Published for SISSA by Springer

Received: *December 22, 2017*
Accepted: *January 27, 2018*
Published: *February 22, 2018*

The massless integer superspin multiplets revisited

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Abstract: We propose a new off-shell formulation for the massless $\mathcal{N} = 1$ supersymmetric multiplet of integer superspin s in four dimensions, where $s = 2, 3, \ldots$ (the $s = 1$ case corresponds to the gravitino multiplet). Its gauge freedom matches that of the superconformal superspin-s multiplet described in arXiv:1701.00682. The gauge-invariant action involves two compensating multiplets in addition to the superconformal superspin-s multiplet. Upon imposing a partial gauge fixing, this action reduces to the one describing the so-called longitudinal formulation for the massless superspin-s multiplet. Our new model is shown to possess a dual realisation obtained by applying a superfield Legendre transformation. We present a non-conformal higher spin supercurrent multiplet associated with the new integer superspin theory. This fermionic supercurrent is shown to occur in the Fayet-Sohnius model for a massive $\mathcal{N} = 2$ hypermultiplet. We also give a new off-shell realisation for the massless gravitino multiplet.

Keywords: Supergravity Models, Superspaces, Supersymmetry and Duality

ArXiv ePrint: 1711.11364

Dedicated to Professor Ulf Lindström on the occasion of his 70th birthday

https://doi.org/10.1007/JHEP02(2018)137

Contents

1 Introduction

In $\mathcal{N} = 1$ supersymmetric field theory in four dimensions, a massless multiplet of (half) integer superspin $\hat{s} > 0$ describes two ordinary massless fields of spin $\hat{s}$ and $\hat{s} + \frac{1}{2}$. Such a supermultiplet is often denoted $\big(\hat{s}, \hat{s} + \frac{1}{2}\big)$. The three lowest superspin values, $\hat{s} = \frac{1}{2}, 1$ and $\frac{3}{2}$, correspond to the vector, gravitino and supergravity multiplets, respectively. It follows from first principles that the sum of two actions for free massless spin-$\hat{s}$ and spin-$\big(\hat{s} + \frac{1}{2}\big)$ fields should possess an on-shell supersymmetry. This means that there is no problem of constructing on-shell massless higher superspin multiplets, with $\hat{s} > \frac{3}{2}$, for it is only necessary to work out the structure of supersymmetry transformations. The latter task was completed first by Curtright [1] who made use of the (Fang-)Fronsdal actions [2, 3], and soon after by Vasiliev [4] who employed his frame-like reformulation of the (Fang-)Fronsdal models pioneered in [4]. Applications of the on-shell higher spin supermultiplets presented in [1, 4] are rather limited. In particular, they do not allow one to construct supermultiplets containing conserved higher spin currents that have to be off-shell, like the so-called supercurrent multiplet [5] containing the energy-momentum tensor and the supersymmetry current. To obtain such higher spin supercurrents, off-shell realisations for the massless higher superspin multiplets are required, and these are nontrivial to construct.[1]

The problem of constructing gauge off-shell formulations for the massless higher superspin multiplets was solved in the early 1990s in the case of Poincaré supersymmetry [9, 10].[2] For each superspin $\hat{s} > \frac{3}{2}$, half-integer [9] and integer [10], these publications provided two dually equivalent off-shell actions formulated in $\mathcal{N} = 1$ Minkowski superspace. At the component level, each of the two superspin-$\hat{s}$ actions [9, 10] reduces, *upon* imposing a

[1] Early attempts to construct such off-shell realisations [6, 7] were unsuccessful, as was explained in detail in [8].

[2] The results obtained in [9, 10] are reviewed in [11].

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Wess-Zumino-type gauge and eliminating the auxiliary fields, to a sum of the spin-$\hat{s}$ and spin-$\big(\hat{s}+\frac{1}{2}\big)$ actions [2, 3]. The massless higher superspin theories of [9, 10] were generalised to the case of anti-de Sitter supersymmetry in [8].

The non-supersymmetric higher spin theories of [2, 3] and their supersymmetric counterparts of half-integer superspin [9] share one common feature. For each of them, the action is formulated in terms of a (super)conformal gauge (super)field coupled to certain compensators. Such a description does not yet exist for the massless supermultiplets of integer superspin $\hat{s} \geq 2$. One of the goals of this paper is to provide such a formulation by properly generalising the off-shell supersymmetric actions given in [10]. We now make these points more precise.

Given an integer $s \geq 2$, the conformal spin-s field [12, 13] is described by a real potential[3] $h_{\alpha_1\dots\alpha_s\dot{\alpha}_1\dots\dot{\alpha}_s} = h_{(\alpha_1\dots\alpha_s)(\dot{\alpha}_1\dots\dot{\alpha}_s)} \equiv h_{\alpha(s)\dot{\alpha}(s)}$ with the gauge freedom

$$\delta h_{\alpha_1\dots\alpha_s\dot{\alpha}_1\dots\dot{\alpha}_s} = \partial_{(\alpha_1(\dot{\alpha}_1}\lambda_{\alpha_2\dots\alpha_s)\dot{\alpha}_2\dots\dot{\alpha}_s)}\,, \tag{1.1a}$$

for an arbitrary real gauge parameter $\lambda_{\alpha_1\dots\alpha_{s-1}\dot{\alpha}_1\dots\dot{\alpha}_{s-1}}=\lambda_{(\alpha_1\dots\alpha_{s-1})(\dot{\alpha}_1\dots\dot{\alpha}_{s-1})}\equiv\lambda_{\alpha(s-1)\dot{\alpha}(s-1)}$. In addition to the gauge field $h_{\alpha(s)\dot{\alpha}(s)}$, the massless spin-$s$ action [2] also involves a real compensator $h_{\alpha(s-2)\dot{\alpha}(s-2)}$ with the gauge transformation[4]

$$\delta h_{\alpha_1\dots\alpha_{s-2}\dot{\alpha}_1\dots\dot{\alpha}_{s-2}} = \partial^{\beta\dot{\beta}}\lambda_{\beta\alpha_1\dots\alpha_{s-2}\dot{\beta}\dot{\alpha}_1\dots\dot{\alpha}_{s-2}}\,. \tag{1.1b}$$

In the fermionic case, the conformal spin-$\big(s+\frac{1}{2}\big)$ field [12, 13] is described by a potential $\psi_{\alpha(s+1)\dot{\alpha}(s)}$ and its conjugate $\bar{\psi}_{\alpha(s)\dot{\alpha}(s+1)}$ with the gauge freedom

$$\delta\psi_{\alpha_1\dots\alpha_{s+1}\dot{\alpha}_1\dots\dot{\alpha}_s} = \partial_{(\alpha_1(\dot{\alpha}_1}\xi_{\alpha_2\dots\alpha_{s+1})\dot{\alpha}_2\dots\dot{\alpha}_s)}\,, \tag{1.2a}$$

for an arbitrary gauge parameter $\xi_{\alpha(s)\dot{\alpha}(s-1)}$. In addition to the gauge fields $\psi_{\alpha(s+1)\dot{\alpha}(s)}$ and $\bar{\psi}_{\alpha(s)\dot{\alpha}(s+1)}$, the massless spin-$\big(s+\frac{1}{2}\big)$ action [3] also involves two compensators $\psi_{\alpha(s-1)\dot{\alpha}(s)}$ and $\psi_{\alpha(s-1)\dot{\alpha}(s-2)}$ and their conjugates, with the following gauge transformations

$$\begin{aligned}\delta\psi_{\alpha_1\dots\alpha_{s-1}\dot{\alpha}_1\dots\dot{\alpha}_s} &= \partial^{\beta}{}_{(\dot{\alpha}_1}\xi_{\beta\alpha_1\dots\alpha_{s-1}\dot{\alpha}_2\dots\dot{\alpha}_s)}\,, &(1.2\text{b})\\ \delta\psi_{\alpha_1\dots\alpha_{s-1}\dot{\alpha}_1\dots\dot{\alpha}_{s-2}} &= \partial^{\beta\dot{\beta}}\xi_{\beta\alpha_1\dots\alpha_{s-1}\dot{\beta}\dot{\alpha}_1\dots\dot{\alpha}_{s-2}}\,. &(1.2\text{c})\end{aligned}$$

