)}) = m_{(b)}^{\max} + \tilde{M}_c^{(b)}, \quad (5.20)$$

where $m_{(a)}^{\max}$ and $m_{(b)}^{\max}$ are given by eq. (3.25).

In figure 15 we present our numerical results in this on-shell scenario. The mass spectrum is fixed as: $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 200$ GeV, $M_i^{(a)} = M_i^{(b)} = 550$ GeV and $M_p = 1$ TeV, and we still do not include the effects of any upstream momentum. Figure 15(a) shows the three-dimensional view of the M_{T2} endpoint function $M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$, which exhibits three different sets of ridges, which are more easily seen in the gradient plot of figure 15(b). As usual, the green dot marks the true children masses. Figure 15(b)

shows that the ridgeline separating the two balanced regions B and B' does go through the green dot and thus reveals a relationship between the two children masses, leaving the ridgeline parameter θ as the only remaining unknown degree of freedom. However, unlike the off-shell case of section 5.1, now there is no special point on this ridgeline, and we cannot completely pin down the masses by the ridge method. Thus, in order to determine all masses in the problem, one must use an additional piece of information, for example the visible invariant mass endpoint (3.25) or the P_{UTM} invariance method suggested in section 3.3.3.

6 Summary and conclusions

Cosmological observations hint towards the existence of one or more hypothetical dark matter particles. The start of the Large Hadron Collider may offer an unique opportunity to produce and study dark matter in a high-energy experimental laboratory. Unfortunately, the dark matter signatures at colliders always involve missing transverse energy. Such events will be quite challenging to fully reconstruct and/or interpret. Most previous studies have made (either explicitly or implicitly) the assumption that each event has two *identical* missing particles. Our main point in this paper is that this assumption is unnecessary, and by suitable modifications of the existing analysis techniques one can in principle test both the number and the type of missing particles in the data. Our proposal here was to modify the Cambridge M_{T2} variable [50] by treating each children mass as an independent input parameter. In this approach, one obtains the M_{T2} endpoint $M_{T2(max)}$ as a function of the two children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$, and proceeds to study its properties. The two most important features of the thus obtained function $M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$, identified in this paper, were the following:

- The function $M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)})$ exhibits a ridge structure (i.e. a gradient discontinuity), as illustrated with specific examples in figures 6, 9, 13 and 15. The point corresponding to the correct children masses always lies on a ridgeline, thus the ridgelines provide a model-independent constraint among the children masses, just like the M_{T2} endpoint provides a model-independent constraint on the masses of the child(ren) and the parent.
- In general, the M_{T2} endpoint function also depends on the value of the upstream transverse momentum in the event: $M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, P_{\text{UTM}})$. However, the P_{UTM} dependence disappears completely for precisely the right values of the children masses, as seen in the examples of figures 7(b) and 10(b). This provides a second, quite general and model-independent, method for measuring the *individual* particle masses in such missing energy events.

Before we conclude, we shall discuss a few other possible applications of the asymmetric M_{T2} idea, besides the examples already considered in the paper.

1. *Invisible decays of the next-to-lightest particle.* Most new physics models introduce some new massive and neutral particle which plays the role of a dark matter candidate. Often the very same models also contain other, heavier particles, which for

