Yukawa sector of minimal SO(10) unification

K.S. Babu,[a] **Borut Bajc**[b] **and Shaikh Saad**[a]

[a] *Department of Physics, Oklahoma State University,*
Stillwater, OK, 74078 U.S.A.

[b] *Jožef Stefan Institute,*
Ljubljana, 1000 Slovenia

E-mail: babu@okstate.edu, borut.bajc@ijs.si, shaikh.saad@okstate.edu

ABSTRACT: We show that in SO(10) models, a Yukawa sector consisting of a real 10_H, a real 120_H and a complex 126_H of Higgs fields can provide a realistic fit to all fermion masses and mixings, including the neutrino sector. Although the group theory of SO(10) demands that the 10_H and 120_H be real, most constructions complexify these fields and impose symmetries exterior to SO(10) to achieve predictivity. The proposed new framework with *real* 10_H and *real* 120_H relies only on SO(10) gauge symmetry, and yet has a limited number of Yukawa parameters. Our analysis shows that while there are restrictions on the observables, a good fit to the entire fermion spectrum can be realized. Unification of gauge couplings is achieved with an intermediate scale Pati-Salam gauge symmetry. Proton decay branching ratios are calculable, with the leading decay modes being $p \to \overline{\nu}\pi^+$ and $p \to e^+\pi^0$.

KEYWORDS: Beyond Standard Model, GUT, Neutrino Physics, Quark Masses and SM Parameters

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Contents

1 Introduction

Grand unified theories (GUTs) [1–3] based on the gauge group SO(10) [4, 5] are very attractive candidates to unify the strong, weak and electromagnetic forces into a single force, as well as to shed light on some of the open questions of the Standard Model (SM). Quarks and leptons of each family are unified into a single irreducible representations of SO(10) group, the 16-dimensional spinor, which also contains the right-handed neutrino. The presence of the right-handed neutrino makes the seesaw mechanism [6–15] for generating small neutrino masses very compelling in these theories. Since SO(10) gauge symmetry is automatically anomaly-free [4, 5], it provides a nice explanation for the miraculous cancelation of anomalies that occurs within each fermion family. The observed quantization of electric charges is also understood in these theories owing to their non-Abelian nature. Unifying all fermions into a single multiplet gives us the hope of understanding some aspects of the flavor puzzle in these theories. Unification of gauge couplings occurs naturally at an energy scale of $\sim 10^{15-16}$ GeV [16–29], as SO(10) admits an intermediate symmetry group — unlike theories based on SU(5) which must break directly to the SM. It is of course well known that if supersymmetry is assumed to be present in its minimal version at the TeV scale, one-step breaking of SO(10) directly down to the SM can be realized at an energy

scale of 2×10^{16} GeV [30–33]. The focus of this paper is, however, SO(10) theories without the assumption of supersymmetry.

We wish to inquire what an economic Yukawa sector would look like in renormalizable SO(10) theories. This may appear to be a well understood issue, but as we suggest here, this question has not been properly resolved. Economy may be viewed as having the least number of Higgs fields as well as Yukawa parameters while being realistic. Assuming that there are no new fermions beyond the three families of chiral 16s[1] the answer to this question may be found in the group theory of fermion bilinears:

$$16 \times 16 = 10_s + 120_a + 126_s. \tag{1.1}$$

Here the subscripts s and a stand for symmetric and antisymmetric components (in family space). The 10 and the 120 are real representations in SO(10), while the 126 is complex. The most general renormalizable Yukawa couplings in SO(10) theories then would take the form

$$\mathcal{L}_{\text{yuk}} = 16_F(Y_{10}^i 10_{\text{H}}^i + Y_{120}^j 120_{\text{H}}^j + Y_{126}^k \overline{126}_{\text{H}}^k)16_F. \tag{1.2}$$

Here the index i takes values $i = 1, 2, \ldots n_{10}$ where n_{10} is the number of 10_{H} fields employed, and similarly the index $j = 1, 2, \ldots n_{120}$ and $k = 1, 2, \ldots n_{126}$ with n_{120} and n_{126} being the number of 120_{H} and $\overline{126}_{\text{H}}$ present in the theory. The Yukawa coupling matrices Y_{10}^i and Y_{126}^k are 3×3 complex symmetric matrices in family space, while Y_{120}^j are complex antisymmetric matrices. We wish to identify the smallest possible set of $\{n_{10}, n_{120}, n_{126}\}$ that would lead to a realistic spectrum of quark and lepton masses as well as mixing angles. This set will turn out to be the choice $n_{10} = n_{120} = n_{126} = 1$, as we shall see. This result is satisfying, as it suggests that nature has utilized each possible Higgs field for fermion mass generation exactly once, without any replication.

Before establishing this assertion, which will be done in the next section, let us note that *a complex* 10 can be constructed from *two real* 10s in SO(10): $10_c = (10_1 + i10_2)/\sqrt{2}$. Similarly, *a complex* 120_c may be constructed from *two real* 120s. In these cases, the Yukawa couplings will involve terms of the type $16_F 10_c 16_F$ as well as $16_F 10_c^* 16_F$ with completely independent Yukawa coupling matrices, and similarly for the 120_c field. It is possible to assign a charge exterior to SO(10) to these fields — such as the Peccei-Quinn U(1) motivated on other grounds — so that the Yukawa couplings contain only the $16_F 10_c 16_F$ term, and not the $16_F 10_c^* 16_F$ term. These restricted class of Yukawa couplings in SO(10) have been studied extensively [36–44]. While interesting, the predictions of such models are those of SO(10) $\times$ G where G is a symmetry exterior to SO(10), and not of the true grand unified symmetry SO(10) itself. Our inquiry relates to the minimal Yukawa sector in theories where only the SO(10) gauge symmetry plays a role.

It should be noted that in theories which assume supersymmetry (SUSY), which is not the focus of the present work, chiral superfields are necessarily complex, thus requiring the complexification of 10 and 120 Higgs fields. Holomorphy of the superpotential would imply that the coupling $16_F 10_c^* 16_F$ is not present simultaneously with the superpotential

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[1]If vector-like fermions belonging to $16 + \overline{16}$ (or other real representations) with GUT scale masses exist and mix with the chiral 16s, new possibilities are available, see for e.g., refs. [34, 35].

term $16_F 10_c 16_F$. These models share some of the features of non-SUSY models based on $SO(10) \times U(1)_{PQ}$, although the renormalization group evolution of the fermion mass parameters between the weak scale and the GUT scale would be different in the two classes of theories. Supersymmetric SO(10) models have been studied extensively, and it has been shown that economic models where only a (complex) 10_H and a $\overline{126}_H$ couple to fermions can be predictive and consistent with all fermion masses and mixings [39, 42, 44–53]. If the additional Higgs fields needed for symmetry breaking are restricted to a 126_H and a 210_H, split supersymmetry may be required for consistency [52]. Alternatively, a (complex) 120_H may be introduced to relax some of the restrictions imposed by the symmetry breaking sector [54–57].[2] Our goal in this paper is to identify the analog of the minimal SUSY SO(10) Yukawa sector, but for SO(10) theories without supersymmetry.

The rest of the paper is organized as follows. In section 2 we present our proof that the economic Higgs sector will have $n_{10} = n_{120} = n_{126} = 1$. In section 3 we analyze the predictions of this model for quark and lepton masses and mixings. Here we present our numerical study which shows full consistency with experimental data. In section 4 we present the constraints on these models from the unification of gauge couplings; in section 5 we calculate the proton decay branching ratios. In section 6 we conclude. Four appendices contain a discussion on how the fine-tuning is achieved, technical details on the proof of economic Yukawa sector as well as the best fit parameters for the minimal model.

