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Constraints from LFV processes in the Higgs triplet model

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Abstract: Constraints from lepton flavor violating processes are translated into lower bounds on $v_\Delta m_{H^{\pm\pm}}$ in the Higgs Triplet Model by considering correlations through the neutrino mass matrix. It is shown that $\mu \to e\gamma$, rare τ decays (especially, $\tau \to \bar{\mu}ee$), and the muonium conversion can give a more stringent bound on $v_\Delta m_{H^{\pm\pm}}$ in some parameter regions than the bound from $\mu \to \bar{e}ee$ which is expected naively to give the most stringent one. We consider the cases of suppressed $\mu \to \bar{e}ee$ not only for CP-conserving sets of phases but also for arbitrary values.

Keywords: Higgs Physics, Rare Decays, Beyond Standard Model, Neutrino Physics

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Contents

1 Introduction

In the standard model of the particle physics (SM), neutrinos are massless particles due to the absence of right-handed neutrinos ν_R. The simplest way to give masses to three neutrinos is to add three ν_R similarly to other fermions, which corresponds to six additional degrees of freedom (three ν_R and three $\overline{\nu_R}$) to the SM. In the Higgs triplet model (HTM) [1–3] which we deal with in this article, a complex $\mathrm{SU}(2)_L$ triplet scalar with the hypercharge $Y = 2$ is introduced to the SM in order to have neutrino masses. This model can be regarded as one of the simplest extension of the SM because the number of new degrees of freedom is six in this model also.

The triplet Higgs boson field with hypercharge $Y = 2$ can be parameterized by

$$\Delta \equiv \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \tag{1.1}$$

where the neutral component has a vacuum expectation value (VEV) $v_\Delta = \sqrt{2}\,\langle \Delta^0 \rangle$. The constraint on the rho parameter, $\rho_0 = 1.0004^{+0.0027}_{-0.0007}$ at 2σ CL (page 137 of [4]), gives an upper limit $v_\Delta/v \lesssim 0.01$ where $v = 246\,\mathrm{GeV}$ is the VEV of the doublet Higgs field, which corresponds to $v_\Delta \lesssim 3\,\mathrm{GeV}$. There is no stringent bound from quark sector on triplet Higgs bosons because they do not couple to quarks. The interaction of the Higgs triplet with lepton doublets $L_\ell \equiv (\nu_{\ell L}, \ell_L)^T$ $(\ell = e, \mu, \tau)$ is given by

$$\mathcal{L}_{\text{triplet-Yukawa}} = h_{\ell\ell'}\, \overline{L_\ell^c}\, i\tau_2 \Delta L_{\ell'} + \mathrm{H.\,c.} \tag{1.2}$$

The symmetric matrix $h_{\ell\ell'}$ is coupling strength, $\tau_i (i = 1\text{–}3)$ denote the Pauli matrices, and the superscript c is used for fields with the charge conjugation.

The coupling $h_{\ell\ell'}$ has a direct relation to the neutrino mass matrix $m_{\ell\ell'}$ in the flavor basis through v_Δ as

$$h_{\ell\ell'} = \frac{1}{\sqrt{2}v_\Delta}\left(U_{\text{MNS}}^* \,\text{diag}(m_1, m_2 e^{-i\varphi_1}, m_3 e^{-i\varphi_2})\, U_{\text{MNS}}^\dagger\right)_{\ell\ell'} \equiv \frac{1}{\sqrt{2}v_\Delta} m_{\ell\ell'}. \tag{1.3}$$

The mass eigenvalues m_i are taken to be real positive values. We define $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and refer to the case of $\Delta m_{31}^2 > 0$ ($\Delta m_{31}^2 < 0$) as the normal (inverted) mass ordering. Here neutrinos are required to be Majorana particles,[1] and φ_1 and φ_2 are the Majorana phases [3, 5, 6] defined in an interval of $[0, 2\pi)$. The Maki-Nakagawa-Sakata matrix [7] of the neutrino mixing[2] is parameterized as

$$U_{\text{MNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.4}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$, and δ is the Dirac phase. The ranges are chosen as $0 \le \theta_{ij} \le \pi/2$ and $0 \le \delta < 2\pi$. According to current constraints from neutrino oscillation experiments [9–19], we use the following values in this article

$$\Delta m_{21}^2 = 7.6 \times 10^{-5}\,\text{eV}^2, \qquad |\Delta m_{31}^2| = 2.4 \times 10^{-3}\,\text{eV}^2, \tag{1.5}$$
$$\sin^2 2\theta_{12} = 0.87, \qquad \sin^2 2\theta_{23} = 1, \tag{1.6}$$
$$\sin^2 2\theta_{13} < 0.14\,. \tag{1.7}$$

The absolute scale of the neutrino mass is constrained by tritium beta decay measurements as $m_\nu \le 2.3\,\text{eV}$ (95% CL) [20] and by cosmological observations as $\sum m_i < 0.61\,\text{eV}$ (95% CL) or $\sum m_i < 1.3\,\text{eV}$ (WMAP only, 95% CL) [21, 22].

The HTM has seven physical Higgs bosons which are two CP-even neutral bosons h^0 (lighter) and H^0 (heavier), a CP-odd neutral one A^0, a pair of singly charged bosons $H^\pm$, and a pair of doubly charged bosons $H^{\pm\pm}$. A characteristic particle of the HTM is $H^{\pm\pm}$ and it has been searched at the collider experiment. The limit on the mass of $H^{\pm\pm}$, $m_{H^{\pm\pm}} \ge 112 \to 150\,\text{GeV}$, has been given by searches for $H^{\pm\pm} \to \ell\ell'$ at Tevatron Run II [23–26]. If $H^{\pm\pm} \to \ell\ell'$ is observed in the future at the Tevatron and/or the Large Hadron Collider at CERN, the decay branching ratio will give some important information on the model (e.g., on the neutrino mass matrix) [27–32].

These Higgs bosons contribute to many lepton flavor violating (LFV) processes. Experimental searches for $\mu \to \bar{e}ee$ etc. put upper bounds on $|h_{\mu e}||h_{ee}|/m_{H^{\pm\pm}}^2$ etc. (See e.g. [33]), where $m_{H^{\pm\pm}}$ is the mass of $H^{\pm\pm}$. The couplings $h_{\ell\ell'}$ are, however, not free in the HTM because they relate directly to the neutrino mass matrix $m_{\ell\ell'}$ as shown in (1.3). Previous works for dependences of LFV processes on the parameters in $m_{\ell\ell'}$ can be found

[1] In general, even if the Higgs triplet exists, neutrinos can be Dirac particles by adding ν_R also and requiring lepton number conservation which results in $v_\Delta = 0$. In the HTM we use, neutrino masses are assumed to be given solely by v_Δ and neutrinos are Majorana particles by definition.

