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Global half-BPS $\mathrm{AdS}_2 \times S^6$ solutions in Type IIB

David Corbino, Eric D'Hoker, Justin Kaidi and Christoph F. Uhlemann

Mani L. Bhaumik Institute for Theoretical Physics,
Department of Physics and Astronomy, University of California,
Los Angeles, CA 90095, U.S.A.

E-mail: corbino@physics.ucla.edu, dhoker@physics.ucla.edu, jkaidi@physics.ucla.edu, uhlemann@physics.ucla.edu

Abstract: We investigate half-BPS Type IIB supergravity solutions with spacetime geometry $\mathrm{AdS}_2 \times S^6$ warped over a Riemann surface Σ. The general local solution was obtained in earlier work in terms of two holomorphic functions $\mathcal{A}_\pm$ on Σ. In the first part of this paper we seek global solutions corresponding to the near-horizon behavior of (p, q)-string junctions. We identify the type of singularity in $\mathcal{A}_\pm$ needed at the boundary of Σ to match the solutions locally onto the classic (p, q)-string solution. We construct solutions with multiple (p, q)-strings, but the existence of geodesically complete solutions remains unsettled. In a second part we construct multi-parameter families of non-compact globally regular and geodesically complete solutions with asymptotic regions and an AdS_2 throat which caps off smoothly.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence, Superstring Vacua

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1 Introduction

The classification of supergravity solutions containing a factor of anti-de Sitter space is a topic of long-standing interest in the construction of holographic duals to conformal field theories. With the maximal number of 32 supersymmetries, the classic supergravity solutions $\mathrm{AdS}_5 \times S^5$, $\mathrm{AdS}_4 \times S^7$, and $\mathrm{AdS}_7 \times S^4$ — invariant respectively under the Lie superalgebras $\mathrm{PSU}(2,2|4), \mathrm{OSp}(8|4;\mathbb{R})$, and $\mathrm{OSp}(8^*|4)$ — served as prototypes for the AdS/CFT correspondence [1]. A classification of sub-superalgebras of these maximally supersymmetric algebras containing 16 supersymmetries was developed in [2], and many of the predicted solutions have since been constructed.

Motivated by brane considerations [3], progress was recently made on the construction of supergravity solutions [4–6] which are holographic duals to five-dimensional superconformal field theories. The superconformal algebra in five dimensions is unique and given by a particular real form of the exceptional Lie superalgebra $F(4)$ with maximal bosonic subalgebra $SO(2,5) \oplus SO(3)$. This is not a sub-superalgebra of any of the three maximal superalgebras. The corresponding Type IIB supergravity solutions have a spacetime of the form $AdS_6 \times S^2$ warped over a Riemann surface with boundary Σ, and are specified in terms of two locally holomorphic functions $\mathcal{A}_\pm$ on Σ. Globally regular and geodesically complete solutions sourced by the charges p, q of the complex three-form field strength of Type IIB were shown to provide fully back-reacted geometries for the near-horizon region of general (p,q) five-brane webs [7–9].

By double analytic continuation from Minkowski signature AdS_6 and AdS_2 to Euclidean signature S^6 and S^2, the existence of half-BPS $AdS_6 \times S^2$ solutions suggests the existence of half-BPS solutions with spacetime $AdS_2 \times S^6$. Indeed, general local solutions of this form were constructed in [10]. As in the case of $AdS_6 \times S^2$, the solutions are given in terms of two locally holomorphic functions $\mathcal{A}_\pm$ on a Riemann surface Σ [10]. Their symmetry group is given by a different real form of the Lie superalgebra $F(4)$, with maximal bosonic superalgebra $SO(1,2) \oplus SO(7)$, which is again not a sub-superalgebra of any of the three maximal supersymmetric algebras.

The purpose of this paper is two-fold. In a first part, we shall investigate the existence of global $AdS_2 \times S^6$ solutions sourced by seven-form charges p, q, which are naturally associated with (p,q)-strings. We shall examine the emergence of (p,q)-string web solutions [11–13] in the near-horizon limit. Although the supergravity fields of the $AdS_2 \times S^6$ solutions differ from those of the $AdS_6 \times S^2$ solutions merely by certain sign reversals, these simple differences make the construction of globally regular $AdS_2 \times S^6$ solutions intricate and technically difficult. While we shall succeed in producing solutions with multiple (p,q)-strings in the near-horizon limit, the geodesic completeness of such solutions remains unsettled. In a second part we shall study solutions independently from any string junction interpretation. We present multi-parameter families of globally regular solutions, which have asymptotic regions where spacetime decompactifies and an $AdS_2 \times S^6$ "throat" that caps off smoothly.

While the superconformal algebra $F(4)$ is unique in five-dimensions, there exist four distinct superconformal algebras with 16 supersymmetries in the one-dimensional theories holographically dual to an AdS_2 factor [14]. As mentioned above, the real form of $F(4)$ with maximal bosonic subalgebra $SO(2,1) \oplus SO(7)$ is studied in this paper. $OSp(4^*|4)$ enters the holographic dual to a Wilson line constructed in [15], while holographic duals to the remaining cases $SU(1,1|4)$ and $OSp(8|2,\mathbb{R})$ are currently being investigated.

Supersymmetric AdS_2 solutions have been studied in a wide variety of contexts, including AdS_2/CFT_1 dualities [16, 17], relations with the SYK model [18–21], for compactifications of higher-dimensional field theories [22–25], and as black hole near-horizon geometries. Further studies of AdS_2 solutions in Type IIA and M-theory can be found in [26–31].

The remainder of the paper is organized as follows. In section 2 we review the local $AdS_2 \times S^6$ solutions of [10]. In section 3 we construct an Ansatz suitable for string junction solutions and show that they match locally to the classic (p,q)-string solution. In section 4 we construct multi-parameter families of globally regular solutions. In section 5, we briefly study the T-duals of the D0-F1-D8 system in massive Type IIA supergravity [30]. For the case of $\mathrm{AdS}_6 \times S^2$, a subset of the solutions were found to correspond to Abelian and non-Abelian T-duals of the D4-D8 system [6, 32, 33]. On the other hand, we will find that the T-duals of the analogous D0-F1-D8 system are not of the form $\mathrm{AdS}_2 \times S^6$. We conclude with a discussion in section 6.

2 Local solution and regularity conditions

We summarize the local form of Type IIB supergravity solutions with 16 supersymmetries and spacetime of the form $\mathrm{AdS}_2 \times S^6$ warped over a Riemann surface Σ obtained in [10], and discuss the conditions for physical positivity and regularity of the supergravity fields. The local solutions are invariant under the real form of the exceptional Lie superalgebra $F(4)$ which has maximal bosonic subalgebra $\mathrm{SO}(1,2) \oplus \mathrm{SO}(7)$.

2.1 Supergravity fields

Invariance under $\mathrm{SO}(1,2) \oplus \mathrm{SO}(7)$ dictates the general form of the supergravity fields of the solutions. All Fermi fields vanish and the spacetime metric takes the form,

$$ds^2 = f_2^2 ds^2_{\mathrm{AdS}_2} + f_6^2 ds^2_{S^6} + 4\rho^2 |dw|^2 \tag{2.1}$$

The five-form field strength vanishes $F_{(5)} = 0$ and the three-form field strength $F_{(3)}$ and its Poincaré dual $F_{(7)}$ are given by [10],

$$\begin{aligned} F_{(3)} &= dC_{(2)} & \qquad C_{(2)} &= \mathcal{C}\ \mathrm{vol}_{\mathrm{AdS}_2} \\ F_{(7)} &= dC_{(6)} & \qquad C_{(6)} &= \mathcal{M}\ \mathrm{vol}_{S^6} \end{aligned} \tag{2.2}$$

Throughout, w is a local complex coordinate on Σ while f_2, f_6, and ρ are real-valued functions on Σ. The fields $\mathcal{C}$, $\mathcal{M}$, and the axion-dilaton $B = (1+i\tau)/(1-i\tau)$ are complex-valued functions on Σ. The line elements $ds^2_{\mathrm{AdS}_2}$,$ds^2_{S^6}$ and the volume forms $\mathrm{vol}_{\mathrm{AdS}_2}$,$\mathrm{vol}_{S^6}$ are for maximally symmetric AdS_2 and S^6 with unit radius.

The solutions are parametrized by two locally holomorphic functions $\mathcal{A}_\pm$ and expressed conveniently in terms of the composite quantities κ^2, $\mathcal{G}$, and T given in terms of $\mathcal{A}_\pm$ by,

$$\begin{aligned} \kappa^2 &= -|\partial_w \mathcal{A}_+|^2 + |\partial_w \mathcal{A}_-|^2 & \qquad \partial_w \mathcal{B} &= \mathcal{A}_+ \partial_w \mathcal{A}_- - \mathcal{A}_- \partial_w \mathcal{A}_+ \\ \mathcal{G} &= |\mathcal{A}_+|^2 - |\mathcal{A}_-|^2 + \mathcal{B} + \bar{\mathcal{B}} & \qquad T &= \frac{1-R}{1+R} = \left(1 + \frac{2|\partial_w \mathcal{G}|^2}{3\kappa^2 \mathcal{G}}\right)^{\frac{1}{2}} \end{aligned} \tag{2.3}$$

Note that $\kappa^2 = -\partial_w \partial_{\bar{w}} \mathcal{G}$. By construction [10], the functions $\kappa^2, \mathcal{G}$, and R are real. Furthermore, R is non-negative so that T is real and satisfies $T \in [-1, 1]$. In terms of these composites, the metric functions are given by,

$$f_2^2 = \frac{1}{9}\left(\frac{-6\mathcal{G}}{T^3}\right)^{\frac{1}{2}} \qquad f_6^2 = (-6\mathcal{G}T)^{\frac{1}{2}} \qquad \rho^2 = \kappa^2 \left(\frac{T}{-6\mathcal{G}}\right)^{\frac{1}{2}} \tag{2.4}$$