We now recall the structure of the off-shell higher spin supermultiplets. Given a half-integer superspin $\hat{s} = s+\frac{1}{2}$, with $s = 2, 3, \dots$, the superconformal multiplet introduced in [14] is described by a real unconstrained prepotential $H_{\alpha(s)\dot{\alpha}(s)}$ possessing the gauge transformation law[5]

$$\delta H_{\alpha_1\dots\alpha_s\dot{\alpha}_1\dots\dot{\alpha}_s} = \bar{D}_{(\dot{\alpha}_1}\Lambda_{\alpha_1\dots\alpha_s\dot{\alpha}_2\dots\dot{\alpha}_s)} - D_{(\alpha_1}\bar{\Lambda}_{\alpha_2\dots\alpha_s)\dot{\alpha}_1\dots\dot{\alpha}_s}\,, \tag{1.3}$$

[3] All tensor (super)fields encountered in this paper are completely symmetric with respect to their undotted spinor indices, and separately, with respect to their dotted indices. We use the notation $V_{\alpha(s)\dot{\alpha}(t)} := V_{\alpha_1\cdots\alpha_s\dot{\alpha}_1\cdots\dot{\alpha}_t} = V_{(\alpha_1\cdots\alpha_s)(\dot{\alpha}_1\cdots\dot{\alpha}_t)}$ and $V^{\alpha(s)\dot{\alpha}(t)}U_{\alpha(s)\dot{\alpha}(t)} := V^{\alpha_1\cdots\alpha_s\dot{\alpha}_1\cdots\dot{\alpha}_t}U_{\alpha_1\cdots\alpha_s\dot{\alpha}_1\cdots\dot{\alpha}_t}$. Parentheses denote symmetrisation of indices; the undotted and dotted spinor indices are symmetrised independently. Indices sandwiched between vertical bars (for instance, $|\gamma|$) are not subject to symmetrisation.

[4] For a review of the (Fang-)Fronsdal models [2, 3] in the two-component spinor notation used in this paper, see e.g. [11].

[5] In the $s = 1$ case, the transformation law (1.3) corresponds to linearised conformal supergravity [15].

with unconstrained gauge parameter $\Lambda_{\alpha(s)\dot\alpha(s-1)}$. In addition to the gauge superfield $H_{\alpha(s)\dot\alpha(s)}$, each of the massless superspin-$\big(s+\frac{1}{2}\big)$ actions constructed in [9] contains a compensating multiplet. In one case, the compensating multiplet is described by a longitudinal linear superfield $G_{\alpha(s-1)\dot\alpha(s-1)}$ (and its conjugate $\bar G_{\alpha(s-1)\dot\alpha(s-1)}$) constrained by

$$\bar D_{(\dot\alpha_1} G_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_s)} = 0 \Longrightarrow \bar D^2 G_{\alpha(s-1)\dot\alpha(s-1)} = 0\,, \tag{1.4}$$

with the gauge transformation

$$\begin{aligned}\delta G_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_{s-1}} &= -\frac{1}{2}\bar D_{(\dot\alpha_1}\bar D^{|\dot\beta|}D^\beta\Lambda_{\beta\alpha_1\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1})\dot\beta}\\ &\quad +\mathrm{i}(s-1)\bar D_{(\dot\alpha_1}\partial^{\beta|\dot\beta|}\Lambda_{\beta\alpha_1\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1})\dot\beta}\,.\end{aligned} \tag{1.5}$$

In the other formulation, the compensating multiplet is described by a transverse linear superfield $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ (and its conjugate $\bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$) constrained by

$$\bar D^{\dot\beta}\,\Gamma_{\alpha(s-1)\dot\beta\dot\alpha(s-2)} = 0 \Longrightarrow \bar D^2\Gamma_{\alpha(s-1)\dot\alpha(s-1)} = 0\,, \tag{1.6}$$

with the gauge transformation

$$\delta_\Lambda\Gamma_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_{s-1}} = -\frac{1}{4}\bar D^{\dot\beta}D^2\bar\Lambda_{\alpha_1\dots\alpha_{s-1}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}}\,. \tag{1.7}$$

Finally, in the case of an integer superspin $\hat s = s$, with $s = 2, 3, \dots$, the superconformal multiplet introduced in [14] is described by an unconstrained prepotential $\Psi_{\alpha(s)\dot\alpha(s-1)}$ and its complex conjugate with the gauge transformation given by eq. (2.5a) below, with unconstrained gauge parameters $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$ and $\zeta_{\alpha(s)\dot\alpha(s-2)}$. The prepotential $\Psi_{\alpha(s)\dot\alpha(s-1)}$ naturally occurs in the longitudinal formulation for the massless superspin-s multiplet [10]. However, the gauge transformation of $\Psi_{\alpha(s)\dot\alpha(s-1)}$ given in [10] differs from eq. (2.5a). The difference is that the parameter $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$ in [10] is not unconstrained, but instead is given by (2.10). In this paper we propose a new off-shell formulation for the massless higher integer superspin multiplet with the following properties: (i) the gauge freedom of $\Psi_{\alpha(s)\dot\alpha(s-1)}$ is given by (2.5a); and (ii) the longitudinal formulation of [10] emerges upon imposing a gauge condition.

This paper is organised as follows. In section 2 we present the new formulation for the massless superspin-s multiplet. Its dual version is described in section 3. In section 4 we introduce non-conformal higher spin supercurrents associated with the gauge massless superspin-s multiplets. Section 5 is devoted to computing the higher spin supercurrents that originate in the massive $\mathcal{N}=2$ hypermultiplet model. Concluding comments are given in section 6, including a brief discussion of the off-shell models for the massless gravitino multiplet.

