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Stringy instanton effects in $\mathcal{N} = 2$ gauge theories

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Abstract: We study the non-perturbative effects induced by stringy instantons on $\mathcal{N} = 2$ SU(N) gauge theories in four dimensions, realized on fractional D3 branes in a $\mathbb{C}^3/\mathbb{Z}_3$ orientifold. The stringy instantons, corresponding to D(–1) branes that occupy a node of the orientifold quiver diagram where no D3 brane is present, have the right content of zero-modes to produce non-perturbative terms in the four-dimensional effective action. In the SU(2) theory these terms have the same structure for all instanton numbers and yield a series of non-perturbative corrections to the prepotential. We explicitly compute these corrections up to instanton number $k = 5$ using localization methods.

Keywords: D-branes, Solitons Monopoles and Instantons, Brane Dynamics in Gauge Theories

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1 Introduction and motivations

The study of the non-perturbative regime of supersymmetric gauge theories has always attracted great interest (for reviews see, for example, [1–3]). In the last decade remarkable progress in this field has been achieved using string inspired methods, i.e. realizing the gauge theories on the world-volume of space-filling D-branes embedded in supersymmetric string compactifications and introducing the non-perturbative corrections by means of localized branes, like D-instantons or totally wrapped Euclidean branes [4–9] (for a recent review see, for instance, [10]). This stringy setup has allowed to reproduce in a nice and unified framework many different features and results of the standard instanton calculus for supersymmetric gauge theories, like for instance the ADHM construction [11], the classical instanton profile and the non-perturbative corrections to prepotentials or superpotentials in various models.

On the other hand, the observation that instantons can be described as branes within branes has paved the way to several interesting generalizations corresponding to instanton configurations that do not admit a standard gauge theory interpretation but still have a natural realization in terms of D-branes. We shall refer to this type of configurations as "exotic" or "stringy" instantons as opposed to the "ordinary" gauge instantons. The

latter correspond to localized branes that share with the space-filling branes all features except their dimensionality. In the simplest setups where the four-dimensional gauge theory is engineered with D3 branes, the ordinary instantons are described by D(–1) branes of the same kind, while in more general string compactifications where the gauge sector is realized on D$(3+p)$ branes wrapped on a p-cycle $\mathcal{C}$, the ordinary instantons correspond to Euclidean D$(p-1)$ branes totally wrapped on $\mathcal{C}$. Different types of D(–1) branes (for example with different Chan-Paton structures), or Euclidean branes wrapped on cycles $\mathcal{C}' \neq \mathcal{C}$ correspond, instead, to stringy instantons that do not have a clear field-theory interpretation, at least from a four-dimensional point of view.[1] Despite this fact, or maybe precisely for this fact, the stringy instantons have recently attracted much interest since they can generate novel types of interactions which are perturbatively forbidden and whose strength is not linked to the gauge theory scale. This feature is very welcome in the search for semi-realistic string scenarios for the physics beyond the Standard Model where a hierarchy between various Majorana masses and Yukawa couplings is expected. Indeed, in some specific contexts the stringy instantons have been indicated as possible sources of neutrino masses [13–15], of certain Yukawa couplings in GUT models [16], or of non-perturbative contributions that may be relevant for moduli stabilization [17, 18]. Other interesting applications of stringy instantons can be found in [19–37].

From a conformal field theory point of view, in the ordinary gauge instanton configurations the mixed open strings suspended between the instantonic and the space-filling branes have four directions with mixed Neumann-Dirichlet (ND) boundary conditions, and possess massless excitations in the Neveu-Schwarz sector which describe the size and gauge orientation of field theoretical instanton solutions. On the other hand, in the exotic cases the mixed open strings either have extra twisted directions besides the four ND space-time directions, or are characterized by different types of Chan-Paton factors at their end-points. As a consequence, the bosonic moduli corresponding to the instanton size are missing and certain fermionic zero-modes become difficult to saturate. These unwanted fermionic zero-modes must be either removed by appropriate orientifold projections [19, 20], or lifted with fluxes [21, 29, 30] or with other mechanisms [24, 32].

Parallel to these developments, the application of localization techniques to the computation of the instanton partition functions, originally pioneered by N. Nekrasov [38–40], has remarkably boosted the multi-instanton calculus in gauge theories far beyond the results obtained in the past with standard methods, and many non-perturbative phenomena can now be put in a framework amenable of a proper mathematical treatment. Recently, these localization techniques have been successfully applied also to multi-instantons of exotic type yielding results that are in perfect agreement with those expected from the heterotic/Type I′ duality [41–43] or from F-theory considerations [44]. It is therefore fair to say that also the stringy multi-instanton calculus is now on a rather solid ground.

In all examples of exotic multi-instantons considered up to now, the gauge theory is realized either on the world-volume of D7 branes [41, 42] or on systems of D7 and

[1] Some stringy instantons configurations have a nice field-theory interpretation in eight dimensions as shown in [12].

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D3 branes [43, 44]; therefore, part of the results that have been obtained so far have necessarily an eight-dimensional interpretation due to the presence of the D7 branes. In this paper, instead, we consider a gauge sector made entirely of D3 branes so that the results we get have only a four-dimensional character. In particular, we investigate the gauge theory engineered with stacks of fractional D3 branes in a $\mathbb{C}^3/\mathbb{Z}_3$ orientifold of type IIB preserving $\mathcal{N}=2$ supersymmetry in four dimensions, and study the corresponding stringy multi-instanton configurations along the lines already discussed in [19] for the 1-instanton case. More specifically, we analyze a configuration of fractional D3 branes that realizes an $\mathcal{N}=2$ SU(N) theory in four dimensions with a hypermultiplet in the symmetric representation, and then introduce exotic instantons by adding stacks of D(–1) branes on the nodes of the $\mathbb{C}^3/\mathbb{Z}_3$ quiver diagram that are not occupied by the D3 branes. In this way the mixed open strings stretched between the D3 and the D(–1) branes have only fermionic charged zero-modes, a typical feature of the exotic instantons. Furthermore, the orientifold projection removes the dangerous neutral fermionic zero-modes we alluded to above, so that the stringy instantons have the right content of zero-modes to provide non-vanishing contributions to the D3 brane effective action. We have computed such non-perturbative effects with the same localization methods [38–40] used to find the gauge instanton terms in the $\mathcal{N}=2$ super Yang-Mills theory predicted by the Seiberg-Witten curve [45, 46]. However, due to the different structure of the moduli space of the stringy instantons and of the corresponding moduli integrals, the non-perturbative terms we obtain are of a novel type.

This paper is organized as follows. In section 2 we review the main features of the fractional D3 branes in the $\mathbb{C}^3/\mathbb{Z}_3$ orientifold and of the $\mathcal{N}=2$ gauge theory living on their world-volume. In section 3 we introduce unoriented fractional D-instantons, focusing then in section 4 on the exotic configurations, on their moduli spectrum and on the cohomological properties of their moduli action. In section 5 we explicitly evaluate the moduli integrals for the SU(2) theory, and derive the non-perturbative corrections to the effective prepotential induced by the stringy instantons up to instanton number $k=5$. Finally, in section 6 we summarize our results and present our conclusions. Several technical details that are useful to reproduce some of the computations of the main text are collected in the appendix.

2 D3 branes in the $\mathbb{C}^3/\mathbb{Z}_3$ orientifold

We consider fractional D3 branes in a $\mathbb{C}^3/\mathbb{Z}_3$ orientifold and study the non-perturbative effects produced by fractional D-instantons along the lines discussed in [19]. Even though this is quite standard, we briefly recall the main features of this orientifold model in order to be self-contained.

We place both the D3's and the D(–1)'s at the orbifold singularity, and parametrize the world-volume directions of the D3's by the first four string coordinates, as shown in table 1. In the six-dimensional "internal" space orthogonal to the D3 branes we introduce three complex coordinates

$$z^1 = x^4 + \mathrm{i}x^5\,, \qquad z^2 = x^6 + \mathrm{i}x^7\,, \qquad z^3 = x^8 + \mathrm{i}x^9\,, \tag{2.1}$$

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	0	1	2	3	4	5	6	7	8	9
D3	−	−	−	−	×	×	×	×	×	×
D(−1)	×	×	×	×	×	×	×	×	×	×

Table 1. D brane arrangement. The symbols − and × denote respectively Neumann and Dirichlet boundary conditions for the open strings attached to the branes.

on which the $\mathbb{Z}_3$ orbifold action can be naturally defined. Denoting by g the generator of $\mathbb{Z}_3$ such that $g^3 = 1$, we take

$$g \;:\; \begin{pmatrix} z^1 \\ z^2 \\ z^3 \end{pmatrix} \;\to\; \begin{pmatrix} \xi\, z^1 \\ \xi^{-1}\, z^2 \\ z^3 \end{pmatrix} \tag{2.2}$$

where $\xi = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$. Since one of the complex coordinates does not transform, this $\mathbb{Z}_3$ action breaks half of the supersymmetries of the original ten-dimensional background and therefore leads to $\mathcal{N} = 2$ theories on the world-volume of the fractional D3 branes.

Notice that the action (2.2) can be interpreted as a rotation of $+\frac{2\pi \mathrm{i}}{3}$ in the z^1-plane combined with a rotation of $-\frac{2\pi \mathrm{i}}{3}$ in the z^2-plane. Thus, g can be represented by

$$R(g) = \mathrm{e}^{+\frac{2\pi \mathrm{i}}{3} J_1}\, \mathrm{e}^{-\frac{2\pi \mathrm{i}}{3} J_2} \tag{2.3}$$

where J_i is the generator of the rotations in the z^i-plane in the vector representation. This expression is particularly useful to define the orbifold action on spin-fields and, more generally, on fields carrying spinor indices. To this aim, in fact, it is enough to take (2.3) with the generators J_i in the spinor representation. As a consequence of the $4+6$ splitting of the ten-dimensional space-time induced by the D3 branes, the "Lorentz" group[2] SO(10) is broken to $\mathrm{SO}(4) \times \mathrm{SO}(6)$, and thus any ten-dimensional spinor decomposes accordingly. For example an anti-chiral spinor Λ decomposes as

$$\left(\Lambda^{\alpha A}\,,\, \Lambda_{\dot\alpha A}\right) \tag{2.4}$$

where α ($\dot\alpha$) are chiral (anti-chiral) spinor indices of SO(4), and the lower (upper) indices A are chiral (anti-chiral) spinor indices of SO(6). Upon using the explicit expression for the SO(6) spinor weights, from (2.3) we can easily deduce that

$$g \;:\; \begin{pmatrix} \Lambda^{\alpha---} \\ \Lambda^{\alpha++-} \\ \Lambda^{\alpha+-+} \\ \Lambda^{\alpha-++} \end{pmatrix} \;\to\; \begin{pmatrix} \Lambda^{\alpha---} \\ \Lambda^{\alpha++-} \\ \xi\, \Lambda^{\alpha+-+} \\ \xi^{-1}\, \Lambda^{\alpha-++} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \Lambda_{\dot\alpha+++} \\ \Lambda_{\dot\alpha--+} \\ \Lambda_{\dot\alpha-+-} \\ \Lambda_{\dot\alpha+--} \end{pmatrix} \;\to\; \begin{pmatrix} \Lambda_{\dot\alpha+++} \\ \Lambda_{\dot\alpha--+} \\ \xi^{-1}\, \Lambda_{\dot\alpha-+-} \\ \xi\, \Lambda_{\dot\alpha+--} \end{pmatrix} . \tag{2.5}$$

This action shows that only half of the spinor components are invariant under the orbifold action, thus leading to $\mathcal{N} = 2$ supersymmetry as anticipated above.