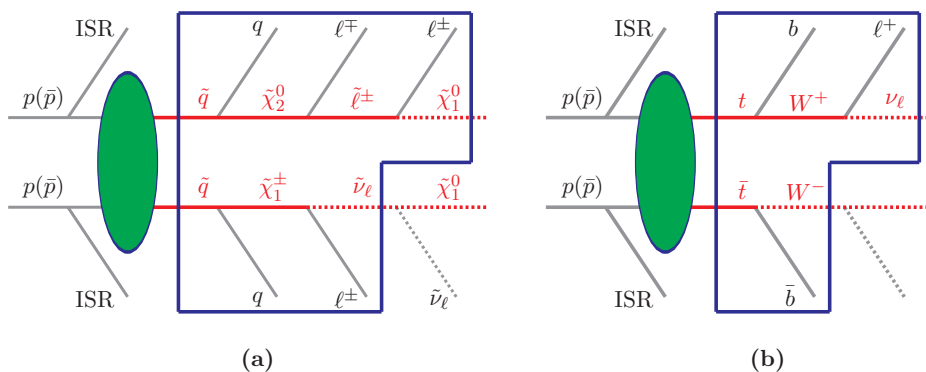


Figure 16. Event topology for the two examples discussed in section 6. The black solid lines represent SM particles which are visible in the detector while red solid lines represent particles at intermediate stages. The missing particles are denoted by dotted lines. (a) Squark pair production with decay chains terminating in two different invisible particles ($\tilde{\chi}_1^0$ and $\tilde{\nu}_\ell$, correspondingly). In this case $\tilde{\nu}_\ell$ decays invisibly. (b) The subsystem M_{T2} variable applied to $t\bar{t}$ events. The W -boson in the lower leg is treated as a child particle and can decay either hadronically or leptonically.

collider purposes behave just like a dark matter candidate: they decay invisibly and result in missing energy in the detector. For example, in supersymmetry one may find an invisibly decaying sneutrino $\tilde{\nu}_\ell \rightarrow \nu_\ell \tilde{\chi}_1^0$, in UED one finds an invisibly decaying KK neutrino $\nu_1 \rightarrow \nu \gamma_1$, etc. These scenarios can easily generate an asymmetric event topology. For example, consider the strong production of a squark (\tilde{q}) pair, as illustrated in figure 16(a). One of the squarks subsequently decays to the second lightest neutralino $\tilde{\chi}_2^0$, which in turn decays to the lightest neutralino $\tilde{\chi}_1^0$ by emitting two SM fermions $\tilde{\chi}_2^0 \rightarrow \ell^+ \ell^- \tilde{\chi}_1^0$ (or $\tilde{\chi}_2^0 \rightarrow jj \tilde{\chi}_1^0$). The other squark decays to a chargino $\tilde{\chi}_1^\pm$, which then decays to a sneutrino as $\tilde{\chi}_1^\pm \rightarrow \ell^\pm \tilde{\nu}_\ell$. Since $\tilde{\nu}_\ell$ can only decay invisibly, we obtain the asymmetric event topology outlined with the blue box in figure 16(a). The two squarks are the parents, the lightest neutralino $\tilde{\chi}_1^0$ is the first child, and the sneutrino $\tilde{\nu}_\ell$ is the second child.

2. *Applying M_{T2} to an asymmetric subsystem.* One can also apply the M_{T2} idea even to events in which there is only one (or even no) missing particles to begin with. Such an example is shown in figure 16(b), where we consider $t\bar{t}$ production in the dilepton or semi-leptonic channel. In the first leg we can take $b\ell$ as our visible system and the neutrino ν_ℓ as the invisible particle, while in the other leg we can treat the b -jet as the visible system and the W -boson as the child particle. In this case, there still should be a ridge structure revealing the true t , W and ν masses.
3. *Multi-component dark matter.* Of course, the model may contain two (or more) different genuine dark matter particles [105–113], whose production in various combinations will inevitably lead at times to asymmetric event topologies.

In conclusion, our work shows that the M_{T2} concept can be easily generalized to decay chains terminating in two different daughter particles. Nevertheless, the methods discussed in this paper allow to extract all masses involved in the decays, at least as a

matter of principle. We believe that such methods will prove extremely useful, if a missing energy signal of new physics is seen at the Tevatron or the LHC.

