Drag force in a hot non-relativistic, non-commutative Yang-Mills plasma

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ABSTRACT: We apply the standard technique of Null Melvin Twist to the non-extremal (D1, D3) bound state configuration of type IIB string theory. Under a particular decoupling limit, such configuration represents the gravity dual of the non-relativistic, non-commutative Yang-Mills theory at a finite temperature. We then use the AdS/CFT and the string probe approach to compute the drag force on an external quark moving through such a hot non-relativistic, non-commutative YM plasma. We discuss various limiting cases to show the interplay between the non-relativistic as well as the non-commutative effect of the general drag force expression.

KEYWORDS: Brane Dynamics in Gauge Theories, D-branes, AdS-CFT Correspondence

Contents

1 Introduction

The AdS/CFT correspondence (or its generalizations) [1–4] holographically relates weakly coupled string theory in a particular background to a strongly coupled ('t Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N \gg 1$) relativistic conformal field theory (or gauge theory). This is useful as it gives a computational handle on the otherwise hard to access strongly coupled gauge theory from supergravity. Indeed, such a strong-weak duality has been, in recent years, proved to be instrumental for a better understanding of various transport properties of certain strongly coupled systems like quark gluon plasma (QGP) produced in heavy ion collision (for reviews see [5–9]). Apart from nuclear physics strongly coupled CFT's also appear in atomic and condensed matter physics and more recently the holographic ideas have been applied to these systems as well. For example, peculiar strong coupling behavior like quantum Hall effect, Nernst effect, high temperature superconductors, and quantum phase transitions in certain strongly correlated electron systems can be understood at least qualitatively by using the holographic dual descriptions involving gravity [10–18].

In the examples mentioned above the CFT's were mainly of relativistic in nature. However, for the application to most condensed matter systems, it is useful to find holographic descriptions of CFT's which are non-relativistic [19–21]. These systems sometimes can be produced in the laboratory and indeed there exist such a strongly coupled non-relativistic system, namely, the cold fermions at unitarity (for review see [22, 23]) which can be understood using gravity/NRCFT correspondence, if the proper gravity dual for this system can be found [24–27]. Motivated by the possible realizations of strongly coupled CFT's in the laboratory there have been attempts to construct the gravity duals in the form of non-relativistic branes in string theory [28]. The near horizon geometry of these branes will have the isometry same as the non-relativistic conformal (Schrodinger) symmetry of the boundary theory and using the gravity/NRCFT correspondence one can get a handle on the non-relativistic strongly coupled CFT from the weakly coupled string theory or gravity.

In this paper we construct the non-relativistic non-extremal (D1, D3) bound state solution of type IIB string theory. We will use the standard procedure of Null Melvin Twist [26–29] to construct such a solution. A particular low energy limit, known as the

decoupling limit, of a stack of coincident (D1, D3) brane bound state system gives rise to a non-commutative Yang-Mills (NCYM) theory on the boundary [30–32]. It is known that the D1-branes in the world-volume of D3-branes in the decoupling limit produces a large magnetic or B-field asymptotically and this is the source of a space-space non-commutativity in the world-volume directions of D3-brane [30]. The same decoupling limit for the stack of coincident non-relativistic (D1, D3) bound state system will give rise to a non-relativistic, NCYM theory on the boundary. We will compute the drag force [5, 33] experienced by an external quark moving through this background of hot non-relativistic, NCYM plasma. In this picture an external quark is represented by the end point of a fundamental string attached to the boundary carrying a fundamental charge under a gauge group and is infinitely massive [34–37]. The external quark loses its energy as the string attached to it trails back and imparts a drag force on it. We will compute this drag force when the quark moves along one of the non-commutative directions for a sufficiently long time. We find that when the boundary theory is both non-relativistic and non-commutative, it is difficult to write the expression of the drag force in a closed form. So, we will get the expression in various limiting cases to show the interplay of the non-relativistic and non-commutative effect. When the parameter characterizing the non-commutativity is small, we find that there is no upper bound for the velocity. On the other hand, when the non-commutativity parameter is large the velocity of the quark can not be arbitrarily large in contrast to what is expected of a non-relativistic theory. We will express the drag force in terms of the parameters of the YM theory, namely, the 't Hooft parameter, the temperature, the chemical potential and the non-commutativity parameter in the various limiting cases. Finally, we will formally integrate the drag force expression to compute the momentum or energy loss [33] of the quark moving in the hot non-relativistic NCYM theory.

This paper is organized as follows. In the next section we discuss the Null Melvin Twist on the non-extremal (D1, D3) bound state system of type IIB string theory and also the decoupling limit. In section 3, we calculate the drag force on a heavy quark moving through the hot non-relativistic NCYM plasma and discuss the various limits to understand the general drag force expression in the various corners of the solution space. We conclude in section 4.