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The functions $\mathcal{C}$ and $\mathcal{M}$ parametrizing the two- and six-form potentials are given by,

$$\mathcal{C} = -\frac{2i}{3}\left(\frac{U}{3T^2} - \bar{\mathcal{A}}_- - \mathcal{A}_+\right)$$
$$\mathcal{M} = 80(\mathcal{W}_+ + \bar{\mathcal{W}}_-) - 12\mathcal{G}\,U + 20(\mathcal{A}_+ + \bar{\mathcal{A}}_-)\Big(2|\mathcal{A}_+|^2 - 2|\mathcal{A}_-|^2 - 3\mathcal{G}\Big) \tag{2.5}$$

where U and $\mathcal{W}_\pm$ are defined by,

$$\kappa^2 U = \overline{\partial_w \mathcal{G}}\partial_w \mathcal{A}_+ + \partial_w \mathcal{G}\overline{\partial_w \mathcal{A}_-} \qquad\qquad \partial_w \mathcal{W}_\pm = \mathcal{A}_\pm \partial_w \mathcal{B} \tag{2.6}$$

The axion-dilaton scalar field is given by,

$$B = -\frac{\partial_w \mathcal{A}_+ \partial_{\bar{w}} \mathcal{G} + R\,\partial_{\bar{w}} \bar{\mathcal{A}}_- \partial_w \mathcal{G}}{R\,\partial_{\bar{w}} \bar{\mathcal{A}}_+ \partial_w \mathcal{G} + \partial_w \mathcal{A}_- \partial_{\bar{w}} \mathcal{G}} \tag{2.7}$$

The global $\mathrm{SU}(1,1)$ symmetry transformations of the Type IIB supergravity fields are induced by the following transformations of $\mathcal{A}_\pm$ under the group $\mathrm{SU}(1,1) \otimes \mathbb{C}$,

$$\mathcal{A}_+ \to \mathcal{A}'_+ = +u\mathcal{A}_+ - v\mathcal{A}_- + a$$
$$\mathcal{A}_- \to \mathcal{A}'_- = -\bar{v}\mathcal{A}_+ + \bar{u}\mathcal{A}_- + \bar{a} \tag{2.8}$$

where $\mathrm{SU}(1,1)$ is parametrized by $u, v \in \mathbb{C}$ with $|u|^2 - |v|^2 = 1$ and the complex shift parameter a has the effect of producing gauge transformations in $\mathcal{C}$ and $\mathcal{M}$ only.

2.2 Positivity and regularity conditions

Minkowski signature of the ten-dimensional spacetime metric imposes the positivity conditions $f_2^2, f_6^2, \rho^2 > 0$ which require $\kappa^2 > 0$ and $\mathcal{G}T < 0$, assuming that all square roots of positive real arguments in (2.4) are taken to be positive (see section 7 of [10]). Without loss of generality, we may choose the branch $T > 0$ for the square root in (2.3), so that $0 < R < 1$. As a result, the positivity conditions become,

$$\kappa^2 > 0 \qquad\qquad \mathcal{G} < 0 \qquad\qquad 0 < R < 1 \tag{2.9}$$

Regularity of the supergravity fields of the solutions in the interior of Σ requires that the inequalities of (2.9) be obeyed strictly. If Σ has a non-empty boundary $\partial\Sigma$, then geodesic completeness of the solutions requires that the six-sphere shrinks to zero size $f_6 \to 0$ at the boundary, while the radius of AdS_2 remains finite. Since we have $f_6^2/f_2^2 = 9T^2$ this means that $T \to 0$ and $R \to 1$ as the boundary is being approached. Regularity of the solution at the boundary then requires the following behavior as $r \equiv 1 - R \to 0$,

$$\kappa^2 = \mathcal{O}(r) \qquad\qquad \mathcal{G} = \mathcal{O}(r^3) \qquad\qquad \partial_w \mathcal{G} = \mathcal{O}(r^2) \tag{2.10}$$

The explicit expression for R in terms of $\kappa^2\mathcal{G}$ and $\partial_w \mathcal{G}$ in (2.3) furthermore requires,

$$\frac{\kappa^2 \mathcal{G}}{|\partial_w \mathcal{G}|^2} \to -\frac{2}{3} \tag{2.11}$$

Note that the boundary condition $\partial_w \mathcal{G} = 0$ on $\partial\Sigma$ is stronger than the corresponding condition for the AdS_6 case, where $(\partial_w + \partial_{\bar{w}})\mathcal{G} = 0$ was sufficient [6].

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2.3 Realizing the regularity conditions at the boundary $\partial\Sigma$

The boundary conditions discussed in section 2.2 can be realized naturally by imposing a conjugation condition on the holomorphic functions $\mathcal{A}_\pm$ on $\partial\Sigma$. We shall take $\partial\Sigma$ to consist of only a single connected boundary component, though the construction may be easily generalized to the case when $\partial\Sigma$ has several components. We may map the boundary $\partial\Sigma$ to the real line by a Schwarz-Christoffel transformation, which is piecewise conformal. Let $w, \bar{w}$ be local complex coordinates in terms of which a boundary segment is given by $w = \bar{w}$. The conjugation condition is then given by,

$$\overline{\mathcal{A}_\pm(\bar{w})} = \mathcal{A}_\mp(w) \tag{2.12}$$

This condition readily implies $\kappa^2 = 0$ on $\partial\Sigma$ and, noting that we have,

$$\partial_w \mathcal{G}(w,\bar{w}) = (\overline{\mathcal{A}_+(w)} - \mathcal{A}_-(w))\partial_w \mathcal{A}_+(w) + (\mathcal{A}_+(w) - \overline{\mathcal{A}_-(w)})\partial_w \mathcal{A}_-(w) \tag{2.13}$$

it also implies $\partial_w \mathcal{G} = 0$ on $\partial\Sigma$. Consequently, $\mathcal{G}$ is constant along each boundary segment and can be made to vanish on any one single segment by fixing the integration constant implicit in the definitions of $\mathcal{B}$ and $\mathcal{G}$. The behaviors near the boundary in (2.10) are implied by the relations between $\mathcal{G}$, $\partial_w \mathcal{G}$, and κ^2 via differentiation, which in turn imply (2.11).

We conclude this section by drawing a comparison between the boundary conditions for the $\mathrm{AdS}_2 \times S^6$ case studied here and the boundary conditions for the $\mathrm{AdS}_6 \times S^2$ case studied in [8]. The conjugation relation between the differentials resulting from (2.12) differs from the analogous condition for the differentials in the $\mathrm{AdS}_6 \times S^2$ solutions of [8] only by a sign. More importantly, it was sufficient in [8] to implement a conjugation condition on the differentials $\partial_w \mathcal{A}_\pm$ to ensure $(\partial_w + \partial_{\bar{w}})\mathcal{G}|_{\partial\Sigma} = 0$, whereas here we impose the conjugation relation on the functions $\mathcal{A}_\pm$ themselves in order to implement the stronger condition $\partial_w \mathcal{G}|_\Sigma = 0$.

Furthermore, the conjugation condition of (2.12) is incompatible with the presence of logarithmic branch cuts in $\mathcal{A}_\pm$ starting at branch points on the boundary $\partial\Sigma$ and with branch cuts along the boundary. Suppose that we have a branch point at $w = 0$,

$$\mathcal{A}_\pm(w) = \mathcal{A}_\pm^{(0)}(w) + \mathcal{A}_\pm^{(1)}(w)\ln w \qquad\qquad \overline{\mathcal{A}_\pm^{(i)}(\bar{w})} = \mathcal{A}_\mp^{(i)}(w) \tag{2.14}$$

where $\mathcal{A}_\pm^{(0)}(w)$ and $\mathcal{A}_\pm^{(1)}(w)$ are regular and single-valued in a neighborhood of $w = 0$. Upon encircling $w = 0$ counterclockwise, $\mathcal{A}_\pm \to \mathcal{A}_\pm + i\pi\mathcal{A}_\pm^{(1)}$. This is compatible with (2.12) and the assumed conjugation properties of $\mathcal{A}_\pm^{(i)}$ only if $\mathcal{A}_\pm^{(1)}$ is zero as a function. Hence such branch cuts are ruled out, contrary to the case of $\mathrm{AdS}_6 \times S^2$ where they were crucial ingredients in the construction of the global solutions.

3 Towards string junction solutions

In this section we determine the behavior needed for the functions $\mathcal{A}_\pm$ to source the seven-form charges associated with (p,q)-strings. We shall show that, in addition to reproducing the charges, $\mathcal{A}_\pm$ with this behavior correctly reproduces the metric, axion-dilaton, and two-form fields of the near-horizon limit of the classic (p,q)-string solution, provided we carry out a certain coordinate inversion to be explained below. Though we will be able to write down the functions $\mathcal{A}_\pm$ producing (p,q)-string charges at multiple points on $\partial\Sigma$, the question of whether these supergravity solutions are actually geodesically complete for some choices of the parameters remains unsettled.

3.1 Realizing the charge and the S^7 of the (p,q)-string solution

To realize a (p,q)-string charge in an $\mathrm{AdS}_2 \times S^6$ supergravity solution, we begin by determining the behavior of the functions $\mathcal{A}_\pm$ near a point $b \in \partial\Sigma$ where a (p,q)-string charge resides. A first ingredient is that the supergravity fields should be regular in a neighborhood of the point b with b itself removed, and the seven-form should support (p,q) charge. A second ingredient is the fact that the classic (p,q)-string solution exhibits a round S^7 in the directions transverse to the string. Noting that the metric function f_6^2 vanishes on $\partial\Sigma$, we conclude that the S^7 is realized by a fibration of S^6 over a curve in Σ which begins and ends on $\partial\Sigma$. The angular dependence required to realize this fibration smoothly will constrain the functions $\mathcal{A}_\pm$.

Consider a point $b \in \partial\Sigma$ and local complex coordinates $w, \bar{w}$ which vanish at this point. Regularity and single-valuedness of the supergravity fields f_2^2, f_6^2, ρ^2, and B near $w=0$ require $\mathcal{A}_\pm$ to be single-valued in a neighborhood of $w=0$, just as was the case for $\mathrm{AdS}_6 \times S^2$ solutions. The extra condition that the factor $d\mathcal{C}$ in $F_{(3)}$ be residue-free at $w=0$ ensures the absence of five-brane charges and excludes logarithmic branch cuts emanating from $w=0$. Thus, we shall assume that $\mathcal{A}_\pm$ has a Laurent expansion in w at $w=0$. While $\mathcal{A}_\pm$ and $\mathcal{B}+\bar{\mathcal{B}}$ are single-valued near $w=0$, this set-up still allows the factor $d\mathcal{M}$ of $F_{(7)}$ to have a non-zero residue and thus to carry a non-zero (p,q)-string charge.