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A The asymmetric M_{T2} in the limit of infinite P_{UTM}

In this appendix we revisit our previous two examples from sections 4.1 and 4.2, this time considering the infinitely large P_{UTM} limit [97]. While this situation is impossible to achieve in a real experiment, its advantage is that it can be treated by analytical means. In the $P_{\text{UTM}} \rightarrow \infty$ limit, the “decoupling argument” of ref. [97] holds, and one finds the following analytical expression for the M_{T2} endpoint as a function of the two test children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$:

$$M_{T2(max)}(\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}, \infty) = \begin{cases} \sqrt{M_p^2 - (M_c^{(a)})^2 + (\tilde{M}_c^{(a)})^2}, & \text{if } (\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \in \mathcal{R}_1, \\ \sqrt{M_p^2 - (M_c^{(b)})^2 + (\tilde{M}_c^{(b)})^2}, & \text{if } (\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \in \mathcal{R}_2, \\ \frac{\tilde{M}_c^{(b)}}{M_c^{(b)}} M_p, & \text{if } (\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \in \mathcal{R}_3, \\ \frac{\tilde{M}_c^{(a)}}{M_c^{(a)}} M_p, & \text{if } (\tilde{M}_c^{(a)}, \tilde{M}_c^{(b)}) \in \mathcal{R}_4, \end{cases} \quad (\text{A.1})$$

where the four defining regions \mathcal{R}_i , ($i = 1, \dots, 4$) are shown in figure 17 and are defined as follows:

$$\mathcal{R}_1 : \tilde{M}_c^{(b)} < \sqrt{(M_c^{(b)})^2 - (M_c^{(a)})^2 + (\tilde{M}_c^{(a)})^2} \wedge \tilde{M}_c^{(a)} < M_c^{(a)}, \quad (\text{A.2})$$

$$\mathcal{R}_2 : \sqrt{(M_c^{(b)})^2 - (M_c^{(a)})^2 + (\tilde{M}_c^{(a)})^2} < \tilde{M}_c^{(b)} < M_c^{(b)}, \quad (\text{A.3})$$

$$\mathcal{R}_3 : M_c^{(b)} < \tilde{M}_c^{(b)} \wedge \tilde{M}_c^{(a)} < \left(\frac{M_c^{(a)}}{M_c^{(b)}}\right) \tilde{M}_c^{(b)}, \quad (\text{A.4})$$

$$\mathcal{R}_4 : M_c^{(a)} < \tilde{M}_c^{(a)} \wedge \tilde{M}_c^{(b)} < \left(\frac{M_c^{(b)}}{M_c^{(a)}}\right) \tilde{M}_c^{(a)}. \quad (\text{A.5})$$

Since the functional expression for $M_{T2(max)}$ within each region \mathcal{R}_i is different, there is in general a gradient discontinuity when crossing from one region into the next. Therefore, the ridges on the $M_{T2(max)}$ hypersurface will appear along the common boundaries of the four regions \mathcal{R}_i . Let us denote by L_{ij} the boundary between regions \mathcal{R}_i and \mathcal{R}_j . As indicated

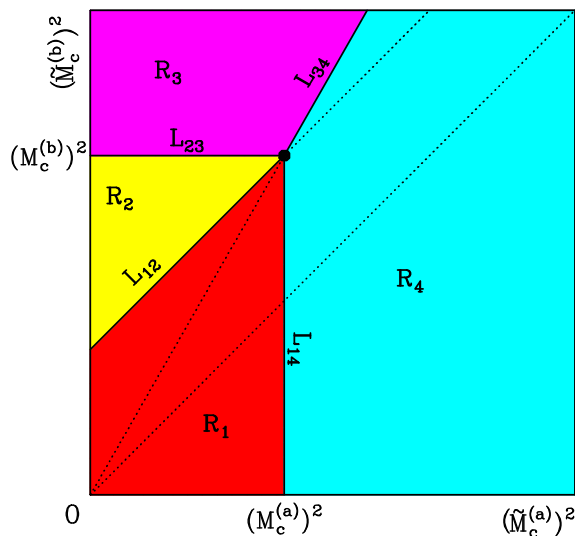


Figure 17. The parameter plane of test children masses squared, divided into the four different regions \mathcal{R}_i used to define the M_{T2} endpoint function (A.1). Their common boundaries L_{ij} are parametrically defined in eqs. (A.6)–(A.9). The black dot corresponds to the true values of the children masses.

