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An update on moduli stabilization with antibrane uplift

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ABSTRACT: It was recently shown that in warped compactifications based on a Klebanov-Strassler throat there is a light complex structure field, governing the size of the throat and the redshift at its tip. We show that after uplift of the cosmological constant by an anti-D3 brane at the tip of the throat, the contribution to supersymmetry breaking coming from the new light field is large. We work out the mass scales, in particular the condition for this field to be heavier than the Kähler modulus. We check that for the range of parameters relevant for the destabilization we find agreement with de Sitter swampland conjecture. Adding matter fields on distant branes, we discuss the effects on supersymmetry breaking in the observable sector. A hierarchically small scale of supersymmetry breaking translates generically into large values of localized D3 charges in the manifold.

KEYWORDS: Compactification and String Models, Supergravity Models, Supersymmetry Breaking, Superstring Vacua

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1 Introduction

The KKLT construction of moduli stabilization [1] relies on a three step procedure. In the first step, fluxes in the internal manifold stabilize all moduli fields, except the overall volume (Kähler) modulus ρ .¹ In a second step, nonperturbative effects like stringy instantons or gaugino condensation on D7 branes stabilize ρ in anti-de Sitter space. Finally, an anti-D3 brane is introduced which breaks supersymmetry.² The second and third step were debated over the years [9–21] and whereas there is not yet a consensus over the last step, there are recent positive results indicating the validity of the second step [22–29].

Recently [30, 31] the first step was addressed in the Klebanov-Strassler (KS) deformed conifold construction, which contains all ingredients needed in the KKLT construction. It was shown that one of the complex structure fields, called S in what follows, which governs the size of the KS throat, is much lighter than previously thought. Its scalar potential is therefore shallow and it is significantly modified by the uplifting anti-D3 brane. Not destabilizing the throat requires a minimal value of one of the flux quanta M [30–32],

$$g_s M^2 \geq (6.8)^2 q, \tag{1.1}$$

¹Here we consider only models with one Kähler modulus, i.e. $h^{1,1} = 1$. In more general terms only the complex structure moduli get stabilized by fluxes.

²Actually realizing it nonlinearly, similar to perturbative string constructions of the “Brane Supersymmetry” type [2–6]), whose nonlinear supersymmetric actions were constructed in [7, 8].

where g_s is the string coupling and q is the number of antibranes. In this paper we consider the most favorable case $q = 1$.³

The purpose of the present letter is to investigate in more detail the consequences for the KKLT construction: the resulting vacuum structure and mass scales, various contributions to supersymmetry breaking and the needed localized D3 charge in the internal space which produce physically motivated hierarchies. In section 2 we review the effective action for the light complex structure field S in the KS geometry and the mechanism behind the potential destabilization of the throat, once one adds the antibrane uplift. Section 3 proposes a 4d supergravity description of the system including the KKLT sector of moduli stabilization and discusses the vacuum structure and supersymmetry breaking. We use a manifestly supersymmetric four-dimensional supergravity description and describe the uplift via a nilpotent chiral multiplet in supergravity [33–38]. In section 4 we add matter fields and study the effects of supersymmetry breaking, from a 4d perspective and, alternatively, from a higher-dimensional one. We conclude with some comments and a short appendix.

2 The effective action of the Klebanov-Strassler warped compactification

The traditional KKLT construction of moduli stabilization [1] is based on warped compactifications of Calabi-Yau manifolds, with a constant dilaton, five and three-form fluxes [39]. The background metric and five-form flux are

$$\begin{aligned} ds^2 &= H^{-1/2} ds_4^2 + H^{1/2} ds_6^2, \\ F_5 &= (1 + *) \text{vol}_4 \wedge dH^{-1} \equiv *F_5 + \mathcal{F}_5, \end{aligned} \tag{2.1}$$

where H is the warp factor and ds_6^2 is the unwarped metric of the internal manifold. As argued in [40], one can interpret this manifold as a throat-type region of strong warping, analogous to Randall-Sundrum type models [41, 42], glued to a compact Calabi-Yau space.

In the region of strong warping the local internal geometry is that of the deformed conifold, defined by its embedding into \mathbb{C}^4 , $\sum_{a=1}^4 \omega_a^4 = S$. The deformation parameter S is the complex structure modulus whose absolute value corresponds to the size of the 3-sphere at the tip of the cone. The other complex structure moduli Z^I come from the “UV” geometry. We thus have $h^{2,1} + 1$ A-cycles:

$$\int_A \Omega_3 = S, \quad \int_{A_I} \Omega_3 = Z^I \tag{2.2}$$

where $I = 0, \dots, h^{2,1} - 1$. We assume that the prepotential splits according to

$$F(S, Z^I) = F_{cf}(S) + F_{UV}(Z^I), \tag{2.3}$$

³It can be argued that there could be further corrections to the potential for S which become relevant if one goes far away from the original, supersymmetric minimum. These corrections could potentially be so important that the bound (1.1) gets modified significantly or even becomes invalid. Resolving these issues is beyond the scope of this paper. Instead, we assume that the results of [30, 31] are correct. Moreover, we will mostly work in a regime where (1.1) is satisfied and it is therefore reasonable to expect that further corrections to the potential are subleading in the region of interest.

where F_{cf} is the prepotential of the deformed conifold and the “UV prepotential,” F_{UV} , does not explicitly depend on S . We thus have

$$\int_B \Omega_3 = F_S = \frac{S}{2\pi i} \left(\log \frac{\Lambda_0^3}{S} + 1 \right) + F_S^0, \quad \int_{B_I} \Omega_3 = F_I, \quad (2.4)$$

where F_S and F_I are the derivatives of F with respect to S and Z_I respectively, and F_S^0 depends on the details of the compactification manifold, but is independent of S . The cutoff Λ_0 corresponds to the transition between the highly warped region, modeled as a KS throat, and (relatively unwarped) rest of the compact Calabi-Yau manifold.

The 3-form fluxes on the 3-cycles are⁴

$$\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} F_3 &= M\alpha + M^I \alpha_I - M_I \beta^I, \\ \frac{1}{(2\pi)^2 \alpha'} H_3 &= -K\beta + K^I \alpha_I - K_I \beta^I. \end{aligned} \quad (2.5)$$

where α_I, β^I are Poincare duals to the cycles B_I, A^I and we have singled out the RR flux on the S^3 cycle at the tip of the throat, M , and its NSNS partner K . These are the fluxes responsible for the deformation of the conifold by the parameter S .

The throat region is that of the Klebanov-Strassler (KS) solution [43], with the six-dimensional metric of the deformed conifold⁵

$$ds_6^2 = \frac{|S|^{2/3}}{2} \mathcal{K}(\mathcal{T}) \left[\frac{1}{3\mathcal{K}^3(\mathcal{T})} \left(d\mathcal{T}^2 + (g^5)^2 \right) + \sinh^2(\mathcal{T}/2) \left((g^1)^2 + (g^2)^2 \right) + \cosh^2(\mathcal{T}/2) \left((g^3)^2 + (g^4)^2 \right) \right], \quad (2.6)$$

where g^i is an orthogonal basis of one-forms on the base of the cone and

$$\mathcal{K}(\mathcal{T}) = \frac{(\sinh(2\mathcal{T}) - 2\mathcal{T})^{1/3}}{2^{1/3} \sinh \mathcal{T}}. \quad (2.7)$$

The warp factor of the KS solution is

$$H(\mathcal{T}) = 2^{2/3} \frac{g_s (\alpha' M)^2 (\rho + \bar{\rho})}{|S|^{4/3}} I(\mathcal{T}) \quad (2.8)$$

where

$$I(\mathcal{T}) = \int_{\mathcal{T}}^{\infty} dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}. \quad (2.9)$$

The UV cutoff Λ_0 where the solution is glued to the compact Calabi-Yau solution is such that the total NSNS flux over the B cycle is K , according to (2.5):

$$K = \frac{1}{(2\pi)^2 \alpha'} \int_B H_3 = \frac{1}{(2\pi)^2 \alpha'} \int_{\mathcal{T} \leq \mathcal{T}_0} \int_{S^2} H_3, \quad \Lambda_0^2 = \frac{3}{2^{5/3}} |S|^{2/3} e^{2\mathcal{T}_0/3}. \quad (2.10)$$

⁴The setup only requires one type of flux on each cycle.