2 Economic Yukawa sector in SO(10)

In this section we establish the assertion that $n_{10} = n_{120} = n_{126} = 1$ is the economic choice of Yukawa sector in non-supersymmetric SO(10) theories. This corresponds to choosing one real 10_H, one real 120_H and a complex $\overline{126}_H$ of Higgs bosons that have Yukawa couplings with the three chiral families of 16_F. An additional Higgs filed belonging to 45_H, 54_H or 210_H would be needed for completing the symmetry breaking. These fields, however, do not have Yukawa couplings with the 16_F, and the precise choice is not so important for now. A proof of our assertion would require that the choice $n_{10} = n_{120} = n_{126} = 1$ leads to a realistic fermion spectrum, and no other simpler choice exists consistent with realism. The former part of the proof is delegated to section 3 where we perform a numerical analysis of this economic Yukawa sector; here we address the latter part.

If only one Higgs field among 10_H, 120_H and $\overline{126}_H$ is present in a theory, there would be no flavor mixing — as the Yukawa coupling matrix of this single Higgs field can be diagonalized using an SO(10) rotation. Thus at least two Higgs fields are needed for realistic fermion spectrum. One of the fields used must be a 126_H, since it gives large Majorana masses to the righ-handed neutrinos directly. This field also plays a role in the symmetry breaking sector, as it breaks SO(10) down to SU(5), reducing the rank. One could consider replacing the 126_H with a 16_H which can play a similar role in rank reduction. In such a case the right-handed neutrino can acquire a large Majorana mass via the two-loop Witten diagram [63] involving gauge boson and scalar loops. The induced Majorana mass can be

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[2]Symmetries external to SO(10) have also been applied in the context of renormalizable SUSY SO(10) with some success in explaining the fermion spectrum. See for example refs. [58–62].

estimated [64] to be of order

$$M_{\nu^c} \approx \left(\frac{\alpha_{10}}{4\pi}\right)^2 Y_{10} \frac{v_R^2}{M_{\text{GUT}}} \tag{2.1}$$

where α_{10} is the SO(10) gauge coupling, Y_{10} is the Yukawa coupling of 10_{H}, and v_R is the $B-L$ breaking vacuum expectation value (VEV) of the 16_{H}. In a nonsupersymmetric SO(10) theory v_R is well below the GUT scale for consistency with gauge coupling unification, with its range being $v_R \approx (10^{11}\text{–}10^{14})\,\text{GeV}$ depending on the surviving intermediate symmetry. M_{ν^c} is then of order $10^8\,\text{GeV}$ or less, which is too small to reproduce the correct order of magnitude for the light neutrino masses.[3]

Keeping one 126_{H} field in the theory, we seek if a realistic fermion spectrum can be generated with the addition of a second Higgs field. This turns out to be not possible. If the second Higgs field is a 126_{H}, the mass relations $m_\tau = -3m_b, m_\mu = -3m_s$ and $m_e = -3m_d$ will result at the GUT scale, which are inconsistent with observations. The ratio m_τ/m_b is found to be about 1.7 at the GUT scale (with small input errors) when the low energy mass parameters are evolved up to the GUT scale using SM renormalization group equations. We found that this ratio is more realistically in the range (1.4–1.7), when intermediate scale threshold effects arising from the right-handed neutrino sector and the gauge bosons of $SU(4)_c$ are included. Each of the two threshold effects causes a decrease in the ratio m_τ/m_b at the GUT scale. We conclude that the relation $m_\tau = 3m_b$ is clearly excluded. The relation $m_\mu = 3m_s$ is not too far off (our RGE evolution shows the ratio m_μ/m_s to be about 4 at the GUT scale), while $m_e = 3m_d$ is off by an order of magnitude. Thus a minimal Yukawa sector consisting of two copies of $\overline{126}_{\text{H}}$ is not realistic.

If the second Higgs field is a real 10_{H}, two complex symmetric Yukawa matrices can be written down, one with the 10_{H}, and one with the $\overline{126}_{\text{H}}$. However, the Higgs doublet in the 10_{H} is self-conjugate, and is contained in the $(2, 2, 1)$ representation of the Pati-Salam subgroup $SU(2)_L \times SU(2)_R \times SU(4)_c$. This field can be written as

$$\Phi^* = \tau_2 \Phi \tau_2 \Rightarrow \Phi = \begin{pmatrix} \phi_0 & \phi^+ \\ -\phi^- & \phi_0^* \end{pmatrix}. \tag{2.2}$$

In general, if the (1,1) element of Φ is independent from the (2, 2) element, we can denote their respective vacuum expectation values to be v_u and v_d with v_u giving mass to the up-quarks and Dirac neutrinos, while v_d generates down-quark and charged lepton masses. The reality of 10_{H} implies that $v_u = v_d^* \equiv v_{10}$, and thus the ratio $r = |v_u/v_d| = 1$. With $r = 1$, the needed splitting between the top and bottom quark masses cannot be achieved. Note that $r = 1$ is a special case of the general $SO(10) \times U(1)_{PQ}$ models with $v_u \neq v_d^*$. Such models have been studied, which find the phenomenological requirement $r \sim m_t/m_b$. A three generation analysis of fermion masses and mixings with a complex 10_{H} in ref. [42] shows that a realistic fit requires $r \sim 70$, which is well outside of the prediction of $r = 1$ in the case of real 10_{H}. Thus we conclude that one $\overline{126}_{\text{H}}$ and one real 10_{H} is not realistic [41].

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[3]This issue with the Witten mechanism may be resolved in split supersymmetry, where $v_R = M_{\text{GUT}}$ [64]. The SUSY particle masses should be of order the GUT scale to prevent additional suppression factor of $M_{\text{SUSY}}/M_{\text{GUT}}$ in eq. (2.1), which may cause a problem with generating a Higgs boson mass of 125 GeV [65].

What about using one $\overline{126}_H$ and one 120_H? As shown in ref. [41], this case also cannot reproduce fermion masses correctly. The ratio m_t/m_b comes out to be of order one, rather than the phenomenological value of ~ 70. In addition, as we shall show, this model predicts the GUT scale mass ratio $m_\tau/m_b \simeq 3$, with any deviation of order $m_s/m_b \sim 5\%$. As already noted, the ratio $m_\tau/m_b = (1.4\text{–}1.7)$ at the GUT scale in SO(10) models under discussion. Thus we conclude that only two Higgs fields being responsible for Yukawa couplings cannot be realistic.

When three Higgs fields are introduced, the choice of one 10_H, one 120_H and one $\overline{126}_H$ appears attractive, as there in no replication here. This choice can indeed lead to a realistic fermion mass spectrum, as we elaborate in the next section. There would be two complex symmetric Yukawa coupling matrices in this case, along with one complex antisymmetric matrix. If an alternative choice of one $\overline{126}_H$ and two copies of 120_H can lead to a realistic spectrum, that would have less parameters with one symmetric and two antisymmetric Yukawa matrices. However, as we show in appendix B, this choice would lead to the relation $m_\tau = 3m_b$ with corrections of order 5%, even when one allows for large off-diagonal contributions to the mass matrices from the 120_H. Models with one $\overline{126}_H$ and two copies of 10_H would be realistic; however, these models would have three complex symmetric Yukawa matrices which have more parameters compared to the case of one 10_H, one 120_H and one $\overline{126}_H$. This completes the first part of the proof that $n_{10} = n_{120} = n_{126} = 1$ is the economic choice for the Yukawa sector. To complete the proof we establish in the next section that this choice is indeed realistic.

3 Realistic fermion spectrum with minimal Yukawa sector

As argued in the previous section, the minimal Yukawa sector of SO(10) makes use of one real 10_H, one real 120_H and one complex $\overline{126}_H$ of Higgs bosons that couple to the three families of fermions in the 16_F representation. Here we proceed to establish the consistency of such a theory with observed fermion masses and mixings.

