Chiral generations on intersecting 5-branes in heterotic string theory

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ABSTRACT: We show that there exist two **27** and one **$\overline{27}$** of E_6, net one $D = 4$, $\mathcal{N} = 1$ chiral matter supermultiplet as zero modes localized on the intersection of two 5-branes in the $E_8 \times E_8$ heterotic string theory. The smeared intersecting 5-brane solution is used via the standard embedding to construct a heterotic background, which provides, after a compactification of some of the transverse dimensions, a five-dimensional Randall-Sundrum II like brane-world set-up in heterotic string theory. As a by-product, we present a new proof of anomaly cancellation between those from the chiral matter and the anomaly inflow onto the brane without small instanton.

Contents

1 Introduction

How the Standard Model emerges in string theory is a long-standing question. In early days of string theory, the heterotic string theory [1–3] was considered as a promising candidate for the fundamental theory which would provide a basis for model building. Its miraculous anomaly cancellation allows only two choices (that is, $E_8 \times E_8$ and SO(32)) of a consistent gauge group, and in Calabi-Yau compactifications (including orbifold and other 1/4 supersymmetric compactifications in a broad sense) there appear variety of four-dimensional supersymmetric standard-model-like theories with chiral generations. The problem is, however, that the number of such possible compactifications seems too large [4] to find natural necessity for our world to be as observed, despite the remarkable uniqueness of the original theory.

In the late last century, a conceptually different approach was proposed to realize a four-dimensional world by using D-branes in type II string theories. The key observation is that two intersecting D-branes can support chiral fermions at the intersection [5]. Since then many intersecting D-brane models have been built and discussed so far. We refer to the articles [6, 7] for a review of these developments. Also, inspired by the discovery of D-branes, brane-world models have also been extensively studied as a possible solution to the hierarchy problem and in terms of cosmological model building [8–10].

In this paper, we propose a new brane-world set-up for E_6 GUT model building by using intersecting 5-branes in *heterotic string theory*. The 5-branes in heterotic string theory are,

of course, not D-branes. They are NS5-branes [11–14], and unlike D-branes, they are not described by open strings. What makes them hard to deal with is that, near the core of the solution, the geometry is not AdS but an infinite throat where the dilaton diverges linearly. Nevertheless, we can identify what low-energy excitations are on the brane by investigating zero modes of the supergravity solution [15, 16]. It has been known for some time that on a symmetric 5-brane [15, 16] there are 30 $D = 6$, $\mathcal{N} = 1$ supermultiplet as zero modes in either of $E_8 \times E_8$ or SO(32) heterotic string theory. In fact, as we explain in section 2, they can be regarded as certain Nambu-Goldstone modes associated with various spontaneously broken symmetries of the theory. Therefore, we may expect that, as pions are effectively described by the chiral model without detailed knowledge of QCD, the zero modes on the heterotic 5-branes may also provide enough information for low-energy model building, even though their microscopic description (such as little string theory) is not fully understood. The existence of chiral zero modes is also consistent with the anomaly cancellation against an anomaly inflow from the bulk.

In order to examine the zero modes on the intersecting system, we first construct an intersecting 5-brane solution in the $E_8 \times E_8$ heterotic string theory by the so-called standard embedding in the known smeared intersecting NS5-brane solution of type II theories. We then study the zero modes of the relevant Dirac operator on this background. We show that there exist three localized chiral zero modes, two of which are in the **27** representation of E_6, and one in the $\overline{\mathbf{27}}$ representation. They give rise to net *one* **27** of massless chiral fermions in the four-dimensional spacetime. Therefore, still being a toy model, this is the first example of a brane set-up in heterotic string theory that supports four-dimensional chiral matter fermions transforming as an E_6 gauge multiplet.[1]

There is a good reason why we should study NS5-branes as a set-up for particle-physics model building: They are T-dual to noncompact Calabi-Yau manifolds obtained by blowing up an isolated singularity [17, 18]. For instance, parallel NS5-branes are known to be T-dual to a multi-center Taub-NUT, or an A_n singularity which is obtained as a limit of a Taub-NUT. Similarly, a system of two intersecting 5-branes is known to be T-dual to a deformed conifold [19–23]. Therefore, the intersecting 5-brane background in heterotic string theory may be regarded as a T-dual to a heterotic "compactification" on the deformed conifold. While there are a variety of compact Calabi-Yau manifolds with complicated structures, singularities may occur on a moduli space of *any* compact Calabi-Yau, and the local structure of a singularity is universal and can be simple, no matter what the rest of the manifold is. Therefore, the idea is that if a realistic GUT could be realized on such a singularity, it would mean that our universe is not just a coincidence, as *every* compact Calabi-Yau has a chance to realize the GUT on a part of it.

This work is a first step toward a brane realization of a realistic E_6 GUT model in string theory. The remainder of this paper is organized as follows: In section II, we give a brief review of known 5-brane solutions in type II and heterotic supergravity theories. In section III, we present a new proof of anomaly cancellation between those from the chiral

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[1]This corrects the statement made in an earlier version of [53], in which it was erroneously conjectured that the three supermultiplets would be of the same chirality.

matter on the brane and the anomaly inflow into the brane in the $E_8 \times E_8$ heterotic theory. In section IV, we construct an intersecting solution in the heterotic theory, and compute explicitly the zero modes of the Dirac operator to find net $2 - 1 = 1$ set of chiral zero modes transforming as the **27** representation of E_6. The last section is devoted to conclusions and discussion.

2 Review of 5-brane solutions in heterotic string theory

We will focus on the $E_8 \times E_8$ heterotic string theory. The string-frame bosonic supergravity Lagrangian is given, to $O(\alpha')$ [24, 25], as

$$\mathcal{L} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left\{ R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\partial_M \phi)^2 \right.$$
$$\left. -\alpha' \left(\frac{1}{30} \text{Tr}(F_{MN} F^{MN}) - R_{MNAB}(\omega_+) R^{MNBA}(\omega_+) \right) \right\}. \quad (2.1)$$

The convention we use in this paper is basically the one used in Callan-Harvey-Stromenger's original paper [15, 16], and [25], to which the reader is also referred for the comparison with other articles such as [24].

As already seen in the above effective Lagrangian, particular combinations of the spin connection ω and the antisymmetric three-form H play different roles in different places [24, 25]. In (2.1), the R^2 term is the Riemann square made of the combination

$$\omega_+ \equiv \omega + H. \quad (2.2)$$

This combination also appears in the higher order terms in the effective action, and in the Bianchi identity for the H field in the presence of flux:

$$dH = \alpha' \left(\text{tr}(R(\omega_+) \wedge R(\omega_+)) - \frac{1}{30} \text{Tr}(F \wedge F) \right). \quad (2.3)$$

On the other hand, another combination

$$\omega_- \equiv \omega - H \quad (2.4)$$

is relevant for the lowest order SUSY variation of the gravitino:

$$\delta\psi_M = \left(\partial_M + \frac{1}{4} \omega_{-M}{}^{AB} \Gamma_{AB} \right) \epsilon. \quad (2.5)$$

Finally, the Dirac operator of the gaugino equation of motion has a combination $\omega - \frac{1}{3} H$ as will be seen in a moment. The relations among the above three spin connections are discussed in [26].

2.1 The neutral solution

In the absence of the nonabelian gauge field, the following configurations solve the leading order equations of motion:

$$
\begin{aligned}
g_{ij} &= \eta_{ij} \quad (i,j = 0,1,\ldots,5), \\
g_{\mu\nu} &= e^{2\phi}\delta_{\mu\nu} \quad (\mu,\nu = 6,\ldots,9), \\
e^{2\phi} &= e^{2\phi_0} + \frac{n\alpha'}{x^2}, \\
H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}{}^\rho \partial_\rho \phi,
\end{aligned}
\tag{2.6}
$$

where

$$
x^2 \equiv \sum_{\mu=6}^{9}(x^\mu)^2. \tag{2.7}
$$

$\epsilon^{\mu\nu\lambda\rho}$ is the (undensitized) completely antisymmetric tensor. All other components of H vanish.

