Published for SISSA by Springer

Received: *December 20, 2015*
Revised: *February 2, 2016*
Accepted: *February 6, 2016*
Published: *February 18, 2016*

Approaching Minimal Flavour Violation from an $\mathrm{SU}(5) \times S_4 \times \mathrm{U}(1)$ SUSY GUT

Maria Dimou,[a] Stephen F. King[a] and Christoph Luhn[b]

[a] *School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, U.K.*

[b] *Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Straße 3, 57068 Siegen, Germany*

E-mail: md1e10@soton.ac.uk, king@soton.ac.uk, christoph.luhn@uni-siegen.de

Abstract: We show how approximate Minimal Flavour Violation (MFV) can emerge from an SU(5) Supersymmetric Grand Unified Theory (SUSY GUT) supplemented by an $S_4 \times \mathrm{U}(1)$ family symmetry, which provides a good description of all quark and lepton (including neutrino) masses, mixings and CP violation. Assuming a SUSY breaking mechanism which respects the family symmetry, we calculate in full explicit detail the low energy mass insertion parameters in the super-CKM basis, including the effects of canonical normalisation and renormalisation group running. We find that the very simple family symmetry $S_4 \times \mathrm{U}(1)$ is sufficient to approximately reproduce the effects of low energy MFV.

Keywords: Beyond Standard Model, Supersymmetric Standard Model, GUT, Discrete Symmetries

ArXiv ePrint: 1511.07886

doi:10.1007/JHEP02(2016)118

JHEP02(2016)118

Contents

1 Introduction

The mystery of flavour has been with us from the discovery of the muon in 1936 to the discovery of neutrino mass and mixing in 1998. The Standard Model (SM), extended to include neutrino mass, is described by at least 26 parameters, of which no less than 20 are flavour parameters: 10 from the quark sector and at least 10 from the lepton sector. At least two of these parameters are related to CP violation in the quark and lepton sectors, although the latter has not yet been definitively observed.

A lot of effort has been put into trying to understand the flavour structure of the SM (for reviews see e.g. [1–5]). Its peculiar features include hierarchical charged fermion masses, with the down-type quark and charged lepton masses showing a similar pattern which differs from that of the up-type quarks, while neutrinos are significantly lighter than all other particles. Flavour mixing in the lepton sector has turned out to be much larger than in the quark sector, and the number of generations is not explained.

Following the award of the 2015 Nobel Prize for "the discovery of neutrino oscillations which shows that neutrinos have mass", we still have no more understanding of flavour than back in 1936 when Rabi famously asked of the muon "who ordered that?". Part of the reason for this impasse is the failure of experiment to measure any flavour and CP violation beyond that expected in the SM. The problem is that the SM is not a theory of flavour and, as such, provides no understanding of the origin or nature of flavour.

In the absence of any observed beyond SM flavour and CP violation, a sort of "straw man" ansatz for flavour has emerged known as Minimal Flavour Violation (MFV) [6–8] in which all flavour and CP-violating transitions are postulated to originate in the SM Yukawa matrices so that they are governed by the CKM matrix. The formulation of MFV in an effective field theory involving a high-energy $SU(3)^5$ flavour symmetry, broken only by the Yukawa matrices, allows higher-dimensional operators which can contribute considerably to flavour observables [9–11]. Going beyond an effective field theory description, it is possible to implement the idea of MFV in a renormalisable theory by introducing new heavy fermions. In such a setup, the flavour symmetry is broken by scalar fields whose Vacuum Expectation Values (VEVs) are related to the Yukawa matrices in an inverse way [12–16]. Although this differs from the standard MFV approach, where the fundamental flavour breaking fields are linearly related to the Yukawa matrices, it does reproduce MFV phenomenologically by predicting very SM-like flavour and CP violation, which is of course exactly what is observed.

When considering extensions of the SM, such as Supersymmetry (SUSY) softly broken at the TeV scale, then in general large deviations from SM flavour and CP violation are expected. SUSY models include one-loop diagrams that lead to Flavour Changing Neutral Current (FCNC) processes such as e.g. $b \to s\gamma$ and $\mu \to e\gamma$ at rates which are proportional to the size of the off-diagonal elements of the scalar mass matrices, when the latter have been rotated to the super-CKM (SCKM) basis where the Yukawa matrices are diagonal [17]. These SUSY contributions are tamed in the Constrained Minimal Supersymmetric Standard Model (CMSSM) which postulates that, at the high energy scale, the SUSY breaking squark and slepton mass squared matrices are proportional to the unit matrix and the trilinear A-terms are additionally aligned with the Yukawa matrices, resulting in an (approximate) MFV-like structure at low energy [17].

In the framework of Grand Unified Theories (GUTs), the embedding of the SM fermions into GUT multiplets does not allow to implement the $SU(3)^5$ flavour symmetry of MFV. However, in GUTs based on SU(5) [18] or the Pati-Salam group $SU(4) \times SU(2) \times SU(2)'$ [19, 20], it is possible to introduce an $SU(3)^2$ flavour symmetry instead, and this has been shown to lead to sufficient suppression of flavour violation [21–23]. Considering SUSY GUTs, the CMSSM framework always provides a safe haven from unwanted flavour violation, although CP violation in the form of Electric Dipole Moments (EDMs) remains a challenge [17]. However, with SUSY and SUSY GUTs, the real challenge is to justify the assumptions of MFV or the CMSSM, while at the same time providing a realistic explanation of quark and lepton (including neutrino) masses, mixing and CP violation. This non-trivial balancing act is what concerns us in this paper.

The discovery of neutrino mass and mixing has spurred a lot of work aiming to describe flavour in terms of a family symmetry of some kind, in particular discrete non-Abelian family symmetry [1–5]. It was realised early on that in such models, the idea of spontaneous flavour and CP violation could effectively tame the flavour and CP problems of the SM [24, 25] without any *ad hoc* assumptions about MFV or the CMSSM. The main point is that the same family symmetry introduced to understand the Yukawa sector will also automatically control the flavour structures of the soft SUSY breaking sector. The only requirement is that the SUSY breaking hidden sector must respect the family symmetry, which means that the family (and CP) symmetry breaking scale must be below the mass scale of the messengers which mediate SUSY breaking to the visible sector. SUSY breaking in the framework of supergravity provides one attractive example for such a situation.

The idea of using family symmetry to solve the SUSY favour and CP problems has been fully explored in the framework of an SU(3) family symmetry [25–27], where it was shown that the flavons that spontaneously break family and CP symmetry will perturb the SUSY breaking sector, leading to tell-tale signatures of flavour and CP violation beyond MFV or the CMSSM. Unfortunately, these signatures which were expected to appear in Run1 of the LHC [28] did not in fact materialise, and indeed the allowed parameter space has been much reduced [29, 30].

In the setup discussed in [26, 27], the extra flavour violation can be understood as follows. At leading order, the CMSSM is enforced by the SU(3) family symmetry acting on the squark and slepton mass squared matrices. However the fact that SU(3) is broken by flavons, as it must be to generate the quark and lepton masses, means that flavons appearing in the Kähler potential will give important contributions to the kinetic terms, requiring extra canonical normalisation [31, 32]. Since SUSY breaking also originates from the Kähler potential, the flavons will also modify the couplings of squarks and sleptons to the fields with SUSY breaking F-terms. The resulting corrections to the soft mass squared matrices from unity will be similar to the corrections of the corresponding Kähler metrics, yet both are not aligned due to independent coefficients of the relevant operators. Likewise, the trilinear soft SUSY breaking A-terms will replicate the flavour structure of the Yukawa matrices prior to canonical normalisation, but exact alignment is not realised. All of this occurs at the high scale. Additional flavour violation is generated by renormalisation group (RG) running down to low energy, taking into account the seesaw mechanism [33–36] which will involve thresholds at an intermediate scale, see e.g. [37, 38].

In this paper we show how approximate MFV can emerge from an SU(5) SUSY GUT, supplemented by an $S_4 \times \mathrm{U}(1)$ family symmetry [39, 40], which provides a good description of all quark and lepton (including neutrino) masses, mixings and CP violation. Assuming that SUSY breaking respects the family symmetry, we calculate in full detail the low energy mass insertion parameters in the SCKM basis. We include the effects of canonical normalisation as well as RG running. Remarkably, due to the peculiar flavour structure of the model, we find that the small family symmetry $S_4 \times \mathrm{U}(1)$ is sufficient to reproduce the effects of low energy MFV much more accurately than the previous SU(3) family symmetry model.

2 Trimaximal $S_4 \times \mathrm{SU}(5)$ model

In this section, we present the basic ingredients of the supersymmetric model of flavour proposed in [40]. It is capable of correctly describing a sizable reactor neutrino mixing angle θ^l_{13} by generating a neutrino mass matrix of trimaximal form. The model represents a modification of an earlier tri-bimaximal model [39] with only minor changes. Being formulated in a supersymmetric SU(5) grand unified framework, the matter superfields fall into the $\mathbf{10}$ and $\bar{\mathbf{5}}$ representations,

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u^c_G & u^c_B & -u_R & -d_R \\ u^c_G & 0 & -u^c_R & -u_B & -d_B \\ -u^c_B & u^c_R & 0 & -u_G & -d_G \\ u_R & u_B & u_G & 0 & -e^c \\ d_R & d_B & d_G & e^c & 0 \end{pmatrix} \quad \text{and} \quad F = (d^c_R \; d^c_B \; d^c_G \; e \; -\nu)\,, \tag{2.1}$$

where the superscript c denotes charge conjugation of the right-handed superfields. Table 1 lists the matter, Higgs and flavon superfields together with their transformation properties under the imposed $\mathrm{SU}(5) \times S_4 \times \mathrm{U}(1)$ symmetry. Details of the non-Abelian finite group S_4 are provided in appendix A. The $\bar{\mathbf{5}}$-plets, labelled by F, are assigned to a triplet representation of S_4, while the $\mathbf{10}$-plets are split into an S_4 doublet T for the first two generations and an S_4 singlet T_3 for the third generation. In addition, right-handed neutrinos N are introduced transforming in the same S_4 triplet representation as F. The SU(5) Higgs fields $H_{\mathbf{5}}$, $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ are all S_4 singlets. Note that each of these GUT Higgs representations contains an $\mathrm{SU}(2)_L$ Higgs doublet. Therefore, the low energy doublet H_u originates from

Field	T_3	T	F	N	H_5	$H_{\bar{5}}$	$H_{\overline{45}}$	Φ^u_2	$\widetilde{\Phi}^u_2$	Φ^d_3	$\widetilde{\Phi}^d_3$	Φ^d_2	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1	η
SU(5)	$\mathbf{10}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}^{(\prime)}$
U(1)	0	5	4	-4	0	0	1	-10	0	-4	-11	1	8	8	8	7

Table 1. The matter, Higgs and flavon superfields of the model in [40] together with their transformation properties under the imposed $\mathrm{SU}(5) \times S_4 \times \mathrm{U}(1)$ symmetry.

$H_{\mathbf{5}}$, while H_d arises from a linear combination of $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ [17, 41, 42].[1] In addition, we introduce a number of flavon fields Φ^f_ρ, which are labelled by the corresponding S_4 representation ρ as well as the fermion sector f to which they couple at leading order (LO). Two flavons, Φ^u_2 and $\tilde{\Phi}^u_2$, generate the LO up-type quark mass matrix. Three flavon multiplets, Φ^d_3, $\tilde{\Phi}^d_3$ and Φ^d_2, are responsible for the down-type quark and charged lepton mass matrices. Finally, the right-handed neutrino mass matrix is generated from the flavon multiplets $\Phi^\nu_{3'}$, Φ^ν_2 and Φ^ν_1 as well as the flavon η which is responsible for breaking the tri-bimaximal pattern of the neutrino mass matrix to a trimaximal one at subleading order [40]. The additional U(1) symmetry has been introduced in order to control the coupling of the flavon fields to the matter fields in a way which avoids significant perturbations of the LO flavour structure by higher-dimensional operators. We refer the reader to [39] for more details.

