

Variant supercurrent multiplets

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ABSTRACT: In $\mathcal{N} = 1$ rigid supersymmetric theories, there exist three standard realizations of the supercurrent multiplet corresponding to the (i) old minimal, (ii) new minimal and (iii) non-minimal off-shell formulations for $\mathcal{N} = 1$ supergravity. Recently, Komargodski and Seiberg in arXiv:1002.2228 put forward a new supercurrent and proved its consistency, although in the past it was believed not to exist. In this paper, three new variant supercurrent multiplets are proposed. Implications for supergravity-matter systems are discussed.

KEYWORDS: Supersymmetry and Duality, Supergravity Models, Supersymmetric Effective Theories

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Contents

1	Introduction	1
2	Minimal supercurrents	3
3	Reducible supercurrents	5
4	Discussion	7
A	Superprojectors	8

1 Introduction

The supercurrent multiplet [1] is a supermultiplet containing the energy-momentum tensor and the supersymmetry current, and therefore it is of primary importance in the context of supersymmetric field theories. In complete analogy with the energy-momentum tensor, it is fruitful to view the supercurrent as the source of supergravity [2–4]. Given a linearized off-shell formulation for $\mathcal{N} = 1$ supergravity, the supercurrent conservation equation can be obtained by coupling the supergravity prepotentials to external sources and then demanding the resulting action to be invariant under the linearized supergravity gauge transformations. One of the prepotentials is always the gravitational superfield $H^{\alpha\dot{\alpha}}$ [2] which couples to the supercurrent $J_{\alpha\dot{\alpha}}$. The gravitational superfield is accompanied by a superconformal compensator. The latter is not universal and depends on the supergravity formulation chosen. The source associated with the compensator is sometimes called a multiplet of anomalies, for its components include the trace of the energy momentum tensor and the γ -trace of the supersymmetry current.

In the literature, there exist three standard supercurrent multiplets which correspond to the (i) old minimal, (ii) new minimal and (iii) non-minimal off-shell formulations for $\mathcal{N} = 1$ supergravity (see, e.g., [5] for a review). The Ferrara-Zumino supercurrent [1] is the most well-known multiplet. It is characterized by the conservation equation

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(I)} = D_{\alpha} X, \quad \bar{D}_{\dot{\alpha}} X = 0, \tag{1.1}$$

and corresponds to the old minimal formulation for $\mathcal{N} = 1$ supergravity [6–8] in which the compensator is a chiral scalar σ [9]. On the other hand, the supercurrent corresponding to the new minimal supergravity [10, 11] obeys the conservation law

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(II)} = \chi_{\alpha}, \quad \bar{D}_{\dot{\alpha}} \chi_{\alpha} = 0, \quad D^{\alpha} \chi_{\alpha} = \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}. \tag{1.2}$$

This equation reflects, in particular, the fact that the new minimal compensator, G , is real linear [12–14]. The constraint on G is solved [15] by introducing a chiral spinor potential ψ_α , by the rule

$$G = D^\alpha \psi_\alpha + \bar{D}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}} \psi_\beta = 0. \quad (1.3)$$

It is defined modulo gauge transformations of the form:

$$\delta \psi_\alpha = i \bar{D}^2 D_\alpha K, \quad K = \bar{K}. \quad (1.4)$$

The last equation in (1.2) is simply the manifestation of this gauge symmetry. Finally, the supercurrent for the non-minimal supergravity [16, 17] is discussed briefly in [5], and it is also reviewed, in a form that differs slightly from that given in [5], at the end of section 3.