[2]Since we will be interested in studying instanton corrections, we take a Euclidean signature in space-time.

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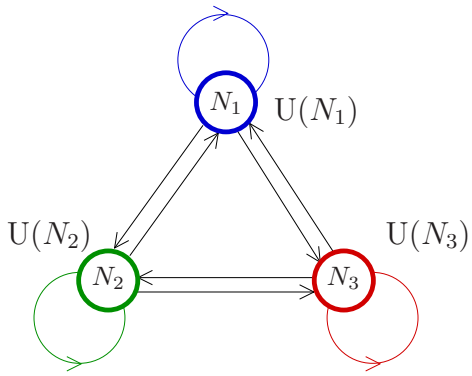

Figure 1. The $\mathbb{C}^3/\mathbb{Z}_3$ un-orientifolded theory corresponding to a configuration of N_1, N_2 and N_3 fractional D3 branes. The lines starting and ending on the same node represent $\mathcal{N}=2$ vector multiplets in the adjoint representation of the $\mathrm{U}(N_i)$ groups. The oriented lines between different nodes represent bi-fundamental chiral multiplets which pair up into $\mathcal{N}=2$ hypermultiplets.

The orbifold group $\mathbb{Z}_3$ has three irreducible representations: $R_1(g)=1$, $R_2(g)=\xi$ and $R_3(g)=\xi^{-1}$. Consequently [47], there are three types of fractional D branes and the associated quiver diagram has three nodes. The number of fractional D3 branes occupying the i-th node which corresponds to the representation $R_i(g)$ is denoted by N_i. A generic open string excitation in this brane system carries a Chan-Paton (CP) factor X that is a $(N_1+N_2+N_3)\times(N_1+N_2+N_3)$ matrix on which the orbifold generator g acts according to

$$g \;:\; X \;\to\; \gamma(g)\, X\, \gamma(g)^{-1}\ . \tag{2.6}$$

Here $\gamma(g)$ is

$$\gamma(g) = \begin{pmatrix} \mathbb{1}_{N_1} & 0 & 0 \\ 0 & \xi\,\mathbb{1}_{N_2} & 0 \\ 0 & 0 & \xi^{-1}\,\mathbb{1}_{N_3} \end{pmatrix} \tag{2.7}$$

with $\mathbb{1}_{N_i}$ denoting the $N_i\times N_i$ identity matrix. This system supports an $\mathcal{N}=2$ gauge theory with group $\mathrm{U}(N_1)\times\mathrm{U}(N_2)\times\mathrm{U}(N_3)$ represented by the quiver diagram of figure 1.

We now enrich our configuration by adding an O3 plane with a world-volume lying along the same four space-time directions as the D3 branes. The action of the orientifold generator Ω on the various open string fields is standard and can be deduced by writing

$$\Omega = \omega\,(-1)^{F_L}\,\mathcal{I}_{456789} \tag{2.8}$$

where ω is the world-sheet parity, F_L the (left) space-time fermion number and $\mathcal{I}_{456789}$ is the reflection in the internal space. On the other hand, the orientifold acts on the CP factors X by means of a matrix $\gamma(\Omega)$ according to

$$\Omega \;:\; X \;\to\; \gamma(\Omega)\, X^T\, \gamma(\Omega)^{-1}\ . \tag{2.9}$$

In the presence of an orbifold the matrix $\gamma(\Omega)$ must satisfy the following consistency condition [47, 48]

$$\gamma(h)\,\gamma(\Omega)\,\gamma(h)^T = \gamma(\Omega) \tag{2.10}$$

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for any h belonging to the orbifold group, which amounts to requiring that the orientifold and orbifold projections commute with each other. The matrix $\gamma(\Omega)$ can be either symmetric or antisymmetric. Here we choose to perform an antisymmetric projection on the D3 branes and denote the corresponding matrix by $\gamma_-(\Omega)$. Taking N_1 to be even and $N_2 = N_3$, we can write

$$\gamma_-(\Omega) = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & 0 & \mathbb{1}_{N_2} \\ 0 & -\mathbb{1}_{N_2} & 0 \end{pmatrix} \tag{2.11}$$

where ϵ is a $N_1 \times N_1$ antisymmetric matrix obeying $\epsilon^2 = -1$. Using (2.6) it is easy to verify that $\gamma_-(\Omega)$ satisfies the consistency condition (2.10).

The bosonic field content on the fractional D3 branes at the singularity follows after implementing the following orbifold and orientifold conditions[3]

$$\mathbf{A}_\mu = \gamma(g)\, \mathbf{A}_\mu\, \gamma(g)^{-1}\,, \qquad \mathbf{A}_\mu = -\gamma_-(\Omega)\, \big(\mathbf{A}_\mu\big)^T\, \gamma_-(\Omega)^{-1}\,, \tag{2.12a}$$

$$\mathbf{\Phi}^I = (\xi)^I\, \gamma(g)\, \mathbf{\Phi}^I\, \gamma(g)^{-1}\,, \quad \mathbf{\Phi}^I = -\gamma_-(\Omega)\, \big(\mathbf{\Phi}^I\big)^T\, \gamma_-(\Omega)^{-1}\,. \tag{2.12b}$$

Here $\mathbf{A}_\mu$ is the gauge vector field along the D3 world-volume directions ($\mu = 0, \ldots, 3$), while $\mathbf{\Phi}^I$ ($I = 1, 2, 3$) are three complex scalars along the three complex directions (2.1). The orbifold part of these conditions forces $\mathbf{A}_\mu$ and $\mathbf{\Phi}^3$ to be 3×3 block diagonal matrices, namely

$$\mathbf{A}_\mu = \begin{pmatrix} A_{\mu(11)} & 0 & 0 \\ 0 & A_{\mu(22)} & 0 \\ 0 & 0 & A_{\mu(33)} \end{pmatrix}\,, \qquad \mathbf{\Phi}^3 = \begin{pmatrix} \Phi^3_{(11)} & 0 & 0 \\ 0 & \Phi^3_{(22)} & 0 \\ 0 & 0 & \Phi^3_{(33)} \end{pmatrix}\,, \tag{2.13}$$

and $\mathbf{\Phi}^1$ and $\mathbf{\Phi}^2$ to have the following off-diagonal structure

$$\mathbf{\Phi}^1 = \begin{pmatrix} 0 & \Phi^1_{(12)} & 0 \\ 0 & 0 & \Phi^1_{(23)} \\ \Phi^1_{(31)} & 0 & 0 \end{pmatrix}\,, \qquad \mathbf{\Phi}^2 = \begin{pmatrix} 0 & 0 & \Phi^2_{(13)} \\ \Phi^2_{(21)} & 0 & 0 \\ 0 & \Phi^2_{(32)} & 0 \end{pmatrix}\,. \tag{2.14}$$

The orientifold conditions impose that $A_{\mu(11)} = \epsilon\big(A_{\mu(11)}\big)^T\epsilon$ and $A_{\mu(22)} = -\big(A_{\mu(33)}\big)^T$, and similarly that $\Phi^3_{(11)} = \epsilon\big(\Phi^3_{(11)}\big)^T\epsilon$ and $\Phi^3_{(22)} = -\big(\Phi^3_{(33)}\big)^T$. The resulting theory is therefore an $\mathrm{USp}(N_1) \times \mathrm{U}(N_2)$ gauge theory, with $\mathbf{A}_\mu$ and $\mathbf{\Phi}^3$ being the bosonic components of the $\mathcal{N} = 2$ adjoint vector multiplet. Sometimes, it is convenient to still denote diagrammatically $A_{\mu(22)}$ and $A_{\mu(33)}$ (as well as $\Phi^3_{(22)}$ and $\Phi^3_{(33)}$) as belonging to different quiver nodes, with the understanding that they should be actually identified in the above way. Most often we will use the simplified notation $A_{\mu(22)} \equiv A_\mu$ and $\Phi^3_{(22)} \equiv \Phi$.

The orientifold projection on the complex fields $\mathbf{\Phi}^1$ and $\mathbf{\Phi}^2$, which represent the bosonic components of the matter superfields, can be done in a similar way and leads to the following relations

$$\Phi^1_{(12)} = -\epsilon\,\big(\Phi^1_{(31)}\big)^T\,, \quad \Phi^1_{(23)} = \big(\Phi^1_{(23)}\big)^T\,, \quad \Phi^2_{(13)} = \epsilon\,\big(\Phi^2_{(21)}\big)^T\,, \quad \Phi^2_{(32)} = \big(\Phi^2_{(32)}\big)^T\,. \tag{2.15}$$

[3]This same analysis can be performed in a straightforward way also in the fermionic sectors.

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field	$\mathrm{USp}(N_1)$	$\mathrm{U}(N_2)$
$\Phi^1_{(12)}$	$\square$	$\overline{\square}$
$\Phi^1_{(31)}$	$\square$	$\square$
$\Phi^1_{(23)}$	$\cdot$	$\square\!\square$
$\Phi^2_{(21)}$	$\square$	$\square$
$\Phi^2_{(13)}$	$\square$	$\overline{\square}$
$\Phi^2_{(32)}$	$\cdot$	$\overline{\square\!\square}$

Table 2. Matter content and associated gauge representations.

With respect to the gauge group $\mathrm{USp}(N_1)\times\mathrm{U}(N_2)$ they belong to the representations given in table 2.

In the following we will consider a D3 brane system with $N_1 = 0$ and $N_2 = N_3 = N$, supporting a four-dimensional gauge theory with group $\mathrm{U}(N)$ and matter in the symmetric representation. Actually, we can neglect the U(1) factor since it is IR free, and thus we will concentrate only on the low-energy dynamics of the $\mathrm{SU}(N)$ part. Note that the complex fields $\Phi^1_{(23)}$ and $\overline{\Phi}^2_{(32)}$, plus their fermionic partners, pair up and build an $\mathcal{N} = 2$ hypermultiplet in the symmetric representation of $\mathrm{SU}(N)$. For such a gauge theory, the 1-loop β-function coefficient is

$$b_1 = N - 2 \ . \tag{2.16}$$

The theory is therefore UV asymptotically free for $N > 2$ and conformal for $N = 2$. The latter case is a non-standard realization of the $\mathcal{N} = 4$ SU(2) superconformal Yang-Mills theory; indeed for SU(2) the symmetric representation coincides with the adjoint, and thus the matter hypermultiplet can be combined with the vector multiplet enhancing the supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 4$. In the following we will see that this realization leads to non-trivial results in the non-perturbative sectors of the theory even in the superconformal case.