The vacuum structure of the flavon fields arises from the F-term alignment mechanism [43, 44]. Introducing a set of so-called driving fields, the corresponding F-term conditions give rise to particular flavon alignments as described in appendix B. To LO, these are given as [39, 40],

$$\frac{\langle\Phi^u_2\rangle}{M} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \phi^u_2\,\lambda^4, \qquad \frac{\langle\tilde{\Phi}^u_2\rangle}{M} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tilde{\phi}^u_2\,\lambda^4, \tag{2.2}$$

$$\frac{\langle\Phi^d_3\rangle}{M} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \phi^d_3\,\lambda^2, \qquad \frac{\langle\tilde{\Phi}^d_3\rangle}{M} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \tilde{\phi}^d_3\,\lambda^3, \quad \frac{\langle\Phi^d_2\rangle}{M} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \phi^d_2\,\lambda\,, \tag{2.3}$$

$$\frac{\langle\Phi^\nu_{3'}\rangle}{M} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \phi^\nu_{3'}\,\lambda^4, \qquad \frac{\langle\Phi^\nu_2\rangle}{M} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \phi^\nu_2\,\lambda^4, \quad \frac{\langle\Phi^\nu_1\rangle}{M} = \phi^\nu_1\,\lambda^4, \quad \frac{\langle\eta\rangle}{M} = \phi^\eta\,\lambda^4, \tag{2.4}$$

where $\lambda \approx 0.225$ is the Wolfenstein parameter [45] and the ϕs are dimensionless order one parameters. Imposing CP symmetry of the underlying theory, all coupling constants can be taken real [46–48], so that CP is broken spontaneously by generally complex values for the ϕs. M denotes a generic messenger scale which is common to all the non-renormalisable

[1] As $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ transform differently under U(1), it is clear that the mechanism which spawns the low energy Higgs doublet H_d must necessarily break U(1). Although the discussion of any details of the SU(5) GUT symmetry breaking (which, e.g., could even have an extra dimensional origin) are beyond the scope of our paper, we remark that a mixing of $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ could be induced by introducing the pair $H^{\pm}_{\mathbf{24}}$ with U(1) charges ± 1 in addition to the standard SU(5) breaking Higgs $H^0_{\mathbf{24}}$.

effective operators and assumed to be around the scale of grand unification. Considering also subleading terms in the flavon potential, these LO vacuum alignments receive corrections which are parameterised by small shifts as discussed in appendix B, and shown explicitly in eq. (B.4). Throughout our calculations, we have taken into account such shifts as well as all other subleading effects. As our LO results for the mass insertion parameters depend solely on the LO structure of the model, we only report the LO analysis in the main part of this paper. When giving explicit expressions, we therefore limit ourselves to showing the leading contributions, omitting additional higher order corrections. We will indicate such approximations by $\approx$ throughout the paper. Finally, the VEVs of the two neutral Higgses are:

$$\upsilon_u = \frac{\upsilon}{\sqrt{1+t_\beta^2}} t_\beta , \qquad \upsilon_d = \frac{\upsilon}{\sqrt{1+t_\beta^2}} , \tag{2.5}$$

where $t_\beta \equiv \tan(\beta) = \frac{\upsilon_u}{\upsilon_d}$ and $\upsilon = \sqrt{\upsilon_u^2 + \upsilon_d^2} = 174\,\text{GeV}$.

3 Kähler potential

A characteristic feature of any effective theory is the presence of non-renormalisable operators which are only constrained by the imposed symmetries. In the context of supersymmetry, this is the case for both the superpotential as well as the Kähler potential. The effective coupling of flavon fields to the Kähler potential gives rise to kinetic terms with a non-canonical Kähler metric $\mathcal{K} \neq \mathbb{1}$,

$$\mathcal{L}_{\text{kin}} \;=\; \mathcal{K}_{ij} \left(\partial_\mu \tilde{f}_i^* \partial^\mu \tilde{f}_j + i\, f_i^* \partial_\mu \bar{\sigma}^\mu f_j \right) , \tag{3.1}$$

where $\tilde{f}$ and f are, respectively, the scalar and fermionic components of a generic chiral superfield $\hat{f}$. In order to extract physically meaningful properties of a model, the kinetic terms have to be brought to a canonical form. The required basis transformation is usually referred to as canonical normalisation [31, 32].

In the context of SU(5), we encounter a Kähler metric for each of the three GUT representations containing the matter fields. We denote these by $\mathcal{K}_T$, $\mathcal{K}_F$ and $\mathcal{K}_N$, respectively. Using the symmetries of table 1, the expansions of these 3×3 matrices in terms of flavon fields can be obtained from

$$\begin{pmatrix} T^\dagger & T_3^\dagger \end{pmatrix} (\mathcal{K}_T - \mathbb{1}) \begin{pmatrix} T \\ T_3 \end{pmatrix} = \sum_n \begin{pmatrix} T^\dagger & T_3^\dagger \end{pmatrix} \begin{pmatrix} c_n^{K_{T_{22}}} (\mathcal{R}_2)_n & c_n^{K_{T_{i3}}} (\mathcal{R}_4)_n \\ \left[c_n^{K_{T_{i3}}} (\mathcal{R}_4)_n \right]^\dagger & c_n^{K_{T_{33}}} (\mathcal{R}_3)_n \end{pmatrix} \begin{pmatrix} T \\ T_3 \end{pmatrix} , \tag{3.2}$$

$$F^\dagger (\mathcal{K}_F - \mathbb{1}) F = \sum_n F^\dagger \left[c_n^{K_F} (\mathcal{R}_1)_n \right] F , \tag{3.3}$$

$$N^\dagger (\mathcal{K}_N - \mathbb{1}) N = \sum_n N^\dagger \left[c_n^{K_N} (\mathcal{R}_1)_n \right] N , \tag{3.4}$$

where the c_n are order one coefficients which we can assume to be real thanks to the imposed CP symmetry. Products of flavon fields which are allowed to couple in the Kähler potential are collected in the tuples $\mathcal{R}_i$, which in turn are unions of tuples $\mathcal{S}_i$. These tuples, which contain all possible combinations of up to eight flavons with a minimum contribution of order λ^8, are defined as

$$\mathcal{R}_1 = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3\,, \qquad \mathcal{R}_2 = \mathcal{S}_1 \cup \mathcal{S}_2\,, \qquad \mathcal{R}_3 = \mathcal{S}_1\,, \qquad \mathcal{R}_4 = \mathcal{S}_4\,, \tag{3.5}$$

where

$$\begin{aligned}
\mathcal{S}_1 = \Bigg\{ & \frac{\Phi_2^d\Phi_2^{d\dagger}}{M^2}, \frac{\Phi_3^d\Phi_3^{d\dagger}}{M^2}, \frac{\tilde\Phi_3^d\tilde\Phi_3^{d\dagger}}{M^2}, \frac{\Phi_2^u\Phi_2^{u\dagger}}{M^2}, \frac{\tilde\Phi_2^u\tilde\Phi_2^{u\dagger}}{M^2}, \frac{(\tilde\Phi_2^u)^2}{M^2}, \frac{\Phi_{3'}^\nu\Phi_{3'}^{\nu\dagger}}{M^2}, \frac{\Phi_2^\nu\Phi_2^{\nu\dagger}}{M^2}, \frac{\Phi_1^\nu\Phi_1^{\nu\dagger}}{M^2}, \frac{\eta\eta^\dagger}{M^2}, \\
& \frac{(\Phi_3^d)^2\Phi_1^\nu}{M^3}, \frac{(\Phi_3^d)^2\Phi_2^\nu}{M^3}, \frac{(\Phi_3^d)^2\Phi_{3'}^\nu}{M^3}, \frac{\Phi_2^d\Phi_2^{d\dagger}\tilde\Phi_2^u}{M^3}, \frac{\Phi_2^{d\dagger}\tilde\Phi_3^{d\dagger}\Phi_2^u}{M^3}, \frac{(\Phi_2^d\Phi_2^{d\dagger})^2}{M^4}, \frac{(\Phi_3^d\Phi_3^{d\dagger})^2}{M^4}, \\
& \frac{\Phi_2^d\Phi_2^{d\dagger}\Phi_3^d\Phi_3^{d\dagger}}{M^4}, \frac{\Phi_2^d\Phi_2^{d\dagger}\tilde\Phi_3^d\tilde\Phi_3^{d\dagger}}{M^4}, \frac{(\Phi_2^d\Phi_2^{d\dagger})^2\tilde\Phi_2^u}{M^5}, \frac{(\Phi_2^d\Phi_2^{d\dagger})^3}{M^6}, \frac{(\Phi_2^d\Phi_2^{d\dagger})^4}{M^8} + \text{all h.c.} \Bigg\},
\end{aligned} \tag{3.6}$$

$$\mathcal{S}_2 = \left\{ \frac{\tilde\Phi_2^u}{M}, \frac{\Phi_1^\nu\Phi_2^{\nu\dagger}}{M^2}, \frac{\Phi_2^{d\dagger}\tilde\Phi_3^{d\dagger}\Phi_2^u}{M^3} + \text{all h.c.} \right\}, \tag{3.7}$$

$$\mathcal{S}_3 = \left\{ \frac{(\Phi_2^d)^4\Phi_3^d}{M^5}, \frac{\Phi_1^\nu\Phi_{3'}^{\nu\dagger}}{M^2}, \frac{\Phi_2^\nu\Phi_{3'}^{\nu\dagger}}{M^2}, \frac{\Phi_3^d\Phi_3^{d\dagger}\tilde\Phi_2^u}{M^3}, \frac{(\Phi_2^d)^5\Phi_2^{d\dagger}\Phi_3^d}{M^7} + \text{all h.c.} \right\}, \tag{3.8}$$

$$\begin{aligned}
\mathcal{S}_4 = \Bigg\{ & \frac{(\Phi_2^d)^5}{M^5}, \frac{\eta(\Phi_2^{d\dagger})^2}{M^3}, \frac{\Phi_2^d\Phi_3^d\Phi_{3'}^\nu}{M^3}, \frac{\Phi_2^d\Phi_3^d(\Phi_3^{d\dagger})^2}{M^4}, \frac{(\Phi_2^{d\dagger})^2\Phi_3^d\Phi_3^{d\dagger}}{M^4}, \frac{(\Phi_2^{d\dagger})^3\Phi_2^\nu}{M^4}, \\
& \frac{(\Phi_2^{d\dagger})^3(\Phi_3^{d\dagger})^2}{M^5}, \frac{\eta\Phi_2^d(\Phi_2^{d\dagger})^3}{M^5}, \frac{(\Phi_2^d)^6\Phi_2^{d\dagger}}{M^7} \Bigg\}.
\end{aligned} \tag{3.9}$$

$\mathcal{S}_1$ and $\mathcal{S}_2$ contain combinations of flavons with U(1) charges that sum up to zero. They can form S_4 invariants when contracted with two doublets or two triplets. Therefore, $\mathcal{S}_1$ and $\mathcal{S}_2$ contribute to $\mathcal{K}_F$, $\mathcal{K}_N$ and the upper-left 2×2 block of $\mathcal{K}_T$ in eq. (3.2). Moreover, the combinations in $\mathcal{S}_1$ can be contracted to S_4 invariants so that they additionally contribute to the lower-right 1×1 block of $\mathcal{K}_T$. $\mathcal{S}_3$ gives further contributions to $\mathcal{K}_F$ and $\mathcal{K}_N$ but not to $\mathcal{K}_T$. Finally, the combinations contained in $\mathcal{S}_4$ have U(1) charges which add up to 5 and allow for S_4 contractions to a doublet. Hence, they contribute to the off-diagonal upper-right block of $\mathcal{K}_T$. We remark that the effects of the operators involving the flavon field η are independent of its S_4 transformation properties as a $\mathbf{1}$ or $\mathbf{1}'$.