2 New formulation

Given a positive integer $s \geq 2$, we propose to describe the massless superspin-s multiplet in terms of the following superfield variables: (i) an unconstrained prepotential

$\Psi_{\alpha(s)\dot\alpha(s-1)}$ and its complex conjugate $\bar\Psi_{\alpha(s-1)\dot\alpha(s)}$; (ii) a real superfield $H_{\alpha(s-1)\dot\alpha(s-1)} = \bar H_{\alpha(s-1)\dot\alpha(s-1)}$; and (iii) a complex superfield $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ and its conjugate $\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}$, where $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ is constrained to be transverse linear,[6]

$$\bar D^{\dot\beta}\Sigma_{\alpha(s-1)\dot\beta\dot\alpha(s-3)} = 0\,. \tag{2.1}$$

In the $s=2$ case, for which (2.1) is not defined, $\Sigma_{\alpha(2)}$ is instead constrained to be complex linear,

$$\bar D^2\Sigma_{\alpha(2)} = 0\,. \tag{2.2}$$

The constraint (2.1), or its counterpart (2.2) for $s=2$, can be solved in terms of a complex unconstrained prepotential $Z_{\alpha(s-1)\dot\alpha(s-1)}$ by the rule

$$\Sigma_{\alpha(s-1)\dot\alpha(s-2)} = \bar D^{\dot\beta} Z_{\alpha(s-1)(\dot\beta\dot\alpha_1\dots\dot\alpha_{s-2})}\,. \tag{2.3}$$

This prepotential is defined modulo gauge transformations

$$\delta_\xi Z_{\alpha(s-1)\dot\alpha(s-1)} = \bar D^{\dot\beta}\xi_{\alpha(s-1)(\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1})}\,, \tag{2.4}$$

with the gauge parameter $\xi_{\alpha(s-1)\dot\alpha(s)}$ being unconstrained.

The gauge freedom of $\Psi_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}}$ is chosen to coincide with that of the superconformal superspin-s multiplet [14], which is

$$\delta_{\mathfrak{V},\zeta}\Psi_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}} = \frac{1}{2}D_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_1\dots\dot\alpha_{s-1}} + \bar D_{(\dot\alpha_1}\zeta_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1})}\,, \tag{2.5a}$$

with unconstrained gauge parameters $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$ and $\zeta_{\alpha(s)\dot\alpha(s-2)}$. The $\mathfrak{V}$-transformation is defined to act on the superfields $H_{\alpha(s-1)\dot\alpha(s-1)}$ and $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ as follows

$$\delta_{\mathfrak{V}} H_{\alpha(s-1)\dot\alpha(s-1)} = \mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)} + \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)}\,, \tag{2.5b}$$

$$\delta_{\mathfrak{V}}\Sigma_{\alpha(s-1)\dot\alpha(s-2)} = \bar D^{\dot\beta}\bar{\mathfrak{V}}_{\alpha(s-1)\dot\beta\dot\alpha(s-2)} \Longrightarrow \delta_{\mathfrak{V}} Z_{\alpha(s-1)\dot\alpha(s-1)} = \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)}\,. \tag{2.5c}$$

The longitudinal linear superfield

$$G_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} := \bar D_{(\dot\alpha_1}\Psi_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s)}\,, \qquad \bar D_{(\dot\alpha_1}G_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s+1})} = 0 \tag{2.6}$$

is invariant under the ζ-transformation (2.5a) and varies under the $\mathfrak{V}$-transformation as

$$\delta_{\mathfrak{V}} G_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} = \frac{1}{2}\bar D_{(\dot\alpha_1}D_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,. \tag{2.7}$$

[6] In general, complex tensor superfields $\Gamma_{\alpha(r)\dot\alpha(t)}$ and $G_{\alpha(r)\dot\alpha(t)}$ are called transverse linear and longitudinal linear, respectively, if the constraints $\bar D^{\dot\beta}\Gamma_{\alpha(r)\dot\beta\dot\alpha(t-1)} = 0$ and $\bar D_{(\dot\beta}G_{\alpha(r)\dot\alpha_1\dots\dot\alpha_t)} = 0$ are satisfied. The former constraint is defined for $t\neq 0$; it has to be replaced with the standard linear constraint, $\bar D^2\Gamma_{\alpha(r)} = 0$, for $t=0$. The latter constraint for $t=0$ is the chirality condition $\bar D_{\dot\beta}G_{\alpha(r)} = 0$.

It may be checked that the following action

$$\begin{aligned}
S^{\parallel}_{(s)} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, \Big\{ &\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
&+\frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(D^\beta \bar D^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar D^{\dot\beta} D^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \Big) \\
&+2\bar G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \frac{s}{s+1}\Big(G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \bar G^{\alpha(s)\dot\alpha(s)} \bar G_{\alpha(s)\dot\alpha(s)} \Big) \\
&+\frac{s-1}{4s} H^{\alpha(s-1)\dot\alpha(s-1)} \big(D_{\alpha_1} \bar D^2 \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar D_{\dot\alpha_1} D^2 \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}} \big) \\
&+\frac{1}{s} \Psi^{\alpha(s)\dot\alpha(s-1)} \big(D_{\alpha_1} \bar D_{\dot\alpha_1} - 2\mathrm{i}(s-1)\partial_{\alpha_1\dot\alpha_1} \big) \Sigma_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1}} \\
&+\frac{1}{s} \bar\Psi^{\alpha(s-1)\dot\alpha(s)} \big(\bar D_{\dot\alpha_1} D_{\alpha_1} - 2\mathrm{i}(s-1)\partial_{\alpha_1\dot\alpha_1} \big) \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_s} \\
&+\frac{s-1}{8s} \Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)} D^2 \Sigma_{\alpha(s-1)\dot\alpha(s-2)} - \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)} \bar D^2 \bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)} \Big) \\
&-\frac{1}{s^2} \bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta} \Big(\frac{1}{2}(s^2+1) D^\beta \bar D_{\dot\beta} + \mathrm{i}(s-1)^2 \partial^\beta{}_{\dot\beta} \Big) \Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)} \Big\} \qquad (2.8)
\end{aligned}$$

is invariant under the gauge transformations (2.5). By construction, the action is also invariant under (2.4).

The $\mathfrak{V}$-gauge freedom (2.5) may be used to impose the condition

$$\Sigma_{\alpha(s-1)\dot\alpha(s-2)} = 0\,. \qquad (2.9)$$

In this gauge, the action (2.8) reduces to that describing the longitudinal formulation for the massless superspin-s multiplet [10]. The gauge condition (2.9) does not fix completely the $\mathfrak{V}$-gauge freedom. The residual gauge transformations are generated by

$$\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)} = D^\beta L_{(\beta\alpha_1\dots\alpha_{s-1})\dot\alpha(s-1)}\,, \qquad (2.10)$$

with the parameter $L_{\alpha(s)\dot\alpha(s-1)}$ being an unconstrained superfield. With this expression for $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$, the gauge transformations (2.5a) and (2.5b) coincide with those given in [10]. Our consideration implies that the action (2.8) indeed provides an off-shell formulation for the massless superspin-s multiplet.

Instead of choosing the condition (2.9), one can impose an alternative gauge fixing

$$H_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \qquad (2.11)$$

In accordance with (2.5b), in this gauge the residual gauge freedom is described by

$$\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{i}\mathfrak{R}_{\alpha(s-1)\dot\alpha(s-1)}\,, \qquad \bar{\mathfrak{R}}_{\alpha(s-1)\dot\alpha(s-1)} = \mathfrak{R}_{\alpha(s-1)\dot\alpha(s-1)}\,. \qquad (2.12)$$

The action (2.8) includes a single term which involves the 'naked' gauge field $\Psi_{\alpha(s)\dot\alpha(s-1)}$ and not the field strength $G_{\alpha(s)\dot\alpha(s)}$, the latter being defined by (2.6) and invariant under the ζ-transformation (2.5a). This is actually a BF term, for it can be written in two different forms

$$\frac{1}{s}\int d^4x d^2\theta d^2\bar\theta\, \Psi^{\alpha(s)\dot\alpha(s-1)}\left(D_{\alpha_1}\bar D_{\dot\alpha_1} - 2i(s-1)\partial_{\alpha_1\dot\alpha_1}\right)\Sigma_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1}}$$
$$= -\frac{1}{s+1}\int d^4x d^2\theta d^2\bar\theta\, G^{\alpha(s)\dot\alpha(s)}\left(\bar D_{\dot\alpha_1} D_{\alpha_1} + 2i(s+1)\partial_{\alpha_1\dot\alpha_1}\right) Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s}\,. \quad (2.13)$$

The former makes the gauge symmetry (2.4) manifestly realised, while the latter turns the ζ-transformation (2.5a) into a manifest symmetry.

