

Defect CFT techniques in the 6d $\mathcal{N} = (2, 0)$ theory

Nadav Drukker, Malte Probst and Maxime Trépanier

*Department of Mathematics, King's College London,
The Strand, London WC2R 2LS, United Kingdom*

E-mail: nadav.drukker@gmail.com, mltprbst@gmail.com,
trepanier.maxime@gmail.com

ABSTRACT: Surface operators are among the most important observables of the 6d $\mathcal{N} = (2, 0)$ theory. Here we apply the tools of defect CFT to study local operator insertions into the 1/2-BPS plane. We first relate the 2-point function of the displacement operator to the expectation value of the bulk stress tensor and translate this relation into a constraint on the anomaly coefficients associated with the defect. Secondly, we study the defect operator expansion of the stress tensor multiplet and identify several new operators of the defect CFT. Technical results derived along the way include the explicit supersymmetry transformations of the stress tensor multiplet and the classification of unitary representations of the superconformal algebra preserved by the defect.

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Contents

1	Introduction	1
2	Displacement multiplet	4
2.1	Superconformal Ward identity	5
3	Stress tensor correlators	6
3.1	Stress tensor multiplet	7
3.2	Defect without insertions	9
3.3	Defect with an insertion	10
4	Relation to anomaly coefficients	11
4.1	Displacement operator	12
4.2	Stress tensor	13
5	Defect operator expansion	15
5.1	Representations of $\mathfrak{osp}(4^* 2) \oplus \mathfrak{osp}(4^* 2)$	15
5.2	Unitary multiplets of $\mathfrak{osp}(4^* 2)$	16
5.3	The stress tensor dOE	18
5.4	Constraints from supersymmetry	20
6	Conclusion	22
A	Conventions	24
A.1	Spinors and γ -matrices	24
B	Weyl anomaly for surface operators	25
C	Conformal Ward identities for defect correlators	26
C.1	Defect without insertions	26
C.2	Defect with an insertion	28
D	Algebras	30
D.1	The algebra $\mathfrak{osp}(8^* 4)$	30
D.2	The subalgebra $\mathfrak{osp}(4^* 2) \oplus \mathfrak{osp}(4^* 2)$	31
D.2.1	Subalgebra as an embedding inside $\mathfrak{osp}(8^* 4)$	32

1 Introduction

A natural class of observables of the $\mathcal{N} = (2, 0)$ 6d SCFT is that of surface operators [1]. These operators share many properties with the much studied Wilson loops of gauge theories: they are extended objects which can enjoy superconformal symmetry [2], in some cases have a holographic description [3], and in the abelian $\mathcal{N} = (2, 0)$ theory admit a field realisation as the integral of the 2-form B field, akin to a gauge connection [4–8]. These

similarities suggest that some of the methods which have proven successful in the study of Wilson loops can be applied to the $\mathcal{N} = (2, 0)$ theory as well, providing a window into its dynamics.

In this paper we apply the framework of defect CFT to the surface operators of the $\mathcal{N} = (2, 0)$ theory. We adopt the approach of the conformal bootstrap program [9–14] and use the symmetries preserved by the surface operators to constrain their correlators with other bulk operators as well as local operator insertions on the surface. One of the virtues of this description is that it does not rely on a field realisation and therefore is applicable to the nonabelian theory.

We focus on 1/2-BPS defects because they preserve the largest amount of symmetry. These are surface operators defined over a plane and expected to be labeled by a representation of the *ADE* group of the $\mathcal{N} = (2, 0)$ theory [15–17]. We consider local operator insertions into the defect, the simplest example encoding an infinitesimal geometric deformation of the plane itself. Because the plane preserves superconformal symmetry, the correlators of local operator insertions are constrained and obey the axioms of a dCFT—the 2- and 3-point functions are fixed up to a small set of numbers defining the dCFT, which make up the dCFT data.

Explicitly, consider a correlator involving such a surface operator V . While translating the plane along parallel directions leaves the correlator invariant, translations in directions transverse to the plane do not. Instead, the stress tensor receives a contribution from a contact term localised on the defect (at $x = 0$):

$$\partial_\mu T^{\mu m}(\sigma, x) V = V[\mathbb{D}^m(\sigma)]\delta^{(4)}(x). \tag{1.1}$$

The index $\mu = 1, \dots, 6$ runs over all spacetime coordinates, while $m = 1, \dots, 4$ are the coordinates transverse to the plane. We use the notation $V[\hat{\mathcal{O}}(\sigma)]$ to denote the planar surface operator with a defect operator $\hat{\mathcal{O}}$ inserted at a point σ on the plane.

Equation (1.1) is an operator equation, so it holds inside correlation functions. It defines \mathbb{D} , known as the displacement operator. In addition, because V preserves some supersymmetries, the displacement operator sits in a multiplet containing also contact terms for the divergence of the broken super- and R-current, which we label \mathbb{Q} and \mathbb{O} , respectively.

It turns out that these defect operators enjoy a very favorable position: not only are they highly constrained by the residual symmetry (which includes the 2d rigid superconformal symmetry), but they also correspond to interesting physical quantities [18, 19].¹ Indeed it is easy to show that, as a consequence of (1.1), the insertion of a displacement operator \mathbb{D} corresponds to small deformations of the plane, and thus captures the shape dependence of surface operators.

This paper revolves around two correlators that capture physical properties of the defect. The first one is the 2-point function of displacement operators. Using the residual conformal symmetry of the plane and reading the conformal dimension $\Delta_{\mathbb{D}} = 3$ from (1.1),

¹Note that the dCFT is not expected to contain a conserved stress tensor [20] and the rigid conformal symmetry is not necessarily enhanced to Virasoro symmetry.

the 2-point function is constrained up to a single coefficient $C_{\mathbb{D}}$ to be

$$\langle V[\mathbb{D}^m(\sigma)\mathbb{D}^n(0)] \rangle = \frac{C_{\mathbb{D}}\delta^{mn}}{\pi^2|\sigma|^6}. \tag{1.2}$$

Notice however that, unlike most operators, the normalisation of \mathbb{D} is already fixed by the normalisation of $T^{\mu\nu}$ from (1.1), so that $C_{\mathbb{D}}$ is part of the data characterising the dCFT [21] (the factor π^2 is for convenience).

The second is the stress tensor, which in the presence of the defect acquires an expectation value. Both the components of the tensor along the defect T^{ab} and orthogonal to it T^{mn} can have a nonzero 1-point function, and they are fixed by conformal invariance up to an arbitrary coefficient h_T to be

$$\langle T^{ab}(\sigma, x)V \rangle = \frac{h_T\eta^{ab}}{\pi^3x^6}, \quad \langle T^{mn}(\sigma, x)V \rangle = -\frac{h_T(\delta^{mn} - 2x^m x^n/x^2)}{\pi^3x^6}. \tag{1.3}$$

$T(\sigma, x)$ is inserted at a distance x from the defect, and obviously the correlators do not depend on the coordinate σ by translation invariance along the plane. $\eta^{ab} = \text{diag}(-1, 1)$ is the Minkowski metric.

In theories with only conformal invariance the coefficients h_T and $C_{\mathbb{D}}$ are independent quantities [22], but in theories with enough supersymmetries one can use superconformal Ward identities to relate them [23]. For our surface operators we show in section 3 that

$$h_T = \frac{3C_{\mathbb{D}}}{80}. \tag{1.4}$$

To derive this result, we obtain the transformations of the stress tensor multiplet under supersymmetry (3.7), which is also an important result of section 3.

Analogous relations between h_T and $C_{\mathbb{D}}$ were first derived using the same techniques for the 1/2-BPS Wilson loops of 4d $\mathcal{N} = 2$ theories [23] and the 1/6-BPS bosonic loops of ABJM [24], proving the conjecture of [25, 26]. A similar analysis was also applied recently to surface operators in 4d $\mathcal{N} = 1$ theories [27]. All these different examples show how the language of dCFT is a powerful and universal tool to study superconformal defects.

More than simply equating different constants, the relation (1.4) has an important physical consequence. Recall that surface operators in CFTs typically have a conformal anomaly, which manifests itself as a divergence in the expectation value. The anomaly density \mathcal{A}_{Σ} is the sum of conformal invariants [28, 29] and can be written as

$$\mathcal{A}_{\Sigma} = \frac{1}{4\pi} \left[aR^{\Sigma} + b_1 \text{tr} \tilde{\Pi}^2 + b_2 \text{tr} W + c(\partial n)^2 \right], \tag{1.5}$$

where the invariants R^{Σ} , $\text{tr} \tilde{\Pi}^2$, $\text{tr} W$, $(\partial n)^2$ are local quantities depending on the embedding of the surface (see appendix B for a review), while the coefficients a , b_1 , b_2 and c are known as anomaly coefficients and depend on the specifics of the theory and operator in question.

In section 4 we relate the coefficients b_1 , b_2 , c to $C_{\mathbb{D}}$, h_T and an additional constant $C_{\mathbb{O}}$ to be introduced in (2.3). In the language of anomaly coefficients, the result (1.4) along

with the relative normalisations (2.6) of the operators in the displacement multiplet can be stated as

$$c = -b_1/2, \quad b_1 = -b_2. \tag{1.6}$$

We emphasize that these identities are a consequence of supersymmetry and hold for any 1/2-BPS operator of the $\mathcal{N} = (2, 0)$ theory and for any *ADE* group. In particular, the second identity agrees with the explicit holographic calculations of [8, 30, 31] and was conjectured to come from supersymmetry in [27]. The two remaining anomaly coefficients a and b_1 were calculated at $N = 1$ in [8] and for $N > 1$ using holographic entanglement entropy in the presence of surface operators [32–35], and the superconformal index [36].

Finally, in section 5 we expand our scope and consider the analog of the operator product expansion but for bulk operators in the presence of a defect—the defect operator expansion (dOE) [37, 38]. This expansion gives a representation of bulk operators near the defect in terms of insertions of defect operators. To understand what these defect operators are more generally, we classify unitary multiplets of the algebra preserved by the defect. We then look at operators in the stress tensor multiplet and determine the short multiplets arising in their dOE. We find a new marginal defect operator, which we associate with the RG flow between the nonsupersymmetric and 1/2-BPS surface operator discussed in [31].²

In addition to this result, we find that the defect operator expansion provides a useful framework and makes the constraints imposed by the preserved symmetries manifest. In fact, in section 5.4 we use the dOE and representation theory to give a different perspective on the relation (1.4). Unlike in section 3, where (1.4) follows from a technical calculation, we are able to conclude directly that h_T and $C_{\mathbb{D}}$ must be related. This suggests a strategy for determining the minimal amount of supersymmetry required in order for the conjecture of [25], which relates these coefficients in the case of supersymmetric Wilson loops, to hold (see also [19] and references therein for a similar conjecture in the context of entanglement entropy).

Some auxiliary results are collected in appendices. appendix A summarises our conventions and the gamma matrices used throughout the paper. appendix B provides a short review of the Weyl anomaly for surface operators. In appendix C we show how to constrain correlators containing both bulk and defect operators using conformal symmetry. appendix D reviews the 2 algebras used in this paper: the $\mathfrak{osp}(8^*|4)$ symmetry of the bulk theory and the $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$ symmetry preserved by the defect.

Note added. In the last stages of preparation of this paper, we were made aware by [41] that the classification of unitary multiplets of $\mathfrak{osp}(4^*|2)$ (here presented in section 5.2) had previously been worked out in the context of line operators [42].

2 Displacement multiplet

As far as defect operators go, the displacement operator is pretty universal. As (1.1) suggests, any defect breaking translation symmetry contains that defect operator. For this

²This is analogous to the flow of Wilson line operators introduced in [39, 40].

reason, it has appeared in many contexts: the prototypical example is the 1/2-BPS Wilson line in $\mathcal{N} = 4$ SYM, where the study of deformations and operator insertions was initiated in [43], but many other examples have been studied over the years and follow the general analysis of [22].

In the case of $\mathcal{N} = (2, 0)$, we are mostly interested in the multiplet which contains the displacement operator. Of the full superconformal algebra $\mathfrak{osp}(8^*|4)$, the 1/2-BPS plane preserves a 2d conformal algebra $\mathfrak{so}(2, 2)_{\parallel}$ in the directions parallel to the plane, along with rotations of the transverse directions $\mathfrak{so}(4)_{\perp}$ and an $\mathfrak{so}(4)_R$ R-symmetry. In addition, it also preserves half the supersymmetries \mathbb{Q}_+ (and $\tilde{\mathbb{S}}_+$) such that $\mathbb{Q}_+ V = 0$. These are obtained by a half-rank projector $\mathbb{Q}_+ = \Pi_+ \mathbb{Q}$ whose explicit definition can be found in (D.12). The preserved generators form an $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$ subalgebra [44], detailed in appendix D.2.

Importantly, in direct analogy to (1.1), the Ward identities associated to the remaining broken super- and R-symmetries also receive contributions localised on the defect, which give rise to defect operators \mathbb{Q} and \mathbb{O}^i , encoding the nontrivial response of the defect to the broken generators. Explicitly, the conservation laws associated with the R-current j and the supercurrent J are broken as follows:

$$\begin{aligned} \partial_{\mu} T^{\mu m} V &= V[\mathbb{D}^m] \delta^{(4)}(x), \\ \partial_{\mu} (\Pi_{-} J^{\mu}) V &= V[\mathbb{Q}] \delta^{(4)}(x), \\ \partial_{\mu} j^{\mu i 5} V &= V[\mathbb{O}^i] \delta^{(4)}(x). \end{aligned} \tag{2.1}$$

In this equation, $i = 1, \dots, 4$ is the R-symmetry index of $\mathfrak{so}(4)_R$. The spinor indices of $J^{\mu}_{\alpha\dot{\alpha}}$ and $\mathbb{Q}_{\alpha\dot{\alpha}}$ are suppressed and follow the conventions outlined in appendix A (see however footnote 4). For the definition of Π_{-} , see (D.12).

As mentioned previously, the (nonabelian) theory does not have a known field realisation, so we cannot write these operators in terms of fundamental fields. We can however derive some of their properties purely from representation theory. The full multiplet as derived in appendix D.2.1 reads

$$\begin{aligned} \delta_{+} \mathbb{D}_m &= \frac{1}{2} \varepsilon_{+} \gamma_{am} \partial^a \mathbb{Q}, \\ \delta_{+} \mathbb{Q} &= 2\varepsilon_{+} \gamma_m \mathbb{D}^m - 2\varepsilon_{+} \gamma_a \check{\gamma}_{i5} \partial^a \mathbb{O}^i, \\ \delta_{+} \mathbb{O}_i &= -\frac{1}{2} \varepsilon_{+} \check{\gamma}_{i5} \mathbb{Q}. \end{aligned} \tag{2.2}$$

$\delta_{+} = \varepsilon_{+} \mathbb{Q}_{+}$ is a variation with respect to the preserved supercharges and $\varepsilon_{+} = \varepsilon_{+} \Pi_{+}$.