Next, we determine the order of the pole in $\mathcal{A}_\pm$ by requiring a smooth S^6 slicing of S^7. We shall assume that $\mathcal{A}_\pm$ has a pole at $w=0$ of order at most $p-1$,

$$\mathcal{A}_\pm(w) = \frac{\alpha_\pm}{w^{p-1}} + \frac{\beta_\pm}{w^{p-2}} + \frac{\gamma_\pm}{w^{p-3}} + \cdots \tag{3.1}$$

The coefficients are constrained by (2.12), so that $\bar{\alpha}_\pm = \alpha_\mp$ and likewise for $\beta_\pm, \gamma_\pm$, which forces the orders of the poles in $\mathcal{A}_\pm$ to coincide with one another. Whether a smooth 7-cycle is formed around the pole at $w=0$ can be inferred from the ratio f_6^2/ρ^2. In terms of polar coordinates $w = re^{i\theta}$ near the pole, the metric (2.1) may be written as,

$$ds^2 = f_2^2 ds^2_{\mathrm{AdS}_2} + 4\rho^2 \left(dr^2 + r^2 d\theta^2 + \frac{f_6^2}{4\rho^2} ds^2_{S^6} \right) \tag{3.2}$$

A smooth cycle is formed if f_6^2/ρ^2 is positive for $\theta \in (0,\pi)$ and approaches zero quadratically as $\theta \to 0$ and $\theta \to \pi$. For $p=2$ no smooth 7-cycle is formed. For $p \geq 3$ we find,

$$\frac{f_6^2}{4\rho^2} = -\frac{3\mathcal{G}}{2\kappa^2} = 3r^2 \, \frac{(2p-3)\sin\theta - \sin(2p\theta - 3\theta)}{2(p-1)(p-2)(2p-3)\sin\theta} + \mathcal{O}(r^3) \tag{3.3}$$

For $p=3$ we find $f_6^2/4\rho^2 \approx r^2 \sin^2\theta$, giving rise to a round S^7 from S^6 and the polar part of the metric on Σ. For integer $p>3$, a smooth 7-cycle is formed which is not a round S^7. We conclude that the poles in $\mathcal{A}_\pm$ must be double, with $p=3$.

3.2 Supergravity fields near a double pole in $\mathcal{A}_\pm$

To obtain the supergravity fields near a double pole in $\mathcal{A}_\pm$, we need the Laurent expansions of these functions to order w^3,

$$\mathcal{A}_\pm = \frac{\alpha_\pm}{w^2} + \frac{\beta_\pm}{w} + \gamma_\pm + \delta_\pm w + \epsilon_\pm w^2 + \chi_\pm w^3 + \mathcal{O}(w^4) \tag{3.4}$$

along with the conjugation condition implied by (2.12) so that $\bar{\alpha}_\pm = \alpha_\mp$, etc. The first regularity condition is that $\kappa^2 > 0$ in the interior of Σ. The leading behavior of κ^2 is obtained from (3.4),

$$\kappa^2 = \frac{2\zeta_{\alpha\beta}\,\mathrm{Im}\,w}{|w|^6} + \mathcal{O}(|w|^{-3}) \qquad\qquad \zeta_{\alpha\beta} = 2i(\alpha_+\beta_- - \alpha_-\beta_+) \tag{3.5}$$

The conjugation conditions imply that $\zeta_{\alpha\beta}$ is real, and positivity of κ^2 in the upper half-plane requires $\zeta_{\alpha\beta} > 0$. In addition, the function $\mathcal{G}$, and hence $\mathcal{B}+\bar{\mathcal{B}}$, must be single-valued. Computing $\mathcal{B}$ in terms of (3.4), we find,

$$\mathcal{B} = \frac{i\zeta_{\alpha\beta}}{6w^3} + \frac{i\zeta_{\alpha\gamma}}{2w^2} + \frac{i(3\zeta_{\alpha\delta}+\zeta_{\beta\gamma})}{2w} - 2i(2\zeta_{\alpha\epsilon}+\zeta_{\beta\delta})\ln w + \mathcal{O}(|w|) \tag{3.6}$$

with $\zeta_{\alpha\gamma}$ etc. defined in analogy with $\zeta_{\alpha\beta}$. Single-valuedness of $\mathcal{B}+\bar{\mathcal{B}}$ requires the purely imaginary coefficient of $\ln w$ to vanish,

$$2\zeta_{\alpha\epsilon} + \zeta_{\beta\delta} = 0 \tag{3.7}$$

The functions $\mathcal{G}$ and T, in terms of which the metric functions f_2^2, f_6^2, ρ^2 are given by (2.4), take the following form near $w = 0$,

$$\mathcal{G} \approx -\frac{4\zeta_{\alpha\beta}(\mathrm{Im}\,w)^3}{3|w|^6} \qquad T^2 \approx 4\xi|w|^2(\mathrm{Im}\,w)^2 \qquad -\xi = \frac{2\zeta_{\alpha\chi}+\zeta_{\beta\epsilon}}{\zeta_{\alpha\beta}} + \frac{\zeta_{\alpha\delta}^2}{\zeta_{\alpha\beta}^2} \tag{3.8}$$

The condition $\zeta_{\alpha\beta} > 0$, which already guaranteed $\kappa^2 > 0$, is seen to also guarantee that $\mathcal{G} < 0$, as is indeed required by the regularity of the supergravity solution. In addition, we impose the requirement $\xi > 0$ to render T positive.

With these conditions fulfilled, the behavior of the functions f_2^2, f_6^2, ρ^2, and $\mathcal{C}$ near the pole is given as follows in terms of polar coordinates $w = re^{i\theta}$ near $w = 0$,

$$\rho^2 \approx \frac{\zeta_{\alpha\beta}^{1/2}\xi^{1/4}}{r^{5/2}} \qquad f_6^2 \approx 4r^2\sin^2\theta\,\rho^2 \qquad f_2^2 \approx \frac{\zeta_{\alpha\beta}^{1/2}}{9\xi^{3/4}r^{9/2}} \qquad \mathcal{C} \approx \frac{-2i\alpha_+}{9\xi r^6} \tag{3.9}$$

The complex axion-dilaton field τ, for $\alpha_+ \neq \alpha_- = \bar{\alpha}_+$, is given by,

$$\mathrm{Re}\,(\tau) \approx \frac{\mathrm{Re}\,(\alpha_+)}{\mathrm{Im}\,(\alpha_+)} \qquad\qquad \mathrm{Im}(\tau) \approx \frac{\xi^{1/2}\zeta_{\alpha\beta}}{4\,\mathrm{Im}\,(\alpha_+)^2}r^3 \tag{3.10}$$

Finally, the presence of string charge at the pole may be verified by examining the potential $\mathcal{M}$ for the seven-form field strength $F_{(7)}$. By inspection of (2.5) we see that all terms in $\mathcal{M}$

are single-valued by construction, except for the contributions from the locally holomorphic functions $\mathcal{W}_\pm$, whose behavior near $w = 0$ is given as follows,

$$\mathcal{W}_\pm(w) = \mathcal{W}^s_\pm(w) - \frac{3}{2}i\Big((3\zeta_{\alpha\chi} + \zeta_{\beta\epsilon})\beta_\pm + \zeta_{\alpha\delta}\delta_\pm - \zeta_{\alpha\beta}\chi_\pm\Big)\ln w \tag{3.11}$$

where $\mathcal{W}^s_\pm$ denotes the single-valued part. Therefore, as w encircles the pole at $w = 0$ counterclockwise in Σ by a 180° degree arc, the potential $\mathcal{M}$ shifts as follows,

$$\mathcal{M} \to \mathcal{M} + 240\pi\Big((3\zeta_{\alpha\chi} + \zeta_{\beta\epsilon})\beta_+ + \zeta_{\alpha\delta}\delta_+ - \zeta_{\alpha\beta}\chi_+\Big) \tag{3.12}$$

The shift in $\mathcal{M}$ gives the integral of the seven-form field strength over the S^7, producing a formula for the p, q string charges in terms of the coefficients of the Laurent series,

$$p + iq = \int_{S^7} F_{(7)} = 80\pi^5\Big((3\zeta_{\alpha\chi} + \zeta_{\beta\epsilon})\beta_+ + \zeta_{\alpha\delta}\delta_+ - \zeta_{\alpha\beta}\chi_+\Big) \tag{3.13}$$

where we have used $\mathrm{vol}(S^7) = \pi^4/3$. Note that the dependence of the string charges p, q on the coefficients of the Laurent series is trilinear, in contrast with the $\mathrm{AdS}_6 \times S^2$ case where the five-brane charges had linear dependence.

3.3 Satisfying the regularity conditions near a double pole

The various positivity and regularity conditions derived in the preceding subsection may be satisfied simultaneously. To see this, we use the $\mathrm{SU}(1,1)$ symmetry of supergravity to rotate $\alpha_\pm$ to be real, and furthermore scale it to 1 without loss of generality. The conditions then reduce to the following relations,

$$\zeta_{\alpha\beta} = 4\,\mathrm{Im}\,(\beta_+) > 0\,, \quad \mathrm{Im}\,(\epsilon_+) = -\frac{1}{8}\zeta_{\beta\delta}\,, \quad \xi = -\frac{(\mathrm{Im}\,\delta_+)^2}{(\mathrm{Im}\,\beta_+)^2} - \frac{8\,\mathrm{Im}\,\chi_+ + \zeta_{\beta\epsilon}}{4\,\mathrm{Im}\,\beta_+} > 0 \tag{3.14}$$

For given $\mathrm{Re}\,(\beta_+)$, $\mathrm{Im}\,(\beta_+) > 0$, δ_+, and ε_+, we may always choose $\mathrm{Im}\,(-\chi_+)$ large enough to satisfy the remaining condition $\xi > 0$. The expression for the charge p is unenlightening, but the charge q takes the simple form $q = -320\pi^5\,\xi\,(\mathrm{Im}\,\beta)^2$ and must be negative.

We conclude this subsection with a remark on a no-go result derived in [10]. Assuming certain regularity conditions on κ^2 and $\mathcal{G}$, it was argued that $\kappa^2 > 0$ and $\mathcal{G} < 0$ cannot both be realized for compact Σ with boundary. This argument was based on an integral representation for $\mathcal{G}$, obtained by solving the differential relation $\kappa^2 = -\partial_w\partial_{\bar{w}}\mathcal{G}$,

$$\mathcal{G}(w) = H(w) + \frac{1}{\pi}\int_\Sigma d^2z\; G(w,z)\kappa^2(z) \tag{3.15}$$

with harmonic H and G the Green's function on Σ. The functions κ^2 and $\mathcal{G}$ obtained here circumvent this no-go result, because they are too singular at the pole to allow for the integral representation (3.15). Indeed, the singularity in κ^2 is not integrable against the Green function, as may be seen from the form of κ^2 and $\mathcal{G}$ given in (3.5) and (3.8). This shows that the assumptions entering the argument of section 7.3.2 of [10] do not hold here.