Making use of (2.13) leads to a different representation for the action (2.8). It is

$$\begin{aligned}
S^{\parallel}_{(s)} = \left(-\frac{1}{2}\right)^s \int d^4x d^2\theta d^2\bar\theta\, \Big\{ &\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
&+ \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)}\left(D^\beta \bar D^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar D^{\dot\beta} D^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\right) \\
&+ 2\bar G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \frac{s}{s+1}\left(G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \bar G^{\alpha(s)\dot\alpha(s)} \bar G_{\alpha(s)\dot\alpha(s)}\right) \\
&+ \frac{s-1}{4s} H^{\alpha(s-1)\dot\alpha(s-1)}\left(D_{\alpha_1} \bar D^2 \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar D_{\dot\alpha_1} D^2 \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}\right) \\
&- \frac{1}{s+1} G^{\alpha(s)\dot\alpha(s)}\left(\bar D_{\dot\alpha_1} D_{\alpha_1} + 2i(s+1)\partial_{\alpha_1\dot\alpha_1}\right) Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s} \\
&+ \frac{1}{s+1} \bar G^{\alpha(s)\dot\alpha(s)}\left(D_{\alpha_1} \bar D_{\dot\alpha_1} + 2i(s+1)\partial_{\alpha_1\dot\alpha_1}\right) \bar Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s} \\
&+ \frac{s-1}{8s}\left(\Sigma^{\alpha(s-1)\dot\alpha(s-2)} D^2 \Sigma_{\alpha(s-1)\dot\alpha(s-2)} - \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)} \bar D^2 \bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\right) \\
&- \frac{1}{s^2} \bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta}\left(\frac{1}{2}(s^2+1) D^\beta \bar D_{\dot\beta} + i(s-1)^2 \partial^\beta{}_{\dot\beta}\right)\Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)} \Big\}\,.
\end{aligned} \quad (2.14)$$

3 Dual formulation

The theory with action (2.14) possesses a dual formulation that can be obtained by applying the duality transformation introduced in [9, 10]. In general, it works as follows. Suppose we have a supersymmetric field theory formulated in terms of a longitudinal linear superfield $G_{\alpha(t)\dot\alpha(s)}$ and its conjugate $\bar G_{\alpha(s)\dot\alpha(t)}$, and the action has the form

$$S[G_{\alpha(t)\dot\alpha(s)}, \bar G_{\alpha(s)\dot\alpha(t)}] = \int d^4x d^2\theta d^2\bar\theta\, \mathcal{L}\left(G_{\alpha(t)\dot\alpha(s)}, \bar G_{\alpha(s)\dot\alpha(t)}\right)\,, \quad (3.1)$$

where $\mathcal{L}(G, \bar G)$ is an algebraic function of its arguments. We now associate with this theory a first-order model of the form

$$S_{\text{first-order}} = \int d^4x d^2\theta d^2\bar\theta\, \left\{\mathcal{L}\left(U_{\alpha(t)\dot\alpha(s)}, \bar U_{\alpha(s)\dot\alpha(t)}\right) + \left(\Gamma^{\alpha(t)\dot\alpha(s)} U_{\alpha(t)\dot\alpha(s)} + \text{c.c.}\right)\right\}\,, \quad (3.2)$$

where $U_{\alpha(t)\dot\alpha(s)}$ is a complex unconstrained superfield, and the Lagrange multiplier $\Gamma_{\alpha(t)\dot\alpha(s)}$ is transverse linear. Varying $S_{\text{first-order}}$ with respect to the Lagrange multiplier gives

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$U_{\alpha(t)\dot\alpha(s)} = G_{\alpha(t)\dot\alpha(s)}$, and then $S_{\text{first-order}}$ reduces to the original action (3.1). On the other hand, we can consider the equation of motion for $U^{\alpha(t)\dot\alpha(s)}$, which is

$$\frac{\partial}{\partial U^{\alpha(t)\dot\alpha(s)}}\mathcal{L}\left(U_{\beta(t)\dot\beta(s)},\bar U_{\beta(s)\dot\beta(t)}\right) + \Gamma_{\alpha(t)\dot\alpha(s)} = 0\,. \tag{3.3}$$

We assume that (3.3) can be solved to express $U_{\beta(t)\dot\beta(s)}$ in terms of $\Gamma_{\alpha(t)\dot\alpha(s)}$ and its conjugate. Plugging this solution back into (3.2) gives a dual action

$$S_{\text{dual}}[\Gamma_{\alpha(t)\dot\alpha(s)},\bar\Gamma_{\alpha(s)\dot\alpha(t)}] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\mathcal{L}_{\text{dual}}\left(\Gamma_{\alpha(t)\dot\alpha(s)},\bar\Gamma_{\alpha(s)\dot\alpha(t)}\right)\,. \tag{3.4}$$

In the $t = s = 0$ case, the above duality transformation coincides with the so-called complex linear-chiral duality [16] which plays a fundamental role in the context of off-shell supersymmetric sigma models with eight supercharges [17, 18].

We now associate with our theory (2.14) the following first-order action[7]

$$\begin{aligned} S_{\text{first-order}} &= S^{\parallel}_{(s)}[U,\bar U,H,Z,\bar Z] \\ &\quad + \left(\frac{-1}{2}\right)^s \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\left(\frac{2}{s+1}\Gamma^{\alpha(s)\dot\alpha(s)}U_{\alpha(s)\dot\alpha(s)} + \text{c.c.}\right)\,, \end{aligned} \tag{3.5}$$

where $S^{\parallel}_{(s)}[U,\bar U,H,Z,\bar Z]$ is obtained from the action (2.14) by replacing $G_{\alpha(s)\dot\alpha(s)}$ with an unconstrained complex superfield $U_{\alpha(s)\dot\alpha(s)}$, and $\Gamma_{\alpha(s)\dot\alpha(s)}$ is a transverse linear superfield,

$$\bar D^{\dot\beta}\Gamma_{\alpha(s)\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}} = 0\,. \tag{3.6}$$

As discussed above, the first-order model introduced is equivalent to the original theory (2.14). The action (3.5) is invariant under the gauge ξ-transformation (2.4) which acts on $U_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ by the rule

$$\delta_\xi U_{\alpha(s)\dot\alpha(s)} = 0\,, \tag{3.7a}$$

$$\delta_\xi \Gamma_{\alpha(s)\dot\alpha(s)} = \bar D^{\dot\beta}\left\{\frac{s+1}{2(s+2)}\bar D_{(\dot\beta}D_{\alpha_1}\xi_{\alpha_2\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s)} + \mathrm{i}(s+1)\partial_{\alpha_1(\dot\beta}\xi_{\alpha_2\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s)}\right\}\,. \tag{3.7b}$$

The first-order action (3.5) is also invariant under the gauge $\mathfrak{V}$-transformation (2.5b) and (2.5c), which acts on $U_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ as

$$\delta_{\mathfrak{V}} U_{\alpha(s)\dot\alpha(s)} = \frac{1}{2}\bar D_{(\dot\alpha_1}D_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,, \tag{3.8a}$$

$$\delta_{\mathfrak{V}} \Gamma_{\alpha(s)\dot\alpha(s)} = 0\,. \tag{3.8b}$$

[7]The specific normalisation of the Lagrange multiplier in (3.5) is chosen to match that of [8, 10].