Recently Komargodski and Seiberg [18], motivated by earlier discussions of the supercurrent multiplets in theories with Fayet-Iliopoulos terms [19–21], introduced a new supercurrent with the following conservation law

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(IV)} = \chi_\alpha + D_\alpha X, \quad \bar{D}_{\dot{\alpha}} \chi_\alpha = \bar{D}_{\dot{\alpha}} X = 0, \quad D^\alpha \chi_\alpha = \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}}. \quad (1.5)$$

As pointed out in [18], such a supercurrent had been considered in the past in ref. [22] where it had been ruled out as not having a conserved energy-momentum tensor. The conclusion of [22] was shown in [18] to be incorrect by explicit component calculations. In fact, the consistency of eq. (1.5) follows from the earlier analysis of supercurrents given in [21] (see the discussion in section 3 below).

In this note, three new variant supercurrent multiplets are proposed. Our consideration is based on the results of [23] where a classification of off-shell (3/2, 2) supermultiplets, or linearized supergravity models, is given. Such models are described in terms of the gravitational superfield $H_{\underline{a}} := H_{\alpha\dot{\alpha}}$ and some compensator(s). The latter may occur in one of the following disguises: (i) a chiral scalar σ , $\bar{D}_{\dot{\alpha}} \sigma = 0$; (ii) a real linear superfield G , $\bar{G} - G = \bar{D}^2 G = 0$, which is the gauge-invariant field strength of a chiral spinor potential, eq. (1.3); (iii) another real linear superfield

$$F = D^\alpha \rho_\alpha + \bar{D}_{\dot{\alpha}} \bar{\rho}^{\dot{\alpha}}, \quad \bar{D}_{\dot{\alpha}} \rho_\beta = 0, \quad (1.6)$$

possessing a different supergravity transformation law; (iv) a combination of such compensators (say, a complex linear compensator Γ , which emerges in non-minimal supergravity, can be represented as $\Gamma = \sigma + G + iF$). The linearized supergravity transformations are:

$$\delta H_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} L_\alpha - D_\alpha \bar{L}_{\dot{\alpha}}, \quad (1.7a)$$

$$\delta \sigma = -\frac{1}{12} \bar{D}^2 D^\alpha L_\alpha, \quad (1.7b)$$

$$\delta G = \frac{1}{4} (D^\alpha \bar{D}^2 L_\alpha + \bar{D}_{\dot{\alpha}} D^2 \bar{L}^{\dot{\alpha}}) \implies \delta \psi_\alpha = \frac{1}{4} \bar{D}^2 L_\alpha, \quad (1.7c)$$

$$\delta F = \frac{i}{12} (D^\alpha \bar{D}^2 L_\alpha - \bar{D}_{\dot{\alpha}} D^2 \bar{L}^{\dot{\alpha}}) \implies \delta \rho_\alpha = \frac{i}{12} \bar{D}^2 L_\alpha. \quad (1.7d)$$

Here the gauge parameter L_α is an unconstrained spinor superfield. The analysis carried out in [23] results in the following different models for linearized supergravity: (i) three minimal

realizations with 12 + 12 off-shell degrees of freedom; (ii) three reducible realizations with 16 + 16 components; (iii) one non-minimal formulation with 20 + 20 components. These seven supergravity models lead to different supercurrents. We discuss most of these models and associated supercurrents, with the exception of the non-minimal case for which we do not have anything new to say. It is useful to formulate the linearized supergravity actions in terms of special $\mathcal{N} = 1$ superprojectors [24, 25]; all relevant information about these superprojectors is collected in the appendix.

2 Minimal supercurrents

It is natural to begin our analysis by considering the supercurrents corresponding to the three minimal formulations with 12 + 12 off-shell degrees of freedom [23].