3 D-instantons in the $\mathbb{C}^3/\mathbb{Z}_3$ orientifold

We now briefly discuss the D-instantons in the $\mathbb{C}^3/\mathbb{Z}_3$ orientifold introduced in the previous section. The most general instanton configuration is realized by putting k_1 D(–1) branes on node 1, k_2 D(–1)'s on node 2 and k_3 D(–1)'s on node 3 with $k_2 = k_3$. A generic open string excitation stretching between two D-instantons will therefore have a CP factor Y which is a $(k_1 + 2k_2) \times (k_1 + 2k_2)$ matrix. On it the $\mathbb{Z}_3$ orbifold generator g acts by means of a matrix $\gamma'(g)$ which has the same form as $\gamma(g)$ in (2.6) but with N_i replaced with k_i. The orientifold action on the D(–1) CP factors is instead different with respect to the D3 case [19, 48]. Indeed, the consistency with the antisymmetric matrix (2.11) chosen for the D3 branes requires to transform the D(–1) CP factors with a symmetric matrix $\gamma_+(\Omega)$ according to

$$\Omega \ : \quad Y \ \to \ \gamma_+(\Omega)\, Y^T\, \gamma_+(\Omega)^{-1} \tag{3.1}$$

where[4]

$$\gamma_+(\Omega) = \begin{pmatrix} \mathbb{1}_{k_1} & 0 & 0 \\ 0 & 0 & \mathbb{1}_{k_2} \\ 0 & \mathbb{1}_{k_2} & 0 \end{pmatrix} . \tag{3.2}$$

Adopting an ADHM-inspired notation, we can organize the bosonic excitations in the Neveu-Schwarz sector of the open strings suspended between two D-instantons in a four-dimensional vector $\mathbf{a}_\mu$ and three complex scalars $\boldsymbol{\chi}^I$, which are subject to the following conditions

$$\mathbf{a}_\mu = \gamma'(g)\, \mathbf{a}_\mu\, \gamma'(g)^{-1} , \qquad \mathbf{a}_\mu = +\gamma_+(\Omega)\, \big(\mathbf{a}_\mu\big)^T\, \gamma_+(\Omega)^{-1} , \tag{3.3a}$$

$$\boldsymbol{\chi}^I = (\xi)^I\, \gamma'(g)\, \boldsymbol{\chi}^I\, \gamma'(g)^{-1} , \quad \boldsymbol{\chi}^I = -\gamma_+(\Omega)\, \big(\boldsymbol{\chi}^I\big)^T\, \gamma_+(\Omega)^{-1} . \tag{3.3b}$$

The plus sign in the orientifold action on $\mathbf{a}_\mu$ is due to the fact that now the first four directions labeled by μ are of Dirichlet type. Implementing the constraints (3.3) we obtain

$$\mathbf{a}_\mu = \begin{pmatrix} a_{\mu(11)} & 0 & 0 \\ 0 & a_{\mu(22)} & 0 \\ 0 & 0 & a_{\mu(33)} \end{pmatrix} , \qquad \boldsymbol{\chi}^3 = \begin{pmatrix} \chi^3_{(11)} & 0 & 0 \\ 0 & \chi^3_{(22)} & 0 \\ 0 & 0 & \chi^3_{(33)} \end{pmatrix} , \tag{3.4}$$

with

$$a_{\mu(11)} = \big(a_{\mu(11)}\big)^T , \quad a_{\mu(22)} = \big(a_{\mu(33)}\big)^T , \quad \chi^3_{(11)} = -\big(\chi^3_{(11)}\big)^T , \quad \chi^3_{(22)} = -\big(\chi^3_{(33)}\big)^T , \tag{3.5}$$

and

$$\boldsymbol{\chi}^1 = \begin{pmatrix} 0 & \chi^1_{(12)} & 0 \\ 0 & 0 & \chi^1_{(23)} \\ \chi^1_{(31)} & 0 & 0 \end{pmatrix} , \qquad \boldsymbol{\chi}^2 = \begin{pmatrix} 0 & 0 & \chi^2_{(13)} \\ \chi^2_{(21)} & 0 & 0 \\ 0 & \chi^2_{(32)} & 0 \end{pmatrix} , \tag{3.6}$$

with

$$\chi^1_{(12)} = -\big(\chi^1_{(31)}\big)^T , \quad \chi^1_{(23)} = -\big(\chi^1_{(23)}\big)^T , \quad \chi^2_{(13)} = -\big(\chi^2_{(21)}\big)^T , \quad \chi^2_{(32)} = -\big(\chi^2_{(32)}\big)^T . \tag{3.7}$$

The conditions (3.5) imply that the symmetry group on the D-instantons is $\mathrm{SO}(k_1)\times\mathrm{U}(k_2)$, with the orthogonal factor referring to the first node of the quiver and the unitary factor to the remaining two nodes that are identified with each other under the orientifold projection.

This analysis can be easily extended also to the fermionic excitations of the Ramond sector. We will provide some details on this in the following sections. Here, instead, we dwell on the fact that depending on whether or not the D-instanton occupies a quiver node populated also by a stack of D3 branes, it represents an ordinary gauge instanton or a stringy instanton. Referring to the $\mathrm{SU}(N)$ theory of the previous section, which corresponds to a D3 brane configuration of type $(N_1, N_2) = (0, N)$, a D-instanton configuration of type $(k_1, k_2) = (0, k)$ describes a gauge instanton with instanton number k and instanton group $\mathrm{U}(k)$. On the other hand, a D-instanton configuration of type $(k_1, k_2) = (k, 0)$ describes a stringy instanton with charge k and instanton group $\mathrm{SO}(k)$.[5] All this is summarized in table 3.

[4]Notice that, differently from N_1, k_1 does not need to be even.

[5]The occurrence of an orthogonal symmetry in the instanton sector of a theory with a unitary gauge group is a clear signal of the exotic character of the stringy instantons.

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	D3's	$\oplus$	D(–1)'s	gauge group	instanton group
gauge instantons	$(0, N)$	$\oplus$	$(0, k)$	SU(N)	U(k)
stringy instantons	$(0, N)$	$\oplus$	$(k, 0)$	SU(N)	SO(k)

Table 3. D3 and D(–1) brane configurations and their associated symmetry groups corresponding to gauge and exotic instantons.

The most general D-instanton configuration for our SU(N) gauge theory is therefore a superposition of gauge and stringy instantons. In the following sections we will discuss in detail the spectrum of moduli for the stringy instantons, and compute explicitly their contributions to the gauge effective action for $N = 2$. The analysis for $N > 2$ will be presented in a separate publication [49].

4 Stringy instantons

We now describe in more detail the stringy instanton configurations and thus consider a system made of a stack of k D(–1) branes placed on node 1 of the quiver diagram and two stacks of N D3 branes placed on nodes 2 and 3 and identified with each other under the orientifold action.

4.1 Moduli spectrum

The open strings excitations with at least one end-point on the D-instantons can be distinguished into neutral and charged ones, which we are going to analyze in turn.

Neutral sector. The neutral sector contains the modes of the open strings starting and ending on the D-instantons which are therefore uncharged under the gauge group of the D3 branes. Since in this configuration there is only one stack of instantonic branes on node 1, the CP factors of the neutral moduli have only one non-zero entry, i.e. the (11) component which is a $k \times k$ matrix. Since the complex scalars $\boldsymbol{\chi^1}$ and $\boldsymbol{\chi^2}$ do not have a (11) component as is clear from eq. (3.6), we can set $\boldsymbol{\chi^1} = \boldsymbol{\chi^2} = 0$. Furthermore, for the moduli $\mathbf{a}_\mu$ and $\boldsymbol{\chi}^3$ which do have a diagonal (11) component in their CP factors, we can simplify the notation and put

$$a_{\mu(11)} \equiv a_\mu = \big(a_\mu\big)^T , \qquad \chi^3_{(11)} \equiv \chi = -\big(\chi\big)^T . \tag{4.1}$$

As far as the fermionic moduli are concerned, we see from the spinor transformation properties (2.5) that only the components with indices $(\alpha - -)$, $(\alpha + +-)$, $(\dot\alpha + ++)$ and $(\dot\alpha - -+)$ are invariant under the $\mathbb{Z}_3$ orbifold action. Therefore, in the configuration we are now considering, these are the only components that can have a (11) entry in their CP factors and can then survive the orbifold projection. Adopting an ADHM inspired notation, we denote them as $M^{\alpha a}$ and $\lambda_{\dot\alpha a}$ where the upper index a takes the values $(- - -)$ and $(++-)$, while the lower index a takes the values $(+++)$ and $(--+)$. Also these fermionic moduli are $k \times k$ matrices and on them the orientifold projection acts according to

$$M^{\alpha a} = +\big(M^{\alpha a}\big)^T , \quad \lambda_{\dot\alpha a} = -\big(\lambda_{\dot\alpha a}\big)^T . \tag{4.2}$$

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These rules are a consequence of the fact that the matrix $\gamma_+(\Omega)$ restricted to the (11) block of a CP factor is simply the identity and that the orientifold generator (2.8) acting on a ten-dimensional spinor effectively measures its chirality in the first four directions, from which the signs in (4.2) immediately follow.

Charged sector. The charged sector contains the modes of the open strings which have one end-point on the D-instantons and one on the D3 branes, and which are charged under the gauge group created by the latter. Since in the exotic configuration the D-instantons sit on node 1 while the D3 branes occupy nodes 2 and 3, the CP factors for the 3/(–1) strings and the (–1)/3 strings have, respectively, the following structure

$$\begin{pmatrix} 0 & 0 & 0 \\ \star & 0 & 0 \\ \star & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & \star & \star \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{4.3}$$

It is easy to realize that both such CP factors transform non-trivially under the orbifold generator g represented by the matrices $\gamma(g)$ and $\gamma'(g)$. Thus, the only charged states surviving the orbifold projection are those whose vertex operators transform under g in such a way to compensate the phase acquired by their CP factors. In the Neveu-Schwarz sector, due to the mixed Neumann-Dirichlet boundary conditions, the GSO projected physical vertex operators carry an anti-chiral spinor index in the first four directions but are singlets in the internal directions where the orbifold acts. Thus, these bosonic vertex operators do not acquire any phase under g and cannot survive the orbifold projection for the above argument. The absence of bosonic charged moduli is a typical signal of the exotic nature of these instanton configurations. On the other hand, in the Ramond sector, the GSO projected physical vertex operators are anti-chiral spinors in the six internal directions and two of their components, namely those with indices $(+\,-\,+)$ and $(-\,+\,+)$, transform non-trivially under g as one can see from (2.5), and can survive the orbifold projection. Being more explicit and adopting again an ADHM inspired notation, the physical charged moduli of the 3/(–1) sector are

$$\boldsymbol{\mu}^{+-+} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mu & 0 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\mu}^{-++} = \begin{pmatrix} 0 & 0 & 0 \\ \mu' & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.4}$$

where both μ and μ' are $N \times k$ matrices. The physical moduli in the (–1)/3 sector, corresponding to open strings with opposite orientation, are related to those of the 3/(–1) sector through the orientifold action. In our case we have

$$\begin{aligned} \bar{\boldsymbol{\mu}}^{+-+} &= \gamma_+(\Omega)\big(\boldsymbol{\mu}^{+-+}\big)^T\gamma_-(\Omega)^{-1} = \begin{pmatrix} 0 & +\mu^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \\ \bar{\boldsymbol{\mu}}^{-++} &= \gamma_+(\Omega)\big(\boldsymbol{\mu}^{-++}\big)^T\gamma_-(\Omega)^{-1} = \begin{pmatrix} 0 & 0 & -\mu'^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \end{aligned} \tag{4.5}$$

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4.2 Moduli action

As shown in [8, 9] the moduli action can be obtained from open string disk-amplitudes involving all moduli listed above. Such action can be expressed as the sum of three parts,

$$S = S_1 + S_2 + S_3\,, \tag{4.6}$$

with

$$S_1 = \frac{1}{g_0^2}\,\mathrm{tr}\left\{-\frac{1}{4}\big[a^\mu, a^\nu\big]\,\big[a_\mu, a_\nu\big] - \big[a_\mu, \chi\big]\,\big[a^\mu, \overline{\chi}\big] + \frac{1}{2}\,\big[\overline{\chi},\,\chi\big]\,\big[\overline{\chi}, \chi\big]\right\}, \tag{4.7a}$$

$$S_2 = \frac{1}{g_0^2}\,\mathrm{tr}\left\{2\,\lambda_{\dot\alpha a}\big[a^\mu, M_\beta{}^a\big]\,(\overline{\sigma}_\mu)^{\dot\alpha\beta} - \mathrm{i}\,\lambda_{\dot\alpha a}\big[\chi, \lambda^{\dot\alpha a}\big] - 2\mathrm{i}\,M^{\alpha a}\big[\overline{\chi}, M_{\alpha a}\big]\right\}, \tag{4.7b}$$

$$S_3 = \frac{1}{g_0^2}\,\mathrm{tr}\left\{-\,\mathrm{i}\,\mu^T\mu'\,\chi\right\} \tag{4.7c}$$

corresponding, respectively, to quartic, cubic and mixed interactions. Here the trace is over the SO(k) indices and g_0 is the coupling constant of the zero-dimensional Yang-Mills theory on the D(-1) branes, which is related to the string coupling constant g_s through the relation

$$g_0^2 = \frac{g_s}{4\pi^3\alpha'^2}\,. \tag{4.8}$$

All moduli appearing in this action have canonical scaling dimensions, namely the bosons have dimension of (length)$^{-1}$ and the fermions dimension of (length)$^{-3/2}$. More standard ADHM-dimensions can be obtained absorbing suitable powers of g_0, but we refrain from doing this.