The solution (2.6) may be regarded as representing the NS5-branes stacked on top of each other in both type IIA and type IIB theories. It has a nonzero axion charge

$$
\frac{1}{2\pi^2}\int_{S^3} H = n\alpha'. \tag{2.8}
$$

n must be an integer. This is an everywhere smooth solution; $x = 0$ is an apparent singularity as is verified by the coordinate transformation $t \equiv \ln x^2$ [15, 16]. The scalar curvature and Riemann square (in the string frame) are both everywhere finite:

$$
R = \frac{6}{n\alpha'} \cdot \frac{1}{(1+\frac{x^2}{\rho^2})^3}, \tag{2.9}
$$

$$
R_{ABCD}R^{ABCD} = \frac{12}{n^2\alpha'^2} \cdot \frac{1 + 4\frac{x^2}{\rho^2} + 8(\frac{x^2}{\rho^2})^2}{(1+\frac{x^2}{\rho^2})^6}, \tag{2.10}
$$

where $\rho^2 \equiv e^{-2\phi_0}n\alpha'$. The supergravity analysis is trusted if the string coupling is small enough, and the metric varies slow enough:

$$
e^{2\phi} \ll 1, \quad R \ll \alpha'^{-1}. \tag{2.11}
$$

They are satisfied if

$$
e^{2\phi_0} \ll 1, \quad \rho \ll |x|. \tag{2.12}
$$

When considered in heterotic string theory later, the parameter ρ corresponds to the size of the instanton. Therefore, a small instanton means that the string coupling is everywhere large, and some nonperturbative phenomenon is known to occur [27, 28]. Even though $e^{-\phi_0}$ is large, the dilaton becomes large if one gets closer than the instanton size to the brane. However, a close relative of the symmetric 5-brane has been obtained [29] as a certain double scaling limit of a non-extremal solution, and it is known to have, as

a part of its near-horizon geometry, a two-dimensional black hole rather than a linear-dilaton throat geometry. CFT models inspired by this solution have been constructed [30]. (The worldsheet approach for 5-branes was originally mentioned in the second reference of [11–13].)

The zero modes on this solution are a six-dimensional *chiral* $(2,0)$ matter supermultiplet in the IIA case, and *nonchiral* $(1,1)$ supermultiplet in the IIB case [15, 16]. This flip of the chirality may be understood as a consequence of the T-duality to the ADE singularities.

2.2 The symmetric solution

Next we include a nonabelian gauge field in heterotic string theory. It is well known that in order for the anomaly cancellation mechanism to work, the Bianchi identity must be modified as we saw in (2.3) as to

$$dH = \alpha' \left(\text{tr}(R(\omega_+) \wedge R(\omega_+)) - \frac{1}{30}\text{Tr}(F \wedge F) \right), \tag{2.13}$$

where Tr is the trace in the adjoint representation of $E_8 \times E_8$ or SO(32).

Since the neutral solution satisfies $dH = 0$ except at the brane position $x^\mu = 0$ where the magnetic 5-brane charge is located, it remains a solution in heterotic theory only if the right hand side vanishes. The most common way to achieve this is to set ω_+ equal to the gauge connection A. This may be called the "standard" embedding, but the point is that, in the presence of nonzero H flux, what is embedded in the gauge connection is not simply the spin connection ω, but the particular combination $\omega + H$. What is nice about this embedding is that some corrections of the supersymmetry variations to higher orders in α' vanish [24].

The spin connections computed in the neutral background (2.6) are found to be

$$\omega_\mu{}^\alpha{}_\beta = (\delta_\mu^\alpha \delta_\beta^\nu - \delta_{\mu\beta}\delta^{\alpha\nu})\partial_\nu \phi. \tag{2.14}$$

All other components vanish. The H field is written as[2]

$$H_\mu{}^\alpha{}_\beta = -\epsilon^\alpha{}_\beta{}^{\gamma\delta}\,\delta_{\mu\gamma}\delta_\delta{}^\nu\partial_\nu \phi. \tag{2.15}$$

Therefore ω_+ is given by

$$\omega_{+\mu}{}^{\alpha\beta} \equiv (\omega + H)_\mu{}^{\alpha\beta}$$
$$= 2\rho^2\sigma^{\alpha\beta}{}_{\mu\lambda} \cdot \frac{x^\lambda}{x^2(x^2 + \rho^2)}, \tag{2.16}$$

where

$$\sigma^{\alpha\beta}{}_{\mu\lambda} \equiv \delta_\mu^\alpha \delta_\lambda^\beta - \frac{1}{2}\epsilon^{\alpha\beta}{}_{\gamma\delta}\,\delta_\mu^\gamma \delta_\lambda^\delta, \tag{2.17}$$

$$\rho^2 \equiv e^{-2\phi_0}n\alpha'. \tag{2.18}$$

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[2]In these expressions no vielbein appears because the metric is diagonal and the scale factors cancel in the present case.

The tensor $\sigma^{\alpha\beta}{}_{\mu\lambda}$ is anti-self-dual:

$$\frac{1}{2}\epsilon^{\alpha\beta}{}_{\gamma\delta}\,\sigma^{\gamma\delta}{}_{\mu\nu} = -\sigma^{\alpha\beta}{}_{\mu\nu}. \tag{2.19}$$

Thus the SO(4) connection ω_+ is anti-self-dual, which means that the structure group of the bundle is reduced to SU(2) ("SU(2) structure" [31]). We then identify

$$A_\mu^{\alpha\beta} = \omega_{+\mu}{}^{\alpha\beta}, \tag{2.20}$$

and assume all other components to be zero. This is the symmetric solution [15, 16]. In this way, a part of the gauge connection acquires a nonzero background in an SU(2) subalgebra of $E_8 \times E_8$.

This is a supersymmetric configuration; a different combination

$$\begin{aligned}
\omega_{-\mu}{}^{\alpha\beta} &\equiv (\omega - H)_\mu{}^{\alpha\beta} \\
&= 2\rho^2 \hat{\sigma}^{\alpha\beta}{}_{\mu\lambda} \cdot \frac{x^\lambda}{x^2(x^2 + \rho^2)}, \\
\hat{\sigma}^{\alpha\beta}{}_{\mu\lambda} &\equiv \delta_\mu^\alpha \delta_\lambda^\beta + \frac{1}{2}\epsilon^{\alpha\beta}{}_{\gamma\delta}\,\delta_\mu^\gamma \delta_\lambda^\delta
\end{aligned} \tag{2.21}$$

is a *self-dual* connection, and hence belongs also to a (different) SU(2) subalgebra of SO(4). This ensures that there is a Killing spinor for the gravitino SUSY variation

$$\delta\psi_M = \left(\partial_M + \frac{1}{4}\omega_{-M}{}^{AB}\Gamma_{AB}\right)\epsilon. \tag{2.22}$$

