

Giant magnons and spiky strings in the Schrödinger/dipole-deformed CFT correspondence

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ABSTRACT: We construct semi-classical string solutions of the Schrödinger $Sch_5 \times S^5$ space-time, which is conjectured to be the gravity dual of a non-local dipole-deformed CFT. They are the counterparts of the giant magnon and spiky string solutions of the undeformed $AdS_5 \times S^5$ to which they flow when the deformation parameter is turned off. They live in an S^3 subspace of the five-sphere along the directions of which the B -field has non-zero components, having also extent in the Sch_5 part of the metric. Finally, we speculate on the form of the dual field theory operators.

KEYWORDS: AdS-CFT Correspondence, Duality in Gauge Field Theories, Integrable Field Theories, Space-Time Symmetries

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1 Introduction

Integrability is the powerful tool behind the recent advances in understanding the gauge-gravity correspondence in the planar limit (for a review see [1]). On the gauge theory side, diagonalizing the matrix of the anomalous dimensions can be performed through Bethe ansatz techniques [2] since the anomalous dimension matrix can be mapped to the Hamiltonian of an integrable spin chain. The magnons play the role of the asymptotic states and are considered as the fundamental single particle excitations propagating on a certain BMN vacuum. Integrability of the theory implies that any scattering process factorizes in a scattering between two magnons. As a result, the role of integrability is instrumental in determining the spectrum of $\mathcal{N} = 4$ SYM for any value of the coupling.¹

On the string side, the strong coupling dual to the field theory magnons are the so-called giant magnons [11–15]. These are semi-classical solutions of string theory on $AdS_5 \times S^5$ and are described by open strings moving in a subspace of the sphere S^5 with finite angular extent. Their dispersion relation is the strong coupling limit of the exact dispersion relation obtained from the gauge theory analysis [16]. Similar classical string configurations with finite angular amplitude that wind infinitely many times around the orthogonal angular direction are called single spiky strings [17–19]. Their gauge theory interpretation needs further clarification.

Over the last years there is a lot of attention on the integrable deformations of the AdS/CFT. Two important examples of such deformations are the β -deformations [20, 21]

¹Recently, progress has also been made in the calculation of higher point correlation functions using integrability. For an incomplete list of work in this direction see [3–10] and references therein.

for which the magnon solution was calculated in [22] and the non-commutative theories [23]. Recent activity is coming from the null dipole deformation of the $\mathcal{N} = 4$ SYM, which is special since it has minimal non-locality along one of its light-like directions. The dual gravity background is known as a Schrödinger space-time, it was first worked out in [24–26] and can be obtained by performing an abelian T-duality along one of the isometries of the five-sphere followed by a shift along a light-like direction of the AdS_5 boundary and a second T-duality along the dual coordinate of the sphere. On the field theory side, the deformation is realised by introducing the appropriate \star -product among the fields of the $\mathcal{N} = 4$ Lagrangian. Generically, the \star -product used definition of the deformed theory can be identified with a corresponding Drinfeld-Reshetikhin twist of the underlying integrable structure [27, 28].

Integrability issues of the dipole-deformed CFT were discussed in detail in [29].² The deformation is realized as a Drinfeld-Reshetikhin twist [34, 35] of the $\mathcal{N} = 4$ integrable structure. In [29] the authors provided a test of the Schrödinger holography by matching the anomalous dimensions of long gauge theory operators with the strong coupling prediction of certain BMN-like strings (see also [36]). While the present paper was being written the work [37] appeared. The authors have calculated the dispersion relation of a giant magnon-like solution in $Sch_5 \times S^5$. However, their solution is fundamentally different from ours since it is point-like in the Schrödinger space-time part of the metric unlike the ones presented here.³

In this paper we construct semi-classical string solutions living in the Schrödinger $Sch_5 \times S^5$ spacetime, which is conjectured to be the gravity dual of a certain non-local dipole-deformed CFT. Those solutions are the counterparts of the giant magnon and the single spike of the undeformed $AdS_5 \times S^5$, since in the zero deformation limit they become the ordinary giant magnon and single spike of the original $AdS_5 \times S^5$ background. We find those solutions in an S^3 subspace of the five-sphere along the direction of which the B -field has non-zero components. Furthermore, the solutions presented here are not point like but also extend in the Sch_5 part of the metric. For the dyonic giant magnon the dispersion relation is given by

$$\sqrt{E^2 - \frac{\lambda}{4\pi^2} L^2 M^2 - J_1} = \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \tag{1.1}$$

where the angle difference $\Delta\varphi_1$ is identified with the momentum p carried by the excitation, while for the single spike is given by

$$J_1^2 - J_2^2 = \frac{\lambda}{\pi^2} \sin^2 \left[\frac{\pi}{\sqrt{\lambda}} E - \frac{1}{2} L M - \frac{1}{2} \Delta\varphi_1 \right]. \tag{1.2}$$

The paper is organised as follows: in section 2 we present the ansatz for the solution and discuss the boundary conditions corresponding either to the giant magnon or to the single

²Integrability of the σ -model at the classical level was discussed in [30–33], where the Schrödinger spacetime was realized as a particular Yang-Baxter deformation of the $AdS_5 \times S_5$ superstring.