With no symmetry other than the gauge symmetry of SO(10) imposed, the most general Yukawa interactions of the model can be written down as

$$\mathcal{L}_{\text{yuk}} = 16_F(Y_{10}10_H + Y_{120}120_H + Y_{126}\overline{126}_H)16_F. \tag{3.1}$$

Here Y_{10} and Y_{126} are complex symmetric Yukawa matrices, while Y_{120} is a complex antisymmetric matrix. Under the Pati-Salam subgroup $G_{\text{PS}} \equiv \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_c$, these fields decompose as

$$16 = (2, 1, 4) + (1, 2, \overline{4}) \tag{3.2}$$

$$10 = (2, 2, 1) + (1, 1, 6) \tag{3.3}$$

$$120 = (2, 2, 1) + (1, 1, 10) + (1, 1, \overline{10}) + (3, 1, 6) + (1, 3, 6) + (2, 2, 15) \tag{3.4}$$

$$126 = (1, 1, 6) + (3, 1, 10) + (1, 3, \overline{10}) + (2, 2, 15). \tag{3.5}$$

The 10_H has one SM doublet Higgs field contained in the bidoublet $(2,2,1)$, while the 120_H has two SM Higgs doublets, one each belonging to $(2,2,1)$ and $(2,2,15)$. The reality

condition for the (2,2,1) from 10_H is listed in eq. (2.2), while those from the 120_H would imply $v_u^{(1)} = v_d^{(1)*} \equiv v_{120}^{(1)}$ and $v_u^{(15)} = v_d^{(15)*} \equiv v_{120}^{(15)}$ with the superscripts (1) and (15) denoting the (2,2,1) and the (2,2,15) fragments. The 126_H contains two SM Higgs fields contained in the complex bidoublet (2,2,15) fragment, which is not subject to the reality condition. We denote the up-type and down-type electroweak VEVs of the $\overline{126}_H$ as v_{126}^u and v_{126}^d respectively. Note also that the (1,3,10) fragment of $\overline{126}_H$ contains a SM singlet field which generates large Majorana masses for the right-handed neutrinos once it acquires a VEV.

The up-quark, down-quark, charged leptons, Dirac neutrino and Majorana neutrino mass matrices derived from eq. (3.1)can be now written down:

$$M_U = v_{10}Y_{10} + v_{126}^u Y_{126} + (v_{120}^{(1)} + v_{120}^{(15)})Y_{120}, \tag{3.6}$$

$$M_D = v_{10}^* Y_{10} + v_{126}^d Y_{126} + (v_{120}^{(1)*} + v_{120}^{(15)*})Y_{120}, \tag{3.7}$$

$$M_E = v_{10}^* Y_{10} - 3v_{126}^d Y_{126} + (v_{120}^{(1)*} - 3v_{120}^{(15)*})Y_{120}, \tag{3.8}$$

$$M_{\nu_D} = v_{10}Y_{10} - 3v_{126}^u Y_{126} + (v_{120}^{(1)} - 3v_{120}^{(15)})Y_{120}, \tag{3.9}$$

$$M_{\nu_{R,L}} = v_{R,L}Y_{126}. \tag{3.10}$$

Now defining

$$D = v_{10}Y_{10}, \qquad A = (v_{120}^{(1)} + v_{120}^{(15)})Y_{120}, \qquad S = v_{126}^u Y_{126}, \tag{3.11}$$

$$r_1 = \frac{v_{126}^d}{v_{126}^u}, \qquad r_2 = \frac{v_{120}^{(1)*} - 3v_{120}^{(15)*}}{v_{120}^{(1)} + v_{120}^{(15)}}, \qquad e^{i\phi} = \frac{v_{120}^{(1)*} + v_{120}^{(15)*}}{v_{120}^{(1)} + v_{120}^{(15)}}, \qquad c_{R,L} = \frac{v_{R,L}}{v_{126}^u}, \tag{3.12}$$

and going into a phase convention where v_{10} is real (this can be done by an $SU(2)_L$ rotation), we get

$$M_U = D + S + A, \tag{3.13}$$

$$M_D = D + r_1 S + e^{i\phi}A, \tag{3.14}$$

$$M_E = D - 3r_1 S + r_2 A, \tag{3.15}$$

$$M_{\nu_D} = D - 3S + r_2^* e^{i\phi}A, \tag{3.16}$$

$$M_{\nu_{R,L}} = c_{R,L}S. \tag{3.17}$$

These matrices are written in a basis $f_i M_{ij} f_j^c$. The light neutrino mass matrix, obtained from the see-saw formula, is given by

$$M_N = M_{\nu_L} - M_{\nu_D}M_{\nu_R}^{-1}M_{\nu_D}^T. \tag{3.18}$$

Without loss of generality one can choose a basis where S is real, positive and diagonal. In this basis, S would have 3 real parameters while D has 6 complex parameters. Since the matrix A is antisymmetric, it has 3 complex parameters. There are 4 additional complex parameters in $r_{1,2}, c_{R,L}$ and one phase ϕ. An overall phase either from c_L or c_R will be irrelevant in the matrix M_N. Altogether there are then 16 real parameters and 13 phases. With these parameters one should fit 18 observables: 6 quark masses, 3 quark mixing

angles, 1 CKM phase, 3 charged lepton masses, 2 neutrino mass squared differences, and 3 mixing angles in the neutrino sector. If we assume dominance of either type-I or type-II seesaw, then the parameter set is reduced by 1 magnitude and 1 phase. Although the number of model parameters is larger than the number of observables, it is nontrivial to find an acceptable fit owing to the fact that 12 or 13 parameters are phases which cannot be manipulated much.

The type-II contribution to the light neutrino mass matrix originates in the model from terms such as $10_{\mathrm{H}}^2 126_{\mathrm{H}}^2$ in the scalar potential. When decomposed into the the Pati-Salam symmetry group, this term would contain terms of the type $(3, 1, 10)(2, 2, 1)^2(1, 3, \overline{10})$. When the singlet VEV of $(1, 3, \overline{10})$ and the doublet VEV of $(2, 2, 1)$ are inserted in this term, a linear term in $(3, 1, 10)$ would result, which leads to an induced VEV for its neutral component: $v_L \sim v_R v^2/M_{\mathrm{GUT}}^2$. We note that with the right-handed neutrino mass given as in (3.10), the mass of the (X', Y') gauge bosons which are outside of SU(5) but mediate proton decay is given as $M_{X',Y'} = \sqrt{2}gv_R$, where g is the SO(10) gauge coupling.

3.1 Numerical analysis of the fermion masses and mixings

In this section we discuss the procedure we follow for the numerical analysis to the fermion masses and mixings and present our fit results. For optimization purpose we do a χ^2-analysis. The pull and χ^2-function are defined as:

$$P_i = \frac{O_{i\,\mathrm{th}} - E_{i\,\mathrm{exp}}}{\sigma_i}, \tag{3.19}$$

$$\chi^2 = \sum_i P_i^2, \tag{3.20}$$

where σ_i represent experimental 1σ uncertainty and $O_{i\,\mathrm{th}}$, $E_{i\,\mathrm{exp}}$ and P_i represent the theoretical prediction, experimental central value and pull of observable i. We fit the values of the observables at the GUT scale, $M_{\mathrm{GUT}} = 2 \times 10^{16}\,\mathrm{GeV}$. To get the GUT scale values of the observables we take the central values at the M_Z scale from table 1 of ref. [66]. With this input we do the renormalization group equation (RGE) running of the Yukawa couplings [67, 68] and the CKM parameters [69] within the SM up to the GUT scale. For the associated one sigma uncertainties of the observables at the GUT scale, we keep the same percentage uncertainty with respect to the central value of each quantity as that at the M_Z scale. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects. All these inputs are presented in table 1. The RGE running factors for the Yukawa couplings $\eta_i = y_i(M_{\mathrm{GUT}})/y_i(M_Z)$ and for the CKM mixing angles $\eta_{ij}^{\mathrm{CKM}} = \theta_{ij}^{\mathrm{CKM}}(M_{\mathrm{GUT}})/\theta_{ij}^{\mathrm{CKM}}(M_Z)$ are taken to be:

$$(\eta_u, \eta_c, \eta_t) = (0.382, 0.382, 0.434) \tag{3.21}$$

$$(\eta_d, \eta_s, \eta_b) = (0.399, 0.399, 0.348) \tag{3.22}$$

$$(\eta_e, \eta_\mu, \eta_\tau) = (0.967, 0.967, 0.967) \tag{3.23}$$

$$(\eta_{12}^{\mathrm{CKM}}, \eta_{23}^{\mathrm{CKM}}, \eta_{13}^{\mathrm{CKM}}) = (1.000, 1.154, 1.154). \tag{3.24}$$