On the other hand, the gaugino SUSY variation reads

$$\delta\chi^{\alpha\beta} = -\frac{1}{4}\Gamma^{MN}F_{MN}^{\alpha\beta}\epsilon, \tag{2.23}$$

where the SO(4) matrix indices α, β are now understood as the SU(2) gauge indices. The field strength $F_{MN}^{\alpha\beta}$ involves the connection ω_+ due to the embedding, and not ω_-. However, there is a following identity between the Riemann tensor made of the connection ω_+ and that made of ω_-:

$$R(\omega_+)_{MNPQ} = R(\omega_-)_{PQMN} + (dH)_{MNPQ}. \tag{2.24}$$

Therefore, in the background where dH vanishes, the gaugino variation (2.23) amounts to

$$\begin{aligned}
\delta\chi^{\alpha\beta} &= -\frac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}^{\alpha\beta}\epsilon \\
&= -\frac{1}{4}\Gamma^{\mu\nu}R(\omega_+)_{\mu\nu}{}^{\alpha\beta}\epsilon \\
&= -\frac{1}{4}\Gamma^{\gamma\delta}R(\omega_-)^{\alpha\beta}{}_{\gamma\delta}\epsilon.
\end{aligned} \tag{2.25}$$

Thus the Killing spinor ϵ for the gravitino variation (2.22) is automatically the Killing spinor for the gaugino variation (2.23). (The dilatino variation equation must be checked separately.) The SU(2) gauge connection $A_\mu^{\alpha\beta}$ (2.20) satisfies the lowest-order equation of motion

$$\partial_\nu\left(\sqrt{-g}e^{-2\phi}F^{\mu\nu\alpha\beta}\right) + \sqrt{-g}e^{-2\phi}\left([A_\nu,\ F^{\mu\nu}]^{\alpha\beta} - H^{\mu\nu\rho}F_{\nu\rho}^{\alpha\beta}\right) = 0 \tag{2.26}$$

as expected.

2.3 Zero modes on the symmetric 5-brane

Let us consider zero modes existing on the symmetric 5-brane solution (2.6) with (2.20) [15, 16]. The obvious bosonic zero modes are the four translation moduli, and the instanton size ρ modulus. Besides, there are other zero modes coming from infinitesimal global gauge rotations of the instanton: By construction, the gauge fields have nonzero vacuum expectation values in the four-dimensional space transverse to the 5-brane. They belong to an SU(2) subalgebra of one of E_8. The centralizer of SU(2) in E_8 is E_7, and the adjoint **248** is decomposed into a sum of representations of $E_7 \times$ SU(2) as

$$\mathbf{248} = (\mathbf{133}, \mathbf{1}) \oplus (\mathbf{56}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{3}). \tag{2.27}$$

133, the adjoint of E_7, does nothing on the SU(2) background, while the other $56 \times 2 + 1 \times 3 = 115$ generators rotate the background, and hence give rise to zero modes. Thus, in all, there are $4 + 1 + 115 = 120$ bosonic zero modes on this background. Since the symmetric solution is half BPS, they together with their superpartners constitute 30 $D = 6$, $\mathcal{N} = 1$ hypermultiplets. The existence of the fermionic zero modes have also been confirmed by the index theorem [32].

These zero modes can be regarded as Nambu-Goldstone modes associated with various spontaneously broken symmetries of the theory [33]. Indeed, the four position moduli above are the Nambu-Goldstone modes coming from the spontaneous broken translational invariance due to the presence of the 5-brane. The size modulus corresponds to the broken scale invariance. The remaining 115 moduli are also thought of as coming from how the SU(2) subalgebra is embedded in the whole E_8 Lie algebra; by "standard embedding" we mean we choose *some* SU(2) in E_8 and set the gauge connection for *this* SU(2) to be equal to the (generalized) spin connection. But the choice of such an SU(2) is arbitrary, and the original E_8 symmetry is spontaneously broken. Incidentally, this way of counting reproduces the correct instanton-number dependence of the dimensions of instanton moduli in flat space, for all gauge groups, obtained by the index theorem [34].

But there is a puzzle here: Why aren't they absorbed into the gauge bosons by the Higgs mechanism? The gauge bosons in the transverse dimensions can be viewed as adjoint Higgs fields from the brane, and the standard embedding amounts to giving vev's to these Higgs fields. Then small fluctuations around the vev's are Nambu-Goldstone modes, which are completely gauged away to leave, in ordinary gauge theories, a Proca Lagrangian for massive vector fields. This is the standard Higgs mechanism in the textbook, and it is interpreted to mean that the Nambu-Goldstone modes are "eaten" by the gauge bosons to be their longitudinal degrees of freedom. So why are there such extra zero-mode degrees of freedom left on the brane, other than those used as a part of massive vector bosons in the bulk?

The resolution to this problem lies[3] in the apparent breakdown of the gauge invariance due to the Green-Schwarz counterterm $BX_8 \sim -dBX_7$. In eliminating the small fluctuations around the vev, both B and X_7 also get transformed by the gauge transformation. The contribution from the variation of B is compensated by the one-loop anomaly

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[3]We are grateful to H. Kawai, H. Kunitomo and N. Ohta for discussions on this issue.

in the bulk [35], while that from X_7 vanishes if there are no magnetic source of the B field $d^2 B = 0$. In the present case, however, there *is* such a source $d^2 B \propto \delta^4(x^\mu)$, and therefore the gauge variation of X_7 gives rise to a change of the field configurations on the brane. Thus gauge transformations can *not completely* eliminate the fluctuations of the "Higgs", but local fluctuations are left on the brane.[4]

This phenomenon is known as anomaly inflow [36, 37], and the change of the brane action is cancelled by, again, the one-loop effect of chiral fermions on the brane, which are the superpartners of the bosonic zero modes. The gauge invariance of the total quantum action is thus restored. In the next section, we will show the precise arithmetic of the cancellation.[5]

3 Anomaly inflow and cancellation

We will show that the 30 hypermultiplets, 28 ($=56$ half-hypermultiplets) in the **56** representation of E_7 and two singlets ($=4$ half-hypermultiplets), precisely cancel the inflows of the tangent bundle, E_7 gauge and mixed anomalies via the Green-Schwarz mechanism. The cancellation of anomalies on the gauge 5-brane [15, 16] in heterotic string theory was already discussed in [38, 39]. Here we give a somewhat different proof of cancellation than theirs in the case of the $E_8 \times E_8$ symmetric 5-brane. Although they should be basically the same, ours is closely parallel to Mourad [40] and appears to be simpler. In particular, we do not need to consider any current at infinity. We ignore the normal bundle connection and write out only terms consisting of the tangent bundle and gauge connections.

The relevant anomaly polynomials are

$$
\begin{aligned}
I_8^{singlet} &= \left. \frac{1}{2} \hat{A}(T\Sigma) \right|_8 \times 4 \\
&= \frac{2}{5760}(-4p_2 + 7p_1^2), \\
I_8^{\mathbf{56}} &= \left. \frac{1}{2} \hat{A}(T\Sigma) \mathrm{tr}_{\mathbf{56}} e^{iF} \right|_8 \\
&= \frac{28}{5760}(-4p_2 + 7p_1^2) + \frac{1}{96} p_1^2 \mathrm{tr}_{\mathbf{56}} F^2 + \frac{1}{48} \mathrm{tr}_{\mathbf{56}} F^4,
\end{aligned}
$$

(3.1)

(3.2)

and

$$
\begin{aligned}
X_8 &= \frac{1}{24} \left(\frac{1}{8} \mathrm{tr} R^4 + \frac{1}{32} (\mathrm{tr} R^2)^2 - \frac{1}{240} \mathrm{tr} R^2 \mathrm{Tr}_{\mathbf{248}} F^2 + \frac{1}{24} \mathrm{Tr}_{\mathbf{248}} F^4 - \frac{1}{7200} (\mathrm{Tr}_{\mathbf{248}} F^2)^2 \right) \\
&= \frac{1}{192}(-4p_2 + 7p_1^2) + \frac{1}{2880} p_1^2 \mathrm{Tr}_{\mathbf{248}} F^2 + \frac{1}{576} \mathrm{Tr}_{\mathbf{248}} F^4 - \frac{1}{24 \cdot 7200} (\mathrm{Tr}_{\mathbf{248}} F^2)^2.
\end{aligned}
$$

(3.3)

---

[4]In contrast, zero modes coming from an abelian gauge field in other theories (such as type IIA theory [15, 16] and $D = 5$ supergravity [54]) are not pure gauge rotations.