⁵Note that taking \mathcal{T} and g^i to be dimensionless requires the deformation parameter S to be of dimension $(length)^3$.

On a compact manifold the Bianchi identity for the five-form flux leads to the tadpole cancelation condition forcing the total D3-charge of the solution to be zero.

$$MK + M^I K_I - M_I K^I + Q_3^{\text{loc}} = 0, \tag{2.11}$$

where the charge of localized D3-brane and O3-plane sources is⁶

$$Q_3^{\text{loc}} = N_{\text{D3}} - \frac{1}{4} N_{\text{O3}}. \tag{2.12}$$

The tadpole condition (2.11) leads to an upper bound on the product of fluxes allowing the cancelation of C_4 flux

$$MK \leq |Q_3^{\text{loc}}|. \tag{2.13}$$

Recently it was shown [30, 31] that in the strongly warped region of the Klebanov-Strassler compactification, the light complex structure field S can be destabilized by the $\overline{\text{D3}}$ uplift. The potential for the complex structure modulus S involves the fluxes M and K , while it depends on the other fluxes only indirectly through the axion-dilaton τ , whose vev is determined by all fluxes. Furthermore, unlike the other “bulk” moduli, the potential for S is highly affected by the warp factor. Its functional form, in the Einstein frame, derived in [44, 45] is

$$\begin{aligned} V_{\text{KS}} &= \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(\rho + \bar{\rho})^3} \left[c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s (\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2 \\ &\simeq \frac{\pi^{3/2} |S|^{4/3}}{\kappa_{10} c' (\alpha' M)^2 (\rho + \bar{\rho})^3} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2, \end{aligned} \tag{2.14}$$

where in the last line we used the approximation of strong warping. Moreover, g_s is the stabilized vev of the dilaton, $\rho + \bar{\rho} = (\text{Vol}_6)^{3/2}$, c denotes the constant value of the warp factor at the UV and will not be relevant here, whereas the constant c' , multiplying the term coming solely from the warp factor, denotes an order one coefficient, whose approximate numerical value was determined in [44] to be $c' \approx 1.18$. The potential (2.14) has a supersymmetric minimum

$$S_{\text{KS}} = \Lambda_0^3 e^{-\frac{2\pi K}{g_s M}}, \tag{2.15}$$

which is exponentially small for appropriate values of the fluxes (M, K) . Since the field S has mass dimension -3 , whereas the corresponding gauge theory condensate Z has dimension 3, one can write the potential in terms of Z in the following way. Writing the 10d metric in the form

$$ds^2 = e^{2A} ds_4^2 + e^{-2A} t^{1/2} ds_6^2, \tag{2.16}$$

where the volume of the internal space is parametrized in terms of $t = \rho + \bar{\rho}$. The relation between the 10d and the 4d Newton constant is

$$\frac{1}{\kappa_4^2} = \frac{V_w}{\kappa_{10}^2}, \quad \text{where} \quad V_w = \int d^6 y \sqrt{g_6} e^{-4A}, \tag{2.17}$$

⁶There can be also an additional contribution to Q_3^{loc} coming from D7-branes and O7-branes.

where V_w is a fiducial volume. Using the relation $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ and redefining the S field according to

$$S = \left(2^{3/4} \pi^{1/2} \alpha'\right)^3 Z, \tag{2.18}$$

one arrives at the 4d scalar potential in the Einstein frame⁷

$$V_{\text{KS}} = \frac{|Z|^{4/3}}{c' M^2 (\rho + \bar{\rho})^2} \left| \frac{M}{2\pi i} \log \frac{\Lambda^3}{Z} + i \frac{K}{g_s} \right|^2, \tag{2.19}$$

where we have redefined $\Lambda_0 \rightarrow \Lambda$ analogously to (2.18) and from now on one sets $\kappa_4 = 1$. The scale Λ in (2.19) has now mass dimension one. Later on one will define a more canonical dimension-one field $Z \sim Y^3$.

An anti-D3 brane at the tip of the throat uplifts the KS potential (2.19). The contribution to the potential is determined from

$$S_{\text{D3}} = S_{\text{DBI}} + S_{\text{CS}} = -T_3 \int d^4x \sqrt{-g_4} [1 + \mathcal{O}(\alpha'^2)] \pm T_3 \int C_4, \tag{2.20}$$

where the sign in front of the second term is determined by the charge of the brane, and T_3 is given by $T_3 = \frac{1}{(2\pi)^3 \alpha'^2}$. For the D3-brane in a background given by (2.1), the DBI and the CS pieces of the action cancel each other. Hence, for the $\overline{\text{D3}}$ -brane they add up and one finds

$$V_{\overline{\text{D3}}} = -2T_3 C_4 = \frac{2}{(2\pi)^3 \alpha'^2} H^{-1}. \tag{2.21}$$

Using the warp factor given in (2.8) and turning into the 4d Einstein frame, one finally obtains

$$V_{\overline{\text{D3}}} = \frac{1}{\pi(\rho + \bar{\rho})^2} \frac{2^{1/3}}{I(\mathcal{T})} \frac{|Z|^{4/3}}{g_s M^2}. \tag{2.22}$$

The $I(\mathcal{T})$, defined in (2.9) is a monotonically decreasing function. Therefore, a $\overline{\text{D3}}$ -brane has minimal energy if it is placed at the tip of the throat. For later convenience we introduce a constant $c'' = \frac{2^{1/3}}{I(0)} \approx 1.75$.

With these notations and in the highly warped region, the total potential takes the form

$$V_{\text{KS+uplift}} = \frac{|Z|^{4/3}}{(\rho + \bar{\rho})^2 c' M^2} \left\{ \left| \frac{M}{2\pi i} \log \frac{\Lambda^3}{Z} + i \frac{K}{g_s} \right|^2 + \frac{c' c''}{\pi g_s} \right\}. \tag{2.23}$$

The minimum of the potential with the uplift can be found analytically to be given by [30]

$$Z_0 = e^{-\frac{3}{4} \left(1 - \sqrt{1 - \frac{64\pi c' c''}{9g_s M^2}}\right)} \Lambda^3 e^{-\frac{2\pi K}{g_s M}}, \tag{2.24}$$

which clearly displays the disappearance of the non-trivial minimum for small values of the fluxes $g_s M^2 < (64\pi c' c'')/9$, leading to the condition (1.1).

⁷We take also into account the change of power $1/(\rho + \bar{\rho})^3 \rightarrow 1/(\rho + \bar{\rho})^2$ due to the warping, as argued for in [46].

Some caution is however required in using the scalar potential (2.23).⁸ In the KS solution the S field is not a modulus, but is fixed to its supersymmetric value (2.15). Replacing the warp factor with its S -dependence as in (2.22) is well justified only at the minimum and might not be trusted far from it. So the potential (2.23) might not be really trusted “off-shell” far from its KS minimum. For large values of M -flux, the shift in the vev induced by the antibrane uplift is small and the potential should be reliable close to the new minimum. For smaller values of the flux, below to the destabilization point (1.1), the runaway behavior depends crucially on the form of the potential for small values of S , far away from the old supersymmetric minimum. Therefore, it cannot be excluded that there are significant corrections to the potential which modify or invalidate the bound (1.1). On the other hand, the results of [32] provide further evidence for the validity of (1.1) and thus also of the potential (2.23). There, a similar bound was obtained from the numerical construction of Klebanov-Strassler black holes [47] which goes beyond the probe approximation and takes the full back reaction on the geometry into account. Moreover, [83] compared the analysis of [30, 31] with the radion stabilization in RS scenarios by the Goldberger-Wise formalism and found qualitative agreement.