2.1 Superconformal Ward identity

The 2-point functions of these operators is easy to find. Both \mathbb{D} and \mathbb{O} transform as scalars with respect to the 2d conformal symmetry, while \mathbb{Q} is a spinor. Their conformal dimensions can also be read from (2.1) and are $\Delta_{\mathbb{D}} = 3$, $\Delta_{\mathbb{Q}} = 5/2$ and $\Delta_{\mathbb{O}} = 2$. Consequently, using the preserved bosonic symmetries, their 2-point functions are (up to some

arbitrary coefficients $C_{\mathbb{D}}, C_{\mathbb{Q}}, C_{\mathbb{O}}$)

$$\begin{aligned} \langle V[\mathbb{D}^m(\sigma)\mathbb{D}^n(0)] \rangle &= \frac{C_{\mathbb{D}}\delta^{mn}}{\pi^2|\sigma|^6}, \\ \langle V[\mathbb{Q}(\sigma)\mathbb{Q}(0)] \rangle &= \frac{C_{\mathbb{Q}}(\gamma_a\sigma^a\Pi_-)}{\pi^2|\sigma|^6}, \\ \langle V[\mathbb{O}_i(\sigma)\mathbb{O}_j(0)] \rangle &= \frac{C_{\mathbb{O}}\delta_{ij}}{\pi^2|\sigma|^4}. \end{aligned} \tag{2.3}$$

As \mathbb{Q} is a 2d spinor, its 2-point function should be written in terms of the corresponding 2d gamma matrices. In order to emphasize the relation between the respective symmetry algebras in 6d and 2d, we write these matrices as blocks of their 6d counterparts obtained by the projector Π_- .

We can now relate $C_{\mathbb{O}}$ and $C_{\mathbb{Q}}$ to $C_{\mathbb{D}}$ using superconformal Ward identities associated to the preserved supersymmetries. Apply the supersymmetry transformations (2.2) to the vanishing correlator $\langle V[\mathbb{Q}_{\beta\check{\beta}}\mathbb{O}_i] \rangle$ to find

$$-\frac{1}{2}(\check{\gamma}_{i5})_{\check{\alpha}}^{\check{\gamma}} \langle V[\mathbb{Q}_{\beta\check{\beta}}\mathbb{Q}_{\alpha\check{\gamma}}] \rangle = 2(\gamma_a\check{\gamma}_{j5}\Pi_-c\Omega)_{\alpha\beta\check{\alpha}\check{\beta}} \partial^a \langle V[\mathbb{O}^j\mathbb{O}_i] \rangle. \tag{2.4}$$

Substituting the explicit 2-point functions (2.3), we obtain the linear relation $C_{\mathbb{Q}} = -16C_{\mathbb{O}}$. In the same fashion, the Ward identity associated to $\langle V[\mathbb{Q}_{\beta\check{\beta}}\mathbb{D}_m] \rangle$ leads to

$$2(\gamma_n\Pi_-c\Omega)_{\alpha\check{\alpha}\beta\check{\beta}} \langle V[\mathbb{D}^n\mathbb{D}_m] \rangle = -\frac{1}{2}(\gamma_{am})_{\alpha}^{\gamma} \partial^a \langle V[\mathbb{Q}_{\beta\check{\beta}}\mathbb{Q}_{\gamma\check{\alpha}}] \rangle, \tag{2.5}$$

which serves to relate $C_{\mathbb{D}}$ to $C_{\mathbb{Q}}$. Altogether, we find that the normalisations of the 2-point functions obey

$$C_{\mathbb{D}} = -C_{\mathbb{Q}} = 16C_{\mathbb{O}}. \tag{2.6}$$

3 Stress tensor correlators

Some of the most important operators in any theory are the stress tensor and its multiplet. In the presence of the 1/2-BPS defect, their expectation values are highly constrained by the residual symmetry: typically the $\mathfrak{so}(2,2)_{\parallel} \oplus \mathfrak{so}(4)_{\perp} \oplus \mathfrak{so}(4)_R$ bosonic subalgebra of preserved symmetries is powerful enough to fix them up to a constant (see e.g. (1.3)).

In addition to the constraints imposed by conformal symmetry, supersymmetry relates correlators of different operators in the same multiplet. Adapting the strategy of [23, 24, 27], the key to deriving (1.4) is to focus on the correlator $\langle T^{\mu\nu}(x)V[\mathbb{D}^m(\sigma)] \rangle$, which is entirely fixed in terms of the constants $C_{\mathbb{D}}$ and h_T [22]. The kinematics of that correlator admit 2 independent tensor structures with their own coefficient. They are related to $C_{\mathbb{D}}$ by taking the divergence

$$\partial_{\mu} \langle T^{\mu m} V[\mathbb{D}^n] \rangle = \langle V[\mathbb{D}^m\mathbb{D}^n] \rangle \propto C_{\mathbb{D}}, \tag{3.1}$$

and to h_T by integrating the displacement operator over the surface, which simply translates the defect

$$\int_{\mathbb{R}^2} d^2\sigma \langle T^{\mu\nu}(0, x) V[\mathbb{D}^m(\sigma)] \rangle = \partial^m \langle T^{\mu\nu}(0, x) V \rangle \propto h_T. \quad (3.2)$$

We stress that this does not provide in itself a relation between $C_{\mathbb{D}}$ and h_T , as can be checked using the explicit form of the correlators (see equation (6.2) of [19]).

Instead, to derive the relation, we should use superconformal Ward identities to relate this correlator to $\langle O^{i5} V[\mathbb{O}^j] \rangle$, where O is the superconformal primary of the stress tensor multiplet. Because the latter admits only a single tensor structure, this would imply that $C_{\mathbb{D}}$ and h_T are related.

In order to derive this result, we need the explicit supersymmetry transformations of the stress tensor multiplet, which are summarised in (3.7). We also need the 1-point functions of the stress tensor appearing on the right-hand side of (3.2), which are derived in section 3.2 (the 2-point functions of the displacement multiplet are given in (2.3)). Then, we use the supersymmetric Ward identities associated with correlators of the form $\langle \mathcal{O} V[\hat{\mathcal{O}}] \rangle$ to derive (1.4).

3.1 Stress tensor multiplet

We begin by obtaining explicit supersymmetry transformations for the stress tensor multiplet, whose content is derived from representation theory and can be found in [45], where it is presented as a massless graviton multiplet (see also [46, 47] for an overview of superconformal multiplets in various dimensions).

The primaries of any multiplet are labelled by their transformation under Lorentz symmetry $[j_1, j_2, j_3]_{\mathfrak{su}(4)}$, R-symmetry $(R_1, R_2)_{\mathfrak{sp}(2)}$ as well as their conformal dimension Δ .³ In the notation of [46], the stress tensor multiplet is the $D_1[0, 0, 0]_4^{(0,2)}$ multiplet (with representations written as $[j_1, j_2, j_3]_{\Delta}^{(R_1, R_2)}$). Its primaries are

- $T^{\mu\nu}$, the stress tensor ($[0, 2, 0]_6^{(0,0)} = \mathbf{20}$). It contains a null state, since $\partial_\mu T^{\mu\nu} = 0$, and has $20 - 6$ degrees of freedom.
- $J_{\alpha\dot{\alpha}}^\mu$, the supercurrent ($[1, 1, 0]_{11/2}^{(1,0)} = \mathbf{20} \cdot \mathbf{4}$). It also has a null state $\partial_\mu J_{\alpha\dot{\alpha}}^\mu = 0$, satisfies $(\bar{\gamma}_\mu)_{\dot{\alpha}}^\beta J_{\beta\dot{\beta}}^\mu = 0$, and contains $80 - 16$ degrees of freedom.⁴
- $j^{\mu[IJ]}$, the R-current ($[0, 1, 0]_5^{(2,0)} = \mathbf{6} \cdot \mathbf{10}$). It has a null state $\partial_\mu j^{\mu IJ} = 0$, and contains $60 - 10$ degrees of freedom.
- $H_{\mu\nu\rho}^I$, a self-dual 3-form ($[2, 0, 0]_5^{(0,1)} = \mathbf{10} \cdot \mathbf{5}$) containing 50 degrees of freedom.

³These Dynkin labels are related to the usual $\mathfrak{so}(1, 5)$ and $\mathfrak{so}(5)$ labels by

$$[j_1, j_2, j_3]_{\mathfrak{su}(4)} = [j_2, j_1, j_3]_{\mathfrak{so}(1,5)}, \quad (R_1, R_2)_{\mathfrak{sp}(2)} = (R_2, R_1)_{\mathfrak{so}(5)}.$$

⁴Note that J transforms in the $[1, 1, 0]$ irrep. Since the tensor product of a vector and a chiral spinor decomposes into $[1, 1, 0] \oplus [0, 0, 1]$, we can write J with indices μ and α , provided we project out the antichiral spinor by requiring $(\bar{\gamma}_\mu)_{\dot{\alpha}}^\beta J_{\beta\dot{\beta}}^\mu = 0$.

- $\chi_{\alpha\check{\alpha}}^I$, a fermion $([1, 0, 0]_{9/2}^{(1,1)} = \mathbf{4} \cdot \mathbf{16})$ satisfying $(\check{\gamma}_I)_{\check{\alpha}}^{\check{\beta}} \chi_{\beta\check{\beta}}^I = 0$ and containing 64 degrees of freedom.
- O^{IJ} , a scalar $([0, 0, 0]_4^{(0,2)} = \mathbf{14})$ with 14 degrees of freedom. It is the superprimary of the multiplet.

Together with their descendants, these form an on-shell multiplet with 128 bosonic operators (and a matching number of fermionic operators).

In addition to the operator content, we need below the explicit supersymmetry transformations, which have not been calculated before to the best of our knowledge. These can be obtained in a variety of ways (e.g. oscillator constructions [45] and superspace transformations [48, 49]), but here we simply list the terms allowed by Lorentz and R-symmetry and fix the coefficients by requiring closure of the algebra, i.e. imposing that on every operator $\{Q, Q\} \Phi = 2P\Phi$. Importantly, imposing this condition is made easy because we already know the operator content.

We start from the superprimary O^{IJ} . Since Q transforms as $[1, 0, 0]_{1/2}^{(1,0)}$, we know from representation theory that the product QO can contain

$$[1, 0, 0]_{9/2}^{(1,2)} \oplus [1, 0, 0]_{9/2}^{(1,1)}, \tag{3.3}$$

but as $[1, 0, 0]_{9/2}^{(1,2)}$ does not appear in the multiplet, we remove it. The remaining term $[1, 0, 0]_{9/2}^{(1,1)}$ can be constructed explicitly and is fixed up to a constant c_1

$$Q_{\alpha\check{\alpha}} O^{IJ} = c_1 (\check{\gamma}^{IJ} \chi^J)_{\alpha\check{\alpha}}. \tag{3.4}$$

The transformation of χ is more complicated but the same analysis leads to

$$Q_{\alpha\check{\alpha}} \chi_{\beta\check{\beta}}^I = c_2 (\gamma^{\mu\nu\rho})_{\alpha\beta} (\check{\gamma}^{IJ} + 4\delta^{IJ})_{\check{\alpha}\check{\beta}} H_{\mu\nu\rho}^J + c_3 (\gamma_\mu)_{\alpha\beta} (\check{\gamma}^{IJK} + 3\delta^{IJ} \check{\gamma}^K)_{\check{\alpha}\check{\beta}} j_{JK}^\mu + d_1 (\gamma^\mu)_{\alpha\beta} (\check{\gamma}^J)_{\check{\alpha}\check{\beta}} \partial_\mu O^{IJ}. \tag{3.5}$$

It is easy to check that

$$\{Q_{\alpha\check{\alpha}}, Q_{\beta\check{\beta}}\} O^{IJ} = 2c_1 d_1 (\gamma^\mu)_{\alpha\beta} \Omega_{\check{\alpha}\check{\beta}} \partial_\mu O^{IJ}, \tag{3.6}$$

so the algebra closes provided $c_1 d_1 = 1$ (we identify $P_\mu = \partial_\mu$, see (C.1)).

We can proceed this way for the full multiplet and build the supersymmetry transformations. Checking for closure of the algebra becomes a tedious (if straightforward) task

and is not very illuminating, so we omit the details. The end result is (with $\delta = \varepsilon^{\alpha\dot{\alpha}}\mathbf{Q}_{\alpha\dot{\alpha}}$)

$$\begin{aligned} \delta T^{\mu\nu} &= \frac{1}{2}\varepsilon\gamma^{\rho(\mu}\partial_\rho J^{\nu)}, \\ \delta J^\mu &= 2\varepsilon\gamma_\nu T^{\mu\nu} + \frac{2c_2}{5c_3}\left(6\eta^{\rho\mu}\left(\gamma^{\nu\sigma\lambda} + 3\eta^{\sigma\nu}\gamma^\lambda\right) - \eta^{\mu\nu}\gamma^{\rho\sigma\lambda}\right)\check{\gamma}_I\partial_\nu H_{\rho\sigma\lambda}^I \\ &\quad + \frac{1}{10}\varepsilon\left(\gamma^{\mu\nu\rho} - 4\eta^{\mu\rho}\gamma^\nu\right)\check{\gamma}^{IJ}\partial_\nu j_{\rho IJ}, \\ \delta j_{IJ}^\mu &= -\frac{1}{2}\varepsilon\check{\gamma}_{IJ}J^\mu + \frac{1}{5c_3}\varepsilon\gamma^{\mu\nu}\partial_\nu\check{\gamma}_{[I\chi J]}, \\ \delta H_{\mu\nu\rho}^I &= \frac{c_3}{8c_2}\varepsilon\check{\gamma}^J\gamma_{[\mu\nu}J_{\rho]} + \frac{1}{120c_2}\varepsilon\gamma_\sigma\bar{\gamma}_{\mu\nu\rho}\partial^\sigma\chi^I, \\ \delta\chi^I &= c_2\varepsilon\gamma^{\mu\nu\rho}\left(\check{\gamma}^{IJ} + 4\delta^{IJ}\right)H_{\mu\nu\rho}^J + c_3\varepsilon\gamma_\mu\left(\check{\gamma}^{IJK} + 3\delta^{IJ}\check{\gamma}^K\right)j_{JK}^\mu \\ &\quad + \frac{1}{c_1}\varepsilon\gamma^\mu\check{\gamma}^J\partial_\mu O^{IJ}, \\ \delta O^{IJ} &= c_1\varepsilon\check{\gamma}^{(I}\chi^{J)}. \end{aligned} \tag{3.7}$$

There are still some arbitrary constants c_i that remain unfixed and can be absorbed into the normalisations of O , χ and H . On the other hand, the normalisation of the conserved currents must match that of the algebra, so these operators cannot be rescaled. This can be seen by checking that the variation of the currents reproduces the corresponding commutator in (D.3). For example, the variation of j^μ computed using (3.7) is

$$\int \mathbf{Q}_{\alpha\dot{\alpha}} j_{IJ}^0 d^5x = -\frac{1}{2}\int \left(\check{\gamma}_{IJ}J^0\right)_{\alpha\dot{\alpha}} d^5x = -\frac{1}{2}\left(\check{\gamma}_{IJ}\mathbf{Q}\right)_{\alpha\dot{\alpha}}, \tag{3.8}$$

which is indeed the correct normalisation for the commutator $[\mathbf{Q}_{\alpha\dot{\alpha}}, \mathbf{R}_{IJ}]$ of (D.3).