2 Null Melvin twist on non-extremal (D1, D3) bound state solution

The non-extremal (D1, D3) bound state configuration of type IIB string theory is given as [38–40],

$$ds^2 = H^{-\frac{1}{2}} \left[-f(dx^0)^2 + (dx^1)^2 + \left(\frac{H}{F} \right) \left((dx^2)^2 + (dx^3)^2 \right) \right] + H^{\frac{1}{2}} \left[f^{-1} dr^2 + r^2 d\Omega_5^2 \right]$$

$$e^{2\phi} = g_s^2 \frac{H}{F}$$

$$B_{[2]} = \tan\theta F^{-1}dx^2 \wedge dx^3, \qquad A_{[2]} = -\frac{1}{g_s}\sin\theta\coth\varphi F^{-1}dx^0 \wedge dx^1$$

$$F_{[5]} = -\frac{1}{g_s}\cos\theta\coth\varphi\left(\frac{H}{F}\right)\partial_r H^{-1}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dr \qquad (2.1)$$

where the various functions appearing in the solution (2.1) are defined as,

$$f = 1 - \frac{r_0^4}{r^4}, \quad H = 1 + \frac{r_0^4\sinh^2\varphi}{r^4}, \quad F = 1 + \frac{r_0^4\cos^2\theta\sinh^2\varphi}{r^4} \qquad (2.2)$$

Note that the metric in the above is given in the string frame. The dilaton ϕ is non-constant and g_s is the string coupling. The 5-form field strength[1] and $A_{[2]}$ tell us the presence of D3 and D1 branes in the solution respectively. The non-zero NSNS $B_{[2]}$ field is due the non-threshold nature of the bound state (D1, D3). It is clear from the solution (2.1) that D3-branes are lying along $x^1, \ldots, x^3$, whereas D1-branes are lying along x^1. The angle θ measures the relative numbers of D3-branes and D1-branes and is defined as, $\cos\theta = N/\sqrt{N^2 + M^2}$, where N is the number of D3-branes and M is the number of D1-branes per unit co-dimension two-volume transverse to the D1-branes. Also in the above φ is the boost parameter and r_0 is the radius of the horizon of non-extremal or black (D1, D3)-brane solution.

Eq. (2.1) represents the relativistic (D1, D3) solution. The corresponding non-relativistic solution can be obtained by applying the standard procedure of the so-called Null Melvin Twist [26–29] or by taking a Penrose limit or a TsT transformation [41] to the relativistic (D1, D3) solution. The procedure of Null Melvin Twist generates a new solution in eight steps starting from the original relativistic solution. So, starting from the black (D1, D3) bound state solution given in (2.1), we first apply boost along x^1-direction, T-dualize the boosted isometric x^1-direction, twist a local one-form in a transverse compact direction, T-dualize back along x^1, boost back along x^1 and then take a scaling limit. In the light-cone coordinates $t = (x^1 + x^0)/\sqrt{2}$, $\xi = (x^1 - x^0)/\sqrt{2}$ the final solution takes the form,

$$ds^2 = \frac{H^{-\frac{1}{2}}}{K}\left[\left\{-\left(2r^2\beta^2 f + \frac{g}{2}\right)dt^2 - \frac{g}{2}d\xi^2 + (1+f)dtd\xi\right\}\right. \qquad (2.3)$$
$$\left. + \left(\frac{H}{F}\right)K\left((dx^2)^2 + (dx^3)^2\right)\right] + H^{\frac{1}{2}}\left[f^{-1}dr^2 + r^2\left(\frac{1}{K}(d\chi + \mathcal{A})^2 + ds^2_{\mathrm{P}^2}\right)\right]$$

Here $K = 1 - \beta^2 r^2 g(r)$ and $g(r) = -r_0^4/r^4$. We have introduced a one form $\mathcal{A}$ by $d\mathcal{A} = J$, with J, the Kahler form on the complex projective space P^2 and $ds^2_{\mathrm{P}^2}$ is the metric on the complex projective space P^2. The part of the metric $(1/K)(d\chi + \mathcal{A})^2 + ds^2_{\mathrm{P}^2}$ is the metric on the squashed 5-sphere with the squashing parameter K. The other fields are given as,

$$e^{2\phi} = g_s^2\left(\frac{H}{F}\right)\frac{1}{K}$$

$$B_{[2]} = \frac{r^2\beta}{\sqrt{2}K}(d\chi + \mathcal{A}) \wedge [(1+f)dt + (1-f)d\xi] + \frac{\tan\theta}{F}dx^2 \wedge dx^3$$