In this paper we will not try to resolve these issues and will take the validity of the potential (2.23) as a working assumption. Moreover, we mostly work in the regime of large enough $g_s M^2$ where the shift in S induced by the $\overline{D3}$ brane is small. It is therefore reasonable to assume that in the vicinity of the $\overline{D3}$ minimum possible corrections to the potential are subleading. Notice, that even in this regime of parameters the effects of the warp factor on the potential [44, 45] are crucial for our following analysis. Neglecting them corresponds to setting $c' = 0$ and results in a significantly different mass spectrum.

3 Four-dimensional supergravity description and moduli stabilization

Adding the nonperturbative term generated by stringy instantons or gaugino condensation amounts to adding to the perturbative superpotential the KKLT-like terms $W_{\text{KKLT}} = W_0 + Ae^{-a\rho}$. The KKLT sector by itself will be defined by

$$\mathcal{K} = -3 \log(\rho + \bar{\rho}), \quad W_{\text{KKLT}} = W_0 + Ae^{-a\rho}, \quad (3.1)$$

which leads to the KKLT potential

$$V_{\text{KKLT}} = \frac{1}{(\rho + \bar{\rho})^3} \left\{ \frac{(\rho + \bar{\rho})^2}{3} \left| \partial_\rho W_{\text{KKLT}} - \frac{3}{\rho + \bar{\rho}} W_{\text{KKLT}} \right|^2 - 3|W_{\text{KKLT}}|^2 \right\}, \quad (3.2)$$

which has an AdS minimum. We will show that, under some mild assumptions, the KKLT sector does not affect the conifold destabilization mechanism we found. Before showing this, we remind the reader that the uplift can also be described in a manifestly supersymmetric formalism using nonlinear supersymmetry with a nilpotent goldstino superfield [33–38, 48–52]. Therefore, if there is a mass gap we should be able to describe the whole action in

⁸We thank A. Hebecker and L. Martucci for useful discussions on this point.

terms of a supergravity action. Introducing a nilpotent superfield X , it is indeed possible to write the Kähler potential at the perturbative level as

$$\begin{aligned} \mathcal{K} &= -3 \log \left(\rho + \bar{\rho} - \frac{|X|^2}{3} - \frac{\xi |Z|^{2/3}}{3} \right) - \log(-i(\tau - \bar{\tau})) + c|Z|^2 \left(\log \frac{\Lambda^3}{|Z|} + 1 \right), \\ W &= W_0 + Ae^{-a\rho} + \frac{M}{2\pi i} Z \left(\log \frac{\Lambda^3}{Z} + 1 \right) + K\tau Z + \frac{1}{M} \sqrt{\frac{c''}{\pi}} Z^{2/3} \tau X, \end{aligned} \quad (3.3)$$

where we introduced the (flux-dependent) constant $\xi = 9c'g_sM^2$. W_0 denotes the vev of the bulk or “UV” superpotential. In what follows, we assume the dilaton to be stabilized $\tau = \frac{i}{g_s}$. The field X satisfies the nilpotent constraint $X^2 = 0$. It contains the goldstino G localized on the antibrane and generates the Volkov-Akulov nonlinear supersymmetric Lagrangian. The solution of the constraint, in superspace language is

$$X = \frac{GG}{2F_X} + \sqrt{2}\theta G + F_X\theta^2, \quad (3.4)$$

where θ is the fermionic superspace coordinate. The simplest recent string theory examples of Volkov-Akulov nonlinear supersymmetric actions consists of putting a stuck $\overline{D3}$ antibrane on top of an $O3_-$ plane [53], which reduces the (anti)brane localized degrees of freedom to only the goldstino [48–52]. Similar construction, much like the original string vacua with “brane supersymmetry breaking” [2–6] also generate a nonlinear realization of supersymmetry on the antibranes, as shown explicitly in [7, 8]. The nilpotent constraint eliminates the scalar partner of the goldstino, keeping the auxiliary field F_X . Consequently, the scalar potential is computed from the usual supergravity potential, by setting at the end $X = 0$. The last term in the superpotential reproduces the antibrane uplift, redshifted by the S-dependent prefactor. Note that the nilpotent goldstino formalism is valid as long as $F_X \neq 0$. In the example we consider (3.5), we find⁹ that $F_X = \frac{i}{g_s M} \sqrt{\frac{c''}{\pi}} Z^{2/3}$ and, since $\langle Z \rangle \neq 0$, the formalism is indeed valid. The stronger the warping the smaller the supersymmetry breaking. We expect in principle a maximum value of the warping also from the requirement that states decoupled by the supersymmetry breaking to be heavy enough.

This Kähler potential should be understood as an Z -expansion of the general Kähler potential derived in [54] (see also [55–57]), reproducing the metric $G_{S\bar{S}}$ of [44, 45].

A naive integration-out of Z would produce an effective constant

$$W_{0,\text{eff}} = W_0 + \frac{M}{2\pi i} \left(1 + \sqrt{1 - \frac{64\pi c' c''}{9g_s M^2}} \right) Z_0. \quad (3.5)$$

It is convenient in what follows to work with a dimension 1-field Y instead of the dimension 3 one Z . We introduce the convenient definitions

$$\begin{aligned} Y &= \left(\frac{\xi}{3} \right)^{1/2} Z^{1/3} = \left(\frac{3c'g_sM^2}{2^{3/2}\pi\alpha'^2} \right)^{1/2} S^{1/3}, \\ \tilde{M} &= \left(\frac{3}{\xi} \right)^{3/2} M, \quad \tilde{K} = \left(\frac{3}{\xi} \right)^{3/2} K, \quad \tilde{\Lambda}_0 = \left(\frac{\xi}{3} \right)^{1/2} \Lambda, \quad \tilde{c}'' = \frac{3}{\xi g_s M} \sqrt{\frac{c''}{\pi}}. \end{aligned} \quad (3.6)$$

⁹The factor i can be removed by a redefinition of the field X .

With these definitions, freezing the dilaton and in the region of strong warping, where the perturbative Kähler potential for S is negligible, the effective action (3.3) becomes

$$\begin{aligned}\mathcal{K} &= -3 \log \left(\rho + \bar{\rho} - \frac{|X|^2}{3} - |Y|^2 \right), \\ W &= W_0 + Ae^{-a\rho} + \frac{\tilde{M}}{2\pi i} Y^3 \left(3 \log \frac{\tilde{\Lambda}_0}{Y} + 1 \right) + \frac{i\tilde{K}}{g_s} Y^3 + \tilde{c}'' Y^2 X.\end{aligned}\quad (3.7)$$

Therefore, the SUGRA scalar potential of the model (3.7) can be written in the form

$$V = \frac{1}{r^2} \left\{ 3|Y|^4 \left| \frac{3\tilde{M}}{2\pi i} \log \frac{\tilde{\Lambda}_0}{Y} + i\frac{\tilde{K}}{g_s} \right|^2 + |\tilde{c}'' Y^2|^2 + \left| \partial_\rho W - \frac{3}{\rho + \bar{\rho}} W_{\text{eff}} \right|^2 - \frac{3}{\rho + \bar{\rho}} |W_{\text{eff}}|^2 \right\}, \quad (3.8)$$

where we defined

$$W_{\text{eff}} = W_0 + Ae^{-a\rho} + \frac{\tilde{M}}{2\pi i} Y^3 \quad \text{and} \quad r = \rho + \bar{\rho} - |Y|^2. \quad (3.9)$$

The first two terms in (3.8) recover the potential $V_{\text{KS+uplift}}$ displayed in (2.23) coming from the fluxes and the $\overline{\text{D3}}$ uplift, written in terms of the dimension-one field Y and in the small Y limit. The last two terms in the potential (3.8) reduce, for $Y = 0$, to the KKLT potential for ρ .