The linearized action for old minimal supergravity is well-known (see, e.g., [5]) and has the form:

$$\mathcal{S}^{(I)} = \int d^8z \left\{ H^{\underline{a}\square} \left(-\frac{1}{3}\Pi_0^L + \frac{1}{2}\Pi_{3/2}^T \right) H_{\underline{a}} - i(\sigma - \bar{\sigma})\partial_{\underline{a}}H^{\underline{a}} - 3\sigma\bar{\sigma} \right\}. \quad (2.1)$$

We introduce couplings to external sources,

$$\mathcal{S}^{(I)} \longrightarrow \mathcal{S}^{(I)} - \frac{1}{2} \int d^8z H^{\alpha\dot{\alpha}} J_{\alpha\dot{\alpha}} - \frac{3}{2} \left\{ \int d^6z \sigma X + \text{c.c.} \right\}, \quad (2.2)$$

and require invariance under the transformations (1.7a) and (1.7b). Then, it is a two-line calculation to show that $J_{\alpha\dot{\alpha}}$ and X have to obey the equation (1.1).

Given a chiral scalar Ξ , the supercurrent and the multiplet of anomalies can be transformed as

$$\delta J_{\alpha\dot{\alpha}} = \frac{1}{2}[D_{\alpha}, \bar{D}_{\dot{\alpha}}](\Xi + \bar{\Xi}) = i\partial_{\alpha\dot{\alpha}}(\Xi - \bar{\Xi}), \quad \delta X = \frac{1}{4}\Xi, \quad \bar{D}_{\dot{\alpha}}\Xi = 0 \quad (2.3)$$

without changing the conservation equation (1.1). At the nonlinear supergravity level, such an improvement corresponds (see, e.g., [26]) to the possibility of adding to the action a ‘non-minimal’ term of the form

$$\int d^8z E^{-1}(\Xi + \bar{\Xi}), \quad \bar{D}_{\dot{\alpha}}\Xi = 0, \quad (2.4)$$

which is a generalization of $R\varphi^2$ in field theory in curved space.

Next, consider the linearized action for new minimal supergravity (see, e.g., [26])

$$\mathcal{S}^{(II)} = \int d^8z \left\{ H^{\underline{a}\square} \left(-\Pi_{1/2}^T + \frac{1}{2}\Pi_{3/2}^T \right) H_{\underline{a}} + \frac{1}{2}G[D_{\alpha}, \bar{D}_{\dot{\alpha}}]H^{\underline{a}} + \frac{3}{2}G^2 \right\}, \quad (2.5)$$

where the real linear compensator G should be represented in the form (1.3) implying gauge invariance (1.4). Coupling it to external sources and imposing invariance under the gauge transformations (1.4), (1.7a) and (1.7c) leads to the supercurrent (1.2).

As is well known (see, e.g., [18] for a recent discussion), there exists a natural ambiguity in the definition of $J_{\alpha\dot{\alpha}}^{(\text{II})}$ and χ_α . Given a U(1) *current superfield*, J , which is real linear and contains a conserved vector among its components, the transformation

$$\delta J_{\alpha\dot{\alpha}}^{(\text{II})} = [D_\alpha, \bar{D}_{\dot{\alpha}}]J, \quad \delta\chi_\alpha = \frac{3}{2}\bar{D}^2 D_\alpha J, \quad J - \bar{J} = \bar{D}^2 J = 0 \quad (2.6)$$

preserves the conservation equation (1.2).

The supercurrent (1.2) can be related to the Ferrara-Zumino one, eq. (1.1), if the chiral spinor χ^α can be represented as

$$\chi_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad \bar{V} = V, \quad (2.7)$$

for some well-defined real scalar V . Then we can introduce

$$J_{\alpha\dot{\alpha}}^{(\text{I})} := J_{\alpha\dot{\alpha}}^{(\text{II})} + \frac{1}{6}[D_\alpha, \bar{D}_{\dot{\alpha}}]V, \quad X := -\frac{1}{12}\bar{D}^2 V. \quad (2.8)$$

It is easy to see that $J_{\alpha\dot{\alpha}}^{(\text{I})}$ and X obey the conservation equation (1.1).