The quartic interaction terms among the a_μ's can be disentangled by means of the three auxiliary fields D_c ($c = 1, 2, 3$), so that S_1 can be rewritten in the following way

$$S_1' = \frac{1}{g_0^2}\,\mathrm{tr}\left\{\frac{1}{2}D_cD^c - \frac{1}{2}D_c\overline{\eta}^c_{\mu\nu}\,\big[a^\mu, a^\nu\big] - \big[a_\mu, \chi\big]\,\big[a^\mu, \overline{\chi}\big] + \frac{1}{2}\big[\overline{\chi},\,\chi\big]\,\big[\overline{\chi},\,\chi\big]\right\} \tag{4.9}$$

where $\overline{\eta}^c_{\mu\nu}$ are the anti-self dual 't Hooft symbols. Indeed, eliminating the auxiliary fields through their algebraic equations

$$D^c = \frac{1}{2}\,\overline{\eta}^c_{\mu\nu}[a^\mu, a^\nu]\,, \tag{4.10}$$

one can see that S_1' reduces to S_1.

Another useful rewriting concerns the cubic action (4.7b). It is obtained by making suitable combinations among the components of the fermionic moduli that correspond to a "topological twist" in which the internal spinor index a is identified with a space-time spinor index $\dot\beta$. More explicitly, this identification leads to

$$\begin{aligned}\lambda_{\dot\alpha a} \;&\to\; \lambda_{\dot\alpha\dot\beta} \equiv \frac{1}{2}\,\epsilon_{\dot\alpha\dot\beta}\,\eta + \frac{\mathrm{i}}{2}\,(\tau^c)_{\dot\alpha\dot\beta}\,\lambda_c\,,\\ M^{\alpha a} \;&\to\; M^{\alpha\dot\beta} \equiv \frac{1}{2}\,M_\mu\,(\sigma^\mu)^{\alpha\dot\beta}\,.\end{aligned} \tag{4.11}$$

In this way the original Lorentz group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gets replaced by the "twisted" version $\mathrm{SU}(2) \times \mathrm{SU}(2)'$ where $\mathrm{SU}(2) = \mathrm{SU}(2)_L$ and $\mathrm{SU}(2)' = \mathrm{diag}\left(\mathrm{SU}(2)_R, \mathrm{SU}(2)_I\right)$ with $\mathrm{SU}(2)_I$ being the internal R-symmetry group of the $\mathcal{N} = 2$ theory.

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With the definitions (4.11), the cubic action S_2 can be rewritten as follows

$$S_2' = \frac{1}{g_0^2}\,\mathrm{tr}\left\{\eta\,[a_\mu, M^\mu] + \lambda_c\,[a^\mu, M^\nu]\,\overline{\eta}^c_{\mu\nu} - \frac{\mathrm{i}}{2}\,\eta\,[\chi,\eta] - \frac{\mathrm{i}}{2}\,\lambda_c\,[\chi,\lambda^c] - \mathrm{i}M_\mu\,[\overline{\chi}, M^\mu]\right\}\,. \tag{4.12}$$

Finally, it is also convenient to replace the mixed action (4.7c) with

$$S_3' = \frac{1}{g_0^2}\,\mathrm{tr}\left\{-\,\mathrm{i}\,\mu^T\mu'\,\chi + h^T h'\right\} \tag{4.13}$$

where h and h' are charged auxiliary fields which do not interact with any other modulus. Even if this replacing looks trivial, it is nevertheless useful for reasons that will become clear in a moment.

The total action

$$S' = S_1' + S_2' + S_3' \tag{4.14}$$

is invariant under the D-instanton group SO(k) and the D3 brane gauge group SU(N). It is also invariant under the "twisted" Lorentz group SU(2) × SU(2)′ under which a_μ and M_μ transform in the $(\mathbf{2},\mathbf{2})$, λ_c and D_c in the $(\mathbf{1},\mathbf{3})$, and all the remaining moduli χ, $\overline{\chi}$, η, μ, μ', h and h' are singlets. Furthermore, the action (4.14) is invariant under the following BRST-like transformations

$$\begin{aligned}
&Qa^\mu = M^\mu\,, \quad QM^\mu = \mathrm{i}\,[\chi, a^\mu]\,,\\
&Q\lambda_c = D_c\,, \quad\;\; QD_c = \mathrm{i}\,[\chi,\lambda_c]\,,\\
&Q\,\overline{\chi} = -\mathrm{i}\eta\,, \quad\;\; Q\eta = -\,[\chi,\overline{\chi}]\,, \quad Q\chi = 0\,,\\
&Q\mu = h\,, \quad\;\; Qh = \mathrm{i}\,\mu\,\chi\,,\\
&Q\mu' = h'\,, \quad\;\; Qh' = \mathrm{i}\,\mu'\,\chi\,.
\end{aligned} \tag{4.15}$$

The BRST charge Q is the "singlet" component of the supercharges $Q_{\dot\alpha a}$ that arises after the topological twist that identifies a with $\dot\beta$, namely

$$Q \equiv Q_{\dot\alpha\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\,. \tag{4.16}$$

Note that Q is nilpotent up to an infinitesimal SO(k) transformation parametrized by χ. Indeed, on any modulus we have

$$Q^2\,\bullet = T_{\mathrm{SO}(k)}(\chi)\,\bullet\,, \tag{4.17}$$

where $T_{\mathrm{SO}(k)}(\chi)$ denotes an infinitesimal SO(k) rotation with parameter χ in the appropriate representation. According to (4.15), all moduli except χ form BRST doublets of the type (Ψ_0,Ψ_1) such that $Q\,\Psi_0 = \Psi_1$ and whose properties are collected in table 4.

By exploiting the above properties and using the invariance under SO(k), one can easily show that the total action (4.14) is Q-exact; indeed

$$S' = Q\,\Xi\,, \tag{4.18}$$

with

$$\Xi = \frac{1}{g_0^2}\,\mathrm{tr}\left\{\mathrm{i}M^\mu\,[\overline{\chi}, a_\mu] - \frac{1}{2}\,\overline{\eta}^c_{\mu\nu}\lambda_c[a^\mu, a^\nu] + \frac{1}{2}\,\lambda_c D^c - \frac{1}{2}\,[\chi,\overline{\chi}]\eta + \mu^T h'\right\}\,. \tag{4.19}$$

(Ψ_0, Ψ_1)	SO(k)	SU(N)	SU(2) × SU(2)′
(a_μ, M_μ)	$\square\square$	$\mathbf{1}$	$(\mathbf{2},\mathbf{2})$
(λ_c, D_c)	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$(\mathbf{1},\mathbf{3})$
$(\overline{\chi}, \eta)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$(\mathbf{1},\mathbf{1})$
(μ, h)	$\square$	$\overline{\mathbf{N}}$	$(\mathbf{1},\mathbf{1})$
(μ', h')	$\square$	$\overline{\mathbf{N}}$	$(\mathbf{1},\mathbf{1})$

Table 4. Moduli in the stringy instanton configuration organized as BRST pairs and their transformation properties under the various symmetry groups.

Since the scaling dimension of the BRST charge is (length)$^{-1/2}$, the dimensions of the components (Ψ_0, Ψ_1) of any BRST doublet are of the form (length)$^{\Delta}$ and (length)$^{\Delta-1/2}$. Thus, recalling that a fermionic variable and its differential have opposite dimensions, the measure on the instanton moduli space

$$d\mathcal{M}_k \equiv d\chi \prod_{(\Psi_0,\Psi_1)} d\Psi_0\, d\Psi_1 \tag{4.20}$$

has the total dimension

$$(\text{length})^{-\frac{1}{2}k(k-1)+\frac{1}{2}n_b-\frac{1}{2}n_f} \ . \tag{4.21}$$

Here, the first term in the exponent accounts for the unpaired modulus χ in the anti-symmetric representation of SO(k), while n_b (n_f) denotes the number of BRST multiplets whose lowest components Ψ_0 are bosonic (fermionic). From table 4 it is not difficult to verify that $n_b = \frac{5}{2}\,k^2 + \frac{3}{2}\,k$ and $n_f = \frac{3}{2}\,k^2 - \frac{3}{2}\,k + 2kN$, so that the measure (4.20) has dimension

$$(\text{length})^{k(2-N)} = (\text{length})^{-kb_1} \tag{4.22}$$

where b_1 is the coefficient of the 1-loop β-function for our gauge theory, given in (2.16). The negative sign in the exponent of (4.22) is another hallmark of the intrinsically stringy nature of the instanton configuration we are considering.[6] However, in the conformal $N = 2$ case which we will discuss in detail in the following section also the exotic instanton measure (4.20) is dimensionless and thus one expects that some non-perturbative contributions may be seen also in the effective field theory. In section 5 we will explicitly see that this is indeed what happens.

4.3 Deformed moduli action

To obtain the non-perturbative contributions induced by the stringy instantons, it is necessary to generalize the moduli action (4.18) and fully exploit all symmetries of the instanton moduli space, which are the gauge group SO(k) on the k D(–1)'s, the gauge group SU(N) on the N D3 branes and the "twisted" Lorentz group SU(2) × SU(2)′.

[6] For the usual gauge theory instantons the dimension of the moduli measure is (length)$^{+kb_1}$, see for instance [2] for a general discussion.