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3.4 Matching with the classic (p, q)-string solutions

The classic (p, q)-string solutions of Type IIB supergravity constructed in [34] are labeled by a pair of integers (q_1, q_2) which characterize the charges. The metric, two-form, and axion-dilaton $\tau = \chi + ie^{-\phi}$ are given by,

$$
\begin{aligned}
ds^2 &= A_q^{-3/4} ds^2_{\mathbb{R}^{1,1}} + A_q^{1/4}\left(dy^2 + y^2 ds^2_{S^7}\right) & \tau &= \frac{q_1\chi_0 - q_2|\tau_0|^2 + iq_1 e^{-\phi_0}A_q^{1/2}}{q_1 - q_2\chi_0 + iq_2 e^{-\phi_0}A_q^{1/2}} \\
B^{(i)}_{01} &= \left(\mathcal{M}_0^{-1}\right)_{ij} q_j \Delta_q^{-1/2} A_q^{-1} & A_q &= 1 + \frac{\alpha_q}{y^6}
\end{aligned}
\tag{3.16}
$$

The asymptotic values of the axion-dilaton are given by $\tau_0 = \chi_0 + ie^{-\phi_0}$, and we have,

$$
\alpha_q = \Delta_q^{1/2} Q \qquad \Delta_q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}^t \mathcal{M}_0^{-1} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \qquad \mathcal{M} = e^{\phi}\begin{pmatrix} |\tau|^2 & \chi \\ \chi & 1 \end{pmatrix}
\tag{3.17}
$$

As $y \to \infty$ we recover flat spacetime $\mathbb{R}^{1,9}$. The near-horizon limit corresponds to $y^6 \ll \alpha_q$, so that the first term in A_q may be neglected in this limit and we have simply $A_q(y) \to \alpha_q / y^6$. The supergravity fields take the following form,

$$
\begin{aligned}
ds^2 &= \frac{y^{\frac{9}{2}}}{\alpha_q^{3/4}} ds^2_{\mathbb{R}^{1,1}} + \frac{\alpha_q^{1/4}}{y^{\frac{3}{2}}}(dy^2 + y^2 ds^2_{S^7}) \qquad \tau = \frac{q_1\chi_0 - q_2|\tau_0|^2 + iq_1 e^{-\phi_0}\sqrt{\alpha_q}/y^3}{q_1 - q_2\chi_0 + iq_2 e^{-\phi_0}\sqrt{\alpha_q}/y^3} \\
B^{(i)}_{01} &= \left(\mathcal{M}_0^{-1}\right)_{ij} q_j \Delta_q^{-1/2} \frac{y^6}{\alpha_q}
\end{aligned}
\tag{3.18}
$$

In this limit, $y \to 0$ corresponds to the location of the string, but this is a strong coupling limit since the dilaton blows up there. The limit $y \to \infty$ corresponds to the other end of the throat which is also a strong coupling limit. Clearly, identifying the coordinate r of (3.9) with y does not lead to a match between the supergravity fields of the $\mathrm{AdS}_2 \times S^6$ solutions and the supergravity fields of the classic (p, q)-string solution to Type IIB. However, if we perform a coordinate inversion on y in the string solution by setting,

$$
y = L/r \tag{3.19}
$$

then the supergravity fields of the string solution in terms of r are given by,

$$
\begin{aligned}
ds^2 &= \frac{L^{\frac{9}{2}} ds^2_{\mathbb{R}^{1,1}}}{\alpha_q^{3/4} r^{9/2}} + \frac{L^{\frac{1}{2}}\alpha_q^{1/4}}{r^{5/2}}(dr^2 + r^2 ds^2_{S^7}) \qquad \tau = \frac{q_1\chi_0 - q_2|\tau_0|^2 + iq_1 e^{-\phi_0}\sqrt{\alpha_q} r^3/L^3}{q_1 - q_2\chi_0 + iq_2 e^{-\phi_0}\sqrt{\alpha_q} r^3/L^3} \\
B^{(i)}_{01} &= \left(\mathcal{M}_0^{-1}\right)_{ij} q_j \Delta_q^{-1/2} \frac{L^6}{\alpha_q r^6}
\end{aligned}
\tag{3.20}
$$

which perfectly match with the $\mathrm{AdS}_2 \times S^6$ solution provided we identify the parameters,

$$
L^3 = \frac{\zeta_{\alpha\beta}}{3} \qquad\qquad \alpha_q = \xi\,(3\zeta^2_{\alpha\beta})^{\frac{2}{3}} \tag{3.21}
$$

and a corresponding identification for the flux field. Note that the worldvolume for the $\mathrm{AdS}_2 \times S^6$ solution is AdS_2, whereas for the classic string solution it is $\mathbb{R}^{1,1}$. The inversion in the identification (3.19) may play a role in the physical interpretation of potential global solutions.

3.5 Multiple (p, q) charge solutions on the upper half-plane

In the previous subsection, we have shown that a double pole in the functions $\mathcal{A}_\pm$ on the boundary $\partial\Sigma$ produces supergravity fields which may be identified locally, i.e. in a finite neighborhood of the pole, with the supergravity fields of the classic (p, q)-string solution. Here we shall extend this construction to the case of multiple double poles which are all located on the boundary $\partial\Sigma$. For simplicity, we shall consider the case where Σ has the topology of the upper half-plane, for which the boundary is the real line. Hence we shall consider functions $\mathcal{A}_\pm$ with N double poles, located at points $p_\ell \in \mathbb{R}$ for $\ell = 1, \cdots, N$.

To make further progress, we shall assume that $\mathcal{A}_\pm$ are rational functions of w and that $w = \infty$ is a regular point (which may always be achieved by conformal mapping). The functions may therefore be decomposed into partial fractions in w as follows,

$$\mathcal{A}_\pm = \mathcal{A}_\pm^{(0)} + \sum_{\ell=1}^{N} \left(\frac{Y_\pm^\ell}{(w - p_\ell)^2} + \frac{Z_\pm^\ell}{w - p_\ell} \right) \tag{3.22}$$

where $Y_-^\ell = \bar{Y}_+^\ell$, $Z_-^\ell = \bar{Z}_+^\ell$, and $\mathcal{A}_\pm^{(0)}$ are complex parameters which are independent of w. This Ansatz implements the reflection condition (2.12), as a result of which κ^2 and $\mathcal{G}$ vanish on $\partial\Sigma = \mathbb{R}$. It remains to enforce the positivity requirement $\kappa^2 > 0$ everywhere in the interior of the upper half-plane, which in particular requires that $\partial\mathcal{A}_-$ has no zeros in the upper half-plane. We also need the condition that the function $\mathcal{B} + \bar{\mathcal{B}}$ be single-valued.

An alternative formulation starts from the differentials $\partial\mathcal{A}_\pm$, which have a triple pole at each $w = p_\ell$. We may easily enforce the conditions that the zeros of $\partial\mathcal{A}_+$ and $\partial\mathcal{A}_-$ all be located in the upper and lower half-planes, respectively, by the following parametrization (analogous to the parametrization used for the AdS_6 case in [8]),

$$\partial_w \mathcal{A}_\pm = P_\pm(w) \prod_{\ell=1}^{N} \frac{1}{(w - p_\ell)^3} \qquad P_+(w) = \prod_{n=1}^{3N-2} (w - s_n) \qquad P_-(w) = \prod_{n=1}^{3N-2} (w - \bar{s}_n) \tag{3.23}$$

with $\mathrm{Im}\,(s_n) > 0$. In order to integrate to single-valued functions $\mathcal{A}_\pm$ and $\mathcal{B} + \bar{\mathcal{B}}$, the differentials $\partial\mathcal{A}_\pm$ must have vanishing residues at p_ℓ, while the imaginary part of the residue of the differential $\partial_w\mathcal{B}$ must also vanish,

$$\mathrm{Res}(\partial_w \mathcal{A}_\pm)\Big|_{w=p_\ell} = 0 \qquad\qquad \mathrm{Res}(\partial_w \mathcal{B})\Big|_{w=p_\ell} \in \mathbb{R} \tag{3.24}$$

The counting of parameters shows that, for a given arrangement of poles p_ℓ, there are $3N - 2$ complex zeros, subject to $3N - 3$ real residue conditions. Thus parameter counting allows for the existence of large families of solutions. While it is clear that the positivity and regularity conditions are satisfied in the neighborhood of each pole, and that the supergravity fields match onto a classical (p, q)-string solution in the near-horizon limit, it is unclear how to ensure regularity throughout the upper half-plane. The solutions found numerically thus far have all been geodesically incomplete, and this includes the cases with one and two charges. The situation will be discussed explicitly for the case of three charges in the next subsection.