³In addition to this the dispersion relation of [37] when expressed solely in terms of the conserved quantities takes the form $\frac{(E - \sqrt{\mu^2 S^2 + J_+^2 + 4J_\phi^2})^2 E^2}{E^2 - (\mu^2 S^2 + J_+^2)} = T^2 \sin^2 \frac{p}{2}$ which is completely different from ours (1.1).

spike case. After writing the solution we calculate the angular amplitudes. In section 3 we focus on the giant magnon case by finding the finite combinations of the conserved charges and extracting the dispersion relation. In section 4 we provide the finite combination of global charges and the dispersion relation for the single spiky string solution. In section 5 we summarise and discuss about the possible form of the field theory operators that are dual to the giant magnon solutions in this dipole-deformed CFT. Furthermore, we comment on a possible, exact in λ , dispersion relation of the giant magnon solution. Our proposed relation (5.4) when expanded agrees, in the BMN limit, with the one-loop result that is obtained by solving the Baxter equation. Finally, an appendix contains the details of the 10-dimensional background as well as the proof that our solution lives in a consistent truncation of the full background.

2 Giant magnons and single spiky strings on $Sch_5 \times S^3$

We consider the following truncation of the 10d $Sch_5 \times S^5$ metric⁴ on the sphere S^3 , which as we explain in appendix A is consistent

$$ds^2 = - \left(1 + \frac{\mu^2}{Z^4} \right) dT^2 + \frac{1}{Z^2} (2dTdV + dZ^2) + \frac{1}{4} \left[d\theta^2 + \sin^2 \theta d\phi^2 + (d\psi - \cos \theta d\phi)^2 \right] \quad (2.1)$$

where $\psi \in [0, 4\pi)$, $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$ are the angular coordinates of the three dimensional sphere and their ranges. The B-field is given by the following expression

$$B = \frac{\mu}{2Z^2} dT \wedge (d\psi - \cos \theta d\phi). \quad (2.2)$$

In order to compare the dispersion relation of the magnon with the standard result from the literature [11], we have to perform the following change of variables

$$\theta = 2\eta \quad \psi = \varphi_1 + \varphi_2 \quad \phi = \varphi_1 - \varphi_2 \quad (2.3)$$

where $\eta \in [0, \pi/2]$, $\varphi_1 \in [0, 2\pi)$ and $\varphi_2 \in [0, 2\pi)$ are the ranges of those variables.

We now introduce the following ansatz to find classical solutions

$$\begin{aligned} T &= \kappa \tau + T_y(y) & V &= \alpha \tau + V_y(y) & Z &= Z_0 \\ [3pt] \theta &= \theta_y(y) & \psi &= \omega_\psi \tau + \Psi_y(y) & \phi &= \omega_\phi \tau + \Phi_y(y) \end{aligned} \quad (2.4)$$

where we have defined the variable y as

$$y \equiv c\sigma - d\tau. \quad (2.5)$$

Additionally κ , α , Z_0 , ω_ψ and ω_ϕ are constants, while $T_y(y)$, $V_y(y)$, $\theta_y(y)$, $\Psi_y(y)$ and $\Phi_y(y)$ are functions of σ and τ (in the combination $c\sigma - d\tau$) that we will determine through the equations of motion and the Virasoro constraints.

⁴We have set $\ell = 1$ and $\alpha' = 1$.

2.1 Equations of motion

Applying the ansatz (2.4) on the equation of motion that is coming from the variation of (A.1) along the direction V , we end up with an equation for the function $T_y(y)$. It is possible to integrate this differential equation and fully determine the function

$$T_y(y) = \frac{y}{c^2 - d^2} (A_V Z_0^2 - d \kappa) \quad (2.6)$$

where A_V is the integration constant.

The equation of motion coming from the variation of (A.1) along the direction T , written in terms of y , can be integrated once, providing $V'_y(y)$ in terms of $\theta_y(y)$ and constants as follows

$$V'_y(y) = \frac{1}{c^2 - d^2} \left[A_V (\mu^2 + Z_0^4) - A_T Z_0^2 - \alpha d + \frac{1}{2} c \mu (\omega_\psi - \omega_\phi \cos \theta_y) \right] \quad (2.7)$$

where A_T is the integration constant.

In the same fashion, the equations of motion coming from the variation of (A.1) along the directions ψ and ϕ (in terms of y) can be integrated to express $\Psi'_y(y)$ and $\Phi'_y(y)$ in terms of $\theta_y(y)$ and constants as follows

$$\Psi'_y(y) = \frac{1}{c^2 - d^2} \left[\frac{4}{\sin^2 \theta_y} (A_\psi + A_\phi \cos \theta_y) - d \omega_\psi - \frac{2 c \kappa \mu}{Z_0^2} \right] \quad (2.8)$$

$$\Phi'_y(y) = \frac{1}{c^2 - d^2} \left[\frac{4}{\sin^2 \theta_y} (A_\psi \cos \theta_y + A_\phi) - d \omega_\phi \right] \quad (2.9)$$

where A_ψ and A_ϕ are the integration constants.