3.1 Vacuum structure and mass scales

The scalar potential (3.8) has an almost decoupled structure, between the KS and the KKLT sector. In the case that one of the moduli is significantly heavier than the other, this implies that the decoupled KS+uplift and the KKLT+uplift minima will be a good zeroth order approximation. The later ones are determined as

$$\left. \frac{\partial V_{\text{KS+uplift}}}{\partial Y} \right|_{Y_0} = 0, \quad \left. \frac{\partial V_{\text{KKLT+uplift}}}{\partial \rho} \right|_{\rho_0} = 0. \quad (3.10)$$

The explicit values are

$$\begin{aligned}Y_0 &= e^{-\frac{2\pi\epsilon_0}{3\tilde{M}}} \tilde{\Lambda}_0 e^{-\frac{2\pi\tilde{K}}{3g_s\tilde{M}}} \leftrightarrow Z_0 = e^{-\frac{3}{4} \left(1 - \sqrt{1 - \frac{64\pi c' c''}{9g_s M^2}} \right)} \Lambda_0^3 e^{-\frac{2\pi K}{g_s M}}, \\ [a(\rho_0 + \bar{\rho}_0) + 5] Ae^{-a\rho_0} &= -3W_0.\end{aligned}\quad (3.11)$$

The constant ϵ_0 above is given by

$$\begin{aligned}\epsilon_0 &= \frac{3\tilde{M}}{2\pi} \log \frac{\tilde{\Lambda}_0}{Y_0} - \frac{\tilde{K}}{g_s} = \frac{1}{4} \left(\frac{3\tilde{M}}{2\pi} - \sqrt{\left(\frac{3\tilde{M}}{2\pi} \right)^2 - \frac{48\tilde{c}''^2}{9}} \right) \\ &= \frac{3}{8\pi M^2 (3c'g_s)^{3/2}} \left(1 - \sqrt{1 - \frac{64\pi c' c''}{9g_s M^2}} \right),\end{aligned}\quad (3.12)$$

and measures the deviation of the minimum of S_0 from the supersymmetric one (2.15) before the uplift. Notice that the higher the flux M the smaller the deviation of the vev of Z from its supersymmetric value. The main result of [30] was that the existence of a non-trivial minimum for S implies a minimum value for the M flux

$$g_s M^2 \geq M_{\min}^2 = \frac{64\pi c' c''}{9} \simeq (6.8)^2. \quad (3.13)$$

The cancelation of the cosmological constant after uplift translates into the tuning

$$\left(|\tilde{c}''|^2 + 3\epsilon_0^2\right) |Y_0|^4 \simeq \frac{3|W_0|^2}{\rho_0 + \bar{\rho}_0} \rightarrow W_0 \simeq \epsilon_0(\rho_0 + \bar{\rho}_0)^{1/2} Y_0^2, \quad (3.14)$$

where in the last estimate we neglected $|\tilde{c}| \ll \epsilon_0$, valid for not extremely large values of the flux. Notice that an uplift to zero cosmological constant requires $W_0 \sim Z_0^{2/3}$ which is much larger (in the strong warping regime) than the last term in (3.5), induced by integrating-out the field Z . The mass of the moduli fields are then readily computed. We denote by $m_{Y,\pm}$ the two mass eigenstates of the complex field Y . At the leading order one finds

$$\begin{aligned} m_{Y,+}^2 &= \frac{\tilde{M}}{4\pi} \frac{Y_0^2}{\rho_0 + \bar{\rho}_0} = \frac{Z_0^{2/3}}{8\pi\rho_0(3c'g_s)^{1/2}}, & m_{Y,-}^2 &= m_{Y,+}^2 \sqrt{1 - \frac{M_{\min}^2}{g_s M^2}}, \\ m_{\rho\bar{\rho}}^2 &= \frac{a^2 W_0^2}{\rho_0 + \bar{\rho}_0}, & m_{\rho\bar{\rho}}^2 &\ll m_{Y,\pm}^2. \end{aligned} \quad (3.15)$$

After imposing the cancelation of the vacuum energy, the ratio of the moduli masses is given by

$$\frac{m_{\rho}^2}{m_Y^2} \sim \left(1 - \sqrt{1 - \frac{M_{\min}^2}{g_s M^2}}\right)^2 \frac{a^2 (3c')^{5/2} \rho_0 Z_0^{2/3}}{32\pi^2 g_s^{1/2}}. \quad (3.16)$$

The Z modulus is therefore heavier than ρ provided that

$$Z_0 \ll \left[\frac{32\pi^2 g_s^{1/2}}{\left(1 - \sqrt{1 - \frac{M_{\min}^2}{g_s M^2}}\right)^2 (3c')^{5/2}} \right]^{\frac{3}{2}} \frac{1}{(a^2 \rho_0)^{3/2}}. \quad (3.17)$$

This is also the condition of validity for the (quasi) decoupling of the KS and KKLT sectors, that was our starting assumption in finding the vacuum structure. One can a posteriori check that the shifts in the minima after coupling the KS+uplift and the KKLT sector are parametrically $\delta Y/Y_0 \sim \delta\rho/\rho_0 \sim Y_0\sqrt{\rho_0}$, which are small precisely when (3.17) is fulfilled. The constraint (3.17) is easy to satisfy for large fluxes. The strongest constraint arises for small fluxes. For example, close to the critical value (3.13) and in the case of stringy instantons $a = 2\pi$, (3.17) gives roughly $Z_0 \ll g_s^{3/4}/(3\rho_0)^{3/2}$. Since

$$\frac{2\pi K}{g_s M} = \frac{2\pi MK}{g_s M^2} \leq \frac{2\pi |Q_3^{\text{loc}}|}{(6.8)^2}, \quad (3.18)$$

a given value of the warp factor translates into a minimum value of the localized D3 charge Q_3^{loc} , according to

$$|Q_3^{\text{loc}}| \geq \frac{(6.8)^2}{2\pi} |\log Z_0|. \quad (3.19)$$

For example, typical KKLТ values $\rho_0 \sim 20 - 50$ and for $\Lambda_0 \sim M_P$ imply $Z_0 < 10^{-2}$ to satisfy (3.17), which would imply $|Q_3^{\text{loc}}| \geq 40 - 50$. As another example, the gravitino mass is given by

$$m_{3/2} \simeq \frac{W_0}{(\rho_0 + \bar{\rho}_0)^{3/2}} \sim \frac{\epsilon_0 Y_0^2}{\rho_0}, \quad (3.20)$$

where the last estimate is order of magnitude only. TeV values of the gravitino mass $m_{3/2} \sim \text{TeV}$ would require $Z_0 \sim 10^{-21}$, which translates into $|Q_3^{\text{loc}}| \geq 350 - 400$. Whereas this is possible in F-theory, it is challenging to construct explicit examples with such large localized D3 charges [58].