[2] We took the definition of the mixing $\nu_\ell = \sum_i U_{\ell i}\nu_i$ according to page 517 of [4] although another definition $\nu_\ell = \sum_i U_{\ell i}^*\nu_i$ is used for example, in [8] and on page 163 of [4]. In latter definition, we need to take complex conjugate in the middle equation of (1.3).

in [8, 34, 35]. In this article, we consider in detail the correlation of upper bounds on $|h^*_{ij}h_{kl}|/m^2_{H^{\pm\pm}}$ from new physics searches and deal with them as lower bounds on $v_\Delta m_{H^{\pm\pm}}$.

2 Lower bound on $v_\Delta m_{H^{\pm\pm}}$

2.1 Constraint from the muon anomalous magnetic dipole moment

Let us consider first the anomalous magnetic dipole moment (MDM) of muon, $a_\mu \equiv (g-2)/2$. The muon anomalous MDM has been measured very precisely [36] as

$$a_\mu^{\rm exp} = 11659208.0(6.3)\times 10^{-10}, \tag{2.1}$$

where the number in parentheses shows 1σ uncertainty. On the other hand, the SM predicts

$$a_\mu^{\rm SM}[\tau] = 11659193.2(5.2)\times 10^{-10}, \tag{2.2}$$
$$a_\mu^{\rm SM}[e^+e^-] = 11659183.4(4.9)\times 10^{-10}, \tag{2.3}$$

where the hadronic contributions to $a_\mu^{\rm SM}[\tau]$ and $a_\mu^{\rm SM}[e^+e^-]$ were calculated [37, 38] by using data of hadronic τ decay and e^+e^- annihilation to hadrons, respectively (See also [39–41, 44, 45]). The deviations of the SM predictions from the experimental result are given by

$$\Delta a_\mu[\tau] \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}[\tau] = 14.8(8.2)\times 10^{-10}, \tag{2.4}$$
$$\Delta a_\mu[e^+e^-] \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}[e^+e^-] = 24.6(8.0)\times 10^{-10}. \tag{2.5}$$

These values of $\Delta a_\mu[\tau]$ and $\Delta a_\mu[e^+e^-]$ correspond to 1.8σ and 3.1σ deviations from SM predictions, respectively.

New contributions to a_μ at the 1-loop level in the HTM come mainly from $H^{\pm\pm}$ and $H^\pm$. The Yukawa interactions of $H^\pm$, which are mixtures of doublet and triplet Higgs bosons, and of $H^{\pm\pm}$ $(=\Delta^{\pm\pm})$ are written by

$$\mathcal{L}^{H^\pm,H^{\pm\pm}}_{\text{triplet-Yukawa}} = -\sqrt{2}\frac{v}{\sqrt{v^2+2v_\Delta^2}}(U^T_{\rm MNS}h)_{i\ell}\,\overline{\nu_i^c}P_L\ell H^+ - h_{\ell\ell'}\overline{\ell^c}P_L\ell' H^{++} + \text{H.c.}, \tag{2.6}$$

where $P_L\equiv(1-\gamma^5)/2$ and ν_i represent mass eigenstates of Majorana neutrinos which satisfy conditions $\nu_i=\nu_i^c$. The 1-loop contribution of $H^\pm$ through the triplet Yukawa interaction[3] is calculated as

$$a_\mu^{H^\pm} = \frac{m_\mu^2}{8\pi^2 m_{H^\pm}^2}\frac{v^2}{v^2+2v_\Delta^2}\sum_i (h^\dagger U^*_{\rm MNS})_{\mu i}(U^T_{\rm MNS}h)_{i\mu}\int_0^1 dt\,\frac{-t^2(1-t)}{R^\mu_{H^\pm}t^2+(1-R^\mu_{H^\pm}-R^i_{H^\pm})t+R^i_{H^\pm}} \tag{2.7}$$

$$\simeq -\frac{\langle m^2\rangle_{\mu\mu}}{96\pi^2}\frac{m_\mu^2}{v_\Delta^2 m_{H^\pm}^2}, \tag{2.8}$$

[3]The contribution through m_μ/v is ignored because it is suppressed by v_Δ^2/v^2.

and the $H^{\pm\pm}$ contribution is given by

$$a_\mu^{H^{\pm\pm}} = \frac{m_\mu^2}{8\pi^2 m_{H^{\pm\pm}}^2} \sum_\ell (h^\dagger)_{\mu\ell} h_{\ell\mu} \int_0^1 dt \left[\frac{-4t^2(1-t)}{R^\mu_{H^{\pm\pm}} t^2 + (1 - R^\mu_{H^{\pm\pm}} - R^\ell_{H^{\pm\pm}})t + R^\ell_{H^{\pm\pm}}} \right.$$
$$\left. + \frac{-2t^2(1-t)}{R^\mu_{H^{\pm\pm}} t^2 + (R^\ell_{H^{\pm\pm}} - R^\mu_{H^{\pm\pm}} - 1)t + 1} \right] \quad (2.9)$$
$$\simeq -\frac{\langle m^2 \rangle_{\mu\mu}}{12\pi^2} \frac{m_\mu^2}{v_\Delta^2 m_{H^{\pm\pm}}^2}. \quad (2.10)$$

Here we have defined

$$R^a_b \equiv \frac{m_a^2}{m_b^2}, \quad (2.11)$$
$$\langle m^2 \rangle_{\ell\ell'} \equiv \Big(U_{\rm MNS}\, {\rm diag}(m_1^2, m_2^2, m_3^2)\, U_{\rm MNS}^\dagger \Big)_{\ell\ell'} = 2v_\Delta^2 (h^\dagger h)_{\ell\ell'}. \quad (2.12)$$

Note that $\langle m^2 \rangle_{\ell\ell'}$ does not depend on Majorana phases and $\langle m^2 \rangle_{\ell\ell}$ is positive definite. Thus $a_\mu^{\rm HTM} \equiv a_\mu^{H^\pm} + a_\mu^{H^{\pm\pm}}$ is negative definite though Δa_μ is positive. The minus sign of the contributions from Higgs triplets has been known [33, 46–50] but it does not seem to be dealt with appropriately in the translation to a constraint on coupling $h_{\ell\ell'}$. There seems to be confusions also about the combination $(h^\dagger h)_{\mu\mu}$. Sometimes the combination was written as $(h_{\mu\mu})^2$ or $(h^2)_{\mu\mu}$, for which sign of $a_\mu^{\rm HTM}$ can be flipped, and then it seemed as if it were possible to obtain a (finite) constraint on $h_{\ell\ell'}$. Actually, any value of $h_{\ell\ell'}$ can not fit $\Delta a_\mu[e^+e^-]$ ($\Delta a_\mu[\tau]$) at 3.1σ (1.8σ) because of the wrong sign of $a_\mu^{\rm HTM}$.

Concerning only on the sign, the 1-loop contribution from H^0 can have the right sign to explain Δa_μ (See [51] for the case in the type II two Higgs doublet model (2HDM-II)). However, the contribution has a suppression with v_Δ^2/v^2 in the HTM because Δ^0 does not couple with charged leptons at the tree level.[4] Although in the Barr-Zee type [52, 53] 2-loop diagrams the right-sign contribution of A^0 can be important in some models like the 2HDM-II [54] and the minimal supersymmetric standard model (MSSM) [55], such a situation does not happen in the HTM because couplings of $A^0 (\simeq {\rm Im}(\Delta^0))$ with quarks and charged leptons are also suppressed by v_Δ/v.