The equation of motion coming from the variation of Z will give us the following constraint for the constant Z_0

$$A_T A_V Z_0^4 - A_V^2 Z_0^6 + c \kappa (\alpha c - 2 A_\psi \mu) = 0. \quad (2.10)$$

Substituting the expressions (2.6), (2.7), (2.8) & (2.9) into the first Virasoro constraint (A.3), we obtain the following expression for $\theta'_y(y)$

$$\begin{aligned} (\theta'_y)^2 = & \frac{4}{(c^2 - d^2)^2} \left[\frac{c^2 + d^2}{d} (A_\psi \omega_\psi + A_\phi \omega_\phi - \kappa A_T + \alpha A_V) - A_V Z_0^2 (A_V Z_0^2 - 2 A_T) \right. \\ & - \frac{c^2}{4} \left(\omega_\psi^2 - 2 \omega_\psi \omega_\phi \cos \theta_y + \omega_\phi^2 - 4 \kappa^2 + \frac{8 \alpha \kappa}{Z_0^2} \right) - c \mu A_V (\omega_\psi - \omega_\phi \cos \theta_y) \\ & \left. - \frac{4}{\sin^2 \theta_y} (A_\psi^2 + 2 A_\psi A_\phi \cos \theta_y + A_\phi^2) - \mu^2 A_V^2 + \frac{4 c \kappa \mu}{Z_0^2} A_\psi \right]. \quad (2.11) \end{aligned}$$

Using (2.11) it is possible to prove that the equation of motion coming from the variation along the direction θ is satisfied. Finally, substituting the expressions (2.6), (2.7), (2.8), (2.9) and (2.11) into the second Virasoro constraint (A.3), we obtain the following algebraic relation

$$A_\psi \omega_\psi + A_\phi \omega_\phi - \kappa A_T + \alpha A_V = 0. \quad (2.12)$$

From the analysis above it is clear that once we solve (2.11) we can substitute the solution for $\theta_y(y)$ in equations (2.7), (2.8) and (2.9), in order to integrate them and obtain $V_y(y)$, $\Psi_y(y)$ and $\Phi_y(y)$ respectively.

The next step in the direction of solving (2.11) is to introduce a new function, namely $u(y)$, as follows

$$u \equiv \cos^2 \left(\frac{\theta_y}{2} \right). \quad (2.13)$$

Using (2.11) we can write the differential equation for $u(y)$

$$\frac{(u')^2}{2} + \mathcal{W}(u) = 0 \quad \text{with} \quad \mathcal{W}(u) = -2 (\beta_6 u^3 + \beta_4 u^2 + \beta_2 u + \beta_0) \quad (2.14)$$

where

$$\beta_6 = - \frac{c \omega_\phi (c \omega_\psi + 2 A_V \mu)}{(c^2 - d^2)^2} \quad (2.15)$$

$$\begin{aligned} \beta_4 = & \frac{1}{4(c^2 - d^2)^2} \left[-\frac{4}{d} (c^2 + d^2) (A_\psi \omega_\psi + A_\phi \omega_\phi - \kappa A_T + \alpha A_V) \right. \\ & + c^2 \left(\omega_\psi^2 + 6 \omega_\phi \omega_\psi + \omega_\phi^2 - 4 \kappa^2 + \frac{8 \alpha \kappa}{Z_0^2} \right) + 4 A_V c \mu (\omega_\psi + 3 \omega_\phi) \\ & \left. + 4 \mu^2 A_V^2 + 4 A_V Z_0^2 (A_V Z_0^2 - 2 A_T) - \frac{16 c \kappa \mu A_\psi}{Z_0^2} \right] \end{aligned} \quad (2.16)$$

$$\begin{aligned} \beta_2 = & \frac{1}{4(c^2 - d^2)^2} \left[\frac{4}{d} (c^2 + d^2) (A_\psi \omega_\psi + A_\phi \omega_\phi - \kappa A_T + \alpha A_V) \right. \\ & - c^2 \left((\omega_\psi + \omega_\phi)^2 - 4 \kappa^2 + \frac{8 \alpha \kappa}{Z_0^2} \right) - 4 A_V c \mu (\omega_\psi + \omega_\phi) \\ & \left. - 4 \mu^2 A_V^2 - 4 A_V Z_0^2 (A_V Z_0^2 - 2 A_T) + \frac{16 c \kappa \mu A_\psi}{Z_0^2} - 16 A_\phi A_\psi \right] \end{aligned} \quad (2.17)$$

$$\beta_0 = - \frac{(A_\psi - A_\phi)^2}{(c^2 - d^2)^2}. \quad (2.18)$$

Notice that $u(y) \in [0, 1]$.

2.2 Boundary conditions

The next important step in order to solve equation (2.14) is to impose the correct boundary conditions. Here we are interested in a string configuration that will extend between $\theta = \pi$ which defines an equator of S^3 and some other angle θ_0 that will be determined dynamically from the equation of motion for θ (or equivalently u). From the requirement of finiteness for Ψ'_y, Φ'_y at $\theta = \pi$ we are led to the following first constraint

$$A_\psi = A_\phi. \quad (2.19)$$

Requiring the function T_y from equation (2.6) to vanish, we fix the constant A_V

$$A_V = \frac{d \kappa}{Z_0^2}. \quad (2.20)$$

Since the $\theta = \pi$ is an end point for the string configuration, we impose that θ'_y and V'_y at $\theta = \pi$ are zero and not just finite (as Ψ'_y, Φ'_y). Combining those two conditions with (2.10) and (2.19) it is possible to fully determine the three remaining constants, namely A_T, Z_0 and A_ϕ . The expressions for the constants A_T and Z_0 are the following

$$A_T = \frac{1}{2 Z_0^4} \left[c \mu Z_0^2 (\omega_\psi + \omega_\phi) + 2 d (\kappa \mu^2 + \kappa Z_0^4 - \alpha Z_0^2) \right]$$

$$Z_0^2 = \frac{2 d^2 \kappa \mu^2}{c \mu \left[4 A_\phi - d (\omega_\psi + \omega_\phi) \right] - 2 \alpha c^2 + 2 \alpha d^2}. \quad (2.21)$$

