Charm CP violation and the electric dipole moments from the charm scale

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ABSTRACT: The reported CP asymmetry in $D \to K^+K^-/\pi^+\pi^-$ is argued to be too large to naturally fit the SM. If so, a new source of CP violation is implied in the $\Delta C = 1$ sector with a milliweak strength. CP-odd interactions in the flavor-diagonal sector are strongly constrained by the EDMs placing severe limitations on the underlying theory. While the largest effects usually come directly from the effective operators generated at the New Physics energy scale, they are strongly model-dependent. Yet the interference of the CP-odd forces manifested in D decays with the conventional CP-even $\Delta C = 1$ weak interaction at the charm scale also generates d_n at a certain level. It has been argued that the d_n in the SM is largely generated via such an interference, with mild KM-specific additional suppression. The reported CP asymmetry is expected to generate d_n through such effects of 30 to 100 times larger than in the SM, or even higher in certain, yet not fully natural, scenarios. In the SM the charm-induced loop-less $|d_n|$ is expected around 10^{-31} e·cm. On the technical side, we present a compact Ward-identity-based derivation of the induced scalar pion-nucleon coupling in the presence of the CP-odd interactions, which appears once the latter include the right-handed light quarks.

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Contents

1 Introduction

Recent hints at possible direct CP violation in singly Cabibbo-suppressed D meson decays have caused some excitement, since they may be the first direct indication for physics beyond the Standard Model (SM). The LHCb collaboration observes a difference of time-integrated CP asymmetries [1]

$$\Delta a_{\rm CP} = a_{\rm CP}(D^0 \to K^+ K^-) - a_{\rm CP}(D^0 \to \pi^+ \pi^-) = -(0.82 \pm 0.21 \pm 0.11)\%, \qquad (1.1)$$

a result preliminarily confirmed by CDF [2]:

$$\Delta a_{\rm CP} = -(0.62 \pm 0.21 \pm 0.10)\%. \qquad (1.2)$$

The CP asymmetry is defined according to

$$a_{\rm CP}(D^0 \to f) = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to \overline{f})} \qquad (1.3)$$

Since the both final states are positive CP eigenstates in strong interactions, $\Delta a_{\rm CP}$ evidently roots in a direct CP asymmetry.

As argued below, an effect of this magnitude solely in the framework of the SM may not be rigorously excluded, yet it would require a strong enhancement of certain decay matrix elements. Such a loophole definitely deserves further scrutiny. In this paper we

analyze immediate consequences of the assumption that the reported effect is due to a new source of CP violation, beyond the CKM mechanism.

In the present paper we focus on the impact of new CP-odd forces on the electric dipole moments, in particular on one of the neutron, d_n. The observation at LHCb and CDF assumes CP violation in $|\Delta C| = 1$ amplitudes. A more general model should embed this into a full flavor framework, hopefully highlighting a certain underlying symmetry. It would allow one to obtain predictions for other sensitive processes as well, in particular for B and K decays, where some tensions have also been noted in certain cases.

We do not aim at working out any realistic model, or discuss general features in such a framework here. We only note that a flavor-diagonal CP violation of the size reported in the decays $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ is incompatible with the current limits on the electric dipole moment of the neutron. At the same time, we find that if the new CP-odd forces show up at low energies only in $|\Delta C| = 1$ interactions, the neutron EDM is still well below the current limit, although it should be significantly enhanced, by more than an order of magnitude, compared to the SM. A concrete dynamic realization of New Physics would generically imply the contributions to d_n of both types. Considering them separately is legitimate, since the contributions coming from totally different energy scales are physically distinct and may exhibit cancellations only accidentally.

In the following section we briefly review the CP violation in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$ within the SM and introduce new $|\Delta C| = 1$ interactions as a source of the enhanced CP violation. In section 3 we examine d_n in the SM and describe the loop-free mechanism to generate it at the charm scale. The elaborated estimates of the associated nucleon matrix elements indicate that it yields d_n around $10^{-31}\,\mathrm{e \cdot cm}$ and may well constitute the principal contribution in the SM. The same analysis is then adapted to new BSM-mediated CP-odd amplitudes to estimate the corresponding effect on the neutron EDM. Section 4 summarizes the study. Appendix derives the induced CP-odd scalar pion-nucleon coupling by generalizing the Goldberger-Treiman relation and applying in QCD the current algebra technique.

2 CP violation in $D^0 \to K^+ K^-$ and $D^0 \to \pi^+ \pi^-$

2.1 Charm CP violation in the standard model

For the charm decays considered hereafter we have the effective weak interaction

$$
\mathcal{L} = -\frac{G_F}{\sqrt{2}} \left[\frac{V_{cs}V_{us}^* - V_{cd}V_{ud}^*}{2} \left([\bar{c}\Gamma^\mu u][\bar{s}\Gamma_\mu s] - [\bar{c}\Gamma^\mu u][\bar{d}\Gamma_\mu d] \right) \right.
$$
$$
\left. - \frac{1}{2} V_{cb}V_{ub}^* \left([\bar{c}\Gamma^\mu u][\bar{s}\Gamma_\mu s] + [\bar{c}\Gamma^\mu u][\bar{d}\Gamma_\mu d] - 2[\bar{c}\Gamma^\mu u][\bar{b}\Gamma_\mu b] \right), \right] + \mathrm{H.c.} \quad (2.1)
$$
$$
= -\frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \left[o_1 - \frac{1}{2} r_{\mathrm{SM}} e^{-i\gamma} o_2 \right] + \mathrm{H.c.},
$$

$$
[\bar{c}\Gamma^\mu u][\bar{q}\Gamma_\mu q] \equiv (\bar{c}\gamma^\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)u), \qquad \Gamma_\alpha = \gamma_\alpha(1-\gamma_5),
$$

where we have used the CKM unitarity, and color indices are assumed to be contracted within parentheses. The phase γ is practically equal to the corresponding angle of the

Unitarity Triangle, while

$$r_{\text{SM}} = \left| \frac{V_{cb}^* V_{ub}}{V_{cs}^* V_{us}} \right| \simeq 7.5 \times 10^{-4}. \tag{2.2}$$

CP violation in the SM is quantified by the imaginary part of the invariant product of four CKM mixing elements describing the relative phase between the coefficients for o_1 and o_2,

$$\Delta = \text{Im}\, V_{cs}^* V_{us} V_{cd} V_{ud}^* = \text{Im}\, (V_{cb}^* V_{cd})^* (V_{ub}^* V_{ud}), \tag{2.3}$$

numerically $\Delta \simeq 3.3 \cdot 10^{-5}$.

The operators o_1 and o_2 are a U-spin triplet and a U-spin singlet, respectively. Their interference induces the CP violation in the SM in the $\Delta C = 1$ sector. In what follows we discard possible CP violation in $\bar{D} - D$ mixing since it drops out from the asymmetry difference Δa_{CP}.

The decay amplitudes in $D^0 \to f$ with $f = \bar{f}$ are given by ($f = K^+ K^-$ or $f = \pi^+ \pi^-$)

$$A(D^0 \to f) = -i \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \left[m_1^{(f)} - \frac{1}{2} r_{\text{SM}} e^{-i\gamma} m_2^{(f)} \right] \tag{2.4}$$

where $m_i^{(f)} = \langle f | o_i | D^0 \rangle$ are the reduced amplitudes, in general complex due to the strong interaction in the final state. The corresponding phases are generically referred to as $\delta_i^{(f)}$; they are equal for the decays of D and $\bar{D}$. Since the o_1 amplitudes strongly dominate, $r_{\text{SM}} \ll 1$, the CP asymmetry takes a simple form

$$a_{\text{CP}}(D^0 \to f) = -r_{\text{SM}} \sin\gamma \left| \frac{m_2^{(f)}}{m_1^{(f)}} \right| \sin\delta_{21}^{(f)}, \qquad r_{\text{SM}} \sin\gamma \simeq \frac{\Delta}{\sin^2\theta_C \cos^2\theta_C} \simeq 0.70 \cdot 10^{-3}, \tag{2.5}$$

where $\delta_{21}^{(f)} = \delta_2^{(f)} - \delta_1^{(f)}$ is the phase difference between the two hadronic matrix elements.

The two final states $K^+ K^-$ and $\pi^+ \pi^-$ are components of the same U-spin triplet. Therefore in the SU(3) limit $\delta_{21}^{\pi^+\pi^-} = \delta_{21}^{K^+K^-} + \pi$ would hold and

$$|\Delta a_{\text{CP}}| \simeq 2|a_{CP}(D^0 \to K^+ K^-)| \simeq 2|a_{CP}(D^0 \to \pi^+ \pi^-)|. \tag{2.6}$$

However, U-spin symmetry is significantly violated; one concludes from the decay rates

$$\Gamma(D^0 \to P\bar{P}) = \frac{G_F^2}{32\pi M_D} \sin^2\theta_C \cos^2\theta_C \sqrt{1 - \frac{4M_P^2}{M_D^2}} \, |m_1^{(P\bar{P})}|^2 \tag{2.7}$$

that

$$|m_1^{K^+K^-}| \simeq 0.456 \,\text{GeV}^3, \qquad |m_1^{\pi^+\pi^-}| \simeq 0.252 \,\text{GeV}^3. \tag{2.8}$$

We expect even larger potential SU(3) breaking in the phases of the amplitudes. This is consistent with the analysis of the $K^\pm \pi^\mp$ modes [3].

The values in eq. (2.8) reasonably agree with the simplest factorization estimate

$$m_1^{K^+K^-} \simeq i f_+^{D \to K}(M_K^2) f_K (M_D^2 - M_K^2), \quad m_1^{\pi^+\pi^-} \simeq -i f_+^{D \to \pi}(M_\pi^2) f_\pi (M_D^2 - M_\pi^2) \tag{2.9}$$

(the straightforward color renormalization factors have been omitted from the full expression). It even yields the right scale for the SU(3)-breaking [4], although the literal ratio of the amplitudes tends to fall short of 1.81 in eq. (2.8).

The matrix elements of o_2 determining the amplitudes $m_2^{K^+K^-}$ and $m_2^{\pi^+\pi^-}$ are not directly known. If estimated using factorization, one evidently obtains the values close to m_1 in eqs. (2.9), (2.8), with the additional minus sign for $m_2^{\pi^+\pi^-}$. However, the conventional factorization accounts only for the valence contributions. In a valence approximation, on the other hand, the same term in both o_1 and o_2 — with s-quarks for $D^0 \to K^+K^-$ and with d-quarks for $D^0 \to \pi^+\pi^-$, respectively — contribute. Consequently, no CP-asymmetry is generated in a valence approximation: the two strong amplitudes come from the same underlying operators and their strong phases coincide.

Strictly speaking, any valence approximation should only be applied to the operators normalized at a low scale. The evolution of the operators o_1 and o_2 above m_b is identical, yet generates additional terms for o_2 below it due to Penguin diagrams [5, 6]. These in general have different strong phases. However, the new operators come with small loop-induced coefficients, while we do not expect their matrix elements to be enhanced. Therefore, we neglect these effects.

As the starting point we assume that the magnitudes of $m_2^{K^+K^-}$ and $m_2^{\pi^+\pi^-}$ may be approximated using factorization, yet allow for arbitrary FSI phases relative to m_1. This amounts to having the ratio of the amplitudes in eq. (2.5) about unity. We then end up with

$$|a_{\mathrm{CP}}(D^0 \to K^+K^-)| \approx 0.7 \cdot 10^{-3} |\sin \delta_{21}^{K^+K^-}|,$$

$$|a_{\mathrm{CP}}(D^0 \to \pi^+\pi^-)| \approx 0.7 \cdot 10^{-3} |\sin \delta_{21}^{\pi^+\pi^-}|, \tag{2.10}$$

and the sign of the two asymmetries may naturally be opposite. Therefore, the expected scale for $|\Delta a_{\mathrm{CP}}|$ in the SM is a few times 10^{-4} up to $1.5 \cdot 10^{-3}$ — provided the both FSI phase shifts are optimal. This is still about five times smaller than what is reported by LHCb.