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To this aim, we begin by considering the interactions among the instanton moduli and the gauge fields propagating on the world-volume of the D3 branes, which we combine into an $\mathcal{N}=2$ chiral superfield $\Phi(x,\theta)$. Such interactions can be easily obtained by computing mixed disk amplitudes involving both vertex operators for moduli and vertex operators for dynamical fields, as discussed in detail in [9, 50] for analogous D(–1)/D3 brane systems. In the present case the result of such computations is

$$\frac{1}{g_0^2}\,\mathrm{tr}\left\{\mathrm{i}\,\mu^T\Phi(x,\theta)\,\mu'\right\} \tag{4.23}$$

which has to be added to the moduli action (4.18). For our later purposes it is enough to focus on the dependence on the vacuum expectation value

$$\phi=\langle\Phi(x,\theta)\rangle\,, \tag{4.24}$$

and hence we will consider the following modified mixed action

$$S_3'(\phi)=S_3'+\frac{1}{g_0^2}\,\mathrm{tr}\left\{\mathrm{i}\,\mu^T\phi\,\mu'\right\}\,. \tag{4.25}$$

Another kind of deformation concerns the inclusion of a non-trivial background to fully exploit the Euclidean Lorentz symmetry in the four space-time directions. This is usually called the Ω-background deformation [38–40] which, in our stingy context, can be realized by turning on a non-trivial Ramond-Ramond 3-form flux as discussed in detail in [50] and more recently in [51] where the equivalence between the Ω-background and the Ramond-Ramond flux has been shown in full generality. More specifically, we introduce a Ramond-Ramond 3-form flux of the type $F_{\mu\nu z^3}$, i.e. with two indices along the 4-dimensional world-volume of the D3 branes and one holomorphic index in the internal direction left invariant by the $\mathbb{Z}_3$ orbifold. It is not difficult to realize that such a field strength survives the orientifold projection under $\omega\,(-1)^{F_L}\,\mathcal{I}_{456789}$, since $F_{\mu\nu z^3}$ is even under the world-sheet parity ω (like any other RR 3-form field strength), odd under $(-1)^{F_L}$ (like any field of the Ramond-Ramond sector) and odd under the inversion $\mathcal{I}_{456789}$ (like any field with only one index in the internal directions). From now on, we denote $F_{\mu\nu z^3}$ simply as $\mathcal{F}_{\mu\nu}$ and parametrize it in terms of the 't Hooft symbols as follows

$$\mathcal{F}_{\mu\nu}=-\frac{\mathrm{i}}{2}\,\bar{f}_c\,\eta^c_{\mu\nu}-\frac{\mathrm{i}}{2}\,f_c\,\overline{\eta}^c_{\mu\nu}\,, \tag{4.26}$$

with $\bar{f}_c$ and f_c belonging, respectively, to the representations $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ of $\mathrm{SU}(2)\times\mathrm{SU}(2)'$. Furthermore, for reasons that will become apparent in the following, we also turn on the component of the Ramond-Ramond 3-form field-strength with an anti-holomorphic index in the internal space, i.e. $F_{\mu\nu\bar{z}^3}\equiv\overline{\mathcal{F}}_{\mu\nu}$. We then compute mixed disk amplitudes with insertions of $\mathcal{F}_{\mu\nu}$ and $\overline{\mathcal{F}}_{\mu\nu}$ to obtain their couplings with the instanton moduli. The results of these calculations, which are performed as explained in detail in [29, 41, 50], are new terms in the moduli action that can be accounted by replacing the quartic and cubic terms given in (4.9) and (4.12) as follows

$$\begin{aligned}
S_1'\to S_1'(\mathcal{F},\overline{\mathcal{F}})&=S_1'+\frac{1}{g_0^2}\mathrm{tr}\left\{\mathcal{F}^{\mu\nu}a_\nu\,[\overline{\chi},a_\mu]+\mathrm{i}\,\overline{\mathcal{F}}^{\mu\nu}a_\mu\,[\chi,a_\nu]-\mathrm{i}\,\overline{\mathcal{F}}^{\mu\nu}a_\mu\mathcal{F}_{\nu\rho}a^\rho\right\}\,,\\
S_2'\to S_2'(\mathcal{F},\overline{\mathcal{F}})&=S_2'+\frac{1}{g_0^2}\mathrm{tr}\left\{-\frac{1}{2}\epsilon_{cde}\,\lambda^c\lambda^d f^e-f_c\,\lambda^c\eta+\mathrm{i}\,f_c\,D^c\overline{\chi}+\overline{\mathcal{F}}_{\mu\nu}M^\mu M^\nu\right\}\,.
\end{aligned} \tag{4.27}$$

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Then, the full moduli action in the presence of Ramond-Ramond fluxes $\mathcal{F}_{\mu\nu}$ and $\overline{\mathcal{F}}_{\mu\nu}$, and of a vacuum expectation value ϕ for the adjoint scalar of the gauge multiplet, is given by

$$S'(\mathcal{F},\overline{\mathcal{F}},\phi) = S'_1(\mathcal{F},\overline{\mathcal{F}}) + S'_2(\mathcal{F},\overline{\mathcal{F}}) + S'_3(\phi)\ . \tag{4.28}$$

This action is still BRST exact, but with respect to a modified BRST charge Q'. Indeed, taking

$$\begin{aligned}
&Q'a^\mu = M^\mu\,, \quad Q'M^\mu = \mathrm{i}\,[\chi,a^\mu] - \mathrm{i}\,\mathcal{F}^{\mu\nu}a_\nu\,,\\
&Q'\lambda_c = D_c\,, \quad Q'D_c = \mathrm{i}\,[\chi,\lambda_c] + \epsilon_{cde}\,\lambda^d f^e\,,\\
&Q'\overline{\chi} = -\mathrm{i}\,\eta\,, \quad Q'\eta = -\,[\chi,\overline{\chi}]\ , \quad Q'\chi = 0\,,\\
&Q'\mu = h\,, \quad Q'h = \mathrm{i}\,\mu\,\chi - \mathrm{i}\,\phi\,\mu\,,\\
&Q'\mu' = h'\,, \quad Q'h' = \mathrm{i}\,\mu'\,\chi - \mathrm{i}\,\phi\,\mu'\,,
\end{aligned} \tag{4.29}$$

one can check that

$$S'(\mathcal{F},\overline{\mathcal{F}},\phi) = Q'\,\Xi' \tag{4.30}$$

where

$$\Xi' = \Xi + \frac{1}{g_0^2}\mathrm{tr}\left\{\mathrm{i}\,f_c\,\lambda^c\overline{\chi} + \overline{\mathcal{F}}_{\mu\nu}a^\mu M^\nu\right\} \tag{4.31}$$

with Ξ defined in (4.19). The deformed BRST charge Q' is nilpotent up to (infinitesimal) transformations of all the symmetry groups of the system; indeed we have

$$Q'^2\,\bullet = T_{\mathrm{SO}(k)}(\chi)\,\bullet\, - T_{\mathrm{SU}(N)}(\phi)\,\bullet\, + T_{\mathrm{SU}(2)\times\mathrm{SU}(2)'}(\mathcal{F})\bullet\,, \tag{4.32}$$

where $T_{\mathrm{SO}(k)}(\chi)$, $T_{\mathrm{SU}(N)}(\phi)$ and $T_{\mathrm{SU}(2)\times\mathrm{SU}(2)'}(\mathcal{F})$ are infinitesimal transformations of $\mathrm{SO}(k)$, $\mathrm{SU}(N)$ and $\mathrm{SU}(2)\times\mathrm{SU}(2)'$, parametrized respectively by χ, ϕ and $\mathcal{F}$, in the appropriate representation. Note that $\overline{\mathcal{F}}_{\mu\nu}$ appears only in Ξ' but not in Q'; hence the variation of $S'(\mathcal{F},\overline{\mathcal{F}},\phi)$ with respect to $\overline{\mathcal{F}}_{\mu\nu}$ is Q'-exact. This fact implies that the instanton partition function does not depend on $\overline{\mathcal{F}}_{\mu\nu}$, which can therefore be set to the most convenient value for the calculations. For later purposes it is useful to rewrite the moduli action in the following more explicit way

$$\begin{aligned}
S'(\mathcal{F},\overline{\mathcal{F}},\phi) = \frac{1}{g_0^2}\,\mathrm{tr}\,\Big\{&\eta\,[a_\mu,M^\mu] + \lambda^c\,[a^\mu,M^\nu]\,\bar\eta^c_{\mu\nu} - \frac{\mathrm{i}}{2}\eta\,[\chi,\eta] - \mathrm{i}\,M^\mu\,[\overline{\chi},M_\mu]\\
&- \frac{1}{2}D_c\,\bar\eta^c_{\mu\nu}\,[a^\mu,a^\nu] - [a_\mu,\overline{\chi}]\,[a^\mu,\chi] + \frac{1}{2}\,[\overline{\chi},\chi]\,[\overline{\chi},\chi] + \mathcal{F}^{\mu\nu}a_\nu\,[\overline{\chi},a_\mu]\\
&- \frac{1}{2}\lambda_c\,Q'^2\lambda^c + \frac{1}{2}D_c\,D^c - \mu^T Q'^2\mu' + h^T h' - f_c\,\lambda^c\eta\\
&+ \mathrm{i}\,f_c\,D^c\,\overline{\chi} + \overline{\mathcal{F}}^{\mu\nu}a_\mu\,Q'^2 a_\nu + \overline{\mathcal{F}}^{\mu\nu}M_\mu M_\nu\Big\}\ .
\end{aligned} \tag{4.33}$$

To this action we should add the classical part

$$S_{\mathrm{cl}} = -2\pi\mathrm{i}\tau\,k = \frac{2\pi\mathrm{i}}{g_s}\,k \tag{4.34}$$

which represents the topological normalization of the pure D(–1) disk amplitude with multiplicity k and no moduli insertions [9, 52]. If a non-zero vacuum expectation value for the Ramond-Ramond scalar C_0 is present, τ is promoted to the usual combination $\tau = C_0 + \frac{\mathrm{i}}{g_s}$.

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5 Non-perturbative effective action from stringy instantons

To obtain the non-perturbative contributions to the D3 brane effective action induced by the stringy instantons, we need to compute the partition function[7]

$$Z_k = \mathcal{N}_k \int d\mathcal{M}_k \, \mathrm{e}^{-S'(\mathcal{F},\overline{\mathcal{F}},\phi)} \tag{5.1}$$

where $\mathcal{N}_k$ is a normalization that contains also the appropriate power of the scale factor needed to compensate for the dimensions of the moduli measure $d\mathcal{M}_k$. For $N = 2$, which is the case we will consider in detail, the normalization $\mathcal{N}_k$ is simply a numerical factor because in this case the moduli measure is dimensionless (see eq. (4.22)). We now evaluate the integrals in (5.1) in the semiclassical approximation, which due to the BRST structure of the instanton action actually turns out to be exact. One way to see this is to rescale the BRST doublets in the following way [41]

$$\begin{aligned} &(a_\mu, M_\mu) \to \frac{1}{x}\,(a_\mu, M_\mu)\,, \quad (\overline{\chi}, \eta) \to \frac{1}{x}\,(\overline{\chi}, \eta)\,, \\ &(\lambda_c, D_c) \to x^2\,(\lambda_c, D_c)\,, \quad (\mu, h) \to x^2\,(\mu, h)\,, \quad (\mu', h') \to x^2\,(\mu', h')\,, \end{aligned} \tag{5.2}$$

and the anti-holomorphic background as

$$\overline{\mathcal{F}}_{\mu\nu} \to z\,\overline{\mathcal{F}}_{\mu\nu}\ . \tag{5.3}$$

The partition function Z_k does not depend on x and z; indeed x only appears through a change of integration variables which leaves the measure $d\mathcal{M}_k$ invariant, while z is introduced through $\overline{\mathcal{F}}_{\mu\nu}$ which only appears inside the gauge fermion Ξ' as shown in (4.31). Thus, we can choose these parameters to simplify as much as possible the structure of Z_k. In particular, taking the limit

$$x \to \infty\,, \quad z \to \infty \qquad \text{with} \quad \frac{z}{x^2} \to \infty\,, \tag{5.4}$$

the moduli action (4.33) reduces to

$$\begin{aligned} S'(\mathcal{F},\overline{\mathcal{F}},\phi) = \mathrm{tr}\,\Big\{ &-\frac{s}{2}\lambda_c\,Q'^2\lambda^c + \frac{s}{2}D_c\,D^c - s\,\mu^T Q'^2\mu' + s\,h^T h' - t\,f_c\,\lambda^c\eta \\ &+ \mathrm{i}\,t\,f_c\,D^c\,\overline{\chi} + u\,\overline{\mathcal{F}}^{\mu\nu}a_\mu\,Q'^2 a_\nu + u\,\overline{\mathcal{F}}^{\mu\nu}M_\mu M_\nu\Big\} \ + \cdots\ . \end{aligned} \tag{5.5}$$

Here we have introduced the coupling constants

$$s = \frac{x^4}{g_0^2}\,, \quad t = \frac{x}{g_0^2}\,, \quad u = \frac{z}{x^2\,g_0^2}\,, \tag{5.6}$$

which all tend to ∞ due to (5.4), and have denoted with dots the terms of the first two lines of (4.33) which are subleading in this limit. The integrals over the moduli can now be easily computed.