in figure 17, each L_{ij} is a straight line in the parameter space of the children test masses squared and is given by

$$L_{12} : (\tilde{M}_c^{(b)})^2 = (M_c^{(b)})^2 - (M_c^{(a)})^2 + (\tilde{M}_c^{(a)})^2, \tilde{M}_c^{(a)} \leq M_c^{(a)}; \tag{A.6}$$

$$L_{23} : \tilde{M}_c^{(b)} = M_c^{(b)}, \tilde{M}_c^{(a)} \leq M_c^{(a)}; \tag{A.7}$$

$$L_{34} : \tilde{M}_c^{(b)} = \frac{M_c^{(b)}}{M_c^{(a)}} \tilde{M}_c^{(a)}, \tilde{M}_c^{(a)} \geq M_c^{(a)}; \tag{A.8}$$

$$L_{14} : \tilde{M}_c^{(a)} = M_c^{(a)}, \tilde{M}_c^{(b)} \leq M_c^{(b)}. \tag{A.9}$$

As seen in figure 17, all four lines L_{ij} meet at the true children mass point $\tilde{M}_c^{(a)} = M_c^{(a)}$, $\tilde{M}_c^{(b)} = M_c^{(b)}$, where in turn the M_{T2} endpoint $M_{T2(max)}$ gives the true parent mass M_p , in accordance with eq. (3.16).

With those preliminaries, we are now in a position to revisit our two examples from sections 4.1 and 4.2. Figures 18 and 19 are the corresponding analogues of figures 6 and 9 in the case of infinite P_{UTM} . Comparing with our earlier results, we notice both quantitative and qualitative changes in the ridge structure. First, the smooth ridge in figure 6(b) (figure 9(b)) has now been deformed into two straight line segments, one horizontal (L_{23}) and the other vertical (L_{14}), which meet at an angle of 90° precisely at the true values of the children masses. More importantly, figures 18 and 19 now exhibit another pair of ridges L_{12} and L_{34} (plotted in red in figures 18(b) and 19(b)), which were absent from the earlier figures in section 4. The system of four ridges seen in figures 18(a) and 19(a) is very similar to the crease structure observed in ref. [97]. We thus confirm the result of ref. [97]

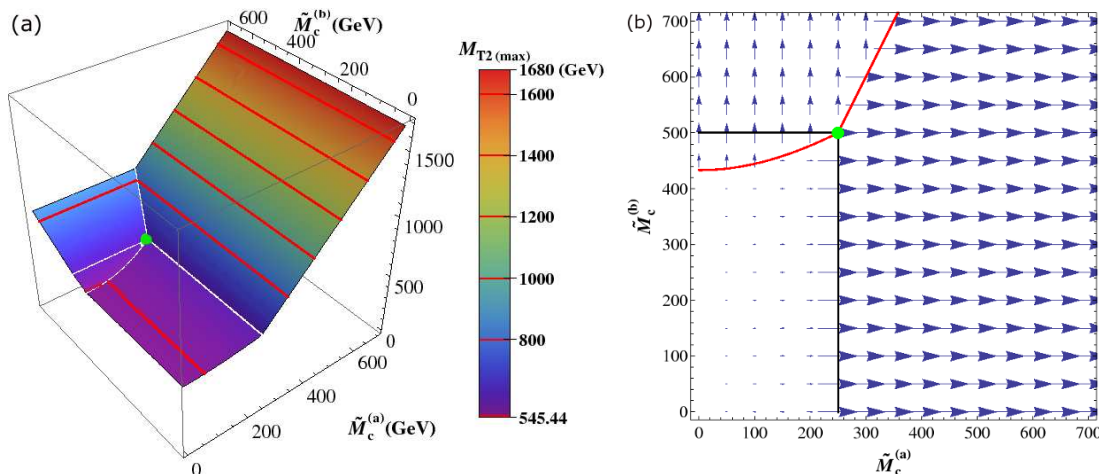


Figure 18. The same as figure 6 but for $P_{UTM} \rightarrow \infty$.