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3.6 Three charges

In this final subsection, we analyze the case of three double poles in $\mathcal{A}_\pm$, and thus three (p,q)-string charges. In order to conveniently exploit as much symmetry of the configuration as possible, we shall work on the unit disc with complex coordinates $z, \bar z$ rather than on the upper half-plane with complex coordinates $w, \bar w$. The conjugation condition (2.12) on the disc becomes $\overline{\mathcal{A}_\pm(1/\bar z)} = \mathcal{A}_\mp(z)$, and we may exploit SU(1,1) symmetry of the unit disc to map the positions of the poles to $1, \varepsilon, \varepsilon^2$ where ε is a non-trivial cube root of unity. The differentials and polynomials of (3.23) then take the form,

$$\partial_z \mathcal{A}_\pm(z) = \frac{P_\pm(z)}{(z^3-1)^3} \qquad P_+(z) = \sum_{k=0}^{7} c_k z^k \qquad P_-(z) = \sum_{k=0}^{7} \bar c_{7-k} z^k \tag{3.25}$$

The vanishing of the residues of $\partial_z \mathcal{A}_\pm$ at the poles gives two complex linearly independent relations between the coefficients c_k,

$$\begin{aligned} c_3 &= 5c_0 + 2c_6 \\ c_4 &= 5c_7 + 2c_1 \end{aligned} \tag{3.26}$$

while the vanishing of the imaginary part of the residues of $\partial_z \mathcal{B}$ gives two independent real relations, which may be combined into one complex relation between the coefficients c_k,

$$\begin{aligned} 0 = &\, 27|c_7|^2 + 9|c_6|^2 - 2|c_5|^2 + 2|c_2|^2 - 9|c_1|^2 - 27|c_0|^2 \\ &- 18\bar c_7 c_6 + 21\bar c_2 c_7 - 9\bar c_1 c_7 - 36\bar c_7 c_1 - 3\bar c_6 c_2 \\ &+ 9\bar c_0 c_6 + 36\bar c_6 c_0 + 3\bar c_5 c_1 - 21\bar c_0 c_5 + 18\bar c_1 c_0 \end{aligned} \tag{3.27}$$

where we have eliminated c_3, c_4 using (3.26). Global regularity and geodesic completeness requires furthermore that we have $\kappa^2 > 0, \mathcal{G} < 0$, and T real. The condition $\kappa^2 > 0$ in the interior of the disc requires that all the zeros of $P_+(z)$ be in the interior of the disc, which implies that all the zeros of $P_-(z)$ will be outside the disc. We have not been able to solve this condition in any general form, nor numerically for any particular choice of parameters c_k. However, we have also not been able to show convincingly that no solutions exist. The cases with 4 poles have also been explored, but the complexity of the conditions required is then even more involved. In the absence of these results, we are left only with solutions with (p,q)-string charges which are not geodesically complete.

4 Non-compact globally regular solutions

In this section, we shall study solutions independently from a string junction interpretation and construct families of globally regular and geodesically complete solutions with asymptotic regions where spacetime decompactifies.

4.1 Poles in the interior of Σ

In the case of AdS$_6$ solutions, poles in the interior of Σ were not compatible with the regularity conditions [8]. In contrast, due to κ^2 and $\mathcal{G}$ having opposite signs, poles in the interior of Σ can be realized for AdS$_2$ solutions. Positivity of κ^2 requires that at any singular point the divergence in $\partial_w \mathcal{A}_-$ be at least as strong as the one in $\partial_w \mathcal{A}_+$. We start with the case where $\partial_w \mathcal{A}_+$ is subleading with respect to $\partial_w \mathcal{A}_-$ and we shall generalize afterwards.

With $\mathcal{A}_+$ subleading to $\mathcal{A}_-$, the leading behavior of the composite quantities is given by the behavior of $\mathcal{A}_-$ as follows,

$$\kappa^2 \approx |\partial_w \mathcal{A}_-|^2 \qquad \mathcal{G} \approx -|\mathcal{A}_-|^2 \qquad \partial_w \mathcal{G} \approx -\bar{\mathcal{A}}_- \partial_w \mathcal{A}_- \tag{4.1}$$

As the pole is approached this yields $T^2 \approx \frac{1}{3}$. The metric functions, axion-dilaton scalar B, and the functions $\mathcal{C}$ and $\mathcal{M}$ parametrizing the two- and six-form potentials become,

$$\begin{aligned} f_2^2 &\approx \sqrt[4]{\frac{4}{27}}\, |\mathcal{A}_-| & f_6^2 &\approx \sqrt[4]{12}\, |\mathcal{A}_-| & \rho^2 &\approx \frac{1}{\sqrt[4]{4 \cdot 27}} \frac{|\partial_w \mathcal{A}_-|^2}{|\mathcal{A}_-|} \\ B &\approx -\left(2-\sqrt{3}\right) \frac{\bar{\mathcal{A}}_-}{\mathcal{A}_-} & \mathcal{C} &\approx \frac{4i}{3} \bar{\mathcal{A}}_- & \mathcal{M} &\approx 8|\mathcal{A}_-|^2 \bar{\mathcal{A}}_- \end{aligned} \tag{4.2}$$

In particular, we have $|B| < 1$, such that $\mathrm{Im}\,(\tau)$ is non-zero and finite. Assuming that $\mathcal{A}_-$ has a pole of order n with a complex coordinate z centered on the pole, we have,

$$\mathcal{A}_- \approx \frac{a}{z^n} \qquad \partial_w \mathcal{A}_- \approx -\frac{na}{z^{n+1}} \tag{4.3}$$

Since $\mathrm{Im}\,(\tau)$ is finite and non-zero, the metric in string-frame is related to the Einstein-frame metric by a finite rescaling. The Einstein-frame metric near the pole becomes,

$$ds^2 \approx \frac{\sqrt{2}|a|}{3^{3/4}|z|^n} \left(ds^2_{\mathrm{AdS}_2} + 3ds^2_{S^6} + 2n^2 \left| \frac{dz}{z} \right|^2 \right) \tag{4.4}$$

The proper distance between the point $z = 0$ and any other point on Σ is infinite. This suggests a change of coordinates on Σ to $u = 1/z$,[1] such that the near-pole region corresponds to $|u| \gg 1$. The metric becomes,

$$ds^2 \approx \frac{\sqrt{2}|a|}{3^{3/4}} \left(|u|^n ds^2_{\mathrm{AdS}_2} + 3|u|^n ds^2_{S^6} + 2n^2 \left| u^{n/2-1} du \right|^2 \right) \tag{4.5}$$

At the pole, the radii of AdS$_2$ and S^6 diverge. The geometry decompactifies; it is regular and asymptotically conical. For $n = 1$ it approaches the asymptotic region of a cone with deficit angle π, for $n = 2$ there is no deficit angle, and for $n > 2$ it has an excess angle. The singularity at the apex of the cone $u = 0$ is not a problem since this form of the metric is only valid in the regime $|u| \gg 1$.

The two- and six-form RR potentials are given by,

$$C_{(2)} \approx \frac{4i}{3} \bar{a} \left(\frac{\bar{u}}{u} \right)^{n/2} |u|^n \,\mathrm{vol}_{\mathrm{AdS}_2} \qquad C_{(6)} \approx 8|a|^2 \bar{a} \left(\frac{\bar{u}}{u} \right)^{n/2} |u|^{3n} \,\mathrm{vol}_{S^6} \tag{4.6}$$

The $|u|^n$ and $|u|^{3n}$ divergences in $\mathcal{C}$ and $\mathcal{M}$, respectively, combine with the volume forms on unit-radius AdS$_2$ and S^6 to the volume forms that are naturally associated with the AdS$_2$

[1]This transformation will be part of the SL$(2, \mathbb{C})$ and SL$(2, \mathbb{R})$ automorphisms of the sphere and the upper half plane, respectively, in the examples to be discussed below.

and S^6 factors of radius proportional to $|u|^{n/2}$ in the metric (4.5). The remaining coefficient functions in $C_{(2)}$ and $C_{(6)}$ are finite and regular. With $u = re^{i\theta}$, the axion-dilaton scalar asymptotes to,

$$\tau \approx \frac{\sin(2n\theta) + i\sqrt{3}}{2 - \cos(2n\theta)} \tag{4.7}$$

showing again that the dilaton is finite at the pole, with non-trivial dependence on the angular coordinate. The entire solution is thus regular at the pole.

The asymptotic metric has an additional U(1) isometry, acting as phase transformations on z and u. This symmetry is broken by the remaining fields τ, $C_{(2)}$, and $C_{(6)}$ to a $\mathbb{Z}_n$ symmetry acting as $z \to e^{2\pi i/n} z$, or $u \to e^{-2\pi i/n} u$, which leaves $\mathcal{A}_-$ in (4.3) and the entire solution invariant. Whether this symmetry in the asymptotic region extends to a symmetry of the full solution depends on the precise form of $\mathcal{A}_\pm$.

Finally, we note that this regularity analysis generalizes to the case where $\mathcal{A}_+$ and $\mathcal{A}_-$ both have poles, of the form

$$\mathcal{A}_+ \approx \frac{b}{z^n} \qquad\qquad \mathcal{A}_- \approx \frac{a}{z^n} \tag{4.8}$$

with $|a| > |b|$. An SU(1,1) transformation (2.8) with $v = bu/a$ and arbitrary u such that $|u|^2 = |a|^2/(|a|^2 - |b|^2)$ then sets $b = 0$, and the analysis reduces to the one presented above. The Einstein-frame metric is invariant under SU(1,1), so the previous discussion applies directly. Axion-dilaton scalar and RR-potentials transform but remain regular, and the action of the $\mathbb{Z}_n$ symmetry is unchanged.

4.2 Solutions for Σ without boundary

Having found local regularity in the vicinity of an interior pole, we now try to embed this in a globally regular solution. We consider a compact Riemann surface Σ of genus g without boundary, and investigate the positivity and regularity conditions on $\mathcal{A}_\pm$ for such a surface. Since there is no boundary, there is no need for a conjugation relation between $\mathcal{A}_\pm$. The number of zeros M and the number of poles N of the meromorphic 1-form differentials $\partial_w \mathcal{A}_\pm$ on Σ are related to the genus g by the Riemann-Roch theorem,

$$M = N + 2g - 2 \tag{4.9}$$

For surfaces with $g > 1$ the differentials $\partial_w \mathcal{A}_\pm$ necessarily have zeros in Σ. To keep κ^2 non-negative, any zero of $\partial_w \mathcal{A}_-$ must also be a zero of $\partial_w \mathcal{A}_+$. However, this means that κ^2 vanishes at these zeros, generally leading to solutions with conical singularities. These zeros can be avoided for genus zero surfaces without boundary, as we now discuss, as well as for solutions with boundary, to be discussed in the next subsection.