[7] Here, for simplicity we have omitted the exponential of minus the classical instanton action, $\mathrm{e}^{2\pi\mathrm{i}\tau k}$; we will restore these factors later on.

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To evaluate these integrals we choose the external background $\mathcal{F}_{\mu\nu}$ along the Cartan directions of $\mathrm{SU}(2) \times \mathrm{SU}(2)'$, namely in (4.26) we take

$$f_c = f\,\delta_{c3}\,, \quad \bar{f}_c = \bar{f}\,\delta_{c3}\,, \tag{5.7}$$

so that

$$\mathcal{F} = -\frac{\mathrm{i}}{2}\,\bar{f}\,\eta^3 - \frac{\mathrm{i}}{2}\,f\,\overline{\eta}^3 = -\frac{\mathrm{i}}{2}\begin{pmatrix} 0 & (\bar{f}+f) & 0 & 0 \\ -(\bar{f}+f) & 0 & 0 & 0 \\ 0 & 0 & 0 & (\bar{f}-f) \\ 0 & 0 & -(\bar{f}-f) & 0 \end{pmatrix}\,. \tag{5.8}$$

When the choice (5.7) is inserted in (5.5), the fermion η only appears in the term proportional to $(f\,\lambda_3\,\eta)$. Thus, the integration over η and λ_3 can be performed simultaneously producing a factor of $t\,f$, and all other terms containing λ_3 can be neglected. On the other hand, the boson $\overline{\chi}$ only appears in the term proportional to $(f\,D_3\,\overline{\chi})$, so that the Gaussian integration over D_3 and $\overline{\chi}$ produces a factor of $1/(t\,f)$. In the end the integral over the BRST quartet formed by λ_3, D_3, η and $\overline{\chi}$ simply produces a numerical constant which we absorb in the overall normalization factor $\mathcal{N}_k$ of the instanton partition function.

Once this is done, we are left with the integrals over the BRST pairs (a_μ, M_μ), (μ, h), (μ', h') and $(\lambda_{\hat{c}}, D_{\hat{c}})$ with $\hat{c} = 1, 2$, plus of course the integral over χ. The integrals over the BRST pairs are all Gaussian in the semiclassical limit we are considering, and can be easily performed yielding

$$\begin{aligned} &\int (d\lambda_{\hat{c}} dD_{\hat{c}})\; \mathrm{e}^{\mathrm{tr}\,\{\frac{s}{2}\lambda_{\hat{c}}\,Q'^2\lambda^{\hat{c}} - \frac{s}{2}D_{\hat{c}}\,D^{\hat{c}}\}} \times \int (d\mu dh)\,(d\mu' dh')\; \mathrm{e}^{\mathrm{tr}\,\{s\,\mu^T Q'^2 \mu' - s\,h^T h'\}} \\ &\times \int (da_\mu dM_\mu)\; \mathrm{e}^{-\mathrm{tr}\,\{u\,\overline{\mathcal{F}}^{\mu\nu} a_\mu\,Q'^2 a_\nu + u\,\overline{\mathcal{F}}^{\mu\nu} M_\mu M_\nu\}} \;\sim\; \mathcal{P}(\chi) \times \mathcal{R}(\chi) \times \frac{1}{\mathcal{Q}(\chi)}\,. \end{aligned} \tag{5.9}$$

In the last step we have defined

$$\mathcal{P}(\chi) \equiv \mathrm{Pf}_{\left(\begin{smallmatrix}\square\\\square\end{smallmatrix},\mathbf{1},(\mathbf{1},\mathbf{3})'\right)}\left(Q'^2\right)\,, \tag{5.10a}$$

$$\mathcal{R}(\chi) \equiv \det{}_{\left(\square,\overline{\mathbf{N}},(\mathbf{1},\mathbf{1})\right)}\left(Q'^2\right)\,, \tag{5.10b}$$

$$\mathcal{Q}(\chi) \equiv \det{}^{1/2}_{\left(\square\square,\mathbf{1},(\mathbf{2},\mathbf{2})\right)}\left(Q'^2\right)\,, \tag{5.10c}$$

where the labels on the Pfaffian and determinants specify the representations on which Q'^2 acts,[8] and neglected all numerical factors that are absorbed in the overall normalization. Thus, the k-instanton partition function is given in terms of the (super) determinant of Q'^2 evaluated at the fixed points of Q' in agreement with the localization formulas [38, 53, 54], and can be expressed in the following form

$$Z_k = \mathcal{N}_k \int \Big\{\frac{d\chi}{2\pi\mathrm{i}}\Big\}\,\frac{\mathcal{P}(\chi)\,\mathcal{R}(\chi)}{\mathcal{Q}(\chi)}\,. \tag{5.11}$$

[8] In the first line of eq. (5.10), $(\mathbf{1},\mathbf{3})'$ means that the component of the BRST pair (λ_c, D_c) along the null weight must not be considered, since it has been already integrated out with the quartet mechanism.

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Notice that, as we anticipated above, the result does not depend on the anti-holomorphic background $\overline{\mathcal{F}}_{\mu\nu}$, nor on the coupling constants s, t and u.

Since the integrand in (5.11) is singular when the denominator $\mathcal{Q}(\chi)$ vanishes and tends to one when $\chi \to \infty$, the integral over χ is naively divergent and must be suitably defined to make sense. Here we follow exactly the same prescription of ref. [55], which has already been tested for the stringy instanton calculus in several explicit examples [41–44]. In particular, we cure the singularities along the integration path by giving the zeroes of $\mathcal{Q}(\chi)$ a small positive imaginary part moving them in the upper-half complex plane, and regulate the divergence at infinity by interpreting the χ-integral as a contour integral.

5.1 Explicit results for small instanton numbers

We will now derive the explicit expression of the partition function for low instanton numbers in the SU(2) theory. The case of SU(N) will be considered in a separate publication [49].

5.1.1 $k = 1$

The 1-instanton partition function Z_1 is particularly simple: in fact, for $k = 1$ there are no λ_c's and no χ's, so that the factor $\mathcal{P}(\chi)$ is not generated and no contour integral has to be evaluated. Furthermore, for $k = 1$ we simply have

$$\begin{aligned} \mathcal{R}(\chi) &\propto \det \phi \,, \\ \mathcal{Q}(\chi) &\propto \det{}^{1/2} \mathcal{F} \propto E_1 E_2 \equiv \mathcal{E} \,, \end{aligned} \tag{5.12}$$

where we have defined

$$E_1 = \frac{f + \bar{f}}{2} \,, \quad E_2 = \frac{f - \bar{f}}{2} \,, \tag{5.13}$$

and neglected all numerical factors. Absorbing the latter into the overall normalization, we eventually find

$$Z_1 = \mathcal{N}_1 \, \frac{\det \phi}{\mathcal{E}} \,. \tag{5.14}$$

Notice that the factor $1/\mathcal{E}$ in the above result can be interpreted as the regulated volume of the four-dimensional $\mathcal{N} = 2$ superspace [38, 50], since for $k = 1$ the moduli a_μ and M_μ are identified with the superspace coordinates.

5.1.2 $k > 1$

In this case, in order to perform the integration over the χ's we exploit the SO(k) invariance of the integrand in (5.11) and, at the price of introducing a Vandermonde determinant $\Delta(\chi)$, bring the χ's to the Cartan subalgebra, whose generators we denote as $H^i_{\mathrm{SO}(k)}$, i.e.

$$\chi \;\; \to \;\; \vec{\chi} \cdot \vec{H}_{\mathrm{SO}(k)} \;\; = \sum_{i=1}^{\mathrm{rank}\,\mathrm{SO}(k)} \chi_i \, H^i_{\mathrm{SO}(k)} \,. \tag{5.15}$$

Then the partition function becomes

$$Z_k = \mathcal{N}_k \int \prod_i \left(\frac{d\chi_i}{2\pi \mathrm{i}} \right) \Delta(\vec{\chi}) \, \frac{\mathcal{P}(\vec{\chi}) \, \mathcal{R}(\vec{\chi})}{\mathcal{Q}(\vec{\chi})} \,. \tag{5.16}$$

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Again, we have absorbed all numerical factors produced by the "diagonalization" of χ into a redefinition of the normalization coefficient $\mathcal{N}_k$. Furthermore, without any loss of generality we can assume that also the vacuum expectation value ϕ belongs to the Cartan direction of SU(2), namely

$$\phi = \frac{\varphi}{2}\,\tau^3 \ . \tag{5.17}$$

Let us now consider the 2-instanton partition function. As shown in detail in appendix A, for $k = 2$ we have

$$\begin{aligned} &\mathcal{P}(\vec{\chi}) \propto -(E_1 + E_2)\,, \quad \mathcal{R}(\vec{\chi}) \propto \left(\chi^2 + \det\phi\right)^2\,, \\ &\mathcal{Q}(\vec{\chi}) \propto \mathcal{E}\prod_{A=1}^{2}(2\chi - E_A)(2\chi + E_A)\,, \quad \Delta(\vec{\chi}) = 1\,, \end{aligned} \tag{5.18}$$

so that

$$Z_2 = -\mathcal{N}_2\,\frac{E_1 + E_2}{\mathcal{E}}\int\frac{d\chi}{2\pi\mathrm{i}}\,\frac{\left(\chi^2 + \det\phi\right)^2}{(4\chi^2 - E_1^2)(4\chi^2 - E_2^2)}\ . \tag{5.19}$$

As we mentioned in the previous subsection, the χ-integral must be understood as a contour integral in the upper-half complex plane and the singularities at the zeroes of the denominator in (5.19) are avoided by giving the deformation parameters E_A a small positive imaginary part, according to the prescriptions of refs. [55]. In particular, we choose

$$\mathrm{Im}\,E_1 > \mathrm{Im}\,E_2 > \mathrm{Im}\,\frac{E_1}{2} > \mathrm{Im}\,\frac{E_2}{2} > 0\ . \tag{5.20}$$

Evaluating the residues, we finally obtain

$$Z_2 = \frac{\mathcal{N}_2}{4\,\mathcal{E}^2}\,\det^2\phi - \frac{\mathcal{N}_2}{8\,\mathcal{E}}\,\det\phi - \frac{\mathcal{N}_2}{64\,\mathcal{E}}\left[(E_1^2 + E_2^2) + \mathcal{E}\right]\ . \tag{5.21}$$

The calculation for $k = 3$ proceeds in the same way, even if it is algebraically a bit more involved. Some technical details are given in appendix A; here we simply quote the final result, namely

$$Z_3 = \frac{\mathcal{N}_3}{12\,\mathcal{E}^3}\,\det^3\phi - \frac{\mathcal{N}_3}{8\,\mathcal{E}^2}\,\det^2\phi - \frac{\mathcal{N}_3}{192\,\mathcal{E}^2}\left[3(E_1^2 + E_2^2) - 5\mathcal{E}\right]\,\det\phi\ . \tag{5.22}$$

The explicit expressions for Z_4 and Z_5 can be obtained as well and they are given in appendix A. Since they are rather cumbersome, we refrain from writing them here; however, we report the terms with the highest power of $\mathcal{E}$ in the denominator, namely

$$Z_4 = \frac{\mathcal{N}_4}{48\,\mathcal{E}^4}\,\det^4\phi + \cdots\,, \tag{5.23a}$$

$$Z_5 = \frac{\mathcal{N}_5}{240\,\mathcal{E}^5}\,\det^5\phi + \cdots\,, \tag{5.23b}$$

which will be useful for the subsequent calculations.