Yukawa Couplings & CKM parameters	$\mu = M_Z$	$\mu = M_{\mathrm{GUT}}$
$y_u/10^{-6}$	6.65 ± 2.25	2.54 ± 0.86
$y_c/10^{-3}$	3.60 ± 0.11	1.37 ± 0.04
y_t	0.9860 ± 0.00865	0.428 ± 0.003
$y_d/10^{-5}$	1.645 ± 0.165	6.56 ± 0.65
$y_s/10^{-4}$	3.125 ± 0.165	1.24 ± 0.06
$y_b/10^{-2}$	1.639 ± 0.015	0.57 ± 0.005
$y_e/10^{-6}$	2.79475 ± 0.0000155	2.70341 ± 0.00270
$y_\mu/10^{-4}$	5.89986 ± 0.0000185	5.70705 ± 0.00570
$y_\tau/10^{-2}$	1.00295 ± 0.0000905	0.97020 ± 0.00097
$\theta_{12}^{\mathrm{CKM}}$	0.22735 ± 0.00072	0.22739 ± 0.0006
$\theta_{23}^{\mathrm{CKM}}/10^{-2}$	4.208 ± 0.064	4.858 ± 0.06
$\theta_{13}^{\mathrm{CKM}}/10^{-3}$	3.64 ± 0.13	4.202 ± 0.13
δ^{CKM}	1.208 ± 0.054	1.207 ± 0.054

Table 1. Values of observables at M_Z scale from ref. [66]. Here experimental central values with associated 1σ uncertainties are quoted. The masses of fermions are given by the relations $m_i = v\, y_i$ with $v = 174.104\,\mathrm{GeV}$. The corresponding values at the GUT scale are obtained by RGE evolution. For the associated one sigma uncertainties of the observables at the GUT scale, we keep the same percentage uncertainty with respect to the central value of each quantity as that at the M_Z scale. For the charged lepton Yukawa couplings at the GUT scale, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects.

The low scale inputs as shown in table 2 in the neutrino sector are taken from ref. [70]. For neutrino observables, we run the RGE for the dimension five operator from low scale to the v_R scale [71, 72] and use these new values during the fitting produce. For this running purpose, we have assumed hierarchical structure of the neutrinos and used the approximations $m_2 = \sqrt{\Delta m_{\mathrm{sol}}^2}$ and $m_3 = \sqrt{\Delta m_{\mathrm{atm}}^2}$. The running values of the observables at the high scale depend on the scale v_R, this is why we present the neutrino mass squared differences resulting from running in table 3 at the relevant scale v_R corresponding to two different fits (type-I dominance and type-I+II case), while all the other inputs are at $M_{\mathrm{GUT}} = 2 \times 10^{16}\,\mathrm{GeV}$.

In SO(10) GUT models such as the one we are considering, the $(3,3)$ entry of the Dirac neutrino Yukawa coupling matrix $Y_{\nu D}$ is expected to be of the order of unity, and thus RGE corrections proportional to $Y_{\nu D}$ can be important in the momentum range $M_{\nu^c} \leq \mu \leq M_{\mathrm{GUT}}$. This effect could have a sizeable contribution to the tau lepton mass only, since for the first and second generation Dirac Yukawa couplings turn out to be small. Including

Quantity	Central Value
$\Delta m^2_{\text{sol}}/10^{-5}\,\text{eV}^2$	7.56±0.24
$\Delta m^2_{\text{atm}}/10^{-3}\,\text{eV}^2$	2.41±0.08
$\sin^2\theta^{\text{PMNS}}_{12}/10^{-1}$	3.08±0.17
$\sin^2\theta^{\text{PMNS}}_{23}/10^{-1}$	3.875±0.225
$\sin^2\theta^{\text{PMNS}}_{13}/10^{-2}$	2.41±0.25

Table 2. Observables in the neutrino sector used in our fits taken from ref. [70].

this effect of the heavy right-handed neutrinos thresholds, the Dirac neutrino mass matrix gets modified at the GUT scale as

$$M'_{\nu D} = \left[1 - \frac{3}{2(16\pi^2)}Y_{\nu D}\log\left(\frac{M_{\text{GUT}}}{c_R S}\right)Y^\dagger_{\nu D}\right]M_{\nu D}, \qquad (3.25)$$

while the modified charged lepton mass matrix becomes

$$M'_E = \left[1 + \frac{3}{2(16\pi^2)}Y'_{\nu D}\log\left(\frac{M_{\text{GUT}}}{c_R S}\right)Y'^\dagger_{\nu D}\right]M_E. \qquad (3.26)$$

To be clear, the tau lepton mass decreases in going from the ν_R mass scale to the GUT scale due to the Dirac neutrino Yukawa correction. In the fitting procedure it was thus M'_E from eq. (3.26) to be compared to the experimental values at M_{GUT}, while in (3.18) $M_{\nu D}$ has been replaced by $M'_{\nu D}$ (3.25). Notice that M_E and $Y_{\nu D} = M_{\nu D}/v$ in eqs. (3.26) and (3.25) are defined in eqs. (3.15) and (3.16).

We investigate three different scenarios, type-I dominance, type-II dominance and the general scenario where both contributions are present, type-I+II. The fit results corresponding to our numerical analysis is presented in table 3. We found good solutions for both type-I and type-I+II with total $\chi^2 = 0.45$ and 0.004 respectively, but not for type-II scenario (the total $\chi^2 \sim 1000$ in this case). For the type-II case, our numerical analysis shows that, for the best fit, the worst fitted quantity corresponds to Δm^2_{sol} that comes out to be $\sim 10^3$ times smaller (with pull ~ -32) than the experimental data. The other discrepancy is of the quantity $\theta^{\text{PMNS}}_{23}$ that is $\sim$1.5 times smaller compared to the experimental central value. With these fit results the predictions of the model for these two scenarios are listed in table 4. The parameter set corresponding to these best fit results are presented in appendices C and D for type-I and type-I+II cases respectively. We conclude that the model gives an excellent fit to all observables in the fermion sector. This completes our proof of the minimality of the Yukawa sector in SO(10) models.

From the best fit results presented in table 3 it is clear that the $SU(2)_R$ breaking scale v_R for the type-I and type- I+II solutions are very different. In type-I seesaw scenario, the scale $v_R \sim 10^{15}\,\text{GeV}$ is just of the right size in order to reproduce the light neutrino masses. For both these seesaw scenarios, the $(3,3)$ element of the matrix S in eq. (3.16) is much larger than the $(3,3)$ element of the matrix D, while $A_{33} = 0$. Similar dominance