3.2 Defect without insertions

Among the operators of the stress tensor multiplet, some can acquire an expectation value in the presence of V . For the stress tensor, this happens when $h_T \neq 0$ in (1.3), and we can similarly constrain the 1-point functions of the other operators. This computation is done explicitly in appendix C and the only nonvanishing correlators are

$$\begin{aligned} \langle T^{ab}V \rangle &= \frac{h_T\eta^{ab}}{\pi^3x^6}, & \langle T^{mn}V \rangle &= -\frac{h_T}{\pi^3x^6}\left(\delta^{mn} - 2\frac{x^m x^n}{x^2}\right), \\ \langle H_{01m}^5V \rangle &= \frac{h_H x_m}{\pi^3x^6}, & \langle H_{lmn}^5V \rangle &= -\frac{h_H\varepsilon_{lmnp}x^p}{\pi^3x^6}, \\ \langle O^{55}V \rangle &= \frac{h_O}{\pi^3x^4}, & \langle O^{ij}V \rangle &= -\frac{h_O\delta^{ij}}{4\pi^3x^4}, \end{aligned} \tag{3.9}$$

where h_O , h_H , and h_T are as yet undetermined constants. They are however related by the supersymmetry transformations (3.7) derived above. Specifically, consider the Ward identities associated with the preserved supersymmetries $\mathbf{Q}^+ = \Pi_+\mathbf{Q}$ (with the projector

Π_+ defined in (D.12))

$$\begin{aligned}
 0 &= \langle \mathbf{Q}_{\alpha\check{\alpha}}^+(\chi_{\beta\check{\beta}}^5 V) \rangle = -4 \left(12c_2 h_H + \frac{h_O}{c_1} \right) \frac{[\Pi_+ \gamma_m x^m \check{\gamma}^5]_{\alpha\check{\alpha}\beta\check{\beta}}}{\pi^3 x^6}, \\
 0 &= \langle \mathbf{Q}_{\alpha\check{\alpha}}^+(J_{\beta\check{\beta}}^a V) \rangle = 2 \left(h_T + \frac{36c_2}{5c_3} h_H \right) \frac{[\Pi_+ \gamma^a]_{\alpha\check{\alpha}\beta\check{\beta}}}{\pi^3 x^6}.
 \end{aligned} \tag{3.10}$$

These equations fix

$$h_O = -12c_1 c_2 h_H = \frac{5}{3} c_1 c_3 h_T, \tag{3.11}$$

and the correlators in (3.9) are fixed up to a single constant h_T .

3.3 Defect with an insertion

We are now in a position to derive the result (1.4) by relating $\langle O^{i5} V[\mathbb{O}^j] \rangle$ to $\langle T^{am} V[\mathbb{D}^n] \rangle$ using superconformal Ward identities. There are two Ward identities to consider, $\langle \mathbf{Q}^+ \chi V[\mathbb{O}] \rangle = 0$ and $\langle \mathbf{Q}^+ J V[\mathbb{D}] \rangle = 0$, but one can check that they yield the same constraint, so we present only the first one.

The correlators we need are derived in appendix C by using the constraints of conformal symmetry. Importantly, the correlators $\langle OV[\mathbb{O}] \rangle$, $\langle \chi V[\mathbb{Q}] \rangle$ and $\langle HV[\mathbb{O}] \rangle$ are related to h_T by integrated relations like (3.2), while $\langle jV[\mathbb{O}] \rangle$ is related to $C_{\mathbb{D}}$ by (3.1), as we show below. They are

$$\begin{aligned}
 \langle O^{i5} V[\mathbb{O}^j] \rangle &= \frac{C_{O\mathbb{O}} \delta^{ij}}{x^2 (\sigma^2 + x^2)^2}, & \langle \chi_{\alpha\check{\alpha}}^5 V[\mathbb{Q}_{\beta\check{\beta}}] \rangle &= \frac{C_{\chi\mathbb{Q}} [\check{\gamma}^5 (\gamma_a \sigma^a + \gamma_m x^m) \Pi_- c \Omega]_{\alpha\beta\check{\alpha}\check{\beta}}}{x^2 (x^2 + \sigma^2)^3}, \\
 \langle j_a^{i5} V[\mathbb{O}^j] \rangle &= \frac{C_{j\mathbb{O}} \delta^{ij} \sigma_a}{x^2 (\sigma^2 + x^2)^3}, & \langle j_m^{i5} V[\mathbb{O}^j] \rangle &= \frac{C_{j\mathbb{O}} \delta^{ij} (x^2 - \sigma^2) x_m}{2x^4 (\sigma^2 + x^2)^3}, \\
 \langle H_{01m}^i V[\mathbb{O}^j] \rangle &= \frac{C_{H\mathbb{O}} \delta^{ij} x_m}{x^2 (\sigma^2 + x^2)^3}, & \langle H_{lmn}^i V[\mathbb{O}^j] \rangle &= \frac{C_{H\mathbb{O}} \delta^{ij} \varepsilon_{lmnp} x^p}{x^2 (\sigma^2 + x^2)^3}.
 \end{aligned} \tag{3.12}$$

Explicitly, the Ward identity is

$$\begin{aligned}
 0 &= \langle \mathbf{Q}_{\alpha\check{\alpha}}^+(\chi_{\beta\check{\beta}}^5 V[\mathbb{O}^i]) \rangle \\
 &= 6c_2 [\Pi_+ \gamma^{01m} (\check{\gamma}^5_J + 4\delta_J^5)]_{\alpha\check{\alpha}\beta\check{\beta}} \langle H_{01m}^J V[\mathbb{O}^i] \rangle + 6c_2 [\Pi_+ \gamma^{lmn} (\check{\gamma}^5_J + 4\delta_J^5)]_{\alpha\check{\alpha}\beta\check{\beta}} \langle H_{lmn}^J V[\mathbb{O}^i] \rangle \\
 &\quad + 3c_3 [\Pi_+ \gamma^\mu \check{\gamma}_j]_{\alpha\check{\alpha}\beta\check{\beta}} \langle j_\mu^{5j} V[\mathbb{O}^i] \rangle + \frac{1}{c_1} [\Pi_+ \gamma^\mu \check{\gamma}_J]_{\alpha\check{\alpha}\beta\check{\beta}} \partial_\mu \langle O^{5J} V[\mathbb{O}^i] \rangle \\
 &\quad + \frac{1}{2} (\check{\gamma}^{i5})_{\check{\alpha}} \check{\gamma} \langle \chi_{\beta\check{\beta}}^5 V[\mathbb{Q}_{\alpha\check{\alpha}}] \rangle.
 \end{aligned} \tag{3.13}$$

Plugging in the explicit forms of these correlators (3.12), and demanding that the terms proportional to $\gamma_a \sigma^a$ vanish, we obtain a linear relation

$$0 = 3c_3 C_{j\mathbb{O}} + \frac{4}{c_1} C_{O\mathbb{O}} - C_{\chi\mathbb{Q}}. \tag{3.14}$$

The terms proportional $\gamma_m x^m$ give the same constraint.

Next, recall that \mathbb{O} and \mathbb{Q} respectively encode the action of a broken infinitesimal R-symmetry or supersymmetry variation. Therefore we can relate

$$0 = \langle \mathbb{R}_{j5}(O^{i5}(x)V) \rangle = \delta^{ij} \langle O^{55}(x)V \rangle - \langle O^{ij}(x)V \rangle + \int d^2\sigma \langle O^{i5}(0,x)V[\mathbb{O}^j](\sigma) \rangle. \quad (3.15)$$

Using (3.9) and (3.12), we obtain

$$C_{O\mathbb{O}} = -\frac{5}{4\pi^4} h_O = -\frac{25c_1c_3}{12\pi^4} h_T. \quad (3.16)$$

A slightly more involved but entirely analogous calculation yields

$$C_{\chi\mathbb{Q}} = -\frac{5 \cdot 8}{3\pi^4} h_T, \quad C_{H\mathbb{O}} = \frac{5c_3}{36c_2\pi^4} h_T. \quad (3.17)$$

Finally, $C_{j\mathbb{O}}$ is related to the normalisation of the displacement operator multiplet by (2.1)

$$\partial_\mu \langle j^{\mu i5}(\sigma, x)V[\mathbb{O}^j(0)] \rangle = \langle V[\mathbb{O}^i(0)\mathbb{O}^j(\sigma)] \rangle \delta^{(4)}(x). \quad (3.18)$$

Plugging the correlator of $j^{\mu i5}$ and \mathbb{O}^j into the right hand side and integrating against a test function allows us to fix

$$C_{j\mathbb{O}} = -\frac{1}{\pi^4} C_{\mathbb{O}} = -\frac{1}{16\pi^4} C_{\mathbb{D}}. \quad (3.19)$$

Combining the above results into (3.14), we obtain

$$\frac{c_3}{\pi^4} (3C_{\mathbb{O}} - 5h_T) = 0 \quad \implies \quad h_T = \frac{3C_{\mathbb{O}}}{5} = \frac{3C_{\mathbb{D}}}{80}, \quad (3.20)$$

which proves (1.4).

4 Relation to anomaly coefficients

In this section we explore the consequences of the relation between the coefficients $C_{\mathbb{D}}$ and h_T (1.4) for physical observables. These pieces of dCFT data appear in the Weyl anomaly of surface operators as defined in (1.5), and as we show below the relations (2.6) and (3.20) relate the anomaly coefficients as (1.6).

The relation between correlators and anomaly coefficients is not specific to 2d defects in the $\mathcal{N} = (2, 0)$ theory, but applies for any surface operator in a CFT. The anomaly coefficient b_1 was first shown to be related to $C_{\mathbb{D}}$ in [19], while the relation between b_2 and h_T was obtained in [18, 19]. Here we review their derivation and apply it to surface operators in the (2,0) theory to prove $c = -b_1/2$, $b_1 = -b_2$.

In a slightly different direction, the anomaly coefficients also feature notably in entanglement entropy in 4d [50] and were discussed in the entanglement entropy literature, see [19] and references therein.

4.1 Displacement operator

In order to isolate the contribution of $C_{\mathbb{D}}$ to the anomaly coefficients, we separately switch on each of the terms in (1.5). Since the displacement operator generates geometric deformations, one expects that inserting sufficiently many \mathbb{D}^m into the planar surface operator V leads to a logarithmic divergence in the expectation value, signalling a conformal anomaly associated to the curvature of the surface. Similarly, inserting \mathbb{O}^i to sufficient order will allow us to access the anomaly coefficient c associated with deformations in R-symmetry space.

To make this relation precise, we formally write deformations of the 1/2 BPS plane in terms of operator insertions

$$V_{\xi,\omega} = \exp \left[\int d^2\sigma \xi_m(\sigma) P^m + \omega_i(\sigma) R^{i5} \right] V. \tag{4.1}$$

Here $P^m = \int d^4x \partial_\mu T^{\mu m}$ generates translations transverse to the defect, while R-symmetry rotations are generated by $R^{i5} = \int d^4x \partial_\mu j^{\mu i5}$. For constant parameters ξ, ω , the currents can be freely integrated and we recover the standard action of the charges P^m and R^{i5} .

equation (4.1) is generally a complicated expression involving contact terms like (1.1), but also contact terms from P^m acting on defect operators and possibly other operators from the OPE. We can calculate its expectation value to quadratic order by expanding the exponential and noting that the 1-point functions of defect operators vanish:

$$\log \langle V_{\xi,\omega} \rangle - \log \langle V \rangle = \frac{1}{2} \int_{\mathbb{R}^2 \times \mathbb{R}^2} \left(\langle V[\mathbb{D}_m \mathbb{D}_n] \rangle \xi^m \xi^n + \langle V[\mathbb{O}_i \mathbb{O}_j] \rangle \omega^i \omega^j \right) d^2\sigma d^2\sigma' + \text{cubic}. \tag{4.2}$$

We can discard $\log \langle V \rangle$ since for the 1/2-BPS plane in a flat background, all anomaly terms vanish separately. Since the anomaly is quadratic in ξ and ω , it is related to the two point functions written here and we can safely ignore the higher order terms in the expansion.

To extract the anomaly coefficients, we study the UV divergence of the integrals in (4.2). The relevant correlators are found in (1.2) and (2.3). Fixing σ , the σ' integral can be evaluated explicitly by Taylor expanding $\xi^m(\sigma')$ and $\omega^i(\sigma')$ around σ . Starting with the second integrand and using $\tau = \sigma' - \sigma$,

$$\begin{aligned} & \frac{1}{2} \int_{\mathbb{R}^2} \langle V[\mathbb{O}_i(\sigma) \mathbb{O}_j(\sigma')] \rangle \omega^i(\sigma) \omega^j(\sigma') d^2\sigma' \\ &= \frac{C_{\mathbb{O}}}{2\pi^2} \int_{\mathbb{R}^2} \frac{\delta_{ij}}{|\tau|^4} \omega^i(\sigma) \left[\omega^j(\sigma) + \tau^a \partial_a \omega^j(\sigma) + \frac{1}{2} \tau^a \tau^b \partial_a \partial_b \omega^j(\sigma) + \mathcal{O}(\tau^3) \right] d^2\tau. \end{aligned} \tag{4.3}$$

While this integral leads to power law singularities as well, a logarithmic divergence arises only from the term quadratic in τ . We adopt polar coordinates $\tau^a = \tau e^a$ where e^a are orthonormal vectors parametrised by an angle φ . Using the identities

$$\int d\varphi e^a e^b = \pi \eta^{ab}, \quad \int d\varphi e^a e^b e^c e^d = \frac{\pi}{4} (\eta^{ab} \eta^{cd} + \eta^{ac} \eta^{bd} + \eta^{ad} \eta^{bc}), \tag{4.4}$$

and dropping all but the logarithmic divergence, we obtain

$$\frac{C_{\mathbb{O}}}{4\pi^2} \pi \eta^{ab} \int_{\epsilon} \frac{\tau^3 d\tau}{\tau^4} \omega^i(\sigma) \partial_a \partial_b \omega^i(\sigma) = \frac{C_{\mathbb{O}}}{4\pi} (\partial\omega)^2 \log \epsilon. \tag{4.5}$$

To leading order, the R-symmetry transformation in (4.1) takes the 1/2-BPS plane to a surface operator with $\partial_a n^i(\sigma) = \partial_a \omega^i$, so we can read the anomaly coefficient as

$$c = C_{\mathbb{O}}. \quad (4.6)$$

The logarithmic divergence of the first integrand in (4.2) can be evaluated in a similar way, and arises only from the fourth order in the Taylor expansion of ξ^n

$$\begin{aligned} & \frac{1}{2} \int_{\mathbb{R}^2} \langle V[\mathbb{D}_m \mathbb{D}_n] \rangle \xi^m \xi^n d^2 \sigma' \\ &= \frac{C_{\mathbb{D}}}{2\pi^2} \int_{\mathbb{R}^2} \frac{\delta_{mn}}{|\tau|^6} \xi^m(\sigma) \left[\dots + \frac{1}{24} \tau^a \tau^b \tau^c \tau^d \partial_a \partial_b \partial_c \partial_d \xi^n(\sigma) + \mathcal{O}(\tau^5) \right] d^2 \tau. \end{aligned} \quad (4.7)$$

Performing the angular integral with (4.4) leads to

$$\frac{C_{\mathbb{D}}}{48\pi^2} \frac{3\pi^2}{4} \int_{\epsilon} \frac{\tau^5 d\tau}{\tau^6} \xi_m(\sigma) (\partial^2)^2 \xi^m(\sigma) = -\frac{C_{\mathbb{D}}}{64\pi} \partial^a \partial^b \xi_m(\sigma) \partial_a \partial_b \xi^m(\sigma) \log \epsilon. \quad (4.8)$$

This is the trace of the second fundamental form squared of the deformed surface (see (B.3)), which can be rewritten using the Gauss-Codazzi equation (B.4) as

$$\partial^a \partial^b \xi_m \partial_a \partial_b \xi^m = \mathbb{I}^2 = 2 \operatorname{tr} \tilde{\mathbb{I}}^2 + R^{\Sigma} - \operatorname{tr} W. \quad (4.9)$$

Since we are on flat space, the Weyl tensor vanishes. The volume form for the deformed surface gets corrected, but to leading order in ξ does not affect the calculation. Therefore the contribution of this term to the anomaly density is

$$-\frac{C_{\mathbb{D}}}{64\pi} \int_{\Sigma} \left(2 \operatorname{tr} \tilde{\mathbb{I}}^2 + R^{\Sigma} \right) \operatorname{vol}_{\Sigma} \log \epsilon. \quad (4.10)$$

Note that the integral of R^{Σ} vanishes for small deformations of the plane. It therefore does not contribute to the anomaly, and we find

$$b_1 = -C_{\mathbb{D}}/8. \quad (4.11)$$

Using (2.6) along with (4.6) and (4.11) we find a relation for the anomaly coefficients

$$c = -b_1/2. \quad (4.12)$$

4.2 Stress tensor

The relation between b_2 and h_T is derived in a similar fashion, but instead of deforming the surface itself, we can relate the insertion of a stress tensor to a change in the background geometry.⁵ The expectation value of the planar surface operator now receives a contribution from the metric variation:

$$\langle V \rangle_{\eta+\delta g} = \langle V \rangle_{\eta} - \frac{1}{2} \int_{\mathbb{R}^2 \times \mathbb{R}^4} \delta g_{\mu\nu}(\sigma, x) \langle T^{\mu\nu} V \rangle_{\eta} d^2 \sigma d^4 x + \mathcal{O}(\delta g^2). \quad (4.13)$$

⁵In the same way one can show that the bulk anomaly coefficients are related to the 2- and 3-point functions of the stress tensor [51].