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5.2 The non-perturbative prepotential

From the partition functions Z_k computed above, we define the "grand-canonical" instanton partition function

$$\mathcal{Z} = \sum_{k=0}^{\infty} Z_k \, \mathrm{e}^{2\pi \mathrm{i} \tau k} = \sum_{k=0}^{\infty} Z_k \, q^k \tag{5.24}$$

where we have set $Z_0 = 1$ and $q \equiv \exp(2\pi \mathrm{i} \tau)$. To obtain the non-perturbative D3 brane effective action induced by the stringy instantons and remove the disconnected contributions, we have to take the logarithm of $\mathcal{Z}$. Notice that the partition functions Z_k have been computed by integrating over *all* moduli, including the instanton "center-of-mass" coordinates and their superpartners playing the rôle of the superspace coordinates. In the absence of the Ramond-Ramond deformations these zero-modes do not appear in the moduli action and the integration over them would diverge. In the presence of deformations, instead, this integration yields a factor of $1/\mathcal{E}$ (as is clearly seen from the $k = 1$ result (5.14)), and thus to extract the integral over the centered moduli only, it is sufficient to multiply $\log \mathcal{Z}$ by $\mathcal{E}$. Having done so, we can promote the vacuum expectation value ϕ appearing in $\mathcal{Z}$ to the full fledged dynamical superfield $\Phi(x, \theta)$ and, after removing the Ramond-Ramond deformations, we finally obtain the non-perturbative contributions to the D3 brane effective action, namely

$$S^{(\mathrm{n.p.})} = \int d^4x \, d^4\theta \, F^{(\mathrm{n.p.})}\big(\Phi(x, \theta)\big) \tag{5.25}$$

where the "prepotential" $F^{(\mathrm{n.p.})}(\Phi)$ is

$$F^{(\mathrm{n.p.})}(\Phi) = \mathcal{E} \, \log \mathcal{Z} \Big|_{\phi \to \Phi, E_A \to 0} \, . \tag{5.26}$$

Expanding in powers of q, we have

$$F^{(\mathrm{n.p.})}(\Phi) = \sum_{k=1}^{\infty} F_k \, q^k \, \Big|_{\phi \to \Phi, E_A \to 0} \tag{5.27}$$

where the first few coefficients are

$$\begin{aligned}
F_1 &= \mathcal{E} Z_1 \, , \\
F_2 &= \mathcal{E} Z_2 - \frac{F_1^2}{2\mathcal{E}} \, , \\
F_3 &= \mathcal{E} Z_3 - \frac{F_2 F_1}{\mathcal{E}} - \frac{F_1^3}{6\mathcal{E}^2} \, , \\
F_4 &= \mathcal{E} Z_4 - \frac{F_3 F_1}{\mathcal{E}} - \frac{F_2^2}{2\mathcal{E}} - \frac{F_2 F_1^2}{2\mathcal{E}^2} - \frac{F_1^4}{24\mathcal{E}^3} \, , \\
F_5 &= \mathcal{E} Z_5 - \frac{F_4 F_1}{\mathcal{E}} - \frac{F_3 F_2}{\mathcal{E}} - \frac{F_3 F_1^2}{2\mathcal{E}^2} - \frac{F_2^2 F_1}{2\mathcal{E}^2} - \frac{F_2 F_1^3}{6\mathcal{E}^3} - \frac{F_1^5}{120\mathcal{E}^4} \, .
\end{aligned} \tag{5.28}$$

The prepotential $F^{(\mathrm{n.p.})}(\Phi)$ must be well-defined when the closed-string deformations are turned off, and thus all coefficients F_k must be finite in the limit $E_A \to 0$. From (5.28) and the expressions of Z_k we derived in the previous subsection, we see that the F_k's contain

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singular terms diverging as $1/\mathcal{E}^{k-1}$, $1/\mathcal{E}^{k-2}, \cdots$ for $E_A \to 0$. Imposing the cancellation of the most divergent terms of F_k fixes the overall normalization $\mathcal{N}_k$ but, once this choice is made, no freedom is left and all the remaining divergences must disappear. Verifying that this happens is a very strong check on our results and the consistency of the whole procedure.

For $k = 1$, from eq. (5.14) we have directly

$$F_1 = \mathcal{N}_1 \det \phi \,. \tag{5.29}$$

This is the same result obtained in [19]. For $k = 2$, from eqs. (5.28) and (5.21) we find

$$F_2 = \left(\frac{\mathcal{N}_2}{4} - \frac{\mathcal{N}_1^2}{2}\right) \frac{\det^2 \phi}{\mathcal{E}} - \frac{\mathcal{N}_2}{8} \det \phi - \frac{\mathcal{N}_2}{64} \left[(E_1^2 + E_2^2) + \mathcal{E}\right] \,. \tag{5.30}$$

If we choose

$$\mathcal{N}_2 = 2\mathcal{N}_1^2 \,, \tag{5.31}$$

the most divergent term disappears, and we are left with

$$F_2 = -\frac{\mathcal{N}_1^2}{4} \det \phi - \frac{\mathcal{N}_1^2}{32} \left[(E_1^2 + E_2^2) + \mathcal{E}\right] \tag{5.32}$$

which is indeed finite when $E_A \to 0$. We then proceed in the same way at the next order, $k = 3$. Using eq. (5.22) and inserting the above expressions for F_1 and F_2 in (5.28), we find

$$F_3 = \left(\frac{\mathcal{N}_3}{12} - \frac{\mathcal{N}_1^3}{6}\right) \frac{\det^3 \phi}{\mathcal{E}^2} + \dots \,, \tag{5.33}$$

so that we have to choose

$$\mathcal{N}_3 = 2\mathcal{N}_1^3 \,. \tag{5.34}$$

Once this is done, all other divergences in F_3 cancel and we are simply left with

$$F_3 = \frac{\mathcal{N}_1^3}{12} \det \phi \,. \tag{5.35}$$

For $k = 4$, we use the partition function Z_4 given in appendix A and again require the cancellation of most divergent term in the resulting expression for F_4 following from eq. (5.28). This fixes $\mathcal{N}_4 = 2\mathcal{N}_1^4$. Using this, we then find

$$F_4 = -\frac{\mathcal{N}_1^4}{32} \det \phi - \frac{\mathcal{N}_1^4}{256} \left[(E_1^2 + E_2^2) + \mathcal{E}\right] \,, \tag{5.36}$$

which has a finite limit when $E_A \to 0$. In the case $k = 5$, having computed Z_5 along the lines described in appendix A, the cancellation of the highest divergence in F_5 leads to $\mathcal{N}_5 = 2\mathcal{N}_1^5$, after which we get

$$F_5 = \frac{\mathcal{N}_1^5}{80} \det \phi \,. \tag{5.37}$$

Making the replacement $\phi \to \Phi(x, \theta)$ and taking the limit $E_A \to 0$ in the above results, we finally obtain from the non-perturbative prepotential of the SU(2) gauge theory induced by the stringy instantons. Up to instanton number $k = 5$, our findings are summarized in

$$F^{(\text{n.p.})}(\Phi) = -\operatorname{Tr} \Phi^2 \left(\frac{\mathcal{N}_1}{2} q - \frac{\mathcal{N}_1^2}{8} q^2 + \frac{\mathcal{N}_1^3}{24} q^3 - \frac{\mathcal{N}_1^4}{64} q^4 + \frac{\mathcal{N}_1^5}{160} q^5 \dots\right) , \tag{5.38}$$

where we made use of the relation $\det \phi = -\frac{1}{2} \operatorname{Tr} \phi^2$ which easily follows from eq. (5.17).

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6 Summary and conclusions

The detailed analysis presented in the previous sections shows that the stringy instantons have the right content of zero-modes to produce non-perturbative terms in the $\mathcal{N} = 2$ SU(N) theories in four dimensions realized with fractional D3 branes in a $\mathbb{C}^3/\mathbb{Z}_3$ orientifold. For the SU(2) model such terms have been explicitly computed using localization methods up to instanton number $k = 5$, and have been shown to provide non-perturbative corrections to the effective prepotential of the theory.

It is worth to remark that these results are unconventional from a purely field-theory point of view, but are quite natural in the stringy approach to the instanton calculus. In fact, the exotic instantons in our model are fractional D(–1) branes that occupy quiver nodes where no D3 branes are present but, apart from this feature, they are completely standard D-instantons, possessing their "own life" independently of the existence of an underlying gauge theory. What is non-standard, however, is the content of their moduli space: indeed, the charged zero modes corresponding to the mixed open strings stretching between the stringy instantons and the gauge branes are only fermionic, and the neutral zero-modes corresponding to open strings with both end-points on the stringy instantons are in representations of orthogonal groups even if the gauge groups are unitary. This is to be contrasted with what happens for the ordinary instantons in theories with unitary gauge groups, where the charged zero-modes are both bosonic and fermionic and the neutral zero-modes fall into representations of unitary groups if the gauge group is unitary. These differences result in a different structure of the moduli integral and in a different scaling dimension of the integration measure on moduli space. For the SU(2) model the integration measure turns out to be dimensionless (see eq. (4.22)) and thus the prepotential of the theory can receive contributions from exotic configurations with any instanton number.[9] Notice that in this SU(2) model the supersymmetry is enhanced at tree-level from $\mathcal{N} = 2$ to $\mathcal{N} = 4$, because the SU(2) symmetric representation in which the hypermultiplet transforms is equivalent to the adjoint representation. Therefore, this model can be regarded as a non-conventional realization of an $\mathcal{N} = 4$ SU(2) super Yang-Mills theory in four dimensions. As is well known, the usual gauge instantons in this case do not contribute to the (quadratic) effective action; on the contrary, as we have explicitly shown, the stringy instantons do. Furthermore, since they only correct the prepotential, the above supersymmetry enhancement is lost at the non-perturbative level. From our results up to instanton number $k = 5$ (see eq. (5.38)), it is very natural to conjecture that the stringy instanton series of the SU(2) theory can be resummed into

$$F^{(\text{n.p.})}(\Phi) = -\text{Tr}\,\Phi^2\,\log\left(1 + \frac{\mathcal{N}_1}{2}\,q\right) \tag{6.1}$$

where the non-vanishing constant $\mathcal{N}_1$ can be fixed by a careful analysis of the normalization of the 1-instanton partition function. This seems to suggest that the stringy instantons induce a non-perturbative redefinition of the gauge coupling constant of logarithmic type,

[9] This is similar to what happens with ordinary instantons in the $\mathcal{N} = 2$ SU(2) gauge theory with four fundamental flavors.

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so that the SU(2) prepotential appears classical in terms of the new coupling. We notice that a similar logarithmic redefinition of the coupling constant has been observed in a conformal $SO(4) \simeq SU(2) \times SU(2)$ theory realized with D7 branes on a $\mathbb{C}^2/\mathbb{Z}_2$ singularity in the presence of exotic instantons [42]. It would be interesting to understand whether such non-perturbative redefinitions may have some deeper meaning.

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A Details on the D-instanton computations

The expressions of the functions $\mathcal{P}(\chi)$, $\mathcal{R}(\chi)$, $\mathcal{Q}(\chi)$ defined in (5.10) and appearing in the integrand of the instanton partition function (5.11), can be expressed in terms of the weights of the relevant representations of the instantonic symmetry group $SO(k)$, of the twisted Lorentz group $SU(2) \times SU(2)'$ and of the gauge group SU(2) . For convenience we recall the form of these weight vectors.