We take Σ to be the sphere and assume that infinity is a regular point of $\mathcal{A}_\pm$. Single-valued differentials $\partial_w \mathcal{A}_\pm$ without zeros can only have one double pole. Implementing the condition $\kappa^2 > 0$ on $\mathcal{A}_\pm$ and its differentials, we find,

$$\mathcal{A}_\pm = a_\pm - \frac{b_\pm}{w - p} \qquad\qquad \partial_w \mathcal{A}_\pm = \frac{b_\pm}{(w-p)^2} \qquad\qquad |b_+| < |b_-| \tag{4.10}$$

An $\mathrm{SU}(1,1)\otimes\mathbb{C}$ transformation (2.8) allows us to set $b_+ = a_+ = 0$, so that we have,

$$\kappa^2 = |\partial_w \mathcal{A}_-|^2 \qquad \mathcal{G} = \mathcal{G}_0 - |\mathcal{A}_-|^2 \qquad T^2 = 1 - \frac{2\,|\mathcal{A}_-|^2}{3\,(|\mathcal{A}_-|^2 - \mathcal{G}_0)} \tag{4.11}$$

with integration constant $\mathcal{G}_0$. Manifestly, κ^2 is positive and $\mathcal{G}$ is negative throughout Σ provided $\mathcal{G}_0 < 0$. With this choice we have $1/3 < T^2 < 1$, such that the regularity condition for R is satisfied as well. Since κ^2, $\mathcal{G}$, and T^2 are invariant under the $\mathrm{SU}(1,1)\otimes\mathbb{C}$ transformations (2.8), this implies that the entire class of solutions (4.10) is regular for appropriate choices of $\mathcal{G}_0$. With the analysis of section 4.1 for the pole, we have thus realized globally regular $\mathrm{AdS}_2 \times S^6$ solutions. The geometry decompactifies at the pole and approaches the asymptotic region of a cone with deficit angle π. The axion-dilaton scalar B and the functions $\mathcal{C}$ and $\mathcal{M}$ are single-valued for all solutions (4.10), and therefore carry no charges.

The Ansatz (4.10) has one complex parameter in each of $a_\pm$, $b_\pm$, p, and an additional real parameter in $\mathcal{G}_0$, making for 11 real parameters. Subtracting 3 complex parameters for the $\mathrm{SL}(2,\mathbb{C})$ automorphisms on the sphere leaves a total of 5 real parameters. The $\mathrm{SU}(1,1)\otimes\mathbb{C}$ duality transformations (2.8) map the class of functions (4.10) into itself.

4.3 Solutions for Σ with boundary

The presence of a boundary for Σ gives greater flexibility for the distribution of zeros and poles of $\mathcal{A}_\pm$ compatible with the positivity and regularity conditions. We take Σ to be the upper half-plane with the real line as its boundary. Assuming that the differentials $\partial_w \mathcal{A}_\pm(w)$ are rational functions of w, the condition $\kappa^2 > 0$ may be solved by an electrostatics problem [8, 9] and the solution is given by,

$$\frac{\partial_w \mathcal{A}_+}{\partial_w \mathcal{A}_-} = \lambda_0^2 \prod_n \frac{w - s_n}{w - \bar{s}_n} \tag{4.12}$$

where s_n are points in the upper half-plane with $\mathrm{Im}\,(s_n) > 0$ and λ_0 is a constant. In the case of AdS_6 solutions, no poles in the interior of Σ were allowed, which forced us to assign all points s_n to be zeros of $\partial_w \mathcal{A}_+$ and by conjugation all points $\bar{s}_n$ to be zeros of $\partial_w \mathcal{A}_-$. However, common poles on the real line were allowed. In contrast, in the case of AdS_2 solutions we can allow for poles in the interior of Σ. To have single-valued $\mathcal{A}_\pm$ functions, the poles must be of order at least two. Thus, we may distribute the points s_n amongst the zeros of $\partial_w \mathcal{A}_+$ and the poles of $\partial_w \mathcal{A}_-$, and allow for additional zeros and poles on the real axis common to $\mathcal{A}_\pm$. The general form of the differentials is then as follows,

$$\begin{aligned}
\partial_w \mathcal{A}_+ &= \prod_{k=1}^{N} \frac{1}{(w - \bar{t}_k)^{\nu_k}} \times \prod_{m=1}^{N_u} (w - u_m) & \qquad N_u &= \sum_{k=1}^{N} \nu_k - 2 \\
\partial_w \mathcal{A}_- &= \prod_{k=1}^{N} \frac{1}{(w - t_k)^{\nu_k}} \times \prod_{m=1}^{N_u} (w - \bar{u}_m) & \qquad \mathrm{Im}\,(t_k), \mathrm{Im}\,(u_m) &\geq 0
\end{aligned} \tag{4.13}$$

The regularity conditions required to have a single-valued $\mathcal{A}_\pm$ and $\mathcal{G}$ are given as follows,

$$\mathrm{Res}(\partial_w \mathcal{A}_-)\Big|_{w=t_k} = 0 \qquad\qquad \mathrm{Res}(\partial_w \mathcal{B})\Big|_{w=t_k} \in \mathbb{R} \tag{4.14}$$

for $k = 1, \ldots, N$. In addition one has to ensure that $\mathcal{G} < 0$ throughout the interior of Σ with $\mathcal{G} = 0$ on $\partial\Sigma$ and that $0 < T^2 < 1$.[2] This construction introduces a large number of parameters and we show in the following subsections that regular solutions exist.

If the conditions (4.14) are satisfied, the axion-dilaton scalar and two-form potential do not have branch points at $w = t_k$ since $\mathcal{A}_\pm$ are single-valued. In the six-form potential the terms involving $\mathcal{W}_\pm$ in $\mathcal{M}$ are potentially non-single-valued. However, due to the conjugation condition relating $\mathcal{A}_\pm$, the residues of $\partial_w \mathcal{W}_+$ at $w = t_k$ and of $\partial_{\bar w} \bar{\mathcal{W}}_-$ at $\bar w = \bar t_k$ are related. As a result, even though individual terms in $\mathcal{M}$ may have logarithmic branch cuts, the total monodromy of $\mathcal{M}$ vanishes. There are thus no apparent brane charges at the poles t_k.

4.4 Minimal solutions on the disc

The minimal non-trivial case is $N = 1$ with $\nu_1 = 2$,

$$\partial_w \mathcal{A}_+ = \frac{\bar\sigma}{(w - \bar t_1)^2} \qquad\qquad \partial_w \mathcal{A}_- = \frac{\sigma}{(w - t_1)^2} \tag{4.15}$$

The functions $\mathcal{A}_\pm$ are given by

$$\mathcal{A}_+ = \mathcal{A}_+^0 - \frac{\bar\sigma}{w - \bar t_1} \qquad\qquad \mathcal{A}_- = \mathcal{A}_-^0 - \frac{\sigma}{w - t_1} \tag{4.16}$$

It will be convenient to map this solution to the disc, $\Sigma = \{z \in \mathbb{C} \,\big|\, |z|^2 \leq 1\}$. With redefined constants $\varsigma = -\sigma/(t_1 - \bar t_1)$ and $\tilde{\mathcal{A}}_+^0 = \mathcal{A}_+^0 - \bar\varsigma$, $\tilde{\mathcal{A}}_-^0 = \mathcal{A}_-^0 - \varsigma$, the functions $\mathcal{A}_\pm$ are then,

$$\mathcal{A}_+ = \tilde{\mathcal{A}}_+^0 + \bar\varsigma z \qquad\qquad \mathcal{A}_- = \tilde{\mathcal{A}}_-^0 + \frac{\varsigma}{z} \qquad\qquad z = \frac{w - t_1}{w - \bar t_1} \tag{4.17}$$

They satisfy $\overline{\mathcal{A}_\mp(1/\bar z)} = \mathcal{A}_\pm$, which is the analog of the conjugation condition (2.12) on the disc. For κ^2 and $\mathcal{G}$ we find, with a suitable choice of the integration constant in $\mathcal{B}$ to ensure $\mathcal{G}|_{\partial\Sigma} = 0$ and using that $\tilde{\mathcal{A}}_+^0$ and $\tilde{\mathcal{A}}_-^0$ are related by conjugation,

$$\kappa^2 = |\varsigma|^2 \left(\frac{1}{|z|^4} - 1 \right) \qquad\qquad \mathcal{G} = |\varsigma|^2 \left(|z|^2 - \frac{1}{|z|^2} - 2 \ln |z|^2 \right) \tag{4.18}$$

Manifestly, the positivity and regularity conditions $\kappa^2 > 0$ and $\mathcal{G} < 0$ are obeyed in the interior of Σ, and both functions vanish on the boundary. Moreover, we have,

$$\frac{\kappa^2 \mathcal{G}}{|\partial_z \mathcal{G}|^2} = \frac{1 + |z|^2}{(1 - |z|^2)^3} \left(|z|^4 - 1 - 2|z|^2 \ln |z|^2 \right) \tag{4.19}$$

This is a smooth function which monotonically increases from -1 at $|z| = 0$ to $-2/3$ at $|z| = 1$, as required by the boundary conditions. Therefore, these functions $\mathcal{A}_\pm$ yield a solution which is regular everywhere. At the pole in $\mathcal{A}_-$, we have that $\mathcal{A}_+$ is finite. By

[2] For the AdS_6 case, regularity of κ^2 together with the boundary condition for $\mathcal{G}$ automatically implied the full set of regularity conditions. This is not the case for AdS_2.

the arguments of section 4.1, the geometry decompactifies at this point and approaches the asymptotic region of a cone with deficit angle π.

Taking into account the conjugation relation between $\mathcal{A}_+$ and $\mathcal{A}_-$, the ansatz in (4.16) has 6 real parameters. The integration constant in $\mathcal{G}$ is fixed by the boundary condition. Subtracting 3 degrees of freedom for redundancies due to the $\mathrm{SL}(2,\mathbb{R})$ automorphisms of the upper half-plane leaves 3 free real parameters. We note that the $\mathrm{SU}(1,1)\times\mathbb{C}$ duality transformations (2.8) do not map the class of functions in (4.16) into itself; the shifts do, but $\mathrm{SU}(1,1)$ transformations with $v\neq 0$ do not. The functions in (4.16) are therefore representatitves of entire $\mathrm{SU}(1,1)$ orbits of non-trivial solutions.

In figure 1 we plot the metric factors, axion-dilaton, and two-form fields for a minimal solution on the disc. In order to obtain these plots, we have chosen $\varsigma = \tilde{\mathcal{A}}_+^0 = \frac{i}{2}$ in (4.17). Because the metric factors all take the same qualitative form, we have only included the plot for f_2^2.