Finally substituting in the condition $\theta'_y(\pi) = 0$ the value of the constants A_V, A_T and Z_0 we end up with a second order algebraic equation for A_ϕ with solutions that characterize the classical string configurations of the giant magnon and the single spike

$$A_\phi = \frac{1}{4 \mu} \left[2 c \alpha + d \mu (\omega_\psi + \omega_\phi) \right] \quad \text{giant magnons} \quad (2.22)$$

$$A_\phi = \frac{c \alpha}{2 \mu} \quad \text{single spiky strings.} \quad (2.23)$$

Combining the values of the constants we have already determined and the Virasoro constraint (2.12), we calculate the values of κ^2 for the giant magnon and the single spike

$$\kappa^2 = \frac{4 \alpha^2 + \mu^2 (\omega_\psi + \omega_\phi)^2}{4 \mu^2} \quad \text{giant magnons} \quad (2.24)$$

$$\kappa^2 = \frac{\alpha^2}{\mu^2} \quad \text{single spike strings.} \quad (2.25)$$

It should be mentioned that taking the limit $Z_0 \rightarrow \infty$ from (2.21) (that corresponds to the limit of zero deformation $\mu \rightarrow 0$) and using the corresponding expression for the A_ϕ ((2.22) for the magnon and (2.23) for the spike) we determine the value of the constant α . Substituting this value in (2.22) and (2.23) we obtain the value of the constant A_ϕ in the undeformed case that is analyzed in [14]

$$\alpha = 0 \quad \Rightarrow \quad A_\phi = \frac{1}{4} d (\omega_\psi + \omega_\phi) \quad \text{giant magnons} \quad (2.26)$$

$$\alpha = \frac{c \mu (\omega_\psi + \omega_\phi)}{2 d} \quad \Rightarrow \quad A_\phi = \frac{1}{4} \frac{c^2 (\omega_\psi + \omega_\phi)}{d} \quad \text{single spike strings.} \quad (2.27)$$

Substituting (2.19), (2.20), (2.21), (2.22) and (2.23) in the definitions for the coefficients β_0 (2.18) and β_2 (2.17) of the potential \mathcal{W} in (2.14) we conclude that

$$\beta_0 = \beta_2 = 0 \quad (2.28)$$

while the coefficients β_4 (2.16) and β_6 (2.15) obey the following relation

$$\beta_4 + \beta_6 = - \frac{4 A_\phi^2}{(c^2 - d^2)^2}. \quad (2.29)$$

In the following, since the quantity $c^2 - d^2$ is positive for the magnons and negative for the spikes, we introduce the quantity v with value $v \equiv d/c$ for the magnons and $v \equiv c/d$ for the spikes. Also we define $\Omega \equiv \omega_\psi/\omega_\phi$.

In order to determine the dispersion relations for the giant magnons and the single spikes we need to solve the differential equation (2.14), which after using the boundary conditions reduces to

$$\frac{(u')^2}{2} + \mathcal{W}(u) = 0 \quad \text{with} \quad \mathcal{W}(u) = -2u^2(\beta_4 + \beta_6 u). \quad (2.30)$$

2.3 Solution and some useful integrals

In this subsection we will discuss the solution of equation (2.30) with $\beta_4 > 0$ and $\beta_6 < 0$, in such a way that $\beta_4 + \beta_6 < 0$ and (2.29) is satisfied.⁵ Since the internal sphere S^3 is not deformed by the presence of μ the solution is simple and given by the following

$$u(y) = \frac{\beta_4}{|\beta_6|} \frac{1}{\cosh^2(\sqrt{\beta_4} y)}. \quad (2.31)$$

In the calculation of the conserved quantities for the giant magnon and the single spike we will need the following two definite integrals

$$\mathcal{I}_1 \equiv \int_{-\infty}^{+\infty} u(y) dy = \frac{2\sqrt{\beta_4}}{|\beta_6|} \quad (2.32)$$

$$\mathcal{I}_2 \equiv \int_{-\infty}^{+\infty} \frac{u(y)}{1-u(y)} dy = \frac{2}{\sqrt{|\beta_4 + \beta_6|}} \arcsin\left(\sqrt{\frac{\beta_4}{|\beta_6|}}\right). \quad (2.33)$$

2.4 Angular amplitudes of the solution

In order to compute the dispersion relations for the giant magnon and the single spike it is essential to construct and compute the angular amplitudes along the directions of ψ and ϕ . For this we need to integrate equations (2.8) and (2.9) over $y \in (-\infty, \infty)$. Rewriting those equations in terms of the variable u through (2.13) and using (2.19) we arrive to the following simple expressions

$$\Psi'_y = \frac{1}{c^2 - d^2} \left(\frac{2A_\phi}{1-u} - d\omega_\psi - \frac{2c\kappa\mu}{Z_0^2} \right) \quad (2.34)$$

$$\Phi'_y = \frac{1}{c^2 - d^2} \left(\frac{2A_\phi}{1-u} - d\omega_\phi \right). \quad (2.35)$$

From the analysis of the previous subsection 2.3 we know that for the solution $u = u(y)$, given in (2.31), the integral of $1/(1-u)$ diverges and therefore both $\Delta\psi$ and $\Delta\phi$ are divergent quantities. Motivated by the archetypical giant magnon dispersion relation of [11], we introduce the following combination

$$\Delta\varphi_1 \equiv \frac{\Delta\psi + \Delta\phi}{2} = \int_{-\infty}^{+\infty} \frac{\Psi'_y + \Phi'_y}{2} dy \quad (2.36)$$

that remains finite for the case of the giant magnons while it diverges for the case of the single spikes.