Accommodating the central value in eq. (1.1) within the SM thus implies at least a five-fold enhancement of the U-spin singlet amplitude mediated by o_2, or even a ten-fold if the asymmetry is dominated by one of the two modes and/or the strong phase shifts are not optimal. Moreover, this must happen for a non-valence part of the amplitude.

Although at the moment the possibility of a sufficiently strong enhancement of the U-scalar amplitude in the D decay within conventional QCD dynamics cannot be rigorously ruled out, we view this possibility as contrived. The confirmation of the asymmetry Δa_{CP} at the currently observed level, in particular studying its share among the two channels, would be a strong evidence for the new CP-violating dynamics in the charm sector. At the same time, the strength of this conclusion crucially depends on the actual amount of the excess over our expectations. An eventual value around or somewhat below -0.3%, while still not smoothly accommodated in the SM, *per se* would make the case for new sources of CP-violation in D decays significantly weaker. A similar conclusion was inferred from a more detailed analysis by the authors of ref. [3].

2.2 Charm CP violation through new physics

In what follows we adopt the assumption that the reported CP asymmetry roots in new CP-odd interactions. Within this hypothesis we will not attempt to stretch the uncertainties due to the QCD interaction to as strong extent and rather apply an educated judgment elaborated in weak decays so far; we then examine the consequences for the electric dipole moments. Neither we focus on the extreme values of parameters maximizing the CP asymmetry. Consequently, we will gauge our expectations on an assumption that, speaking generally, the new source of CP violation produces an *order of magnitude* stronger CP-odd amplitude in $D \to K^+K^-$ or in $D \to \pi^+\pi^-$ decays than in the SM.

Turning to NP, we make a relatively safe assumption that the New Physics-induced amplitude is small compared to the SM one m_1; this is obvious for the CP-odd NP part, and is applied also to its CP-even component. Then the asymmetry is given by the sum of the pure NP-induced asymmetry and the SM one, for either channels. Keeping in mind the conclusions of section 2.1 we neglect the SM contribution altogether, and have

$$a_{\mathrm{CP}}(D^0 \to f) = -2 \left| \frac{\mathrm{Im}\, g_{\mathrm{NP}}\, m_{\mathrm{NP}}^{(f)}}{m_1^{(f)}} \right| \sin \delta_{\mathrm{NP}}^{(f)}, \tag{2.11}$$

where $g_{\mathrm{NP}}\, m_{\mathrm{NP}}^{(f)}$ denotes the New Physics amplitude (in units of $G_F \sin\theta_C \cos\theta_C / \sqrt{2}$) and $\delta_{NP}^{(f)}$ is its strong phase relative to $m_1^{(f)}$ of the SM.[1]

The value of the new couplings g_{NP} depends on the convention chosen to parameterize the BSM amplitudes (we tacitly anticipate using effective local operators to describe them). If we assume a 'natural' normalization of the operators where $|m_{NP}^{(f)}| \approx |m_1^{(f)}|$ holds for the reduced amplitudes, we arrive at a ballpark estimate for the CP-odd coupling:

$$|\mathrm{Im}\, g_{\mathrm{NP}}| \sim (2 \div 5) \cdot 10^{-3}, \tag{2.12}$$

allowing for the generic unsuppressed strong phase differences as the educated guess about QCD dynamics in charm. The new CP-odd forces must therefore be of a 'milliweak' strength, according to the venerable terminology in CP violation. Their strength is in general about 10 times larger than what one estimates in the SM, cf. $r_{\mathrm{SM}} \sin\gamma$ in eq. (2.5). Specifically, to accommodate the 'direct' CP-asymmetry eq. (1.1) one needs

$$0.55 \frac{\mathrm{Im}\, g_{\mathrm{NP}}\, m_{\mathrm{NP}}^{K^+K^-}}{10^{-3}\,\mathrm{GeV}^3} \sin \delta_{\mathrm{NP}}^{K^+K^-} + \frac{\mathrm{Im}\, g_{\mathrm{NP}}\, m_{\mathrm{NP}}^{\pi^+\pi^-}}{10^{-3}\,\mathrm{GeV}^3} \sin \delta_{\mathrm{NP}}^{\pi^+\pi^-} = \frac{-\Delta a_{\mathrm{CP}}}{8.2 \cdot 10^{-3}}. \tag{2.13}$$

Proceeding to the induced CP-odd effects in other observables requires specifying the nature of new interactions. Below the charm scale we have the realm of light hadrons including flavor-diagonal processes with stable hadrons and the decays of strange particles. CP-odd effects there are highly constrained; therefore we discard these, and relegate new sources to heavy particles. Their effect at low energies is described by local operators classified over the canonic dimension, with the lowest-dimension potentially dominating.

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[1] We assume $\mathrm{Im}\, g_{\mathrm{NP}} > 0$ and include the possible sign into $\delta_{NP}^{(f)}$. Let us also clarify that the phase convention required for CP conjugation is defined in such a way that the a_1 amplitude in the SM is CP-even.

A flavor-diagonal CP violation (for instance, the induced QCD θ-term unless it is offset by a Peccei-Quinn-type mechanism) with a coupling of the size commensurate with eq. (2.12) is by far excluded by electric dipole moments, in particular of the neutron. This likewise applies to the four-quark operators — they would generate d_n in the ball park of 10^{-22} e·cm. Consequently, the flavor structure of the new CP-odd interaction must have vanishing flavor-diagonal components in the light sector. This property must replicate itself at the loop level, which strongly suggests it to apply to the heavy flavors alike. We do not analyze here the consequences of this requirement for various classes of the BSM models, but rather note that this may be a hint at a structure of the underlying flavor interactions antisymmetric in respect to the generation indices. Yet even when postulating such a property, nontrivial constraints may follow at the loop level in view of the large gap between the scale of the potential effect on the EDMs and of their experimental limits. The loop-induced effects and the related renormalization of the effective low-energy operators may strongly depend on a particular class of models, see, e.g. refs. [7, 8]; this lies outside the scope of our analysis.

We therefore concentrate on the most direct consequences of the presence of new $\Delta C = 1$ CP-odd amplitudes and describe them by the effective operators of dimension 5 and 6. Most of them are four-quark operators. This appears representative enough. The reason, as argued below, is that a significant — and probably the dominant — piece of d_n in the Standard Model likewise originates from the same underlying effect: the interference of the CP-odd and CP-even weak $|\Delta C|=1$ amplitudes in the nucleon. The SM bears only mild additional model-specific suppressions; these may, or may not be vitiated by the new CP-odd $|\Delta C|=1$ interaction, depending on particular details. Consequently, we typically obtain a 30- to 100-fold enhancement of d_n compared to the SM.

The number of appropriate $D=6$ operators (they can be both scalar or pseudoscalar) is quite large since they may differ in the chiral, color and light-flavor content. We first note that the scalar operators do not affect either of the D decays in question, yet they do generate d_n. Therefore, d_n could have been further enhanced if the scalar NP operators dominate. This possibility can be effectively eliminated experimentally by studying the similar CP-asymmetries including the parity-even final states in decays of D mesons, and we will not dwell on it any further.

To substantiate the consideration we pick out ad hoc a few operators of interest:

$$O_1 = em_c \bar{c} \, i\sigma_{\alpha\beta} F^{\alpha\beta} \gamma_5 u \,, \qquad\qquad O_2 = g_s m_c \bar{c} \, i\sigma_{\alpha\beta} G^{\alpha\beta} \gamma_5 u \,,$$
$$O_3 = [\bar{c}\Gamma_\mu u]([\bar{s}\Gamma^\mu s] + [\bar{d}\Gamma^\mu d])\,, \qquad O_4 = (\bar{c}\gamma_\mu(1+\gamma_5)u)\,(\bar{d}\gamma^\mu(1-\gamma_5)d) \tag{2.14}$$

and put

$$\mathcal{L}_{\rm np} = -\frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \sum_k c_k O_k, \tag{2.15}$$

with c_k dimensionless. The first two operators are the unique quark bilinears. Operator O_3 has been picked since it evidently represents the CP-odd operator o_2 of the SM — yet with an arbitrarily inflated coefficient. Consequently, we would roughly expect

$$|\mathrm{Im}\, c_3| \approx 10 \cdot \frac{1}{2} r_{\rm SM} \tag{2.16}$$

if O_3 is the only New-Physics source of CP violation.

The operator O_4 is an example with a different chiral content for both charm and light quarks and differs also in color and flavor. Operator O_2, like O_3 is a U-spin singlet. For the sake of definiteness we assume in what follows that the direct CP asymmetry is largely seen in the $\pi^+\pi^-$ mode having a numerically smaller SM CP even amplitude, eq. (2.8).

The O_4 matrix element can be estimated with simple factorization yielding

$$\langle \pi^+\pi^-|O_4|D\rangle \approx -i\, f_\pi f_+^{D\to\pi}(0)M_D^2\, \frac{1}{N_c}\frac{2m_\pi^2}{(m_u+m_d)m_c}. \tag{2.17}$$

It would have shown a relative enhancement if charm mass scale were lower, while would have been suppressed for larger m_c. For actual quark masses the corresponding factor is not too far from unity. This amplitude is color suppressed, therefore the factorization is not expected to be a good approximation — yet it makes explicit the expected qualitative features required for the scale estimates.

The amplitudes for operators O_1 and O_2 cannot be estimated by simple vacuum insertion. Keeping in mind that both are color-allowed we use instead a "rule of thumb" for our estimates

$$\frac{\langle \pi^+\pi^-|O'|D\rangle}{\langle \pi^+\pi^-|O|D\rangle} \approx \sqrt{\frac{\Gamma_{O'}^{\text{part}}}{\Gamma_O^{\text{part}}}} \tag{2.18}$$

and set, as the reference, the operator O to be the 'valence' part of o_1, $(\bar{c}\Gamma_\mu d)(\bar{d}\Gamma_\mu u)$ (in fact, its P-odd part only). In other words, the fraction of the decay events into the exclusive $\pi\pi$ final state is assumed the same in the decays mediated by O and O'. Since charm mass lies in the intermediate domain, there must be no large kinematic factors floating around.

For O_2, the total decay width mediated by $m_c\bar{u}g_s i\sigma_{\mu\nu}G^{\mu\nu}\gamma_5 c$ is

$$\Gamma_{\sigma G} = \alpha_s m_c^5 \frac{N_c^2-1}{N_c}, \tag{2.19}$$

and the resulting estimate reads

$$\langle \pi^+\pi^-|m_c\bar{u}g_s i\sigma_{\mu\nu}G^{\mu\nu}\gamma_5 c|D\rangle \approx i\, 4\pi g_s\sqrt{3}\, f_\pi f_+^{D\to\pi}(0)M_D^2. \tag{2.20}$$

(The amplitude proportional to only the first power of g_s reflects the fact that we are not yet in the asymptotic heavy quark regime.)

In the case of photonic O_1 the partonic rate itself describes the probability of a different process, $D\to\gamma+$hadrons. Instead we need the similar partonic rate for the photon conversion into a d-quark pair:

$$\Gamma_{\text{conv}} = \int \frac{d\lambda^2}{\lambda^2}\, \Gamma_{\sigma F}(\lambda^2)\cdot\frac{\alpha}{3\pi}N_c\, q_d^2\,, \quad \Gamma_{\sigma F}(\lambda^2) = \frac{e^2 m_c^5}{2\pi}\left(1-\frac{\lambda^2}{m_c^2}\right)^2\left(1+\frac{\lambda^2}{2m_c^2}\right)\theta(m_c^2-\lambda^2). \tag{2.21}$$

The kinematic integral equals to $\ln m_c^2/m^2 - \frac{4}{3} + \ldots$; the lower cutoff can be taken at m around $400\,\text{MeV}$, to match the overall hadronic polarization contribution to charge renormalization. Then the integral turns out numerically close to unity. Using this as a counterpart to eq. (2.19) we get

$$\langle \pi^+\pi^-|m_c\bar{u}e\, i\sigma_{\mu\nu}F^{\mu\nu}\gamma_5 c|D\rangle \approx i8\sqrt{2}\,\pi\alpha\, q_d\, f_\pi f_+^{D\to\pi}(0)M_D^2\,, \tag{2.22}$$

where the small deviation of the explicit log factor from unity has been neglected.