[5]The anomaly inflow argument in the next section is *not* intended to establish the net chirality in *four* dimensions, but only to confirm that the gauge rotation moduli are certainly not absorbed by the Higgs mechanism.

Since the gauge symmetry is broken from E_8 to E_7, we rewrite the traces in the representations of E_8 to those of E_7. The following formulas are useful [41]:

$$\text{Tr}_{\mathbf{248}}F^2 = (\text{Tr}_{\mathbf{133}} + 2\text{tr}_{\mathbf{56}})F^2$$
$$= 5\text{tr}_{\mathbf{56}}F^2, \tag{3.4}$$

$$\text{Tr}_{\mathbf{248}}F^4 = (\text{Tr}_{\mathbf{133}} + 2\text{tr}_{\mathbf{56}})F^4$$
$$= \frac{1}{4}(\text{tr}_{\mathbf{56}}F^2)^2. \tag{3.5}$$

$$\text{Tr}_{\mathbf{133}}F^2 = 3\text{tr}_{\mathbf{56}}F^2,$$
$$\text{Tr}_{\mathbf{133}}F^4 = \frac{1}{6}(\text{tr}_{\mathbf{56}}F^2)^2,$$
$$\text{tr}_{\mathbf{56}}F^4 = \frac{1}{24}(\text{tr}_{\mathbf{56}}F^2)^2. \tag{3.6}$$

Therefore

$$X_8 = \frac{1}{192}(-4p_2 + 7p_1^2) + \frac{1}{24^2}p_1^2\text{tr}_{\mathbf{56}}F^2 + \frac{1}{4\cdot24^2}(\text{tr}_{\mathbf{56}}F^2)^2 - \frac{1}{12\cdot24^2}(\text{tr}_{\mathbf{56}}F^2)^2. \tag{3.7}$$

They add up to

$$I_8^{singlet} + I_8^{\mathbf{56}} - X_8 = \frac{1}{3\cdot24^2}(3p_1 + \text{tr}_{\mathbf{56}}F^2)(12p_1 + \text{tr}_{\mathbf{56}}F^2). \tag{3.8}$$

Note that the number (thirty) of hypermultiplets is precisely the one which can cancel out the p_2 term, otherwise the sum of anomalies does not factorize and the Green-Schwarz mechanism does not apply. Since

$$12p_1 + \text{tr}_{\mathbf{56}}F^2 = 6\left(-\text{tr}R^2 + \frac{1}{30}\text{Tr}_{\mathbf{248}}F^2\right), \tag{3.9}$$

which is proportional to the anomalous part of the heterotic Bianchi identity, the sum (3.8) is cancelled by introducing a Green-Schwarz counterterm on the brane as in [40].

	0	1	2	3	4	5	6	7	8	9
5-brane1	○					○	○	○	○	○
5-brane2	○			○	○			○	○	○

Table 1. Dimensions in which the 5-branes stretch.

4 Intersecting 5-branes in heterotic string theory

4.1 Intersecting neutral 5-branes

We will now consider a system of two intersecting NS5-branes. We start with the neutral smeared solution [42, 43]:

$$
ds^2 = \sum_{i,j=0,7,8,9} \eta_{ij} dx^i dx^j + h(x^1)^2 \sum_{\mu,\nu=1,2} \delta_{\mu\nu} dx^\mu dx^\nu + h(x^1) \sum_{\mu,\nu=3,4,5,6} \delta_{\mu\nu} dx^\mu dx^\nu,
$$
$$
e^{2\phi} = h(x^1)^2,
$$
$$
H_{\mu\nu\lambda} = \begin{cases} \frac{h'}{2} \ (= \frac{\xi |x^1|'}{2}) & \text{if } (\mu,\nu,\lambda) = (2,3,4),(2,5,6) \text{ or their even permutation,} \\ -\frac{h'}{2} \ (= -\frac{\xi |x^1|'}{2}) & \text{if } (\mu,\nu,\lambda) = (2,4,3),(2,6,5) \text{ or their even permutation,} \\ 0 & \text{otherwise,} \end{cases} \tag{4.1}
$$

where

$$
h(x^1) = h_0 + \xi |x^1|. \tag{4.2}
$$

All other components of H_{MNL} vanish. h_0 and ξ are real constants. The prime $'$ denotes the differentiation with respect to x^1, and $|x^1|'$ is therefore a step function. This is a solution to equations of motion of the leading-order NSNS-sector Lagrangian in type II theories:

$$
\mathcal{L}_{NS} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left(R(\omega) - \frac{1}{3} H_{MNP} H^{MNP} + 4(\partial_M \phi)^2 \right). \tag{4.3}
$$

The solution describes a pair of intersecting NS5-branes stretching in dimensions as shown in table 1. These branes are delocalized in the x^2, x^3, x^4, x^5 and x^6 directions. Consequently, the solution depends only on x^1, and hence the name "smeared solution".

4.2 Brane tension and the harmonic function

The coefficient ξ in the definition of the harmonic function $h(x^1)$ is related to the tension of the brane. To see this, let us consider Einstein's equation in the Einstein frame:

$$
(R_E)_A{}^B - \frac{1}{2} \delta_A{}^B R_E - T_A{}^B = \begin{cases} 0 & (A,B = 1,2), \\ +\frac{h''}{2h^{\frac{5}{2}}} & (A,B = 3,4,5,6), \\ +\frac{h''}{h^{\frac{5}{2}}} & (A,B = 0,7,8,9), \end{cases} \tag{4.4}
$$
$$
-T_A{}^B = -\frac{1}{2} \partial_A \phi \partial^B \phi - e^{-\phi} H_{ACD} H^{BCD} + \frac{1}{2} \delta_A^B \left(\frac{1}{2} (\partial\phi)^2 + \frac{1}{3} e^{-\phi} H^2 \right). \tag{4.5}
$$

The fact that the right hand side of (4.4) does not vanish implies that the action must include δ-function like brane-energy terms:

$$\mathcal{L}_E = \frac{1}{2\kappa^2}\sqrt{-G}R_E + \mathcal{L}_E(\phi, H) - V\sqrt{-G_{\text{5-brane1}}} \prod_{\mu'=1,2,3,4} \delta(x^{\mu'})$$
$$- V\sqrt{-G_{\text{5-brane2}}} \prod_{\mu''=1,2,5,6} \delta(x^{\mu''}), \qquad (4.6)$$

where V is the brane tension. $\mathcal{L}_E(\phi, H)$ is the Lagrangian for the ϕ and H fields in the Einstein frame, which contributes to the energy-momentum tensor $T_A{}^B$ in (4.4). The brane metrics are defined as

$$(G_{\text{5-brane1}})_{i'j'} = G_{i'j'}(x^{\mu'} = 0) \quad (i', j' = 0, 5, 6, 7, 8, 9;\ \mu' = 1, 2, 3, 4),$$
$$(G_{\text{5-brane2}})_{i''j''} = G_{i''j''}(x^{\mu''} = 0) \quad (i'', j'' = 0, 3, 4, 7, 8, 9;\ \mu'' = 1, 2, 5, 6). \qquad (4.7)$$

What we see here is a no-cosmological-constant analogue of the Randall-Sundrum (RS) models [9, 10], and the intersecting nature of the solution is reflected in the two different brane-energy terms. After delocalizations:

$$\prod_{\mu'=2,3,4} \delta(x^{\mu'}) \to 1,$$
$$\prod_{\mu''=2,5,6} \delta(x^{\mu''}) \to 1, \qquad (4.8)$$

the inclusion of these terms matches (4.4) if

$$\xi = -\kappa^2 V h_0^{\frac{5}{2}}. \qquad (4.9)$$

Since $e^\phi = h(x^1)$, the sign of ξ strongly affects the dilaton profile. (If $\xi = 0$, the solution is reduced to a flat Minkowski space.) We consider the following two cases separately:

- If $\xi > 0$ as in figure 1, the brane tension is negative. It is doubtful whether such an object may consistently exist in heterotic string theory. Also, if $\xi > 0$, the string coupling becomes stronger as one goes away from the branes, which is puzzling. Thus we consider another option.