When calculating the Kähler metric from the expressions of eqs. (3.2)–(3.4), it is important to take into account all invariant S_4 contractions of two matter fields with a given product of flavons.

3.1 Kähler metric with LO corrections

It is straightforward though tedious to determine the matrices $\mathcal{K}_T$, $\mathcal{K}_F$ and $\mathcal{K}_N$ from eqs. (3.2)–(3.4). Keeping only the LO corrections to the unit matrix, we find for the $\mathbf{10}$ of SU(5)

$$\mathcal{K}_T - \mathbb{1} \approx \begin{pmatrix} (k_5+k_1)\,\lambda^2 & k_2\,\lambda^4 & k_4\, e^{-i\theta_4^k}\lambda^6 \\ \cdot & (k_5-k_1)\,\lambda^2 & k_3\, e^{-i\theta_3^k}\lambda^5 \\ \cdot & \cdot & k_6\,\lambda^2 \end{pmatrix}, \tag{3.10}$$

where k_i denote real order one coefficients, and θ_i^k are phases associated with the generally complex flavon VEVs. Here and throughout our paper, the dots in the lower-left corner of the matrix represent the complex conjugates of the corresponding entries in the upper-right part of the matrix. The operator $T^\dagger \Phi_2^d \Phi_2^{d\dagger} T/M^2$ gives rise to the parameters k_1 and k_5 through different S_4 contractions, while k_6 is due to $T_3^\dagger \Phi_2^d \Phi_2^{d\dagger} T_3/M^2$. Being associated with $T^\dagger \tilde{\Phi}_2^u T/M$, the parameter k_2 carries no phase factor because $\tilde{\phi}_2^u \in \mathbb{R}$, cf. appendix B. Finally, the (13) and (23) elements originate from $T^\dagger \eta (\Phi_2^{d\dagger})^2 T_3/M^3$ and $T^\dagger (\Phi_2^d)^5 T_3/M^5$, respectively. Making use of the phases of the LO flavon VEVs, given explicitly in eq. (B.2), we can write the phases of eq. (3.10) as

$$\theta_4^k = \theta_3^d - \theta_2^d \qquad \text{and} \qquad \theta_3^k = -5\theta_2^d \,, \tag{3.11}$$

where θ_2^d and θ_3^d are the phases of the LO VEVs ϕ_2^d and ϕ_3^d, respectively.

Analogously, we obtain the matrix $\mathcal{K}_F$,[2]

$$\mathcal{K}_F - \mathbb{1} \approx \begin{pmatrix} 2K_1 & K_3 & K_3 \\ \cdot & K_2 - K_1 & K_3 \\ \cdot & \cdot & -(K_2 + K_1) \end{pmatrix} \lambda^4 \,, \tag{3.12}$$

where $K_i \in \mathbb{R}$. The parameters on the diagonal, K_1 and K_2, originate from different contractions of the term $F^\dagger \Phi_3^d \Phi_3^{d\dagger} F/M^2$. The off-diagonal elements, parametrised by K_3, are derived from the operator $F^\dagger \tilde{\Phi}_2^u F/M$ and are real due to $\tilde{\phi}_2^u \in \mathbb{R}$. Hence the LO correction of $\mathcal{K}_F$ from unity is given by a real matrix.

The corresponding Kähler metric $\mathcal{K}_N$ for the right-handed neutrinos is identical to $\mathcal{K}_F$ up to a difference in the order one coefficients of the individual corrections. We thus have

$$\mathcal{K}_N - \mathbb{1} \approx \begin{pmatrix} 2K_1^N & K_3^N & K_3^N \\ \cdot & K_2^N - K_1^N & K_3^N \\ \cdot & \cdot & -(K_2^N + K_1^N) \end{pmatrix} \lambda^4 \,, \tag{3.13}$$

where the coefficients K_i^N are again real.

3.2 Canonical normalisation

The expansion of the Kähler potentials in terms of flavon insertions leads to non-canonical kinetic terms. In order to bring the Kähler potential back to its canonical form, a non-unitary transformation has to be applied on the matter superfields. This procedure is known as canonical normalisation (CN) [31, 32], and introduces the 3×3 matrices P_A which transform the matter superfields $A = T, F, N$ as $A = P_A^{-1} A'$ so that

$$(P_A^\dagger)^{-1} \mathcal{K}_A P_A^{-1} = \mathbb{1} \quad \Longrightarrow \quad \mathcal{K}_A = P_A^\dagger P_A \,. \tag{3.14}$$

[2] There are also flavour universal λ^2 and λ^4 contributions to the diagonal elements of $\mathcal{K}_F$ which, however, do not effect our LO results.

JHEP02(2016)118

A prescription for deriving the matrices P_A can be found in appendix C.1. To LO, they take the simple form

$$P_T \approx \begin{pmatrix} 1 & \frac{k_2}{2}\lambda^4 & \frac{k_4}{2}e^{-i\theta_4^k}\lambda^6 \\ \cdot & 1 & \frac{k_3}{2}e^{-i\theta_3^k}\lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \qquad P_{F(N)} \approx \begin{pmatrix} 1 & \frac{K_3^{(N)}}{2}\lambda^4 & \frac{K_3^{(N)}}{2}\lambda^4 \\ \cdot & 1 & \frac{K_3^{(N)}}{2}\lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}. \tag{3.15}$$

In the following sections we study the structure of the Yukawa as well as the soft supersymmetry breaking sectors. The CN transformations of eq. (3.15) have to be applied to these before aiming at a physical interpretation of the resulting patterns.

4 Yukawa sector after CN

In this section, we study the fermionic sector of the model, completing the analysis of [39, 40] by including the effects of canonical normalisation. Our parametrisation differs slightly from the one used in [39, 40] as, in this work, we do not absorb any of the higher order corrections to the mass matrices or the flavon VEVs into the associated leading order terms. See appendix B for more details.

4.1 Charged fermions

4.1.1 Up-type quarks

The Yukawa matrix of the up-type quarks can be constructed by considering all the possible combinations of a product of flavons with TTH_5 for the upper-left 2×2 block, with TT_3H_5 for the $(i3)$ elements, and with $T_3T_3H_5$ for the (33) element. The operators which generate a contribution to the Yukawa matrix of order up to and including λ^8 are

$$\begin{aligned} &y_t T_3 T_3 H_5 + \frac{1}{M} y_1^u TT\Phi_2^u H_5 + \frac{1}{M^2} y_2^u TT\Phi_2^u\tilde{\Phi}_2^u H_5 \\ &+ \frac{1}{M^3} y_{3,4}^u T_3T_3(\Phi_3^d)^2\Phi_{2,3'}^\nu H_5 + \frac{1}{M^5} y_5^u TT(\Phi_2^d)^2(\Phi_3^d)^3 H_5 + \frac{1}{M^5} y_6^u TT_3(\Phi_2^d)^3(\Phi_3^d)^2 H_5\,, \end{aligned} \tag{4.1}$$

where the parameters y_t and y_i^u are real order one coefficients. Inserting the flavon VEVs and expanding the S_4 contractions of eq. (4.1) using the Clebsch-Gordan coefficients given for instance in [39], yields the up-type Yukawa matrix at the GUT scale

$$\mathcal{Y}_{\rm GUT}^u \approx \begin{pmatrix} y_u e^{i\theta_u^y}\lambda^8 & 0 & 0 \\ 0 & y_c e^{i\theta_c^y}\lambda^4 & z_2^u e^{i\theta_2^{zu}}\lambda^7 \\ 0 & z_2^u e^{i\theta_2^{zu}}\lambda^7 & y_t \end{pmatrix}, \tag{4.2}$$

where the relation to the flavon VEVs, cf. eqs. (2.2)–(2.4) as well as appendix B, is given by

$$y_u e^{i\theta_u^y} = y_2^u\phi_2^u\tilde{\phi}_2^u + y_1^u\delta_{2,1}^u, \qquad y_c\, e^{i\theta_c^y} = y_1^u\phi_2^u, \qquad z_2^u e^{i\theta_2^{zu}} = y_6^u(\phi_2^d)^3(\phi_3^d)^2\,. \tag{4.3}$$

Applying the phases of the LO flavon VEVs as given in eq. (B.2), we moreover have

$$\theta_u^y = \theta_c^y = 2\theta_2^d + 3\theta_3^d, \qquad \theta_2^{zu} = 3\theta_2^d + 2\theta_3^d, \tag{4.4}$$

where we have also used the fact that the shift $\delta^u_{2,1}$ of the flavon VEV $\langle\Phi^u_2\rangle$ in the first component is of order λ^8 and proportional to $(\phi^d_2)^2(\phi^d_3)^3$, cf. eq. (B.5). It is worth noting that the (12), (13) and (21), (31) elements of eq. (4.2) remain zero up to order λ^8.

Changing to the basis with canonical kinetic terms, we calculate $(P_T^{-1})^T\mathcal{Y}^u_{\rm GUT}P_T^{-1}$. For convenience we also apply an extra phase redefinition on the right-handed superfields,

$$Q_u = \mathrm{diag}(e^{i\theta^y_u}, e^{i\theta^y_u}, 1). \tag{4.5}$$

As a result we obtain the up-type quark Yukawa matrix in the canonical basis,

$$Y^u_{\rm GUT} \approx \begin{pmatrix} y_u\,\lambda^8 & -\frac{1}{2}k_2\,y_c\,\lambda^8 & -\frac{1}{2}k_4\,y_t e^{i\theta^k_4}\,\lambda^6 \\ -\frac{1}{2}k_2\,y_c\lambda^8 & y_c\,\lambda^4 & -\frac{1}{2}k_3\,y_t e^{i\theta^k_3}\lambda^5 \\ -\frac{1}{2}k_4\,y_t e^{i(\theta^k_4-\theta^y_u)}\,\lambda^6 & -\frac{1}{2}k_3\,y_t e^{i(\theta^k_3-\theta^y_u)}\lambda^5 & y_t \end{pmatrix}. \tag{4.6}$$

Compared to eq. (4.2), the canonical normalisation has significantly modified the off-diagonal entries: the texture zeros are filled in; moreover, the (23) and (32) elements feature a reduced λ-suppression.