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{\text{GUT}}$)	Fitted values (type-I) (at $\mu = M_{\text{GUT}}$)	pulls (type-I)	Fitted values (type-I+II) (at $\mu = M_{\text{GUT}}$)	pulls (type-I+II)
$m_u/10^{-3}$	0.442 ± 0.149	0.444	0.009	0.442	-0.0002
m_c	0.238 ± 0.007	0.238	-0.002	0.238	0.0001
m_t	74.51 ± 0.65	74.52	0.009	74.52	-0.005
$m_d/10^{-3}$	1.14 ± 0.11	1.14	-0.0002	1.14	-0.00006
$m_s/10^{-3}$	21.58 ± 1.14	21.60	0.007	21.59	0.0001
m_b	0.994 ± 0.009	0.994	0.002	0.994	0.000005
$m_e/10^{-3}$	0.470692 ± 0.000470	0.470674	-0.03	0.470675	-0.003
$m_\mu/10^{-3}$	99.3658 ± 0.0993	99.3618	-0.04	99.3621	-0.003
m_τ	1.68923 ± 0.00168	1.68925	0.01	1.68925	0.001
$|V_{us}|/10^{-2}$	22.54 ± 0.06	22.54	0.002	22.54	0.00008
$|V_{cb}|/10^{-2}$	4.856 ± 0.06	4.856	0.001	4.856	0.0007
$|V_{ub}|/10^{-2}$	0.420 ± 0.013	0.420	-0.007	0.420	-0.0001
δ_{CKM}	1.207 ± 0.054	1.207	0.01	1.207	0.005
$\Delta m^2_{\text{sol}}/10^{-4}(\text{eV}^2)$	1.29 ± 0.04 $(1\times10^{15}\text{GeV})$ 1.27 ± 0.04 $(7.3\times10^{12}\text{GeV})$	1.27	-0.48	1.27	0.04
$\Delta m^2_{\text{atm}}/10^{-3}(\text{eV}^2)$	4.12 ± 0.13 $(1\times10^{15}\text{GeV})$ 4.05 ± 0.13 $(7.3\times10^{12}\text{GeV})$	4.06	-0.46	4.06	0.04
$\sin^2\theta_{12}^{\text{PMNS}}$	0.308 ± 0.017	0.308	-0.01	0.308	0.00001
$\sin^2\theta_{23}^{\text{PMNS}}$	0.387 ± 0.0225	0.387	-0.01	0.387	-0.00006
$\sin^2\theta_{13}^{\text{PMNS}}$	0.0241 ± 0.0025	0.0241	0.01	0.0241	-0.0003

Table 3. Best fit values of the observables correspond to $\chi^2 = 0.45$ and 0.004 for **type-I** and **type-I+II** scenarios respectively for 18 observables. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects. The neutrino mass squared differences are fitted at the v_R scale, which for our solutions are $\sim 1 \times 10^{15}\,\text{GeV}$ and $\sim 7.3 \times 10^{12}\,\text{GeV}$ for **type-I** and **type-I+II** respectively. Here the v_R scale is determined by using the relation $v_R = c_R v_{126}^u$ given in eq. (3.12), we have taken $v_{126}^u = 174.104\,\text{GeV}$. One should note that due to the right-handed neutrino threshold corrections the charged lepton mass matrix gets modified and is given in eq. (3.26). The fitted masses for the charged leptons presented in this table are the eigenvalues of this modified matrix, M_E'. The effect of the right-handed neutrinos is to decrease the tau lepton mass in going from ν_R scale to the GUT scale. For the fits presented in the table, the actual fitted mass of the tau lepton is $m_\tau = 1.617\,\text{GeV}$ $(1.573\,\text{GeV})$ at the GUT scale for the **type-I** (**type-I+II**) scenario, which matches correctly with the input value when the right-handed neutrino threshold correction is taken into account. For **type-II** scenario, we have not found any acceptable solution as mentioned in the text.

| Quantity	Predicted Value (**type-I**)	Predicted Value (**type-I+II**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{1.51 \cdot 10^{-4}, 1.12 \cdot 10^{-2}, 6.47 \cdot 10^{-2}\}$	$\{1.02 \cdot 10^{-2}, 1.52 \cdot 10^{-2}, 6.55 \cdot 10^{-2}\}$
$\{\delta^{\mathrm{PMNS}}, \alpha_{21}^{\mathrm{PMNS}}, \alpha_{31}^{\mathrm{PMNS}}\}$	$\{2.81°, 169.61°, 27.25°\}$	$\{-150.82°, -136.92°, -106.50°\}$
$\{m_{\mathrm{cos}}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{7.61 \cdot 10^{-2}, 5.05 \cdot 10^{-3}, 2.13 \cdot 10^{-3}\}$	$\{9.10 \cdot 10^{-2}, 1.30 \cdot 10^{-2}, 4.09 \cdot 10^{-3}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{1.04 \cdot 10^{5}, 1.23 \cdot 10^{12}, 4.34 \cdot 10^{14}\}$	$\{6.14 \cdot 10^{6}, 1.12 \cdot 10^{10}, 3.14 \cdot 10^{12}\}$

Table 4. Predictions of the minimal non-SUSY SO(10) model for **type-I** and **type-I+II** scenarios. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{\mathrm{cos}} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

is realized for the $(2,2)$ element. Thus there are no cancellations involved. Note that the off-diagonal elements of the matrices D and A are comparable. On the other hand, the best fit solution corresponding to the type-I+II seesaw scenario, v_R is two orders of magnitude smaller compared to the type-I seesaw case, hence some cancellation is realized between the type-I and type-II contributions to reproduce the correct neutrino masses. However, in the latter scenario solutions in the neighborhood of the best fit solution for type-I, where there are no such cancellations, are permitted. No other obvious fine tuning is needed to reproduce the observables in the fermion sector from the setup given by eqs.(3.13)–(3.17).

4   Gauge coupling unification

As is well known, the three gauge couplings of the SM do not unify at a common scale. SO(10) models provide a way to achieve coupling unification by virtue of an intermediate scale. In our proposed framework, the first stage of symmetry breaking can be achieved by employing a real 45_H, or a real 54_H or a real 210_H, along with a complex 126_H. Employing 45_H Higgs would require relying on the quantum corrections in the Higgs potential [26, 27, 29], while there is no such problem with the use of 210_H. In both cases the discrete D Parity symmetry would be broken at the GUT scale [20]. The intermediate gauge symmetry may be $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ when a 210_H is used, while it is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(3)_c \times \mathrm{U}(1)_{B-L}$ if the 45_H is used. Alternatively, a 54_H can break SO(10) down to Pati-Salam symmetry preserving D parity. In this case the unification scale tends to be lower, of order $2 \times 10^{15}\,\mathrm{GeV}$, if threshold effects arising from the scalar multiplets are ignored. This can potentially be in conflict with proton decay limits. It has been recently shown in ref. [28] that symmetry breaking with a 54_H and a 126_H can lead to higher values of M_{GUT} consistent with proton lifetime, when threshold effects are properly included. Here we present for completeness our results on the unification of gauge couplings assuming the intermediate symmetry to be $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_c$ with or without D parity.

Since breaking SO(10) gauge group by 54_H Higgs preserves the discrete parity, that demands the equality of the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ gauge couplings ($g_L = g_R$) at the PS intermediate scale. The low energy data completely determines the value of this scale as
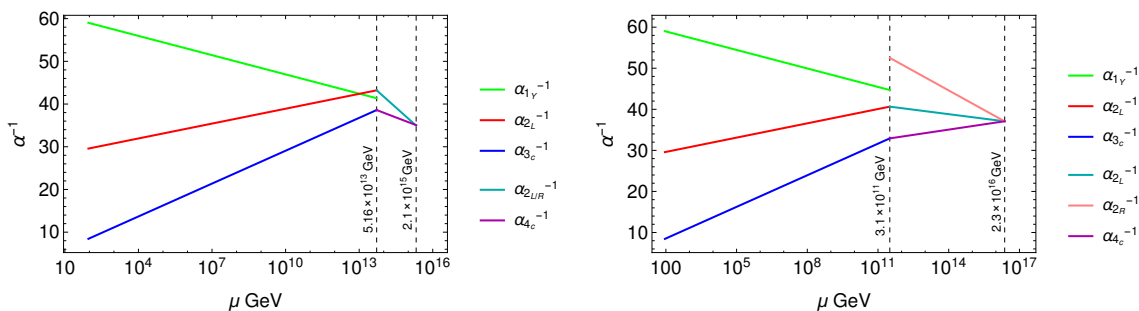
Figure 1. 1-loop gauge coupling running of the three SM gauge couplings from low scale to intermediate PS scale and from PS scale to GUT scale for minimal non-SUSY SO(10) model. The left plot corresponds to the case when the GUT symmetry is broken by 54_H Higgs that leaves the discrete symmetry $g_L = g_R$ unbroken. The right plot is for the case when 54_H is replaced by 210_H Higgs that does not preserve the discrete symmetry.

well as the GUT scale with the assumption of survival hypothesis [73–75]. The one-loop beta function coefficients for the evolution of the U(1), $SU(2)_L$ and $SU(3)_c$ gauge couplings are $b_i = \{41/10, -19/6, -7\}$ [76]. To determine the intermediate scale, we use the low energy values from ref. [66]: $\alpha_1^{-1}(M_Z) = 59.02$, $\alpha_2^{-1}(M_Z) = 29.57$ and $\alpha_3^{-1}(M_Z) = 8.44$ (only the central values are quoted here). Then from the intermediate scale to the GUT scale we run the RGEs with one-loop coefficients $b_i = \{67/6, 67/6, 10/3\}$ for the group G_{224} that determines the GUT scale. The existence of the multiplets $(2, 2, 1) \subset 10_H$, $(2, 2, 1) + (2, 2, 15) \subset 120_H$ and $(2, 2, 15) + (3, 1, 10) + (1, 3, \overline{10}) \subset 126_H$ is assumed at the intermediate scale while the rest of the multiplets are assumed to have GUT scale mass following the survival hypothesis. One-loop running of the RGEs of the gauge couplings are shown in figure 1 (left plot).