- If $\xi < 0$ as in figure 2, the brane has a positive tension. The function $h(x^1)$, and hence the string coupling, is now convex upwards in x^1. It decreases linearly from a positive value h_0, to necessarily cross the x^1 axis, where the string coupling becomes zero. Beyond that point, $h(x^1)$ becomes negative, which is inconsistent. Thus we identify this point as the "end of the world"; one can send this point infinitely far away[6] by the coordinate transformation

$$z = -\text{sign}(x^1)\log\frac{h(x^1)}{h_0}, \qquad (4.10)$$

---

[6]Of course, this is just a change of a coordinate, and hence does not change the geodesic distance. Also it is not smooth at $x^1 = 0$ ($z = 0$), and gives rise to an extra delta function in the second derivative.
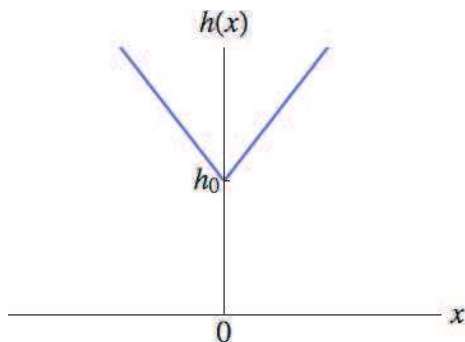
Figure 1. $h(x)$ with $\xi > 0$. The brane tension is negative. Also, the string coupling becomes stronger as one goes away from the branes.
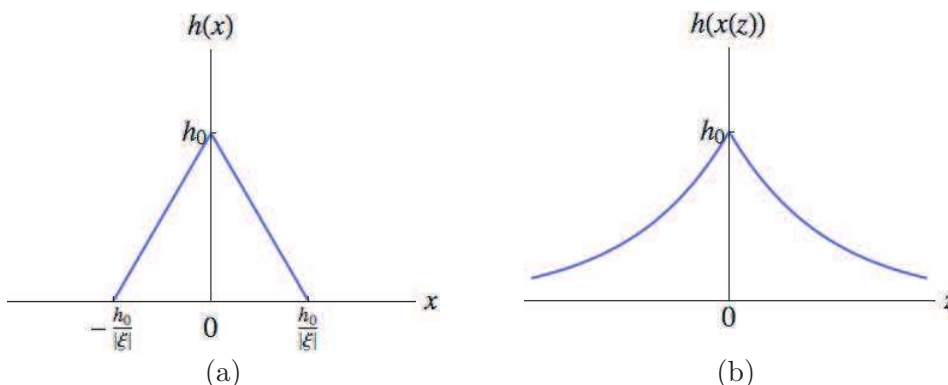

(a)   (b)

Figure 2. $h(x)$ with $\xi < 0$. (a) The brane has a positive tension. The string coupling decreases linearly from a positive value h_0, to necessarily cross the x^1 axis, where the string coupling becomes zero. We identify this point as the "end of the world". (b) By a change of the coordinate the points $x = \pm \frac{h_0}{|\xi|}$ are mapped to $z = \pm \infty$. The profile of $h(x(z))$ becomes similar to the warp factor of the RS II model.

where z is the new coordinate. Then the function $h(x^1)$, which is the string coupling and a typical warp factor for the relatively transverse dimensions, is expressed simply as

$$h = h_0 \, e^{-|z|}. \tag{4.11}$$

Apparently, this looks similar to the RS II model [10], but there are the following differences: The first is that we have no bulk cosmological constant. Instead, we have the dilaton and axion fields (and also the nonabelian gauge fields after the standard embedding) which balance gravity. Secondly, as we see in a moment, there exist chiral zero modes on the branes, which are in the **27** representation of E_6. This is not an assumption but a logical consequence of string theory. The final difference is in the warp factor. Unlike the RS models, our four-dimensional metric is not warped at all in the string frame.[7] It would

---

[7]More curiously, although the branes have a positive tension as we have derived (4.9), the 4D metric is inversely warped (like near the negative tension brane in the RS I model [9]) in the Einstein frame.

be interesting to examine whether gravity or gauge field is localized, but in this paper we will focus only on the localization of chiral fermions.

4.3 Intersecting 5-branes in heterotic string theory

We now construct an intersecting solution in the $E_8 \times E_8$ heterotic string theory by the standard embedding, similarly to the previous parallel brane case.

The (generalized) spin connections of the neutral intersecting background are computed as

$$(\omega \pm H)_{\mu=1}{}^{\alpha}{}_{\beta} = 0,$$

$$(\omega \pm H)_{\mu=2}{}^{\alpha}{}_{\beta} = \frac{h'}{h} \begin{pmatrix} & -1 & & \\ 1 & & \pm\frac{1}{2} & \\ & \mp\frac{1}{2} & & \pm\frac{1}{2} \\ & & \mp\frac{1}{2} & \end{pmatrix},$$

$$(\omega \pm H)_{\mu=3}{}^{\alpha}{}_{\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & -1 & \mp 1 \\ 1 & & & \\ & \pm 1 & & \end{pmatrix},$$

$$(\omega \pm H)_{\mu=4}{}^{\alpha}{}_{\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & \pm 1 & -1 \\ & \mp 1 & & \\ 1 & & & \end{pmatrix},$$

$$(\omega \pm H)_{\mu=5}{}^{\alpha}{}_{\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & & -1 \\ & & & \mp 1 \\ 1 & & & \\ & \pm 1 & & \end{pmatrix},$$

$$(\omega \pm H)_{\mu=6}{}^{\alpha}{}_{\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & & -1 \\ & & \pm 1 & \\ & & & \mp 1 \\ 1 & & & \end{pmatrix}. \tag{4.12}$$

The gauge connections are obtained by identifying

$$A_{\mu}^{\alpha\beta} = (\omega + H)_{\mu}{}^{\alpha\beta}. \tag{4.13}$$

The result is

$$A_{\mu=1}^{\alpha\beta} = 0,$$

$$A_{\mu=2}^{\alpha\beta} = \frac{h'}{h} \begin{pmatrix} -s & & \\ & \frac{1}{2}s & \\ & & \frac{1}{2}s \end{pmatrix} = \frac{h'}{h} \left(-\frac{3\lambda_3 + \sqrt{3}\lambda_8}{4} \right) \otimes s,$$

$$A_{\mu=3}^{\alpha\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & -\mathbf{1} & \\ \mathbf{1} & & \\ & & \end{pmatrix} = \frac{h'}{2h^{\frac{3}{2}}} \left(-i\lambda_2 \right) \otimes \mathbf{1},$$

$$A_{\mu=4}^{\alpha\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & -s & \\ -s & & \\ & & \end{pmatrix} = \frac{h'}{2h^{\frac{3}{2}}} \left(-\lambda_1 \right) \otimes s,$$

$$A_{\mu=5}^{\alpha\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & -\mathbf{1} \\ & & \\ \mathbf{1} & & \end{pmatrix} = \frac{h'}{2h^{\frac{3}{2}}} \left(-i\lambda_5 \right) \otimes \mathbf{1},$$

$$A_{\mu=6}^{\alpha\beta} = \frac{h'}{2h^{\frac{3}{2}}} \begin{pmatrix} & & -s \\ & & \\ -s & & \end{pmatrix} = \frac{h'}{2h^{\frac{3}{2}}} \left(-\lambda_4 \right) \otimes s, \tag{4.14}$$

where λ_i's $(i = 1, \ldots, 8)$ are the Gell-Mann matrices and $\mathbf{1} \equiv \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$, $s \equiv i\sigma_2 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$.