⁵We do not consider the cases with $\beta_4 \leq 0$ since they do not satisfy the boundary conditions. See also the discussion and analysis in [14] for the zero deformation case.

3 Giant magnons

In this section we calculate the conserved charges for the giant magnon solution and construct the finite linear combinations of those conserved quantities that will lead us to the dispersion relation. The four conserved charges, coming from the derivatives of the Polyakov action with respect to $\partial_\tau T$, $\partial_\tau V$, $\partial_\tau \psi$ and $\partial_\tau \phi$, for the giant magnon solution (that is characterized by the value (2.22) for A_ϕ) become

$$\frac{p_T}{T} = \frac{1}{2\mu\Omega} \sqrt{4\alpha^2\Omega^2 + \mu^2(1+\Omega)^2\omega_\psi^2} \quad (3.1)$$

$$\frac{p_V}{T} = \frac{\alpha}{\mu^2} \quad (3.2)$$

$$\frac{p_\psi}{T} = \frac{(1+\Omega)\omega_\psi}{4\Omega} - \frac{\omega_\psi}{2(1-v^2)\Omega} u(y) \quad (3.3)$$

$$\frac{p_\phi}{T} = \frac{(1+\Omega)\omega_\psi}{4\Omega} - \frac{\mu\omega_\psi + 2\alpha v}{2\mu(1-v^2)} u(y) \quad (3.4)$$

$$\Delta\varphi_1 = \frac{2\alpha\Omega + \mu v(1+\Omega)\omega_\psi}{2c\mu(1-v^2)\Omega} \mathcal{I}_2. \quad (3.5)$$

The four charges⁶ \mathcal{E} , \mathcal{M} , \mathcal{J}_ψ and \mathcal{J}_ϕ that are obtained by integrating the momenta in (3.1), (3.2), (3.3) and (3.4), respectively diverge because of the constant term and only $\Delta\varphi_1$ converges, since the integral in (2.33) converges. Taking inspiration from the dispersion relation of the undeformed giant magnon, it is possible to construct linear combination of conserved quantities that will lead us to the following dispersion relation

$$\left(\sqrt{\mathcal{E}^2 - \mu^2\mathcal{M}^2} - \mathcal{J}_1\right)^2 - \mathcal{J}_2^2 = 4\sin^2\frac{\Delta\varphi_1}{2} \quad (3.6)$$

where we have used that

$$\mathcal{J}_1 = (\mathcal{J}_\psi + \mathcal{J}_\phi) \quad \& \quad \mathcal{J}_2 = (\mathcal{J}_\psi - \mathcal{J}_\phi). \quad (3.7)$$

4 Single spikes

In this section we calculate the conserved charges for the single spike solution, in the same fashion as we did for the giant magnons. The four conserved charges for the single spike solution (that is characterized by the value (2.23) for A_ϕ) become

$$\frac{p_T}{T} = \frac{\alpha}{\mu v} \quad (4.1)$$

$$\frac{p_V}{T} = \frac{2\alpha - \mu v(1+\Omega)\omega_\phi}{2\mu^2 v} \quad (4.2)$$

$$\frac{p_\psi}{T} = \frac{v\omega_\phi}{2(1-v^2)} u(y) \quad (4.3)$$

$$\frac{p_\phi}{T} = \frac{2\alpha - \mu v\omega_\phi}{2\mu(1-v^2)} u(y) \quad (4.4)$$

$$\Delta\varphi_1 = \frac{(1+\Omega)\omega_\phi}{2d} \mathcal{I}_3 - \frac{1}{2d} \left(\frac{2\alpha v}{\mu(1-v^2)} + (1+\Omega)\omega_\phi \right) \mathcal{I}_2 \quad (4.5)$$

⁶Here and in the following we use the notation $\mathcal{E} \equiv E/T$, $\mathcal{M} \equiv M/T$, $\mathcal{J}_\psi \equiv J_\psi/T$ and $\mathcal{J}_\phi \equiv J_\phi/T$.

where \mathcal{I}_3 is a non-convergent integral.⁷ Taking inspiration from the dispersion relation of the undeformed single spike in S^3 of [18], it is possible to construct linear combinations of conserved quantities that will lead us to the following dispersion relation

$$\frac{1}{4} (\mathcal{J}_1^2 - \mathcal{J}_2^2) = \sin^2 \left[\frac{1}{2} (\mathcal{E} - \mu \mathcal{M} - \Delta\varphi_1) \right]. \tag{4.7}$$

5 Discussion

In this paper we have derived semi-classical string solutions living in the Schrödinger $Sch_5 \times S^5$ spacetime which is conjectured to be the gravity dual of the non-local CFT coming under the name null dipole CFT. Our solutions are the counterparts of the giant magnon and single spike solutions of the undeformed $AdS_5 \times S^5$ background. This claim is supported by the fact that in the limit where the deformation parameter $\mu \rightarrow 0$ our solutions become those of the giant magnon and single spike solutions of the original $AdS_5 \times S^5$ background. The same is true when the string solution, and as a result the dual operator, does not carry momentum along the X^- direction, i.e. $M = 0$. The solutions presented here live in an S^3 subspace of the five-sphere along the directions of which the B -field has non-zero components. We have explicitly checked that the submanifold we have used is a consistent sector of classical string theory on $Sch_5 \times S^5$.