From this figure 1 one sees that the GUT scale is $\sim 2 \times 10^{15}$ GeV, which is about a factor of 2.5 smaller compared to what is needed to save the theory from the experimental proton decay limit $\tau_p \gtrsim 1.29 \times 10^{34}$ yrs [77]. Certainly the assumption made that all scalar particles have a common mass at the assumed scale is too restrictive: the Higgs multiplets are likely to have non-degenerate mass spectrum with masses scattered around each scale under consideration. If one includes this threshold correction arising from the Higgses, the unification scale can be raised as shown in an explicit calculation in ref. [28]. There is no strict guideline, however, on how much the mass spectrum may be scattered; this would lead to significant uncertainty in proton lifetime estimate. As we show in the next section, the branching ratios for proton decay are much more stable and can be used to test these theories.

If instead of a 54_H a 210_H is used to break the GUT symmetry, then the unification scale is naturally raised to about 2×10^{16} GeV. This is because D parity is broken by the VEV of 210_H, and as a result, with the assumption of survival hypothesis, the intermediate scale scalar spectrum is left-right asymmetric. Although 210_H allows for other intermediate symmetries, here we focus on the Pati-Salam symmetry. The gauge coupling evolution with the PS intermediate symmetry is presented in figure 1 (right plot). Following survival

hypothesis, we consider the multiplets $(2, 2, 1) \subset 10_{\mathrm{H}}$, $(2, 2, 1) \subset 120_{\mathrm{H}}$ and $(2, 2, 15) + (1, 3, \overline{10}) \subset 126_{\mathrm{H}}$ at the intermediate scale with the rest of the multiplets lying at the GUT scale. With these multiplets, the one-loop RGE coefficients are $b_i = \{2, 26/3, -7/3\}$ for the group G_{224}. This plot clearly shows that the GUT scale can be raised by about an order of magnitude compared to the 54_{H} scenario and one does not need to rely on the threshold correction to save the theory from rapid proton decay. It should be noted that the scenario with 210_{H} has a drawback that the intermediate scale is relatively low $\sim 10^{11}$ GeV, which does not fit the right-handed neutrino mass spectrum as well as the 54_{H} model where this scale is around $(10^{13}$–$10^{14})$ GeV. A look at the heaviest right-handed neutrino mass from table 4 suggests that the case of type-I seesaw prefers symmetry breaking by a 54_{H}, while the type I + type II scenario can accommodate breaking by a 210_{H}.

5 Proton decay branching ratios

In non-SUSY SO(10) models, proton decay mediated by the gauge bosons are the most important. The lifetime of the proton is extremely sensitive to the superheavy gauge bosons masses $(M_{(X,Y)})$ since the lifetime goes as $\tau_p \sim M_{(X,Y)}^4/(g^4 m_p^5)$, where m_p is the proton mass and g is the unified gauge coupling. As noted in the previous section, there is a large uncertainty in the determination of M_X from low energy data, owing to unknown high scale threshold effects. On the other hand, proton decay branching ratios are less sensitive to these threshold effects, and so we focus on the predictions of the model for branching ratios.

The gauge bosons of SO(10) belong to the adjoint 45. The decomposition of this field under the SM gauge symmetry is given by:

$$45 = (1, 1, 1) + (1, 1, 0) + (1, 1, -1) + (1, 3, 0) + \left(3, 2, \frac{1}{6}\right) + \left(3, 2, -\frac{5}{6}\right) + \left(\overline{3}, 2, \frac{1}{6}\right) + \left(\overline{3}, 2, \frac{5}{6}\right)$$
$$+ (1, 1, 0) + \left(3, 1, \frac{2}{3}\right) + \left(\overline{3}, 1, -\frac{2}{3}\right) + (8, 1, 0). \tag{5.1}$$

The gauge bosons responsible for proton decay are the $(X, Y)(3, 2, -5/6)$ and (X', Y') $\cdot (3, 2, 1/6)$. The gauge interaction Lagrangian of these bosons with the fermions in the current eigenstate basis is given by [78]:

$$\mathcal{L}_{\mathrm{int}} = \frac{g}{\sqrt{2}} \{ [-\overline{e}_L \gamma^\mu \overline{X}_\mu^i d_{Li}^C] + [\overline{\nu}_L \gamma^\mu \overline{Y}_\mu^i d_{Li}^C] + [\overline{d}_{Li} \gamma^\mu \overline{X}_\mu^i e_L^C + \epsilon_{ijk} \overline{u}_L^{Ck} \gamma^\mu \overline{X}_\mu^i u_L^j]$$
$$+ [-\overline{u}_{Li} \gamma^\mu \overline{Y}_\mu^i e_L^C + \epsilon_{ijk} \overline{u}_L^{Ck} \gamma^\mu \overline{Y}_\mu^i d_L^j] + [-\epsilon_{ijk} \overline{d}_L^{Ck} \gamma^\mu \overline{X}_\mu'^i d_L^j] + [-\overline{u}_{Ri} \gamma^\mu \overline{X}_\mu'^i \nu_R^C]$$
$$+ [-\overline{u}_{Li} \gamma^\mu \overline{X}_\mu'^i \nu_L^C] + [\epsilon_{ijk} \overline{d}_L^{Ck} \gamma^\mu \overline{Y}_\mu'^i u_L^j] + [-\overline{u}_{Ri} \gamma^\mu \overline{Y}_\mu'^i e_R^+] + [-\overline{d}_{Li} \gamma^\mu \overline{Y}_\mu'^i \nu_L^C] + \mathrm{h.c.}\}, \tag{5.2}$$

where i, j, k are color indices and we have suppressed the family indices and SU(2)$_L$ indices.

The resulting $d = 6$ effective operators of the form $QQQL$ responsible for proton decay can be constructed from this Lagrangian [79, 80]:

$$\mathcal{O}_I^{B-L} = k_1^2 \, \epsilon_{ijk} \, \epsilon_{\alpha\beta} \, \overline{u_{iaL}^C} \, \gamma^\mu \, Q_{j\alpha aL} \, \overline{e_{b\,L}^C} \, \gamma_\mu \, Q_{k\beta bL}; \tag{5.3}$$

$$\mathcal{O}_{II}^{B-L} = k_1^2 \, \epsilon_{ijk} \, \epsilon_{\alpha\beta} \, \overline{u_{iaL}^C} \, \gamma^\mu \, Q_{j\alpha aL} \, \overline{d_{kbL}^C} \, \gamma_\mu \, L_{\beta bL}; \tag{5.4}$$

$$\mathcal{O}_{III}^{B-L} = k_2^2 \, \epsilon_{ijk} \, \epsilon_{\alpha\beta} \, \overline{d_{iaL}^C} \, \gamma^\mu \, Q_{j\beta aL} \, \overline{u_{kbL}^C} \, \gamma_\mu \, L_{\alpha bL}; \tag{5.5}$$

$$\mathcal{O}_{IV}^{B-L} = k_2^2 \, \epsilon_{ijk} \, \epsilon_{\alpha\beta} \, \overline{d_{iaL}^C} \, \gamma^\mu \, Q_{j\beta aL} \, \overline{\nu_{b\,L}^C} \, \gamma_\mu \, Q_{k\alpha bL}. \tag{5.6}$$