4.1.2 Down-type quarks and charged leptons

The Yukawa matrices of the down-type quarks and the charged leptons can be deduced from the superpotential operators

$$\begin{aligned} & y^d_1\frac{1}{M}FT_3\Phi^d_3H_{\bar 5} + y^d_2\frac{1}{M^2}(F\tilde\Phi^d_3)_1(T\Phi^d_2)_1H_{\overline{45}} + y^d_5\frac{1}{M^3}(F(\Phi^d_2)^2)_3(T\tilde\Phi^d_3)_3H_{\bar 5} \\ & + y^d_3\frac{1}{M^2}FT_3\Phi^d_3\tilde\Phi^u_2H_{\bar 5} + y^d_4\frac{1}{M^2}FT_3\eta\tilde\Phi^d_3H_{\bar 5} + y^d_6\frac{1}{M^3}FT\Phi^d_2\tilde\Phi^d_3\tilde\Phi^u_2H_{\overline{45}} \\ & + y^d_7\frac{1}{M^5}FT(\Phi^d_2)^2(\Phi^d_3)^3H_{\overline{45}} + y^d_8\frac{1}{M^5}FT_3(\Phi^d_2)^3(\Phi^d_3)^2H_{\overline{45}} + y^d_9\frac{1}{M^6}FT_3(\Phi^d_2)^4(\Phi^d_3)^2H_{\bar 5}\,, \end{aligned} \tag{4.7}$$

where the y^d_i are real order one coefficients. For the operators proportional to y^d_2 and y^d_5, specific contractions have been chosen as described in [39, 40], such that the Gatto-Sartori-Tonin (GST) [49] and Georgi-Jarlskog (GJ) [50] relations are satisfied at LO. For all other operators we do not restrict the contractions to special choices; however, we have checked that in all cases, our LO result can simply be parameterised by an effective coupling constant which is given as a combination of the individual contributions from each contraction. It is worth noting that the operator proportional to y^d_4 is only allowed if η transforms as a trivial singlet under S_4. Separating the contributions of $H_{\bar 5}$ and $H_{\overline{45}}$, the S_4 contractions give rise to

$$\mathcal{Y}_{\bar 5} \approx \begin{pmatrix} 0 & \tilde x_2 e^{i\theta^{\tilde x}_2}\lambda^5 & -\tilde x_2 e^{i\theta^{\tilde x}_2}\lambda^5 \\ -\tilde x_2 e^{i\theta^{\tilde x}_2}\lambda^5 & 0 & \tilde x_2 e^{i\theta^{\tilde x}_2}\lambda^5 \\ z^d_3 e^{i\theta^{zd}_3}\lambda^6 & z^d_2 e^{i\theta^{zd}_2}\lambda^6 & y_b e^{i\theta^y_b}\lambda^2 \end{pmatrix}, \quad \mathcal{Y}_{\overline{45}} \approx \begin{pmatrix} z^d_1 e^{i\theta^{zd}_1}\lambda^8 & 0 & 0 \\ 0 & y_s e^{i\theta^y_s}\lambda^4 & -y_s e^{i\theta^y_s}\lambda^4 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4.8}$$

The parameters in these expressions are related to the flavon VEVs as defined in eqs. (2.2)–(2.4) and appendix B via

$$\begin{aligned} &\tilde x_2 e^{i\theta^{\tilde x}_2} = y^d_5(\phi^d_2)^2\tilde\phi^d_3\,, \quad y_b e^{i\theta^y_b} = y^d_1\phi^d_3\,, \quad z^d_2 e^{i\theta^{zd}_2} = y^d_1\delta^d_{3,3} + y^d_3\phi^d_3\tilde\phi^u_2\,, \quad z^d_3 e^{i\theta^{zd}_3} = y^d_1\delta^d_{3,1}\,, \\ &y_s e^{i\theta^y_s} = y^d_2\phi^d_2\tilde\phi^d_3\,, \quad z^d_1 e^{i\theta^{zd}_1} = y^d_7(\phi^d_2)^2(\phi^d_3)^3 - y^d_6\phi^d_2\tilde\phi^d_3\tilde\phi^u_2\,. \end{aligned} \tag{4.9}$$

Using eqs. (B.2), (B.6), we deduce the following relations for the phases

$$\theta_2^{\tilde{x}} = 3(\theta_2^d + \theta_3^d)\,, \qquad \theta_s^y = \theta_1^{z_d} = 2\theta_2^d + 3\theta_3^d\,, \qquad \theta_b^y = \theta_2^{z_d} = \theta_3^{z_d} = \theta_3^d\,. \tag{4.10}$$

The Yukawa matrices of the down-type quarks and the charged leptons are linear combinations of the two structures in eq. (4.8). Following the construction proposed by Georgi and Jarlskog, we have $\mathcal{Y}^d_{\rm GUT} = \mathcal{Y}_{\bar 5} + \mathcal{Y}_{\overline{45}}$ and $\mathcal{Y}^e_{\rm GUT} = (\mathcal{Y}_{\bar 5} - 3\mathcal{Y}_{\overline{45}})^T$, respectively.

Performing the canonical normalisation on the Yukawa matrices $(P_T^{-1})^T \mathcal{Y}^d_{\rm GUT} P_F^{-1}$ and $(P_F^{-1})^T \mathcal{Y}^e_{\rm GUT} P_T^{-1}$ as well as an additional rephasing of the right-handed superfields by

$$Q_d = Q_e = \mathrm{diag}(e^{i\theta_2^{\tilde{x}}}, e^{i\theta_2^{\tilde{x}}}, e^{i\theta_b^y}), \tag{4.11}$$

we end up with

$$Y^d_{\rm GUT} \approx \begin{pmatrix} e^{i(\theta_1^{z_d} - \theta_2^{\tilde{x}})} z_1^d \lambda^8 & \tilde{x}_2 \lambda^5 & -e^{i(\theta_2^{\tilde{x}} - \theta_b^y)} \tilde{x}_2 \lambda^5 \\ -\tilde{x}_2 \lambda^5 & e^{i(\theta_s^y - \theta_2^{\tilde{x}})} y_s \lambda^4 & -e^{i(\theta_s^y - \theta_b^y)} y_s \lambda^4 \\ e^{-i\theta_2^{\tilde{x}}} \left(z_3^d e^{i\theta_3^{z_d}} - \frac{K_3}{2} e^{i\theta_b^y} y_b \right) \lambda^6 & e^{-i\theta_2^{\tilde{x}}} \left(z_2^d e^{i\theta_2^{z_d}} - \frac{K_3}{2} e^{i\theta_b^y} y_b \right) \lambda^6 & y_b \lambda^2 \end{pmatrix}, \tag{4.12}$$

$$Y^e_{\rm GUT} \approx \begin{pmatrix} -3e^{i(\theta_1^{z_d} - \theta_2^{\tilde{x}})} y_d \lambda^8 & -\tilde{x}_2 \lambda^5 & e^{-i\theta_b^y} \left(z_3^d e^{i\theta_3^{z_d}} - \frac{K_3}{2} e^{i\theta_b^y} y_b \right) \lambda^6 \\ \tilde{x}_2 \lambda^5 & -3\, e^{i(\theta_s^y - \theta_2^{\tilde{x}})} y_s \lambda^4 & e^{-i\theta_b^y} \left(z_2^d e^{i\theta_2^{z_d}} - \frac{K_3}{2} e^{i\theta_b^y} y_b \right) \lambda^6 \\ -\tilde{x}_2 \lambda^5 & 3\, e^{i(\theta_s^y - \theta_2^{\tilde{x}})} y_s \lambda^4 & y_b \lambda^2 \end{pmatrix}. \tag{4.13}$$

We observe that the canonical normalisation modifies the down-type quark and charged lepton Yukawa matrices solely by additional contributions of the same order in the (31), (32) and (13), (23) elements, respectively. Comparing Eq, (4.12) with eq. (4.6) suggests that the CKM mixing is dominated by the diagonalisation of the down-type quark Yukawa matrix. We will explicitly verify this when calculating the SCKM transformations in section 6.

4.2 Neutrinos

4.2.1 Dirac neutrino coupling

Having introduced right-handed neutrinos N in table 1, their Dirac coupling to the left-handed SM neutrinos originates from the superpotential terms

$$\begin{aligned} &y_D F N H_5 + y_1^D \frac{1}{M} F N \tilde{\Phi}_2^u H_5 + y_2^D \frac{1}{M^2} F N (\tilde{\Phi}_2^u)^2 H_5 + y_{3,4,5}^D \frac{1}{M^3} F N (\Phi_3^d)^2 \Phi^\nu_{1,2,3'} H_5 \\ &+ y_6^D \frac{1}{M^5} F N (\Phi_2^d)^4 \Phi_3^d H_5, \end{aligned} \tag{4.14}$$

where y_D and y_i^D are real order one parameters. The corresponding Yukawa matrix is determined as

$$\mathcal{Y}^\nu \approx \begin{pmatrix} y_D & z_2^D e^{i\theta_2^{z_D}} \lambda^6 & z_1^D \lambda^4 \\ z_2^D e^{i\theta_2^{z_D}} \lambda^6 & z_1^D \lambda^4 & y_D \\ z_1^D \lambda^4 & y_D & z_2^D e^{i\theta_2^{z_D}} \lambda^6 \end{pmatrix}, \tag{4.15}$$

with

$$z_1^D = y_1^D \tilde{\phi}_2^u\,, \qquad z_2^D e^{i\theta_2^{z_D}} = y_1^D \tilde{\delta}_{2,1}^u\,, \qquad \theta_2^{z_D} = 4\theta_2^d + \theta_3^d\,. \tag{4.16}$$

Here, the phase can be deduced from eq. (B.5).

Applying the CN transformation $(P_F^{-1})^T \mathcal{Y}^\nu P_N^{-1}$, the corresponding Yukawa matrix in the basis with canonical kinetic terms takes the form

$$Y^\nu \approx \begin{pmatrix} y_D & -\frac{y_D(K_3+K_3^N)}{2}\lambda^4 & \left(z_1^D - \frac{y_D(K_3+K_3^N)}{2}\right)\lambda^4 \\ -\frac{y_D(K_3+K_3^N)}{2}\lambda^4 & \left(z_1^D - \frac{y_D(K_3+K_3^N)}{2}\right)\lambda^4 & y_D \\ \left(z_1^D - \frac{y_D(K_3+K_3^N)}{2}\right)\lambda^4 & y_D & -\frac{y_D(K_3+K_3^N)}{2}\lambda^4 \end{pmatrix}. \tag{4.17}$$

Compared to eq. (4.15), an additional contribution of the same order arises in the (13), (22) and (31) entries. Moreover, the λ-suppression of the (12), (21) and (33) elements is reduced.

4.2.2 Majorana neutrino mass

The mass matrix of the right-handed neutrinos is obtained from the superpotential terms

$$w_{1,2,3} NN\Phi^\nu_{1,2,3'} + w_4 \frac{1}{M} NN\Phi^d_2\eta + w_{5,6,7}\frac{1}{M} NN\tilde{\Phi}^u_2\Phi^\nu_{1,2,3'} + w_8\frac{1}{M^7} NN(\Phi^d_2)^8\,, \tag{4.18}$$

where w_i denote real order one coefficients. This results in a right-handed Majorana neutrino mass matrix $\mathcal{M}_R$ of the form

$$\frac{\mathcal{M}_R}{M} \approx \begin{pmatrix} A+2C & B-C & B-C \\ B-C & B+2C & A-C \\ B-C & A-C & B+2C \end{pmatrix} e^{i\theta_A}\lambda^4 + \begin{pmatrix} 0 & 0 & D \\ 0 & D & 0 \\ D & 0 & 0 \end{pmatrix} e^{i\theta_D}\lambda^5\,, \tag{4.19}$$

with

$$Ae^{i\theta_A} = w_1\phi^\nu_1\,, \quad Be^{i\theta_A} = w_2\phi^\nu_2\,, \quad Ce^{i\theta_A} = w_3\phi^\nu_{3'}\,, \quad De^{i\theta_D} = w_2(\delta^\nu_{2,1}-\delta^\nu_{2,2}) + w_4\,\eta\,\phi^d_2\,. \tag{4.20}$$

According to eqs. (B.2), (B.5), (B.6), the phases are given by

$$\theta_A = -2\theta^d_3\,, \qquad \theta_D = 4\theta^d_2 - \theta^d_3\,. \tag{4.21}$$

The first matrix of eq. (4.19) arises from terms involving only $\Phi^\nu_{1,2,3'}$. As their VEVs respect the tri-bimaximal (TB) Klein symmetry $Z_2^S \times Z_2^U \subset S_4$, this part is of TB form. The second matrix of eq. (4.19), proportional to D, is due to the operator $w_4\frac{1}{M}NN\Phi^d_2\eta$. As the product of both flavon VEVs involved is not an eigenvector of U, half of the TB Klein symmetry is broken at a relative order of λ. The resulting trimaximal TM_2 [51–61] structure can accommodate the sizable value of the reactor neutrino mixing angle θ^l_{13} as explained in [40] in the context of the original model [39].

Performing the CN basis transformation $(P_N^{-1})^T\mathcal{M}_R P_N^{-1}$ does not alter the matrix in eq. (4.19) at the given order, so that $M_R = \mathcal{M}_R + \mathcal{O}(\lambda^6)M$.