4.5 Non-minimal solutions on the disc

Generalizing to a single pole at $t = t_1$ of higher order $\nu = \nu_1 > 2$, we have,

$$\partial_w \mathcal{A}_+ = \frac{\bar\sigma}{(w-\bar t)^\nu}\prod_{k=1}^{\nu-2}(w-u_k) \qquad\qquad \partial_w \mathcal{A}_- = \frac{\sigma}{(w-t)^\nu}\prod_{k=1}^{\nu-2}(w-\bar u_k) \tag{4.20}$$

with $\mathrm{Im}\,(u_k)>0$. Since $\partial_w\mathcal{A}_-$ has no zeros in the upper half-plane, κ^2 is positive throughout Σ. The pole at t can be mapped to $w=i$ by $\mathrm{SL}(2,\mathbb{R})$. By partial fraction decomposition the differentials can then be rewritten as,

$$\partial_w\mathcal{A}_\pm = \sum_{k=2}^{\nu}\frac{Z_\pm^k}{(w\pm i)^k} \qquad\qquad Z_+^k = \bar\sigma h_{\nu-2,k} \qquad\qquad Z_-^k = \overline{Z_+^k} \tag{4.21}$$

where $h_{\nu-2,k}(u_i)$ are k-th order symmetric polynomials in the $\nu-2$ zeroes u_i given by

$$h_{\nu-2,k}(u_i) = (-1)^k \sum_{\ell_1<\cdots<\ell_{k-2}}^{\nu-2}(u_{\ell_1}+i)\dots(u_{\ell_{k-2}}+i) \tag{4.22}$$

The residues of $\partial_w\mathcal{A}_\pm$ at $w=\mp i$ vanish, such that $\mathcal{A}_\pm$ are single-valued and given by,

$$\mathcal{A}_\pm = \mathcal{A}_\pm^0 - \sum_{k=2}^{\nu}\frac{1}{k-1}\frac{Z_\pm^k}{(w\pm i)^{k-1}} \tag{4.23}$$

To ensure that $\mathcal{G}$ is single-valued we compute $\partial_w\mathcal{B}$,

$$\partial_w\mathcal{B} = \mathcal{A}_+^0\partial_w\mathcal{A}_- - \mathcal{A}_-^0\partial_w\mathcal{A}_+ - \sum_{k,\ell=2}^{\nu}\frac{1}{k-1}\left[\frac{Z_+^kZ_-^\ell}{(w+i)^{k-1}(w-i)^\ell} - \frac{Z_-^kZ_+^\ell}{(w-i)^{k-1}(w+i)^\ell}\right] \tag{4.24}$$

The residue at $w=i$ is found to be

$$\mathrm{Res}(\partial_w\mathcal{B})\Big|_{w=i} = -\sum_{k,\ell=2}^{\nu}\frac{2^{2-k-\ell}}{k-1}\binom{k+\ell-3}{k-2}\left[i^{\ell-k}Z_+^kZ_-^\ell + (-i)^{\ell-k}Z_-^kZ_+^\ell\right] \tag{4.25}$$

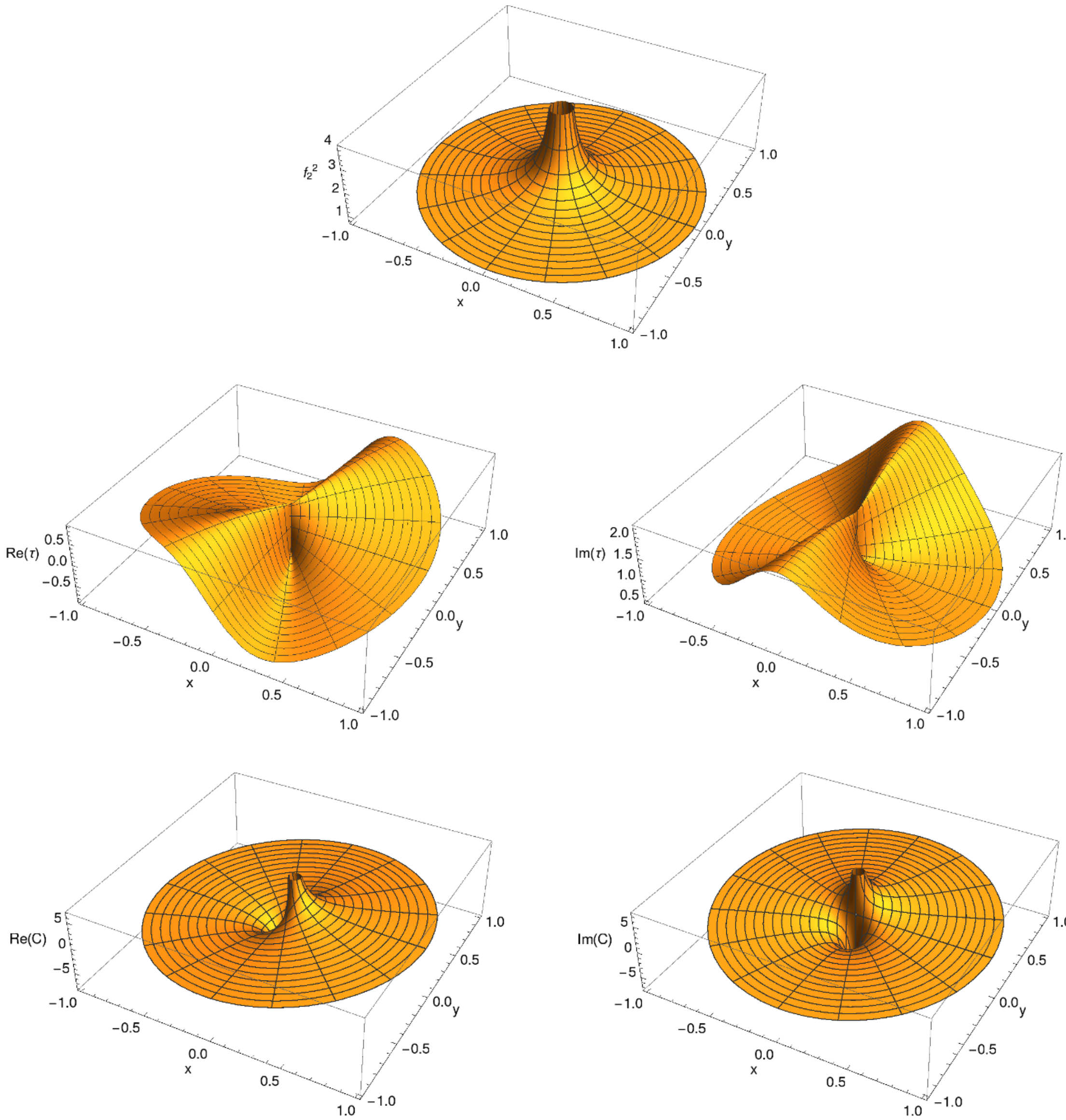

Figure 1. The metric factors, axion-dilaton, and two-form fields for the minimal solution with a single interior pole of order two. We have chosen $\varsigma = \tilde{\mathcal{A}}_+^0 = \frac{i}{2}$ in (4.17). Because the metric factors all take the same qualitative form, we display only f_2^2.

This is real, satisfying (4.14) for any choice of the zeroes u_i. Thus, $\mathcal{G}$ is single-valued for all differentials of the form (4.20). It remains to implement $\mathcal{G} < 0$ and $0 < T^2 < 1$. Explicit investigation shows that this is satisfied e.g. for $\mathcal{A}_\pm^0 = 0$ and $u_i = \ldots = u_{\nu-2} = 2i$ for $\nu \in \{3, 4, 5, 6\}$.

A more symmetric solution can be realized conveniently by working directly on the disc, $\Sigma = \{z \in \mathbb{C} \,\big|\, |z|^2 \leq 1\}$, with the coordinate transformation used in (4.17) and the differentials

$$\partial_z \mathcal{A}_+ = \bar{\varsigma}\left(z^{\nu-2} - \alpha\right) \qquad\qquad \partial_z \mathcal{A}_- = -\frac{\varsigma}{z^2}\left(z^{2-\nu} - \alpha\right) \tag{4.26}$$

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with $0 < \alpha < 1$. This corresponds to a pole at $z = 0$ in $\partial_z \mathcal{A}_-$ and zeros at the $(\nu-2)^{\text{th}}$ roots of α in $\partial_z \mathcal{A}_+$. The functions $\mathcal{A}_\pm$ are

$$\mathcal{A}_+ = \mathcal{A}_+^0 + z\bar{\varsigma}\left(\frac{z^{\nu-2}}{\nu-1} - \alpha\right) \qquad\qquad \mathcal{A}_- = \mathcal{A}_-^0 + \frac{\varsigma}{z}\left(\frac{z^{2-\nu}}{\nu-1} - \alpha\right) \tag{4.27}$$

They satisfy $\overline{\mathcal{A}_\mp(1/\bar{z})} = \mathcal{A}_\pm$. Under the $\mathbb{Z}_{\nu-2}$ generated by $z \to z\exp(\frac{2\pi i}{\nu-2})$ and an appropriate transformation of $\mathcal{A}_\pm^0$, the functions $\mathcal{A}_\pm$ transform by an overall multiplicative phase. These $\mathbb{Z}_{\nu-2}$ transformations leave κ^2, $\mathcal{G}$, and T, and consequently the metric functions invariant. The RR potentials and axion-dilaton scalar transform non-trivially.

The general Ansatz for the differentials (4.20) has ν complex parameters. Adding the integration constants $\mathcal{A}_\pm^0$ and subtracting 3 real degrees of freedom for the $\mathrm{SL}(2,\mathbb{R})$ automorphisms of the upper half-plane leaves $2\nu - 1$ real parameters. These are subject to additional regularity constraints to implement $\mathcal{G} < 0$ and $0 < T^2 < 1$. The examples given above show that these constraints can be satisfied. Since the additional regularity conditions take the form of inequalities, there is indeed a $2\nu-1$ parameter family of solutions for each ν. The $\mathrm{SU}(1,1)$ duality transformations (2.8) with $v \neq 0$ again do not map the set of functions (4.23) into itself, and instead carve out $\mathrm{SU}(1,1)$ orbits of regular solutions. Adding two parameters for the $\mathrm{SU}(1,1)$ transformations with $v \neq 0$ and subtracting one degree for the constant shifts which only produce gauge transformations of $\mathcal{C}$ and $\mathcal{M}$, we arrive at 2ν parameters.

In figure 2, we plot the metric factors, axion-dilaton, and two-form fields for the simplest non-minimal solution on the disc, i.e. the solution with $n = 3$. To obtain these plots, we have chosen $\mathcal{A}_\pm^0 = 0, \sigma = 1$, and $u_1 = 2i$ in (4.23). As in the case of the minimal solutions, the metric factors all have the same qualitative form, and hence we include the result only for f_2^2.