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[1]Note that the 5-form field strength must be made self-dual by adding the hodge-dual $*F_{[5]}$ with the $F_{[5]}$ given above for (2.1) to be a solution of type IIB supergravity, although we have not written it explicitly.

$$A_{[2]} = \frac{1}{g_s F} \sin\theta \coth\varphi \, dt \wedge d\xi$$

$$F_{[5]} = F_{[5]}^{(\text{old})} + \frac{1}{2}\left(B_{[2]}^{(\text{old})} \wedge dA_{[2]}^{(\text{old})} - A_{[2]}^{(\text{old})} \wedge dB_{[2]}^{(\text{old})} \right)$$

$$- \frac{1}{2}\left(B_{[2]}^{(\text{new})} \wedge dA_{[2]}^{(\text{new})} - A_{[2]}^{(\text{new})} \wedge dB_{[2]}^{(\text{new})} \right) \tag{2.4}$$

where $F_{[5]}^{(\text{old})}$, $B_{[2]}^{(\text{old})}$, $A_{[2]}^{(\text{old})}$ are the various fields given in (2.1) and $B_{[2]}^{(\text{new})}$, $A_{[2]}^{(\text{new})}$ are the fields given in (2.4). The solution represented by (2.3) and (2.4) are the non-relativistic non-extremal (D1, D3) solution of type IIB string theory. The extremal solution with the non-relativistic symmetry can be obtained from (2.3) and (2.4) by scaling $r_0 \to 0$ and $\varphi \to \infty$ such that the product $r_0^2 \sinh\varphi$ remains finite. Note that the parameter β, appearing in the solution above can be scaled away in the extremal case by scaling the t and ξ coordinates appropriately. However, this can not be done for the non-extremal case and in this case β is a physical parameter related to the chemical potential of the boundary theory [27].

The non-commutative Yang-Mills (NCYM) decoupling limit [31, 32] is a low energy limit by which we zoom into the region,

$$r_0 < r \sim r_0 \sinh^{\frac{1}{2}}\varphi \cos^{\frac{1}{2}}\theta \ll r_0 \sinh^{\frac{1}{2}}\varphi \tag{2.5}$$

Note that in this region φ is very large, whereas, the angle θ is very close to $\pi/2$. In this approximation,

$$H \approx \frac{r_0^4 \sinh^2\varphi}{r^4}, \qquad \frac{H}{F} \approx \frac{1}{\cos^2\theta(1 + a^4 r^4)} \equiv \frac{h}{\cos^2\theta} \tag{2.6}$$

where,

$$h = \frac{1}{1 + a^4 r^4}, \quad \text{with} \quad a^4 = \frac{1}{r_0^4 \sinh^2\varphi \cos^2\theta} \tag{2.7}$$

Note that in the NCYM limit, the asymptotic value of B_{23} component (from (2.4) we find that this is $\tan\theta$) responsible for creating non-commutativity takes a very large value as $\theta \to \pi/2$. Now with the above approximation (2.6), the metric in (2.3) takes the form,

$$ds^2 = \frac{r^2}{R^2}\frac{1}{K}\left[\left\{ -\left(2r^2\beta^2 f + \frac{g}{2} \right) dt^2 - \frac{g}{2}d\xi^2 + (1 + f)dt d\xi \right\} . \right.$$

$$\left. + hK\left((dx^2)^2 + (dx^3)^2 \right) \right] + \frac{R^2}{r^2}\left[f^{-1}dr^2 + r^2\left(\frac{1}{K}(d\chi + \mathcal{A})^2 + ds_{\text{P2}}^2 \right) \right] \tag{2.8}$$

where $R^2 = r_0^2 \sinh\varphi$ and we have rescaled the coordinates $x^{2,3}$ as $x^{2,3} \to \cos\theta x^{2,3}$. Due to the large magnetic or B-field in $x^{2,3}$-directions, they satisfy the non-commutativity relation $[x^2, x^3] = i\Theta$, where Θ is the non-commutativity parameter. Similarly the other fields[2] in (2.4) can also be rewritten using (2.6), and this will be the holographic dual of non-relativistic NCYM theory. Note that since for the non-relativistic case β is a physical parameter, by setting β to zero, we recover the near horizon metric of the relativistic non-extremal (D1, D3) solution. However, for the extremal case, β can not be put to zero, but should be scaled away before recovering the relativistic limit.

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[2]Since we do not need the other fields in what follows we will not write them explicitly here.