The definite sign of $a_\mu^{\rm HTM}$ is a feature of the simpleness and the predictability of the HTM. In the MSSM in contrast, the contributions from supersymmetric particles to a_μ can have the right sign easily by the appropriate choice of the sign of the Higgs mass parameter μ_H [56, 57]. As the result, the HTM is somehow disfavored by the muon anomalous MDM and it results in a strong constraint on the model. This is also the case for other models (e.g. the Zee-Babu model [58, 59]) which do not have extra neutral Higgs bosons with sizable couplings to charged leptons similarly to the HTM. Of course, the positive Δa_μ does not seem conclusive yet and it does not mean exclusion of the HTM. The difference between $\Delta a_\mu[e^+e^-]$ and $\Delta a_\mu[\tau]$ may indicate existence of new physics in the quark sector which is not modified in the HTM.

[4]The modification of the contribution to $a_\mu^{\rm SM}$ from h^0 is also suppressed by v_Δ^2/v^2.

Hereafter we take $m_{H^{\pm\pm}} = m_{H^{\pm}}$ for simplicity. The large splitting of their masses is disfavored by the constraint on the ρ parameter. Once we fix the neutrino mass matrix, muon anomalous MDM and LFV processes are interpreted as lower bounds on $v_\Delta m_{H^{\pm\pm}}$. Figure 1 shows the lower bounds with respect to the confidence level in a unit of the standard deviation σ and they are given by constraints on the muon anomalous MDM with e^+e^- data (bold solid red line), the MDM with τ data (solid red line), $\mu \to \bar{e}ee$ (bold dashed green line), $\mu \to e\gamma$ (dashed green line), $\tau \to \bar{\mu}\mu\mu$ (bold dash-dotted blue line), $\tau \to \bar{\mu}ee$ (dash-dotted blue line), and the muonium (μ^+e^-) conversion to the anti-muonium (bold dash-dot-dotted magenta line). Bounds from $\tau \to \bar{\mu}\mu\mu$ and $\tau \to \bar{\mu}ee$ are important in our analysis among six possible $\tau \to \bar{\ell}\ell'\ell''$. Formulae of branching ratios of these LFV decays in the HTM and their current bounds at 90% CL are

$$\mathrm{BR}(\mu \to \bar{e}ee) = \frac{|m_{\mu e}|^2 |m_{ee}|^2}{16 G_F^2 v_\Delta^4 m_{H^{\pm\pm}}^4} < 1.0 \times 10^{-12} \text{ [60]}, \tag{2.13}$$

$$\mathrm{BR}(\mu \to e\gamma) = \frac{27\alpha |\langle m^2 \rangle_{e\mu}|^2}{256\pi G_F^2 v_\Delta^4 m_{H^{\pm\pm}}^4} < 1.2 \times 10^{-11} \text{ [61]}, \tag{2.14}$$

$$\mathrm{BR}(\tau \to \bar{\mu}\mu\mu) = \frac{|m_{\tau\mu}|^2 |m_{\mu\mu}|^2}{16 G_F^2 v_\Delta^4 m_{H^{\pm\pm}}^4} \mathrm{BR}(\tau \to \mu\bar{\nu}_\mu\nu_\tau) < 3.2 \times 10^{-8} \text{ [62]}, \tag{2.15}$$

$$\mathrm{BR}(\tau \to \bar{\mu}ee) = \frac{|m_{\tau\mu}|^2 |m_{ee}|^2}{16 G_F^2 v_\Delta^4 m_{H^{\pm\pm}}^4} \mathrm{BR}(\tau \to \mu\bar{\nu}_\mu\nu_\tau) < 2.0 \times 10^{-8} \text{ [62]}, \tag{2.16}$$

where $\mathrm{BR}(\tau \to \mu\bar{\nu}_\mu\nu_\tau) = 17\%$, $\alpha = 1/137$ stands for the fine structure constant, and $G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$ denotes the Fermi coupling constant. The effective Lagrangian for the muonium conversion is

$$\mathcal{L}_{M\overline{M}} = 2\sqrt{2} G_{M\overline{M}} \left(\overline{\mu}\gamma^\rho P_L e\right)\left(\overline{\mu}\gamma_\rho P_L e\right) = 4\sqrt{2} G_{M\overline{M}} \left(\overline{\mu} P_R \mu^c\right)\left(\overline{e^c} P_L e\right). \tag{2.17}$$

The formula of the coupling $G_{M\overline{M}}$ in the HTM and current constraint at 90% CL for that are

$$\left(\frac{|G_{M\bar{M}}|}{G_F}\right)^2 = \frac{|m_{ee}|^2 |m_{\mu\mu}|^2}{128 G_F^2 v_\Delta^4 m_{H^{\pm\pm}}^4} < (3.0 \times 10^{-3})^2 \text{ [63]}. \tag{2.18}$$

In figure 1, parameters of the neutrino mass matrix are fixed by (1.5), (1.6), and the following values as an example: $m_1 = 0$, $\sin^2 2\theta_{13} = 0$, $\varphi_1 = \varphi_2 = 0$. With these values of parameters, we have $\langle m^2 \rangle_{\mu\mu} = 1.2 \times 10^{-3}\,\mathrm{eV}^2$, $|m_{\mu e}|^2 |m_{ee}|^2 = 6.4 \times 10^{-11}\,\mathrm{eV}^4$, $|\langle m^2 \rangle_{e\mu}|^2 = 6.3 \times 10^{-10}\,\mathrm{eV}^4$, $|m_{\tau\mu}|^2 |m_{\mu\mu}|^2 = 3.5 \times 10^{-7}\,\mathrm{eV}^4$, $|m_{\tau\mu}|^2 |m_{ee}|^2 = 3.6 \times 10^{-9}\,\mathrm{eV}^4$, and $|m_{ee}|^2 |m_{\mu\mu}|^2 = 5.9 \times 10^{-9}\,\mathrm{eV}^4$. Bounds (2.13)–(2.16) and (2.18) at 90% CL are translated naively into $x\sigma$ CL bounds by multiplying $x/1.64$ because 90% CL corresponds to 1.64σ. Below around 1.8σ (3.1σ), the muon anomalous MDM $\Delta a_\mu[\tau]$ ($\Delta a_\mu[e^+e^-]$) gives the strongest constraint on the HTM but it becomes weaker rapidly than other constraints at higher confidence levels. Hereafter, we take $\Delta a_\mu[\tau]$ and concentrate ourselves on 2σ CL in order to avoid qualitative disagreement with $\Delta a_\mu[\tau]$ in the HTM.