3.2 Contribution to supersymmetry breaking

The contribution of various fields to supersymmetry breaking is encoded in the auxiliary fields, in terms of which one can write the scalar potential as

$$V = \mathcal{K}_{I\bar{J}} F^I F^{\bar{J}} - 3m_{3/2}^2, \quad \text{where} \quad F^I = e^{\mathcal{K}/2} \mathcal{K}^{I\bar{J}} \overline{D_{\bar{J}} W}. \quad (3.21)$$

Using the vev's obtained in the previous section, one finds

$$D_X W = \frac{1}{g_s M} \sqrt{\frac{c''}{\pi}} Z_0^{2/3}, \quad D_Y W \simeq -3i\epsilon_0 Y_0^2, \quad D_\rho W \simeq -\frac{6W_0}{a(\rho_0 + \bar{\rho}_0)^2}. \quad (3.22)$$

Even if the fields Y and ρ mix in the Kähler potential, it can be shown that it is a good approximation to neglect the mixing and to define individual contributions to SUSY breaking, according to

$$f_I \equiv e^{\mathcal{K}/2} (\mathcal{K}^{I\bar{I}})^{1/2} \overline{D_{\bar{I}} W}, \quad V \simeq \sum_I |f_I|^2 - 3m_{3/2}^2. \quad (3.23)$$

Then (3.22) leads to

$$f_X \sim f_Y \sim m_{3/2}, \quad f_\rho \sim \frac{m_{3/2}}{a(\rho_0 + \bar{\rho}_0)}. \quad (3.24)$$

It is interesting that the conifold field $Y \sim Z^{1/3}$ has a large contribution to supersymmetry breaking, at the same order as the one of the uplift (nilpotent) field X . Both contributions are localized at the tip of the throat. Notice that a contribution to supersymmetry breaking of the complex structure field Z could be interpreted as an effective generation of an (1, 2) flux. On the other hand, the contribution to supersymmetry breaking of the Kähler modulus ρ , which propagates across the whole bulk, is suppressed by a factor of order $1/(a\rho_0)$. This will have consequences on the transmission of supersymmetry breaking into the matter sector.

3.3 Comments on the dS swampland conjecture

It was recently conjectured [59–61] that in all controlled compactifications

$$|\nabla V| \geq aV \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -a'V, \quad (3.25)$$

where $a, a' > 0$ are $\mathcal{O}(1)$ numbers. The second condition is not constraining in our case, therefore we discuss the first one. Since the KS and the KKLT sector are approximately decoupled in the regime $m_\rho \ll m_Y$, we can concentrate on the KS plus the uplift sector, assuming the Kähler modulus ρ is stabilized, described by

$$\mathcal{L}_Y = \frac{3}{\rho + \bar{\rho}} |\partial Y|^2 - V_{\text{KS+uplift}}, \quad (3.26)$$

where the appropriate scalar potential is given in (2.23). We therefore compute

$$\frac{|\nabla_Y V|}{V} = \frac{\sqrt{G^{Y\bar{Y}} \partial_Y V \partial_{\bar{Y}} V}}{V}. \quad (3.27)$$

If $\sqrt{g_s}M \geq M_{\text{min}}$, there is a dS minimum and the dS conjecture is violated. If this is realized in string theory depends on the existence of compactifications with large localized D3 charges [58]. Following the same steps and arguments as in [30], another check can be performed for small flux $\sqrt{g_s}M < M_{\text{min}}$, where the dS minimum disappear. One finds

$$\frac{|\nabla_Y V|}{V} \geq \frac{2}{|Y|} \sqrt{\frac{\rho + \bar{\rho}}{3}} \left(1 - \frac{\sqrt{g_s}M}{M_{\text{min}}}\right). \quad (3.28)$$

This is generically satisfied in the limit of strong warping. It is however amusing to notice that by imposing $a \sim 1$ one obtains a condition parametrically of the type (3.17), although the two conditions apply to different cases.

On the other hand, a sufficient (but not necessary) condition to satisfy the dS conjecture would be to select large enough values of the (0, 3) flux parameter W_0 to forbid an uplift to positive vacuum energy. Using the results and notations from section 3.1, this condition reads

$$W_0 \geq \epsilon_0 (\rho_0 + \bar{\rho}_0)^{1/2} Y_0^2. \quad (3.29)$$

4 Adding matter: soft terms

The Klebanov-Strassler throat can generate an exponential hierarchy for the scale of supersymmetry breaking in the observable sector. The matter fields Q_i are defined as usual by the properties $\langle Q_i \rangle = 0$, $F^i = 0$. The 4d supergravity lagrangian contains hidden sector (moduli) fields called T_α in what follows ($T_\alpha = X, Y, \rho$ in our case), coupled to the matter fields Q_i . The Kähler potential and superpotential are defined by an expansion in powers of the matter fields

$$\begin{aligned} \mathcal{K} &= \hat{\mathcal{K}}(T_\alpha, \bar{T}_\alpha) + K_{i\bar{j}}(T_\alpha, \bar{T}_\alpha) Q^i \bar{Q}^{\bar{j}} + \frac{1}{2} \left(Z_{ij}(T_\alpha, \bar{T}_\alpha) Q^i Q^j + \text{h.c.} \right), \\ W &= \hat{W}(T_\alpha) + \frac{1}{2} \tilde{\mu}_{ij}(T_\alpha) Q^i Q^j + \frac{1}{3} \tilde{Y}_{ijk}(T_\alpha) Q^i Q^j Q^k + \dots \end{aligned} \quad (4.1)$$

The low-energy softly broken supersymmetric lagrangian is defined by the superpotential and soft scalar potential

$$\begin{aligned}
 W_{\text{eff}} &= \frac{1}{2}\mu_{ij}Q^iQ^j + \frac{1}{3}Y_{ijk}Q^iQ^jQ^k, \\
 \mathcal{L}_{\text{soft}} &= -m_{ij}^2q^iq^{\bar{j}} - \left(\frac{1}{2}B_{ij}q^iq^{\bar{j}} + \frac{1}{3}A_{ijk}q^iq^{\bar{j}}q^k + \frac{1}{2}M_a\lambda_a\lambda_a + \text{h.c.} \right),
 \end{aligned} \tag{4.2}$$

where $Y_{ijk} = e^{K/2}\tilde{Y}_{ijk}$. For zero cosmological constant, the general tree-level expressions for soft terms and supersymmetric μ -terms in 4d supergravity, are given by [62–66]

$$\begin{aligned}
 m_{ij}^2 &= m_{3/2}^2(G_{i\bar{j}} - G^\alpha G^{\bar{\beta}} R_{i\bar{j}\alpha\bar{\beta}}) = m_{3/2}^2 K_{i\bar{j}} - F^\alpha F^{\bar{\beta}} R_{i\bar{j}\alpha\bar{\beta}}, \\
 \mu_{ij} &= m_{3/2} \nabla_i G_j = e^{\frac{K}{2}} \tilde{\mu}_{ij} + m_{3/2} Z_{ij} - F^{\bar{\alpha}} \partial_{\bar{\alpha}} Z_{ij}, \\
 A_{ijk} &= m_{3/2}^2 (3\nabla_i G_j + G^\alpha \nabla_i \nabla_j \nabla_k G_\alpha) \\
 &= (m_{3/2} - F^\alpha \partial_\alpha \log m_{3/2}) Y_{ijk} + F^\alpha \partial_\alpha Y_{ijk} - 3F^\alpha \Gamma_{\alpha(i}^l Y_{ljk)}, \\
 B_{ij} &= m_{3/2}^2 (2\nabla_i G_j + G^\alpha \nabla_i \nabla_j G_\alpha) = 2m_{3/2}^2 Z_{ij} - m_{3/2} F^{\bar{\alpha}} \partial_{\bar{\alpha}} Z_{ij} \\
 &\quad + m_{3/2} F^\alpha (\partial_\alpha Z_{ij} - \Gamma_{\alpha i}^k Z_{kj} - \Gamma_{\alpha j}^k Z_{ki}) - F^\alpha F^{\bar{\beta}} (Z_{i\bar{j}\alpha\bar{\beta}} - \Gamma_{\alpha i}^k Z_{k\bar{j}\bar{\beta}} - \Gamma_{\alpha j}^k Z_{k\bar{i}\bar{\beta}}) \\
 &\quad - e^{\frac{K}{2}} \tilde{\mu}_{ij} m_{3/2} + e^{\frac{K}{2}} F^\alpha (\partial_\alpha \tilde{\mu}_{ij} + \frac{1}{2} \hat{K}_\alpha \tilde{\mu}_{ij} - \Gamma_{\alpha i}^l \tilde{\mu}_{lj} - \Gamma_{\alpha j}^l \tilde{\mu}_{il}), \\
 M_{1/2}^a &= \frac{1}{2} g_a^2 F^\alpha \partial_\alpha f_a,
 \end{aligned} \tag{4.3}$$