In this equation, the subscript $\langle \bullet \rangle_g$ means the expectation value is calculated on a curved background metric g .

Since the insertion of a stress tensor sources a metric perturbation of linear order δg , we can only reproduce the anomaly to that order, which, expanding (1.5), is

$$\mathcal{A}|_{\delta g} = \frac{1}{4\pi} \left[-\frac{b_2}{10} \left(\partial_p^2 \delta^{mn} - \partial^m \partial^n \right) \delta g_{mn} + \frac{3b_2}{20} \eta^{ab} \partial_p^2 \delta g_{ab} + \partial_a(\dots) \right]. \quad (4.14)$$

These two terms are respectively associated to $\langle T^{mn} V \rangle$ and $\langle T^{ab} V \rangle$ in (4.13), and the total derivative drops out of the integral over the plane.

Using (3.9), we can evaluate the first term of (4.13). The logarithmic divergence arises as

$$\begin{aligned} \int_{\mathbb{R}^4} \delta g_{mn} \langle T^{mn} V \rangle d^4 x &= -\frac{h_T}{\pi^3} \int_{\mathbb{R}^4} d^4 x \delta g_{mn}(\sigma, x) \frac{\delta^{mn} - 2x^m x^n / x^2}{x^6} \\ &= -\frac{h_T}{\pi^3} \int_{\mathbb{R}^4} \frac{d^4 x}{x^6} \left(\dots + \frac{1}{2} \partial_{pq} \delta g_{mn}|_{x=0} x^p x^q + \dots \right) \left(\delta^{mn} - 2 \frac{x^m x^n}{x^2} \right). \end{aligned} \quad (4.15)$$

In the second step we expanded $\delta g(x)$ in a Taylor series and dropped powers of x not contributing to the anomaly. We again switch to spherical coordinates $x^m = r e^m$ and take note of the 4d analogue of (4.4)

$$\int \text{vol}_{S^3} e^m e^n = \frac{\pi^2}{2} \delta^{mn}, \quad \int \text{vol}_{S^3} e^m e^n e^p e^q = \frac{\pi^2}{12} (\delta^{mn} \delta^{pq} + \delta^{mp} \delta^{nq} + \delta^{mq} \delta^{np}). \quad (4.16)$$

The integral then becomes

$$-\frac{2\pi^2 h_T}{2\pi^3} \frac{2}{3} \int_{\epsilon} \frac{dr}{r} \partial_{pq} \delta g_{mn}|_{x=0} (\delta^{mn} \delta^{pq} - \delta^{mp} \delta^{nq}) = \frac{1}{4\pi} \log \epsilon \left[\frac{2h_T}{3} \left(\partial_p^2 \delta^{mn} - \partial^m \partial^n \right) \delta g_{mn} \right]_{x=0}. \quad (4.17)$$

Comparing against (4.14), we identify

$$h_T = \frac{3b_2}{10}. \quad (4.18)$$

The calculation for $\langle T^{ab} V \rangle$ is similar and gives the same result.

With expressions for b_1, b_2, c in terms of $C_{\mathbb{D}}$ and h_T in hand, we can finally translate the result of the previous section (3.20) into a constraint on the anomaly coefficients, and find

$$b_2 = -b_1, \quad (4.19)$$

as claimed.

A direct consequence of this relation (together with (4.12)) is that one only needs to calculate two nontrivial surface operators to calculate all the independent anomaly coefficients, for instance the sphere and cylinder.

5 Defect operator expansion

A useful tool in dCFT is the defect operator expansion (dOE), also known as the bulk-defect operator product expansion [37, 38] (see [13] for a recent review of some dCFT techniques, including the dOE, in the context of the CFT bootstrap program). This is a convergent expansion representing bulk operators in terms of insertions of defect operators

$$\mathcal{O}_i(\sigma, x)V = \sum_k \frac{C_{ik}^V(x, \partial_\sigma)}{x^{\Delta_i - \hat{\Delta}_k}} V[\hat{\mathcal{O}}_k(\sigma)], \quad (5.1)$$

where the sum is over defect primaries. The differential operators $C_{ik}^V(x, \partial_\sigma)$ are fixed by conformal symmetry. Their exact form can be obtained from the corresponding bulk-defect 2-point function of \mathcal{O}_i and $\hat{\mathcal{O}}_k$ by equating

$$\langle \mathcal{O}_i(\sigma, x)V[\hat{\mathcal{O}}_k(0)] \rangle = \sum_j \frac{C_{ij}^V(x, \partial_\sigma)}{x^{\Delta_i - \hat{\Delta}_j}} \langle V[\hat{\mathcal{O}}_j(\sigma)\hat{\mathcal{O}}_k(0)] \rangle = \frac{1}{x^{\Delta_i - \hat{\Delta}_k}} C_{ik}^V(x, \partial_\sigma) \frac{C_{\hat{\mathcal{O}}_k}}{\sigma^{2\hat{\Delta}_k}}, \quad (5.2)$$

where we denote by $C_{\hat{\mathcal{O}}_k}$ the numerator of the 2-point function of $\hat{\mathcal{O}}_k$. Explicit expressions for C_{ik}^V can be found in [22, 52], but are not needed in this paper.

The list of defect primaries appearing on the right-hand side of (5.1) can include the defect operators of section 3 (namely the defect identity and the displacement operator multiplet), but it certainly includes more defect operators. This can be viewed as a consequence of the associativity of the OPE: since (5.1) maps bulk operators to defect operators and is valid in any correlator, all the CFT data of the bulk operators must be encoded, in some way, in the OPE of defect operators. Hence there must be at least as many defect degrees of freedom as bulk degrees of freedom.

Here we initiate the study of these other defect operators. We first classify the unitary multiplets of defect operators in sections 5.1 and 5.2. This allows us to find the decomposition of the stress tensor multiplet in multiplets of the preserved algebra, see figures 1 and 2.

After this detour into representation theory, we write the leading terms in the dOE for some operators and discuss the appearance of a new marginal operator. We finally comment on constraints imposed by supersymmetry and show how the dOE sheds light on the derivation of section 3.

5.1 Representations of $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$

Defect operators sit in multiplets of the algebra preserved by the defect. For the 1/2-BPS plane V , the preserved algebra consists of 2 copies of $\mathfrak{osp}(4^*|2)$, so we are interested in constructing representations of $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$. The formulation of the algebra as a 2d superconformal algebra is reviewed in the appendix D.2, along with its embedding inside the bulk algebra $\mathfrak{osp}(8^*|4)$.

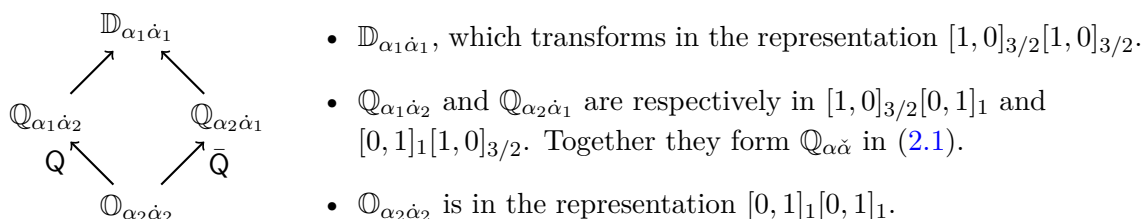
As usual, we can label primaries by their representation under the bosonic subalgebra, which here is

$$[\mathfrak{sl}(2) \oplus \mathfrak{su}(2)_\perp \oplus \mathfrak{su}(2)_R] \oplus [\mathfrak{sl}(2) \oplus \mathfrak{su}(2)_\perp \oplus \mathfrak{su}(2)_R]. \quad (5.3)$$

The corresponding labels are $[r_1, r_2]_h [\bar{r}_1, \bar{r}_2]_{\bar{h}}$, with r_1 and r_2 the Dynkin labels for $\mathfrak{su}(2)_\perp$ and $\mathfrak{su}(2)_R$, and h the conformal twist and labels representations of $\mathfrak{sl}(2)$. The labels \bar{r}_1 , \bar{r}_2 and \bar{h} are similar, but for the second subalgebra. We note that while (5.3) is equivalent to $\mathfrak{so}(2, 2)_\parallel \oplus \mathfrak{so}(4)_\perp \oplus \mathfrak{so}(4)_R$, the factorisation above in terms of 2 algebras is dictated by supersymmetry, see D.2 for more details. The joint representation has conformal dimension $\hat{\Delta} = h + \bar{h}$ and spin $s = h - \bar{h}$.

The simplest nontrivial example of a multiplet of $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$ is the familiar displacement multiplet of section 2. Unlike our previous treatment however, here we label operators according to (5.3). In order to match that decomposition, we can express the superprimary $\mathbb{O}^i \sim (\tilde{\gamma}^i)^{\alpha_2 \dot{\alpha}_2} \mathbb{O}_{\alpha_2 \dot{\alpha}_2}$ in spinor indices. In this notation, the indices $\alpha = 1, 2$ are all $\mathfrak{su}(2)$ indices. We use α_1, β_1, \dots for $\mathfrak{su}(2)_\perp$ and α_2, β_2, \dots for $\mathfrak{su}(2)_R$; similarly for the second set of $\mathfrak{su}(2)$'s, but with dotted indices.

The values of h and \bar{h} can also be read from (2.1), they are $h = \bar{h} = 1$ (\mathbb{O} is a scalar of dimension 2). The representation of \mathbb{O} is therefore $[0, 1]_1 [0, 1]_1$. Acting with \mathbb{Q} and $\bar{\mathbb{Q}}$ (which transform respectively as $[1, 1]_{1/2} [0, 0]_0$ and $[0, 0]_0 [1, 1]_{1/2}$), one can build the full multiplet:



The structure of the multiplet as a product of two representations of $\mathfrak{osp}(4^*|2)$ is apparent in the diagram above. Under the action of \mathbb{Q} , the operators transform as two multiplets of $\mathfrak{osp}(4^*|2)$, for instance the lower diagonal is

$$\mathbb{Q}_{\alpha_1 \alpha_2} \mathbb{O}_{\beta_2 \dot{\beta}_2} = c \epsilon_{\alpha_2 \beta_2} \mathbb{Q}_{\alpha_1 \dot{\beta}_2}, \quad \mathbb{Q}_{\alpha_1 \alpha_2} \mathbb{Q}_{\beta_1 \dot{\beta}_2} = i c^{-1} \epsilon_{\alpha_1 \beta_1} \partial \mathbb{O}_{\alpha_2 \dot{\beta}_2}, \quad (5.4)$$

which is easily obtained from an ansatz as in section 3.1 (the constant c is arbitrary). This is the simplest representation of $\mathfrak{osp}(4^*|2)$ and it contains the weights $[0, 1]_1$ and $[1, 0]_{3/2}$. Because it is ubiquitous, it is convenient to introduce some notation here and denote it $B[0, 1]$, in anticipation of the results of section 5.2.

5.2 Unitary multiplets of $\mathfrak{osp}(4^*|2)$

Since the algebra preserved by the defect factorises, we now turn our focus to general multiplets of a single copy of $\mathfrak{osp}(4^*|2)$. Importantly, we can classify allowed multiplets by working out the constraints imposed by unitarity.⁶ This follows the method described in [53] used to classify multiplets in superconformal theories for $d \geq 3$.

The idea is the following. In radial quantisation, any operator \mathcal{O} defines a corresponding state $|\mathcal{O}\rangle$. While $|\mathcal{O}\rangle$ has positive norm (by assumption), there is no guarantee that the norm of all the other states of the multiplet is also positive, as required by unitarity.

⁶These results appeared previously in [42]. We thank Pedro Liendo for pointing this out.

Demanding that negative norm states are absent from the multiplet leads to a lower bound on the conformal dimension of the superprimary $h \geq h_A$. In particular, as we show below, at $h = h_A$ (5.7) some states become null, and the corresponding multiplets are the short multiplets A . In addition, we find yet shorter multiplets B with superprimary of conformal dimension h_B (5.8).

Consider the state $|\mathcal{O}\rangle$ of a superprimary operator in the representation $[r_1, r_2]_h$. Unitarity constrains the states $Q|\mathcal{O}\rangle$ to satisfy

$$\|Q|\mathcal{O}\rangle\|^2 = \langle\mathcal{O}|\{S, Q\}|\mathcal{O}\rangle = \langle\mathcal{O}|D_+ + \sigma^i T_{(1)}^i - 2\sigma^j T_{(2)}^j|\mathcal{O}\rangle \geq 0, \quad (5.5)$$

where we use $Q_{\alpha_1\alpha_2}^\dagger = S^{\alpha_1\alpha_2}$ and the anticommutator (D.11), written in terms of $\mathfrak{su}(2)_\perp$ and $\mathfrak{su}(2)_R$ generators $T_{(1,2)}^i$. We suppress the indices of Q and $|\mathcal{O}\rangle$, but the constraint should hold for any choice of Q , $|\mathcal{O}\rangle$, and linear combinations thereof.

The matrix elements $\langle s|\sigma^i T^i|s\rangle$ are bounded by the eigenvalues of $\sigma^i T^i$. Since σ^i is the fundamental representation, the product $\sigma^i T^i$ can be decomposed as $[1] \otimes [r] = [r-1] \oplus [r+1]$, for both r_1 and r_2 . The eigenvalues are expressed in terms of the quadratic Casimirs $C_2(j) = j(j+2)/4$ (using e.g. equation (2.38) of [53]), so that (5.5) takes the form

$$h \geq -(C_2(j_1) - C_2(1) - C_2(r_1)) + 2(C_2(j_2) - C_2(1) - C_2(r_2)), \quad (5.6)$$

with j_1 and j_2 taking any values in $r_1 \pm 1$ and $r_2 \pm 1$. This assumes that both $r_1 > 0$ and $r_2 > 0$, otherwise the tensor product decomposition is simply $[1] \otimes [0] = [1]$ and $j = 1$.

For $r_1 > 0$, we then find that the strongest bound on the scaling dimension implied by (5.6) is

$$h \geq h_A = 1 + \frac{r_1}{2} + r_2. \quad (5.7)$$

For $r_1 = 0$, we should instead take $j_1 = 1$ and we obtain

$$h \geq h_B = r_2, \quad \text{if } r_1 = 0. \quad (5.8)$$

If these bounds are saturated, a subset of states become null and may be consistently removed from the multiplet.

While (5.7) and (5.8) are necessary conditions for unitarity, there could be, in principle, additional states whose norm becomes null (or negative), imposing further restrictions on h . It would be tedious to perform the above calculation for all states, but fortunately the conditions under which a representation is reducible (but not necessarily unitary) are listed by Kac in [54] (see also [55]). These match precisely the values obtained for the 4 choices of j_1 and j_2 in (5.6), which indicates that there are no further constraints.