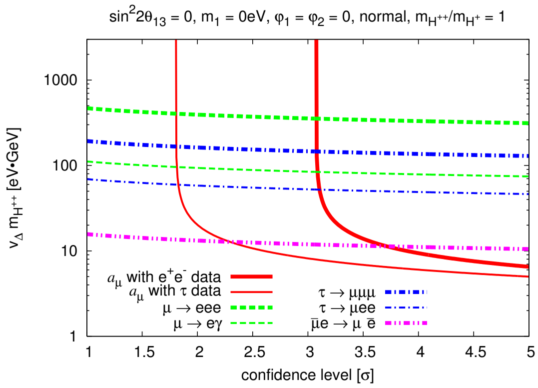

Figure 1. Lower bounds on $v_\Delta m_{H^{\pm\pm}}$ given by constraints on muon anomalous MDM and LFV processes as functions of the confidence level. All parameters in the neutrino mass matrix are fixed as an example (See the text for the values) for the normal mass ordering. We take $m_{H^{\pm\pm}} = m_{H^\pm}$ for simplicity.

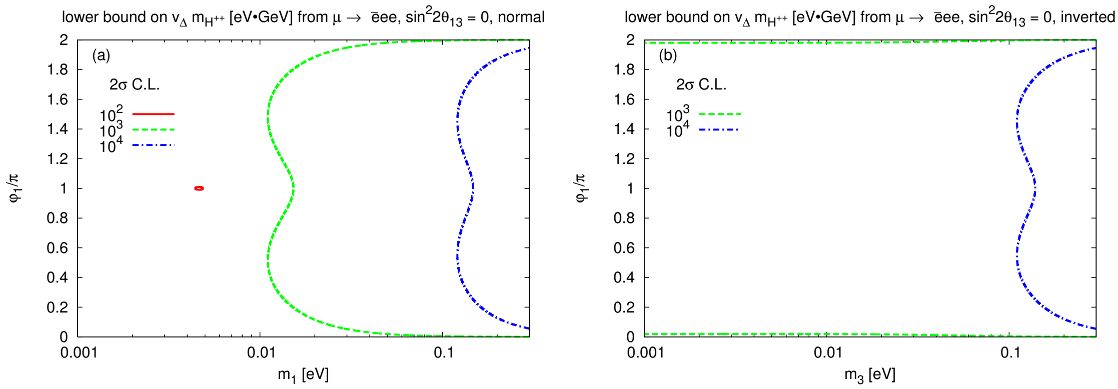

Figure 2. Contours of the lower bounds on $v_\Delta m_{H^{\pm\pm}}$[eV$\cdot$GeV] given by $\mu \to \bar{e}ee$. (a) for the normal mass ordering. (b) for the inverted mass ordering.

2.2 Constraints in the case of BR($\mu \to \bar{e}ee$) $\neq 0$

In most of parameter space, the strongest lower bound on $v_\Delta m_{H^{\pm\pm}}$ is given by $\mu \to \bar{e}ee$ as expected naively from the strong constraint on its branching ratio (2.13). Figures 2(a) and (b) show contours of the bounds with $\theta_{13} = 0$ for the normal and inverted mass ordering, respectively. Note that BR($\mu \to \bar{e}ee$) does not depend on δ and φ_2 for $\theta_{13} = 0$. Although the bound on $v_\Delta m_{H^{\pm\pm}}$ from $\mu \to \bar{e}ee$ is relatively weak for small m_1 in figure 2(a), bounds from other LFV processes are weaker than that. It is shown also that $\varphi_1 \simeq 0$ makes the bound from $\mu \to \bar{e}ee$ weak for both of mass orderings. We focus on the case of $\varphi_1 = 0$ in the next paragraph. In figure 2(a) there is a special point at $\varphi_1 = \pi$ and $m_1 = s_{12}^2\sqrt{\Delta m_{21}^2}/\sqrt{\cos 2\theta_{12}} \simeq 4.6 \times 10^{-3}\,$eV where the bound vanishes because of $m_{ee} = 0$. Such cases of BR($\mu \to \bar{e}ee$) $= 0$ are discussed in the next subsection.

In figure 3, m_1-dependences of bounds from $\Delta a_\mu[\tau]$ and LFV processes are presented for the normal mass ordering at $\varphi_1 = 0$ where the bound from $\mu \to \bar{e}ee$ is relatively weak.

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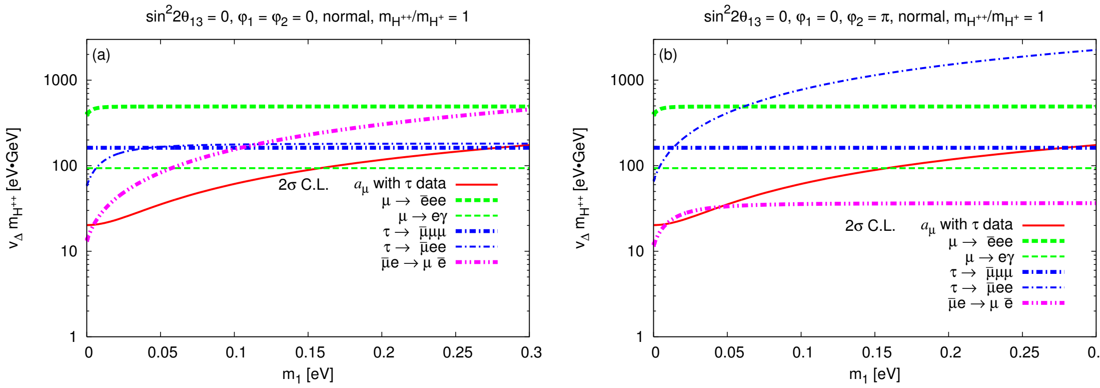

Figure 3. Lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for $\varphi_1 = 0$ in the normal mass ordering. (a) for $\varphi_2 = 0$. (b) for $\varphi_2 = \pi$.

Figures 3(a) and (b) are obtained for $\varphi_2 = 0$ and π, respectively. Other parameters are the same values as ones in figure 1. It is seen in figure 3(a) that $\mu \to \bar{e}ee$ still gives the most stringent bound for $\varphi_1 = \varphi_2 = 0$ and $m_1 \lesssim 0.3\,\text{eV}$ although the bound from the muonium conversion gets close to that for large m_1. If we accept $m_1 \gtrsim 0.3\,\text{eV}$, the bound from the muonium conversion can be stronger than the bound from $\mu \to \bar{e}ee$. On the other hand, figure 3(b) shows that the bound from $\tau \to \bar{\mu}ee$ can be more stringent than the one from $\mu \to \bar{e}ee$ for $m_1 \gtrsim 0.06\,\text{eV}$. This is because a parameter set $(\theta_{13}, \varphi_1, \varphi_2) = (0, 0, \pi)$ in the region of $\Delta m_{ij}^2/m_1^2 \ll 1$ gives