where α, β are hidden sector supersymmetry breaking indices, f_a is the gauge kinetic function for the gauge group factor G_a and some basic definitions are given in the appendix. For geometrical separation in the internal space (sequestering) or in no-scale like models, tree level soft masses are zero or highly suppressed and one-loop contributions become relevant. One loop contributions to gaugino masses, called anomaly-mediated contributions in [67, 68], are given in general by [69]

$$M_{1/2}^a = -\frac{g_a^2}{16\pi^2} \left\{ (3T_G^a - T_R^a) m_{3/2} + (T_G^a - T_R^a) K_\alpha F^\alpha + \frac{2T_R^a}{d_R} (\log \det K_{i\bar{j}, R^a})_\alpha F^\alpha \right\}, \tag{4.4}$$

where T_G^a is the Dynkin index for the adjoint representation of the gauge group and T_R^a , $K_{i\bar{j}, R^a}$ are the Dynkin index for the chiral matter fields in representation R^a and their Kähler metric, respectively. The complete one-loop expression for the other soft terms is more involved [70]. In the limit of small hidden-sector vev's, the one-loop induced anomaly mediated contributions take forms of the type $m_{\text{soft}} \sim (bg^2)/(16\pi^2)m_{3/2}$, depending on beta functions b (and anomalous dimensions) of the low-energy spectrum.

On general grounds, for matter fields which are not sequestered from the KS throat, one expects soft masses of order $m_{3/2}$. For fields far away from the throat, from the four-dimensional perspective, one could anticipate soft terms generated by the Kähler modulus contribution to supersymmetry breaking F^ρ and one-loop anomaly-mediated contributions. These two contributions are similar in size, as emphasized in [71, 72] and parametrically (one-loop) suppressed compared to the gravitino mass.

4.1 Distant D3 “matter” branes

One test of the action (3.3) is to add distant, from the throat, D3 “matter” branes. Denoting by Q_i the (distant) D3 brane superfields, the tree-level action in type IIB orientifolds including them was derived in [75, 76]. In the absence of warping, the Kähler and the complex structure moduli spaces do not talk to each other, except for the holomorphic terms of coefficients called z_{ij} below. The new ingredient from the warping is that the main contribution to the complex structure field Z Kähler potential changes drastically (in the absence of matter fields) according to the previous sections, whereas the perturbative contribution is negligible. Taking this into account, we arrive at the following effective action

$$\begin{aligned} \mathcal{K} &= -3 \log \left(\rho + \bar{\rho} - \frac{|X|^2}{3} - \frac{\xi|Z|^{2/3}}{3} - |Q_i|^2 - z'_{ij}(\bar{Z}Q_iQ_j + \text{h.c.}) \right) \\ &= -3 \log \left(\rho + \bar{\rho} - \frac{|X|^2}{3} - |Y|^2 - |Q_i|^2 - z_{ij}(\bar{Y}^3Q_iQ_j + \text{h.c.}) \right) \equiv -3 \log r, \end{aligned} \quad (4.5)$$

$$W = W_1(\rho, Y, X) + W_2(Q_i).$$

Notice that (4.5) has a sequestered form, except the terms in z_{ij} . Using the appendix, it can be shown that in this case the Riemann tensor satisfies the identity

$$R_{i\bar{j}\alpha\bar{\beta}} = \frac{1}{3}G_{i\bar{j}}G_{\alpha\bar{\beta}}, \quad (4.6)$$

where $\alpha, \beta = X, Z, \rho$ are moduli indices and i, j are matter indices, such that the tree-level scalar soft masses m_{ij}^2 in (4.3) vanish after reinforcing the cancelation of the cosmological constant $G^\alpha G_\alpha = 3$. The tree-level A-terms can be written more explicitly as

$$A_{ijk} = m_{3/2}^2 \left[3G_{ijk} + G^\alpha (\partial_\alpha G_{ijk} - \Gamma_{i\alpha}^m G_{jkm} - \Gamma_{j\alpha}^m G_{ikm} - \Gamma_{k\alpha}^m G_{ijm}) \right] \quad (4.7)$$

and they turn out to also vanish for zero cosmological constant.

The tree-level supersymmetric masses in our case are given by

$$\mu_{ij} = m_{3/2} (\partial_i G_j - \Gamma_{ij}^\alpha G_\alpha) = -2z_{ij}\bar{y}^2 (2\bar{y}W_T + 3W_y) e^{\frac{\kappa}{2}} \sim \mathcal{O} \left(\frac{Y_0^4}{\rho_0^{3/2}} \right) \sim \mathcal{O} \left(\frac{Z_0^{4/3}}{\rho_0^{3/2}} \right). \quad (4.8)$$

This result is in qualitative agreement with the action obtained from the compactification worked out in [75, 76]. It is however important to remember that our effective action is only valid in the leading order in an expansion in powers of $Z^{2/3}$. The leading order contribution to the μ -terms is small and subleading and probably a mass of order $Z^{4/3}$ is quantitatively not fully under control. The $B\mu$ -like terms B_{ij} in (4.3) are of order $\mathcal{O}(Y^6) \sim \mathcal{O}(Z^2)$, but probably their computation is also not fully reliable. It is also possible that the coefficients z_{ij} are warped down such that the real values of μ_{ij}, B_{ij} is even smaller than the one we estimated here.

The tree-level action of the distant D3 branes seem to realize therefore, at the leading order in a power expansion of $S^{2/3}$, an approximate sequestered case [67, 68] where all tree-level “soft terms” for matter D3 fields are zero (or very small). However, it was

argued in [73, 74] that sequestering is broken by the coupling of D3 branes to the gaugino condensates on D7 branes or on Euclidian D3 branes providing the nonperturbative superpotential for the Kähler modulus ρ . This effect, coming from threshold corrections, breaks the sequestered form of the superpotential (4.5), by generating terms of the type $w(Q)e^{-a\rho}$. Such terms generate soft terms, but not non-holomorphic soft scalar masses. On the other hand, it was shown in [71–74] that small values of the latter, suppressed with respect to the gravitino mass, are also generated by the antibrane uplift. In our manifestly supersymmetric nilpotent formalism, such terms can be generated by breaking sequestering in the Kähler potential. Breaking sequestering and agreement with previous results require therefore a modification of our effective action (4.5) by

$$\Delta K = \frac{D|X|^2\bar{Q}_iQ_i}{r^4}, \quad \Delta W = w(Q_i)e^{-a\rho}, \quad (4.9)$$

where D is a constant and $w(Q_i)$ a holomorphic gauge invariant function of matter chiral fields. The form of ΔK , which should be treated as a small perturbation of the Kähler potential in (4.5), is determined by matching the resulting generated non-holomorphic scalar masses to previous results, $m_0^2 \sim m_{3/2}^2/(ar)^2$ for the canonically normalized matter field [71–74]. The resulting soft terms are suppressed and similar in size to the anomaly mediated terms discussed below. It would be interesting to understand from first principles the term ΔK breaking sequestering.

Notice that the $\overline{D3}$ brane interacts via loops of open strings, or equivalently, by tree-level exchange of closed strings, with distant D3 branes. This is a higher-dimensional tiny branes-antibrane interaction which is actually a one-loop effect, that has to be added to the effective action above. The sequestering is approximative therefore from several viewpoints: the threshold effects on D7 branes (or Euclidian D3 branes) and the antibrane-D3 brane distant interactions. In addition, there are small distant branes mass terms related to the contribution to supersymmetry breaking of the conifold field Z . They could maybe have an interpretation since $F^Z \neq 0$ has an effect similar to the generation of an effective (1, 2) flux.