There exists one more minimal 12/12 formulation for linearized supergravity, which was proposed a few years ago [27]. The corresponding action is

$$\mathcal{S}^{(\text{III})} = \int d^8z \left\{ H^{\underline{a}} \square \left(\frac{1}{3}\Pi_{1/2}^L + \frac{1}{2}\Pi_{3/2}^T \right) H_{\underline{a}} + F \partial_{\underline{a}} H^{\underline{a}} + \frac{3}{2}F^2 \right\}. \quad (2.9)$$

Here F is a real linear superfield that should be treated, similarly to G , as the gauge invariant field strength of a chiral spinor superfield, eq. (1.6). Coupling this model to external sources and imposing invariance under the gauge transformations (1.7a) and (1.7d), one derives a new supercurrent characterized by the conservation equation:

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(\text{III})} = i\eta_\alpha, \quad \bar{D}_{\dot{\alpha}} \eta_\alpha = 0, \quad D^\alpha \eta_\alpha = \bar{D}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}}. \quad (2.10)$$

Here the last equation expresses the fact that the chiral spinor potential associated with F must appear in the action only via the gauge invariant field strength F .

In complete analogy with the new minimal supercurrent, there is a natural ambiguity in the definition of $J_{\alpha\dot{\alpha}}^{(\text{III})}$ and η_α . Given a U(1) *current superfield* \mathbb{J} , i.e. a real linear superfield, the transformation

$$\delta J_{\alpha\dot{\alpha}}^{(\text{III})} = \partial_{\alpha\dot{\alpha}} \mathbb{J}, \quad \delta\eta_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \mathbb{J}, \quad \mathbb{J} - \bar{\mathbb{J}} = \bar{D}^2 \mathbb{J} = 0 \quad (2.11)$$

preserves the conservation equation (2.10).

The supercurrent (2.10) can be related to the Ferrara-Zumino one, eq. (1.1), if η_α can be represented in the form:

$$\eta_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \mathbb{V}, \quad \bar{\mathbb{V}} = \mathbb{V}, \quad (2.12)$$

for some well defined real scalar superfield \mathbb{V} . If we now define

$$J_{\alpha\dot{\alpha}}^{(\text{I})} := J_{\alpha\dot{\alpha}}^{(\text{III})} - \partial_{\alpha\dot{\alpha}} \mathbb{V}, \quad X := -\frac{i}{4}\bar{D}^2 \mathbb{V}, \quad (2.13)$$

then $J_{\alpha\dot{\alpha}}^{(\text{I})}$ and X obey the conservation equation (1.1).

It should be pointed out that the linearized supergravity models (2.1), (2.5) and (2.9) are dually equivalent [23].

3 Reducible supercurrents

Let us turn to the derivation of supercurrents corresponding to the three models with $16 + 16$ off-shell degrees of freedom [23]. As demonstrated in [23], such theories appear to look like a sum of two of the three minimal models discussed in the previous section. Some of these models are linearized versions of 16/16 supergravity [28, 29] which is known to have no fundamental significance — it is just 12/12 supergravity coupled to matter [30].

Consider the type-IV model [23]

$$\begin{aligned} \mathcal{S}^{(IV)} = \int d^8z \left\{ H^a \square \left[+ 8 \left(\alpha - \frac{1}{16} \right) \Pi_0^L - 24 \left(\alpha - \frac{1}{48} \right) \Pi_{1/2}^T + \frac{1}{2} \Pi_{3/2}^T \right] H_a \right. \\ \left. - 12 \left[\left(\alpha - \frac{1}{16} \right) (\sigma + \bar{\sigma}) - \left(\alpha - \frac{1}{48} \right) G \right] [D_\alpha, \bar{D}_{\dot{\alpha}}] H^a \right. \\ \left. + 72 \left(\alpha - \frac{1}{16} \right) \sigma \bar{\sigma} + 36 \left(\alpha - \frac{1}{48} \right) G^2 \right\}, \end{aligned} \quad (3.1)$$

with $\alpha \neq \frac{1}{16}, \frac{1}{48}$ a real parameter. This action is invariant under the gauge transformations (1.7a), (1.7b) and (1.7c). If one adds source terms to $\mathcal{S}^{(IV)}$ for all the prepotentials $H^{\alpha\dot{\alpha}}$, σ and ψ_α and demands invariance under the gauge transformations, one immediately arrives at the conservation equation (1.5).