Eliminating the auxiliary superfields $U_{\alpha(s)\dot\alpha(s)}$ and $\bar U_{\alpha(s)\dot\alpha(s)}$ from (3.5) leads to

$$
\begin{aligned}
S^{\perp}_{(s)} = -\Big(-\frac{1}{2}\Big)^{s}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, \Big\{ &-\frac{1}{8}H^{\alpha(s-1)\dot\alpha(s-1)}D^\beta\bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
&+\frac{1}{8}\frac{s^2}{(s+1)(2s+1)}[D^\beta,\bar D^{\dot\beta}]H^{\alpha(s-1)\dot\alpha(s-1)}[D_{(\beta},\bar D_{(\dot\beta}]H_{\alpha(s-1))\dot\alpha(s-1))} \\
&+\frac{1}{2}\frac{s^2}{s+1}\partial^{\beta\dot\beta}H^{\alpha(s-1)\dot\alpha(s-1)}\partial_{(\beta(\dot\beta}H_{\alpha(s-1))\dot\alpha(s-1))} \\
&+\frac{2\mathrm{i}s}{2s+1}H^{\alpha(s-1)\dot\alpha(s-1)}\partial^{\beta\dot\beta}\Big(\mathbf{\Gamma}_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}-\bar{\mathbf{\Gamma}}_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) \\
&+\frac{2}{2s+1}\bar{\mathbf{\Gamma}}^{\alpha(s)\dot\alpha(s)}\mathbf{\Gamma}_{\alpha(s)\dot\alpha(s)}-\frac{s}{(s+1)(2s+1)}\Big(\mathbf{\Gamma}^{\alpha(s)\dot\alpha(s)}\mathbf{\Gamma}_{\alpha(s)\dot\alpha(s)}+\bar{\mathbf{\Gamma}}^{\alpha(s)\dot\alpha(s)}\bar{\mathbf{\Gamma}}_{\alpha(s)\dot\alpha(s)}\Big) \\
&-\frac{s-1}{2(2s+1)}H^{\alpha(s-1)\dot\alpha(s-1)}\Big(D_{\alpha_1}\bar D^2\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)}-\bar D_{\dot\alpha_1}D^2\Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}\Big) \\
&+\frac{1}{2(2s+1)}H^{\alpha(s-1)\dot\alpha(s-1)}\Big(D^2\bar D_{\dot\alpha_1}\Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}-\bar D^2 D_{\alpha_1}\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)}\Big) \\
&-\mathrm{i}\frac{(s-1)^2}{s(2s+1)}H^{\alpha(s-1)\dot\alpha(s-1)}\partial_{\alpha_1\dot\alpha_1}\Big(D^\beta\Sigma_{\beta\alpha_2\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1}}+\bar D^{\dot\beta}\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\beta\dot\alpha_2\dots\dot\alpha_{s-1}}\Big) \\
&-\frac{s-1}{8s}\Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)}D^2\Sigma_{\alpha(s-1)\dot\alpha(s-2)}-\bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar D^2\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\Big) \\
&+\frac{1}{s^2}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta}\Big(\frac{1}{2}(s^2+1)D^\beta\bar D_{\dot\beta}+\mathrm{i}(s-1)^2\partial^\beta{}_{\dot\beta}\Big)\Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)}\Big\}\,,
\end{aligned}
\tag{3.9}
$$

where we have defined

$$
\mathbf{\Gamma}_{\alpha(s)\dot\alpha(s)} = \Gamma_{\alpha(s)\dot\alpha(s)} - \frac{1}{2}\bar D_{(\dot\alpha_1}D_{(\alpha_1}Z_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)} - \mathrm{i}(s+1)\partial_{(\alpha_1(\dot\alpha_1}Z_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,. \tag{3.10}
$$

We point out that $\mathbf{\Gamma}_{\alpha(s)\dot\alpha(s)}$ is invariant under the gauge transformations (2.4) and (3.7b).

In accordance with (2.5c), the gauge $\mathfrak{V}$-freedom may be used to impose the condition

$$
Z_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{3.11}
$$

In this gauge the action (3.9) reduces to the one defining the transverse formulation for the massless superspin-s multiplet [10]. The gauge condition (3.11) is preserved by residual local $\mathfrak{V}$- and ξ-transformations of the form

$$
\bar D^{\dot\beta}\xi_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{3.12}
$$

Making use of the parametrisation (2.10), the residual gauge freedom is

$$
\delta H_{\alpha(s-1)\dot\alpha(s-1)} = D^\beta L_{\beta\alpha(s-1)\dot\alpha(s-1)} - \bar D^{\dot\beta}\bar L_{\alpha(s-1)\dot\beta\dot\alpha(s-1)}\,, \tag{3.13a}
$$

$$
\delta\Gamma_{\alpha(s)\dot\alpha(s)} = \frac{s+1}{2(s+2)}\bar D^{\dot\beta}\Big\{\bar D_{(\dot\beta}D_{(\alpha_1} + 2\mathrm{i}(s+2)\partial_{(\alpha_1(\dot\beta}\Big\}\bar L_{\alpha_2\dots\alpha_s)\dot\alpha_1\dots\dot\alpha_s)}\,, \tag{3.13b}
$$

which is exactly the gauge symmetry of the transverse formulation for the massless superspin-s multiplet [10].

4 Higher spin supercurrent multiplets

We now make use of the new gauge formulation (2.8), or equivalently (2.14), for the integer superspin-s multiplet to derive non-conformal higher spin supercurrents.