3   Drag force in hot non-relativistic NCYM plasma

Now in order to compute the drag force[3] on an external quark it is convenient to write the metric (2.8) in the original coordinate [48] $t \to (\xi - t)/\sqrt{2}$ and $\xi \to (\xi + t)/\sqrt{2}$ as,

$$
ds^2 = \frac{r^2}{R^2}\frac{1}{K}\left[\left\{-\left(1 + r^2\beta^2\right)f dt^2 + \left(1 - r^2\beta^2 f\right)d\xi^2 + 2r^2\beta^2 f dt d\xi\right\}\right.
$$
$$
\left. + hK\left((dx^2)^2 + (dx^3)^2\right)\right] + \frac{R^2}{r^2}\left[f^{-1}dr^2 + r^2\left(\frac{1}{K}\left(d\chi + \mathcal{A}\right)^2 + ds_{\mathrm{P2}}^2\right)\right] \quad (3.1)
$$

It is clear from (3.1) that by putting β to zero we recover the relativistic limit as expected. Now the dynamics of an external quark moving in this background can be understood from the Nambu-Goto action of the fuandamental string given as,

$$
S = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det(g_{ab})} \quad (3.2)
$$

where g_{ab} is the induced metric on the world-sheet of the fundamental string in the background (3.1) and is given as,

$$
g_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b}G_{\mu\nu} \quad (3.3)
$$

where $G_{\mu\nu}$ is the background metric (3.1) and $\xi^{a,b}$, $a, b = 0, 1$ are the world-sheet coordinates $\tau = \xi^0$ and $\sigma = \xi^1$. We now use the static gauge condition $X^0 \equiv t = \tau$ and $r = \sigma$. The end point of the string is allowed to move along one of the non-commutative directions $X^2 = x$ (say). Then the string embedding is completely specified by the function $x(t, r)$. The action (3.2) then reduces to the form,

$$
S = -\frac{1}{2\pi\alpha'}\int dt dr\left[\frac{1 + r^2\beta^2}{K} + \left(\frac{r^2}{R^2}\right)^2\frac{1 + r^2\beta^2}{K}hf(x')^2 - \frac{h}{f}(\dot{x})^2\right]^{\frac{1}{2}} \quad (3.4)
$$

Here 'overdot' and 'prime' on x denote the derivative with respect to 't' and 'r' respectively. Let us now make a simplifying and reasonable assumption that if we allow a sufficiently long time the string will move with a constant velocity and therefore, $x(t, r) = vt + \zeta(r)$. Substituting this in (3.4), the string action takes the form,

$$
S = -\frac{1}{2\pi\alpha'}\int dt dr\left[\frac{1 + r^2\beta^2}{K} + \left(\frac{r^2}{R^2}\right)^2\frac{1 + r^2\beta^2}{K}hf(\zeta')^2 - \frac{h}{f}v^2\right]^{\frac{1}{2}} \quad (3.5)
$$

Since the Lagrangian density does not contain ζ explicitly, the corresponding momentum must be conserved independent of both r and t and so, we have

$$
\pi_\zeta = \frac{\left(\frac{r^4}{R^4}\right)\frac{1 + r^2\beta^2}{K}hf\zeta'}{\sqrt{\frac{1 + r^2\beta^2}{K} + \left(\frac{r^4}{R^4}\right)\frac{1 + r^2\beta^2}{K}hf(\zeta')^2 - \frac{h}{f}v^2}} = \text{const. independent of } r, t \quad (3.6)
$$

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[3]Drag force in various other backgrounds have been calculated earlier in [42–47].

Solving this equation we obtain,

$$\zeta' = \frac{R^4}{r^4} \frac{\pi_\zeta \sqrt{K}}{\sqrt{1 + r^2 \beta^2} h f} \sqrt{\frac{\frac{f}{h} \frac{1 + r^2 \beta^2}{K} - v^2}{\frac{f}{h} \frac{1 + r^2 \beta^2}{K} - \frac{\pi_\zeta^2 R^4}{r^4 h^2}}} \tag{3.7}$$

Now notice in (3.7) that as r varies from r_0 to ∞, both the numerator and the denominator inside the square root change sign. So, at some r, the expression for ζ' can become imaginary when they have opposite signs. Therefore, the solution (3.7) is not always physically acceptable. To get the physical solution we have to choose the constant π_ζ suitably such that both the numerator and the denominator in the square root change the sign at the same place r_v (say). This fixes the constant π_ζ in the form,

$$\pi_\zeta = \frac{r_v^2}{R^2} \frac{v}{(1 + a^4 r_v^4)} = \frac{r_v^2}{r_0^2 \sinh \varphi (1 + a^4 r_v^4)} \tag{3.8}$$

where r_v is the solution of the equation given by,

$$(1 + r_v^2 \beta^2) \left[a^4 r_v^8 + (1 - a^4 r_0^4 - v^2) r_v^4 - r_0^4 \right] + v^2 \beta^2 r_v^2 \left(r_v^4 - r_0^4 \right) = 0 \tag{3.9}$$