5 T-dual of $\mathrm{AdS}_2 \times S^7$ in Type IIA

In this section, we comment on possible T-duals of a class of AdS_2 solutions in Type IIA. T-duals of a Type IIA solution with geometry AdS_6 warped over a half sphere S^4 describing the D4-D8 system could be recovered as special cases of the general local AdS_6 solution in Type IIB [6, 32, 33]. T-dualizing the Type IIA solution along the S^1 Hopf fiber in S^3 produces a supersymmetric solution in Type IIB, as does non-Abelian T-duality [35]. With the AdS_6 superalgebra being unique, these T-duals had to be contained in the solutions of [6].

For the case of AdS_2, the solutions of [30] describing semi-localized D0-D8-F1 systems in massive Type IIA take the form $\mathrm{AdS}_2 \times S^7$ warped over an interval. One could consider various $\mathrm{U}(1)$ isometries with fixed points in S^7 for T-duality. Recalling that S^{2n+1} is a $\mathrm{U}(1)$ bundle over $\mathbb{CP}^n$, the S^1 fiber is a natural candidate on which to carry out this T-duality. Such T-dualities have been discussed in [36–38]. With the $S^p \times S^q$ slicing of S^n, where $p + q = n - 1$, the S^1 in the $S^1 \times S^5$ slicing may be another candidate. The S^1 Hopf fiber in S^3 could be used in one of the factors of the $S^3 \times S^3$ slicing, or in the Hopf fibration of S^3 over S^4. Non-Abelian T-duality may provide further options. None of these options, however, would produce an S^6 in the T-dual geometry. Moreover, even where

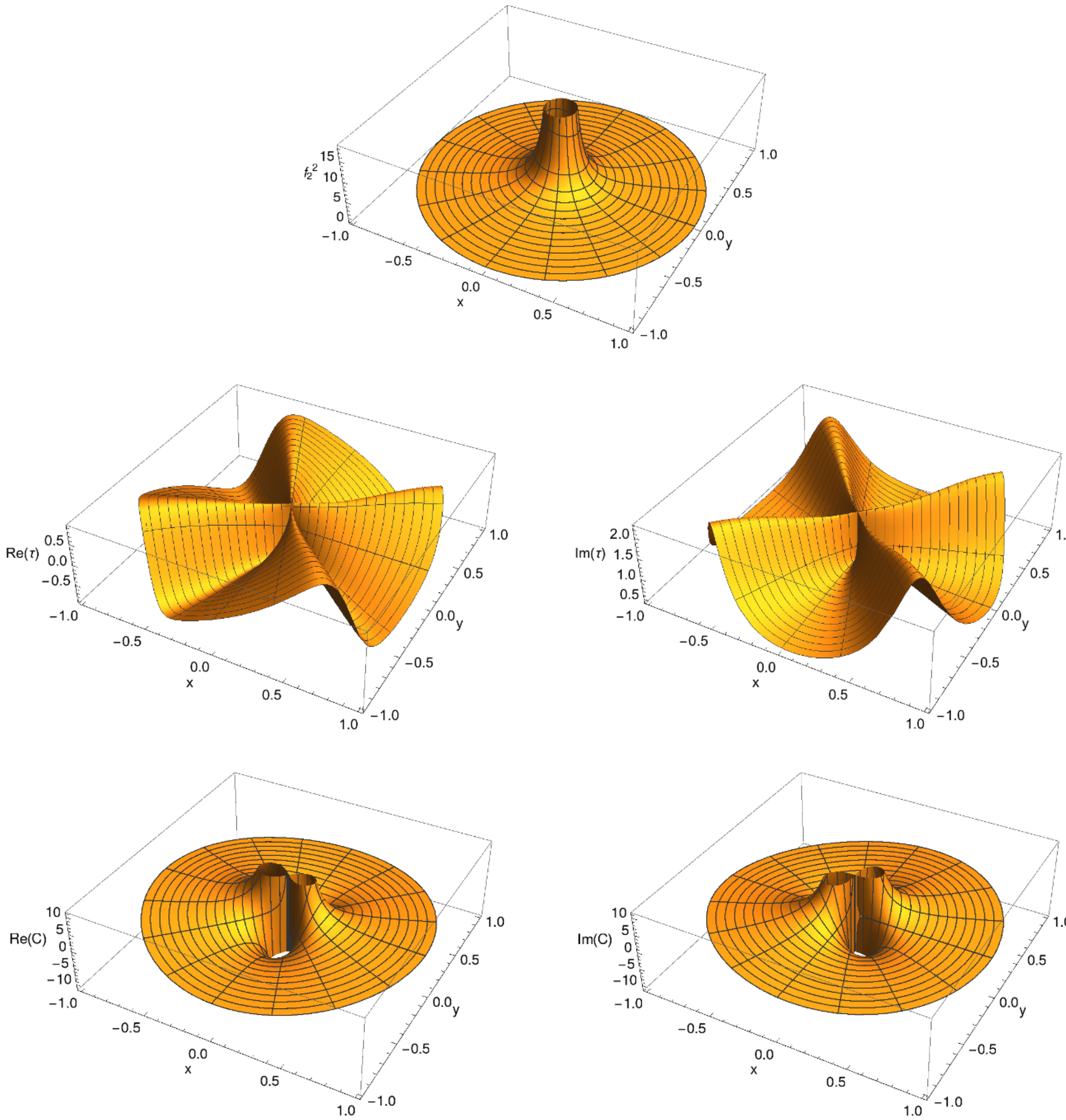

Figure 2. The metric factors, axion-dilaton, and two-form fields for the solution with a single interior pole of order three. We have chosen $\mathcal{A}^0_\pm = 0, \sigma = 1$, and $u_1 = 2i$ in (4.23). Because the metric factors all take the same qualitative form, we display only f_2^2.

a superalgebra with the preserved bosonic symmetries exists, supersymmetry may not be preserved by T-duality along U(1) isometries with fixed points (see, however, [39, 40]).

We discuss the case of $\mathbb{CP}^3$ in the following. We illustrate the massless case, when there are no D8-branes, in the conventions of [30], and consider the semi-localized intersection of D0 and F1. The generalization to the massive case is straightforward. The string-frame metric, dilaton, and two-form field are given by,

$$
\begin{aligned}
ds^2_{10} &= \frac{1}{4} L^2 W^2 \left[ds^2_{\mathrm{AdS}_2} + 4 \left(d\theta^2 + 4 \sin^2\theta ds^2_{S^7} \right) \right] \\
\mathrm{Im}\,(\tau) &= \frac{1}{g_s L W} \qquad\qquad B_2 = -B_0 W^2 \cos\theta \,\mathrm{vol}_{\mathrm{AdS}_2}
\end{aligned}
\tag{5.1}
$$

where $g_s > 0$ and B_0 are constants, $\theta \in [0, \pi]$, $L^2 = \frac{1}{8}\sqrt{Q_{D0}Q_{F1}}$, and $W = (\sin\theta)^{-3/2}$. The F1-string wraps a combination of the AdS_2 radial direction and the θ-direction. T-dualizing along the S^1 fiber in the fibration over $\mathbb{CP}^3$ yields the string-frame metric and dilaton

$$\begin{aligned} ds^2_{10} &= \frac{1}{4}L^2W^2\left[ds^2_{\mathrm{AdS}_2} + 4\sin^2\theta\, ds^2_{\mathbb{CP}^3}\right] + \frac{4}{L^2W^2\sin^2\theta}\left[d\psi^2 + \frac{1}{4}L^4W^4\sin^2\theta d\theta^2\right] \\ \mathrm{Im}\,(\tau) &= \frac{1}{g_s}\sin\theta \end{aligned} \tag{5.2}$$

Such a configuration could naturally arise from Type IIB solutions of the form $\mathrm{AdS}_2 \times \mathbb{CP}^3$ warped over a Riemann surface Σ, where the appropriate superalgebra would be $\mathrm{SU}(1,1|4)$. These solutions are currently under investigation.

6 Discussion

In the first part of this work we have constructed an Ansatz for global Type IIB supergravity solutions with 16 supersymmetries on a space-time of the from $\mathrm{AdS}_2 \times S^6$ warped over the unit disc or equivalently the upper half-plane, which may allow for an identification with string junctions. These solutions circumvent the no-go results of [10], and naturally implement the boundary conditions on $\partial\Sigma$ which impose stronger constraints than in the $\mathrm{AdS}_6 \times S^2$ case. The remaining conditions for regularity and geodesic completeness were reduced to algebraic constraints on the parameters of the Ansatz, whose complete solution remains an open problem. In analogy with the relation of AdS_6 solutions to M5-brane curves [41], one may expect the data $(\Sigma, \mathcal{A}_\pm)$ for solutions corresponding to string junctions to define the holomorphic curve wrapped by the M2-brane in the M-theory uplift of the string junctions [42–45]. As discussed in section 5, the T-duals of Type IIA AdS_2 solutions relating to D0-F1-D8 systems naturally realize a different superalgebra, motivating their further investigation.

In the second part we studied solutions independently from a string junction interpretation. We presented families of non-compact solutions with geometry $\mathrm{AdS}_2 \times S^6$ warped over a punctured sphere or a punctured disc. At the punctures the geometry decompactifies into an asymptotic region. The explicit solutions we have presented are infinite families which all have one asymptotic region. They are labeled by an integer ν, which corresponds to a $\mathbb{Z}_\nu$ symmetry in the asymptotic region, and have 2ν real parameters.

We laid out a systematic construction strategy which may give access to further solutions with similar features. Possible generalizations include solutions with multiple asymptotic regions or Riemann surfaces of different topology. The construction of these AdS_2 solutions involves poles of the functions $\mathcal{A}_\pm$ in the interior of Σ, which would not be compatible with the regularity conditions in the AdS_6 case [8]. This highlights the physical independence of these two cases, despite the similarities in the general local solution to the BPS equations. The AdS_2 solutions share certain features with black hole micro-state geometries [46–51], but we leave their interpretation for future work.

Various further generalizations of the solutions may be possible. For example, one may try to combine the two elements discussed in this paper — namely, to realize the singularities allowing for a local match to (p, q)-string solutions discussed in the first part in the non-compact solutions with asymptotic regions discussed in the second part. The AdS_6 solutions can be generalized to include 7-branes by modifying the functions $\mathcal{A}_\pm$ of a regular solution [9], and it may be possible to generalize the AdS_2 solutions in a similar way. Further solutions may also be constructed by relaxing the regularity conditions, to allow e.g. for smeared branes along the lines of [33].

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