The explicit expressions (4.12) show that both of $\omega_\pm$ are SU(3) connections ("SU(3) structure" [31]). As we did in section II for the symmetric 5-brane, we have embedded ω_+ into the gauge connection A. Then the Bianchi identity is reduced to $dH = 0$, and the solution (4.1) is consistent with it. This time a certain SU(3) piece of the $E_8(\times E_8)$ gauge connection is given a nonzero expectation value. On the other hand, the fact that $\omega_- \in$ SU(3) implies that the Killing spinor equations for the gravitino variation (2.22) as well as, as explained before, the gaugino variation (2.22) have a common single Killing spinor. It can be checked that this also satisfies the equation for the dilatino SUSY variation to lowest order:

$$\delta\lambda = \left(-\frac{1}{4}\Gamma^M\partial_M\phi + \frac{1}{24}\Gamma^{MNP}H_{MNP} \right)\epsilon = 0. \tag{4.15}$$

Thus the background (4.1) together with (4.14) preserve 1/4 of supersymmetries. It also satisfies the equations of motion (2.26) as it should.

4.4 Zero modes as Nambu-Goldstone modes on the intersecting 5-branes

In the previous subsection we have constructed a smeared solution which describes intersecting 5-branes in the $E_8 \times E_8$ heterotic string theory to leading order in α', via the standard embedding, similarly to the way we obtain the symmetric 5-brane. In that case, the connection ω_+ embedded was in SU(2), and the unbroken gauge symmetry was the centralizer E_7. In the present intersecting case, the connection embedded into E_8 is in SU(3), and therefore the unbroken gauge symmetry is E_6. The adjoint representation of E_8 is decomposed into

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{8}) \tag{4.16}$$
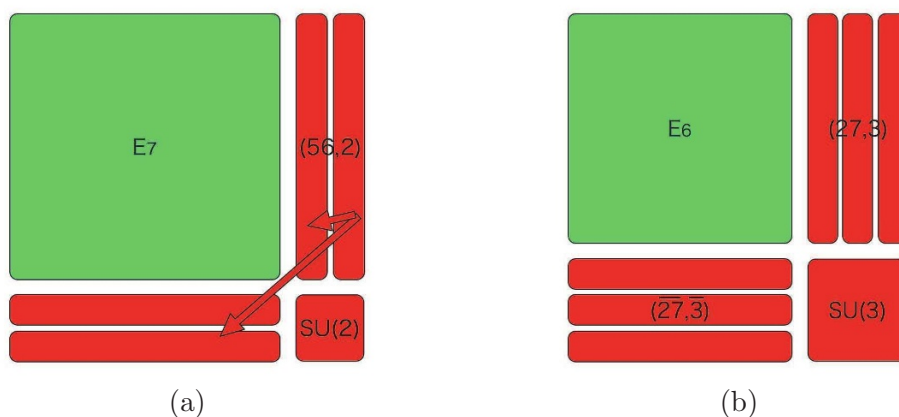
Figure 3. Broken generators which give rise to zero modes. (a) The single 5-brane case. (b) The intersecting case.

as representations of the subalgebra $E_6 \times \mathrm{SU}(3)$. Since the $E_8 \times E_8$ gauge field A_M has by construction a vev in $\mathrm{SU}(3)$, the latter three gauge rotations are the moduli (Figure 3).

Let us focus on the E_6 non-singlet moduli. As we saw in the symmetric 5-brane in the previous sections, spontaneously broken generators in $(\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}})$ give rise to Nambu-Goldstone bosons, each of which has one bosonic degree of freedom. On the other hand, since a $D = 4$, $\mathcal{N} = 1$ chiral supermultiplet needs *two* bosonic degrees of freedom, the Nambu-Goldstone bosons which transform as $\mathbf{27}$ and $\overline{\mathbf{27}}$ must be combined to form a single $\mathcal{N} = 1$ chiral supermultiplet. That is, the E_6 non-singlet moduli form *three* chiral supermultiplets in the $\mathbf{27}$ (or $\overline{\mathbf{27}}$, but not both) representation of E_6.

At first sight, one might think that the argument above would be contradictory to the well-known fact in Calabi-Yau compactifications that the number of chiral generations are determined by the Dirac index, in which the same decomposition (4.16) is used and *one triplet* of zero modes together corresponds to *one* supermultiplet, and is not counted as three. Of course, it is not a contradiction, because what we consider here is not the fermionic zero modes of the Dirac operator, but bosonic zero modes of the gauge fields. As we discussed in the previous sections, they are not removed by gauge transformations, and necessarily exist to cancel the anomaly inflow into each of the two intersecting 5-branes. Each of small gauge rotation generators in $(\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{8})$ is an independent generator and gives rise to an independent zero mode. We also recall that exactly the same way of counting was done in the parallel symmetric 5-brane case, and was indeed consistent with the index analysis [32].

However, it is premature to conclude that these three bosonic zero modes in the $(\mathbf{27}, \mathbf{3})$ representation imply three generations, because we have not yet examined the chiralities of their superpartners. We will do this in the next section. In fact, we will see that one of the three possesses the opposite chirality to that the other two have, and hence there is net one generation.

4.5 Explicit computation of chiral zero modes

The ten-dimensional heterotic gaugino equations of motion reads

$$\not{D}\left(\omega - \frac{1}{3}H, A\right)\chi - \Gamma^M \chi \partial_M \phi + \frac{1}{8}\Gamma^M \gamma^{AB}(F_{AB} + \hat{F}_{AB})\left(\psi_M + \frac{2}{3}\Gamma_M \lambda\right) = 0, \quad (4.17)$$

where

$$D\left(\omega - \frac{1}{3}H, A\right)\chi \equiv \left(\partial_M + \frac{1}{4}\left(\omega_M{}^{AB} - \frac{1}{3}H_M{}^{AB}\right)\Gamma_{AB} + \mathrm{ad}A_M\right)\chi. \quad (4.18)$$

χ is in the adjoint **248** representation of E_8, and $\mathrm{ad}A_M \cdot \chi \equiv [A_M, \chi]$. If $\psi_M = 0$ and $\lambda = 0$, it is simplified to

$$\not{D}\left(\omega - \frac{1}{3}H, A\right)\chi - \Gamma^M \chi \partial_M \phi = 0. \quad (4.19)$$

Further, if we set $\tilde{\chi} \equiv e^{-\phi}\chi$, then this is equivalent to [26]

$$\not{D}\left(\omega - \frac{1}{3}H, A\right)\tilde{\chi} = 0. \quad (4.20)$$

Since there are no nontrivial backgrounds for the four-dimensional $i = 0, 7, 8, 9$ directions,

$$\Gamma^i \partial_i \tilde{\chi} + \Gamma^\mu D_\mu \left(\omega - \frac{1}{3}H, A\right)\tilde{\chi} = 0. \quad (4.21)$$

If $\tilde{\chi} = \tilde{\chi}_{4D} \otimes \tilde{\chi}_{6D}$, the second term is regarded as the mass term for the four-dimensional spinor $\tilde{\chi}_{4D}$. We are interested in the zero modes of this Dirac operator $\Gamma^\mu D_\mu(\omega - \frac{1}{3}H, A)$.