We can substitute the value of π_ζ from (3.8) to (3.7) and integrate to obtain the complete string dynamics. In principle this is possible, but in practice, the difficulty is that the eq. (3.9) is a polynomial equation in r_v of degree ten and in general it is not always possible to solve it analytically. Even if this is possible it is not always guaranteed that the eq. (3.7) can be integrated in a closed form. However, we can formally write down the expression of the drag force from (3.8) as,

$$F = -\frac{1}{2\pi\alpha'} \frac{r_v^2}{R^2} \frac{v}{(1 + a^4 r_v^4)} = -\frac{1}{2\pi\alpha'} \frac{r_v^2}{r_0^2 \sinh \varphi (1 + a^4 r_v^4)} \tag{3.10}$$

Note that by solving r_v from eq. (3.9), we can express the drag force (3.10) in terms of r_0, $\sinh \varphi$, v, β, a and α', the parameters of the string theory or gravity side. Later we will express the drag force expression in terms of the parameters of the boundary gauge theory or non-relativistic NCYM theory. Let us mention here that the parameter β is non-trivial and cannot be scaled away for non-relativistic theory and therefore we will call it the non-relativistic parameter and by setting it to zero, we can recover the relativistic limit. On the other hand the parameter a, is associated with the non-commutativity of the theory and by setting it to zero we can recover the commutative limit. The parameter a was defined before as $a^4 r_0^4 = 1/(\sinh^2 \varphi \cos^2 \theta)$. In the NCYM decoupling limit $\sinh \varphi \sim 1/\alpha'$ and $\cos \theta \sim \alpha'/\Theta$, where Θ is the non-commutativity parameter given earlier and so, as $\alpha' \to 0$, $\sinh \varphi \gg 1$ and $\cos \theta \to 0$ as we mentioned earlier. Therefore, in this decoupling limit $a^4 r_0^4 \sim \Theta^2$. So, a measures the non-commutativity as it is directly related to the non-commutativity parameter.

In order to understand the drag force expression (3.10) more concretely we make some observation from the r_v equation given in (3.9). We note that for the relativistic ($\beta = 0$) and commutative ($a = 0$) case, we get from (3.9)

$$v^2 = \frac{r_v^4 - r_0^4}{r_v^4} \tag{3.11}$$

It is therefore clear that as r_v goes from r_0 to ∞, v varies from 0 to 1 as expected of a relativistic theory. On the other hand, for the commutative ($a = 0$) but non-relativistic ($\beta \neq 0$) case, we get from (3.9),

$$v^2 = \frac{(1 + r_v^2 \beta^2)(r_v^4 - r_0^4)}{r_v^2 (r_v^2 + r_0^4 \beta^2)} \tag{3.12}$$

In this case, as r_v varies from r_0 to ∞, v varies from 0 to ∞, again this is as expected of a non-relativistic theory. Let us next consider the relativistic ($\beta = 0$) but non-commutative ($a \neq 0$) case. We find from (3.9),

$$v^2 = \frac{(r_v^4 - r_0^4)(a^4 r_v^4 + 1)}{r_v^4} \tag{3.13}$$

Here we notice that for $r_v = r_0$, $v = 0$. But since the maximum value v can take for a relativistic theory is 1, r_v can not be arbitrarily large. We can calculate the value of r_v, when $v = 1$ from eq. (3.13) and we find (for large and small non-commutativity)

$$a^4 r_v^4 = a^4 r_0^4 + 1 - \frac{1}{a^4 r_0^4} + \cdots, \qquad \text{when} \quad a r_0 \gg 1 \tag{3.14}$$

$$a^4 r_v^4 = a^2 r_0^2 + \frac{1}{2} a^4 r_0^4 + \cdots, \qquad \text{when} \quad a r_0 \ll 1 \tag{3.15}$$

So, for large non-commutativity r_v is close to r_0, but for small non-commutativity r_v is far away from r_0. Finally, we consider both non-relativistic ($\beta \neq 0$) and non-commutative ($a \neq 0$) case. We find from (3.9),

$$v^2 = \frac{(1 + r_v^2 \beta^2)(r_v^4 - r_0^4)(a^4 r_v^4 + 1)}{r_v^2 (r_v^2 + r_0^4 \beta^2)} \tag{3.16}$$