Here, $k_1 = g_u/(\sqrt{2}M_{(X,Y)})$ and $k_2 = g_u/(\sqrt{2}M_{(X',Y')})$, $Q_L = (u_L, d_L)$ and $L_L = (\nu_L, e_L)$. The indices i, j, k are color indices, a, b are family indices and α, β are SU(2)$_L$ indices. In the physical basis these operators will be modified as:

$$\mathcal{O}(e_\alpha^C, d_\beta) = c(e_\alpha^C, d_\beta) \, \epsilon_{ijk} \, \overline{u_{i\,L}^C} \, \gamma^\mu u_{jL} \, \overline{e_{\alpha\,L}^C} \, \gamma_\mu \, d_{k\beta L}; \tag{5.7}$$

$$\mathcal{O}(e_\alpha, d_\beta^C) = c(e_\alpha, d_\beta^C) \, \epsilon_{ijk} \, \overline{u_{i\,L}^C} \, \gamma^\mu u_{jL} \, \overline{d_{k\beta\,L}^C} \, \gamma_\mu \, e_{\alpha L}; \tag{5.8}$$

$$\mathcal{O}(\nu_l, d_\alpha, d_\beta^C) = c(\nu_l, d_\alpha, d_\beta^C) \, \epsilon_{ijk} \, \overline{u_{i\,L}^C} \, \gamma^\mu \, d_{j\alpha L} \, \overline{d_{k\beta\,L}^C} \, \gamma_\mu \, \nu_{lL}; \tag{5.9}$$

$$\mathcal{O}(\nu_l^C, d_\alpha, d_\beta^C) = c(\nu_l^C, d_\alpha, d_\beta^C) \, \epsilon_{ijk} \, \overline{d_{i\beta\,L}^C} \, \gamma^\mu \, u_{jL} \, \overline{\nu_{l\,L}^C} \, \gamma_\mu \, d_{k\alpha L}; \tag{5.10}$$

where

$$c(e_\alpha^C, d_\beta) = k_1^2 \left[V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta} \left(V_2 V_{UD}^\dagger \right)^{\alpha 1} \right]; \tag{5.11}$$

$$c(e_\alpha, d_\beta^C) = k_1^2 V_1^{11} V_3^{\beta\alpha} + k_2^2 \left(V_4 V_{UD}^\dagger \right)^{\beta 1} \left(V_1 V_{UD} V_4^\dagger V_3 \right)^{1\alpha}; \tag{5.12}$$

$$c(\nu_l, d_\alpha, d_\beta^C) = k_1^2 (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l} + k_2^2 V_4^{\beta\alpha} \left(V_1 V_{UD} V_4^\dagger V_3 V_{EN} \right)^{1l}; \tag{5.13}$$

$$c(\nu_l^C, d_\alpha, d_\beta^C) = k_2^2 \left[\left(V_4 V_{UD}^\dagger \right)^{\beta 1} \left(U_{EN}^\dagger V_2 \right)^{l\alpha} + V_4^{\beta\alpha} \left(U_{EN}^\dagger V_2 V_{UD}^\dagger \right)^{l1} \right]; \alpha = \beta \neq 2. \tag{5.14}$$

In the above V_1, V_2 etc. are mixing matrices defined so that $V_1 = U_L^T U_R$, $V_2 = E_L^T D_R$, $V_3 = D_L^T E_R$, $V_4 = D_L^T D_R$, $V_{UD} = U_R^\dagger D_R$, $V_{EN} = E_R^\dagger N$ and $U_{EN} = E_L^T N$, where U, D, E define the diagonalizing matrices given by

$$U_L^\dagger \, M_U \, U_R = M_U^{\text{diag}} \tag{5.15}$$

$$D_L^\dagger \, M_D \, D_R = M_D^{\text{diag}} \tag{5.16}$$

$$E_L^\dagger \, M_E \, E_R = M_E^{\text{diag}} \tag{5.17}$$

$$N^T \, M_N \, N = M_N^{\text{diag}}. \tag{5.18}$$

Then the partial decay width of the decay $N \to P + \bar{l}$ ($N = p, n$, $P = (\pi, K, \eta)$ and $\bar{l}$ is anti-lepton) is given by:

$$\Gamma(N \to P + \bar{l}) = \frac{m_N}{32\pi} \left[1 - \left(\frac{m_P}{m_N} \right)^2 \right]^2 \left| \sum_I C^I W_0^I(N \to P) \right|^2, \tag{5.19}$$

p decay modes	**type-I**	**type-I+II**
$p \to \overline{\nu} + \pi^+$	49.07%	48.77%
$p \to e^+ \pi^0$	42.57%	35.16%
$p \to \mu^+ K^0$	4.13%	5.12%
$p \to \mu^+ \pi^0$	1.60%	5.62%
$p \to \overline{\nu} K^+$	1.19%	2.64%
$p \to e^+ K^0$	0.99%	2.28%
$p \to e^+ \eta$	0.40%	0.33%
$p \to \mu^+ \eta$	0.01%	0.05%

Table 5. Proton decay branching ratios in minimal non-SUSY SO(10) GUT in **type-I** and **type-I+II** cases. For neutrino final states, we sum over all three flavors.

where the coefficients C^I are given in eqs. (5.11)–(5.14) and the relevant form factors W_0 are obtained by using lattice QCD computations [81]:

$$\langle \pi^0 |(u,d)_R u_L|p\rangle_0 = -0.103, \langle \pi^+ |(u,d)_R u_L|p\rangle_0 = -0.103, \langle K^0 |(u,s)_R u_L|p\rangle_0 = 0.098,$$

$$\langle K^+ |(u,s)_R d_L|p\rangle_0 = -0.054, \langle K^+ |(u,d)_R s_L|p\rangle_0 = -0.093, \langle \eta |(u,d)_R u_L|p\rangle_0 = 0.015.$$

In table 5 we present the $d = 6$ proton decay branching ratios calculated for our best fit parameter sets. We find that the two dominant modes are $p \to \overline{\nu}\pi^+$ and $p \to e^+\pi^0$. A comparison of these modes with those of more general $d = 6$ proton decay studies [80] shows similarity. The near dominance of the $\overline{\nu}\pi^+$ mode may be taken as a test of the Yukawa sector presented here.

6 Conclusion

In this work, we have presented an economic Yukawa sector for SO(10) models. The main feature of this construction is that only the SO(10) symmetry is used to constrain the Yukawa parameters. The Higgs system consists of a *real* 10_H, a *real* 120_H and a complex $\overline{126}_H$ that have Yukawa couplings. In most nonsupersymmetric SO(10) models in the literature symmetries outside of SO(10) — such as a Peccei-Quinn U(1) — are used to constrain the Yukawa sector. That would require the complexification of the *real* 10_H and *real* 120_H. The model presented here deviates from this, and yet is quite constraining. We showed that, with a limited number of Yukawa parameters, a good fit to all fermion masses and mixings, including the neutrino sector is possible. Once the flavor sector is fixed, we are able to calculate the proton decay branching ratios. The dominant decays of the proton are found to be $p \to \overline{\nu}\pi^+$ and $p \to e^+\pi^0$, which may provide partial tests of the model.