The operator appearing in the first line of (3.1) can be rewritten as

$$8 \left(\alpha - \frac{1}{16} \right) \Pi_0^L - 24 \left(\alpha - \frac{1}{48} \right) \Pi_{1/2}^T = \left(\alpha - \frac{1}{48} \right) \left\{ 8 \Pi_0^L - 24 \Pi_{1/2}^T \right\} - \frac{1}{3} \Pi_0^L. \quad (3.2)$$

Using this representation in conjunction with eq. (A.3c), and also setting $\sigma = 0$ in (3.1), one immediately arrives at the linearized action derived in subsection 5.2 of [18].

As shown in [18], there is a freedom in the definition of the triple $(J_{\alpha\dot{\alpha}}^{(IV)}, \chi_\alpha, X)$ appearing in the conservation equation (1.5). Given a real scalar $U = \bar{U}$, the improvement transformation

$$\delta J_{\alpha\dot{\alpha}}^{(IV)} = [D_\alpha, \bar{D}_{\dot{\alpha}}] U, \quad \delta \chi_\alpha = \frac{3}{2} \bar{D}^2 D_\alpha U, \quad \delta X = \frac{1}{2} \bar{D}^2 U \quad (3.3)$$

preserves the defining relations (1.5).

It may happen that applying a finite transformation (3.3) results in $\chi_\alpha = 0$ or $X = 0$, and thus the transformed supercurrent is type-I or type-II, respectively. This is exactly what happens in the case of the free vector multiplet model with a Fayet-Iliopoulos term studied in [19–21].¹ The type-I supercurrent for this model [19]

$$J_{\alpha\dot{\alpha}}^{(I)} = 2W_\alpha \bar{W}_{\dot{\alpha}} + \frac{2}{3} \xi [D_\alpha, \bar{D}_{\dot{\alpha}}] V, \quad X = \frac{1}{3} \xi \bar{D}^2 V \quad (3.4)$$

is not gauge invariant, unlike the type-II supercurrent [20, 21]

$$J_{\alpha\dot{\alpha}}^{(II)} = 2W_\alpha \bar{W}_{\dot{\alpha}}, \quad \chi_\alpha = 4\xi W_\alpha. \quad (3.5)$$

¹In the first version of [20], it was claimed that “no supercurrent supermultiplet exists for globally supersymmetric gauge theories with non-zero Fayet-Iliopoulos terms.” This assertion was shown in [21] to be erroneous. A correct analysis was presented in a revised version of [20].

Here $W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V$ is the chiral field strength of a vector multiplet described by the gauge prepotential V . The pairs $(J_{\alpha\dot{\alpha}}^{(I)}, X)$ and $(J_{\alpha\dot{\alpha}}^{(II)}, \chi_\alpha)$ given are related to each other through a special finite transformation (3.3) with $U \propto \xi V$. Applying instead the same finite transformation but with a different overall coefficient leads to a type-IV supercurrent. Specifically, one can consider the following one-parameter family of supercurrents:

$$J_{\alpha\dot{\alpha}}^{(IV)} = 2W_\alpha \bar{W}_{\dot{\alpha}} + 2k\xi[D_\alpha, \bar{D}_{\dot{\alpha}}]V, \quad \chi_\alpha^{(IV)} = 4(1 - 3\kappa)\xi W_\alpha, \quad X^{(IV)} = \kappa\xi\bar{D}^2 V, \quad (3.6)$$

with κ a numerical coefficient. The supercurrents (3.4) and (3.5) correspond to the choices $\kappa = 1/3$ and $\kappa = 0$, respectively. Transformation (3.6) was essentially behind the analysis in [21].