4.2.3 Effective light neutrino mass matrix

Calculating the effective light neutrino mass matrix which arises via the type I seesaw mechanism $v_u^2\, Y^\nu M_R^{-1}(Y^\nu)^T$, we can parameterise the LO result as

$$m_\nu^{\mathrm{eff}} \approx \frac{y_D^2 v_u^2}{\lambda^4 M}\left[\begin{pmatrix} b^\nu+c^\nu-a^\nu & a^\nu & a^\nu \\ a^\nu & b^\nu & c^\nu \\ a^\nu & c^\nu & b^\nu \end{pmatrix} e^{-i\theta_A} + \begin{pmatrix} 0 & 0 & d^\nu \\ 0 & d^\nu & 0 \\ d^\nu & 0 & 0 \end{pmatrix}\lambda\, e^{i(\theta_D-2\theta_A)}\right], \tag{4.22}$$

with a^ν, b^ν, c^ν and d^ν being functions of the real parameters A, B, C and D. The deviation from tri-bimaximal neutrino mixing is controlled by $d^\nu \propto D$. Due to the three independent LO input parameters ($w_1 \propto A$,$w_2 \propto B$,$w_3 \propto C$), any neutrino mass spectrum can be accommodated in this model. At this order, the canonical normalisation does not modify the effective light neutrino mass matrix as obtained without the CN transformations. Hence, concerning the results on light neutrino masses and mixing, we can simply refer the reader to the corresponding discussion in [40].

5 Soft SUSY breaking sector after CN

Having applied the CN basis transformation of the matter superfields to the Yukawa sector, we now turn to the soft SUSY breaking terms. In the context of the general MSSM with R-parity, these are parameterised as [17]

$$\begin{aligned}-\mathcal{L}_{\rm soft} \supset\ & H_u\tilde{Q}_iA^u_{ij}\tilde{u}^c_j + H_d\tilde{Q}_iA^d_{ij}\tilde{d}^c_j + H_d\tilde{L}_iA^e_{ij}\tilde{e}^c_j + H_u\tilde{L}_iA^\nu_{ij}\tilde{N}_j + \text{h.c.}\\ &+\tilde{Q}^\alpha_i m^2_{Q_{ij}}\tilde{Q}^{\alpha*}_j + \tilde{L}^\alpha_i m^2_{L_{ij}}\tilde{L}^{\alpha*}_j + \tilde{u}^{c*}_i m^2_{u^c_{ij}}\tilde{u}^c_j + \tilde{d}^{c*}_i m^2_{d^c_{ij}}\tilde{d}^c_j + \tilde{e}^{c*}_i m^2_{e^c_{ij}}\tilde{e}^c_j + \tilde{N}^*_i m^2_{N_{ij}}\tilde{N}_j\\ &+m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2\,,\end{aligned} \tag{5.1}$$

and contain trilinear scalar couplings (A-terms) as well as bilinear scalar masses. A tilde indicates the scalar partner $\tilde{f}$ of a SM fermion f. Taking into account the SU(5) framework, we construct the effective soft SUSY breaking operators in this section, assuming that the mechanism of SUSY breaking is practically independent of the family symmetry breaking.

5.1 Trilinear soft couplings

The flavour structure of the trilinear A-terms is similar to the corresponding Yukawa matrices, as both originate from the same set of superpotential terms. In the case of the soft terms, these are coupled to a hidden sector superfield X with independent real order one coupling constants and suppressed by a mass scale M_X. When X develops its SUSY breaking F-term VEV, the scalar components of the Higgs and matter superfields are projected out, thereby generating the trilinear soft terms. There exist in fact extra contributions to the A-terms from superpotential operators involving flavons but no X field. These can be traced back to non-vanishing VEVs for the auxiliary F-components of the flavon fields, which are zero in the SUSY limit but develop a non-trivial value when SUSY breaking terms are included. It turns out that such F-term VEVs are aligned with the LO flavon VEVs in many situations [24, 62]. Hence, these extra contributions to the A-terms do not give rise to new flavour structures.

Defining the mass parameters $m_0 \equiv \langle F_X\rangle/M_X$ and $A_0 \equiv \alpha_0\, m_0$, with α_0 being a real constant, we can obtain the expressions for the trilinear matrices $\mathcal{A}^f_{\rm GUT}/A_0$ by copying the Yukawas matrices of eqs. (4.2), (4.8), (4.15) with different order-one coefficients and phases: $y_f \to a_f$, $\tilde{x}_2 \to \tilde{x}^a_2$, $z^f_i \to z^{f_a}_i$, $y_D \to \alpha_D$ as well as $\theta^y_f \to \theta^a_f$, $\theta^{\tilde{x}}_2 \to \theta^{\tilde{x}_a}_2$, $\theta^{z_f}_i \to \theta^{z_{fa}}_i$. With these replacements, we find

$$\frac{\mathcal{A}^u_{\rm GUT}}{A_0} \approx \begin{pmatrix} a_u\, e^{i\theta^a_u}\lambda^8 & 0 & 0\\ 0 & a_c\, e^{i\theta^a_c}\lambda^4 & z^{u_a}_2\, e^{i\theta^{z_{ua}}_2}\lambda^7\\ 0 & z^{u_a}_2\, e^{i\theta^{z_{ua}}_2}\lambda^7 & a_t \end{pmatrix}, \tag{5.2}$$

and similarly for $\mathcal{A}^d_{\rm GUT}$, $\mathcal{A}^e_{\rm GUT}$ and $\mathcal{A}^\nu$. Applying the CN transformation as well as the rephasing of the right-handed superfields proceeds analogously to the Yukawa sector. The resulting trilinear matrices $A^f_{\rm GUT}/A_0$ in the basis of canonical kinetic terms are thus derived from eqs. (4.6), (4.12), (4.13), (4.17) by simply replacing $y_u \to a_u\, e^{i(\theta^a_u-\theta^y_u)}$, $y_c \to a_c\, e^{i(\theta^a_c-\theta^y_u)}$, $y_t \to a_t$, $y_s \to a_s\, e^{i(\theta^a_s-\theta^y_s)}$, $y_b \to a_b\, e^{i(\theta^a_b-\theta^y_b)}$, $\tilde{x}_2 \to \tilde{x}^a_2\, e^{i(\theta^{\tilde{x}a}_2-\theta^{\tilde{x}}_2)}$, $z^f_i \to z^{fa}_i\, e^{i(\theta^{zfa}_i-\theta^{zf}_i)}$ and $y_D \to \alpha_D$. For example, the up-type quark trilinear matrix takes the form

$$\frac{A^u_{\rm GUT}}{A_0} \approx \begin{pmatrix} a_u\, e^{i(\theta^a_u-\theta^y_u)}\,\lambda^8 & -\frac{1}{2}k_2\, a_c\, e^{i(\theta^a_c-\theta^y_u)}\,\lambda^8 & -\frac{1}{2}k_4\, a_t e^{i\theta^k_4}\,\lambda^6 \\ -\frac{1}{2}k_2\, a_c\, e^{i(\theta^a_c-\theta^y_u)}\lambda^8 & a_c\, e^{i(\theta^a_c-\theta^y_u)}\,\lambda^4 & -\frac{1}{2}k_3\, a_t e^{i\theta^k_3}\,\lambda^5 \\ -\frac{1}{2}k_4\, a_t e^{i(\theta^k_4-\theta^y_u)}\,\lambda^6 & -\frac{1}{2}k_3\, a_t e^{i(\theta^k_3-\theta^y_u)}\,\lambda^5 & a_t \end{pmatrix}. \tag{5.3}$$

5.2 Soft scalar masses

The scalar mass terms of the soft supersymmetry breaking Lagrangian originate from the Kähler potential. Non-renormalisable couplings of the matter superfields to the square $X^\dagger X/M_X^2$ of the SUSY breaking field X generate soft masses when the F-term of X develops a VEV. The structure of the soft mass matrices is therefore similar to the Kähler metric $\mathcal{K}$ of the corresponding GUT multiplet. As for the trilinear soft terms, all order one coefficients are independent of those appearing in $\mathcal{K}$. The scalar masses before canonical normalisation are then obtained from $\mathcal{K}_T$, $\mathcal{K}_F$ and $\mathcal{K}_N$ of eqs. (3.10), (3.12), (3.13) by replacing $k_i \to b_i$, $\theta^k_i \to \theta^b_i$, $K_i \to B_i$ and $K^N_i \to B^N_i$. Moreover, the ones on the diagonal of $\mathcal{K}$ have to be rescaled by a new factor of order one. In the case of the **10** of SU(5), the 2+1 structure requires the introduction of two extra parameters, b_{01} and b_{02}. Explicitly, we get

$$\frac{\mathcal{M}^2_{T_{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01}+(b_5+b_1)\lambda^2 & b_2\lambda^4 & b_4\, e^{-i\theta^k_4}\,\lambda^6 \\ \cdot & b_{01}+(b_5-b_1)\lambda^2 & b_3\, e^{-i\theta^k_3}\,\lambda^5 \\ \cdot & \cdot & b_{02}+b_6\lambda^2 \end{pmatrix}, \tag{5.4}$$

$$\frac{\mathcal{M}^2_{F(N)_{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} B_0^{(N)}+2B_1^{(N)}\lambda^4 & B_3^{(N)}\lambda^4 & B_3^{(N)}\lambda^4 \\ \cdot & B_0^{(N)}+(B_2^{(N)}-B_1^{(N)})\lambda^4 & B_3^{(N)}\lambda^4 \\ \cdot & \cdot & B_0^{(N)}-(B_2^{(N)}+B_1^{(N)})\lambda^4 \end{pmatrix}. \tag{5.5}$$

Performing the transformations to the basis of canonical kinetic terms results in soft scalar mass matrices of the form

$$\frac{M^2_{T_{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01} & (b_2-b_{01}k_2)\lambda^4 & e^{-i\theta^k_4}(b_4-\frac{k_4(b_{01}+b_{02})}{2})\lambda^6 \\ \cdot & b_{01} & e^{-i\theta^k_3}(b_3-\frac{k_3(b_{01}+b_{02})}{2})\lambda^5 \\ \cdot & \cdot & b_{02} \end{pmatrix}, \tag{5.6}$$

$$\frac{M^2_{F(N)_{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} B_0^{(N)} & (B_3^{(N)}-K_3^{(N)})\lambda^4 & (B_3^{(N)}-K_3^{(N)})\lambda^4 \\ \cdot & B_0^{(N)} & (B_3^{(N)}-K_3^{(N)})\lambda^4 \\ \cdot & \cdot & B_0^{(N)} \end{pmatrix}. \tag{5.7}$$

For convenience, we will absorb the order one parameter B_0 into the soft SUSY breaking mass m_0, so that the leading contribution on the diagonal of $M^2_{F_{\rm GUT}}/m_0^2$ is nothing but unity. For the right-handed fields contained in the GUT multiplets, an additional rephasing has to be applied. We will come back to this when calculating the soft terms in the SCKM basis in section 6.2. Notice that we have dropped all λ-suppressed corrections of the diagonal elements. This simplification is justified as FCNC processes are induced by loop diagrams involving the off-diagonal entries of the sfermion mass matrices. The simplification of the diagonal elements in eqs. (5.6), (5.7) does not affect these off-diagonals in our LO analysis, even when going to the SCKM basis.

6 SCKM basis

Predictions relating a theoretical model with its phenomenological implications are typically given in the basis in which the Yukawa matrices are diagonal and positive, corresponding to the physical quark and lepton mass eigenstates. The so-called SCKM basis is the analogue in a supersymmetric framework. Changing to the SCKM basis, all canonically normalised quantities undergo a unitary transformation of the superfields which diagonalises the effective Yukawa couplings in the superpotential. In this basis it is convenient to define a set of dimensionless parameters, known as the "mass insertion parameters", which directly enter the expressions of phenomenological flavour observables.