Let us couple the prepotentials $H_{\alpha(s-1)\dot\alpha(s-1)}$, $Z_{\alpha(s-1)\dot\alpha(s-1)}$ and $\Psi_{\alpha(s)\dot\alpha(s-1)}$ to external sources

$$S^{(s)}_{\rm source} = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta \,\Big\{\Psi^{\alpha(s)\dot\alpha(s-1)}J_{\alpha(s)\dot\alpha(s-1)} - \bar\Psi^{\alpha(s-1)\dot\alpha(s)}\bar J_{\alpha(s-1)\dot\alpha(s)} \\ +H^{\alpha(s-1)\dot\alpha(s-1)}S_{\alpha(s-1)\dot\alpha(s-1)} \\ +Z^{\alpha(s-1)\dot\alpha(s-1)}T_{\alpha(s-1)\dot\alpha(s-1)} + \bar Z^{\alpha(s-1)\dot\alpha(s-1)}\bar T_{\alpha(s-1)\dot\alpha(s-1)}\Big\}\,. \tag{4.1}$$

In order for $S^{(s)}_{\rm source}$ to be invariant under the ζ-transformation in (2.5a), the source $J_{\alpha(s)\dot\alpha(s-1)}$ must satisfy

$$\bar D^{\dot\beta}J_{\alpha(s)\dot\beta\dot\alpha(s-2)} = 0 \Longleftrightarrow D^\beta \bar J_{\beta\alpha(s-2)\dot\alpha(s)} = 0\,. \tag{4.2}$$

Next, in order for $S^{(s)}_{\rm source}$ to be invariant under the transformation (2.4), the source $T_{\alpha(s-1)\dot\alpha(s-1)}$ must satisfy

$$\bar D_{(\dot\alpha_1}T_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_s)} = 0 \Longleftrightarrow D_{(\alpha_1}\bar T_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.3}$$

We see that the superfields $J_{\alpha(s)\dot\alpha(s-1)}$ and $T_{\alpha(s-1)\dot\alpha(s-1)}$ are transverse linear and longitudinal linear, respectively. Finally, requiring $S^{(s)}_{\rm source}$ to be invariant under the $\mathfrak{V}$-transformation (2.5) gives the following conservation equation

$$-\frac{1}{2}D^\beta J_{\beta\alpha(s-1)\dot\alpha(s-1)} + S_{\alpha(s-1)\dot\alpha(s-1)} + \bar T_{\alpha(s-1)\dot\alpha(s-1)} = 0 \tag{4.4a}$$

and its conjugate

$$\frac{1}{2}\bar D^{\dot\beta}\bar J_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + S_{\alpha(s-1)\dot\alpha(s-1)} + T_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{4.4b}$$

As a consequence of (4.3), from (4.4a) we deduce

$$\frac{1}{4}D^2 J_{\alpha(s)\dot\alpha(s-1)} + D_{(\alpha_1}S_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.5}$$

The equations (4.2) and (4.5) describe the conserved current supermultiplet which corresponds to our theory in the gauge (2.9).

Taking the sum of (4.4a) and (4.4b) leads to

$$\frac{1}{2}D^\beta J_{\beta\alpha(s-1)\dot\alpha(s-1)} + \frac{1}{2}\bar D^{\dot\beta}\bar J_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + T_{\alpha(s-1)\dot\alpha(s-1)} - \bar T_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{4.6}$$

The equations (4.2), (4.3) and (4.6) describe the conserved current supermultiplet which corresponds to our theory in the gauge (2.11). As a consequence of (4.3), the conservation equation (4.6) implies

$$\frac{1}{2}D_{(\alpha_1}\Big\{D^{|\beta|}J_{\alpha_2\dots\alpha_s)\beta\dot\alpha(s-1)} + \bar D^{\dot\beta}\bar J_{\alpha_2\dots\alpha_s)\dot\beta\dot\alpha(s-1)}\Big\} + D_{(\alpha_1}T_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.7}$$

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As in [21], it is useful to introduce auxiliary complex variables $\zeta^\alpha \in \mathbb{C}^2$ and their conjugates $\bar\zeta^{\dot\alpha}$. Given a tensor superfield $U_{\alpha(p)\dot\alpha(q)}$, we associate with it the following field on $\mathbb{C}^2$

$$U_{(p,q)}(\zeta,\bar\zeta) := \zeta^{\alpha_1}\dots\zeta^{\alpha_p}\bar\zeta^{\dot\alpha_1}\dots\bar\zeta^{\dot\alpha_q}U_{\alpha_1\dots\alpha_p\dot\alpha_1\dots\dot\alpha_q}\,, \tag{4.8}$$

which is homogeneous of degree (p,q) in the variables ζ^α and $\bar\zeta^{\dot\alpha}$. We introduce operators that increase the degree of homogeneity in the variables ζ^α and $\bar\zeta^{\dot\alpha}$,

$$D_{(1,0)} := \zeta^\alpha D_\alpha\,, \qquad D^2_{(1,0)} = 0\,, \tag{4.9a}$$
$$\bar D_{(0,1)} := \bar\zeta^{\dot\alpha}\bar D_{\dot\alpha}\,, \qquad \bar D^2_{(0,1)} = 0\,, \tag{4.9b}$$
$$\partial_{(1,1)} := 2\mathrm{i}\zeta^\alpha\bar\zeta^{\dot\alpha}\partial_{\alpha\dot\alpha} = -\left\{D_{(1,0)},\bar D_{(1,0)}\right\}\,. \tag{4.9c}$$

We also introduce two *nilpotent* operators that decrease the degree of homogeneity in the variables ζ^α and $\bar\zeta^{\dot\alpha}$, specifically

$$D_{(-1,0)} := D^\alpha\frac{\partial}{\partial\zeta^\alpha}\,, \qquad D^2_{(-1,0)} = 0\,, \tag{4.10a}$$
$$\bar D_{(0,-1)} := \bar D^{\dot\alpha}\frac{\partial}{\partial\bar\zeta^{\dot\alpha}} \qquad \bar D^2_{(0,-1)} = 0\,. \tag{4.10b}$$

Using the notation introduced, the transverse linear condition (4.2) turns into

$$\bar D_{(0,-1)}J_{(s,s-1)} = 0\,, \tag{4.11}$$

while the longitudinal linear condition (4.3) takes the form

$$\bar D_{(0,1)}T_{(s-1,s-1)} = 0\,. \tag{4.12}$$

The conservation equation (4.4a) becomes

$$-\frac{1}{2s}D_{(-1,0)}J_{(s,s-1)} + S_{(s-1,s-1)} + \bar T_{(s-1,s-1)} = 0\,, \tag{4.13}$$

and (4.7) takes the form

$$\frac{1}{2s}D_{(1,0)}\left\{D_{(-1,0)}J_{(s,s-1)} + \bar D_{(0,-1)}\bar J_{(s-1,s)}\right\} + D_{(1,0)}T_{(s-1,s-1)} = 0\,. \tag{4.14}$$

5 Higher spin supercurrents in a massive chiral model

Consider the Fayet-Sohnius model [19, 20] for a free massive hypermultiplet

$$S_{\text{massive}} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\left(\bar\Phi_+\Phi_+ + \bar\Phi_-\Phi_-\right) + \left\{m\int\mathrm{d}^4x\mathrm{d}^2\theta\,\Phi_+\Phi_- + \text{c.c.}\right\}\,, \tag{5.1}$$

where the superfields $\Phi_\pm$ are chiral, $\bar D_{\dot\alpha}\Phi_\pm = 0$, and the mass parameter m is chosen to be positive.