Here also we note that as r_v starts from r_0, v starts from 0. We just mentioned that r_v can not take arbitrary large value when $a \neq 0$, for v to remain less than or equal to 1 (for the relativistic theory). However, we will see that even when $\beta \neq 0$ (i.e. for the non-relativistic theory) r_v can not take arbitrary large value. The reason is that if r_v exceeds the value obtained for the relativistic case given in eqs. (3.14), (3.15), then v will exceed one when we put $\beta = 0$ and this will be unphysical for a relativistic theory. Therefore, we will use the values of r_v given in (3.14) and (3.15) to determine v when $\beta \neq 0$. It can be checked from (3.16) that for $a r_0 \ll 1$, v can be much larger, i.e., $v \gg 1$ (showing the non-relativistic nature of the theory), but for $a r_0 \gg 1$, the maximum value of v is of the order 1. Indeed it can be checked from (3.14) and (3.16) that the value of v is given by,

$$v^2 = 1 + \frac{r_0^2 \beta^2}{a^4 r_0^4 \left(1 + r_0^2 \beta^2\right)} + O\left(\frac{1}{a^8 r_0^8}\right) \tag{3.17}$$

So, for $a^4 r_0^4 \gg 1$, the velocity of the quark v is close to 1 (but the velocity is always greater than 1) as we see from (3.17). So, we will analyse the general r_v equation (3.9) and the drag force (3.10) in four different cases, namely, (i) $v \ll 1$, $a r_0 \ll 1$, (ii) $v \ll 1$, $a r_0 \gg 1$, (iii) $v \gg 1$, $a r_0 \ll 1$, and (iv) $v \sim 1$, $a r_0 \gg 1$.

(i) $v \ll 1$, $ar_0 \ll 1$. In this case (3.9) can be solved to obtain r_v in the following form,

$$a^4 r_v^4 = a^4 r_0^4 (1 + v^2 + v^4) - a^8 r_0^8 v^2 (1 + v^2) + \cdots \tag{3.18}$$

Substituting this in (3.10) we obtain

$$F = -\frac{1}{2\pi\alpha'} \frac{v}{R^2} r_0^2 \left[\left(1 + \frac{1}{2} v^2 \right) - \left(1 + 2v^2 \right) a^4 r_0^4 + \cdots \right] \tag{3.19}$$

This matches exactly with those given in refs. [49, 50] with $v \ll 1$ as it should be in this approximation.

(ii) $v \ll 1$, $ar_0 \gg 1$. In this case by solving (3.9) we obtain r_v in the form,

$$a^4 r_v^4 = a^4 r_0^4 \left[1 + \frac{v^2}{a^4 r_0^4} + \cdots \right] \tag{3.20}$$

Substituting this is (3.10) we obtain,

$$F = -\frac{1}{2\pi\alpha'} \frac{v}{R^2} \frac{r_0^2}{a^4 r_0^4} \left[1 - \frac{v^2 + 2}{2a^4 r_0^4} + \cdots \right] \tag{3.21}$$

This also matches with the drag force expression given in [49, 50] with $v \ll 1$ as expected.

(iii) $v \gg 1$, $ar_0 \ll 1$. In this case we get from (3.9)

$$a^4 r_v^4 = a^2 r_0^2 \left(1 + \frac{a^2 r_0^2}{2} + \cdots \right) \tag{3.22}$$

Substituting these in (3.10) we get,

$$F = -\frac{1}{2\pi\alpha'} \frac{v}{R^2 \beta^2} \left(v^2 + r_0^4 \beta^4 \right) \left(1 - \frac{3}{4} a^2 r_0^2 + \cdots \right) \tag{3.23}$$

(iv) $v \sim 1$, $ar_0 \gg 1$. Eq. (3.9) in this case gives,

$$a^4 r_v^4 = a^4 r_0^4 \left(1 + \frac{1}{a^4 r_0^4} - \frac{1}{a^8 r_0^8} \cdots \right) \tag{3.24}$$

Substituting this in (3.10) we get,

$$F = -\frac{1}{2\pi\alpha'} \frac{v(v^2 - 1)}{R^2} \frac{(1 + r_0^2 \beta^2)}{\beta^2} \left(1 - \frac{3}{2} \frac{1}{a^4 r_0^4} + \cdots \right) \tag{3.25}$$