We therefore conclude that for multiplets satisfying $h \geq h_A$, with h_A given in (5.7), there are no stronger constraints from requiring unitarity at higher levels. Generically, these are long multiplets, and they thus contain $2^4(r_1+1)(r_2+1)$ operators. Multiplets saturating the bound $h = h_A$ have a null state at level one, $\left|[r_1-1, r_2+1]_{h+1/2}\right\rangle$, and their dimension is reduced. The special case $r_1 = 0$ still leads to a unitary multiplet, but in this case the first null state is at level 2.

In the case $h_A > h \geq h_B$ (5.8) however, since h is below the unitarity bound h_A , some states in the multiplet would have a negative norm unless $h = h_B$ exactly: this is an isolated multiplet. It has a null state at level one, $\left| [1, r_2 + 1]_{h+1/2} \right\rangle$.

These short multiplets A and B are important to our discussion. For example, the $B[0, 1]$ multiplet of section 5.1 contains only 2+2 operators, so it is indeed a short multiplet. From the argumentation above, the conformal dimension of its superprimary is thus fixed by unitarity to $h = h_B = 1$, in accordance with (2.1).

The broader question of determining the content of all short multiplets is interesting but lies beyond the scope of this work. However, specific short multiplets play a role in section 5.3, and it is useful to know their content explicitly. It is sufficient for our present purposes to construct some representations heuristically by taking the tensor product decomposition of known multiplets. For instance, taking the product of two $B[0, 1]$ multiplets, the superprimary decomposes into 2 representations $[0, 1] \otimes [0, 1] = [0, 0] \oplus [0, 2]$, so the tensor product gives 2 multiplets, which we identify as

$$B[0, 1] \otimes B[0, 1] = A[0, 0] \oplus B[0, 2]. \tag{5.9}$$

The multiplet $A[0, 0]$ contains the weights $[0, 0]_1, [1, 1]_{3/2}$ and $[2, 0]_2$, while the multiplet $B[0, 2]$ contains $[0, 2]_2, [1, 1]_{5/2}$ and $[0, 0]_3$. Both of these representations appear as defect operators, see figures 1 and 2 below.

5.3 The stress tensor dOE

Having gained some understanding of representations of the preserved algebra, we turn now to the main goal of this section: constructing the dOE (5.1) for the bulk operators of our theory. We focus on operators of the stress tensor multiplet (which should exist in any local quantum field theory), but the same analysis could be applied to other multiplets.

A naive way of thinking about (5.1) is as branching rules for the breaking of symmetry due to the presence of the defect. Indeed, it is natural to decompose, for example, the bulk superprimary O^{IJ} into representations of the preserved R-symmetry O^{55}, O^{i5} and O^{ij} , respectively the representations

$$[0, 0][0, 0], \quad [0, 1][0, 1], \quad [0, 2][0, 2]. \tag{5.10}$$

The dOE (5.1) is particularly simple for a trivial surface defect, where it is just the Taylor expansion of the bulk insertion:

$$O^{55}(x)I = I[O^{55}(0) + x^m \partial_m O^{55}(0) + \dots], \tag{5.11}$$

While this expression merely amounts to a rewriting of the bulk degrees of freedom, the dOE becomes much more interesting if we consider a defect V which interacts with the bulk nontrivially.

A first sign that the dOE for general V contains additional terms is that the bulk operators couple to the defect identity $\mathbf{1}_V$ and the displacement multiplet (cf. for instance (3.9) and (3.12)). It is clear that these operators do not appear in the branching rules and encode additional interactions between bulk and defect degrees of freedom.

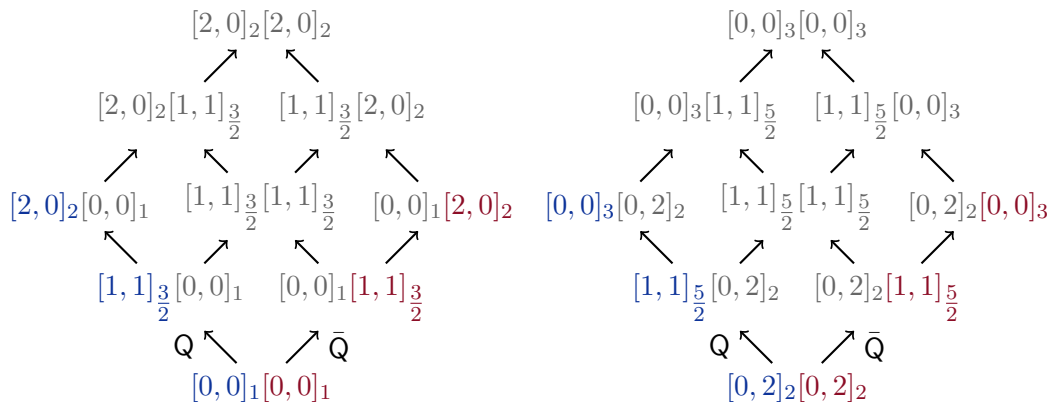


Figure 1. On the left, the $A[0,0]A[0,0]$ multiplet containing $32 + 32$ degrees of freedom. Its superprimary is \hat{O}^{55} . On the right, the $B[0,2]B[0,2]$ multiplet also containing $32+32$ degrees of freedom. Its superprimary is \hat{O}^{ij} .

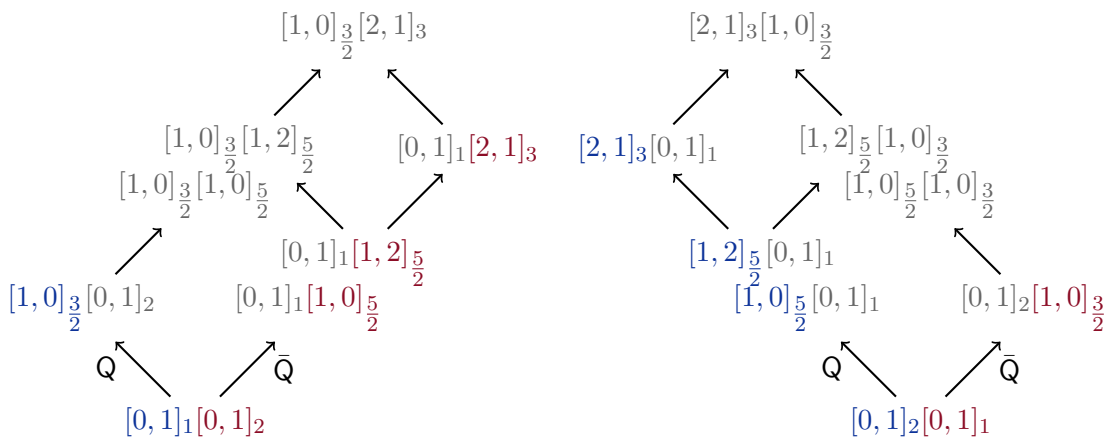


Figure 2. Multiplets $B[0,1]A[0,1]$ and $A[0,1]B[0,1]$. They both contain $32+32$ degrees of freedom.

The second way in which the dOE is interesting is more subtle. The decomposition of operators in terms of the preserved algebra can be performed, as above, for all the operators in the stress tensor multiplet. The resulting representations can be organised in the multiplets of figures 1 and 2 and the displacement multiplet, leading to the branching rules under the breaking of symmetry $\mathfrak{osp}(8^*|4) \rightarrow \mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$. The superprimaries of the multiplets in figure 1 are easily identified as the defect counterparts of the operators O^{55} and O^{ij} by their representation, and with a bit of work this correspondence between bulk and defect operators can be also established for all the other operators.

Observe that the conformal dimension of these defect operators is, in some cases, lower than that of the corresponding bulk operators, leading to singular terms in the dOE. For instance, the dimension of \hat{O}^{55} is 2, whereas the dimension of O^{55} is 4. A similar behavior occurs in the context of Wilson loops in 4d $\mathcal{N} = 4$ SYM, where the 1/2-BPS line operator takes the form

$$W \sim \text{tr} \mathcal{P} \exp i \int (A_\tau + \Phi^6) d\tau. \tag{5.12}$$

In that case, the dOE of the stress tensor superprimary includes a defect operator of dimension 1, which can be understood as the insertion of Φ^6 in the line. Here, we do not have a field realisation of the $\mathcal{N} = (2, 0)$ theory but \hat{O}^{55} plays an analogous role.

Consider then the dOE for O^{55} . From figures 1 and 2 we know some of the defect operators that can appear on the right-hand side of (5.1). This leads to

$$O^{55}(x)V = \frac{1}{x^4}C_{O\mathbf{1}}^V V[\mathbf{1}_V] + \frac{1}{x^2}C_{O\hat{O}}^V(x, \partial_\sigma)V[\hat{O}^{55}] + \frac{x_m}{x^2}C_{O\mathbb{D}}^V(x, \partial_\sigma)V[\mathbb{D}^m] + \dots \quad (5.13)$$

The list of defect operators that may appear in this expansion is constrained by supersymmetry and can be treated systematically, but we do not pursue this direction further.

equation (5.13) can be made more precise. The coefficients of the defect primaries encode the normalisation of bulk-defect correlators as in (5.2): 1-point functions such as (3.9) compute the coefficient of $\mathbf{1}_V$, 2-point functions such as (3.12) capture the coefficients of other defect primaries. Explicitly, $\langle O^{55}(x)V \rangle$ calculates the defect identity component of the dOE, such that

$$C_{O\mathbf{1}}^V = \frac{h_O}{\pi^3}. \quad (5.14)$$

The coefficient of the displacement operator can be found without computing $\langle O^{55}V[\mathbb{D}^m] \rangle$ explicitly, using the fact that the displacement operator is related to the broken translation symmetry. Integrating over the position of \mathbb{D}^m , we can replace it by a derivative:

$$\int d^2\sigma \langle O^{55}(x)V[\mathbb{D}^m(\sigma)] \rangle = -\partial^m \langle O^{55}(x)V \rangle. \quad (5.15)$$

The left hand side is easily computed from (5.13) and related to $C_{\mathbb{D}}$ and $C_{O\mathbb{D}}^V$, while the right hand side is given in terms of h_O . Matching coefficients, we find

$$C_{O\mathbb{D}}^V(x, \partial_\sigma) = \frac{8h_O}{\pi^4 C_{\mathbb{D}}} (1 + \dots). \quad (5.16)$$

By contrast, the coefficient $C_{O\hat{O}}^V$ is not obviously related to the remaining coefficients, and thus an independent piece of dCFT data.

5.4 Constraints from supersymmetry

We conclude this section by sketching an alternative derivation of the results of section 3. It turns out that the dOE provides a simple and elegant way to understand the origin of the linear relations (3.10) and (3.14) without doing explicit calculations, by reframing them in terms of coefficients of displacement primaries in the stress tensor dOE. Indeed, the method we use can in principle be applied far more generally to obtain analogous constraints for the remaining dOE coefficients.

To reproduce these results, consider the dOE of χ^5 . Following the analysis of section 5.3, we decompose χ^5 into representations of the preserved algebra

$$[1, 1][0, 0] \oplus [1, 0][0, 1] \oplus [0, 1][1, 0] \oplus [0, 0][1, 1], \quad (5.17)$$

which we label $\chi_{\alpha_1\alpha_2}^5, \chi_{\alpha_1\dot{\alpha}_2}^5, \chi_{\dot{\alpha}_1\alpha_2}^5, \chi_{\dot{\alpha}_1\dot{\alpha}_2}^5$. We only need the dOE of $\chi_{\alpha_1\dot{\alpha}_2}^5$, which takes the form

$$\chi_{\alpha_1\dot{\alpha}_2}^5 V = \frac{1}{x^2} C_{\chi Q}^V(x, \partial_\sigma) V[\mathbb{Q}_{\alpha_1\dot{\alpha}_2}] + \dots \tag{5.18}$$

Again, there are other terms that could be included in this expansion, but they don't play a role in what follows so we ignore them. We also emphasise that (5.18) is related to the dOE of the stress tensor superprimary by supersymmetry.

We can now proceed as in section 3 and find the constraints imposed by the preserved supersymmetries. Consider first acting with \mathbb{Q} on the bulk operator $\chi_{\alpha_1\dot{\alpha}_2}^5$ to find

$$\mathbb{Q}\chi = H + j + \partial O, \tag{5.19}$$

with some coefficients. (The exact expression can be obtained by restricting (3.7) to the relevant representations of the preserved algebra.) Using the dOE on the right-hand side and focusing on the defect identity component gives

$$(\mathbb{Q}\chi(x))V \sim (H(x) + j(x) + \partial O(x))V \sim \frac{1}{x^5} (C_{H\mathbf{1}} + C_{j\mathbf{1}} + C_{\partial O\mathbf{1}}) V[\mathbf{1}_V] + \dots \tag{5.20}$$

Note that $C_{j\mathbf{1}} = 0$ and $C_{\partial O\mathbf{1}}$ can be obtained from (5.15). We call this the ‘‘bulk’’ channel, since we calculate the action of \mathbb{Q} on χ before taking the dOE.

The expression (5.20) is to be contrasted with the ‘‘defect’’ channel, where we first use (5.18) and then apply \mathbb{Q} . Clearly, since $\mathbf{1}_V$ is not the variation of anything $\mathbf{1}_V \neq \mathbb{Q}(\dots)$, the result does not have an identity component. Consequently, the identity component of (5.20) must vanish as well, giving a linear constraint equivalent to (3.10) relating the normalisations of the stress tensor 1-point functions.

Similarly, (3.14) can be reproduced by focusing on the scalar displacement component of the same equation. The bulk channel gives schematically

$$\mathbb{Q}\chi V \sim \frac{1}{x^3} (C_{H\mathbb{O}}^V + C_{j\mathbb{O}}^V + C_{\partial O\mathbb{O}}^V) V[\mathbb{O}] + \dots \tag{5.21}$$

For the defect channel, we act on (5.18) with \mathbb{Q} . From (5.4), we see that the variation only leads to descendants like $\partial\mathbb{O}$, and no primary. Since equality between defect and bulk channel must hold at the level of each defect operator, we conclude that the contribution of the displacement superprimary \mathbb{O} to the bulk channel must vanish, and we obtain a linear constraint on the dOE coefficients $C_{j\mathbb{O}}^V, C_{H\mathbb{O}}^V, C_{\partial O\mathbb{O}}^V$, which is equivalent to (3.14). These two relations are only the simplest examples of a much larger set of constraints obeyed by the dOE coefficients. Indeed, equating the bulk and defect channel of any supercharge acting on any primary dOE at the level of each defect operator, it is straightforward to derive further such linear relations. These conditions greatly reduce the number of independent coefficients of stress tensor dOE coefficients, until we are left with what we could call a super-dOE, i.e. a set of dOEs which is fully consistent under the preserved supersymmetry.

6 Conclusion

In this paper, we initiate the application of defect CFT techniques to describe surface operators of the 6d $\mathcal{N} = (2, 0)$ theory, that is, we apply the ideas and tools of CFT to study local operator insertions into the 1/2-BPS plane. An important insertion is the displacement operator (1.1) which literally deforms the plane, but there are also other defect operators corresponding to inserting bulk operators near the defect—they are captured by the dOE (5.1).

One of our results is the construction of unitary multiplets of $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$, the algebra preserved by a 1/2-BPS defect, in section 5.2. These multiplets are the building blocks for discussing other aspects of the dCFT, like its spectrum, the OPE of defect operators and the dOE. In this work we focus on the dOE, but it would also be interesting to pursue these other directions, for instance using the tools of conformal bootstrap [13].

There are two important applications of the dOE (5.1) in our analysis: in section 5.3 we use it to find new defect operators and in section 5.4 we sketch how it makes the preserved symmetries manifest.