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In the massless case, $m = 0$, the conserved fermionic supercurrents $J_{\alpha(s)\dot\alpha(s-1)}$ were constructed in [14]. In our notation they read

$$J_{(s,s-1)} = \sum_{k=0}^{s-1}(-1)^k \binom{s-1}{k} \left\{ \binom{s}{k+1} \partial^k_{(1,1)} D_{(1,0)} \Phi_+ \, \partial^{s-k-1}_{(1,1)} \Phi_- \right.$$
$$\left. - \binom{s}{k} \partial^k_{(1,1)} \Phi_+ \, \partial^{s-k-1}_{(1,1)} D_{(1,0)} \Phi_- \right\} . \tag{5.2}$$

Making use of the massless equations of motion, $D^2 \Phi_\pm = 0$, one may check that $J_{(s,s-1)}$ obeys, for $s > 1$, the conservation equations

$$D_{(-1,0)} J_{(s,s-1)} = 0, \qquad \bar D_{(0,-1)} J_{(s,s-1)} = 0\,. \tag{5.3}$$

We will now construct fermionic higher spin supercurrents corresponding to the massive model (5.1). Assuming that $J_{(s,s-1)}$ has the same functional form as in the massless case, eq. (5.2), and making use of the equations of motion

$$-\frac{1}{4} D^2 \Phi_+ + m \bar\Phi_- = 0, \qquad -\frac{1}{4} D^2 \Phi_- + m \bar\Phi_+ = 0, \tag{5.4}$$

we obtain

$$\begin{aligned} D_{(-1,0)} J_{(s,s-1)} &= 2m(s+1) \sum_{k=0}^{s-1} (-1)^{k+1} \binom{s-1}{k} \binom{s}{k} \\ &\qquad \times \left\{ -\frac{s-k}{k+1} \partial^k_{(1,1)} \bar\Phi_- \, \partial^{s-k-1}_{(1,1)} \Phi_- + \partial^k_{(1,1)} \Phi_+ \, \partial^{s-k-1}_{(1,1)} \bar\Phi_+ \right\} \\ &\qquad + 2m(s+1) \sum_{k=1}^{s-1} (-1)^{k+1} \binom{s-1}{k} \binom{s}{k} \frac{k}{k+1} \\ &\qquad \times \partial^{k-1}_{(1,1)} \bar D_{(0,1)} \bar\Phi_- \, \partial^{s-k-1}_{(1,1)} D_{(1,0)} \Phi_- \\ &\qquad + 2m(s+1) \sum_{k=0}^{s-2} (-1)^{k+1} \binom{s-1}{k} \binom{s}{k} \frac{s-1-k}{k+1} \\ &\qquad \times \partial^k_{(1,1)} D_{(1,0)} \Phi_+ \, \partial^{s-k-2}_{(1,1)} \bar D_{(0,1)} \bar\Phi_+ \,. \end{aligned} \tag{5.5}$$

It can be shown that the massive supercurrent $J_{(s,s-1)}$ also obeys (4.11).

We now look for a superfield $T_{(s-1,s-1)}$ such that (i) it obeys the longitudinal linear constraint (4.12); and (ii) it satisfies (4.14), which is a consequence of the conservation equation (4.13). We consider a general ansatz

$$\begin{aligned} T_{(s-1,s-1)} &= \sum_{k=0}^{s-1} c_k \partial^k_{(1,1)} \Phi_- \, \partial^{s-k-1}_{(1,1)} \bar\Phi_- + \sum_{k=0}^{s-1} d_k \partial^k_{(1,1)} \Phi_+ \, \partial^{s-k-1}_{(1,1)} \bar\Phi_+ \\ &\quad + \sum_{k=1}^{s-1} f_k \partial^{k-1}_{(1,1)} D_{(1,0)} \Phi_- \, \partial^{s-k-1}_{(1,1)} \bar D_{(0,1)} \bar\Phi_- + \sum_{k=1}^{s-1} g_k \partial^{k-1}_{(1,1)} D_{(1,0)} \Phi_+ \, \partial^{s-k-1}_{(1,1)} \bar D_{(0,1)} \bar\Phi_+ \,. \end{aligned} \tag{5.6}$$

Condition (i) implies that the coefficients must be related by

$$c_0 = d_0 = 0\,, \qquad f_k = c_k\,, \qquad g_k = d_k\,, \tag{5.7a}$$

while for $k = 1, 2, \ldots s-2$, condition (ii) gives the following recurrence relations:

$$\begin{aligned} c_k + c_{k+1} &= \frac{m(s+1)}{s}(-1)^{s+k}\binom{s-1}{k}\binom{s}{k} \\ &\quad \times \frac{1}{(k+2)(k+1)}\left\{(2k+2-s)(s+1)-k-2\right\}\,, \end{aligned} \tag{5.7b}$$

$$\begin{aligned} d_k + d_{k+1} &= \frac{m(s+1)}{s}(-1)^{k}\binom{s-1}{k}\binom{s}{k} \\ &\quad \times \frac{1}{(k+2)(k+1)}\left\{(2k+2-s)(s+1)-k-2\right\}\,. \end{aligned} \tag{5.7c}$$

Condition (ii) also implies that

$$c_1 = -(-1)^s \frac{m(s^2-1)}{2}\,, \qquad c_{s-1} = -\frac{m(s^2-1)}{s}\,; \tag{5.7d}$$

$$d_1 = -\frac{m(s^2-1)}{2}\,, \qquad d_{s-1} = -(-1)^s\frac{m(s^2-1)}{s}\,. \tag{5.7e}$$

The above conditions lead to simple expressions for c_k and d_k:

$$d_k = \frac{m(s+1)}{s}\frac{k}{k+1}(-1)^k\binom{s-1}{k}\binom{s}{k}\,, \tag{5.8a}$$

$$c_k = -(-1)^s d_k\,, \tag{5.8b}$$

where $k = 1, 2, \ldots s-1$. Now that we have already derived an expression for the trace multiplet $T_{(s-1,s-1)}$, the superfield $S_{(s-1,s-1)}$ can be computed using the conservation equation (4.13). This gives

$$\begin{aligned} S_{(s-1,s-1)} &= -m(s+1)\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{1}{k+1} \\ &\quad \times\left\{\partial^k_{(1,1)}\bar\Phi_-\,\partial^{s-k-1}_{(1,1)}\Phi_- + (-1)^s\partial^k_{(1,1)}\bar\Phi_+\,\partial^{s-k-1}_{(1,1)}\Phi_+\right\}\,. \end{aligned} \tag{5.9}$$

One may verify that $S_{(s-1,s-1)}$ is a real superfield.

6 Concluding comments

To conclude this work, we make several final comments.

The formulation proposed in section 2 can naturally be lifted to the case of anti-de Sitter supersymmetry to extend the results of [8].

The action (2.8) involves the transverse linear compensator $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ and its conjugate $\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}$. These superfields cannot be dualised into a longitudinal linear supermultiplet without destroying the locality of the theory, for the action (2.8) contains terms with derivatives of $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ and $\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}$.