In eqs. (3.19), (3.21), (3.23) and (3.25) we have given the drag force expressions in terms of the parameters of string or gravity theory. Now in order to understand the nature of the force in terms the boundary gauge theory, we have to relate the gravity parameters

with the parameters of the non-relativistic NCYM theory. The temperature of the non-relativistic NCYM theory can be calculated from the Hawking temperature of the non-extremal decoupled gravity configuration given in (3.1) and has the form,

$$T = \frac{1}{\pi r_0 \sinh \varphi} \tag{3.26}$$

Also from the charge of the D3-brane we can calculate,

$$r_0^4 \sinh^2 \varphi = 2\hat{\lambda}\alpha'^2, \tag{3.27}$$

where $\hat{\lambda} = \hat{g}_{YM}^2 N$, is the 't Hooft coupling of the non-relativistic NCYM theory, $\hat{g}_{YM}$ is the NCYM coupling and N is the number of D3-branes and is related to the gauge group $SU(N)$ of the gauge theory. The 't Hooft parameter $\hat{\lambda}$ is related to the corresponding parameter of ordinary YM theory by the scaling of the form, $\lambda = (\alpha'/\Theta)\hat{\lambda}$, where Θ is the non-commutativity parameter [31, 32]. Now using (3.26) and (3.27) we get,

$$\sinh \varphi = \frac{1}{\sqrt{2\hat{\lambda}}\pi^2 T^2 \alpha'}, \quad r_0 = \sqrt{2\hat{\lambda}}\pi T\alpha', \quad \text{and,} \quad a^4 r_0^4 = 2\hat{\lambda}\pi^4 T^4 \Theta^2 \tag{3.28}$$

In obtaining the last expression we have used $a^4 r_0^4 = 1/(\sinh^2 \varphi \cos^2 \theta)$ and $\cos \theta = \alpha'/\Theta$. Using (3.28) we will express the drag force given earlier in (3.19), (3.21), (3.23), (3.25) for various cases in the leading order in terms of the non-relativistic NCYM theory.

In the first case (i) $v \ll 1$, $ar_0 \ll 1$, we get

$$F = -\sqrt{\frac{\hat{g}_{YM}^2 N}{2}}\pi T^2 v \tag{3.29}$$

We can formally express the above expression (3.29) in terms of the momentum p and mass m of the external quark and integrate to find [33, 48],

$$p(t) = p(0)e^{-\pi\sqrt{\frac{\hat{g}_{YM}^2 N}{2}}\frac{T^2}{m}t}, \quad \text{where} \quad p(0) \ll m \tag{3.30}$$

The corresponding energy will be given as,

$$E(t) = E(0)e^{-\pi\sqrt{2\hat{g}_{YM}^2 N}\frac{T^2}{m}t}, \quad \text{where} \quad E(0) \ll m/2 \tag{3.31}$$

The expression of drag force for the case (ii) $v \ll 1$, $ar_0 \gg 1$ can be written using (3.28) in the leading order as,

$$F = -\frac{1}{2\sqrt{2\hat{g}_{YM}^2 N}\pi^3 T^2}\frac{v}{\Theta^2} \tag{3.32}$$

The momentum and energy can be obtained as before and have the forms,

$$p(t) = p(0)e^{-\frac{t}{2\pi^3 m T^2 \sqrt{2\hat{g}_{YM}^2 N}\Theta^2}}, \quad \text{where} \quad p(0) \ll m \tag{3.33}$$

$$E(t) = E(0)e^{-\frac{t}{\pi^3 m T^2 \sqrt{2\hat{g}_{YM}^2 N}\Theta^2}}, \quad \text{where} \quad E(0) \ll m/2 \tag{3.34}$$

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In the above two cases when $v \ll 1$, that is, when the momentum or energy is much less than the quark mass, the quark will lose its momentum or energy exponentially. The relaxation times are different in the two different cases and depend on whether the non-commutativity is small or large. For small non-commutativity the relaxation time does not depend on the non-commutativity parameter in the leading order, but depend directly on the mass of the quark and inversely on the square of the temperature as well as the square-root of the 't Hooft coupling. On the other hand, for large non-commutativity, the dependence on the temperature and the 't Hooft coupling get inverted, but the dependence on the mass remains the same. Also, in this case, the relaxation time depends directly on the square of the non-commutativity parameter. So, for small non-commutativity the non-commutative effect does not show up in the leading order, but it does show up in the leading order for large non-commutativity.