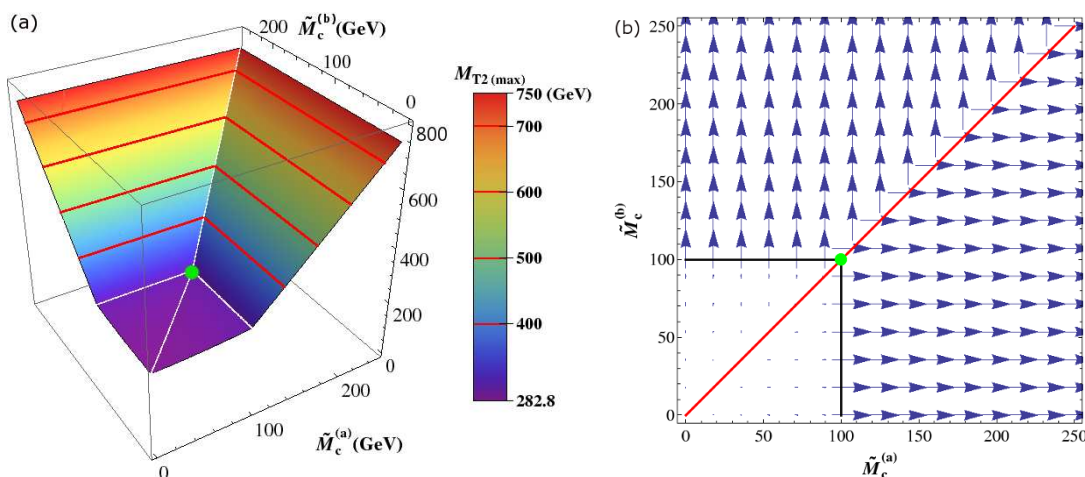


Figure 19. The same as figure 9 but for $P_{UTM} \rightarrow \infty$.

that in the infinite P_{UTM} limit there exist four different ridges, whose common intersection point reveals the true masses of the parent and children particles.

At this point it is instructive to contrast the two sets of ridgelines: L_{23} and L_{14} (shown in figures 18(b) and 19(b) in black) versus L_{12} and L_{34} (shown in figures 18(b) and 19(b) in red). The boundaries L_{23} and L_{14} separate the union of regions \mathcal{R}_1 and \mathcal{R}_2 from the union of regions \mathcal{R}_3 and \mathcal{R}_4 . Along those boundaries, we observe a transition in the configuration of visible momenta which yields the maximum possible value of M_{T2} . More precisely, in regions \mathcal{R}_1 and \mathcal{R}_2 we find that the visible momenta $\vec{p}_T^{(\lambda)}$ for $M_{T2(max)}$ are parallel to the direction of the upstream momentum \vec{P}_{UTM} , while in regions \mathcal{R}_3 and \mathcal{R}_4 we find that $\vec{p}_T^{(\lambda)}$ are anti-parallel to \vec{P}_{UTM} . This fact remains true even at finite values of P_{UTM} , which is why the ridgelines L_{23} and L_{14} could also be seen in the earlier plots from section 4 at finite $P_{UTM} = 1$ TeV.