$$
\begin{aligned}
|m_{\mu e}|^2 |m_{ee}|^2 &\simeq \frac{1}{32} (\Delta m_{21}^2)^2 \sin^2 2\theta_{12} \simeq 1.6 \times 10^{-10}\,\text{eV}^4, & (2.19)\\
|m_{\tau\mu}|^2 |m_{ee}|^2 &\simeq m_1^4, & (2.20)
\end{aligned}
$$

and the large difference between experimental constraints (2.13) and (2.16) can be compensated for $m_1 \gtrsim O(0.1)\,\text{eV}$. In figure 4 the shaded region shows values of Majorana phases for which the bound from $\tau \to \bar{\mu}ee$ becomes more stringent than the one from $\mu \to \bar{e}ee$ at $m_1 = 0.2\,\text{eV}$ for the normal mass ordering. The region is symmetric under a transformation of $(\varphi_1, \varphi_2 - \pi) \to (-\varphi_1, -\varphi_2 + \pi)$ because of $|m_{\ell\ell'}| = |m_{\ell\ell'}^*|$. Although the bound from $\mu \to \bar{e}ee$ is relatively weak for $\varphi_1 \simeq 0$, the bound is still the most stringent one at around $\varphi_2 = 0$ because $\tau \to \bar{\mu}ee$ is also suppressed. If we take nonzero θ_{13} and ignore Δm_{21}^2 for $(\varphi_1, \varphi_2) = (0, \pi)$, eq. (2.19) is rewritten as

$$
|m_{\mu e}|^2 |m_{ee}|^2 \simeq 2 s_{13}^2 m_1^4\,, \tag{2.21}
$$

while eq. (2.20) remains valid. Therefore, the shaded region in figure 4 at around $(\varphi_1, \varphi_2) = (0, \pi)$ exists for $\sin^2 2\theta_{13} \lesssim 10^{-5}$. For the inverted mass ordering, the region where $\tau \to \bar{\mu}ee$ becomes remarkable is almost same as the one in figures 3 and 4 because neutrino masses are almost degenerated in the region. In such a region, we can also expect a signal of $\tau \to \bar{\mu}ee$ in future experiments [64–70] with satisfying the current constraint on $\mu \to \bar{e}ee$.

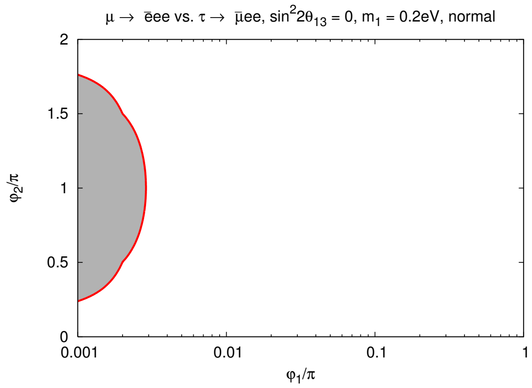

Figure 4. In the shaded region, the bound on $v_\Delta m_{H^{\pm\pm}}$ from $\tau \to \bar{\mu}ee$ is stronger than the one from $\mu \to \bar{e}ee$. We used $m_1 = 0.2\,\mathrm{eV}$ for the normal mass ordering. The shaded region is symmetric under a transformation of $(\varphi_1, \varphi_2 - \pi) \to (-\varphi_1, -\varphi_2 + \pi)$.

2.3 Constraints in cases of BR$(\mu \to \bar{e}ee) = 0$

It has been known that the strong constraint from $\mu \to \bar{e}ee$ can be evaded in the cases of $m_{e\mu} = 0$ [34] and $m_{ee} = 0$ [8]. While it is impossible to have $m_{e\mu} = 0$ with $\theta_{13} = 0$, the case of $m_{ee} = 0$ is possible also for $\theta_{13} = 0$ as we mentioned for figure 2(a). Such cancellations in the HTM are desired also for experiments [64–71] to discover some LFV decays ($\mu \to e\gamma$ etc.) [8, 34] in the future. Figures 5(a)–(d) show results for the case of $m_{e\mu} = 0$ in the normal mass ordering. Four CP conserving sets of Majorana phases are taken for the figures as examples. We use appropriate values of θ_{13} and δ for $m_{e\mu} = 0$, which we call as "magic values" $\theta_{13}^{\mathrm{mgc}}$ and δ^{mgc}, and explicit formulae of them are shown in appendix. For each cases in figure 5, the magic value δ^{mgc} is 0 or π independently of m_1. Although $\langle m^2 \rangle_{e\mu}$ is independent of m_1 and Majorana phases, the bounds from $\mu \to e\gamma$ in figure 5 are not constant with respect to these parameters because $\theta_{13}^{\mathrm{mgc}}$ depends on them. We see in figures 5(a)-(d) that the bound from $\mu \to e\gamma$ is the strongest one for $m_1 \simeq 0$, and this is also the case with any values of φ_1 and φ_2. For the case of figure 5(a), τ decays give the most stringent bound for $0.01\,\mathrm{eV} \lesssim m_1 \lesssim 0.12\,\mathrm{eV}$, and the bound from the muonium conversion becomes the strongest one for $m_1 \gtrsim 0.12\,\mathrm{eV}$. In figure 5(b), the bound from $\tau \to \bar{\mu}ee$ is prominent. The magic θ_{13} in figure 5(b) gives $\sin^2 2\theta_{13}^{\mathrm{mgc}} \simeq 10^{-7}$ for $m_1 = 0.2\,\mathrm{eV}$, and then the remarkable behavior of the bound from $\tau \to \bar{\mu}ee$ can be understood also as a part of the case shown in figure 4 whose shaded region appears for $\sin^2 2\theta_{13} \lesssim 10^{-5}$. In figures 5(c) and (d), the most stringent bound is obtained from $\mu \to e\gamma$. Note that $\sin^2 2\theta_{13}^{\mathrm{mgc}}$ in figures 5(c) and (d) become larger than 0.14 of the CHOOZ bound [19] for $m_1 \gtrsim 0.008\,\mathrm{eV}$, and then we can not have $m_{e\mu} = 0$ for the case.

Figure 6 shows which process gives the most stringent lower bound on $v_\Delta m_{H^{\pm\pm}}$ in a space of Majorana phases by keeping $m_{e\mu} = 0$ for the normal mass ordering. Green circles, blue crosses, and magenta triangles show regions where the strongest bound comes from $\mu \to e\gamma$, $\tau \to \bar{\mu}ee$, and the muonium conversion, respectively. It is impossible to achieve $m_{e\mu} = 0$ outside of the regions because of unacceptably large $\theta_{13}^{\mathrm{mgc}}$, and then it becomes