Similarly, for case (iii) $v \gg 1$, $ar_0 \ll 1$ using (3.28) the expression of drag force in the leading order has the form

$$F = -\frac{v^3 \mu^2}{2\pi \sqrt{2\hat{g}_{YM}^2 N}} \tag{3.35}$$

where we have defined $1/(\beta\alpha') = \mu$, the chemical potential of the non-relativistic NCYM theory. The momentum and the energy in this case have the forms,

$$p(t) = \left[\frac{1}{p(0)^2} + \frac{t\mu^2}{\pi m^3 \sqrt{2\hat{g}_{YM}^2 N}} \right]^{-\frac{1}{2}}, \qquad \text{where} \quad p(0) \gg m \tag{3.36}$$

$$E(t) = \left[\frac{1}{E(0)} + \frac{2t\mu^2}{\pi m^2 \sqrt{2\hat{g}_{YM}^2 N}} \right]^{-1}, \qquad \text{where} \quad E(0) \gg m/2 \tag{3.37}$$

In this case the momentum or the energy loss does not depend on the temperature [48] unlike in the previous two cases. The momentum (or the energy) loss depends directly on the square of the chemical potential and inversely on the cube (square) of the quark mass and the square-root of the 't Hooft coupling. With time they do not decay exponentially as in the previous cases, but the momentum goes as $t^{-1/2}$, whereas the energy goes as t^{-1}.

Finally, the drag force expression for case (iv) $v \sim 1$, $ar_0 \gg 1$ can be written using (3.28) in the leading order as,

$$F = -\sqrt{\frac{\hat{g}_{YM}^2 N}{2}} \pi T^2 C(\hat{g}_{YM}^2 N, \mu, T)(v^3 - v) \tag{3.38}$$

where

$$C(\hat{g}_{YM}^2 N, \mu, T) = \frac{\mu^2 + 2\pi^2 \hat{g}_{YM}^2 N T^2}{2\pi^2 \hat{g}_{YM}^2 N T^2} \tag{3.39}$$

with μ being the chemical potential defined before. Note that the function C tends to unity when $\mu^2 \ll 2\pi^2 \hat{g}_{YM}^2 N T^2$. Also note that when $\mu^2 \gg 2\pi^2 \hat{g}_{YM}^2 N T^2$, the force expression is independent of temperature. It is well-known [49, 50], that the non-commutativity reduces the amount of drag force on the quark moving in a strongly coupled NCYM plasma and

so, it is not surprising that the drag force is very small (since $v \sim 1$, for large non-commutativity $ar_0 \gg 1$). The momentum and energy in this case have the forms,

$$p(t) = m \left[1 - \left(1 - \frac{m^2}{p^2(0)} \right) e^{-\frac{2\pi T^2}{m} \sqrt{\frac{\hat{g}_{YM}^2 N}{2}} C(\hat{g}_{YM}^2 N, \mu, T) t} \right]^{-\frac{1}{2}}, \quad \text{where } p(0) \sim m \quad (3.40)$$

$$E(t) = \frac{m}{2} \left[1 - \left(1 - \frac{m}{2E(0)} \right) e^{-\frac{2\pi T^2}{m} \sqrt{\frac{\hat{g}_{YM}^2 N}{2}} C(\hat{g}_{YM}^2 N, \mu, T) t} \right]^{-1}, \quad \text{where } E(0) \sim \frac{m}{2} \quad (3.41)$$

In this case since $v \sim 1$, both the momentum and the energy loss is also very small due to large non-commutative effects.

4 Conclusion

To summarize, in this paper we have considered the non-extremal (D1, D3) bound state solution of type IIB string theory. In a particular decoupling limit this supergravity configuration is known to describe the holographic dual of relativistic NCYM theory at finite temperature with space-space non-commutativity. By applying the standard technique of Null Melvin Twist we have obtained the non-relativistic version of non-extremal (D1, D3) bound state system. The same decoupling limit in this case, describes the holographic dual of the non-relativistic NCYM theory at finite temperature with space-space non-commutativity. We have computed the drag force on a quark moving through such plasma and along one of the non-commutative directions, by using the AdS/CFT correspondence and the string probe approach. We first computed the drag force in terms of the parameters of the gravity theory and then using the AdS/CFT dictionary expressed it in terms of the parameters of the gauge theory. We found that the general drag force expression can not be written in a closed form. So, to show the various effects we have considered the various corners of the solution space and obtained the drag force expressions in the leading order. We also formally integrated the drag force expression to obtain the momentum as well as the energy loss of the quark in various limits. In particular, we have shown that when the velocity of the quark is small, it loses its energy exponentially with time. The relaxation times are expressed in terms of the parameters of the NCYM theory. When the non-commutative effect is large, the relaxation time depends explicitly on the non-commutativity parameter. We also found that the velocity could be very large when the non-commutative effect is small. In that case the quark loses its energy as inverse power of time. Also the energy loss does not depend on the temperature of the theory unlike in other cases. Finally, we found that when the non-commutative effect is large the velocity can not be arbitrarily large but must be of the order 1. In this case, the energy loss of the quark is very small due to the large non-commutativity.

Acknowledgments

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