The SO(6) gamma matrices in the chiral representation are

$$
\begin{aligned}
\gamma_1 &= \sigma_2 \otimes \mathbf{1} \otimes \mathbf{1}, \\
\gamma_2 &= \sigma_1 \otimes \sigma_1 \otimes \mathbf{1}, \\
\gamma_3 &= \sigma_1 \otimes \sigma_2 \otimes \mathbf{1}, \\
\gamma_4 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_1, \\
\gamma_5 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_2, \\
\gamma_6 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_3.
\end{aligned}
\quad (4.22)
$$

The six-dimensional chiral operator is

$$
\begin{aligned}
\gamma_\sharp &\equiv -i\gamma_1\gamma_2\cdots\gamma_6 \\
&= \sigma_3 \otimes \mathbf{1} \otimes \mathbf{1}.
\end{aligned}
\quad (4.23)
$$

For SO(9, 1) gamma matrices, we take

$$
\begin{aligned}
\Gamma^a &= \gamma_{4D}^a \otimes \mathbf{1}_8 \quad (a = 0, 7, 8, 9), \\
\Gamma^\alpha &= \gamma_{4D}^\sharp \otimes \gamma_\alpha \quad (\alpha = 1, \ldots, 6),
\end{aligned}
\quad (4.24)
$$

where γ_{4D}^a's ($a = 0, 7, 8, 9$) are the ordinary SO(3,1) gamma matrices in the chiral representation:

$$
\begin{aligned}
\gamma_{4D}^0 &= i\sigma_2 \otimes \mathbf{1}, \\
\gamma_{4D}^7 &= \sigma_1 \otimes \sigma_1, \\
\gamma_{4D}^8 &= \sigma_2 \otimes \sigma_2, \\
\gamma_{4D}^9 &= \sigma_3 \otimes \sigma_3, \\
\gamma_{4D}^\sharp &\equiv -i\gamma_{4D}^0 \gamma_{4D}^7 \gamma_{4D}^8 \gamma_{4D}^9 \\
&= \sigma_3 \otimes \mathbf{1}.
\end{aligned}
\tag{4.25}
$$

The ten-dimensional chirality is

$$
\begin{aligned}
\Gamma_{11} &\equiv -\Gamma^0 \Gamma^7 \Gamma^8 \Gamma^9 \cdot \Gamma^1 \cdots \Gamma^6 \\
&= \gamma_{4D}^\sharp \otimes \gamma_\sharp \\
&= (\sigma_3 \otimes \mathbf{1}) \otimes (\sigma_3 \otimes \mathbf{1} \otimes \mathbf{1}).
\end{aligned}
\tag{4.26}
$$

Now we consider the Dirac equation

$$
\Gamma^\mu D_\mu \left(\omega - \frac{1}{3}H, A \right) \tilde{\chi} = 0.
\tag{4.27}
$$

The 16-component SO(9,1) (Majorana-)Weyl spinor χ (or $\tilde{\chi}$) is decomposed in terms of SO(3,1) and SO(6) spinors as

$$
\mathbf{16} = (\mathbf{2}_+, \mathbf{4}_+) \oplus (\mathbf{2}_-, \mathbf{4}_-),
\tag{4.28}
$$

where the subscripts are the SO(3,1) and SO(6) chiralities, $\gamma_{4D}^\sharp$ and $\gamma_\sharp$, respectively. Since $\tilde{\chi}$ is Majorana (but complex in this representation), the $(\mathbf{2}_+, \mathbf{4}_+)$ and $(\mathbf{2}_-, \mathbf{4}_-)$ components are not independent but are transformed each other by a charge conjugation.

As $\Gamma^\mu D_\mu(\omega - \frac{1}{3}H, A)$ is SO(3,1) diagonal, it is enough to consider

$$
\gamma^\mu D_\mu \left(\omega - \frac{1}{3}H, A \right) \tilde{\chi}_{6D} = 0,
\tag{4.29}
$$

with the understanding that each component of $\tilde{\chi}_{6D}$ is accompanied by a two-component SO(3,1) Weyl spinor with a correlated chirality ($\gamma_{4D}^\sharp \gamma_\sharp = +1$).

On the other hand, we are interested in the gaugino zero modes in $(\mathbf{27}, \mathbf{3})$ or $(\overline{\mathbf{27}}, \overline{\mathbf{3}})$ in the decomposition $E_8 \supset E_6 \times \mathrm{SU}(3)$ of $\mathbf{248}$. The gauge connections A_M take only nonzero values in the SU(3) subalgebra, and we look for the zero modes $\tilde{\chi}_{6D}$ transforming as a triplet, either $\mathbf{3}$ or $\overline{\mathbf{3}}$, of SU(3).

Since γ^α's are in the form:

$$
\begin{aligned}
\gamma^1 &= \begin{pmatrix} & -i\mathbf{1} \otimes \mathbf{1} \\ -i\mathbf{1} \otimes \mathbf{1} & \end{pmatrix}, \\
\gamma^{\tilde{\alpha}} &= \begin{pmatrix} & \tilde{\gamma}^{\tilde{\alpha}} \\ \tilde{\gamma}^{\tilde{\alpha}} & \end{pmatrix} \quad (\tilde{\alpha} = 2, \ldots, 6),
\end{aligned}
\tag{4.30}
$$

and $\omega_\mu{}^{\alpha\beta}$, $H_\mu{}^{\alpha\beta}$ and $A_\mu{}^{\alpha\beta}$ all vanish if $\mu = 1$, (4.29) is reduced to two independent differential equations

$$\frac{i}{h}\frac{d}{dx^1}\tilde{\chi}_{6D}^+ + M^+\tilde{\chi}_{6D}^+ = 0, \tag{4.31}$$

$$\frac{i}{h}\frac{d}{dx^1}\tilde{\chi}_{6D}^- - M^-\tilde{\chi}_{6D}^- = 0, \tag{4.32}$$

where $\tilde{\chi}_{6D}^\pm$ is the upper and lower components having definite chiralities:

$$\tilde{\chi}_{6D} = \begin{pmatrix} \tilde{\chi}_{6D}^+ \\ \tilde{\chi}_{6D}^- \end{pmatrix}. \tag{4.33}$$

$\tilde{\chi}_{6D}^+$ ($\tilde{\chi}_{6D}^-$) is a **4** SO(6) Weyl spinor, and each of the four components is a triplet of SU(3). Thus M^+ (M^-) is a ($4 \times 3 =$) 12-by-12 matrix, given explicitly by