The string solutions we have found are open string solutions which have infinite energy and angular momentum. Their end points are situated on the equator of the three-sphere S^3 and for the case of the magnon their angle difference $\Delta\varphi_1$ is to be identified with the momentum p carried by the excitation. In the case of the dyonic giant magnon the dispersion relation is given by

$$\sqrt{E^2 - \frac{\lambda}{4\pi^2} L^2 M^2} - J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}} \tag{5.1}$$

while for the single spike is given by

$$J_1^2 - J_2^2 = \frac{\lambda}{\pi^2} \sin^2 \left[\frac{\pi}{\sqrt{\lambda}} E - \frac{1}{2} L M - \frac{1}{2} \Delta\varphi_1 \right] \tag{5.2}$$

where we have used that

$$\mu = \frac{\sqrt{\lambda}}{2\pi} L \tag{5.3}$$

where L is the parameter defining the \star -product in field theory [29].

Based on (5.1) one may boldly conjecture that the *exact in λ* (in the strict infinite $J \rightarrow \infty$ limit, of course) dispersion relation of a single particle excitation — see also (5.13)

⁷That integral is defined as

$$\mathcal{I}_3 \equiv \int_{-\infty}^{+\infty} \frac{1}{1-u(y)} dy \tag{4.6}$$

and we need to eliminate it by a suitable linear combination of the other conserved quantities.

— in the infinite $J \rightarrow \infty$ limit will be given by⁸

$$\sqrt{E^2 - \frac{\lambda}{4\pi^2} L^2 M^2} - J = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}, \quad J \rightarrow \infty. \quad (5.4)$$

In order to be able to compare with the known field theory results, which are available only at one-loop order, one should take the momentum of the excitation small, i.e. $p = \frac{2\pi n}{J}$ and the coupling to scale in such a way that the quantity λ' will also be small, i.e. $\lambda' = \frac{\lambda}{J^2} \ll 1$ and expand (5.4) around the BMN-like reference state of [29]. Keeping only the linear terms in the λ' expansion we get

$$E = \left(1 + J + \frac{\lambda n^2}{2J^2} + \dots\right) \cdot \left(1 + \frac{\lambda L^2 M^2}{8\pi^2 J^2} + \dots\right), \quad (5.5)$$

where the dots denote terms of order higher or equal to λ'^2 . This prediction should be compared to the one-loop result obtained from the Baxter equation which reads [36]

$$\Delta = E - (J + 1) = \frac{\lambda L^2 M^2}{8\pi^2 J} + \left(\frac{\lambda n^2}{2J^2} + \frac{\lambda L M n}{\pi J^2} + \frac{\lambda L^2 M^2}{8\pi^2 J^2} + \frac{\lambda}{J^2} O(L^4 M^4)\right). \quad (5.6)$$

Comparing (5.5) and (5.6) one easily sees that the string theory prediction correctly reproduces the first, second and fourth term of the field theory result failing to reproduce all other terms. Although (5.4) is conjectured to be exact in the λ expansion it is valid only in the $J \rightarrow \infty$ limit. We now argue that all the terms in (5.6) that can not be reproduced by the string prediction are actually suppressed by powers of $\frac{1}{\sqrt{J}}$. Indeed, in order to be feasible to compare the string result with the field theory perturbative expansion the first term in (5.6) should be small. For this to happen not only $\frac{\lambda}{J^2} \ll 1$ but also $L^2 M^2 J \sim 1$. This can be seen by multiplying and dividing the first term of (5.6) by J in order to express it terms of the effective coupling $\frac{\lambda}{J^2}$. Now the condition $L^2 M^2 J \sim 1$ implies that the quantity LM should scale as $LM \sim \frac{1}{\sqrt{J}}$ in the large J expansion. As a result all terms beyond the second term in (5.6) are suppressed by powers of $\frac{1}{\sqrt{J}}$ and as such can never be obtained from (5.4) which is inherently valid in the infinite angular momentum limit. It would be interesting to calculate finite size correction to the dispersion relation (5.4).

One is thus tempted to interpret the solution with dispersion relation (5.4) as the dispersion relation of single particle excitations which propagate with momentum p along a certain pseudovacuum/reference state, the strong coupling dual of which is the BMN-like point-like string of [29]. The latter resembles the BMN string of the undeformed $AdS_5 \times S^5$ background. In the supegravity limit ($p = 0$) our solution becomes point-like and identical to the one presented in [29].