First, we use it to give the example of how the bulk stress tensor multiplet decomposes into defect multiplets. There are of course the operators \mathbb{D} , \mathbb{Q} and \mathbb{O} of the displacement multiplet, but also other defect multiplets whose operator content is shown in figure 1 and 2. Although we focus on the stress tensor multiplet, this analysis could also be applied to any other multiplet of the $\mathcal{N} = (2, 0)$ theory. In addition to the multiplets presented above, the dOE can include additional terms, and it would be interesting to obtain the selection rules as was done for 4d $\mathcal{N} = 4$ SYM [56], by treating systematically all the superconformal Ward identities.

The important aspect of this decomposition of bulk operators is that it is convergent. In particular, it encodes all the information of the bulk OPE, which opens the possibility of studying the $\mathcal{N} = (2, 0)$ theory from the point of view of a 2d defect CFT. This direction could lead to additional constraints on the bulk theory, since the defect operators are not a trivial rewriting of those in the bulk. This is manifested for instance by the appearance of divergences in the dOE of O^{55} (5.13).

Instead, the dOE captures some important reorganisation of degrees of freedom in the dCFT. For instance, in the expansion of the bulk operator O^{55} (5.13) we find a defect operator which is of dimension 2 and therefore marginal (we expect it to be marginally irrelevant). The analogous expansion of the superprimary of the stress tensor multiplet is well understood in the context of Wilson loops in 4d $\mathcal{N} = 4$ SYM: using the definition of the 1/2-BPS Wilson loop (5.12) the marginal operator there corresponds to inserting Φ^6 into the line defect [39]. Here the interpretation is similar: inserting the analog of \hat{O}^{55} in the non-supersymmetric surface operator triggers an RG flow which comes to a stop when \hat{O}^{55} becomes marginal at the conformal fixed point, which is the 1/2-BPS surface operator. This flow is verified in holography [31] and should hold more generally for all $\mathcal{N} = (2, 0)$ theories.

A second use of the dOE is to make the preserved symmetries manifest. As we sketch in section 5.4, we can explain the origin of the relation between h_T and $C_{\mathbb{D}}$ (1.4) simply

by looking at the structure of multiplets of defect operators. This is to be contrasted with the derivation of section 3, where the relation is the result of a calculation and not obvious from the outset. We believe this approach could shed light on determining the minimal amount of supersymmetry required to prove (1.4), that is whether it also holds for defects of the $\mathcal{N} = (1, 0)$ theory, and more generally what are the necessary conditions to prove the conjecture of [25].

In addition to the methods, the result (1.4) and the corresponding relation between the anomaly coefficients (1.6) are themselves interesting. In the context of Wilson loops, $C_{\mathbb{D}}$ was shown to appear in the Bremsstrahlung function [21] and h_T both in the radiation emitted by a quark undergoing constant acceleration and the entanglement entropy associated with Wilson lines [25]. While these calculations can be generalised to the case of surface operators, they do not give a finite answer: as shown in section 4 respectively inserting displacement operators in the defect or introducing a stress tensor in its vicinity leads to a logarithmic divergence caused by an anomaly. To obtain a finite result, one can define a renormalised surface operator

$$V_\epsilon = \epsilon^{-\int_{\mathcal{A}_\Sigma} d^2\sigma} V, \tag{6.1}$$

so that V_ϵ effectively acquires a dimension (\mathcal{A}_Σ is defined in (1.5)). The interpretation of $C_{\mathbb{D}}$ and h_T are then as the anomaly coefficients b_1 and b_2 , which are the relations (4.11), (4.18) (also (4.6) between c and $C_{\mathbb{0}}$). The net result of the relations between the anomaly coefficients (1.6) is that the anomaly depends on the geometry only through the combination $(H^2 + 4 \text{tr} P) - (\partial n)^2$ (defined in (B.4)), while the anomaly term R^Σ integrates to a topological invariant, the Euler characteristic of the surface Σ . We note that for some classes of BPS operators, H^2 and $(\partial n)^2$ are related and either cancel out or give interesting quantities. A simple example is the uplift of the BPS Wilson loops of [57] for which the anomaly vanishes, but more examples will be presented elsewhere [?].

Finally, there are other interesting directions which we haven't explored in this paper. For the Wilson line, a point of confluence between different techniques is the cusp, whose anomalous dimension at small angles is related to the Bremsstrahlung function [21] and can be calculated using integrability [59–61] and supersymmetric localization [62]. Its analog here are conical singularities which exhibit a peculiar $\log^2 \epsilon$ divergence, see [8, 63–66]. The coefficient of the divergence is entirely fixed by the behavior of the surface near the singularity, so it is natural to consider an operator inserting a conical singularity and to try and find its interpretation in the dCFT.

Another possibility is to study further the OPE for BPS operators. The $\mathcal{N} = (2, 0)$ theory contains a sector isomorphic to a chiral algebra [67] which can be used to calculate for instance the 3-point functions of 1/4-BPS local operators. For 4d $\mathcal{N} = 2$ SCFTs, it was shown in [68] that the supercharges defining the cohomology are compatible with $\mathcal{N} = (2, 2)$ surface defects, and it would be interesting to extend their construction to the $\mathcal{N} = (2, 0)$ theory with 1/2-BPS surface defects. This could lead to exact results for a sector of the dOE and defect OPE.

It would also be interesting to study BPS operators in the context of the AGT correspondence. At large N one can use holography to calculate the expectation values, in the

presence of the defect, of operators in the traceless symmetric representation of $\mathfrak{so}(5)_R$ [69], which contains in particular O^{IJ} in the stress tensor multiplet. Since the AGT correspondence can be used to calculate the expectation value of the stress tensor [36], it might also calculate expectation values for this larger class of operators at finite N .

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A Conventions

We make use of the following indices:

Index	Usage
$\mu = 1, \dots, 6$	6d spacetime coordinates
$m = 1, \dots, 4$	coordinates transverse to the surface x^m
$a = 1, 2$	worldsheet coordinates σ^a
$\alpha (\dot{\alpha}) = 1, \dots, 4$	6d chiral (antichiral) spinors
$I = 1, \dots, 5$	R-symmetry vectors
$i = 1, \dots, 4$	preserved R-symmetry
$\check{\alpha} = 1, \dots, 4$	R-symmetry spinors

We work in Minkowski space with mostly positive signature. Below we detail the properties of gamma matrices arising in the $\mathfrak{osp}(8^*|4)$ algebra. More details can be found in [70] and references therein.

A.1 Spinors and γ -matrices

We suppress contracted spinor indices in the main text. We use the NW-SE spinor index convention, so that

$$\varepsilon\psi \equiv \varepsilon^{\alpha\check{\alpha}}\psi_{\alpha\check{\alpha}}, \tag{A.1}$$

where α is the index of a chiral 6d spinor ($\dot{\alpha}$ for antichiral) and $\check{\alpha}$ that of an R-symmetry spinor. These indices are raised and lowered by the charge conjugation matrices c, Ω , which obey

$$c^\dagger c = c^{\alpha\check{\alpha}}c_{\check{\alpha}\beta} = \delta_\alpha^\beta, \quad c^* c^T = c^{\dot{\alpha}\alpha}c_{\alpha\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}}, \quad \Omega^\dagger \Omega = \Omega^{\check{\alpha}\check{\beta}}\Omega_{\check{\beta}\check{\gamma}} = \delta_{\check{\gamma}}^{\check{\alpha}}. \tag{A.2}$$

We also make use of two types of γ -matrices: 6d chiral $(\gamma_\mu)_\alpha^{\dot{\beta}}$ (antichiral $(\bar{\gamma}_\mu)_{\dot{\alpha}}^\beta$) and 5d $(\check{\gamma}_I)_{\check{\alpha}}^{\check{\beta}}$ associated to R-symmetry. Their algebra is

$$\bar{\gamma}_\mu\gamma_\nu + \bar{\gamma}_\nu\gamma_\mu = 2\eta_{\mu\nu}, \quad \gamma_\mu\bar{\gamma}_\nu + \gamma_\nu\bar{\gamma}_\mu = 2\eta_{\mu\nu}, \quad \{\check{\gamma}_I, \check{\gamma}_J\} = 2\delta_{IJ}. \tag{A.3}$$

The chiral and antichiral representations are related through

$$\bar{\gamma}_\mu^\dagger = \gamma_0 \bar{\gamma}_\mu \gamma_0 = \gamma^\mu, \tag{A.4}$$

and the matrices are antisymmetric

$$(\gamma_\mu c) = -(\gamma_\mu c)^T, \quad (\bar{\gamma}_\mu c^T) = -(\bar{\gamma}_\mu c^T)^T, \quad (\check{\gamma}_I \Omega) = -(\check{\gamma}_I \Omega)^T. \tag{A.5}$$

Because the matrices are chiral, they also satisfy

$$\gamma_{012345} = I, \quad \bar{\gamma}_{012345} = -I, \quad \check{\gamma}_{12345} = I, \tag{A.6}$$

with $\gamma_{\mu\nu\dots\rho} \equiv \gamma_{[\mu} \bar{\gamma}_\nu \dots \gamma_{\rho]}$ the antisymmetrised product of γ -matrices.

A representation of this algebra is given by

$$\begin{aligned} \gamma_0 = \bar{\gamma}_0 = iI_2 \otimes I_2, & \quad \gamma_1 = -\bar{\gamma}_1 = -i\sigma_1 \otimes I_2, & \quad \gamma_2 = -\bar{\gamma}_2 = -i\sigma_2 \otimes I_2, \\ \gamma_3 = -\bar{\gamma}_3 = i\sigma_3 \otimes \sigma_1, & \quad \gamma_4 = -\bar{\gamma}_4 = i\sigma_3 \otimes \sigma_2, & \quad \gamma_5 = -\bar{\gamma}_5 = -i\sigma_3 \otimes \sigma_3, \\ \check{\gamma}_1 = \sigma_1 \otimes \sigma_2, \quad \check{\gamma}_2 = \sigma_2 \otimes \sigma_2, & \quad \check{\gamma}_3 = \sigma_3 \otimes \sigma_2, \quad \check{\gamma}_4 = I_2 \otimes \sigma_1, & \quad \check{\gamma}_5 = I_2 \otimes \sigma_3, \\ c = -c^T = \sigma_1 \otimes i\sigma_2, & \quad \Omega = i\sigma_2 \otimes I_2. \end{aligned} \tag{A.7}$$

B Weyl anomaly for surface operators

Surface operators in CFT typically suffer from UV divergences which make their expectation value ill-defined. Up to power-law divergences (which can be removed by appropriate counterterms) their expectation value takes the form

$$\log \langle V_\Sigma \rangle \sim \log \epsilon \int_\Sigma \text{vol}_\Sigma \mathcal{A}_\Sigma + \text{finite}, \tag{B.1}$$

where ϵ is a regulator, Σ is the surface (in this paper we take the plane) and \mathcal{A}_Σ is known as the anomaly density.

This conformal anomaly is constrained by the Wess-Zumino condition to take the form

$$\mathcal{A}_\Sigma = \frac{1}{4\pi} \left[aR^\Sigma + b_1 \text{tr} \tilde{\Pi}^2 + b_2 \text{tr} W + c(\partial n)^2 \right]. \tag{B.2}$$

The conformal invariants of this expression are

R^Σ : the Ricci scalar on Σ .

$\text{tr} \tilde{\Pi}^2$: the square of the traceless part of the second fundamental form.

$\text{tr} W$: W is the pullback of the Weyl tensor.

$(\partial n)^2$: the norm of the variation of the coupling to R-symmetry.

The exact definition of these invariants in our conventions can be found in appendix B of [8]. Here we use the definition of the second fundamental form in (4.9)

$$\Pi_{ab}^\mu = \left(\partial_a \partial_b x^\lambda + \partial_a x^\rho \partial_b x^\sigma \Gamma_{\rho\sigma}^\lambda \right) (\delta_\lambda^\mu - g_{\kappa\lambda} \partial^c x^\kappa \partial_c x^\mu), \tag{B.3}$$

which for $\Gamma_{\rho\sigma}^\lambda = 0$ and $x^m = \xi^m$ reduces to (4.9). It can be decomposed into its trace, H^μ , and its traceless part, $\tilde{\Pi}_{ab}^\mu$, which are related by the Gauss-Codazzi equation to other invariants

$$\left(H^2 + 4 \text{tr} P \right) = 2R^\Sigma + 2 \text{tr} \tilde{\Pi}^2 - 2 \text{tr} W. \tag{B.4}$$

C Conformal Ward identities for defect correlators

In this appendix, we derive explicit expressions for the structure of the expectation values of stress tensor primaries in the presence of a flat conformal surface defect. Up to overall normalisation constants, which we further constrain in section 3 using supersymmetry, these correlators are completely fixed by the bosonic symmetries (conformal and R-symmetry) preserved by the defect. We consider both defects with an insertion of a single primary of the displacement operator multiplet, and defects without such insertions. For brevity, we do not give an exhaustive list of such correlators and instead focus on those we require in the main text. More specifically, we compute only the expectation values of the primaries in the stress tensor multiplet, and some 2-point functions involving low-level primaries, namely $O^{IJ}, \chi_{\alpha\dot{\alpha}}^I, H_{\lambda\mu\nu}^I$ in the stress tensor, and $\mathbb{O}^i, \mathbb{Q}_{\alpha\dot{\alpha}}$ in the displacement multiplet. The remaining correlators can of course be calculated using the same method.

We proceed in two steps. First, we fix the dependence on σ and x by implementing the Ward identities associated with the conformal symmetry preserved by the defect as well as transverse rotational symmetry. For clarity, in this calculation we suppress the R-symmetry indices of the operators and leave the scaling dimensions general. Indeed, as much of the kinematics is easily generalised to defects of dimension p in arbitrary spacetime dimension $d = p + q$, we state the more general result wherever we can do so without obscuring the results we presently need. Secondly, we fix the R-symmetry tensor structure of these correlators by demanding invariance under the residual $\mathfrak{so}(4)_R$ symmetry. Throughout, we denote generic operators in the bulk \mathcal{O} and on the defect $\hat{\mathcal{O}}$.

Many of the kinematical results have been obtained by different methods in the past. In particular, the embedding space formalism allows for the efficient computation of bosonic correlators [22]. However, it is not straightforwardly applicable to correlators involving fermions.

C.1 Defect without insertions

We want to solve the constraints that the residual conformal symmetry places on expectation values of the form $\langle \mathcal{O}V \rangle$ with \mathcal{O} a bulk operator of scaling dimension Δ . The representation of the conformal algebra (D.1) acting on \mathcal{O} is given in terms of the representation of \mathcal{O} under Lorentz transformations $S_{\mu\nu}$ and is

$$\begin{aligned} P_\mu &= \partial_\mu, & M_{\mu\nu} &= 2x_{[\mu}\partial_{\nu]} + S_{\mu\nu}, & D &= -x^\mu\partial_\mu - \Delta, \\ K_\mu &= x^2\partial_\mu - 2x_\mu(x^\nu\partial_\nu + \Delta) + 2x^\nu S_{\nu\mu}. \end{aligned} \tag{C.1}$$

Treating separately the coordinates along the plane σ^a and transverse x^m , translation invariance on the plane implies that $\langle \mathcal{O}(\sigma, x)V \rangle$ is a function of x^m only. The other Ward identities can be cast into the form:

$$\begin{aligned} 0 &= S_{ab}\langle \mathcal{O}V \rangle, \\ 0 &= (x^m\partial_m + \Delta)\langle \mathcal{O}V \rangle, \\ 0 &= x^m S_{am}\langle \mathcal{O}V \rangle, \\ 0 &= (x_m\partial_n - x_n\partial_m)\langle \mathcal{O}V \rangle + S_{mn}\langle \mathcal{O}V \rangle. \end{aligned} \tag{C.2}$$

These constraints are now straightforwardly solved. We focus on scalars O , vectors j_μ , selfdual 3-forms $H_{\lambda\mu\nu}$ and traceless symmetric 2-tensors $T_{\mu\nu}$, as operators of those types make up the bosonic degrees of freedom of the stress tensor multiplet, while the correlators of fermionic operators with a scalar defect vanish identically.