$$\begin{pmatrix} & M^- \\ M^+ & \end{pmatrix} \equiv \frac{h'}{h^2}\left(\begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{3i}{2} & -\frac{i}{4} & 0 & -\frac{i}{4} \\ 0 & 0 & 0 & 0 & -\frac{i}{4} & -\frac{3i}{2} & -\frac{i}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{i}{4} & -\frac{3i}{2} & -\frac{i}{4} \\ 0 & 0 & 0 & 0 & -\frac{i}{4} & 0 & -\frac{i}{4} & -\frac{3i}{2} \\ \frac{3i}{2} & -\frac{i}{4} & 0 & -\frac{i}{4} & 0 & 0 & 0 & 0 \\ -\frac{i}{4} & \frac{3i}{2} & -\frac{i}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{i}{4} & \frac{3i}{2} & -\frac{i}{4} & 0 & 0 & 0 & 0 \\ -\frac{i}{4} & 0 & -\frac{i}{4} & \frac{3i}{2} & 0 & 0 & 0 & 0 \end{pmatrix} \otimes \mathbf{1}_3 \right.$$
$$\left. + \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{s\lambda_4}{2} & \frac{-s\lambda_1-\lambda_5}{2} & \frac{-2\lambda_2-s\lambda_9}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{\lambda_5-s\lambda_1}{2} & \frac{s\lambda_4}{2} & 0 & \frac{-2\lambda_2-s\lambda_9}{4} \\ 0 & 0 & 0 & 0 & \frac{2\lambda_2-s\lambda_9}{4} & 0 & \frac{s\lambda_4}{2} & \frac{s\lambda_1+\lambda_5}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{2\lambda_2-s\lambda_9}{4} & \frac{s\lambda_1-\lambda_5}{2} & -\frac{s\lambda_4}{2} \\ -\frac{s\lambda_4}{2} & \frac{-s\lambda_1-\lambda_5}{2} & \frac{-2\lambda_2-s\lambda_9}{4} & 0 & 0 & 0 & 0 & 0 \\ \frac{\lambda_5-s\lambda_1}{2} & \frac{s\lambda_4}{2} & 0 & \frac{-2\lambda_2-s\lambda_9}{4} & 0 & 0 & 0 & 0 \\ \frac{2\lambda_2-s\lambda_9}{4} & 0 & \frac{s\lambda_4}{2} & \frac{s\lambda_1+\lambda_5}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{2\lambda_2-s\lambda_9}{4} & \frac{s\lambda_1-\lambda_5}{2} & -\frac{s\lambda_4}{2} & 0 & 0 & 0 & 0 \end{pmatrix}\right), \tag{4.34}$$

where $\lambda_9 \equiv 3\lambda_3 + \sqrt{3}\lambda_8$. In identifying the spin connection as an SU(3) gauge connection, $s = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$ can either be mapped to i, or to $-i$, and depending on this choice, the SU(3) gauge connection matrix becomes one in the **3** representation, or in the $\overline{\mathbf{3}}$ representation.

As we already mentioned, $\tilde{\chi}_{6D}^+$ and $\tilde{\chi}_{6D}^-$ are not independent; we have only to solve the equation (4.31), and the solutions to (4.32) may then be obtained by a charge conjugation. To solve (4.31), we diagonalize M^+ to obtain its eigenvalues. Let $i\lambda$ be an eigenvalue of the *constant* matrix $\left(\frac{h'}{h^2}\right)^{-1}M^+$, and $\psi_\lambda(x^1)$ be the corresponding eigenfunction, then they satisfy

$$\frac{i}{h}\psi_\lambda' + i\lambda\frac{h'}{h^2}\psi_\lambda = 0. \tag{4.35}$$

This is solved to give

$$\psi_\lambda(x^1) = \text{const.}(h(x^1))^{-\lambda}. \tag{4.36}$$

Thus, for each eigenvalue, there exists a zero mode of the Dirac operator. Since ξ is negative for positive tension, if $\lambda < 1$, the mode is localized near $x^1 = 0$, while if $\lambda \geq 1$, it is not localized, being either non-normalizable or localized rather at "infinity" $x^1 = \pm \frac{h_0}{|\xi|}$.

The list of eigenvalues of $\left(\frac{h'}{h^2}\right)^{-1} M^+$ is as follows: If $s = +i$, the eigenvalues are

$$\left\{ 2i, \frac{3i}{2}, \frac{3i}{2}, i, -i, i, i, i, \frac{3i}{2}, \frac{3i}{2}, \frac{7i}{2}, \frac{7i}{2} \right\}, \tag{4.37}$$

while if $s = -i$, they are

$$\left\{ 2i, \frac{3i}{2}, \frac{3i}{2}, i, 2i, 4i, 2i, 2i, -\frac{i}{2}, -\frac{i}{2}, \frac{3i}{2}, \frac{3i}{2} \right\}. \tag{4.38}$$

We can clearly see an asymmetry between (4.37) and (4.38), in particular that the former has only one negative (times imaginary unit) eigenvalue, while the latter has two negative eigenvalues. Assuming that the branes have positive tension so that the function $h(x)$ has the profile shown in figure 2, these are the only modes whose profiles have a peak at $x^1 = 0$ or $z = 0$ in the coordinate (4.10). The same is also true for the original gaugino variable $\chi = h\tilde{\chi}$ (although the modes with $\lambda = +1$ then become constant). This result implies that there are indeed three localized modes, and two of them are in one (say, $(\mathbf{27}, \mathbf{3})$) representation, and the rest belongs to the other ($(\overline{\mathbf{27}}, \overline{\mathbf{3}})$) representation.

5 Conclusions and discussion

In this paper, we have shown that there exist three localized zero modes as $D = 4$, $\mathcal{N} = 1$ supermultiplets on the system of two intersecting 5-branes in the $E_8 \times E_8$ heterotic string theory. By using the standard embedding in the known smeared solution, we have constructed a heterotic background and explicitly solved the Dirac equation on this background. We have found that two of them are in the $\mathbf{27}$ representation of E_6, and one in the $\overline{\mathbf{27}}$ representation. They give rise to net *one* $\mathbf{27}$ of massless chiral fermions in the four-dimensional spacetime. This is the first example of a brane set-up in heterotic string theory that supports, after compactifying some of the transverse dimensions, four-dimensional chiral matter fermions transforming as an E_6 gauge multiplet.[8]

Intuitively, the chirality flip of one of the three zero modes can be understood as follows: the further one goes away from the intersection to the x^3 or x^4 direction along one 5-brane, the less one feels the effect of the other brane, and in the end one would observe as if there were only a single symmetric 5-brane. The gauge connection then becomes smaller than SU(3), and approaches to SU(2). As we have seen in the previous sections, the zero modes on a single 5-brane are 30 six-dimensional supermultiplets, which are of course nonchiral as four-dimensional supermultiplets upon a dimensional reduction. They are regarded as two of the three columns and rows shown figure 3(b), and have opposite chiralities. Similarly, if one goes away from the intersection to the x^5 or x^6 direction, one will observe a reduction of the gauge connection from SU(3) to a different SU(2), and will

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[8]It would be interesting to study this set-up from the dual M-theory point of view.

see, again, a pair of nonchiral zero modes which correspond to different pair of columns and rows in figure 3(b). Therefore, since there are only three sets of zero modes, the chirality of one of them must be opposite to that of the other two.

It is worth mentioning that this chirality flip also agrees with the analysis of Kähler coset sigma models [44, 45]. In general, the dynamics of Nambu-Goldstone modes is described by a low-energy sigma model action constructed as a group coset associated with the corresponding spontaneously broken symmetries. In the $\mathcal{N} = 1$ supersymmetric case, the target space must be Kähler. $E_8/[E_6 \times \mathrm{SU}(3)]$ is not a Kähler coset; no wonder because this is not the moduli space of the intersecting 5-branes (since the adjoint of $\mathrm{SU}(3)$ also belongs to the moduli). On the other hand, there *are* Kähler cosets which contain three **27**'s of E_6. They are $E_8/[E_6 \times \mathrm{SU}(2) \times \mathrm{U}(1)]$ and $E_8/[E_6 \times \mathrm{U}(1)^2]$. It turns out that, in both cases, the chirality of one of three supermultiplets are opposite to the other two.[9] Although neither of them coincides exactly with the moduli space of the intersecting 5-branes, this is just what we have encountered in the present analysis and may be regarded at least as a suggestive fact.

It will be extremely interesting if this set-up could be used to realize the E_6 grand unification scenario [46, 47] by using branes [48–51] in string theory. For this purpose, we need to generalize it to a more realistic brane system which supports three generations. In principle, one could do this by replacing one of the single 5-brane with three 5-branes and consider the intersection with the other 5-brane. This is also suggested by the study of duality between the orbifolded or generalized conifold and a system of intersecting NS5-branes [52]. We hope to report on this issue in the near future.

Acknowledgments

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