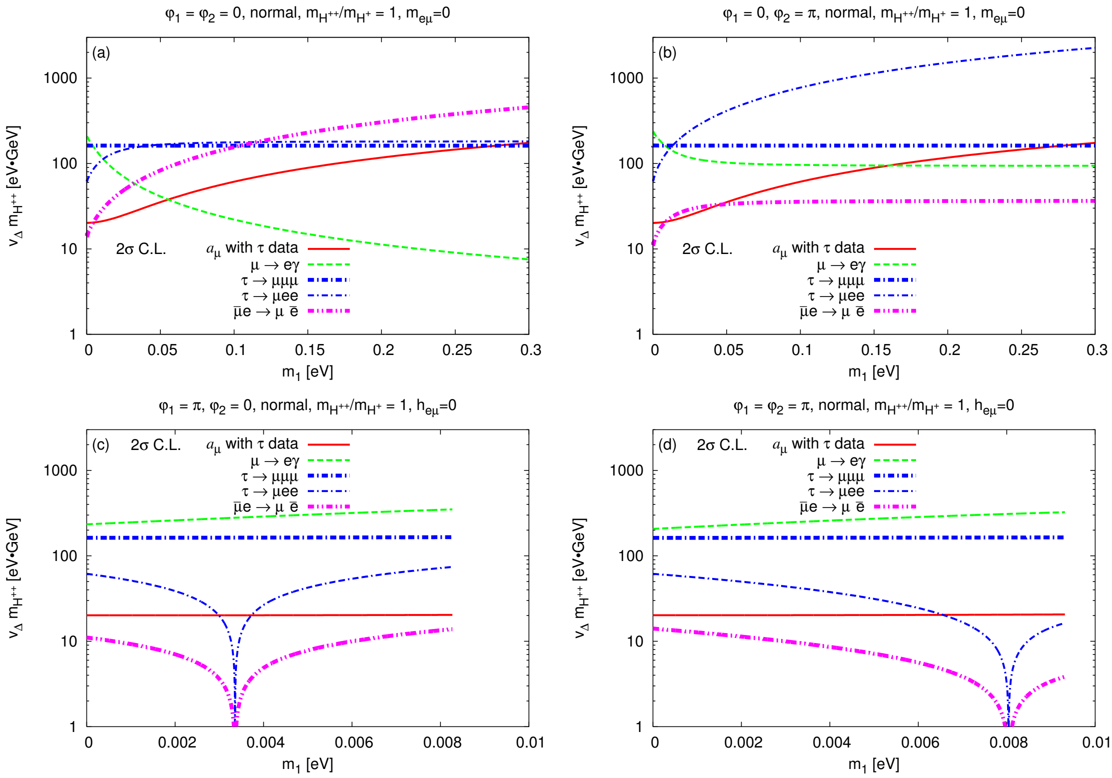

Figure 5. Lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for cases of $m_{e\mu} = 0$ in the normal mass ordering. The value of $\theta_{13}^{\rm mgc}$ varies to keep $m_{e\mu} = 0$ (See appendix). (a) for $(\varphi_1, \varphi_2) = (0, 0), \delta^{\rm mgc} = \pi$. (b) for $(\varphi_1, \varphi_2) = (0, \pi), \delta^{\rm mgc} = 0$. (c) for $(\varphi_1, \varphi_2) = (\pi, 0), \delta^{\rm mgc} = 0$. (d) for $(\varphi_1, \varphi_2) = (\pi, \pi), \delta^{\rm mgc} = \pi$.

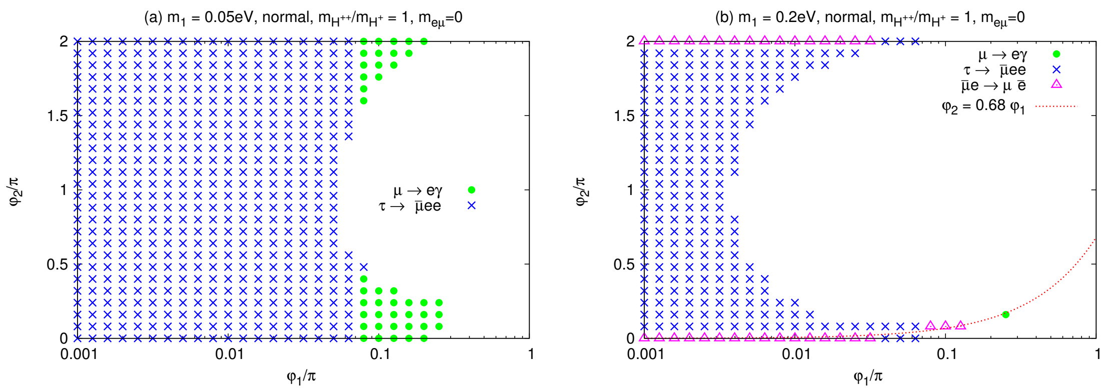

Figure 6. Green circles, blue crosses, and magenta triangles show regions where the strongest lower bound on $v_\Delta m_{H^{\pm\pm}}$ for the case of $m_{e\mu} = 0$ in the normal mass ordering comes from $\mu \to e\gamma$, $\tau \to \bar{\mu}ee$, and the muonium conversion, respectively. The values of $\theta_{13}^{\rm mgc}$ and $\delta^{\rm mgc}$ are functions of Majorana phases (See appendix). We can not have $m_{e\mu} = 0$ outside of these regions. The regions are symmetric under a transformation $(\varphi_1, \varphi_2) \to (-\varphi_1, -\varphi_2)$. (a) with $m_1 = 0.05\,{\rm eV}$. (b) with $m_1 = 0.2\,{\rm eV}$. With $m_1 = 0$, the most stringent bound is given by $\mu \to e\gamma$ for all values of φ_1 and φ_2.

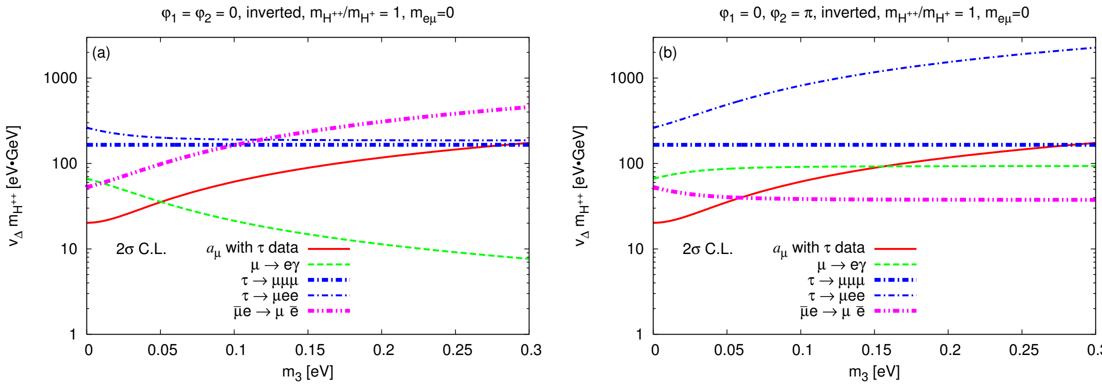

Figure 7. Lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for cases of $m_{e\mu} = 0$ in the inverted mass ordering. The value of $\theta_{13}^{\rm mgc}$ varies to keep $m_{e\mu} = 0$ (See appendix). (a) for $(\varphi_1, \varphi_2) = (0, 0), \delta^{\rm mgc} = 0$. (b) for $(\varphi_1, \varphi_2) = (0, \pi), \delta^{\rm mgc} = 0$.