Weight sets of SO$(2n+1)$: This group has rank n. If we denote by $\vec{e}_i$ the versors in the $\mathbb{R}^n$ weight space, then

- the set of the $2n+1$ weights $\vec{\pi}$ of the vector representation is given by

$$\pm \vec{e}_i\,, \qquad \vec{0} \ \text{ with multiplicity } 1\ ; \tag{A.1}$$

- the set of $n(2n+1)$ weights $\vec{\rho}$ of the adjoint representation (corresponding to the two-index antisymmetric tensor) is the following:

$$\pm \vec{e}_i \pm \vec{e}_j \ (i<j)\,, \qquad \pm \vec{e}_i\,, \qquad \vec{0} \ \text{ with multiplicity } n\ ; \tag{A.2}$$

- the $(n+1)(2n+1)$ weights $\vec{\sigma}$ of the two-index symmetric tensor[10] are

$$\pm \vec{e}_i \pm \vec{e}_j \ (i<j)\,, \qquad \pm \vec{e}_i\,, \qquad \pm 2\vec{e}_i\,, \qquad \vec{0} \ \text{ with multiplicity } n+1\ . \tag{A.3}$$

Weight sets of SO$(2n)$. This group has rank n. If we denote by $\vec{e}_i$ the versors in the $\mathbb{R}^n$ weight space, then

- the set of the $2n$ weights $\vec{\pi}$ of the vector representation is given by

$$\pm \vec{e}_i\ ; \tag{A.4}$$

[10] In fact, this is not an irreducible representation: it decomposes into the $(n+1)(2n+1)-1$ traceless symmetric tensor plus a singlet. One of the $\vec{0}$ weights corresponds to the singlet.

- the set of $n(2n-1)$ weights $\vec{\rho}$ of the two-index antisymmetric tensor is the following:

$$\pm\vec{e}_i \pm \vec{e}_j \ (i<j)\,, \qquad \vec{0} \ \text{ with multiplicity } n\ ; \tag{A.5}$$

- the $n(2n+1)$ weights $\vec{\sigma}$ of the two-index symmetric tensor[11] are

$$\pm\vec{e}_i \pm \vec{e}_j \ (i<j)\,, \qquad \pm 2\vec{e}_i\,, \qquad \vec{0} \ \text{ with multiplicity } n\ . \tag{A.6}$$

Weight sets of SU(2)×SU(2)′. The relevant representations of the twisted Lorentz group are the $(\mathbf{1},\mathbf{3})$ in which the BRST pair (λ_c, D_c) transforms, and the $(\mathbf{2},\mathbf{2})$ in which the BRST pair (a_μ, M_μ) transforms.

- the weights $\vec{\alpha}$ of the $(\mathbf{1},\mathbf{3})$ representation are given by the following two-component vectors

$$(0,\pm 1)\,, \qquad (0,0)\ . \tag{A.7}$$

  In our conventions, the weight $(0,+1)$ is considered to be positive.

- the weights $\vec{\beta}$ of the $(\mathbf{2},\mathbf{2})$ representation are given by the following two-component vectors

$$\big(\pm\,1/2, \pm 1/2\big)\ . \tag{A.8}$$

  The weights $\big(\pm\,1/2, +1/2\big)$ are considered positive in our conventions.

Weight sets of SU(2). In this case, the only relevant SU(2) representation that occurs in our analysis is the fundamental one, for which the two weights $\vec{\gamma}$ is simply given by $\pm 1/2$.

To evaluate the moduli integral and obtain the instanton partition function it is convenient to align the vacuum expectation value ϕ of the chiral multiplet along the Cartan direction of SU(2), and the external Ramond-Ramond background $\mathcal{F}$ along the Cartan directions of $\mathrm{SU}(2)\times\mathrm{SU}(2)'$, namely

$$\phi = \vec{\phi}\cdot\vec{H}_{\mathrm{SU}(2)} \quad \text{and} \quad \mathcal{F} = \vec{f}\cdot\vec{H}_{\mathrm{SU}(2)\times\mathrm{SU}(2)'}\ . \tag{A.9}$$

Comparing with eqs. (5.8) and (5.17), we see that

$$\vec{\phi} = \varphi \quad \text{and} \quad \vec{f} = (\bar{f}, f)\ . \tag{A.10}$$

Furthermore, exploiting the SO(k) invariance, we arrange the χ moduli along the Cartan directions, namely

$$\chi \to \vec{\chi}\cdot\vec{H}_{\mathrm{SO}(k)} = \sum_{i=1}^{n}\chi_i H^i_{\mathrm{SO}(k)}\,, \tag{A.11}$$

at the price of introducing in the integrand a Vandermonde determinant given by

$$\Delta(\vec{\chi}) = \prod_{\vec{\rho}\neq\vec{0}}\vec{\chi}\cdot\vec{\rho} = \begin{cases} \displaystyle\prod_{i<j}\big(\chi_i^2-\chi_j^2\big)^2 & \text{for } k=2n\,, \\[2ex] \displaystyle(-1)^n\prod_{i=1}^{n}\chi_i^2\prod_{j<\ell}\big(\chi_j^2-\chi_\ell^2\big)^2 & \text{for } k=2n+1\ . \end{cases} \tag{A.12}$$

[11] Again, this is not an irreducible representation, since it contains a singlet.

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With all these definitions at hand, we can now give the explicit expressions for the functions $\mathcal{P}(\chi)$, $\mathcal{R}(\chi)$, $\mathcal{Q}(\chi)$. From eq. (5.10a), we have

$$\mathcal{P}(\vec{\chi}) = \prod_{\vec{\rho}} \prod_{\vec{\alpha}}^{+} \left(\vec{\chi} \cdot \vec{\rho} - \vec{f} \cdot \vec{\alpha} \right)$$
$$= \begin{cases} (-f)^n \prod_{i<j}^{n} \left[(\chi_i + \chi_j)^2 - f^2 \right] \left[(\chi_i - \chi_j)^2 - f^2 \right] & \text{for } k = 2n \,, \\ f^n \prod_{i}^{n} (\chi_i^2 - f^2) \prod_{j<\ell}^{n} \left[(\chi_j + \chi_\ell)^2 - f^2 \right] \left[(\chi_j - \chi_\ell)^2 - f^2 \right] & \text{for } k = 2n+1 \,. \end{cases} \tag{A.13}$$

where the product over $\vec{\alpha}$ is limited to the positive weight $(0, +1)$. This is the meaning of the superscript $+$ appearing above. From eq. (5.10b), we have

$$\mathcal{R}(\vec{\chi}) = \prod_{\vec{\pi}} \prod_{\vec{\gamma}} \left(\vec{\chi} \cdot \vec{\pi} - \vec{\phi} \cdot \vec{\gamma} \right) = \begin{cases} \prod_{i=1}^{n} \left(\chi_i^2 + \det \phi \right)^2 & \text{for } k = 2n \,, \\ \det \phi \prod_{i=1}^{n} \left(\chi_i^2 + \det \phi \right)^2 & \text{for } k = 2n+1 \,. \end{cases} \tag{A.14}$$

and finally from eq. (5.10c), we have

$$\mathcal{Q}(\vec{\chi}) = \prod_{\vec{\sigma}} \prod_{\vec{\beta}}^{+} \left(\vec{\chi} \cdot \vec{\sigma} - \vec{f} \cdot \vec{\beta} \right)$$
$$= \begin{cases} \mathcal{E}^n \prod_{A=1}^{2} \prod_{i=1}^{n} (4\chi_i^2 - E_A^2) \prod_{j<\ell} \left[(\chi_j + \chi_\ell)^2 - E_A^2 \right] \left[(\chi_j - \chi_\ell)^2 - E_A^2 \right] & \text{for } k = 2n \,, \\ \mathcal{E}^{n+1} \prod_{A=1}^{2} \left\{ \prod_{i=1}^{n} (\chi_i^2 - E_A^2)(4\chi_i^2 - E_A^2) \times \right. & \\ \left. \qquad \times \prod_{j<\ell} \left[(\chi_j + \chi_\ell)^2 - E_A^2 \right] \left[(\chi_j - \chi_\ell)^2 - E_A^2 \right] \right\} & \text{for } k = 2n+1 \,, \end{cases} \tag{A.15}$$

where again the product over $\vec{\beta}$ is limited to the positive weights.

Using these definitions and recalling from eq. (5.13) that $f = (E_1 + E_2)$, it is easy to find that at instanton number $k = 2$ the partition function (5.16) reads

$$Z_2 = -\mathcal{N}_2 \, \frac{E_1 + E_2}{\mathcal{E}} \int \frac{d\chi}{2\pi \mathrm{i}} \, \frac{\left(\chi^2 + \det \phi \right)^2}{(4\chi^2 - E_1^2)(4\chi^2 - E_2^2)} \,. \tag{A.16}$$

as reported in eq. (5.19) of the main text. Evaluating the χ integral as a contour integral in the upper half complex plane with the pole prescription (5.20), and summing the residues at $\chi = E_A$ and $\chi = E_A/2$ for $A = 1, 2$, we eventually find the result given in eq. (5.21). Proceeding in a similar way, at instanton number $k = 3$ we find

$$Z_3 = -\mathcal{N}_3 \, \frac{\det \phi \, (E_1 + E_2)}{\mathcal{E}^2} \int \frac{d\chi}{2\pi \mathrm{i}} \, \frac{\left(\chi^2 - (E_1 + E_2)^2 \right) \left(\chi^2 + \det \phi \right)^2}{(\chi^2 - E_1^2)(\chi^2 - E_2^2)(4\chi^2 - E_1^2)(4\chi^2 - E_2^2)} \,, \tag{A.17}$$

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from which the result given in eq. (5.22) follows.

We conclude by giving the explicit expressions of the instanton partition functions at $k = 4$ and $k = 5$. They are

$$\begin{aligned} Z_4 = {} & \frac{\mathcal{N}_4}{48\,\mathcal{E}^4}\det{}^4\phi - \frac{\mathcal{N}_4}{16\,\mathcal{E}^3}\det{}^3\phi - \frac{\mathcal{N}_4}{384\,\mathcal{E}^3}\big[3(E_1^2+E_2^2)-19\mathcal{E}\big]\det{}^2\phi \\ & + \frac{\mathcal{N}_4}{256\,\mathcal{E}^2}\big[(E_1^2+E_2^2)-3\mathcal{E}\big]\det\phi + \frac{\mathcal{N}_4}{4096\,\mathcal{E}^2}\big[(E_1^2+E_2^2)-7\mathcal{E}\big]\big[(E_1^2+E_2^2)+\mathcal{E}\big]\,, \end{aligned} \tag{A.18}$$

and

$$\begin{aligned} Z_5 = {} & \frac{\mathcal{N}_5}{240\,\mathcal{E}^5}\det{}^5\phi - \frac{\mathcal{N}_4}{48\,\mathcal{E}^4}\det{}^4\phi - \frac{\mathcal{N}_5}{384\,\mathcal{E}^4}\big[(E_1^2+E_2^2)-13\mathcal{E}\big]\det{}^3\phi \\ & + \frac{\mathcal{N}_5}{768\,\mathcal{E}^3}\big[3(E_1^2+E_2^2)-17\mathcal{E}\big]\det{}^2\phi \\ & + \frac{\mathcal{N}_5}{61440\,\mathcal{E}^3}\big[15(E_1^4+E_2^4)-170(E_1^2+E_2^2)+299\mathcal{E}^2\big]\det\phi\,. \end{aligned} \tag{A.19}$$

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