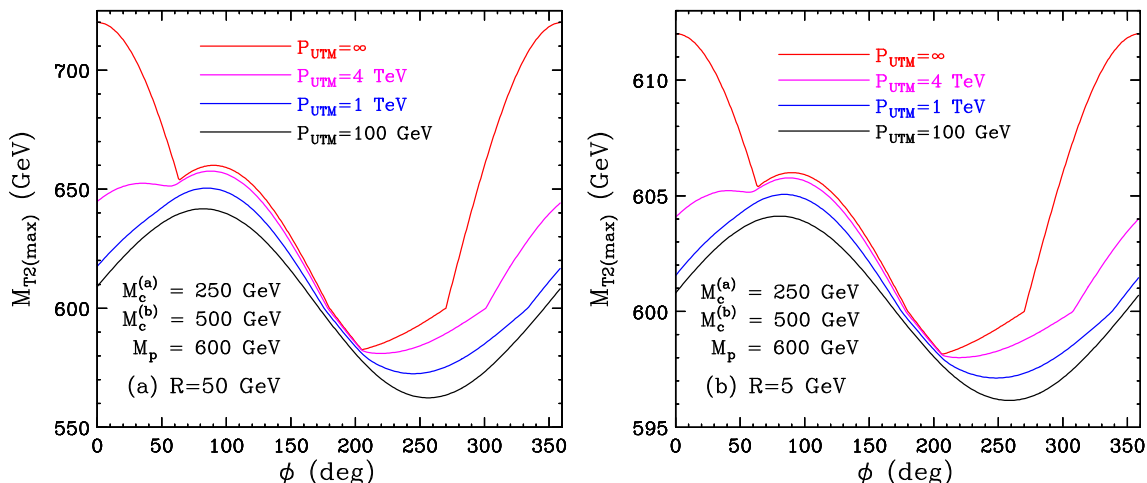


Figure 20. A study of the sharpness of the M_{T2} ridge for the example considered in section 4.1. The event topology is that of figure 3(a) and the mass spectrum is $M_c^{(a)} = 250$ GeV, $M_c^{(b)} = 500$ GeV and $M_p = 600$ GeV. We plot the asymmetric M_{T2} endpoint $M_{T2(max)}(\tilde{M}_c^{(a)}(\phi), \tilde{M}_c^{(b)}(\phi), P_{UTM})$, as a function of the angular variable ϕ parameterizing the circle of radius R defined in eqs. (A.10), (A.11). The radius R of the circle is taken to be $R = 50$ GeV in panel (a) and $R = 5$ GeV in panel (b). We present results for four different choices of the upstream momentum P_{UTM} as labelled in the plot.

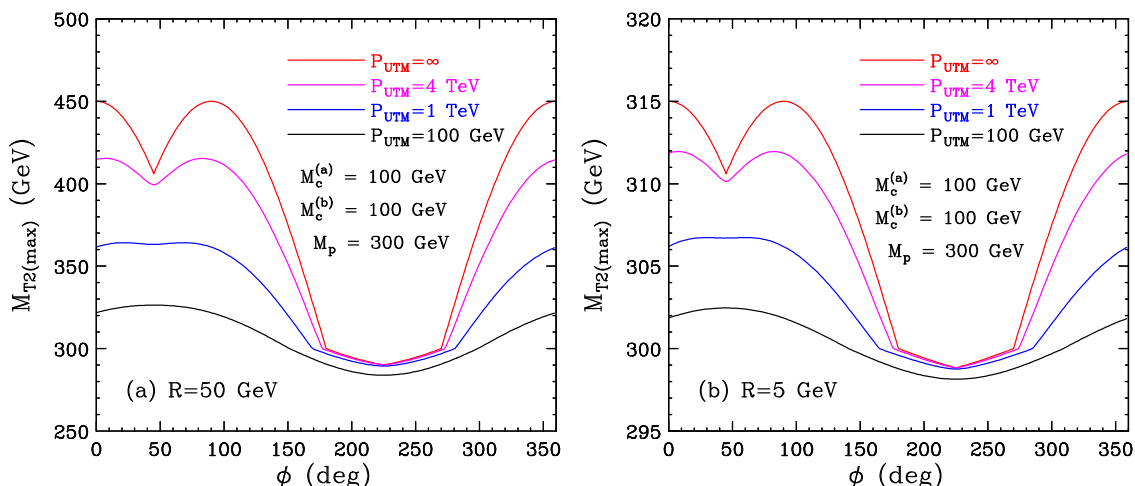


Figure 21. The same as figure 20, but for the example considered in section 4.2, where the input mass spectrum is fixed as $M_c^{(a)} = 100$ GeV, $M_c^{(b)} = 100$ GeV and $M_p = 300$ GeV.