the situations discussed in the previous subsection for BR$(\mu \to \bar{e}ee) \neq 0$. Figure 6(a) is for $m_1 = 0.05\,$eV and (b) is for $m_1 = 0.2\,$eV. For $m_1 = 0$, it is possible to have $m_{e\mu} = 0$ for any values of Majorana phases, and the most stringent bound is always given by $\mu \to e\gamma$ as $v_\Delta m_{H^{\pm\pm}} \gtrsim 200\,eV\cdot$GeV. Values of lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for figure 6(a) from $\mu \to e\gamma$ and $\tau \to \bar{\mu}ee$ vary in $150 \to 350\,$eV$\cdot$GeV and $150 \to 400\,$eV$\cdot$GeV, respectively. We have $v_\Delta m_{H^{\pm\pm}} \gtrsim 300\,eV\cdot$GeV, $\gtrsim 300 \to 1500\,$eV$\cdot$GeV, and $\gtrsim 300\,$eV$\cdot$GeV in figure 6(b) from $\mu \to e\gamma$, $\tau \to \bar{\mu}ee$, and the muonium conversion, respectively. The regions are symmetric under a transformation $(\varphi_1, \varphi_2) \to (-\varphi_1, -\varphi_2)$ because of $|m_{\ell\ell'}| = |m_{\ell\ell'}^*|$. We see that $\varphi_1 \simeq 0$ is preferred to keep $s_{13}^{\rm mgc}$ small for $m_1 \neq 0$ and we can confirm $s_{13}^{\rm mgc} \propto \Delta m_{21}^2$ for $\varphi_1 = 0$ with eq. (A.3) in appendix. It can be found also with eq. (A.3) for $\varphi_1, \varphi_2 \ll 1$ and $\Delta m_{ij}^2 = 0$ that

$$\varphi_2 \simeq \frac{1 + \cos 2\theta_{12}}{2}\,\varphi_1 = 0.68\,\varphi_1 \quad \text{(red dotted line in figure 6(b))} \tag{2.22}$$

is preferred to have a small $s_{13}^{\rm mgc}$. Bounds from $\tau \to \bar{\mu}ee$ and the muonium conversion can be the most stringent one only for $\varphi_1 \lesssim 0.1\pi$. Majorana phases are almost restricted as $\varphi_2 \simeq 0.68\,\varphi_1$ for the case of a strong constraint from the muonium conversion. It is shown that $\mu \to e\gamma$ can be the most stringent bound even for $m_1 \gtrsim 0.008\,$eV (cf. figure 5) because of large $s_{13}^{\rm mgc}$. At the border to the white region in figure 6, we have $\sin^2 2\theta_{13}^{\rm mgc} = 0.14$.

Similarly to figure 5, lower bounds on $v_\Delta m_{H^{\pm\pm}}$ are shown in figure 7 for the inverted mass ordering. Note that $\varphi_1 = \pi$ can not give $m_{e\mu} = 0$ for the mass ordering because $s_{13}^{\rm mgc}$ becomes too large. Very roughly speaking, the results in figures 7(a) and (b) are the same as those in figures 5(a) and (b), respectively. A difference is that $\tau \to \bar{\mu}ee$ can be the most stringent bound at $m_3 = 0$ while it is not the case for $m_1 = 0$ in the normal mass ordering. Figure 8 shows the φ_1-dependence for $m_{e\mu} = 0$ with $m_3 = 0$ where the φ_2-dependence vanishes. For $\varphi_1 \gtrsim 0.1\pi$, $\sin^2 2\theta_{13}^{\rm mgc}$ becomes larger than 0.14. We see that $\mu \to e\gamma$ can give the most stringent bound even for the inverted mass ordering. For $m_3 = 0.2\,$eV, figure 6(b) is almost applicable to see which process gives the most stringent bound.

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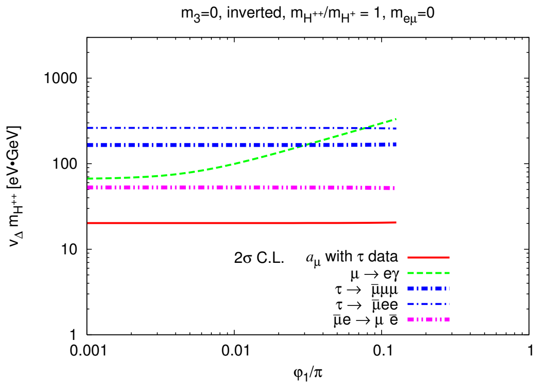

Figure 8. The φ_1-dependence of lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for cases of $m_{e\mu} = 0$ with $m_3 = 0$ in the inverted mass ordering. $\theta_{13}^{\rm mgc}$ and $\delta^{\rm mgc}$ for $m_{e\mu} = 0$ are functions of φ_1 (See appendix).

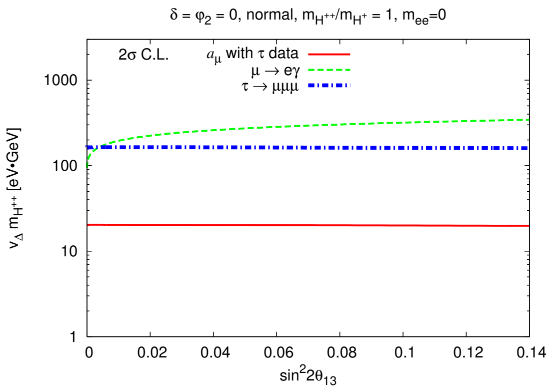

Figure 9. Lower bounds on $v_\Delta m_{H^{\pm\pm}}$ for cases of $m_{ee} = 0$ with $\delta = \varphi_2 = 0$ in the normal mass ordering. Values of $m_1^{\rm mgc}$ and $\varphi_1^{\rm mgc}$ are given by the condition $m_{ee} = 0$ depending on θ_{13} (See appendix).

Figure 9 is the result for the case of $m_{ee} = 0$ which is possible only in the normal mass ordering. Formulae of the magic values $m_1^{\rm mgc}$ and $\varphi_1^{\rm mgc}$ for $m_{ee} = 0$ are shown in appendix. The bound from $\mu \to e\gamma$ is the most stringent one except for $\sin^2 2\theta_{13} \lesssim 0.004$ where the bound from $\tau \to \bar{\mu}\mu\mu$ becomes stronger than that. This is also the case for different values of δ and φ_2.