For a Lorentz scalar O , all $S_{\mu\nu}$ vanish and the conformal Ward identities (C.2) are immediately solved to give

$$\langle O(\sigma, x)V \rangle = \frac{h_O}{x^\Delta}, \tag{C.3}$$

with h_O an as yet undetermined constant.

The transformation law for a vector reads

$$(S_{\mu\nu}j)_\rho = \delta_{\mu\rho}j_\nu - \delta_{\nu\rho}j_\mu, \tag{C.4}$$

which, plugged into (C.2) eventually leads to⁷

$$\langle j_a V \rangle = \langle j_m V \rangle = 0. \tag{C.5}$$

For higher spin bosonic operators, each Lorentz index separately transforms as (C.4). For a 3-form $H_{\lambda\mu\nu}$, the Ward identities (C.2) imply that the only components with non-vanishing expectation value in the presence of V are H_{abm} and H_{lmn} , and furthermore restricts the available terms for their one-point functions to

$$\langle H_{abm}(x)V \rangle \sim \frac{\epsilon_{ab}x_m}{x^{\Delta+1}}, \quad \langle H_{lmn}(x)V \rangle \sim \frac{\epsilon_{lmnp}x^p}{x^{\Delta+1}}. \tag{C.6}$$

In this work, we are concerned with 3-forms which come with a selfduality condition, which serves to relate the proportionality constants in (C.6). We are left with

$$\langle H_{abm}(x)V \rangle = h_H \frac{\epsilon_{ab}x_m}{x^{\Delta+1}}, \quad \langle H_{lmn}(x)V \rangle = h_H \frac{\epsilon_{lmnp}x^p}{x^{\Delta+1}}. \tag{C.7}$$

Lastly, we repeat the same analysis for a symmetric traceless 2-tensor. Exactly the same line of argument as above yields

$$\begin{aligned} \langle T_{ab}(x)V \rangle &= \frac{h_T}{x^\Delta} \delta_{ab}, & \langle T_{am}(x)V \rangle &= 0, \\ \langle T_{mn}(x)V \rangle &= \frac{h_T}{x^{\Delta+2}} \left(2x_m x_n - x^2 \delta_{mn} \right). \end{aligned} \tag{C.8}$$

We are now in a position to construct the correlator of V with any bosonic primary in the stress tensor multiplet. To that end, recall that, under the unbroken $\mathfrak{so}(5)_R$, O^{IJ}

⁷More generally, for a p -dimensional defect in a spacetime of dimension $d = p + q$, one obtains

$$\langle j_a(x)V \rangle = 0, \quad (q-2)\langle j_m(x)V \rangle = 0.$$

Indeed, for $q = 2$, the transverse components of j can take the form

$$\langle j_m(x)V \rangle \sim \frac{\epsilon_{mn}x^n}{x^{\Delta+1}},$$

which is compatible with conservation.

and $H_{\lambda\mu\nu}^I$ transform as a symmetric traceless 2-tensor and a vector, respectively, while the stress tensor $T_{\mu\nu}$ is an R-symmetry singlet.⁸ Without explicitly applying the Ward identities associated with the preserved $\mathfrak{so}(4)_R$, we can fix the R-symmetry structure of the 1-point functions by writing down the available terms and, for O^{IJ} , implementing tracelessness. Plugging in the correct scaling dimensions $\Delta_O = 4$, $\Delta_H = 5$, and $\Delta_T = 6$, we find the only nonvanishing 1-point functions of stress tensor primaries in the presence of V are (3.9).

C.2 Defect with an insertion

We now repeat the above discussion for correlators $\langle \mathcal{O}(\sigma, x)V[\hat{\mathcal{O}}(\sigma')] \rangle$ involving a defect with an insertion of a displacement multiplet primary. The kinematical analysis is more involved than, but technically very similar to, the previous subsection. We use translation invariance to center $\hat{\mathcal{O}}$ at $\sigma' = 0$ and suppress the arguments of $\mathcal{O}(\sigma, x)$. The conformal Ward identities may be cast into the form:

$$\begin{aligned}
 0 &= \left((\sigma_a \partial_b - \sigma_b \partial_a) + \hat{S}_{ab} + S_{ab} \right) \langle \mathcal{O}V[\hat{\mathcal{O}}] \rangle, \\
 0 &= \left((x_m \partial_n - x_n \partial_m) + \hat{S}_{mn} + S_{mn} \right) \langle \mathcal{O}V[\hat{\mathcal{O}}] \rangle, \\
 0 &= \left(\sigma^a \partial_a + x^m \partial_m + \Delta + \hat{\Delta} \right) \langle \mathcal{O}V[\hat{\mathcal{O}}] \rangle, \\
 0 &= \left(2x^m S_{am} + 2\sigma^b S_{ab} + 2\hat{\Delta}\sigma_a + (\sigma^2 + x^2)\partial_a \right) \langle \mathcal{O}V[\hat{\mathcal{O}}] \rangle.
 \end{aligned}
 \tag{C.9}$$

For the simplest case of a scalar \mathbb{O} on the defect and a scalar O in the bulk, (C.9) become particularly simple, and imply⁹

$$\langle O(\sigma, x)V[\mathbb{O}] \rangle = \frac{C_{O\mathbb{O}}}{x^{\Delta-\hat{\Delta}}(\sigma^2 + x^2)^{\hat{\Delta}}},
 \tag{C.10}$$

with $C_{\mathbb{O}O}$ some normalisation constant.

For a defect scalar \mathbb{O} and a bulk vector j_μ we obtain:

$$\begin{aligned}
 \langle j_a(\sigma, x)V[\mathbb{O}] \rangle &= \frac{C_{j\mathbb{O}}\sigma_a}{x^{\Delta-\hat{\Delta}-1}(\sigma^2 + x^2)^{\hat{\Delta}+1}}, \\
 \langle j_m(\sigma, x)V[\mathbb{O}] \rangle &= \frac{C_{j\mathbb{O}}(x^2 - \sigma^2)x_m}{2x^{\Delta-\hat{\Delta}+1}(\sigma^2 + x^2)^{\hat{\Delta}+1}}.
 \end{aligned}
 \tag{C.11}$$

Indeed, these correlators are exactly the same for defects of generic dimension and codimension. It is easily checked that (C.11) is compatible with conservation of j in the bulk if and only if $\Delta = d - 1$ and $\hat{\Delta} = p$, which is indeed satisfied by the displacement superprimary \mathbb{O}^i and the bulk R-symmetry current j_μ^{IJ} . The conservation equation

$$\partial_\mu \langle j^\mu(\sigma, x)V[\mathbb{O}] \rangle = \langle V[\mathbb{O}(\sigma)\mathbb{O}(0)] \rangle,
 \tag{C.12}$$

⁸The R-symmetry current j_μ^{IJ} transforms as an antisymmetric tensor, but as seen above, its 1-point function vanishes identically regardless of the R-symmetry structure.

⁹In particular, inserting for \mathbb{O} the defect identity operator $\mathbf{1}_V$, we recover the form of (C.3), as expected.

then allows us to fix $C_{\mathbb{O}}$ in terms of $C_{\mathbb{O}j}$ in equation (3.19). For the remaining required bosonic correlator, consider a defect scalar \mathbb{O} and a bulk 3-form $H_{\lambda\mu\nu}$. The conformal Ward identities (C.9) imply that the only components of the correlator that do not vanish identically are

$$\begin{aligned}\langle H_{abm}(\sigma, x)V[\mathbb{O}] \rangle &= \frac{h_H \epsilon_{ab} x_m}{x^{\Delta-\hat{\Delta}+1}(\sigma^2+x^2)^{\hat{\Delta}}}, \\ \langle H_{lmn}(\sigma, x)V[\mathbb{O}] \rangle &= \frac{h_H \epsilon_{lmnp} x^p}{x^{\Delta-\hat{\Delta}+1}(\sigma^2+x^2)^{\hat{\Delta}}},\end{aligned}\tag{C.13}$$

where, as for the 1-point function, we have used the selfduality of $H_{\lambda\mu\nu}$ to relate the two normalisation constants. Lastly, we compute the only correlator of fermions that we require in this paper. Consider a bulk chiral spinor χ_α and a defect chiral spinor \mathbb{Q}_α .¹⁰ Their transformation laws are familiar:

$$(S_{\mu\nu}\chi)_\alpha = \frac{1}{2}(\gamma_{\mu\nu})_\alpha{}^\beta \chi_\beta, \quad (S_{ab}\mathbb{Q})_\alpha = \frac{1}{2}(\gamma_{ab})_\alpha{}^\beta \mathbb{Q}_\beta, \quad (S_{mn}\mathbb{Q})_\alpha = \frac{1}{2}(\gamma_{mn})_\alpha{}^\beta \mathbb{Q}_\beta.\tag{C.14}$$

In order to apply the Ward identities (C.9), we expand $\langle \chi_\alpha V[\mathbb{Q}_\beta] \rangle$ in terms of antisymmetrised products of gamma matrices. The only such matrices with the appropriate chirality properties are γ^μ and $\gamma^{\mu\nu\rho}$ (we can omit $\gamma^{\mu\nu\rho\sigma\tau}$ since it is related to γ^μ by duality):

$$\langle \chi_\alpha V[\mathbb{Q}_\beta] \rangle = a_\mu (\gamma^\mu c)_{\alpha\beta} + \frac{1}{3!} b_{\lambda\mu\nu} (\gamma^{\lambda\mu\nu} c)_{\alpha\beta}.\tag{C.15}$$

Writing out and simplifying the conformal Ward identities explicitly then leads to

$$\langle \chi_\alpha(\sigma, x)V[\mathbb{Q}_\beta] \rangle = \frac{c_\chi \mathbb{Q} [(\sigma_a \gamma^a + x_m \gamma^m) c]_{\alpha\beta}}{x^{\Delta-\hat{\Delta}} \sqrt{\sigma^2+x^2}^{1+2\hat{\Delta}}}.\tag{C.16}$$

Having completed the kinematic analysis, we can now restore the R-symmetry structure in order to construct the full bulk-defect 2-point functions. The Ward identities associated with the generators of $\mathfrak{so}(4)_R$ decouple from the kinematics, and therefore take a purely algebraic form (with R, \hat{R} the representations of $\mathcal{O}, \hat{\mathcal{O}}$)

$$0 = (R^{ij} + \hat{R}^{ij}) \langle \mathcal{O} V[\hat{\mathcal{O}}] \rangle.\tag{C.17}$$

Among the bosonic 2-point functions we consider, the only nonvanishing ones are (we again suppress coordinate dependence and Lorentz indices):

$$\langle \mathcal{O}^{i5} V[\mathbb{O}^j] \rangle \sim \delta^{ij}, \quad \langle j^{i5} V[\mathbb{O}^j] \rangle \sim \delta^{ij}, \quad \langle H^i V[\mathbb{O}^j] \rangle \sim \delta^{ij}.\tag{C.18}$$

To restore the correct R-symmetry structure of the fermionic 2-point function, recall that $\chi_{\alpha\check{\alpha}}^I$ transforms in the tensor product of the vector and spinor representation of $\mathfrak{so}(5)_R$ and is subject to a constraint $\check{\gamma}_I \chi^I = 0$, while $\mathbb{Q}_{\alpha\check{\alpha}}$ transforms as an ordinary R-symmetry

¹⁰Since ultimately we are interested in a defect operator defined in terms of a chiral fermionic bulk current, we take \mathbb{Q} to transform as a spinor under both parallel and transverse rotations, and consider only chiral objects.

spinor but obeys a constraint $\Pi_+ \mathbb{Q} = 0$ mixing Lorentz and R-symmetry. Since we only need the correlator involving $\chi_{\dot{\alpha}}^5$, we make the ansatz

$$\langle \chi_{\dot{\alpha}}^5 \mathbb{Q}_{\dot{\beta}} \rangle \sim (\check{\gamma}^5)_{\dot{\alpha}\dot{\beta}}, \quad (\text{C.19})$$

which is indeed compatible with (C.17).

With the kinematical data and R-symmetry structure in hand, we can now assemble the full 2-point functions. Plugging in the correct defect operator scaling dimensions $\Delta_{\mathbb{Q}} = 2$ and $\Delta_{\mathbb{Q}} = 5/2$, we obtain (3.12).

D Algebras

In this appendix we collect some results on the algebras $\mathfrak{osp}(8^*|4)$ and $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$. For a general reference on Lie superalgebra, see [71, 72] and references therein.

D.1 The algebra $\mathfrak{osp}(8^*|4)$

The quaternionic orthosymplectic algebra $\mathfrak{osp}(8^*|4) = D(4, 2)$ is a 6d superconformal algebra containing 38 bosonic and 32 fermionic generators.¹¹ Its bosonic part $\mathfrak{so}(2, 6) \oplus \mathfrak{so}(5)$ contains a 6d conformal algebra

$$\begin{aligned} [M_{\mu\nu}, M_{\rho\sigma}] &= 2\eta_{\sigma[\mu} M_{\nu]\rho} - 2\eta_{\rho[\mu} M_{\nu]\sigma}, & [P_{\mu}, K_{\nu}] &= 2(M_{\mu\nu} + \eta_{\mu\nu} D), \\ [M_{\mu\nu}, P_{\rho}] &= 2P_{[\mu} \eta_{\nu]\rho}, & [M_{\mu\nu}, K_{\rho}] &= 2K_{[\mu} \eta_{\nu]\rho}, \\ [D, P_{\mu}] &= P_{\mu}, & [D, K_{\mu}] &= -K_{\mu}, \end{aligned} \quad (\text{D.1})$$

along with an $\mathfrak{so}(5)$ R-symmetry

$$[R_{IJ}, R_{KL}] = 2\delta_{K[I} R_{J]L} - 2\delta_{L[I} R_{J]K}. \quad (\text{D.2})$$

The fermionic generators \mathbb{Q} and $\bar{\mathbb{S}}$ form a representation under that bosonic algebra and obey

$$\begin{aligned} [M_{\mu\nu}, \mathbb{Q}_{\alpha\dot{\alpha}}] &= -\frac{1}{2}(\gamma_{\mu\nu} \mathbb{Q})_{\alpha\dot{\alpha}}, & [M_{\mu\nu}, \bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}] &= -\frac{1}{2}(\bar{\gamma}_{\mu\nu} \bar{\mathbb{S}})_{\dot{\alpha}\check{\alpha}}, \\ [K_{\mu}, \mathbb{Q}_{\alpha\dot{\alpha}}] &= (\gamma_{\mu} \bar{\mathbb{S}})_{\alpha\dot{\alpha}}, & [P_{\mu}, \bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}] &= (\bar{\gamma}_{\mu} \mathbb{Q})_{\dot{\alpha}\check{\alpha}}, \\ [D, \mathbb{Q}_{\alpha\dot{\alpha}}] &= \frac{1}{2} \mathbb{Q}_{\alpha\dot{\alpha}}, & [D, \bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}] &= -\frac{1}{2} \bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}, \\ [R_{IJ}, \mathbb{Q}_{\alpha\dot{\alpha}}] &= \frac{1}{2} (\check{\gamma}_{IJ} \mathbb{Q})_{\alpha\dot{\alpha}}, & [R_{IJ}, \bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}] &= \frac{1}{2} (\check{\gamma}_{IJ} \bar{\mathbb{S}})_{\dot{\alpha}\check{\alpha}}. \end{aligned} \quad (\text{D.3})$$

Finally, the anticommutator of \mathbb{Q} generates a translation P , while the anticommutator of $\bar{\mathbb{S}}$ generates a special conformal transformation K

$$\begin{aligned} \{\mathbb{Q}_{\alpha\dot{\alpha}}, \mathbb{Q}_{\dot{\beta}\check{\beta}}\} &= 2(\gamma_{\mu} c)_{\alpha\dot{\beta}} \Omega_{\dot{\alpha}\check{\beta}}^{\mu} P^{\mu}, & \{\bar{\mathbb{S}}_{\dot{\alpha}\check{\alpha}}, \bar{\mathbb{S}}_{\dot{\beta}\check{\beta}}\} &= 2(\bar{\gamma}_{\mu} c^T)_{\dot{\alpha}\check{\beta}} \Omega_{\dot{\alpha}\check{\beta}}^{\mu} K^{\mu}, \\ \{\mathbb{Q}_{\alpha\dot{\alpha}}, \bar{\mathbb{S}}_{\dot{\beta}\check{\beta}}\} &= 2 \left[\left(D + \frac{1}{2} \gamma_{\mu\nu} M^{\mu\nu} + \check{\gamma}_{IJ} R^{IJ} \right) c^T \Omega \right]_{\alpha\dot{\beta}\check{\alpha}\check{\beta}}. \end{aligned} \quad (\text{D.4})$$

All the other commutators vanish.