To cross-check the meaningfulness of the estimates we explored alternative ways. For the gluon bilinear O_2 this was relating the gluon field operator to the chromomagnetic (or kinetic) expectation value in heavy mesons, i.e. treating it as fully nonperturbative. This resulted in a different estimate

$$\langle \pi^+ \pi^- | m_c \bar{u} g_s i\sigma_{\mu\nu} G^{\mu\nu} \gamma_5 c | D \rangle \approx i \frac{2\mu_G^2}{f_\pi} f_+^{D\to\pi}(0) M_D^2. \tag{2.23}$$

Assuming numerically that $N_c \alpha_s \simeq 1$ we would get a number only 10% smaller than eq. (2.20). The two totally different estimates led to close values because charm lies in between the light- and the heavy-quark regimes where both perturbative partonic and nonperturbative description can qualitatively be applied.

In the case of O_1 photon cannot per se be nonperturbative; instead one can consider the photon loop with the gluon emitted internally. This effect is proportional to q_u and is therefore physically different. It describes the order-α operator mixing of the electromagnetic O_1 into chromomagnetic O_2. Replacing $\ln \Lambda_{\rm UV}$ by unity we would obtain

$$\langle \pi^+ \pi^- | m_c \bar{u} e \, i\sigma_{\mu\nu} F^{\mu\nu} \gamma_5 c | D \rangle \approx i 2\sqrt{3} \, q_u \, \alpha g_s \, f_\pi f_+^{D\to\pi}(0) M_D^2. \tag{2.24}$$

Comparing this to eq. (2.22) we find the overall factor different:

$$4\sqrt{3\pi\alpha_s} \, q_u \quad \text{vs.} \quad 8\sqrt{2} \, \pi q_d, \tag{2.25}$$

with the former 'direct' contribution expectedly dominating since it does not suffer an additional perturbative loop factor for gluons. This is partially offset by the larger u-quark charge. Altogether the direct photon conversion estimate appears a few times larger, and we adopt eq. (2.22) for the O_1 estimates.

Collecting all the expressions we arrive at

$$|\text{Im} \, c_1| \approx \frac{5.2\cdot 10^{-2}}{|\sin\delta_{\rm FSI}|}, \qquad\qquad |\text{Im} \, c_2| \approx \frac{0.10\cdot 10^{-3}}{|\sin\delta_{\rm FSI}|},$$
$$|\text{Im} \, c_3| \approx \frac{2\cdot 10^{-3}}{|\sin\delta_{\rm FSI}|}, \qquad\qquad |\text{Im} \, c_4| \approx \frac{4.6\cdot 10^{-3}}{|\sin\delta_{\rm FSI}|}, \tag{2.26}$$

assuming that a particular operator is the sole source of the New-Physics CP violation.

3 The neutron EDM

EDMs in general and specifically the neutron EDM d_n are very sensitive probes for physics beyond the SM, in particular for CP violation. As already mentioned, a flavor-diagonal CP violation with a size of coupling found for the new physics operators in the last section would grossly violate the bound, which currently lies at [9]

$$|d_n| \le 2.9\cdot 10^{-26} \, \text{e·cm}. \tag{3.1}$$

In the following we assume that only the $\Delta C = \pm 1$ operators induce the non-SM CP violation, and estimate their effect. First we recapitulate the salient points of the estimates within the Standard Model, where recently a new perspective has been proposed [10]. We will discard the possibility of strong CP violation assuming that the long-standing strong CP problem will find a solution where the QCD θ-parameter is sufficiently close to zero.
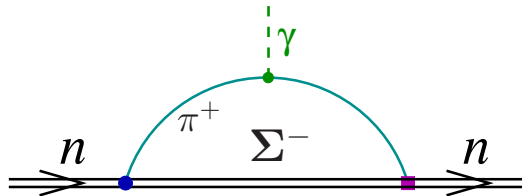
Figure 1. The chirally singular diagram exemplifying the conventional Penguin-based contribution. One of the vertices is the usual CP-conserving weak amplitude while another contains the CP-odd Penguin-mediated operators.

3.1 The neutron EDM in the Standard Model

The estimates of the neutron EDM in the SM have a thirty year long history; the modern perspective can be found in review [11]. EDMs may emerge from the second order on in the weak interaction and are generally proportional to G_F^2.

Motivated by the qualitative success of the constituent quark models in understanding of the properties of hadrons, the early estimates of nucleon EDM focused primarily on the EDMs of quarks d_q. It turned out that for quarks the KM prediction is further suppressed: the sum of all the two-loop diagrams vanishes and d_q emerge first at the three-loop level where an additional loop with at least a gluon must be included [12–14]. On top of this, the quark EDM has to be proportional to the quark mass; this yields an additional suppression for the light quarks. The same applies to the color dipole moments of quarks considered as the simplest induced CP-odd strong force generated through weak interactions at small distances. The unfortunate feature of the quark EDMs is that there is a strong numeric cancellation between the leading logarithmic and the subleading terms in the dominating EDM of the d-quark d_d [15], which makes it difficult to to make a definite prediction beyond an estimate

$$|d_{u,d}| \lesssim 0.5 \cdot 10^{-34}\, \text{e·cm}.$$

It has been noted a while ago [16, 17] that the strong suppression intrinsic to u and d quarks can be vitiated in composite hadronic systems like nucleons. The transition dipole moments changing d-quark into s-quark, electromagnetic or color, are suppressed by the strange quark mass m_s, and such flavor-changing transition without a quark charge change are already in the loop-induced short-distance renormalization of the bare weak interaction due to the so-called Penguin diagrams [5, 6].

It is notoriously difficult to account for the long-range part of the strong interactions generating the neutron EDM [11]. Usually it is considered that the principal effect comes from the diagram in figure 1 having a chiral singularity, or that at least it fairly represents the magnitude of d_n. One of the vertices in the diagram is a conventional CP-conserving $\Delta S = 1$ weak interaction while the second is the CP-odd Penguin-induced amplitude originating from short distances which naturally incorporates heavy quarks, in particular top.

It has recently been argued [10] that there is a complementary mechanism generating d_n to the second order in G_F which does not involve short-distance loop effects and is likewise free from chiral and SU(3) suppression. It scales like $1/m_c^2$ and would fade out

quickly for sufficiently heavy charm, yet it may actually dominate d_n in the SM since charm is marginally heavy in the hadronic mass scale. It originates at the energy scale around m_c due to interference of the conventional $\Delta C = 1$, $\Delta S = 0$ weak amplitudes, much in the same way as the CP-odd D-decay asymmetry discussed in section 2.1. Consequently, this mechanism would be present, with a modified strength, in the BSM scenarios affecting $\Delta a_{\rm CP}$. The analysis of the BSM contributions to d_n presented in section 3.2 parallels the SM case, therefore we remind below the main steps of ref. [10].

The observable CP-odd effects appear in the second order in $\mathcal{L}_w$ and thus are proportional to G_F^2, being embodied in

$$\mathcal{L}_2 = \frac{G_F^2}{2} \int d^4x \, \frac{1}{2} iT \{\mathcal{L}_w(x)\,\mathcal{L}_w(0)\}. \tag{3.2}$$

The generalized GIM-CKM mechanism ensures that the CP-odd piece of $\mathcal{L}_2$ is finite in the local four-fermion approximation. The conventional form of $\mathcal{L}_w$ applies to the high normalization point around M_W. We will neglect, for the most of the consideration, the perturbative gluon corrections, since the effect exists even without loops. This makes the analysis simpler and more transparent.

Descending to a low normalization point we first integrate out top quark and at the second stage, below m_b also the bottom quark. At tree level integrating them out simply means discarding all the terms containing t- or b-quark fields. As a result, we arrive at $\mathcal{L}_2$ generated by a superficially two-family weak Lagrangian

$$\mathcal{L}_w = J_\mu^\dagger J^\mu \ \text{ with } \ J_\mu = V_{cs}\,\bar{c}\Gamma_\mu s + V_{cd}\,\bar{c}\Gamma_\mu d + V_{us}\,\bar{u}\Gamma_\mu s + V_{ud}\,\bar{u}\Gamma_\mu d\,. \tag{3.3}$$

In fact, this is not a true two-family case, since the four V_{kl} do not form a unitary matrix; in particular, it is not CP-invariant. The phases in the four remaining CKM couplings cannot all be removed simultaneously by a redefinition of the four quark fields, as quantified by Δ in eq. (2.3).

Interested in d_n we need to consider only the terms in $\mathcal{L}_2$ that are diagonal in all four quark flavors. Moreover, we can omit explicitly CP-invariant terms of the form of a product of an operator with its conjugated. Only two operators remain after this selection, those proportional to the CKM product $V_{cs}^* V_{us} V_{cd} V_{ud}^*$, and also their Hermitian conjugated. These non-local 8-quark operators include both q and $\bar{q}$ fields for each of the four quark flavors: the CP-odd invariant Δ (as well as CP-violation altogether) vanishes wherever any single CKM matrix element becomes zero. The two terms in $\mathcal{L}_2$ differ by the type, up- or down-, of the quark-antiquark pairs coming off the same weak vertex, see figure 2. The CP-odd amplitudes conventionally considered for d_n are of type a) where one of the weak vertices has $\bar{c}c$ and another $\bar{u}u$, and both have $\Delta S = -\Delta D = 1$. The type b) amplitude has the $\bar{d}d$ and $\bar{s}s$ pairs in the two weak vertices, respectively, while each has $|\Delta C| = 1$. These were routinely omitted.

Considering the nucleon amplitudes we need to eventually integrate out the charm quark field as well. At this point the distinction between the two types of terms becomes important. Where the two charm fields belong to the same four-fermion vertex in the product eq. (3.2), figure 2a, they can be contracted into the short-distance loop yielding,
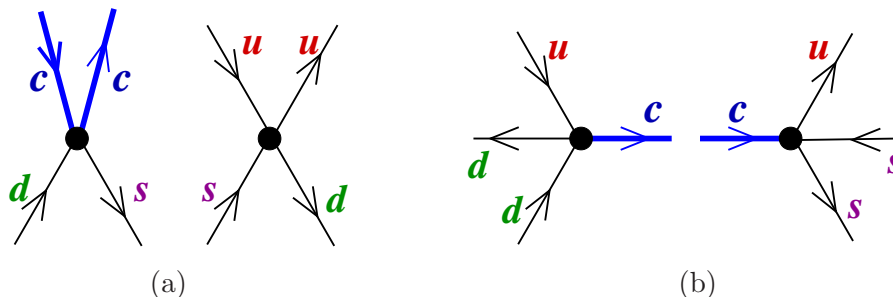
Figure 2. Two types of CP-odd terms. Weak vertices must be off-diagonal in flavor, either for down-type (a) or up-type (b) quark. Solid dots denote the four-quark vertices. Light lines correspond to u, d or s quarks, thicker lines stand for charm.

for instance, the usual perturbative Penguins. These are the conventional source of the long-distance CP-odd effects [16, 17]. The loop cannot be formed for the alternative possibility where c and $\bar{c}$ belong to different $\mathcal{L}_w$, figure 2b since the charm quark must propagate between the two vertices. This is the reason why such contributions were usually discarded.