Let us turn to the type-V model [23]

$$\begin{aligned} \mathcal{S}^{(V)} = \int d^8z \left\{ H^a \square \left[-2\left(\beta - \frac{1}{12}\right)\Pi_0^L - 2\left(\beta - \frac{1}{4}\right)\Pi_{1/2}^L + \frac{1}{2}\Pi_{3/2}^T \right] H_a \right. \\ \left. - 6 \left[i\left(\beta - \frac{1}{12}\right)(\sigma - \bar{\sigma}) + \left(\beta - \frac{1}{4}\right)F \right] \partial_a H^a \right. \\ \left. - 18\left(\beta - \frac{1}{12}\right)\sigma\bar{\sigma} - 9\left(\beta - \frac{1}{4}\right)F^2 \right\}, \end{aligned} \quad (3.7)$$

with $\beta \neq \frac{1}{4}, \frac{1}{12}$ a real parameter. This action is invariant under the gauge transformations (1.7a), (1.7b) and (1.7d). It leads to the supercurrent equation

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(V)} = i\eta_\alpha + D_\alpha X, \quad \bar{D}_{\dot{\alpha}} \eta_\alpha = \bar{D}_{\dot{\alpha}} X = 0, \quad D^\alpha \eta_\alpha = \bar{D}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}}. \quad (3.8)$$

Similarly to the situation of the type-IV supercurrent, there is a freedom in the definition of the triple $(J_{\alpha\dot{\alpha}}^{(V)}, \eta_\alpha, X)$ appearing in the conservation equation (1.5). Given a real scalar $\mathbb{U} = \bar{\mathbb{U}}$, the transformation

$$\delta J_{\alpha\dot{\alpha}}^{(V)} = \partial_{\alpha\dot{\alpha}} \mathbb{U}, \quad \delta \eta_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \mathbb{U}, \quad \delta X = \frac{i}{4}\bar{D}^2 \mathbb{U} \quad (3.9)$$

preserves the conservation equation (3.8).

It remains to consider the type-VI model [23]

$$\begin{aligned} \mathcal{S}^{(VI)} = \int d^8z \left\{ H^a \square \left[-2\left(\gamma - \frac{1}{4}\right)\Pi_{1/2}^L - 6\left(\gamma - \frac{1}{12}\right)\Pi_{1/2}^T + \frac{1}{2}\Pi_{3/2}^T \right] H_a \right. \\ \left. + 3\left(\gamma - \frac{1}{12}\right)G[D_\alpha, \bar{D}_{\dot{\alpha}}]H^a - 6\left(\gamma - \frac{1}{4}\right)F\partial_a H^a \right. \\ \left. + 9\left(\gamma - \frac{1}{12}\right)G^2 - 9\left(\gamma - \frac{1}{4}\right)F^2 \right\}, \end{aligned} \quad (3.10)$$

for a real parameter $\gamma \neq \frac{1}{4}, \frac{1}{12}$. This action is invariant under the gauge transformations (1.7a), (1.7c) and (1.7d). It leads to the conservation law

$$\begin{aligned} \bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(VI)} = \chi_\alpha + i\eta_\alpha, \quad \bar{D}_{\dot{\alpha}} \chi_\alpha = \bar{D}_{\dot{\alpha}} \eta_\alpha = 0, \\ D^\alpha \chi_\alpha - \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = D^\alpha \eta_\alpha - \bar{D}_{\dot{\alpha}} \bar{\eta}^{\dot{\alpha}} = 0. \end{aligned} \quad (3.11)$$