A   A comment on doublet-triplet splitting

As is well known, any grand unified theory has to address the question of making one Higgs doublet light, while its color triplet GUT partner remains superheavy so as to not cause rapid proton decay. This doublet-triplet splitting problem is present in both SUSY and non-SUSY minimal GUTs. If the Higgs doublet mass is not split from the color triplet mass, either the electroweak symmetry would break at the GUT scale, or not break at all, or the light color-triplet would lead to far too fast proton decay. A fine-tuning is necessary to bring the Higgs doublet mass down to the weak scale. In supersymmetric versions, this fine-tuning is done at the tree level, SUSY would guarantee its stability against quantum corrections. In non-supersymmetric SO(10) theories, the tuning must be done after taking account of loop corrections to a very high order. The induced Higgs mass from quantum loops would be at the n-loop level of order $\Delta m_{\mathrm{H}}^{(n)} \sim M_{\mathrm{GUT}}(\alpha/4\pi)^{n/2}$. For $m_{\mathrm{H}} \sim 125\,\mathrm{GeV}$, the tuning must be done after $n = 12$ loop corrections are included. We note that this is nevertheless only one fine-tuning, albeit not easily enforceable by actual calculations. The Hermitian Higgs doublet mass matrix is tuned to have near-zero determinant. In contrast, in minimal SUSY GUTs, the needed tree level tuning requires the determinant of the complex doublet Higgsino mass matrix to be near zero. Recall that in SUSY all mass parameters in the superpotential are complex in general. Such a tuning amounts to two conditions, unlike the non-supersymmetric tuning, which requires only one such condition. Although the Higgs mass can be ensured only after including very high order corrections in non-SUSY SO(10), we find it intriguing that the fine-tuning condition is more minimal here compared to minimal SUSY SO(10).

B   Proof of $m_\tau \simeq 3m_b$ in models with 126_{H} and $2 \times 120_{\mathrm{H}}$

Without loss of generality, we can diagonalize the Yukawa coupling of 126_{H}. We focus on the second and third generation down quarks and charged leptons. Their mass matrices can be written down as

$$M_D = \begin{pmatrix} a & c \\ -c & b \end{pmatrix}, \qquad M_E = \begin{pmatrix} -3a & c' \\ -c' & -3b \end{pmatrix}. \tag{B.1}$$

This form persists even when many of 120_{H} fields are used, with their mass contribution going into the off-diagonal entries differently in M_D and M_E. The exact invariants of these matrices are then

$$m_s^2 + m_b^2 = |a|^2 + |b|^2 + 2\,|c|^2 \tag{B.2}$$

$$m_\mu^2 + m_\tau^2 = 9\left(|a|^2 + |b|^2\right) + 2\,|c'|^2 \tag{B.3}$$

$$m_s m_b = |ab + c^2| \tag{B.4}$$

$$m_\mu m_\tau = |9\,ab + c'^2|\,. \tag{B.5}$$

From these relations it follows that

$$m_\mu^2 + m_\tau^2 = 9\left(m_s^2 + m_b^2\right) + 18\left[\left|c^2 - m_s m_b e^{i\alpha} + \frac{m_\mu m_\tau}{9}e^{i\beta}\right| - |c|^2\right]. \tag{B.6}$$

The undetermined parameter c is bounded by eq. (B.2), and no matter how we vary c, the deviation from 3 in the ratio m_τ/m_b is of order $m_s/m_b \sim 5\%$. The inclusion of the first family is not expected to change considerably this result. This proves that a Higgs sector consisting of one 126_H and two or any number of copies of real 120_H cannot lead to realistic fermion masses.

C Best fit parameters for type-I dominance scenario

In this appendix we present the parameter set corresponding to the χ^2 best fit for the type-I scenario.[4]

$$r_1 = -3.5178190\times10^{-3}-5.1827520\times10^{-3}i \tag{C.1}$$

$$r_2 = -1.0441669+1.6253165\times10^{-1}i \tag{C.2}$$

$$\phi = -7.9459769\times10^{-1} \tag{C.3}$$

$$c_R = 5.8035176\times10^{12} \tag{C.4}$$

$$S = \begin{pmatrix} 1.7926501\times10^{-8} & 0 & 0 \\ 0 & 2.1219581\times10^{-1} & 0 \\ 0 & 0 & 7.4949627\times10^{1} \end{pmatrix} \text{GeV} \tag{C.5}$$

$$D = \begin{pmatrix} 2.8344746\times10^{-4}-5.3097883\times10^{-4}i & 4.501669\times10^{-3}-1.7083332\times10^{-3}i & 6.0343793\times10^{-2}+7.8900202\times10^{-3}i \\ 4.501669\times10^{-3}-1.7083332\times10^{-3}i & 2.7783311\times10^{-2}-1.5722435\times10^{-2}i & 3.0561540\times10^{-1}+9.6327579\times10^{-2}i \\ 6.0343793\times10^{-2}+7.8900202\times10^{-3}i & 3.0561540\times10^{-1}+9.6327579\times10^{-2}i & -4.304058\times10^{-1}+2.4126529\times10^{-1}i \end{pmatrix} \text{GeV} \tag{C.6}$$

$$A = \begin{pmatrix} 0 & -3.9710310\times10^{-3}-1.6550999\times10^{-3}i & -4.0391236\times10^{-2}-4.2504129\times10^{-2}i \\ 3.9710310\times10^{-3}+1.6550999\times10^{-3}i & 0 & -1.7267986\times10^{-1}-3.2019088\times10^{-1}i \\ 4.0391236\times10^{-2}+4.2504129\times10^{-2}i & 1.7267986\times10^{-1}+3.2019088\times10^{-1}i & 0 \end{pmatrix} \text{GeV}. \tag{C.7}$$

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[4]To reproduce the observables presented in table 3 for both the type-I and type-I+II scenarios, one must keep all the significant digits of the parameters presented in these appendices. This high level of accuracy is needed to reproduce the neutrino observables; it is due to the fact that the right-handed neutrino mass spectrum in both cases shows extreme hierarchy among the generations, see table 4. Since this hierarchy between the first and the second generations is extreme, chopping-off digits effects mainly the quantity Δm^2_{sol}.

D Best fit parameters for type-I+II scenario

In this appendix we present the parameter set corresponding to the χ^2 best fit for the type-I+II scenario.

$$r_1 = 4.1628007 \times 10^{-3} - 3.1705843 \times 10^{-3} i \tag{D.1}$$

$$r_2 = -7.4367427 \times 10^{-1} + 3.5915531 \times 10^{-1} i \tag{D.2}$$

$$\phi = -6.4632781 \times 10^{-1} \tag{D.3}$$

$$c_R = 4.2254013 \times 10^{10} \tag{D.4}$$

$$c_L = 1.5155879 \times 10^{-10} - 1.4499546 \times 10^{-11} i \tag{D.5}$$

$$S = \begin{pmatrix} 1.4547716 \times 10^{-4} & 0 & 0 \\ 0 & 2.6693088 \times 10^{-1} & 0 \\ 0 & 0 & 7.4473135 \times 10^{1} \end{pmatrix} \text{GeV} \tag{D.6}$$

$$D = \begin{pmatrix} 4.8953934 \times 10^{-4} - 2.6113522 \times 10^{-4} i & -1.6504521 \times 10^{-5} + 1.1420336 \times 10^{-2} i & -2.151214 \times 10^{-1} + 1.7234983 \times 10^{-2} i \\ -1.6504521 \times 10^{-5} + 1.1420336 \times 10^{-2} i & -2.8562186 \times 10^{-2} + 2.8403787 \times 10^{-2} i & -3.7065300 \times 10^{-1} - 2.0521574 \times 10^{-1} i \\ -2.151214 \times 10^{-1} + 1.7234983 \times 10^{-2} i & -3.7065300 \times 10^{-1} - 2.0521574 \times 10^{-1} i & 3.6722700 \times 10^{-2} + 2.6598904 \times 10^{-1} i \end{pmatrix} \text{GeV} \tag{D.7}$$

$$A = \begin{pmatrix} 0 & 2.518929 \times 10^{-3} - 1.1393329 \times 10^{-2} i & 1.7915567 \times 10^{-1} + 1.1538080 \times 10^{-1} i \\ -2.518929 \times 10^{-3} + 1.1393329 \times 10^{-2} i & 0 & 1.6923025 \times 10^{-1} + 3.6425489 \times 10^{-1} i \\ -1.7915567 \times 10^{-1} - 1.1538080 \times 10^{-1} i & -1.6923025 \times 10^{-1} - 3.6425489 \times 10^{-1} i & 0 \end{pmatrix} \text{GeV} . \tag{D.8}$$

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