One can further speculate on the form of the field theory operators which are dual to our string solutions. As discussed in [29], the Schrödinger/null-dipole CFT correspondence

⁸A more conservative and general but less predictive proposal would be to multiply the $\sin^2 \frac{p}{2}$ by a function depending on the coupling λ , as well as on the deformation parameter L , namely $\sqrt{E^2 - \frac{\lambda}{4\pi^2} L^2 M^2} - J = \sqrt{1 + \frac{f(\lambda, L)\lambda}{\pi^2} \sin^2 \frac{p}{2}}$. This function should have the asymptotics $f(\lambda = \infty, L) = 1$ and should be computed order by order in string or field theory perturbation theory.

can have two equivalent descriptions. One can either consider closed string theory with periodic boundary conditions on the Schrödinger background or, equivalently, open string theory with twisted boundary conditions on the original $AdS_5 \times S^5$ background. On the field theory side the first choice is to employ conformal operators whose elementary constituents are the same as in $\mathcal{N} = 4$ SYM and which transform under local gauge transformations. Their form is as follows

$$\mathcal{O} = \text{tr } \hat{\Phi}_1 \dots \hat{\Phi}_J \quad (5.7)$$

where the hatted fields are those obtained after the Seiberg-Witten map and read

$$\hat{\Phi}(x) = \text{P exp} \int_{x+L_\Phi}^x dv A_-(v) \cdot \hat{\Phi}(x) \cdot \text{P exp} \int_x^{x-L_\Phi} dv A_-(v). \quad (5.8)$$

The integration of the Wilson lines is along the light ray connecting the points x and $x \pm L_\Phi$ and $L_\Phi = \frac{LR_\Phi}{2}$ is half of the dipole length of the field Φ having R-charge R_Φ . For more details one can refer to [29]. The corresponding spin chain is periodic and as a consequence dual to the closed string theory on the deformed background.

An alternative basis which is more convenient in describing the ground state on which the giant magnon propagates is that of the light-ray operators defined by a number of fields located along a light-ray, namely [38]

$$\mathcal{O}(x_1, \dots, x_J) = \text{tr } \tilde{Z}_{x_1}(0) \dots \tilde{Z}_{x_J}(0) \quad (5.9)$$

where we have defined the tilded fields as

$$\tilde{\Phi}_x(0) = \text{P exp} \int_0^x dv A_-(v) \cdot \hat{\Phi}(x) \cdot \text{P exp} \int_x^0 dv A_-(v). \quad (5.10)$$

Notice that the field $\hat{\Phi}(x) \rightarrow U^{-1}(x)\hat{\Phi}(x)U(x)$ transforms under the gauge transformations as a local field situated at the space-time point x while the field $\tilde{\Phi}_x(0) \rightarrow U^{-1}(0)\tilde{\Phi}_x(0)U(0)$ transforms as a local field situated at the space-time point 0.

In this basis of operators the ground state dual to the BMN-like string of [29] will have the form

$$\mathcal{O}_0 = \int \prod_{i=1}^J dx_i \psi_0(x_1, \dots, x_J) \mathcal{O}(x_1, \dots, x_J) \quad (5.11)$$

for a certain wavefunction $\psi_0(z_1, \dots, z_J)$. This state belongs to an $SL(2)$ closed subsector and its eigenvalues for an arbitrary number J of fields Z has been calculated in [29] by using the Baxter equation in this sector. In what follows, we will assume that the (5.11) has a well-defined limit when the number of scalar fields becomes arbitrary large, i.e. when $J \rightarrow \infty$. Our conjecture is that the giant magnon solutions of this work will be dual to the following field theory operator

$$\mathcal{O}_{\text{magnon}} = \int \prod_i dx_i \psi_0(\dots, x_{l-1}, x_l, x_{l+1} \dots) \mathcal{O}_p(\dots x_{l-1}, x_l, x_{l+1} \dots) \quad (5.12)$$

where

$$\mathcal{O}_p(\dots x_{l-1}, x_l, x_{l+1} \dots) = \sum_l e^{ipl} \left(\dots \tilde{Z}_{x_{l-1}}(0) \tilde{\Phi}_{x_l}(0) \tilde{Z}_{x_{l+1}}(0) \dots \right). \quad (5.13)$$

As discussed below (5.10) the fields $\tilde{\Phi}_x(0)$ transform under gauge transformations as if they were all located at the same point 0. As a result (5.13) has striking similarity with the magnon operator in $\mathcal{N} = 4$ SYM where all fields are sitting at the same point too. Let us also mention that l in (5.13) labels the position of the impurity $\tilde{\Phi}_{x_l}(0)$ in the long string of $\tilde{Z}_{x_i}(0)$'s and that the space-time points x_i of (5.13) used in the definition of the tilded fields (5.10) are not necessarily ordered along the light-ray on which they are defined.

In this approach the scalar field $\hat{\Phi}$ is one of the scalars of the three-sphere S^3 and is sitting in the l^{th} site of the infinitely long string of the Z 's. A couple of important comments are in order. Firstly, the operator in (5.12) involves a single excitation Φ and thus is dual to the string solution with $J_2 = J_\psi - J_\phi = 0$ (see also (5.1)). The operator dual to the dyonic magnon of (5.1) is a bound state of J_2 scalar fields propagating coherently with momentum p . Secondly, we should stress that unlike the pseudovacuum (5.11) the operator of (5.13) does not belong to the $SL(2)$ subsector and as a result for a generic impurity one can not use the integrability related Baxter equation to calculate its dimension as has been done in [29] for operators in the aforementioned closed sector. Furthermore, although not apparent the operator of (5.13) contains infinitely many derivatives acting on the scalar fields which are obtained upon Taylor expanding the Wilson lines that are present in the definition of the tilded fields (5.10). Finally, in the limit where one switches the deformation off $\mu \rightarrow 0$ the operator (5.12) becomes the usual $\mathcal{N} = 4$ magnon operator since the wavefunction ψ_0 becomes a product of δ -functions at the same space-time point.