This supercurrent can be related to the Ferrara-Zumino one, eq. (1.1), if the chiral spinors χ_α and η_α are represented as U(1) field strengths²

$$\chi_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V, \quad \bar{V} = V, \quad (3.12a)$$

$$\eta_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha \mathbb{V}, \quad \bar{\mathbb{V}} = \mathbb{V}, \quad (3.12b)$$

for some well-defined real scalars V and \mathbb{V} . Then we can introduce

$$J_{\alpha\dot{\alpha}}^{(I)} := J_{\alpha\dot{\alpha}}^{(VI)} + \frac{1}{6}[D_\alpha, \bar{D}_{\dot{\alpha}}]V - \partial_{\alpha\dot{\alpha}}\mathbb{V}, \quad X := -\frac{1}{12}\bar{D}^2(V + 3i\mathbb{V}). \quad (3.13)$$

It is easy to see that $J_{\alpha\dot{\alpha}}^{(I)}$ and X obey the conservation equation (1.1).

The conservation law (3.11) is, in fact, related to that corresponding to the supercurrent in the non-minimal supergravity (see, e.g., [5]). The latter is

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}}^{(VII)} = -\frac{1}{4}\bar{D}^2 \zeta_\alpha - \frac{1}{4} \frac{n+1}{3n+1} D_\alpha \bar{D}_{\dot{\beta}} \bar{\zeta}^{\dot{\beta}}, \quad D_{(\alpha} \zeta_{\beta)} = 0, \quad (3.14)$$

n is a real parameter, $n \neq -1/3, 0$.³ Setting here $n = -1$ leads to (3.11).

The models (3.1), (3.7) and (3.10) are equivalent, since they are related to each other by superfield duality transformations given in [23]. The real linear superfields G and F can be dualized into a chiral scalar and its conjugate. After doing so, one will end up with a sum of the old minimal action and that for a free chiral scalar (the latter being decoupled from the supergravity prepotentials).

4 Discussion

In this paper we considered six different realizations for the supercurrent multiplet. All of them are consistent, that is contain a conserved energy-momentum tensor and a supersymmetry current. This follows from the fact that all the multiplets were read off from the actions invariant under linearized supergravity transformations, eqs. (1.7a)–(1.7d), generated by an unconstrained parameter $L_\alpha(x, \theta, \bar{\theta})$; the linearized general coordinate and local supersymmetry transformations are part of the gauge freedom. In other words, there is no need to carry out a component analysis of the supercurrent in order to check that the energy-momentum tensor and the supersymmetry current are conserved.

The type-III supergravity formulation, eq. (2.9), possesses quite interesting properties [23]. However, its extension beyond the linearized approximation is not known. This means that the supercurrent multiplets $(J_{\alpha\dot{\alpha}}^{(III)}, \eta_\alpha)$, $(J_{\alpha\dot{\alpha}}^{(V)}, \eta_\alpha, X)$ and $(J_{\alpha\dot{\alpha}}^{(VI)}, \eta_\alpha, \chi_\alpha)$ are of purely academic interest, at least at present.

Komargodski and Seiberg [18] demonstrated that there exist interesting supersymmetric theories⁴ for which the Ferrara-Zumino supercurrent (1.1) is not well defined. They

²Given an unconstrained chiral spinor λ_α , $\bar{D}_{\dot{\beta}} \lambda_\alpha = 0$, it can be represented in the form $\lambda_\alpha = \chi_\alpha + i\eta_\alpha$, where χ_α and η_α are given by eqs. (3.12a) and (3.12b), respectively.

³The constraint on ζ_α in (3.14) is solved by $\zeta_\alpha = D_\alpha W$, for some complex superfield W which is not always a well-defined local operator.