In principle, the SCKM transformation should be performed after electroweak symmetry breaking. The canonically normalised Yukawa, trilinear and soft mass matrices should be evolved from the GUT scale M_{GUT} to the weak scale M_W using the corresponding renormalisation group equations (RGEs). Only at that point, the diagonalisation of the Yukawa matrices should take place, leading to the definition of a SCKM basis. Following this procedure, there is obviously no notion of mass insertion parameters at the scale M_{GUT} as there is no proper definition of the SCKM basis.

An alternative approach which is commonly used consists in diagonalising the Yukawa matrices at (or rather just below) the GUT scale. The so-obtained basis is approximately identical to the SCKM basis provided the RGE contributions to the off-diagonal elements of the Yukawa matrices remain negligible.[3] This is the case as long as the RGE effects can be absorbed into a redefinition of the (unknown) order one coefficients. It is then possible to introduce mass insertion parameters already at M_{GUT}. Their low energy values have to be determined from the corresponding RG evolution. In this work, we will adopt the latter approach as it allows for a semi-analytical study of the relations between the high and low energy parameters by means of a perturbative λ-expansion.

6.1 SCKM transformations

The SCKM transformations are applied on the matter superfields $\hat{f}_{L,R} \to U^f_{L,R}\hat{f}_{L,R}$, where $U^f_{L,R}$ denote unitary 3×3 matrices. These diagonalise the canonically normalised Yukawa matrices Y^f

$$(U^f_L)^\dagger Y^f U^f_R \;=\; \tilde{Y}^f_{\mathrm{diag}}\,, \tag{6.1}$$

[3]For the charged fermion sector, this is a valid approximation thanks to the hierarchical masses of quarks and charged leptons. In the neutrino sector, RGE contributions can be sizable in supersymmetric models with large t_β and a quasi-degenerate neutrino mass spectrum [63, 64]. They are however negligible for small t_β [which is realised in our scenario due to the suppression of the bottom Yukawa coupling by two powers of λ, see eq. (4.12)] and a normal neutrino mass hierarchy [which we assume in the following].

JHEP02(2016)118

where we use the tilde to denote the SCKM basis. The derivation and the explicit form of the unitary transformations can be found in appendix C.2. Applying this change of basis to the Yukawa matrices yields

$$\tilde{Y}^u_{\rm GUT} \approx \begin{pmatrix} y_u\lambda^8 & 0 & 0 \\ 0 & y_c\lambda^4 & 0 \\ 0 & 0 & y_t \end{pmatrix}, \qquad \tilde{Y}^d_{\rm GUT} \approx \begin{pmatrix} \frac{\tilde{x}_2^2}{y_s}\lambda^6 & 0 & 0 \\ 0 & y_s\lambda^4 & 0 \\ 0 & 0 & y_b\lambda^2 \end{pmatrix}, \tag{6.2}$$

$$\tilde{Y}^e_{\rm GUT} \approx \begin{pmatrix} \frac{\tilde{x}_2^2}{3y_s}\lambda^6 & 0 & 0 \\ 0 & 3y_s\lambda^4 & 0 \\ 0 & 0 & y_b\lambda^2 \end{pmatrix}. \tag{6.3}$$

These results, which are valid at the high scale, agree with the LO results derived in [39, 40]. This shows that the canonical normalisation does not affect the LO expressions of the quark and charged lepton masses.

Up to phase convention, the CKM matrix is given by $V_{\rm CKM_{GUT}} = (U^u_L)^T U^{d*}_L$ (see appendix C.2 for explicit expressions). Extracting the mixing angles

$$\sin(\theta^q_{13})_{\rm GUT} \approx \frac{\tilde{x}_2}{y_b}\lambda^3\,, \qquad \tan(\theta^q_{23})_{\rm GUT} \approx \frac{y_s}{y_b}\lambda^2\,, \qquad \tan(\theta^q_{12})_{\rm GUT} \approx \frac{\tilde{x}_2}{y_s}\lambda\,, \tag{6.4}$$

shows that the LO CKM mixing arises purely from the down-type quark sector, incorporating the GST relation [49] $\theta^q_{12} \approx \sqrt{m_d/m_s}$, and agrees with the results obtained in [39, 40]. Concerning the CP violation, we find the Jarlskog invariant [65] to be

$$J^q_{\rm CP_{GUT}} \approx \lambda^7 \frac{\tilde{x}_2^3}{y_b^2 y_s} \sin(\theta^d_2)\,. \tag{6.5}$$

The PMNS matrix is dominated by the trimaximal TM_2 neutrino mixing V_ν which diagonalises the effective light neutrino mass matrix of eq. (4.22). Including the charged lepton corrections, we have $U_{\rm PMNS_{GUT}} = (U^e_L)^T V^*_\nu$ with mixing angles given as

$$\tan(\theta^l_{23})_{\rm GUT} \approx 1 + \lambda\,\frac{d^\nu}{2(a^\nu - c^\nu)}\cos(4\theta^d_2 + \theta^d_3)\,, \tag{6.6}$$

$$\tan(\theta^l_{12})_{\rm GUT} \approx \frac{1}{\sqrt{2}} - \lambda\frac{\tilde{x}_2}{2\sqrt{2}y_s}\cos(\theta^d_2)\,, \tag{6.7}$$

$$\begin{aligned} \sin(\theta^l_{13})_{\rm GUT} \approx \frac{\lambda}{6\sqrt{2}y_s}\Bigg[&\left(\frac{3d^\nu y_s\cos(4\theta^d_2+\theta^d_3) + 2(a^\nu - c^\nu)\tilde{x}_2\cos(\theta^d_2)}{a^\nu - c^\nu}\right)^2 \\ &+ \left(\frac{3d^\nu y_s\sin(4\theta^d_2+\theta^d_3) + 2(a^\nu - b^\nu)\tilde{x}_2\sin(\theta^d_2)}{a^\nu - b^\nu}\right)^2\Bigg]^{\frac{1}{2}}, \end{aligned} \tag{6.8}$$

and a leptonic Jarlskog invariant of the form

$$J^l_{\rm CP_{GUT}} \approx -\frac{\lambda}{36}\left(\frac{2\tilde{x}_2}{y_s}\sin(\theta^d_2) + \frac{3d^\nu}{a^\nu - b^\nu}\sin(4\theta^d_2 + \theta^d_3)\right)\,.$$

6.2 Soft terms in the SCKM basis

In order to obtain the flavour structure of the soft SUSY breaking terms in a basis which is suitable for physical interpretations, we have to apply the SCKM transformations on the canonical trilinear soft couplings and soft scalar masses, cf. section 5. The action of the $U^f_{L,R}$ matrices on the A-terms is identical to the transformation of the Yukawa matrices:

$$(U_L^f)^\dagger A^f_{\rm GUT} U_R^f \;=\; \tilde{A}^f_{\rm GUT}. \tag{6.9}$$

However, due to different order one coefficients, the A-terms remain non-diagonal in the SCKM basis. The soft masses of eqs. (5.6), (5.7) are transformed differently for different components of the SU(5) multiplets. Moreover, we have to associate the mass matrices of the effective soft Lagrangian in eq. (5.1) with $M^2_{T_{\rm GUT}}$ and $M^2_{F_{\rm GUT}}$ and take into account the additional rephasing transformations of the right-handed superfields, see eqs. (4.5), (4.11), that were performed after CN. Then, the soft masses in the SCKM basis are

$$\begin{aligned}
(\tilde{m}^2_u)_{LL{\rm GUT}} &= (U_L^u)^\dagger M^{2\,*}_{T_{\rm GUT}} U_L^u, & (\tilde{m}^2_u)_{RR{\rm GUT}} &= (U_R^u)^\dagger Q_u\, M^2_{T_{\rm GUT}} Q_u^\dagger\, U_R^u, & (6.10)\\
(\tilde{m}^2_d)_{LL{\rm GUT}} &= (U_L^d)^\dagger M^{2\,*}_{T_{\rm GUT}} U_L^d, & (\tilde{m}^2_d)_{RR{\rm GUT}} &= (U_R^d)^\dagger Q_d\, M^2_{F_{\rm GUT}} Q_d^\dagger\, U_R^d, & (6.11)\\
(\tilde{m}^2_e)_{LL{\rm GUT}} &= (U_L^e)^\dagger M^{2\,*}_{F_{\rm GUT}} U_L^e, & (\tilde{m}^2_e)_{RR{\rm GUT}} &= (U_R^e)^\dagger Q_d\, M^2_{T_{\rm GUT}} Q_d^\dagger\, U_R^e. & (6.12)
\end{aligned}$$

We find the following leading order expressions, where the order one coefficients are defined in eqs. (D.4), (D.5). Note that we have absorbed the order one coefficient B_0 into m_0, cf. eq. (5.7), so that $(\tilde{m}^2_d)_{RR{\rm GUT}}/m_0^2$ and $(\tilde{m}^2_e)_{LL{\rm GUT}}/m_0^2$ have 1s on the diagonal.

Up-type quark sector.

$$\frac{\tilde{A}^u_{\rm GUT}}{A_0} \approx \begin{pmatrix} \tilde{a}^u_{11}\lambda^8 & 0 & 0 \\ 0 & \tilde{a}^u_{22}\lambda^4 & e^{i\theta_2^d}\tilde{a}^u_{23}\lambda^7 \\ 0 & e^{i(3\theta_2^d+\theta_3^d)}\tilde{a}^u_{23}\lambda^7 & \tilde{a}^u_{33} \end{pmatrix}, \tag{6.13}$$

$$\frac{(\tilde{m}^2_u)_{LL{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01} & e^{-i\theta_2^d}\tilde{b}_{12}\,\lambda^4 & e^{-i(4\theta_2^d+\theta_3^d)}\tilde{b}_{13}\,\lambda^6 \\ \cdot & b_{01} & e^{-i(7\theta_2^d+2\theta_3^d)}\tilde{b}_{23}\,\lambda^5 \\ \cdot & \cdot & b_{02} \end{pmatrix}, \tag{6.14}$$

$$\frac{(\tilde{m}^2_u)_{RR{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01} & e^{-i\theta_2^d}\tilde{b}_{12}\,\lambda^4 & \tilde{b}_{13}\,\lambda^6 \\ \cdot & b_{01} & e^{i(5\theta_2^d+\theta_3^d)}\tilde{b}_{23}\,\lambda^5 \\ \cdot & \cdot & b_{02} \end{pmatrix}. \tag{6.15}$$

Down-type quark sector.

$$\frac{\tilde{A}^d_{\rm GUT}}{A_0} \approx \begin{pmatrix} \tilde{a}^d_{11}\,\lambda^6 & \tilde{a}^d_{12}\,\lambda^5 & \tilde{a}^d_{12}\,\lambda^5 \\ -\tilde{a}^d_{12}\,\lambda^5 & \tilde{a}^d_{22}\,\lambda^4 & \tilde{a}^d_{23}\,\lambda^4 \\ e^{-i\theta_2^d}\tilde{a}^d_{31}\,\lambda^6 & \tilde{a}^d_{32}\,\lambda^6 & \tilde{a}^d_{33}\,\lambda^2 \end{pmatrix}, \tag{6.16}$$

$$\frac{(\tilde{m}^2_d)_{LL{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01} & \tilde{B}_{12}\,\lambda^3 & e^{i\theta_2^d}\tilde{B}_{13}\,\lambda^4 \\ \cdot & b_{01} & \tilde{B}_{23}\,\lambda^2 \\ \cdot & \cdot & b_{02} \end{pmatrix}, \tag{6.17}$$

$$\frac{(\tilde{m}^2_d)_{RR{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} 1 & e^{i\theta_2^d}\tilde{R}_{12}\,\lambda^4 & -\,e^{i\theta_2^d}\tilde{R}_{12}\,\lambda^4 \\ \cdot & 1 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}. \tag{6.18}$$