From a four-dimensional perspective, one can also contemplate calculating the one-loop anomaly-mediated type contributions. For the effective action (4.5), the one-loop gaugino masses reduce at leading order to the universal term

$$M_{1/2}^a = -\frac{g_a^2}{16\pi^2} (3T_G^a - T_R^a) \left(m_{3/2} + \frac{1}{3}K_\alpha F^\alpha \right) \simeq -\frac{g_a^2}{16\pi^2} (3T_G^a - T_R^a)m_{3/2}. \quad (4.10)$$

4.2 D3 branes-antibrane interactions: a higher-dimensional perspective

In addition to the threshold effects on D7 branes or euclidian D3 branes, the sequestering is broken explicitly by the distant branes-antibrane interactions, which generate an interaction between the antibrane and the distant antibranes [46]. This interaction can be described by a correction to the Kähler potential, breaking the sequestered structure, of the type

$$\Delta \mathcal{K} = \frac{\xi'_z |Z|^{4/3} |X|^2}{r(\mathbf{r}_0 + \mathbf{Q})^4}, \quad (4.11)$$

where $d^2 = \mathbf{r}_0^2$ is the radial distance between the D3 branes and the antibrane. Indeed, for $\mathbf{Q}_i = 0$ this term changes the metric $\mathcal{K}_{X\bar{X}} = (1 + \frac{\xi'_z |Z|^{4/3}}{d^4})^{\frac{1}{r}}$ and changes the scalar potential generated by the uplift sector to

$$V_{\overline{D3}} + V_{D3-\overline{D3}} = \mathcal{K}^{X\bar{X}} e^{\mathcal{K}} |D_X W|^2 \simeq \frac{c'' |Z|^{4/3}}{\pi(\rho + \bar{\rho})^2 g_s M^2} \left(1 - \frac{\xi'_z |Z|^{4/3}}{d^4} \right) \equiv 2T_3 H^{-1}(r_0) \left[1 - \frac{1}{N} \frac{H(r_1)}{H(r_0)} \right], \quad (4.12)$$

where r_0 (r_1) is the radial position of the $\overline{D3}$ brane (matter D3) branes and we performed a leading-order expansion in powers of $Z \ll 1$. An expansion in powers of the matter fields gives the antibrane-distant D3 branes fields interaction

$$V_{D3-\overline{D3}} = -\frac{\xi'_z c'' |Z|^{8/3}}{\pi r^2 d^4 g_s M^2} \left[1 - \frac{4\mathbf{r}_0 \mathbf{Q}}{d^2} - \frac{2\mathbf{Q}^2}{d^2} + \frac{12(\mathbf{r}_0 \mathbf{Q})^2}{d^4} + \dots \right]. \quad (4.13)$$

In the higher-dimensional approach, the second derivative of this scalar potential gives the mass matrix for \mathbf{Q}_i , which lead to tachyonic directions [77]. It is however useful to cast the problem as a correction to the 4d Kähler potential. By using the dimension-1 field Y , it is given by

$$\Delta \mathcal{K} = \frac{\xi' |Y|^4 |X|^2}{r d^4} \left[1 - \frac{4\mathbf{r}_0 \mathbf{Q}}{d^2} - \frac{2\mathbf{Q}^2}{d^2} + \frac{12(\mathbf{r}_0 \mathbf{Q})^2}{d^4} + \dots \right], \quad (4.14)$$

where $\xi' = (3c' g_s M^2)^{-2} \xi'_z$. The linear term signifies that the distant D3 brane does not sit at an extremum, which can be remedied, in case of orientifolds, by adding an image brane at $-\mathbf{r}_0$. The quadratic terms are more conveniently written in a complex basis Φ_i, z_i , $i = 1, 2, 3$, by introducing $\Phi_1 = \frac{1}{\sqrt{2}}(Q_1 + iQ_2)$, $z_1 = \frac{1}{\sqrt{2}}(y_1 + iy_2)$, etc. One gets

$$\Delta \mathcal{K} = \frac{\xi' |Y|^4 |X|^2}{4r |z|^4} \left[1 - \frac{2}{|z|^2} \left(\delta_{ij} - \frac{3z_i \bar{z}_j}{4|z|^2} \right) \bar{\Phi}^i \Phi^j + \frac{3}{4|z|^4} (\bar{z}_i \bar{z}_j \Phi^i \Phi^j + \text{h.c.}) + \dots \right]. \quad (4.15)$$

The non-holomorphic piece $\bar{\Phi}^i \Phi^j$ changes the metric for matter fields and changes the Riemann tensor of the Kähler manifold $R = R^{(0)} + R^{(1)}$. It generates an additional contribution to the soft scalar masses

$$m_{i\bar{j}}^2 = -m_{3/2}^2 |G^X|^2 R_{i\bar{j}X\bar{X}}^{(1)} = -|F^X|^2 R_{i\bar{j}X\bar{X}}^{(1)} = |F^X|^2 \frac{\xi' |Y|^4}{2r |z|^6} \left(\delta_{ij} - \frac{3z_i \bar{z}_j}{4|z|^2} \right), \quad (4.16)$$

which are positive definite and of order $m_0^2 \sim Y_0^8 / (\rho_0^2 |z|^6)$, in agreement with the masses computed from the higher-dimensional potential (4.13). The holomorphic soft masses are then given by

$$B_{ij} = |F^X|^2 \frac{3\xi' |Y|^4}{16r |z|^8} z_i z_j. \quad (4.17)$$

Notice that they are smaller ($\mathcal{O}(Y^8)$) than the tree-level 4d supergravity computation of the previous section. They are of the same order than the non-holomorphic masses $m_{i\bar{j}}^2$

and generate tachyonic masses after diagonalization [77]. However, in a realistic compactification the distant D3 branes have to experience additional projections of the spectrum in order to make it compatible with a MSSM one. For example, it should sit at an orbifold or orientifold singularity. If this is achieved, the only sector that can have B terms is the Higgs sector. A Higgs vev could lead to correct electroweak symmetry breaking if its value is in the TeV range, therefore if the soft terms are in the TeV range. The size of soft terms provided by (4.16)–(4.17) can be re-expressed as a function of the gravitino mass and the distance from the D3 antibrane. Their typical size is then worked out to be of order $m_{soft} \sim \left(\frac{\rho_0}{|z|}\right)^3 \frac{m_{3/2}^2}{M_P}$. If the one-loop contributions would be smaller, intermediate values of $m_{3/2}$ can lead therefore to TeV values for masses. However, as discussed in the previous section, from a four-dimensional perspective, threshold corrections on D7 branes or Euclidian D3 branes and other one-loop corrections are expected to break sequestering and generate suppressed soft terms [71–74] with respect to the gravitino mass, similar in size to anomaly mediated contributions [67, 68]. Their size is typically a one-loop factor times $m_{3/2}$ and will therefore dominate over the (smaller) contributions discussed above. It would be interesting to evaluate from a higher-dimensional perspective, based on anti-brane-distant branes interactions, the one-loop generated soft terms and compare them to the four-dimensional expressions. This is however beyond the scope of the current letter.

Notice that, if one uses some distant D3 branes for inflation, then a scalar potential of the type (4.12) becomes the inflationary energy scale. From the current bounds on the tensor to scalar ratio $r \leq 0.1$, one gets another bound on the acceptable values of Z_0

$$V_{\text{inf}} = V_{\text{D3}-\overline{\text{D3}}} \leq 10^{16} \text{ GeV} \quad \rightarrow \quad \frac{Z_0^{4/3}}{\rho_0^2} \leq 10^{-9}. \quad (4.18)$$

For a KKLT type scenario, this implies $Z_0 \leq 10^{-5}$ and therefore again a largish contribution to the localized D3 flux $|Q_3^{\text{loc}}| > \mathcal{O}(80)$, for $\Lambda_0 \sim M_P$.