Nevertheless, the latter term has an advantage: it does not involve short-distance loops, and has a single charm propagator, although highly virtual in the hadronic scale. Each weak vertex contains a flavorless quark-antiquark pair, but these are light down-type quarks d and s and are not contracted via a perturbative loop; they will go instead into the nucleon wavefunction. The corresponding operator is

$$\frac{G_F^2}{2}\, V_{cs} V_{cd}^* V_{ud} V_{us}^* \int \mathrm{d}^4 x \; iT\{(\bar{d}\Gamma_\mu c)(\bar{u}\Gamma_\mu d)_0 \cdot (\bar{c}\Gamma_\nu s)(\bar{s}\Gamma_\nu u)_x \;+\; \mathrm{H.c.} \tag{3.4}$$

The Hermitian conjugate, apart from complex conjugation of the CKM product, is simply the exchange between s and d, $s \leftrightarrow d$ (this particular property does not hold beyond the SM). For the sake of transparency, we have passed here to the sum and the difference of the two operators in $\mathcal{L}_w$, in terms of eq. (2.1).

As the space separation x in eq. (3.4) is of order $1/m_c$, eliminating charm results in a local OPE; the expansion parameter μ_{hadr}/m_c is not too small and we need to keep a few first terms. The tree-level OPE is particularly simple here and amounts to the series

$$c(0)\bar{c}(x) = \left(\frac{1}{m_c - i\slashed{D}}\right)_{0x} = \frac{1}{m_c}\delta^4(x) + \frac{1}{m_c^2}\delta^4(x)\, i\slashed{D} + \frac{1}{m_c^3}\delta^4(x)\,(i\slashed{D})^2 + \dots \tag{3.5}$$

valid under the T-product. For purely left-handed weak currents in the SM the odd powers of $1/m_c$ in eq. (3.5) are projected out, including the leading $1/m_c$ piece. We then retain only the $1/m_c^2$ term and arrive at the local effective CP-odd Lagrangian[2]

$$\tilde{\mathcal{L}}_- = -i\Delta \frac{G_F^2}{2m_c^2}\, \tilde{O}_{uds}, \tag{3.6}$$

$$\tilde{O}_{uds} = (\bar{u}\Gamma^\mu d)\,(\bar{d}\Gamma_\mu i\slashed{D}\Gamma_\nu s)\,(\bar{s}\Gamma^\nu u) - \{d \leftrightarrow s\}$$

$$= (\bar{u}\Gamma^\mu d)\cdot\left[(\bar{d}\Gamma_\mu i\slashed{D}\Gamma_\nu s)\cdot(\bar{s}\Gamma^\nu u) + (\bar{d}\Gamma_\mu \gamma_\alpha \Gamma_\nu s)\, i\partial^\alpha(\bar{s}\Gamma^\nu u)\right] - \{d \leftrightarrow s\};$$

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[2]We have changed notations compared to ref. [10]: now $\tilde{O}_{uds}$, O_{uds} and O_{uds}^α all include subtraction of the Hermitian conjugated operator.

in the last expression the covariant derivative acts only on the s-quark field immediately following it.

To address the electric dipole moments we need to incorporate the electromagnetic interaction. One photon source lies in the covariant derivative in the operator $\tilde{O}_{uds}$, which includes electromagnetic potential along with the gluon gauge field. It is proportional to the up-type quark electric charge $+\frac{2}{3}$. The corresponding photon vertex is local and is given by the Lorentz-vector six-quark operator which we denote as O^{α}_{uds}:

$$O^{\alpha}_{uds} = (\bar{u}\gamma^{\mu}(1-\gamma_5)s)\,(\bar{s}\gamma_{\mu}\gamma^{\alpha}\gamma_{\nu}(1-\gamma_5)d)\,(\bar{d}\gamma^{\nu}(1-\gamma_5)u) - \{d \leftrightarrow s\}. \qquad (3.7)$$

Another, non-local contribution is the T-product of the pure QCD part of $\tilde{O}_{uds}$

$$O_{uds} = (\bar{u}\gamma^{\mu}(1-\gamma_5)s)\,(\bar{s}\gamma_{\mu}i\slashed{D}\gamma_{\nu}(1-\gamma_5)d)\,(\bar{d}\gamma^{\nu}(1-\gamma_5)u) - \{d \leftrightarrow s\} \qquad (3.8)$$

with the light-quark electromagnetic current.[3] The total photon vertex is thus given by the effective CP-odd Lagrangian

$$A_{\alpha}\mathcal{L}^{\alpha}_{-} = -e\,i\Delta\frac{G_F^2}{m_c^2}A_{\alpha}\left[\frac{2}{3}O^{\alpha}_{uds} + \int \mathrm{d}^4x\,iT\{O_{uds}(0)\,J^{\alpha}_{\mathrm{em}}(x)\}\right], \quad J^{\mu}_{\mathrm{em}} = \sum_q \frac{e_q}{e}\,\bar{q}\gamma^{\mu}q, \quad (3.9)$$

where A_{μ} is the electromagnetic potential and e is the unit charge.

In principle, the local and non-local pieces above correspond to distinct physics: one has photon emitted from distances of order $1/m_c$ while the latter senses long-distance charge distribution. The latter usually dominates, however the specifics of the left-handed weak interactions in the SM make them of the same $1/m_c^2$ order.

An important feature of the considered contribution is that it remains finite in the chiral limit and it does not vanish if d and s quarks become nearly degenerate, at first glance contradicting the origin of the KM mechanism where an additional SU(2) freedom to mix s and d makes the theory CP-invariant at $m_s = m_d$.[4] This in fact is fully consistent, since the external state, the neutron, is explicitly $s \leftrightarrow d$ non-symmetric, and would not stay invariant under the mixing transformation. The same applies, for instance, to the d-quark EDM. In contrast, short-distance effects of light quarks in the loops involve severe GIM-type suppression proportional to the powers of the light quark masses. The quark electric dipole moments as the purely short-distance contributions are explicitly proportional to the corresponding quark mass for chirality reasons.

Let us parenthetically note that there is no formal contradiction between the non-vanishing expression for d_n and the explicit T-invariance at literally $m_s = m_d$ either. T-invariance prohibits dipole moments only for the eigenstates of the Hamiltonian; for instance, non-diagonal dipole moment matrix elements are perfectly allowed. The above considered neutron, a baryon state with strangeness $S = 0$ is such a physical eigenstate only

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[3]In general only the sum of the two terms yields the transverse electromagnetic vertex; however, when projected on the dipole moment Lorentz structures they separately conserve current.

[4]The contribution would vanish if charm and top become degenerate; considering the cases of degenerate bottom and strange quarks, or charm and up makes no sense in this context since it has been assumed as the starting point that $m_b, m_c \gg \mu_{\mathrm{hadr}}$ while u, d and s are light quarks.

as long as the mass splitting $m_s - m_d$ remains much larger than the weak corrections to the hadron masses, since weak interactions violate flavor. Where $m_s - m_d$ becomes of the order of $G_F m_q m_c^2$ mixing between neutron and its strange partners must be accounted for. In this regime the time-violating EDM for a physical state should be distinguished from the conventional d_n; the former would vanish at $m_s = m_d$.

The CP-odd operators contain strange quark fields. This means that the induced effects would vanish in a valence approximation to nucleon where only d and u quarks are active. It is known, however, that even at low normalization point the strange sea in nucleon is only moderately suppressed. The large-N_c perspective on the nucleons paralleling the picture of the baryon as a quantized soliton of the pseudogoldstone meson field [18] makes this explicit: the weight of the operators with strange quarks in the chiral limit is generally determined simply by the operator-specific Clebsh-Gordan coefficients of the SU(3) group.

Such an 'intrinsic strangeness' suppression is specific for the considered mechanism to generate d_n in the SM; the conventional contribution trades it in for the 'intrinsic charm'. Associating the virtual-pair suppression with the strangeness sea in the nucleon is probably a relatively light price to pay. In contrast, the perturbative Penguin effects yield small coefficients whenever considered in the truly short-distance regime.

3.1.1 Matrix elements

The CP-odd operators O_{uds}^α and O_{uds} have high dimension; this is routinely associated with being poorly defined for practical applications. However, these particular operators possess intrinsic symmetry properties, including antisymmetry in respect to $s \leftrightarrow d$, which prohibit mixing with lower-dimension operators, and make them a suitable object for the full-fledged nonperturbative analysis.

The neutron EDM is obtained by evaluating the hadronic operators in eq. (3.9) over the neutron state. Since $\mathcal{L}_-^\mu$ is T-odd, the matrix element vanishes for zero momentum transfer and the linear in q term describes d_n:

$$\langle n(p+q)|\mathcal{L}_-^\mu|n(p)\rangle = d_n \, q_\nu \, \bar{u}_n(p+q) i\sigma^{\mu\nu}\gamma_5 u_n(p). \tag{3.10}$$

Neither of the two matrix elements involved are easy to evaluate, although one may hope that such a contribution may eventually be determined without major ambiguity, including the definitive prediction for the overall sign. Although only the P-violating part of $O_{uds}^{(\mu)}$ contributes, the original form is more compact and makes symmetry explicit.

The contact operator O_{uds}^μ is a product of three left-handed flavor currents; O_{uds} instead of the $\bar{s}d$ current has a flavor non-diagonal left-handed partner of the quark energy-momentum tensor in the chiral limit. Therefore it seems plausible that the required matrix elements can be directly calculated within the frameworks like the Skyrme model [18, 19], or in its dynamic QCD counterpart [20] derived in the large-N_c limit from the instanton liquid approximation.

Lacking presently more substantiated calculations we resort to the simple dimensional estimates. For the local piece we put

$$\langle n(p+q)|O_{uds}^\mu|n(p)\rangle = 2i\mathcal{K}_{uds} \, q_\nu \bar{u}(p+q) i\sigma^{\mu\nu}\gamma_5 u(p). \tag{3.11}$$

The reduced matrix element $\mathcal{K}_{uds}$ has dimension of mass to the fifth power. We estimate it as

$$|\mathcal{K}_{uds}| \approx \kappa \, \mu_{\mathrm{hadr}}^5, \qquad (3.12)$$

where μ_{hadr} is a typical hadronic momentum scale and κ stands for the 'strangeness suppression' to account for the fact that neutron has no valence strange quarks; $\kappa \approx 1/3$ is taken as a typical guess.

Due to the high dimension of the operators the estimate for d_n depends dramatically on the value used for μ_{hadr}. Although the typical momentum of quarks in nucleon is around $600\,\mathrm{MeV}$ or higher, using this as μ_{hadr} would strongly overestimate the effect. Six powers of mass in eq. (3.12) come from the product of two local light quark currents each intrinsically containing factors $N_c/8\pi^2$ when converted into the conventional momentum representation. This is illustrated by the magnitude of the vacuum quark condensate where such a factor effectively reduces μ_{hadr}^3 down to $\sim (250\,\mathrm{MeV})^3$.

To account for such differences we assign a factor of $(0.25\,\mathrm{GeV})^3 \equiv \mu_\psi^3$ to each additional quark current in the product, while the remaining dimension will be made of the powers of μ_{hadr} taken around $500\,\mathrm{MeV}$. Then this contribution to d_n becomes

$$|d_n| = \frac{32}{3} e\,\Delta\,\frac{G_F^2}{m_c^2}|\mathcal{K}_{uds}| \approx 3.3 \cdot 10^{-31} e\cdot\mathrm{cm} \times \kappa \left(\frac{\mu_\psi}{0.25\,\mathrm{GeV}}\right)^6 \left(\frac{0.5\,\mathrm{GeV}}{\mu_{\mathrm{hadr}}}\right), \qquad (3.13)$$

where $\Delta \simeq 3.4 \cdot 10^{-5}$ has been used. An independent enhancement factor may come from summation over the Lorentz indices in the currents, but we neglect it.