Charged lepton sector.

$$\frac{\tilde{A}^e_{\rm GUT}}{A_0} \approx \begin{pmatrix} \frac{1}{3}\tilde{a}^d_{11}\,\lambda^6 & e^{i\theta^d_2}\tilde{a}^d_{12}\,\lambda^5 & \tilde{a}^d_{31}\,\lambda^6 \\ -e^{-i\theta^d_2}\tilde{a}^d_{12}\,\lambda^5 & 3\tilde{a}^d_{22}\,\lambda^4 & \tilde{a}^e_{23}\,\lambda^6 \\ -e^{-i\theta^d_2}\tilde{a}^d_{12}\,\lambda^5 & 3\tilde{a}^d_{23}\,\lambda^4 & \tilde{a}^d_{33}\,\lambda^2 \end{pmatrix}, \tag{6.19}$$

$$\frac{(\tilde{m}^2_e)_{LL{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} 1 & \tilde{R}_{12}\,\lambda^4 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & 1 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{6.20}$$

$$\frac{(\tilde{m}^2_e)_{RR{\rm GUT}}}{m_0^2} \approx \begin{pmatrix} b_{01} & -\,e^{i\theta^d_2}\frac{1}{3}\tilde{B}_{12}\,\lambda^3 & \frac{1}{3}\tilde{B}_{13}\,\lambda^4 \\ \cdot & b_{01} & 3\tilde{B}_{23}\,\lambda^2 \\ \cdot & \cdot & b_{02} \end{pmatrix}. \tag{6.21}$$

7 Mass insertion parameters

In supersymmetry, flavour changing processes are induced by the mismatch of fermion and sfermion mass eigenstates. Having changed the basis of the superfields to the SCKM basis, the Yukawa matrices are diagonal. Thus, the off-diagonal entries of the scalar mass matrices determine the size of the resulting FCNCs. As both the left- and the right-handed fermions have their own scalar partners, there are three types of scalar mass matrices

$$m^2_{\tilde{f}_{LL}} = (\tilde{m}^2_f)_{LL} + \tilde{Y}_f\tilde{Y}_f^\dagger v^2_{u,d}\,, \qquad m^2_{\tilde{f}_{RR}} = (\tilde{m}^2_f)_{RR} + \tilde{Y}_f^\dagger\tilde{Y}_f v^2_{u,d}\,, \qquad m^2_{\tilde{f}_{LR}} = \tilde{A}_f v_{u,d} - \mu\tilde{Y}_f v_{d,u}\,, \tag{7.1}$$

where μ is the higgsino mass which we take to be real. In eq. (7.1), the first contribution on the right-hand sides originates from the soft breaking Lagrangian, while the second term is the supersymmetric F-term contribution to the scalar masses. We note that it is formally possible to define $m^2_{\tilde{f}_{RL}} \equiv (m^2_{\tilde{f}_{LR}})^\dagger$.

From the model building perspective, a convenient measure of flavour violation is provided by a set of dimensionless parameters, known as the mass insertion parameters. These are defined as [66, 67]

$$(\delta^f_{LL})_{ij} = \frac{(m^2_{\tilde{f}_{LL}})_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LL}}\,, \qquad (\delta^f_{RR})_{ij} = \frac{(m^2_{\tilde{f}_{RR}})_{ij}}{\langle m_{\tilde{f}}\rangle^2_{RR}}\,, \qquad (\delta^f_{LR})_{ij} = \frac{(m^2_{\tilde{f}_{LR}})_{ij}}{\langle m_{\tilde{f}}\rangle^2_{LR}}\,, \tag{7.2}$$

where the average masses in the denominators are

$$\langle m_{\tilde{f}}\rangle^2_{AB} = \sqrt{(m^2_{\tilde{f}_{AA}})_{ii}(m^2_{\tilde{f}_{BB}})_{jj}}\,. \tag{7.3}$$

7.1 Mass insertion parameters δ at the GUT scale

Inserting the results of section 6, it is straightforward to calculate the mass insertion parameters at the GUT scale. The full LO expressions are given in appendix D. In the following we only report the flavour structure of the various δs in terms of their λ-suppression.

$$\delta^u_{LL_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{RR_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{LR_{\rm GUT}} \sim \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & \lambda^7 \\ 0 & \lambda^7 & 1 \end{pmatrix}, \tag{7.4}$$

$$\delta^d_{LL_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{RR_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{LR_{\rm GUT}} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda^2 \end{pmatrix}, \tag{7.5}$$

$$\delta^e_{LL_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{RR_{\rm GUT}} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{LR_{\rm GUT}} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}. \tag{7.6}$$

7.2 Effects of RG running

Having calculated the GUT scale mass insertion parameters, it is now necessary to consider their evolution down to the electroweak scale. Only then are we able to compare the predictions of the model to experimental measurements of flavour observables. This evolution is described by the RG equations which are given explicitly in appendix E in the SCKM basis. Technically, we perform the RG running in two stages, first from $M_{\rm GUT}$ to M_R where the right-handed neutrinos are integrated out, and then from M_R to $M_{\rm SUSY} \sim M_W$. In order to derive analytical results, we estimate the effects of the running using the leading logarithmic approximation. As the Yukawa matrices themselves are also affected by the running, it is necessary to apply further basis transformations on the superfields which diagonalise the low energy Yukawas matrices.

Details of the various steps involved in calculating the low energy mass insertion parameters can be found in appendix F. For the down-type squarks and the charged sleptons, the resulting effects can simply be absorbed into new order one coefficients. It is interesting to see that this is not the case for the up-type squarks, where the order of the (13) and (23) elements of δ^u_{LR} gets modified. For completeness, we present the flavour structure of the low energy δs in terms of their λ-suppression, which should be compared to eqs. (7.4)–(7.6).

$$\delta^u_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \cdot & 1 & \lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^u_{LR} \sim \begin{pmatrix} \lambda^8 & 0 & \lambda^7 \\ 0 & \lambda^4 & \lambda^6 \\ 0 & \lambda^7 & 1 \end{pmatrix}, \tag{7.7}$$

$$\delta^d_{LL} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{RR} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^6 & \lambda^2 \end{pmatrix}, \tag{7.8}$$

$$\delta^e_{LL} \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^4 \\ \cdot & 1 & \lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{RR} \sim \begin{pmatrix} 1 & \lambda^3 & \lambda^4 \\ \cdot & 1 & \lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{LR} \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^6 \\ \lambda^5 & \lambda^4 & \lambda^2 \end{pmatrix}. \tag{7.9}$$

8 Conclusion

Despite its tremendous success, the Standard Model of particle physics is widely viewed as the low energy limit of a more fundamental theory. In order to understand the nature of flavour in such extensions of the SM it is necessary to answer the following three questions.

1. Why are there three families of quarks and leptons?

2. How does the structure of fermion masses and mixing arise?

3. Why is the amount of flavour violation induced by new physics so small?

From the phenomenological point of view, the third question is usually addressed by means of *ad hoc* assumptions such as e.g. Minimal Flavour Violation, where all sources of flavour violation are intimately linked to the flavour structure of the Yukawa matrices. However, the concept of MFV is not a theory of flavour as such. Moreover, it does not seem to provide a framework in which the first two questions of the flavour puzzle can be addressed in a satisfactory way.

In this paper, we have investigated the issue of flavour violation within a supersymmetric GUT model of flavour which is based on the simple family symmetry $S_4 \times \mathrm{U}(1)$ [40]. The existence of three families of quarks and leptons is related to the non-Abelian factor of the family symmetry whose triplets are the only faithful irreducible representations. The structure of the Yukawa matrices arises from the breaking of the family symmetry. This aspect was thoroughly studied in [39, 40] where it was shown to provide a good description of all quark and lepton masses, mixings and CP violation.

Applying the family symmetry on the soft SUSY breaking sector, we have worked out the mass insertion parameters which describe the sources of flavour violation beyond the SM. Our calculation relies on the assumption that the SUSY breaking mechanism respects the family symmetry. Working in an expansion in powers of the Wolfenstein parameter λ, we take into account the effect of canonical normalisation as well as renormalisation group running. Our results for the low energy mass insertion parameters are summarised in eqs. (7.7)–(7.9), with the explicit expressions given in appendix F.3. We find that δ^f_{LL} and δ^f_{RR} are approximately equal to the identity with only small off-diagonal entries. Considering the parameters δ^f_{LR} we observe that the diagonal elements feature the same hierarchies as the corresponding diagonal Yukawa matrices $\tilde{Y}^f$, while the off-diagonal elements are strongly suppressed. This shows that our $S_4 \times \mathrm{U}(1)$ SUSY GUT approximately reproduces the effects of low energy MFV, where one would simply impose $\delta^f_{LL} = \delta^f_{RR} = \mathbb{1}$ and $\delta^f_{LR} \propto \tilde{Y}^f$. The phenomenological implications of the deviations form MFV will be discussed quantitatively in a dedicated paper [68], where we will present and discuss the predictions of our model of flavour with respect to a number of different flavour observables in detail.

Acknowledgments

We thank Claudia Hagedorn for helpful discussions throughout this project. MD and SFK acknowledge partial support from the STFC Consolidated ST/J000396/1 grant and the

European Union FP7 ITN-INVISIBLES (Marie Curie Actions, PITN-GA-2011-289442). CL is supported by the Deutsche Forschungsgemeinschaft (DFG) within the Research Unit FOR 1873 "Quark Flavour Physics and Effective Field Theories".

A S_4 and CP symmetry

The non-Abelian finite group S_4 can be defined in terms of the presentation

$$
\begin{aligned}
&S^2 = \mathbb{1}\,, && T^3 = \mathbb{1}\,, && U^2 = \mathbb{1}\,, \\
&(ST)^3 = \mathbb{1}\,, && (SU)^2 = \mathbb{1}\,, && (TU)^2 = \mathbb{1}\,, \quad (STU)^4 = \mathbb{1}\,,
\end{aligned}
$$

where S, T and U denote the generators of the group. Explicit matrix representations are basis dependent. In this work we apply the basis where the T generator is diagonal and complex for the doublet and triplet representations. Defining $\omega = e^{2\pi i/3}$, we have

$$
\begin{aligned}
&\mathbf{1}: && S = 1\,, && T = 1\,, && U = 1\,, \\
&\mathbf{1}': && S = 1\,, && T = 1\,, && U = -1\,, \\
&\mathbf{2}: && S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, && T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, && U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
&\mathbf{3}: && S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, && T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, && U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
&\mathbf{3}': && S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, && T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, && U = +\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
\end{aligned}
$$

The corresponding Clebsch-Gordan coefficients are all real and can be found e.g. in [39].

In addition to the flavour symmetry S_4, we impose the canonical CP symmetry in our theory. As has been discussed in the literature, see e.g. [69–71], the consistent combination of a flavour and a CP symmetry requires certain conditions to be fulfilled; in particular that the subsequent application of a CP, a flavour and a further CP transformation leads to a transformation belonging to the flavour group. The possibility to combine the group S_4 with CP has been explored previously, see e.g. [46, 47, 69, 70]. Here we are interested in combining S_4 symmetry, defined in the above basis, with the canonical CP transformation, i.e. the CP transformation that acts trivially in flavour space with $X_\mathbf{r} = \mathbb{1}$ for all representations $\mathbf{r}$ of S_4. Note that this particular CP transformation $X_\mathbf{r}$ fulfils the constraints of being a unitary and symmetric matrix. Moreover, it represents a consistent choice for a CP transformation (see e.g. [46, 47]), which corresponds to the involutionary automorphism that maps the generators S, T and U in the following way

$$
S \to S\,, \quad T \to T^2 = T^{-1} \quad \text{and} \quad U \to U\,, \tag{A.1}
$$

since S and U are represented by real matrices in our chosen basis, while the generator T is given as a diagonal complex matrix in the two- and three-dimensional representations.