4.3 4d versus higher-dimensional description

It was shown in [31] that Kaluza-Klein states localized on the KS throat have masses parametrically of the same order as the mass of the field Z . Then a full 4d description of the dynamics is probably not an accurate approximation, unless one goes to low energies and integrates the whole KK tower and the field Z . This it difficult to do in practice. On the other hand, the gravitino mass is small enough that a supergravity description should exist at low-energy. It is difficult to make definite statements, but some qualitative remarks go as follows.

From a 4d perspective, assuming that the 4d SUGRA description is valid, one can use the general formulae of soft terms as in the beginning of the previous section. Alternatively, we can use higher-dimensional brane-antibrane forces as a starting point of computing soft terms. At tree-level, we found qualitative agreement for distant (from the throat) D3 branes, which seem to fulfill an approximate sequestering, broken by threshold effects that we commented around (4.9). Additional contributions come from small mass terms related to the contribution to supersymmetry breaking of the conifold field Z , which has

an effect similar to an effective $(1,2)$ flux. As mentioned previously, it is also plausible that the warping is suppressing further the real values of these masses. At one-loop level, 4d anomaly-mediated contributions to soft terms are present, consistent with a diffuse transmission of supersymmetry breaking across the bulk. Their higher-dimensional origin is not clear and it would be very interesting to check them in detail from the higher-dimensional viewpoint, in particular the one-loop contribution to the gaugino masses. We hope to be able to return to this issue elsewhere.

5 Conclusions

We continued the analysis of the KKL_T moduli stabilization with antibrane uplift in the context of the Klebanov-Strassler warped compactification, taking into account the light complex structure field S (Z) identified in [30, 31]. Assuming the validity of the 4d supergravity description and using a manifestly supersymmetric formulation of the uplift via a nilpotent field on the antibrane, we worked out the vacuum structure and physical spectrum, confirming the potential destabilization of the KS throat. The minimal value of the needed flux (1.1), combined with needed redshift for various physical purposes, translates into (relatively) large values of the localized D3 charge Q_3^{loc} , beyond the usual perturbative values $Q_3^{\text{loc}} \leq 16$. Notice that this is not a surprise in itself. Indeed, the gravitational KS solution is valid for $g_s M \gg 1$, which is generically stronger than our destabilization limit $\sqrt{g_s} M \geq 6.8$. However, using just the standard KS validity bound leads to weaker limits on Q_3^{loc} by choosing maximal values ($g_s \sim \mathcal{O}(1)$) of the string coupling, which are reasonable at least in F-theory. The destabilization bound, on the other hand, leads to limits on Q_3^{loc} which are independent on the string coupling and feature generically large numbers. Our viewpoint in this paper is that, whereas it could be difficult to obtain such large localized D3 charges in perturbative type II strings, it is presumably possible in F-theory. In such F-theory models, however, the stabilization of a large number of complex structure moduli is challenging [58].

We studied the consequences of including the light complex structure modulus Z in the low-energy description for the vacuum structure and phenomenology. The effect of the antibrane uplift is that the vev of Z is shifted such that its contribution to supersymmetry breaking is large, comparable with that of the nilpotent field. Secondly, whereas the conifold field is generically heavier than the volume modulus, justifying an integrating out procedure, this is not always the case and there is a condition that the flux and the other parameters have to satisfy. Most of supersymmetry breaking is localized at the tip of the throat, but a small amount of supersymmetry breaking is transmitted across the bulk far from the throat. We studied the effects of the supersymmetry breaking on observable sector fields, in particular on distant D3 branes. Neglecting threshold effects, we found an approximate sequestering, with the only non-zero masses in the observable sector are of μ , $B\mu$ type and very small. Their appearance could be due to the fact that the complex structure modulus Z contributes to supersymmetry breaking, which acts effectively as an $(1,2)$ flux. Comparing the results from a four-dimensional supergravity perspective and from a higher-dimensional one, one finds qualitative agreement at tree-level. Threshold ef-

fects (4.9) are expected to add other contributions breaking sequestering [71–74]. It would be interesting to understand the origin of the term ΔK in (4.9) in the nilpotent formalism, needed in order to match non-holomorphic soft scalar masses with previous results.

It would be clearly interesting to investigate further one-loop soft terms from four dimensional and higher-dimensional perspective and check their compatibility. It would also be interesting to study from a similar viewpoint other models of moduli stabilization and check if the potential throat destabilization still exists. The existence of a critical value of the flux to avoid destabilization could be addressed directly in the dual gauge theory of the KS throat. Another direction to investigate is the search of alternatives, to the $\overline{D3}$ antibrane, uplifts of the vacuum energy. Finally, we commented on the dS swampland conjecture [59–61]. It would be interesting to investigate further this refined model of moduli stabilization from the viewpoint of the other swampland conjectures [78–81] (for an extensive review and references, see [82]).

Note added. While this paper was completed, the paper [83] appeared, which has some overlap with ours and interpret our normalized field Y of section 3 with the radion from a 5d perspective. We thank Lisa Randall for discussions and for sharing her preliminary draft.

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A Supergravity formulae

As well-known, the 4d supergravity action does not depend separately on W and \mathcal{K} , but on the function $G = \mathcal{K} + \log |W|^2$. Some useful formulae for a Kähler space, for which $G_{I\bar{J}} = K_{I\bar{J}} = \partial_I \partial_{\bar{J}} \mathcal{K}$, used in the text and in particular to evaluate (4.3) are

$$\begin{aligned}
 G^I &= G^{I\bar{J}} G_{\bar{J}}, & \Gamma_{I\bar{J}}^K &= G^{K\bar{M}} \partial_I G_{J\bar{M}}, & \nabla_I V_J &= \partial_I V_J - \Gamma_{IJ}^K V_K, \\
 R_{I\bar{J}K\bar{L}} &= \partial_K \partial_{\bar{L}} G_{I\bar{J}} - G^{M\bar{N}} \partial_K G_{I\bar{N}} \partial_{\bar{L}} G_{M\bar{J}}.
 \end{aligned}
 \tag{A.1}$$

In the case of our effective action (4.5), the nontrivial components of the connections are (i, j denote matter fields indices in what follows)

$$\Gamma_{i\rho}^j = -\frac{1}{r} \delta_i^j, \quad \Gamma_{iy}^j = \frac{\bar{y}}{r} \delta_i^j, \quad \Gamma_{ij}^\rho = 4z_{ij} \bar{y}^3, \quad \Gamma_{ij}^y = 6z_{ij} \bar{y}^2.
 \tag{A.2}$$

By using the metric components and (A.1), (A.2), one can easily verify

$$R_{i\bar{j}\alpha\bar{\beta}} = \frac{1}{3} G_{i\bar{j}} G_{\alpha\bar{\beta}},
 \tag{A.3}$$

where $\alpha, \beta = X, Y, \rho$ and i, j are matter fields indices. In the presence of the correction to the Kähler potential (4.9) breaking sequestering, there is a new correction to the Riemann tensor, which becomes, at first order in the correction,

$$R_{i\bar{j}\alpha\bar{\beta}} = \frac{1}{3} \left(1 + \frac{D}{r^2} \delta_{\alpha X} \delta_{\bar{\beta} \bar{X}} \right) G_{i\bar{j}} G_{\alpha\bar{\beta}}. \quad (\text{A.4})$$

Analogously, for the correction in (4.11), the Riemann tensor becomes

$$R_{i\bar{j}\alpha\bar{\beta}} = \frac{1}{3} \left(G_{\alpha\bar{\beta}} - \frac{2\xi'|Y_0|^4}{Nd^6} \delta_{\alpha X} \delta_{\bar{\beta} \bar{X}} \right) G_{i\bar{j}}. \quad (\text{A.5})$$

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