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The hypermultiplet model is $\mathcal{N}=2$ supersymmetric, and therefore its conserved currents should belong to $\mathcal{N}=2$ supermultiplets. In the massless case, $m=0$, we deal with the $\mathcal{N}=2$ Poincaré supersymmetry without central charge on the mass shell. In this case it is easy to embed the bosonic $J_{\alpha(s)\dot\alpha(s)}$ and fermionic $J_{\alpha(s)\dot\alpha(s-1)}$ higher spin supercurrents, which were constructed in [14] for any $s\geq 1$, into $\mathcal{N}=2$ real superfields $\mathbb{J}_{\alpha(s-1)\dot\alpha(s-1)}=\bar{\mathbb{J}}_{\alpha(s-1)\dot\alpha(s-1)}$ introduced in [22] and constrained by

$$D^\beta_i \mathbb{J}_{\beta\alpha(s-2)\dot\alpha(s-1)}=0 \Longleftrightarrow \bar D^{\dot\beta}_i \mathbb{J}_{\alpha(s-1)\dot\beta\dot\alpha(s-2)}=0\,, \qquad i=\underline{1},\underline{2}\,. \tag{6.1}$$

Here D^i_α and $\bar D^{\dot\alpha}_i$ are the spinor covariant derivatives of $\mathcal{N}=2$ Minkowski superspace. Conserved $\mathcal{N}=1$ supercurrent multiplets originate as

$$J_{\alpha(s-1)\dot\alpha(s-1)} := \mathbb{J}_{\alpha(s-1)\dot\alpha(s-1)}|\,, \tag{6.2a}$$

$$J_{\alpha(s)\dot\alpha(s-1)} := D^{\underline{2}}_{\alpha_1}\mathbb{J}_{\alpha_2\dots\alpha_s\dot\alpha(s-1)}|\,, \tag{6.2b}$$

$$J_{\alpha(s)\dot\alpha(s)} := \frac{1}{2}\left(\left[D^{\underline{2}}_{(\alpha_1},\bar D_{\underline{2}(\dot\alpha_1}\right]-\frac{1}{2s+1}\left[D^{\underline{1}}_{(\alpha_1},\bar D_{\underline{1}(\dot\alpha_1}\right]\right)\mathbb{J}_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}|\,, \tag{6.2c}$$

where we have made use of the $\mathcal{N}=1$ projection, $U|:=U(x,\theta^\alpha_i,\bar\theta^j_{\dot\alpha})|_{\theta_{\underline{2}}=\bar\theta^{\underline{2}}=0}$, of any $\mathcal{N}=2$ superfield U.[8] In the $s=1$ case, the relations (6.2) reduce to those in eq. (1.10) of [24].

In the massive case, $m\neq 0$, we deal with the $\mathcal{N}=2$ Poincaré supersymmetry with a constant central charge on the mass shell, and the story becomes pretty subtle. In our previous work [21], we observed that the higher spin supercurrents $J_{\alpha(s)\dot\alpha(s)}$ in the massive chiral model exist only for odd values of s. The same conclusion was also reached in a revised version (v3, 26 Oct.) of ref. [23]. However, the conserved fermionic supercurrents $J_{\alpha(s)\dot\alpha(s-1)}$ constructed in the present paper are realised for all values of $s>1$.

The longitudinal and transverse actions for the massless integer superspin-s multiplet [10] are well defined for $s=1$, in which case they describe two off-shell formulations for the massless gravitino multiplet. However, the action (2.8) is not defined in the $s=1$ case. The point is that the gauge transformation law (2.5a) is not defined for $s=1$. The gauge freedom in the superconformal gravitino multiplet model [14] is

$$\delta\Psi_\alpha=\frac{1}{2}D_\alpha\mathfrak{V}+\zeta_\alpha\,, \qquad \bar D_{\dot\beta}\zeta_\alpha=0\,. \tag{6.3a}$$

This transformation law of Ψ_α coincides with the one occurring in the off-shell model for the massless gravitino multiplet proposed in [25]. In addition to the gauge superfield Ψ_α, this model also involves two compensators, a real scalar H and a chiral scalar Φ, $\bar D_{\dot\alpha}\Phi=0$, with the gauge transformation laws

$$\delta H = \mathfrak{V}+\bar{\mathfrak{V}}\,, \tag{6.3b}$$

$$\delta\Phi = -\frac{1}{2}\bar D^2\bar{\mathfrak{V}}\,. \tag{6.3c}$$

The gauge invariant action of [25] can be written in the form [11]

$$S^{(\mathrm{I})}_{\mathrm{GM}}=S^{\parallel}_{(1,\frac{3}{2})}[\Psi,\bar\Psi,H]-\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\left(\bar\Phi\Phi+\Phi D^\alpha\Psi_\alpha+\bar\Phi\bar D_{\dot\alpha}\bar\Psi^{\dot\alpha}\right)\,, \tag{6.4}$$

[8] In this setting, the $\mathcal{N}=1$ spinor covariant derivatives are identified as $D_\alpha:=D^{\underline{1}}_\alpha$ and $\bar D^{\dot\alpha}:=\bar D^{\dot\alpha}_{\underline{1}}$.

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where $S^{\parallel}_{(1,\frac{3}{2})}[\Psi,\bar{\Psi},H]$ denotes the longitudinal action for the gravitino multiplet, which is obtained from (2.8) by choosing the gauge (2.9) and setting $s=1$. At the component level, this manifestly supersymmetric model is known to describe the Fradkin-Vasiliev-de Wit-van Holten formulation for the gravitino multiplet [26, 27].

There exists a dual formulation for (6.4) that is obtained by performing a superfield Legendre transformation [28]. The dual action given in [28] is

$$S^{(\mathrm{II})}_{\mathrm{GM}} = S^{\parallel}_{(1,\frac{3}{2})}[\Psi,\bar{\Psi},H] + \frac{1}{4}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\left(G + D^\alpha\Psi_\alpha + \bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}}\right)^2 , \tag{6.5}$$

where $G=\bar{G}$ is a real linear superfield, $\bar{D}^2G = D^2G = 0$. The gauge freedom in this theory is given by eqs. (6.3a), (6.3b) and

$$\delta G = -D^\alpha\zeta_\alpha - \bar{D}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}} , \tag{6.6}$$

in accordance with [29]. It may be used to impose two conditions $H=0$ and $G=0$. Then we end up with the Ogievetsky-Sokatchev formulation for the gravitino multiplet [30, 31] (see section 6.9.5 [11] for the technical details).

Actually, there exists one more dual formulation for (6.4) that is obtained by performing the complex linear-chiral duality transformation. It leads to

$$S^{(\mathrm{III})}_{\mathrm{GM}} = S^{\parallel}_{(1,\frac{3}{2})}[\Psi,\bar{\Psi},H] + \frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,(\Sigma + D^\alpha\Psi_\alpha)(\bar{\Sigma} + \bar{D}_{\dot{\alpha}}\bar{\Psi}^{\dot{\alpha}}) , \tag{6.7}$$

where Σ is a complex linear superfield constrained by $\bar{D}^2\Sigma = 0$. The gauge freedom in this theory is given by eqs. (6.3a), (6.3b) and

$$\delta\Sigma = -D^\alpha\zeta_\alpha . \tag{6.8}$$

This gauge freedom does not allow one to gauge away Σ off the mass shell. To the best of our knowledge, the supersymmetric gauge theory (6.7) is a new off-shell realisation for the massless gravitino multiplet.

As shown in [29], the gravitino multiplet actions (6.4) and (6.5) naturally originate upon $\mathcal{N}=2\rightarrow\mathcal{N}=1$ reduction of the linearised superfield action [29] for the off-shell $\mathcal{N}=2$ supergravity with a tensor compensator [32]. The actions (6.4) and (6.5) prove to correspond to different values of the background tensor multiplet [29]. The gravitino multiplet action (6.7) should originate if one linearises the off-shell $\mathcal{N}=2$ supergravity with a tropical compensator [33].

The transverse formulation for the massless gravitino multiplet, which was introduced in [10], is quite mysterious in the sense that it is not contained in any known off-shell formulation for $\mathcal{N}=2$ supergravity.

Acknowledgments

The work of JH is supported by an Australian Government Research Training Program (RTP) Scholarship. The work of SMK is supported in part by the Australian Research Council, project No. DP160103633.

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