3 Conclusion

If we deal with 2σ bounds to avoid the disagreement with $\Delta a_\mu[\tau]$, $v_\Delta m_{H^{\pm\pm}}$ must be greater than $10^3\,$eV$\cdot$GeV in most of parameter space of the HTM in order to satisfy a strong constraint on BR($\mu \to \bar{e}ee$). We found that the bound on $v_\Delta m_{H^{\pm\pm}}$ from $\tau \to \bar{\mu}ee$ becomes more stringent than that from $\mu \to \bar{e}ee$ in a region of $m_1 \gtrsim 0.06\,$eV, $\varphi_1 \lesssim 0.002\pi$, $0.5\pi \lesssim \varphi_2 \lesssim 1.5\pi$, and $\sin^2 2\theta_{13} \lesssim 10^{-5}$. The bound from $\mu \to \bar{e}ee$ can be evaded in cases of $m_{e\mu} = 0$ [34] and $m_{ee} = 0$ [8]. We have considered not only CP-conserving sets of phases but also arbitrary values. In the case of $m_{e\mu}=0$ in the normal mass ordering, the strongest bound is given by $\mu\to e\gamma$ for $m_1\lesssim 0.01\,\mathrm{eV}$ or $\varphi_1\gtrsim 0.1\pi$ and by τ decays (mainly $\tau\to\bar{\mu}ee$) for $m_1\gtrsim 0.01\,\mathrm{eV}$ with $\varphi_1\lesssim 0.1\pi$. On the other hand, $\tau\to\bar{\mu}ee$ gives the strongest bound in the normal mass ordering except for $m_3\simeq 0$ with $\varphi_1\simeq 0.1\pi$ where $\mu\to e\gamma$ gives the bound. For both of the mass orderings with $m_{e\mu}=0$, the muonium conversion gives the most stringent bound if Majorana phases satisfy $\varphi_2\simeq 0.68\,\varphi_1$ for $\varphi_1,\varphi_2\ll 1$ and $m_1\gtrsim 0.1\,\mathrm{eV}$. In the case of $m_{ee}=0$, the strongest bound is obtained from $\mu\to e\gamma$ except for the case of $\sin^2 2\theta_{13}\lesssim 0.004$ where the bound is given by $\tau\to\bar{\mu}\mu\mu$. By looking over all cases, we see that $v_\Delta m_{H^{\pm\pm}}\gtrsim 150\,\mathrm{eV\cdot GeV}$ should be satisfied in the HTM. If $m_{H^{\pm\pm}}$ is measured, the bound can be the lower bound on v_Δ though there remains a possibility of $v_\Delta=0$ for which we can not use the correlation of $h_{\ell\ell'}$ with $\sqrt{2}v_\Delta h_{\ell\ell'}=m_{\ell\ell'}$.

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A Solutions for $m_{e\mu}=0$ and $m_{ee}=0$

We obtained formulae of the magic values of θ_{13} and δ, which give $m_{e\mu}=0$, as

$$\cos\delta^{\rm mgc} = -\frac{c_{12}s_{12}c_{23}C}{s_{23}s_{13}^{\rm mgc}\left\{m_3^2-s_{12}^4m_2^2\sin^2\varphi_1-(s_{12}^2m_2\cos\varphi_1+c_{12}^2m_1)^2\right\}}, \tag{A.1}$$

$$\sin\delta^{\rm mgc} = \frac{c_{12}s_{12}c_{23}D}{s_{23}s_{13}^{\rm mgc}\left\{m_3^2-s_{12}^4m_2^2\sin^2\varphi_1-(s_{12}^2m_2\cos\varphi_1+c_{12}^2m_1)^2\right\}}, \tag{A.2}$$

$$s_{13}^{\rm mgc} = \frac{c_{12}s_{12}c_{23}\sqrt{C^2+D^2}}{s_{23}\left|m_3^2-s_{12}^4m_2^2\sin^2\varphi_1-(s_{12}^2m_2\cos\varphi_1+c_{12}^2m_1)^2\right|}, \tag{A.3}$$

$$C \equiv -c_{12}^2m_1^2+s_{12}^2m_2^2+m_1m_2\cos 2\theta_{12}\cos\varphi_1-m_1m_3\cos\varphi_2+m_2m_3\cos(\varphi_1-\varphi_2), \tag{A.4}$$

$$D \equiv -m_1m_2\sin\varphi_1+m_1m_3\sin\varphi_2+m_2m_3\sin(\varphi_1-\varphi_2), \tag{A.5}$$

where we define $s_{13}^{\rm mgc}\equiv\sin\theta_{13}^{\rm mgc}$. Note that $s_{13}^{\rm mgc}\neq 0$ because it requires $m_2=m_1$. Note also that $s_{13}^{\rm mgc}$ is not always acceptable; For example, $\varphi_1=\pi$ in the inverted mass ordering gives $s_{13}^{\rm mgc}>1$. These results are consistent with $s_{13}^{\rm mgc}$ and $\delta^{\rm mgc}$ used in [8, 34] for $\varphi_1,\varphi_2=0$ or π. Although mixing matrix in [8] is defined as $\nu_\ell=\sum_i U_{\ell i}^*\nu_i$ in stead of $\nu_\ell=\sum_i U_{\ell i}\nu_i$ used in this article, there is no change in formulae of magic values because the difference appears just as the simultaneous flip of signs of all phases.

On the other hand, $m_{ee}=0$ can be achieved only in the normal mass ordering. The magic values of φ_1 and m_1 for $m_{ee} = 0$ are given as functions of s_{13} and $\varphi_2 - 2\delta$ [8] by

$$\sin\varphi_1^{\rm mgc} \equiv -\frac{\sqrt{(m_1^{\rm mgc})^2+\Delta m_{31}^2}}{s_{12}^2\sqrt{(m_1^{\rm mgc})^2+\Delta m_{21}^2}}\, t_{13}^2 \sin(\varphi_2-2\delta), \quad \cos\varphi_1^{\rm mgc} \leq 0, \tag{A.6}$$

$$\begin{aligned}(m_1^{\rm mgc})^2 \equiv{}& \frac{1}{\cos^2 2\theta_{12} - 2\left(s_{12}^4 + c_{12}^4\cos 2(\varphi_2-2\delta)\right) t_{13}^4 + t_{13}^8} \\ &\times\Bigg[s_{12}^4\cos 2\theta_{12}\Delta m_{21}^2 + \left\{ s_{12}^4\Delta m_{21}^2 + \left(s_{12}^4 + c_{12}^4\cos 2(\varphi_2-2\delta)\right)\Delta m_{31}^2 \right\} t_{13}^4 \\ &\qquad - \Delta m_{31}^2 t_{13}^8 - 2c_{12}^2 t_{13}^2\cos(\varphi_2-2\delta)\sqrt{A+B\,t_{13}^4}\Bigg],\end{aligned} \tag{A.7}$$

$$A \equiv \left(s_{12}^4\Delta m_{21}^2 + \cos 2\theta_{12}\Delta m_{31}^2\right) s_{12}^4\Delta m_{21}^2, \tag{A.8}$$

$$B \equiv \left\{ (s_{12}^4 - c_{12}^4\sin^2(\varphi_2-2\delta))\Delta m_{31}^2 - s_{12}^4\Delta m_{21}^2 \right\}\Delta m_{31}^2, \tag{A.9}$$

where we define $t_{13} \equiv s_{13}/c_{13}$.

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