On the other hand, the ridgelines L_{12} and L_{34} shown in red in figures 18(b) and 19(b) are due to the “decoupling argument” [97], which is strictly valid only in the infinite P_{UTM} limit. This is why these ridges become apparent only at very large values of P_{UTM} , and are gradually smeared out at smaller P_{UTM} .

The evolution of the ridge structure as a function of P_{UTM} is shown in figures 20 and 21. In order to compare the sharpness of the four ridges, we choose to vary the test children masses $\tilde{M}_c^{(a)}$ and $\tilde{M}_c^{(b)}$ along a circle centered on their true values and with a fixed radius R . Such a circle is guaranteed to cross all four ridges, and can be parameterized in

terms of an angular coordinate ϕ as follows

$$\tilde{M}_c^{(a)}(\phi) = M_c^{(a)} + R \cos \phi, \tag{A.10}$$

$$\tilde{M}_c^{(b)}(\phi) = M_c^{(b)} + R \sin \phi. \tag{A.11}$$

Then in figure 20 (figure 21) we plot the asymmetric M_{T2} endpoint $M_{T2(max)}(\tilde{M}_c^{(a)}(\phi), \tilde{M}_c^{(b)}(\phi), P_{\text{UTM}})$, as a function of the angular variable ϕ , for the case of mass spectrum I studied in section 4.1 (mass spectrum II studied in section 4.2). The radius R is taken to be $R = 50 \text{ GeV}$ in panels (a) and $R = 5 \text{ GeV}$ in panels (b). We present results for four different choices of the upstream momentum: $P_{\text{UTM}} = 100 \text{ GeV}$ (black lines), $P_{\text{UTM}} = 1 \text{ TeV}$ (blue lines), $P_{\text{UTM}} = 4 \text{ TeV}$ (magenta lines), and $P_{\text{UTM}} = \infty$ (red lines). Notice that the red lines at $P_{\text{UTM}} = \infty$ in figures 20 and 21 are directly correlated to the three-dimensional plots of figures 18 and 19, while the blue lines at $P_{\text{UTM}} = 1 \text{ TeV}$ in figures 20 and 21 are directly correlated to the three-dimensional plots of figures 6 and 9.

Each one of the previously discussed ridges manifests itself as a kink in figures 20 and 21. Indeed, the red lines for $P_{\text{UTM}} = \infty$ reveal four clear kinks, which (from left to right) correspond to the ridgelines L_{34} , L_{23} , L_{12} , and L_{14} . Using eqs. (A.6)–(A.9), it is easy to find the expected location of each kink in the $P_{\text{UTM}} \rightarrow \infty$ limit: $\phi = \{63.4^\circ, 180^\circ, 204.9^\circ, 270^\circ\}$ for figure 20(a), $\phi = \{63.4^\circ, 180^\circ, 206.4^\circ, 270^\circ\}$ for figure 20(b), and $\phi = \{45^\circ, 180^\circ, 225^\circ, 270^\circ\}$ for figures 21(a) and 21(b). However, as the upstream momentum is lowered to more realistic values, the kinks gradually wash out, albeit to a different degree. As anticipated from our earlier results, the smearing effect is quite severe for L_{34} and L_{12} , and by the time we reach $P_{\text{UTM}} = 1 \text{ TeV}$, those two kinks have completely disappeared. On the other hand, L_{23} and L_{14} are affected to a lesser degree by the smearing effect and are still visible at $P_{\text{UTM}} = 1 \text{ TeV}$, but by $P_{\text{UTM}} = 100 \text{ GeV}$ they are essentially gone as well. Notice that the variation in P_{UTM} affects not only the sharpness of the kinks, but also their location. This was to be expected, since we already saw that the shape of the ridge is different at $P_{\text{UTM}} = 1 \text{ TeV}$ and $P_{\text{UTM}} = \infty$: compare the black ridge lines in figures 6(b) and 9(b) to those in figures 18(b) and 19(b). Finally, as a curious fact we notice that the results shown in panels (a) and panels (b) of figures 20 and 21 are approximately related by a simple scaling with a constant factor.

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