The most naive estimates for the d_n induced by the non-local piece in eq. (3.9) would yield a similar dimensional scaling except that no explicit factor $e_c = \frac{2}{3}$ appears: the dimension of the non-local T-product is the same as of O_{uds}^μ itself. Following the more careful way advocated above where we distinguish the mass scale associated with the local product of the quark fields, the result is literally different:

$$|d_n|^{\mathrm{n-loc}} \approx e\,\Delta\,\frac{G_F^2}{m_c^2}32\kappa\,\mu_\psi^9\,\mu_{\mathrm{hadr}}^{-4} \approx 1.2 \cdot 10^{-31} e\cdot\mathrm{cm} \times \kappa \left(\frac{\mu_\psi}{0.25\,\mathrm{GeV}}\right)^9 \left(\frac{0.5\,\mathrm{GeV}}{\mu_{\mathrm{hadr}}}\right)^4; \quad (3.14)$$

numerically the difference is not radical, however.

Alternatively, the non-local contributions can be analyzed focusing on the contributions of the individual intermediate states, usually the lowest in mass. Among them the hidden-strangeness states, including $\bar{K}\Lambda(\Sigma)$ look promising suggesting a way to dynamically estimate the 'intrinsic strangeness' factor κ. In the standard model, however, the corresponding loops are not infrared-enhanced and rather saturate at large virtual mass yielding a result strongly dependent on the assumed cutoff. For the same reason the kaon and the lowest baryon as the intermediate state are not any more remarkable a priori than ordinary resonances.

Ref. [10] considered the contribution of the lowest resonant state, the $\frac{1}{2}^-$ nucleon resonance $N(1535)$ referred to below as $\tilde{N}$, as an alternative estimate of the non-local piece in d_n. In terms of the two hadronic vertices,

$$\langle n(p')|J_{\mathrm{em}}^\mu(0)|\tilde{N}(p)\rangle = -\rho_{\tilde{N}}\bar{u}_n i\sigma^{\mu\nu}\gamma_5 q_\nu u_{\tilde{N}}, \quad \langle \tilde{N}(p')|O_{uds}(0)|n(p)\rangle = 16i\mathcal{N}_{uds}\bar{u}_{\tilde{N}}u_n \quad (3.15)$$
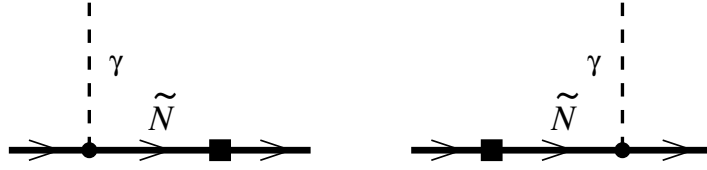
Figure 3. Nonlocal contribution to d_n with the intermediate $\tilde{N}$. Solid block denotes the CP-odd operator O_{uds}.

the sum of the Feynman diagrams in figure 3 gives

$$d_n^{(\tilde{N})} = -e\,\Delta\frac{32G_F^2}{m_c^2}\left(\frac{\rho_{\tilde{N}}\,\mathcal{N}_{uds}}{M_{\tilde{N}} - M_N}\right). \tag{3.16}$$

The electromagnetic vertex estimated from the measured transition $\tilde{N} \to n + \gamma$ becomes $\rho_{\tilde{N}} \approx (0.34 \pm 0.08)\,\text{GeV}^{-1}$. The induced weak CP-odd vertex is estimated in the dimensional way, for the dimension-ten operator O_{uds} yielding

$$|\mathcal{N}_{uds}| \approx \kappa\,\mu_\psi^6\,\mu_{\text{hadr}}. \tag{3.17}$$

Finally this estimate reads

$$|d_n|^{(\tilde{N})} \approx e\,\Delta\frac{32G_F^2}{m_c^2}\kappa\,\mu_\psi^6\,\mu_{\text{hadr}}\,\frac{\rho_{\tilde{N}}}{M_{\tilde{N}} - M_n} \approx 1.4\cdot 10^{-31}\text{e·cm} \times \kappa\left(\frac{\mu_\psi}{0.25\,\text{GeV}}\right)^6\left(\frac{\mu_{\text{hadr}}}{0.5\,\text{GeV}}\right). \tag{3.18}$$

This value is consistent with the direct dimensional estimate of the non-local contribution, in particular considering the fact that the lowest excited state alone may not necessary saturate it. Therefore, in further applications we generally follow the more straightforward estimates paralleling eq. (3.14).

Finally, our estimate for d_n in the SM centers around 10^{-31} e·cm although even the values 5 to 10 times larger may not be excluded.

The natural benchmark for the CKM d_n in the SM evidently lies about

$$d_n \propto \Delta\,G_F^2\mu_{\text{hadr}}^3. \tag{3.19}$$

In the same terms the loop-less contribution considered above is

$$d_n \propto \Delta\,G_F^2\mu_{\text{hadr}}^3\cdot\frac{\mu_{\text{hadr}}^2}{m_c^2}\cdot\kappa. \tag{3.20}$$

The last factor reflects the absence of valence strange quarks in the nucleon. The related suppression is unavoidable for d_n in one form or another; it is natural to think that paying the price by the soft strangeness content in the nucleon state is the minimal burden. Therefore, from this perspective such a contribution appears to bear a mild model-specific suppression, since m_c in practice only moderately exceeds the characteristic hadronic scale μ_{hadr}.

At the same time d_n does not contain parametric chiral enhancement, $\ln \mu_{\text{hadr}}^2/m_\pi^2$ or numerically significant scalar matrix elements possible in the case of generic couplings. This implies also a loss of a potential factor of a few; it may be recovered by the interactions generated beyond the SM.

Throughout the long history of the conventional long-distance contributions to d_n in the SM it has usually been considered [11] that the principal effect comes from the diagram in figure 1 peculiar by showing a chiral singularity. It has been calculated in ref. [17],

$$d_n \approx eG_F^2 \Delta \frac{C_{\text{pert}}\alpha_s}{27\sqrt{2}\pi^3} \ln \frac{m_t^2}{m_c^2} \frac{2|\langle\bar\psi\psi\rangle|\, m_\pi^2}{f_\pi m_s} \tilde{A}(2\alpha-1)g_A \ln \frac{m_K}{m_\pi} \tag{3.21}$$

($\tilde{A}$ is a strong constant parameterizing the conventional CP-even vertex and α a dimensionless ratio of two SU(3) meson-to-baryon axial couplings, while C_{pert} stands for additional perturbative factors). The original authors' estimate was close to $d_n \approx 2 \cdot 10^{-32}$e·cm. It was done in 1981 when even the size of the CKM admixture of the third generation was unknown and was thought to be of the scale of θ_C. The equivalent of the CP-violating parameter Δ likewise was estimated assuming $m_t \approx 30$ GeV, yet a value only 1.5 times larger than known today, see eq. (2.3), was used. At the same time the used log ratio of the t and c quark masses was somewhat smaller. The size of this contribution is now usually cited as $d_n \approx 10^{-32}$ e·cm [11].

The d_n value eq. (3.21) is proportional to α_s/π from the short-distance Penguin loop. It also contains a factor $m_\pi^2 \propto m_q$ compared to the benchmark eq. (3.19), however the overall light-quark mass scaling is remarkable: $m_{u,d}$ enter divided by m_s rather than by μ_{hadr}. Therefore, in the SU(3) chiral limit where all $m_{u,d,s} \to 0$, m_q/m_s fixed it would stay finite.

In practice, however the SU(2) chiral limit $m_{u,d} \to 0$, m_s fixed is more relevant in numeric estimates. In this case eq. (3.21) has an additional factor of the light-quark mass m_q compared to the benchmark value eq. (3.19). This is in agreement with the general fact stated in appendix A: the contribution contains a chiral log and therefore must include an explicit factor of m_q since the SM weak amplitudes do not contain right-handed light quark fields. As emphasized there, it is sufficient to check this for the bare weak vertices.

This illustrates the underlying problem in estimating d_n in the SM: the physically distinct chirally singular contributions have to be m_q-suppressed. The leading-m_q contributions are not related to soft pions and are rather saturated at the loop momenta of the typical hadronic mass scale μ_{hadr}, or by resonances with a significant mass gap. Such effects are generally uncertain and may involve cancellations.

The conventional SM contribution eq. (3.21), therefore, has an additional light-mass suppression $\propto m_q$ on top of the perturbative short-distance factor. Although it is partially offset by numerically large factors accompanying the amplitudes with right-handed light quarks, together with the perturbative loop factor it results in a certain suppression. This may explain the larger number for the loopless EDM which we estimate to be around 10^{-31}e·cm.

3.2 Neutron EDM and a BSM charm CP violation

In order to estimate the effect of the $|\Delta C|\!=\!1$ amplitudes on the neutron EDM we should replace one of the two $\mathcal{L}_w$ in the product $\mathcal{L}_2$ by the New Physics operators. We will assume that we get a reasonable estimate when considering one operator at a time; this basically corresponds to the assumption that in the neutron EDM we do not have a destructive interference absent from the charm decays.

As is clear from the analysis of the SM case, the operator structure obtained upon integrating out charm depends on its chirality in the NP amplitude: in the left-handed case it follows the SM case. Where the charm field is right-handed, only the odd-power terms $1/m_c$, $1/m_c^3\ldots$ survive. In this case the leading term suffers less from the μ_{hadr}/m_c suppression, however it does not include the leading contact photon vertex (the photon operator O_1 is an exception in this respect) which appeared to yield a few times larger contribution, at least within our estimates. The contact photon vertex is then delayed till order $1/m_c^3$. Such a peculiarity introduces certain difference, but in view of the relatively mild numeric power suppression the presence of the nonlocal T-product term to the leading $1/m_c$ order for the right-handed charm does not appear to bring in a notable numeric difference.

The chiral content of the light valence quarks generally makes a bigger difference. The nucleon matrix elements with both left-handed and right-handed fields are usually numerically enhanced as seen on the example of the nucleon σ-term. Moreover, a CP-odd scalar pion-to-nucleon coupling may be induced. Although the CP-nonconservation case is more involved, it can be stated that this vertex at small momentum transfer would be proportional to the light quark masses and therefore negligible in practice unless the New Physics operators include right-handed light quarks, see appendix A.

The contact photon vertex contribution to d_n does not depend on the induced pion-nucleon interaction. The scalar pion vertex $\mathcal{G}_s\,\bar{p}n\pi^-$, on the other hand, generates a chirally enhanced long-distance contribution to the T-product piece with the π^-p intermediate state, described by the diagrams in figure 4:

$$-e\frac{\mathcal{G}_s g_{\pi NN}}{16\pi^2 M_N}\ln\frac{\Lambda^2}{m_\pi^2}\,\bar{u}_n i\sigma_{\mu\nu}q^\nu\gamma_5 u_n = -e\frac{\mathcal{G}_s g_A}{8\pi^2 f_\pi}\ln\frac{\Lambda^2}{m_\pi^2}\,\bar{u}_n\sigma_{\mu\nu}q^\nu i\gamma_5 u_n \qquad (3.22)$$

with Λ the ultraviolet cutoff. In the actual world the chiral $\ln\frac{\mu_{\mathrm{hadr}}^2}{m_\pi^2}$ constitutes a moderate factor about 3 and therefore is remarkable more in the conceptual aspect. However it comes proportional to the large couplings and this makes up for the loop factor. In the few considered examples the overall chiral enhancement roughly offsets the typical suppression (at the same order in $1/m_c$) of the T-product piece relative to the contact photon vertex contribution. Regardless of the details, it is clear that the chiral log per se is a too weak singularity to change dramatically the expected magnitude of d_n.

The effect of the scalar pion-to-nucleon vertex can be more pronounced in atomic EDMs. The pion-mediated nuclear forces are relatively long-range and may be additionally amplified for the isoscalar coupling in heavy atoms.

The current algebra technique allows to unambiguously determine the induced scalar vertex in the chiral limit, see appendix A. Of course, in the general case it would require
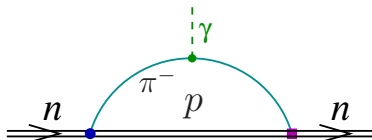
Figure 4. The chirally singular diagram generated by the scalar vertex. One of the vertices is the usual strong CP-conserving pseudoscalar coupling while another is the induced CP-odd scalar vertex.

the nucleon expectation values of similar high-dimension effective operators, currently estimated in a rather crude way. An additional uncertainty would come from possible cancellations due to the second, pole-subtraction term in eq. (A.1). As a rule, aiming only at the overall magnitude, we simply neglect this term.