A number of important questions remain to be answered. It would be interesting to calculate the dimensions of the operators in (5.12) from the field theory side using integrability techniques or even using Feynman diagrams. Additionally, the precise nature of the pseudovacuum on which the excitations propagate should be clarified. Furthermore, it would be very interesting to see how integrability of the theory manifests itself in the scattering of the giant magnon solutions we have found in this work. This may shed some light to the validity of the interpretation of our solutions as single particle asymptotic states whose world-sheet scattering matrix can be subsequently evaluated and compared with the Drinfeld-Reshetikhin twisted S -matrix of the deformed theory. One might be also be tempted to try to use the algebra of the theory in order to compute the exact in the coupling dispersion relation and scattering matrix along the lines of [16]. Finally, it would be nice to evaluate finite size correction to the dispersion relation of giant magnons and spikes.

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A Polyakov action and a consistent truncation

The Polyakov form of the string action is given by the following standard expression⁹

$$S_P = -\frac{T}{2} \int d\tau d\sigma \left(\sqrt{-h} h^{\alpha\beta} g_{\alpha\beta} - \epsilon^{\alpha\beta} b_{\alpha\beta} \right) \tag{A.1}$$

⁹The string tension T is related to the 't Hooft coupling as $T = \frac{\sqrt{\lambda}}{2\pi}$.

where

$$g_{\alpha\beta} = G_{MN} \partial_\alpha x^M \partial_\beta x^N \quad \& \quad b_{\alpha\beta} = B_{MN} \partial_\alpha x^M \partial_\beta x^N \quad (\text{A.2})$$

are the pullbacks of the metric and the B -field on the string worldsheet and the tensor density $\epsilon^{\alpha\beta}$ is defined according to the convention $\epsilon^{01} = 1$. With $h_{\alpha\beta}$ we denote the worldsheet metric and we choose the conformal gauge where $h_{\alpha\beta} = \eta_{\alpha\beta}$. The construction is also supplemented by the Virasoro constraints, which are obtained by differentiating (A.1) with respect to $h_{\alpha\beta}$,

$$G_{MN} (\partial_\tau x^M \partial_\tau x^N + \partial_\sigma x^M \partial_\sigma x^N) = 0 \quad \& \quad G_{MN} \partial_\tau x^M \partial_\sigma x^N = 0. \quad (\text{A.3})$$

The momentum p_M that is canonically conjugate to the coordinate x^M is given by the following expression

$$p_M = \frac{\partial \mathcal{L}}{\partial \dot{x}^M} \quad (\text{A.4})$$

where $\dot{x}^M \equiv \partial_\tau x^M$.

The 10d $Sch_5 \times S^5$ metric is given by the following expression [29, 33]

$$\begin{aligned} ds^2 &= - \left(1 + \frac{\mu^2}{Z^4} \right) dT^2 + \frac{1}{Z^2} \left(2dTdV + dZ^2 - \vec{X}^2 dT^2 + d\vec{X}^2 \right) + ds_{S^5}^2 \\ B_2 &= \frac{\mu}{Z^2} dT \wedge (d\chi + \omega). \end{aligned} \quad (\text{A.5})$$

The metric in the five-sphere is written as an S^1 -fibration over \mathbb{CP}^2

$$ds_{S^5}^2 = (d\chi + \omega)^2 + ds_{\mathbb{CP}^2}^2 \quad \text{with} \quad ds_{\mathbb{CP}^2}^2 = d\eta^2 + \sin^2 \eta (\Sigma_1^2 + \Sigma_2^2 + \cos^2 \eta \Sigma_3^2). \quad (\text{A.6})$$

where the Σ_i ($i = 1, 2, 3$) and ω are defined as follows

$$\begin{aligned} \Sigma_1 &\equiv \frac{1}{2} (\cos \psi d\theta - \sin \psi \sin \theta d\phi) & \Sigma_2 &\equiv \frac{1}{2} (\sin \psi d\theta + \cos \psi \sin \theta d\phi) \\ \Sigma_3 &\equiv \frac{1}{2} (d\psi - \cos \theta d\phi) & \text{and} & \quad \omega &\equiv \sin^2 \eta \Sigma_3. \end{aligned} \quad (\text{A.7})$$

It can be explicitly checked that the following ansatz

$$\begin{aligned} T &= \kappa \tau + T_y(y) & V &= \alpha \tau + V_y(y) & Z &= Z_0 & \vec{X} &= \chi = 0 \\ [3pt] \theta &= \theta_y(y) & \psi &= \omega_\psi \tau + \Psi_y(y) & \phi &= \omega_\phi \tau + \Phi_y(y) & \eta &= \frac{\pi}{2} \end{aligned} \quad (\text{A.8})$$

satisfies the equations of motion coming from (A.1) and the Virasoro constraints from (A.3). While the equations of motion for \vec{X} and η are trivially satisfied, special care should be taken for the equation of motion for χ , since in order to be satisfied we need to use equations (2.8) & (2.9). Equipped by the preceding analysis we set $\vec{X} = \chi = 0$ and $\eta = \frac{\pi}{2}$ in equations (A.5) and (A.6) and obtain the consistent truncation ansatz of (2.1) and (2.2). Finally, we would like to stress that interchanging the ansatz for χ and ψ as follows

$$\chi = \frac{1}{2} (\omega_\psi \tau + \Psi_y(y)) \quad \text{and} \quad \psi = 0 \quad (\text{A.9})$$

lead us to the same differential equations and dispersion relations.

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