⁴Such theories include (i) $\mathcal{N} = 1$ nonlinear sigma-models with a non-exact Kähler form; (ii) models with Fayet-Iliopoulos terms.

also showed that the type-IV supercurrent, eq. (1.5), always exists. Does that mean that it is necessary to develop an off-shell supergravity formulation that automatically leads to the type-IV supercurrent? In our opinion, the answer is *no*. It is well known that any $\mathcal{N} = 1$ supergravity-matter system (including the new minimal and non-minimal supergravity theories) can be realized as a coupling of the old minimal supergravity to matter [26, 31]. Keeping in mind this general result, and the fact that the supercurrent multiplet is the source of supergravity, it is more appropriate to re-formulate the conclusion of [18] in a more positive form: the Ferrara-Zumino supercurrent (1.1) always exists, modulo an improvement transformation of the form:

$$J_{\alpha\dot{\alpha}}^{(I)} \rightarrow J_{\alpha\dot{\alpha}}^{(I)} + [D_\alpha, \bar{D}_{\dot{\alpha}}]U, \quad X \rightarrow X + \frac{1}{2}\bar{D}^2U, \quad \bar{U} = U. \quad (4.1)$$

Such an improvement results in the conservation law (1.5), in which $\chi_\alpha = \frac{3}{2}\bar{D}^2D_\alpha U$.

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A Superprojectors

The gravitational superfield can be represented as a superposition of SUSY irreducible components,

$$H_{\underline{a}} = \left(\Pi_0^L + \Pi_{1/2}^L + \Pi_1^T + \Pi_{1/2}^T + \Pi_{3/2}^T \right) H_{\underline{a}}, \quad (A.1)$$

by making use of the relevant superprojectors [23, 25]

$$\Pi_0^L H_{\underline{a}} = -\frac{1}{32}\square^{-2}\partial_{\underline{a}}\{D^2, \bar{D}^2\}\partial_{\underline{c}}H^{\underline{c}}, \quad (A.2a)$$

$$\Pi_{1/2}^L H_{\underline{a}} = \frac{1}{16}\square^{-2}\partial_{\underline{a}}D^\delta\bar{D}^2D_\delta\partial_{\underline{c}}H^{\underline{c}}, \quad (A.2b)$$

$$\Pi_{1/2}^T H_{\underline{a}} = \frac{1}{3!8}\square^{-2}\partial_{\dot{\alpha}}^\beta[D_\beta\bar{D}^2D^\delta\partial_{(\alpha}^{\dot{\beta}}H_{\delta)\dot{\beta}} + D_\alpha\bar{D}^2D^\delta\partial_{(\beta}^{\dot{\beta}}H_{\delta)\dot{\beta}}], \quad (A.2c)$$

$$\Pi_1^T H_{\underline{a}} = \frac{1}{32}\square^{-2}\partial_{\dot{\alpha}}^\beta\{D^2, \bar{D}^2\}\partial_{(\alpha}^{\dot{\beta}}H_{\beta)\dot{\beta}}, \quad (A.2d)$$

$$\Pi_{3/2}^T H_{\underline{a}} = -\frac{1}{3!8}\square^{-2}\partial_{\dot{\alpha}}^\beta D^\gamma\bar{D}^2D_{(\gamma}\partial_{\alpha}^{\dot{\beta}}H_{\beta)\dot{\beta}}. \quad (A.2e)$$

Here the superscripts L and T denote longitudinal and transverse projectors, while the subscripts 0, 1/2, 1, 3/2 stand for superspin. One can readily express the action in terms of the superprojectors. It is a D-algebra exercise to show

$$D^\gamma\bar{D}^2D_\gamma H_{\underline{a}} = -8\square(\Pi_{1/2}^L + \Pi_{1/2}^T + \Pi_{3/2}^T)H_{\underline{a}}, \quad (A.3a)$$

$$\partial_{\underline{a}}\partial^{\underline{b}}H_{\underline{b}} = -2\square(\Pi_0^L + \Pi_{1/2}^L)H_{\underline{a}}, \quad (A.3b)$$

$$[D_\alpha, \bar{D}_{\dot{\alpha}}][D_\beta, \bar{D}_{\dot{\beta}}]H^{\underline{b}} = +\square(8\Pi_0^L - 24\Pi_{1/2}^T)H_{\underline{a}}. \quad (A.3c)$$

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