Contracting charm propagator we end up with the multi-quark CP-odd operators. The neutron EDM induced by them is evaluated applying the dimensional estimates elaborated for the SM case; they are described in section 3.1. The non-valence strange quarks are neglected here and no factor κ appears.

Considering the quark bilinears, we start with the more natural gluonic O_2. In view of the above mentioned difference in the OPE, we consider separately the cases of O_2^R and O_2^L containing right- and left-handed c-field, respectively, rather than its scalar and pseudoscalar versions.

We start with $O_2^R = m_c \bar{c} g_s i\sigma G(1-\gamma_5)u$. Here charm induces already the $1/m_c$ effect generating d_n via the T-product with the electromagnetic current. The contact term emerges only to order $1/m_c^3$, and we discard it. The corresponding CP-odd operator is

$$O^{(-)} = -i\,\mathrm{Im}\,c_2\,\frac{G_F^2\sin^2\theta_c\cos^2\theta_c}{m_c^0}\left[\bar{u}\,g_s i\sigma_{\alpha\beta}G^{\alpha\beta}\gamma_\mu(1-\gamma_5)d\,\bar{d}\gamma^\mu(1-\gamma_5)u - d\leftrightarrow s\right] + \mathrm{H.c.}$$
(3.23)

For the scale estimate we simply factor out the operator $g_s i\sigma_{\alpha\beta}G^{\alpha\beta}$ and assume it has the value similar to the one in heavy mesons or baryons, $2\mu_G^2 \approx 0.7\,\mathrm{GeV}^2$. A close value is obtained if we use the vacuum condensate $\langle\bar{q}g_s i\sigma Gq\rangle \approx 0.8\,\mathrm{GeV}^2\langle\bar{q}q\rangle$. Discarding strange quarks and applying to the rest our dimensional estimate we get

$$|d_n| \approx \mathrm{Im}\,c_2\,G_F^2\sin^2\theta_c\cos^2\theta_c\,32\mu_G^2\,\frac{(0.25\,\mathrm{GeV})^6}{(\mu_{\mathrm{hadr}})^5}\,\chi_{\mathrm{fl}} \approx 1.1\cdot10^{-26}e\cdot\mathrm{cm}\cdot\mathrm{Im}\,c_2\,\chi_{\mathrm{fl}}\,,\quad(3.24)$$

where $\chi_{\mathrm{fl}} \approx 1\,\mathrm{to}\,2$ is a flavor factor which accounts for the fact that there are two d quarks in the neutron. Using eqs. (2.26) for $\mathrm{Im}\,c_2$ we get

$$|d_n| \approx 10^{-30}\frac{\chi_{\mathrm{fl}}}{|\sin\delta_{\pi^+\pi^-}|}e\cdot\mathrm{cm} \approx 2.3\chi_{\mathrm{fl}}\cdot10^{-30}e\cdot\mathrm{cm}\,.$$
(3.25)

Here and in what follows we assume $|\sin\delta_{\mathrm{FSI}}| \approx 0.5$ as a typical value. Thus we expect an enhancement of roughly a factor of thirty.

Now we turn to $O_2^L = m_c\bar{c}g_s i\sigma G(1+\gamma_5)u$. Here the leading term $1/m_c$ vanishes like in the SM and the $1/m_c$ expansion starts with $1/m_c^2$, yet we have the right-handed u-quark which entails chiral enhancements in the nucleon matrix elements. There are two distinct

contributions to the leading order like in the SM, the contact and non-local. For the latter we obtain

$$O^{(-)} = -i \operatorname{Im} c_2 \frac{2G_F^2 \sin^2\theta_c \cos^2\theta_c}{m_c^1} \times \tag{3.26}$$

$$\times \left[\bar{u}g_s \left(G^{\mu\nu} - i\tilde{G}^{\mu\nu} + G^{\mu\alpha}\sigma_{\alpha\nu} - \sigma^{\mu\alpha}G_{\alpha\nu} + \frac{1}{2}\delta^{\mu\nu}\sigma_{\alpha\beta}G^{\alpha\beta} \right) \times \right.$$

$$\left. \times D_\nu(1-\gamma_5)d\,\bar{d}\gamma_\mu(1-\gamma_5)u - d \leftrightarrow s \right] + \text{H.c.}$$

and the contact electromagnetic vertex is given by

$$O^{\nu(-)} = -i \operatorname{Im} c_2 \frac{2G_F^2 \sin^2\theta_c \cos^2\theta_c}{m_c^1} \times \tag{3.27}$$

$$\times \left[\bar{u}g_s \left(G^{\mu\nu} - i\tilde{G}^{\mu\nu} + G^{\mu\alpha}\sigma_{\alpha\nu} - \sigma^{\mu\alpha}G_{\alpha\nu} + \frac{1}{2}\delta^{\mu\nu}\sigma_{\alpha\beta}G^{\alpha\beta} \right) \times \right.$$

$$\left. \times (1-\gamma_5)d\,\bar{d}\gamma_\mu(1-\gamma_5)u - d \leftrightarrow s \right] + \text{H.c.}$$

It likewise may have a numeric chiral enhancement due to right-handed u-quark, yet no literal chiral log from the pion loop. Our estimate for it reads as

$$|d_n|^{\text{loc}} \approx \frac{2}{3}\operatorname{Im} c_2\, G_F^2 \sin^2\theta_c \cos^2\theta_c \frac{32\mu_G^2}{m_c} \frac{(0.25\,\text{GeV})^3}{\mu_{\text{hadr}}} \chi_{\text{fl}}\,\chi_{\text{scal}}$$

$$\approx 2.5 \cdot 10^{-26}\, e \cdot \text{cm} \cdot \operatorname{Im} c_2\, \chi_{\text{fl}}\,\chi_{\text{scal}}, \tag{3.28}$$

where we, as above, have equated the whole bracket containing the gluon field strength, including g_s, with $2\mu_G^2$; yet another factor χ_{scal} has been added to indicate a possible enhancement of the scalar expectation value (cf. the size of the nucleon σ-term). This is about 3 times larger than in eq. (3.24).

The non-local contribution estimated dimensionally is typically 2.5 to 3 times smaller than the contact one. However here the right-handed u-quark induces the nonvanishing scalar pion-nucleon vertex and the $\pi^- p$ intermediate state yields a chiral log, cf. eq. (3.22). Combined with the current algebra result for the scalar version as described above this enhancement numerically turns out about 3.5, i.e. we get a number close to the contact estimate eq. (3.28).

Thus, we can use for this case the local estimate eq. (3.28) and $\operatorname{Im} c_2$ from eq. (2.26). Then

$$|d_n| \approx 2.5 \cdot 10^{-30} \frac{\chi_{\text{fl}}\,\chi_{\text{scal}}}{|\sin\delta_{\pi^+\pi^-}|}\, e \cdot \text{cm} \approx \chi_{\text{fl}}\,\chi_{\text{scal}} \cdot 5 \cdot 10^{-30}\, e \cdot \text{cm}\,. \tag{3.29}$$

For this chiral structure we get about an 80-fold enhancement compared to the SM.

The photonic operators O_1 are the simplest case since only the contact photon vertex should be considered to the leading order in α. For the case of the right-handed c quark, $O_1^R = em_c \bar{c}i\sigma F(1-\gamma_5)u$ the leading term in the $1/m_c$ expansion yields

$$O^{\nu(-)} = 2i \operatorname{Im} c_1\, G_F^2 \sin^2\theta_c \cos^2\theta_c\, \partial_\mu \left[\bar{u}i\sigma^{\mu\nu}\gamma_\alpha(1-\gamma_5)d\,\bar{d}\gamma^\alpha(1-\gamma_5)u - d \leftrightarrow s \right] + \text{H.c.} \tag{3.30}$$

The matrix elements of the CP-even partner of such an operator may have been estimated in the literature. Applying our standard recipe we get

$$
\begin{aligned}
|d_n| &\approx 16|\mathrm{Im}\, c_1|\, G_F^2 \sin^2\theta_c \cos^2\theta_c\, (0.25\,\mathrm{GeV})^3\, \chi_{\mathrm{fl}} \\
&\approx 3.2\cdot 10^{-26}\, e\!\cdot\!\mathrm{cm}\cdot\mathrm{Im}\, c_1\cdot\chi_{\mathrm{fl}} \\
&\approx 3.4\cdot 10^{-27} e\!\cdot\!\mathrm{cm}\cdot\chi_{\mathrm{fl}}\,.
\end{aligned}
\tag{3.31}
$$

Owing to its nature this operator yields the largest enhancement of all the new physics operators. Nevertheless it is still safe in respect to experimental bounds.

The operator O_1 with the opposite chiralities, $O_1^L = e m_c \bar{c}\sigma F(1+\gamma_5)u$ has a mild suppression by a factor μ_{hadr}/m_c, however it can be enhanced by larger matrix elements appearing with the right-handed u quark. Therefore we expect to have here the same numeric estimate as for O_1^R, within a factor of 0.5 to 2.

Finally we consider the four-quark operator O_4. This is the case of both the leading-order $1/m_c$ contribution and of the chiral enhancement from the light valence quarks in the nucleon. Neglecting the strange quarks we have

$$
O^{(-)} = i\,\mathrm{Im}\, c_4\, \frac{G_F^2 \sin^2\theta_c \cos^2\theta_c}{m_c}\, \left[\bar{d}\gamma^\mu(1-\gamma_5)d\,\bar{u}\gamma_\mu\gamma_\nu(1-\gamma_5)d\,\bar{d}\gamma^\nu(1-\gamma_5)u\right] + \mathrm{h.c.}
\tag{3.32}
$$

The contact photon interaction would come suppressed by two powers of $1/m_c$. On the other hand, the above leading-m_c interaction enjoys a chiral pion loop enhancement in the T-product with J_{em}. Taking the axial charge commutator and neglecting the pole subtraction term in eq. (A.1) the scalar pion vertex becomes

$$
\mathcal{G}_s \bar{u}_p u_n = -i\,\mathrm{Im}\, c_4\, \frac{G_F^2 \sin^2\theta_c \cos^2\theta_c}{m_c f_\pi}\, 2\langle p|\bar{u}\gamma^\mu(1-\gamma_5)d\,\bar{d}\gamma_\mu\gamma_\nu(1+\gamma_5)d\,\bar{d}\gamma^\nu(1-\gamma_5)d|n\rangle_{q=0}\,.
\tag{3.33}
$$

According to our dimensional rules this amounts to

$$
|\mathcal{G}_s| = |\mathrm{Im}\, c_4|\, 8\frac{G_F^2 \sin^2\theta_c \cos^2\theta_c}{m_c f_\pi}\, (0.25\,\mathrm{GeV})^6\, \chi_{\mathrm{scal}}\, \chi_{\mathrm{fl}}^2
\tag{3.34}
$$

and results in

$$
\begin{aligned}
|d_n| &\approx |\mathrm{Im}\, c_4|\, \frac{G_F^2 \sin^2\theta_c \cos^2\theta_c}{\pi^2 f_\pi^2 m_c}\, g_A \ln\frac{\mu_{\mathrm{hadr}}^2}{m_\pi^2}\, (0.25\,\mathrm{GeV})^6\, \chi_{\mathrm{scal}}\, \chi_{\mathrm{fl}}^2 \\
&\approx 6\cdot 10^{-28} e\!\cdot\!\mathrm{cm}\cdot\mathrm{Im}\, c_4\, \chi_{\mathrm{scal}}\, \chi_{\mathrm{fl}}^2\,.
\end{aligned}
\tag{3.35}
$$