¹¹More precisely, it is a real form of $D(4, 2)$ given by $P_{\mu}^{\dagger} = K^{\mu}$ (which also implies $(\mathbb{Q}_{\alpha\dot{\alpha}})^{\dagger} = S^{\alpha\dot{\alpha}}$) and compatible with radial quantisation in Euclidean space. Hermitean generators can be obtained by redefining all generators $P \rightarrow iP$.

Note that this algebra has a natural structure in terms of supermatrices. This point of view, along with its relation to the 6d algebra presented above, is elaborated in [70]. We also note that the $\mathfrak{so}(5)$ generators can be expressed in terms of $\mathfrak{sp}(2)$ generators by the relation

$$\mathbf{U}_{\check{\alpha}\check{\beta}} = \frac{1}{2} (\check{\gamma}_{IJ}\Omega)_{\check{\alpha}\check{\beta}} \mathbf{R}^{IJ}, \quad \mathbf{R}_{IJ} = -\frac{1}{4} \left(\Omega^\dagger \check{\gamma}_{IJ} \right)^{\check{\alpha}\check{\beta}} \mathbf{U}_{\check{\alpha}\check{\beta}}. \quad (\text{D.5})$$

The appropriate commutators are then

$$\begin{aligned} [\mathbf{U}_{\check{\alpha}\check{\beta}}, \mathbf{U}_{\check{\gamma}\check{\delta}}] &= 2\Omega_{\check{\alpha}(\check{\gamma}} \mathbf{U}_{\check{\delta})\check{\beta}} + 2\Omega_{\check{\beta}(\check{\gamma}} \mathbf{U}_{\check{\delta})\check{\alpha}}, \\ [\mathbf{U}_{\check{\alpha}\check{\beta}}, \mathbf{Q}_{\alpha\check{\gamma}}] &= 2\mathbf{Q}_{\alpha(\check{\alpha}} \Omega_{\check{\beta})\check{\gamma}}, \quad [\mathbf{U}_{\check{\alpha}\check{\beta}}, \bar{\mathbf{S}}_{\check{\alpha}\check{\gamma}}] = 2\bar{\mathbf{S}}_{\check{\alpha}(\check{\alpha}} \Omega_{\check{\beta})\check{\gamma}}. \end{aligned} \quad (\text{D.6})$$

D.2 The subalgebra $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$

In the presence of the plane, the original symmetry $\mathfrak{osp}(8^*|4)$ is reduced to the subalgebra $\mathfrak{osp}(4^*|2) \oplus \mathfrak{osp}(4^*|2)$ [44], a real form of $D(2, 1, \alpha) \oplus D(2, 1, \alpha)$ with $\alpha = -1/2$. Each copy of the $\mathfrak{osp}(4^*|2)$ is a (rigid) 1d superconformal algebra, whose bosonic part is

$$\begin{aligned} [\mathbf{P}_+, \mathbf{K}_+] &= 2\mathbf{D}_+, & [\mathbf{D}_+, \mathbf{P}_+] &= \mathbf{P}_+, & [\mathbf{D}_+, \mathbf{K}_+] &= -\mathbf{K}_+, \\ [\mathbf{T}_{(a)}^i, \mathbf{T}_{(b)}^j] &= -i\delta_{(ab)} \varepsilon^{ijk} \mathbf{T}_{(b)}^k, & (a) &= 1, 2. \end{aligned} \quad (\text{D.7})$$

In addition to the 1d conformal algebra, there are 2 additional $\mathfrak{su}(2)$. Together, they form the ‘‘chiral’’ part of the $\mathfrak{so}(2, 2)_\parallel \oplus \mathfrak{so}(4)_\perp \oplus \mathfrak{so}(4)_R$ preserved by the plane, with the ‘‘antichiral’’ part (denoted by a ‘‘-’’ subscript) given by the other $\mathfrak{osp}(4^*|2)$. They are related to the bulk generators by

$$\mathbf{P}_\pm = \frac{1}{2}(\mathbf{P}_0 \pm \mathbf{P}_1), \quad \mathbf{D}_\pm = \frac{1}{2}(\mathbf{D} \pm \mathbf{M}_{01}), \quad \mathbf{K}_\pm = \frac{1}{2}(-\mathbf{K}_0 \pm \mathbf{K}_1), \quad (\text{D.8})$$

where for definiteness we assume that the plane spans the directions $x^{0,1}$. The decomposition of $\mathfrak{so}(4)_{\perp,R}$ is given by the ‘t Hooft symbols

$$\mathbf{T}_{(1)}^{i_1} = \frac{i}{4} \eta_{mn}^{i_1} \mathbf{M}^{mn}, \quad \mathbf{T}_{(2)}^{i_2} = -\frac{i}{4} \eta_{ij}^{i_2} \mathbf{R}^{ij}, \quad (\text{D.9})$$

and similarly for $\bar{\mathbf{T}}$ in terms of the antichiral ‘t Hooft symbols $\bar{\eta}$.

In addition to these generators, the algebra includes supersymmetries $\mathbf{Q}_{\alpha_1\alpha_2}$ and special supersymmetries $\mathbf{S}_{\alpha_1\alpha_2}$ charged under both $\mathfrak{su}(2)$. These satisfy

$$\begin{aligned} [\mathbf{K}_+, \mathbf{Q}_{\alpha_1\alpha_2}] &= -i\mathbf{S}_{\alpha_1\alpha_2}, & [\mathbf{P}_+, \mathbf{S}_{\alpha_1\alpha_2}] &= i\mathbf{Q}_{\alpha_1\alpha_2}, \\ [\mathbf{D}_+, \mathbf{Q}_{\alpha_1\alpha_2}] &= \frac{1}{2}\mathbf{Q}_{\alpha_1\alpha_2}, & [\mathbf{D}_+, \mathbf{S}_{\alpha_1\alpha_2}] &= -\frac{1}{2}\mathbf{S}_{\alpha_1\alpha_2}, \\ [\mathbf{T}_{(1)}^{i_1}, \mathbf{Q}_{\alpha_1\alpha_2}] &= \frac{1}{2}(\sigma^{i_1})_{\alpha_1}{}^{\beta_1} \mathbf{Q}_{\beta_1\alpha_2}, & [\mathbf{T}_{(1)}^{i_1}, \mathbf{S}_{\alpha_1\alpha_2}] &= \frac{1}{2}(\sigma^{i_1})_{\alpha_1}{}^{\beta_1} \mathbf{S}_{\beta_1\alpha_2}, \\ [\mathbf{T}_{(2)}^{i_2}, \mathbf{Q}_{\alpha_1\alpha_2}] &= \frac{1}{2}(\sigma^{i_2})_{\alpha_2}{}^{\beta_2} \mathbf{Q}_{\alpha_1\beta_2}, & [\mathbf{T}_{(2)}^{i_2}, \mathbf{S}_{\alpha_1\alpha_2}] &= \frac{1}{2}(\sigma^{i_2})_{\alpha_2}{}^{\beta_2} \mathbf{S}_{\alpha_1\beta_2}, \end{aligned} \quad (\text{D.10})$$

where σ^i are the Pauli matrices. They anticommute to

$$\begin{aligned} \{Q_{\alpha_1\alpha_2}, Q_{\beta_1\beta_2}\} &= 2i\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2}P_+, & \{S_{\alpha_1\alpha_2}, S_{\beta_1\beta_2}\} &= 2i\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2}K_+, \\ \{Q_{\alpha_1\alpha_2}, S_{\beta_1\beta_2}\} &= 2\left[\epsilon_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2}D_+ + (\sigma^{i_1}\epsilon)_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2}T_{(1)}^{i_1} - 2\epsilon_{\alpha_1\beta_1}(\sigma^{i_2}\epsilon)_{\alpha_2\beta_2}T_{(2)}^{i_2}\right]. \end{aligned} \quad (D.11)$$

The ratio $\alpha = -1/2$ between the coefficients of $T_{(1)}$ and $T_{(2)}$ is a specific case of the exceptional Lie algebra $D(2, 1; \alpha)$ (see [73] for the algebra with general α and its Kac-Moody extension).

The precise embedding of these supercharges inside $Q_{\alpha\check{\alpha}}$ is obtained by restricting to the preserved supercharges $\Pi_+Q = Q$, where the projector is [8]

$$(\Pi_{\pm})_{\alpha\check{\alpha}}^{\beta\check{\beta}} = \frac{1}{2}[1 \pm \gamma_{01}\check{\gamma}_5]_{\alpha\check{\alpha}}^{\beta\check{\beta}}, \quad (\Pi_{\pm})_{\dot{\alpha}\check{\alpha}}^{\dot{\beta}\check{\beta}} = \frac{1}{2}[1 \mp \bar{\gamma}_{01}\check{\gamma}_5]_{\dot{\alpha}\check{\alpha}}^{\dot{\beta}\check{\beta}}, \quad (D.12)$$

which has a different expression acting respectively on chiral and antichiral representations. This projector decomposes as

$$\frac{1}{2}[1 + \gamma_{01}\check{\gamma}_5] = \frac{1}{2}[1 + \gamma_{01}]\frac{1}{2}[1 + \check{\gamma}_5] + \frac{1}{2}[1 - \gamma_{01}]\frac{1}{2}[1 - \check{\gamma}_5], \quad (D.13)$$

which gives, respectively for the two terms, two anticommuting supercharges $\bar{Q}_{\dot{\alpha}_1\dot{\alpha}_2}$ and $Q_{\alpha_1\alpha_2}$. Their chirality is derived from the projector: $(1 + \gamma_{01})$ projects onto the positive chirality component, which is correlated with the positive chirality under $\mathfrak{so}(4)_{\perp}$ since $\gamma_{01} = \gamma_{2345}$.

D.2.1 Subalgebra as an embedding inside $\mathfrak{osp}(8^*|4)$

Lastly, in section 2 and 3 it is convenient to discuss the subalgebra directly within the larger $\mathfrak{osp}(8^*|4)$. Here we decompose some of the commutators of $\mathfrak{osp}(8^*|4)$ into preserved and broken generators directly with the projector. We make use of the following identities

$$\begin{aligned} \Pi_{\pm}^{\dagger} &= \Pi_{\pm}, & (\Pi_{\pm}\mathcal{C})^T &= -\Pi_{\pm}\mathcal{C}^T, \\ [\Pi_{\pm}, \Gamma_a] &= [\Pi_{\pm}, \check{\gamma}_5] = 0, \\ \Pi_{\pm}\Gamma_m &= \Gamma_m\Pi_{\mp}, & \Pi_{\pm}\Gamma_i &= \Gamma_i\Pi_{\mp}. \end{aligned}$$

Note that here we don't differentiate between the action of Q and \bar{Q} for simplicity.

Using these properties, one can easily derive the induced subalgebra and its representation by acting with Π_{\pm} . The only nontrivial part of the preserved algebra is for the supercharges, which now obey

$$\begin{aligned} \{Q_{\alpha\check{\alpha}}^+, Q_{\beta\check{\beta}}^+\} &= 2(\gamma_a\Pi_+c\Omega)_{\alpha\beta\check{\alpha}\check{\beta}}P^a, & \{\bar{S}_{\dot{\alpha}\check{\alpha}}^+, \bar{S}_{\dot{\beta}\check{\beta}}^+\} &= 2(\bar{\gamma}_a\Pi_+c^T\Omega)_{\dot{\alpha}\dot{\beta}\check{\alpha}\check{\beta}}K^a, \\ \{Q_{\alpha\check{\alpha}}^+, \bar{S}_{\dot{\beta}\check{\beta}}^+\} &= 2\left[\left(\check{\gamma}_{ij}R^{ij} + D + \frac{1}{2}\gamma_{mn}M^{mn} + \frac{1}{2}\gamma_{ab}M^{ab}\right)\Pi_+c^T\Omega\right]_{\alpha\beta\check{\alpha}\check{\beta}}. \end{aligned} \quad (D.14)$$

The broken generators satisfy

$$\begin{aligned}
 [Q_{\alpha\check{\alpha}}^+, P_m] &= 0, \\
 \{Q_{\alpha\check{\alpha}}^+, Q_{\beta\check{\beta}}^-\} &= 2(\gamma_m \Pi_- c \Omega)_{\alpha\beta\check{\alpha}\check{\beta}} P^m, \\
 [Q_{\alpha\check{\alpha}}^+, R_{i5}] &= -\frac{1}{2}(\check{\gamma}_{i5} Q^-)_{\alpha\check{\alpha}}, \\
 [Q_{\alpha\check{\alpha}}^+, M_{am}] &= \frac{1}{2}(\gamma_{am} Q^-)_{\alpha\check{\alpha}}, \\
 \{Q_{\alpha\check{\alpha}}^+, \bar{S}_{\beta\check{\beta}}^-\} &= 4\left[\left(\check{\gamma}_{i5} R^{i5} + \frac{1}{2}\gamma_{am} M^{am}\right)\Pi_- c^T \Omega\right]_{\alpha\beta\check{\alpha}\check{\beta}}, \\
 [Q_{\alpha\check{\alpha}}^+, K_m] &= -(\gamma_m \bar{S}^-)_{\alpha\check{\alpha}}.
 \end{aligned}
 \tag{D.15}$$

These transformations are related to (2.2) using (2.1) to write the displacement operator as contact terms in the presence of the defect:

$$R^{i5} V = \int_{\mathbb{R}^2} d^2\sigma V[\mathbb{Q}^i(\sigma)]. \tag{D.16}$$

We can recover the full representation by acting with Q^+ , e.g.,

$$\int_{\mathbb{R}^2} V[Q^+ \mathbb{Q}^i(\sigma)] d^2\sigma = [Q^+, R^{i5}] V = -\frac{1}{2} \check{\gamma}_{i5} Q^- V = -\frac{1}{2} \int_{\mathbb{R}^2} d^2\sigma V[\check{\gamma}_{i5} Q^-(\sigma)]. \tag{D.17}$$

The action of Q^+ on \mathbb{Q} can similarly be read from (D.15), but it misses the descendant. These are fixed instead by requiring closure under the Jacobi identity as in (3.6) (see also for instance the discussion in section 2 of [74]).

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