Using the estimate eq. (2.26) we end up with

$$
|d_n| \approx 5.7\cdot 10^{-30}\, e\!\cdot\!\mathrm{cm}\, \chi_{\mathrm{scal}}\, \chi_{\mathrm{fl}}^2\,.
\tag{3.36}
$$

Should we apply the simple-minded dimensional estimate to the T-product contribution without considering specifically the pion loop or paying attention to the potential chiral enhancement, we would get a somewhat smaller but a consistent value

$$
|d_n| \approx 4\cdot 10^{-28}\, e\!\cdot\!\mathrm{cm}\cdot|\mathrm{Im}\, c_4|\, \chi_{\mathrm{fl}}^2 \approx 4\cdot 10^{-30}\, e\!\cdot\!\mathrm{cm}\, \chi_{\mathrm{fl}}^2\,.
\tag{3.37}
$$

	$i\langle\pi^+\pi^-|O_k|D^0\rangle$	$|\sin\delta_{\text{FSI}}\,\text{Im}\,c_k|$	$|d_n|,\ e\cdot\text{cm}$
O_1	$8\sqrt{2}\pi\alpha\,q_d\,f_\pi f_+^{D\to\pi}(0)M_D^2$	$5.2\cdot10^{-2}$	$4\cdot10^{-27}$
O_2	$4\pi g_s\sqrt{3}\,f_\pi f_+^{D\to\pi}(0)M_D^2$	$1.0\cdot10^{-4}$	$8\cdot10^{-30}$ │ $3\cdot10^{-30}$
O_3	$f_\pi f_+^{D\to\pi}(0)M_D^2$	$2\ \cdot10^{-3}$	10^{-30}
O_4	$f_\pi f_+^{D\to\pi}(0)M_D^2\,\frac{1}{N_c}\frac{2m_\pi^2}{(m_u+m_d)m_c}$	$4.6\cdot10^{-3}$	10^{-29}

Table 1. The estimated D^0 decay amplitudes, the strength of the CP-odd couplings and expected d_n. The two sub-columns for the chromomagnetic operator O_2 correspond to the left-handed (left) and right-handed (right) charm fields, respectively.

Therefore, in the case of O_4 the induced d_n is about 100 times the SM. The origin is evident: O_4 has a color structure unfavorable to $D\to\pi^+\pi^-$. At the same time, the chirality choice is optimal for both charm and light quarks, in the nucleon matrix elements. The combination of the two yields an additional factor of 10 enhancement in our estimates.

For convenience, table 1 summarizes our estimates of d_n in this section along with the values of $\text{Im}\,c_k$ from section 2.2.

3.3 A comment on the atomic EDMs

The atomic size exceeds the nucleon radius by several orders of magnitude, and as a matter of principle they may have larger EDMs; in particular, this applies to paramagnetic atoms. The enhanced EDM, however may originate there mainly through T-violation in the lepton sector, with the electron EDM itself or via the induced contact interaction with the nucleons. Such manifestations of New Physics are not directly associated with the milliweak interaction of quarks and are beyond the subject of the present study.

In diamagnetic atoms like mercury the screening mandated by the Schiff theorem is rather effective and the overall EDM appears to be dominated by the induced isoscalar CP-odd π^0NN coupling affecting non-pointlike electromagnetic potential of the nucleus — yet still at a rather suppressed level,

$$d_{\text{Hg}}\approx\mathcal{G}_s\cdot3.5\cdot10^{-18}\,e\cdot\text{cm},\tag{3.38}$$

see refs. [11, 21]. Using, for instance the estimate eq. (3.34) we can expect for the isoscalar coupling $|\mathcal{G}_s|\approx10^{-15}$. Therefore, as anticipated the diamagnetic atom EDMs, while probably not yet fully competitive in sensitivity with the direct d_n, may become comparable in certain NP scenarios yielding amplitudes with a right-handed light quark, owing to the recent radical improvement [22] in the precision for the ^{199}Hg EDM.

4 Conclusions

The KM mechanism of CP violation in the Standard Model is an instructive example of a realistic phenomenological theory where the dominant contribution to the electric dipole moment of neutron comes not from the effective CP-odd operators of lowest dimension, but

via a nontrivial interplay of different amplitudes at a relatively low energy scale. In the SM this evidently roots in an intricate nature of the CP violation intimately related to flavor dynamics requiring existence of a few generations. It may be interesting to investigate, in general terms, if a similar pattern can naturally fit theories beyond the SM which would describe flavor dynamics at a more fundamental level.

We have argued that in the SM itself with vanishing θ-term the neutron electric dipole moment has natural size about 10^{-31} e·cm and may even exceed this, due to the interference, at the momentum scale around $1\,\text{GeV}$ of the two $\Delta C = 1$, $\Delta S = 0$ weak four-quark amplitudes. This mechanism does not require short-distance loop effects, is finite in the chiral limit and does not depend on the strange- vs. down-quark mass splitting.

The CP-odd direct-type D^0 decay asymmetry reported recently at the level of 10^{-2} does not naturally conform the expectations in the SM, which are typically an order of magnitude smaller. This may be an indication for new CP-odd forces beyond the SM, although such an interpretation should still be viewed cautiously.

If New Physics indeed induce a milliweak CP-odd decay amplitude in charmed particles, it may also be expected to generate, at the NP scale, flavor-diagonal CP-odd interactions in the light hadron sector. The EDMs of nucleons and atoms are extremely sensitive to them, and the existing experimental bounds place strong constraints on the effective interactions seen at low energies. Such low-energy effective operators are model-dependent and their connection to the charm CP violation is indirect, to say the least.

Nevertheless, a certain, possibly subdominant contribution to d_n is generated at the charm energy scale in a direct analogy with the Standard Model. It is fully independent of the effects originating from the NP scale and directly reflects the scale of CP violation in charmed particles. Our analysis suggests that this would increase d_n compared to the SM prediction by more than an order of magnitude: the typical enhancement is between 30 and 100, depending on the chiral, color and flavor composition of the charm NP amplitudes. In an ad hoc case of the CP violation through the electromagnetic $c \to u$ dipole operator alone the neutron EDM can be even as larger as $5 \cdot 10^{-27}$ e·cm. However, the possibility itself for NP to generate such a CP-odd electromagnetic operator but not a similar chromomagnetic one of the commensurate strength, does not look natural.

We conclude that New Physics CP violation in charm at the reported level remains safe in respect to existing strong experimental bounds on EDMs, as long as the direct effects are considered. At the same time it would significantly reduce the gap between the bounds and the expected size of the EDMs, and would make the new generation of the EDM experiments more topical.

In the present analysis we have assumed that a new source of CP violation appears solely in $|\Delta C| = 1$ interactions. Eventually the known flavor dynamics must be embedded in a full picture of flavor together with CP violation at some high scale, where new dynamic fields are also present. Attempts to investigate the new phenomena along these lines have been reported in [7, 8], considering the observed Δa_{CP} e.g. in a supersymmetric framework. This generically induces additional CP violation compared to our scenario, which would modify the impact on the EDMs.

Acknowledgments

A   Generalized Goldberger-Treiman relation and the scalar πNN vertex

CP-odd perturbations in general induce the parity-violating scalar pion-to-nucleon vertex
which may not vanish at small pion momentum. Such amplitudes often play a special
role owing to the small (vanishing in the chiral limit) pion mass. Here we give a compact
current algebra derivation of the corresponding small-momentum limit for a general CP-odd
perturbation operator O^-. We also point out that the induced CP-odd vertex necessarily
vanishes in the chiral limit at zero pion momentum for any operator O^- which does not
involve the right-handed light quarks (their absence may be established at arbitrary chosen
normalization point).

For simplicity of the notations we consider the charge pion. Its amplitude off the
nucleon takes the following form in the limit of vanishing pion momentum:

$$A_{\pi^- NN}(0) = \frac{1}{f_\pi}\left[\langle N|\frac{1}{i}[Q_5^+(0), O^{(-)}]_0|N\rangle - \frac{\langle 0|\frac{1}{i}[Q_5^3(0), O^{(-)}]_0|0\rangle}{-\langle 0|\bar\psi\psi|0\rangle}\langle N|\bar u d(0)|N\rangle\right]. \quad (A.1)$$

where $Q_5^+ = u^\dagger\gamma_5 d$, $Q_5^3 = q^\dagger\frac{\tau^3}{2}\gamma_5 q$ are the axial charge densities and the equal-time commu-
tators marked with the null subscript are calculated according to the standard rules.

To prove it, we first establish a counterpart of the Goldberger-Treiman relation for the
general case where parity can be violated. To this end we consider the exact nucleon matrix
element of the non-singlet light-flavor axial current J_μ (let it be $J_{\mu 5}^+$, for concreteness)

$$\langle N|J_\mu^5|N\rangle = g_A(q^2)\bar\Psi_N\gamma_\mu\gamma_5\Psi_N + b(q^2)\bar\Psi_N\sigma_{\mu\nu}q^\nu\gamma_5\Psi_N + C(q^2)q_\mu\bar\Psi_N\gamma_5\Psi_N$$
$$+ a(q^2)\bar\Psi_N\gamma_\mu\Psi_N + b(q^2)\bar\Psi_N\sigma_{\mu\nu}q^\nu\Psi_N + c(q^2)q_\mu\bar\Psi_N\Psi_N, \quad (A.2)$$

where the last three terms violate parity being induced by $O^{(-)}$. Consequently,

$$\langle N|\partial_\mu J_\mu^5|N\rangle = (2M_N g_A(q^2) + q^2 C(q^2))\bar\Psi_N i\gamma_5\Psi_N + ic(q^2)q^2\bar\Psi_N\Psi_N. \quad (A.3)$$

which gives two relations, for the pseudoscalar and for the scalar structures.

Noether theorem relates the divergence of the current obtained from the quark equa-
tions of motion to the variation of the Lagrangian under the chiral symmetry transforma-
tion; in the case of QCD the variation comes from the conventional light quark mass term
and from the corresponding commutator of the axial charge with $O^{(+)}$ which we denote by

$-i\mathcal{D}^+$:[5]

$$\partial_\mu J_\mu^5 = 2m_q \bar{u} i\gamma_5 d + \mathcal{D}^+, \qquad \mathcal{D}^{(+)} = \frac{1}{i}[Q_5^+(0), O^{(-)}]_0, \tag{A.4}$$

and therefore

$$\langle N|2m_q \bar{u} i\gamma_5 d + \mathcal{D}^{(+)}|N\rangle = (2M_N g_A(q^2) + q^2 C(q^2))\bar{\Psi}_N i\gamma_5 \Psi_N + ic(q^2)q^2\bar{\Psi}_N\Psi_N. \tag{A.5}$$

This equation can be taken in the limit $m_q \to 0$ and to the first order in the $O^{(-)}$ perturbation. The pseudoscalar structure becomes the Goldberger-Treiman relation $f_\pi g_{\pi NN} = 2M_N g_A$ stating the existence of the Goldstone boson through the pole in $C(q^2)$ as long as $M_N g_A(0) \neq 0$. The scalar term dictates that the pion pole residue likewise carries the scalar nucleon vertex proportional to $\langle N|\mathcal{D}^{(+)}|N\rangle$ at zero momentum transfer:

$$A_{\pi NN} = \frac{\langle N|\mathcal{D}^{(+)}(0)|N\rangle'}{f_\pi}. \tag{A.6}$$

$\mathcal{D}^{(+)}$ in eq. (A.4) is just the conventional PCAC commutator. The subtlety is important, however that the matrix element above stands for the *exact* nucleon states rather than for the unperturbed QCD ones as is usually implied when expanding in perturbation; this fact is indicated by the prime in eq. (A.6). The difference becomes important in the chiral limit where the pion mass is parametrically small, as illustrated later.

To bypass this complication we apply a Lagrange multiplier trick, namely consider, instead, the CP-odd perturbation $O_\lambda^{(-)} = O^{(-)} - \lambda(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)$ with an arbitrary λ, and keep m_q nonzero. On one hand, the operator $\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d = 1/m_q \, \partial_\mu J_{\mu 5}^{(3)}$ is the total derivative in QCD and does not change any strong amplitude; hence it can be added to the perturbation for free. On the other hand, λ can be taken such that the perturbation $O_\lambda^{(-)}$ becomes nonsingular in the chiral limit. (In other words, in this case one can safely perform a double expansion in m_q and in $O^{(-)}$.) Since the chiral singularity comes from the pion pole in the correlators, the value of λ is determined by vanishing of the residue $\langle \pi|O_\lambda^{(-)}|0\rangle$, or

$$\lambda = \lim_{m_q \to 0} \frac{\langle 0|O^{(-)}|\pi^0\rangle}{\langle 0|\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d|\pi^0\rangle} = \lim_{m_q \to 0} \frac{\langle 0|\frac{1}{i}[Q_5^3(0), O^{(-)}]_0|0\rangle}{-2\langle 0|\bar{\psi}\psi|0\rangle} \tag{A.7}$$

With this choice of $O_\lambda^{(-)}$ the exact nucleon states in eq. (A.6) enjoy a regular expansion in both m_q and in $O_\lambda^{(-)}$ free from a $1/m_q$ enhancement. Therefore, to the first order in $O_\lambda^{(-)}$ at $m_q \ll \mu_{\text{hadr}}$ we can ignore the difference between the exact and the unperturbed nucleon states in eq. (A.6). The net effect of the resummation of the pole terms then amounts to subtracting from $\mathcal{D}^{(+)}$ the divergence of the current with the mass term $\lambda(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)$ which is equal to $2\lambda\bar{u}d$. This is the relation eq. (A.1).

The presence of a subtraction term beyond the conventional PCAC commutator in the weak transition amplitudes has been appreciated in the context of electroweak calculations in the early 1980s [23, 24], being evident when using σ-models to visualize the chiral symmetry breaking. As noted in the end of this section, it is likewise intuitive in the

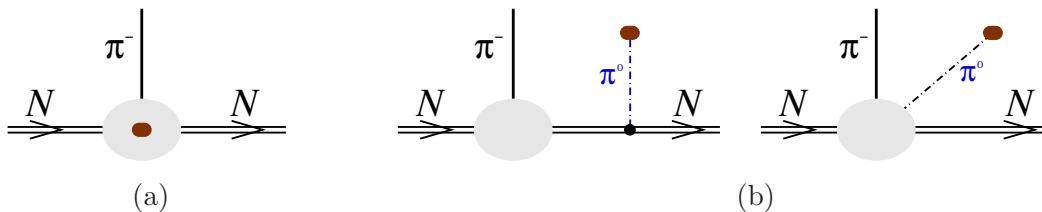---

[5]For simplicity we assume $m_u = m_d$.

Figure 5. Examples of contact **a)** and pole **b)** diagrams for the induced $\pi^- NN$ vertex. The pole diagrams may contain non-singular terms as well and these are included in the contact part of the amplitude. The light shaded blob represents the pion amplitudes off the nucleon, the solid block shows the insertion of the CP-odd operator $O^{(-)}$. The dashed-dotted line is the π^0 propagator with an infinitesimal momentum.

ordinary CP-conserving weak decays, in particular of kaons, without recourse to chiral Lagrangians. Later it was more systematically incorporated in the chiral expansion for many EDM calculations beyond the SM [25].

The above proof, while short and general, may look somewhat mysterious since a finite yet calculable part of the usual PCAC commutator term appears to be miraculously eaten up only as a result of the failure of the conventional chiral expansion. The cover of mystery is removed once the corresponding diagrams are identified and are accurately calculated. This is possible using the double expansion, in $O^{(-)}$ and then in m_q. We illustrate this in what follows.

The pion-nucleon amplitude to the first order in perturbation $O^{(-)}$ has two pieces given by the irreducible and the pole diagrams, respectively, see figure 5. The latter are those which become singular in the limit of $m_q \to 0$ or at vanishing pion momenta. We need to consider them in the kinematics where π^- has a finite momentum yet small compared to the hadronic scale μ_{hadr}, while π^0 has nearly vanishing momentum driven down by small m_q. The pole diagrams in figure 5b have an enhancement $1/m_q$ from the pion propagator at zero momentum. The Adler consistency condition guarantees that the $1/m_q$ enhancement in the pole diagrams is canceled, but it does not protect against the finite piece we are interested in.

The conventional PCAC vertex derived from the axial charge commutator, the first term in eq. (A.1), is just the above contact vertex. To determine the extra finite part we take $O^{(-)} = \bar{u} i\gamma_5 u - \bar{d} i\gamma_5 d$. Using the operator identity

$$\bar{u} i\gamma_5 u - \bar{d} i\gamma_5 d = \frac{1}{m_q} \partial_\mu J^3_{\mu 5}$$

we have

$$\langle N\pi | (\bar{u} i\gamma_5 u - \bar{d} i\gamma_5 d)(0) | N \rangle = i\frac{q_\mu}{m_q} \langle N\pi | J^3_{\mu 5}(0) | N \rangle = 0 \text{ at } q_\mu \to 0. \tag{A.8}$$

This equation is valid for *arbitrary* (even large!) nonzero m_q and arbitrary π^- momentum.

We can examine it to the first two orders in m_q. Since there is a pion propagator pole, the leading constraint is the vanishing of the $1/m_q$ piece. This is the Adler consistency condition: the pole residue proportional to the π^0 emission amplitude at zero momentum vanishes. Vanishing of the $\mathcal{O}(m_q^0)$ term means the cancellation of the two contributions,

the pole and the contact amplitudes; no further terms appear due to the Adler condition established at the previous step. The latter comes from various regular terms not containing poles or kinematic singularities and therefore can be calculated simply at $m_q = 0$ and $p_{\pi^-} \to 0$. The standard PCAC commutator applicable for soft π^- is just this piece. The former contribution is the new chirality-violating terms $\propto m_q$ which spoil the exact Adler cancellation of the amplitude in the chiral limit. Eq. (A.8) fixes it to be exactly minus the contact amplitude:

$$\langle 0|(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)(0)|\pi^0\rangle \cdot \frac{A(N\pi^- \to N\pi^0)}{m_\pi^2} = -\langle N\pi^-|(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)(0)|N\rangle. \qquad (A.9)$$

This relation is exact in the limit of $p_{\pi^0} \to 0$ and is similar in spirit to the Tomozawa-Weinberg formula [26, 27], yet is simpler and differs since only one pion is soft. It actually applies to any hadron state, not only $N\pi^-$.

Now we can go back to the case of a general $O^{(-)}$. It cannot anymore be represented as a total derivative, and the matrix element $\langle N\pi^-|O^{(-)}|N\rangle$ does not vanish. It is still given by the sum of the contact and the pole diagrams. The former would again be given, for soft π^-, by the Goldberger-Treiman commutator; it should be taken over the unperturbed nucleons, since the quark masses are kept nonzero. The latter, the chirally enhanced pole diagrams with the strong vertices corrected at order m_q depend only on m_q but not on $O^{(-)}$, i.e. they are given by QCD proper. The CP-odd operator $O^{(-)}$ enters them only at the tadpole $\langle 0|O^{(-)}(0)|\pi^0\rangle$, figures 5b. Multiplying the tadpole by the strong amplitude $A(N\pi^- \to N\pi^0)$ over m_π^2 from eq. (A.9) we get, for the CP-odd part,

$$\langle N\pi^-|O^{(-)}(0))|N\rangle - \frac{\langle 0|O^{(-)}(0)|\pi^0\rangle}{\langle 0|(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)(0)|\pi^0\rangle} \cdot \langle N\pi^-|(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)(0)|N\rangle. \qquad (A.10)$$

This representation has an advantage of still being valid at arbitrary π^- momentum, yet only to the leading order in m_q (which here means discarding m_q/μ_{hadr}). It clearly conforms to eq. (A.7).

The explicit form of the $N \to N\pi$ amplitudes at arbitrary pion momentum is not known, therefore to have a concrete expression we finally should assume $p_{\pi^-} \ll \mu_{\mathrm{hadr}}$. Using the generalized Goldberger-Treiman relation for the conventional unperturbed nucleon states (m_q is finite now) we expectedly arrive at eq. (A.1). Thus, we have traced how the chiral pole resummation generates the subtraction term exactly in the way anticipated in our original simple derivation.

The additional general observation is useful in view of the left-handed structure of the weak currents in the SM. Namely, for any operator not containing right-handed u- or d-quark fields the induced πNN coupling at zero pion momentum must vanish in the chiral limit $m_{u,d} \to 0$. In the cases where the commutator with the axial charge does not vanish explicitly this implies the vanishing of the corresponding zero-momentum matrix element. This applies to any on-shell amplitude off the hadrons, not only to the nucleon vertex, and is a counterpart of the Adler consistency condition.

The reason is that at $m_{u,d} = 0$ the theory is invariant under the isotriplet right-handed chiral transformation

$$q(x) \to e^{i\frac{\alpha}{2}(1+\gamma_5)\tau^3}q(x);$$

as long as $\mathcal{L}_w$ is free from u_R and d_R this symmetry persists in the full theory including the weak interaction. Since it is spontaneously broken by the conventional strong dynamics, there is an exactly massless (at $m_q = 0$) Goldstone boson, π^0, associated with the corresponding exactly conserved Noether current. This current is evidently a sum of the usually considered axial current and of a flavor-diagonal vector current. Likewise, as in the conventional axial-current case, the corresponding Goldstone vertex off the exact eigenstates vanishes at zero momentum.

The formal derivation is straightforward if one considers, instead of the conventional axial current, the corresponding left-handed current. Its divergence, the analogue of eq. (A.4) vanishes at $m_q = 0$ by virtue of the exact quark field equations of motion, and the generalized Goldberger-Treiman relation eq. (A.5) says that $c(0) = 0$, cf. eq. (A.6). The existence itself of the exact Goldstone boson follows from eq. (A.5) considered in the limit of small nonvanishing m_q with $\mathcal{D} = 0$.

Therefore, any weak pion amplitude vanishes for small pion momentum in the chiral limit unless weak Lagrangian contains right-handed u or d fields. More generally, it may only remain finite if there is no combination of vector and non-anomalous axial transformation that leaves $\mathcal{L}_w$ invariant. Furthermore, the invariance can be checked at arbitrary normalization scale, and usually it is most evident for the bare operators. As an example, the bare $\mathcal{L}_w$ in the Standard Model contains only left-handed fields, but Penguins [5, 6] induce the operators with the right-handed light quarks in the conventionally considered effective renormalized Lagrangian. Nevertheless the zero-momentum pion amplitude vanishes in the chiral limit in the Standard Model.

Unlike the pseudoscalar vertex, the induced CP-violating scalar pion-nucleon vertex describes the Lorentz structure in the amplitude that does not vanish at zero momentum transfer. Therefore, it must vanish in the chiral limit unless the weak interactions include right-handed u or d fields.

Concluding the brief discussion of the application of the current algebra technique we note that the similar methods can be applied, for instance to the usual weak decays, e.g. of kaons or hyperons, both parity-conserving and parity-violating. In the parity-conserving $\Delta S = 1$ decays we may subtract from the weak Lagrangian the scalar operator $\bar{s}d$ with an arbitrary coefficient λ, rewriting it as $\partial_\mu(\bar{s}\gamma_\mu d)/(m_s - m_d)$. This demonstrates that the decay amplitude is not changed regardless of λ. For parity-violating transitions we can subtract $\bar{s}i\gamma_5 d = \partial_\mu(\bar{s}\gamma_\mu\gamma_5 d)/(m_s + m_d)$. This is also useful in establishing the absence of the chiral enhancement in the K-decay amplitudes mediated by composite quark bilinears like $\bar{s}i\sigma_{\mu\nu}G^{\mu\nu}d$, and to elucidate other similar cancellations. We do not expand on the related applications here.

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