Field	X_1^d	$\overline{X}_1^d$	$X_{1'}^{\nu d}$	X_1^u	Y_2^{du}	Y_2^d	Y_2^ν	$Z_{3'}^\nu$	V_0	V_1	V_η	X_1^{new}	$\tilde{X}_{1'}^{\text{new}}$
SU(5)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}^{(\prime)}$	$\mathbf{1}$	$\mathbf{1}'$
U(1)	-2	14	3	10	9	6	-16	-16	0	-8	-7	18	15

Table 2. The transformation properties of the driving fields, as introduced in [40], which serve to align the flavon VEVs.

JHEP02(2016)118

As with all automorphisms of S_4, this is an inner one. In particular, one can check that the automorphism of eq. (A.1) is "class-inverting" [72], i.e. it maps the group element g into the class which includes g^{-1}. This is true, since all automorphisms are inner ones and all classes of S_4 are ambivalent, i.e. the elements g and g^{-1} are in the same class.

With only real Clebsch-Gordan coefficients, a canonical CP symmetry imposed on the theory entails that all coefficients in the (super-)potential are real. Moreover, we observe that the residual symmetry in the neutrino sector at LO comprises the CP symmetry if all three neutrino flavons share the same phase factor. Following the comments of footnote 5 of appendix B, this is the case in our setup, cf. also eqs. (B.1), (B.2), so that the common phase can be factored out of the neutrino mass matrix, leading to an effective LO result which conserves CP. Furthermore, the canonical CP transformation $X_\mathbf{r} = \mathbb{1}$ commutes with the Klein group generated by S and U and thus at LO the residual symmetry is given by the direct product $Z_2 \times Z_2 \times \mathrm{CP}$.

B Vacuum alignment

The vacuum alignment of the flavon fields is achieved by coupling them to a set of so-called driving fields and requiring the F-terms of the latter to vanish. These driving fields, whose transformation properties under the family symmetry are shown in table 2, are SM gauge singlets and carry a charge of $+2$ under a continuous R-symmetry. The flavons and the GUT Higgs fields are uncharged under this $U(1)_R$, whereas the supermultiplets containing the SM fermions (or right-handed neutrinos) have charge $+1$. As the superpotential must have a $\mathrm{U}(1)_R$ charge of $+2$, the driving fields can only appear linearly and cannot have any direct interactions with the SM fermions or the right-handed neutrinos.

The LO alignment of the flavon fields, see eq. (2.2), has been thoroughly discussed in [39, 40]. The particular setup also provides correlations amongst the VEVs. As described in appendix D of [39] and in section 4 of [40],[4] the vanishing of the F-terms of the driving

[4]The introduction of the new flavon field η in [40] favours the exchange of the S_4 doublet driving field V_2, which was introduced in [39], by the S_4 singlet field V_1. Furthermore, the field V_η, transforming in the same representation of S_4 as η, is introduced in order to relate the new flavon field to an explicit mass scale.

fields $X_1^{\rm new}$, $\tilde{X}_{1'}^{\rm new}$, Y_2^ν, $Z_{3'}^\nu$, V_0, V_1 and V_η gives rise to the relations[5]

$$\begin{aligned} &\phi_2^u \sim \phi_2^d\,\tilde{\phi}_3^d\,, && \phi_1^\nu \sim \phi_2^\nu \sim \phi_{3'}^\nu\,, && (\phi_3^d)^2\phi_i^\nu \in \mathrm{Re}\,,\\ &\tilde{\phi}_2^u \sim \frac{\phi_1^\nu}{\phi_2^\nu}\,, && \tilde{\phi}_3^d \sim \phi_2^d\,(\phi_3^d)^3\,, && \phi_{3'}^\nu \sim \frac{\eta}{(\phi_2^d)^3\phi_3^d}\,. \end{aligned} \tag{B.1}$$

Denoting the phase of each flavon VEV ϕ_ρ^f by θ_ρ^f, eq. (B.1) correlates the LO phases as[6]

$$\begin{aligned} &\tilde{\theta}_2^u = 0\,, && \theta_2^u = 2\theta_2^d + 3\theta_3^d\,, && \tilde{\theta}_3^d = \theta_2^d + 3\theta_3^d\,,\\ &\theta^\eta = 3\theta_2^d - \theta_3^d\,, && \theta_{3'}^\nu = \theta_2^\nu = \theta_1^\nu = -2\theta_3^d\,, \end{aligned} \tag{B.2}$$

leaving as free variables only the two phases θ_2^d, θ_3^d, which correspond to the LO VEVs of the two flat superpotential directions: $\langle\Phi_{2,1}^d\rangle$ and $\langle\Phi_{3,2}^d\rangle$ respectively.

In order to find the higher order terms of the flavon VEVs, we start by writing each one of them as a series expansion in λ, up to and including order λ^{12}. For example, the leading operators of the superpotential fix $\langle\Phi_{2,1}^u\rangle/M$ to be zero up to λ^4, while $\langle\Phi_{2,2}^u\rangle/M$ has to be $\phi_2^u\,\lambda^4$ [39]. When considering the subleading operators, the VEVs of $\Phi_{2,1}^u$ and $\Phi_{2,2}^u$ receive corrections (shifts) which we parametrise as

$$\frac{\langle\Phi_2^u\rangle}{M} = \begin{pmatrix} 0 \\ \phi_2^u\,\lambda^4 \end{pmatrix} + \begin{pmatrix} \sum\limits_{n=5}^{12} \delta^u_{2,1_{(n)}}\lambda^n \\ \sum\limits_{n=5}^{12} \delta^u_{2,2_{(n)}}\lambda^n \end{pmatrix}. \tag{B.3}$$

All flavon VEVs are parametrised in a similar manner. The aim is to find the order of λ at which each shift δ has to be non-zero. The computation consists of taking into account all possible operators and solving the F-term conditions resulting from the set of driving field order by order in λ, up to λ^{12}. Each vanishing expression is solved for the lowest order shift involved. At the end, all shifts can be expressed in terms of the LO flavon VEVs. We find

$$\begin{aligned} &\frac{\langle\Phi_2^u\rangle}{M} = \begin{pmatrix} \delta^u_{2,1}\,\lambda^8 \\ \phi_2^u\,\lambda^4 + \delta^u_{2,2}\,\lambda^5 \end{pmatrix}, \quad \frac{\langle\tilde{\Phi}_2^u\rangle}{M} = \begin{pmatrix} \tilde{\delta}^u_{2,1}\,\lambda^6 \\ \tilde{\phi}_2^u\,\lambda^4 + \tilde{\delta}^u_{2,2}\,\lambda^5 \end{pmatrix}, \quad \frac{\langle\eta\rangle}{M} = \phi^\eta\lambda^4 + \delta^\eta\lambda^5,\\ &\frac{\langle\Phi_3^d\rangle}{M} = \begin{pmatrix} \delta^d_{3,1}\,\lambda^6 \\ \phi_3^d\,\lambda^2 \\ \delta^d_{3,3}\,\lambda^6 \end{pmatrix}, \quad \frac{\langle\tilde{\Phi}_3^d\rangle}{M} = \begin{pmatrix} \tilde{\delta}^d_{3,1}\,\lambda^7 \\ -\left(\tilde{\phi}_3^d\,\lambda^3 + \tilde{\delta}^d_{3,2_{(4)}}\,\lambda^4 + \tilde{\delta}^d_{3,2_{(5)}}\,\lambda^5\right) \\ \tilde{\phi}_3^d\,\lambda^3 + \tilde{\delta}^d_{3,2_{(4)}}\,\lambda^4 + \tilde{\delta}^d_{3,3_{(5)}}\,\lambda^5 \end{pmatrix}, \quad \frac{\langle\Phi_2^d\rangle}{M} = \begin{pmatrix} \phi_2^d\,\lambda \\ \delta^d_{2,2}\,\lambda^7 \end{pmatrix},\\ &\frac{\langle\Phi_{3'}^\nu\rangle}{M} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(\phi_{3'}^\nu\lambda^4 + \delta_{3'}^\nu\lambda^5), \quad \frac{\langle\Phi_2^\nu\rangle}{M} = \begin{pmatrix} \phi_2^\nu\,\lambda^4 + \delta^\nu_{2,1}\,\lambda^5 \\ \phi_2^\nu\,\lambda^4 + \delta^\nu_{2,2}\,\lambda^5 \end{pmatrix}, \quad \frac{\langle\Phi_1^\nu\rangle}{M} = \phi_1^\nu\lambda^4 + \delta_1^\nu\lambda^5. \end{aligned} \tag{B.4}$$

Note that the shifts presented in eq. (B.4) are the first non-trivial ones. However, in our calculations of the mass matrices we take into account all shifts up to $\mathcal{O}(\lambda^8)$. It should be

[5]The proportionality constant between $\phi_{3'}^\nu$ and ϕ_2^ν is a square root of an order one real number, which we assume to be positive, such that $\phi_{3'}^\nu$ and ϕ_2^ν have the same phases.

[6]Here and in eq. (B.6), a possible phase shift by π has been ignored as real coefficients can generally be positive or negative.

pointed out that the alignment of $\Phi^\nu_{3'}$ is not perturbed up to order λ^8, so that it preserves the S symmetry to that level. On the other hand, the alignment of Φ^ν_2 is already perturbed at order λ^5 which, however, does not break the S generator as it is nothing but the identity for the doublet representation. Taking into account also CN effects, one can show that $m_\nu^{\rm eff}$ has the form of eq. (4.22) up to $\mathcal{O}(\lambda^7)$.

Eq. (B.4) is in agreement with the discussion presented in section 4 of [40], barring the absorptions of $\delta^u_{2,2}\,\lambda$, $\tilde\delta^u_{2,2}\,\lambda$, $\tilde\delta^d_{3,2_{(4)}}\,\lambda$, $\delta^\nu_{3'}\lambda$, $\delta^\nu_1\lambda$ and $\delta^\eta\,\lambda$ into the corresponding LO VEVs. Being interested in the CP transformation properties of the fields, such absorptions must not be made in the current work, as the phases of shifts and LO VEVs are generally different. In particular, we find the following relations between the shifts and the LO VEVs,

$$\begin{aligned}
&\delta^u_{2,1}\sim(\phi^d_2)^2(\phi^d_3)^3\,,\quad \delta^u_{2,2}\sim(\phi^d_2)^6(\phi^d_3)^4\,,\quad \tilde\delta^u_{2,1}\sim\tilde\delta^u_{2,2}\sim(\phi^d_2)^4\phi^d_3\,,\quad \delta^\eta\sim(\phi^d_2)^7,\\
&\delta^d_{3,1}\sim\delta^d_{3,3}\sim\phi^d_3\,,\quad \tilde\delta^d_{3,1}\sim\phi^d_2\,(\phi^d_3)^3\,,\quad \tilde\delta^d_{3,2_{(4)}}\sim(\phi^d_2)^5(\phi^d_3)^4\,,\quad \tilde\delta^d_{3,3_{(5)}}-\tilde\delta^d_{3,2_{(5)}}\sim(\phi^d_2)^5,\\
&\delta^d_{2,2}\sim(\phi^d_2)^5\phi^d_3\,,\qquad \delta^\nu_{3'}\sim\delta^\nu_{2,1}\sim\delta^\nu_{2,2}\sim\delta^\nu_1\sim\frac{(\phi^d_2)^4}{\phi^d_3}\,.
\end{aligned}\tag{B.5}$$

Similar relations also hold for higher order shifts. Although such shifts have to be taken into account when performing a systematical λ-expansion, their explicit expressions are irrelevant for our phenomenological study.

The phases of the LO shifts can be deduced straightforwardly from eq. (B.5). Denoting the phase of $\delta^f_{\rho,i}$ by $\theta^f_{\rho,i}$ we obtain

$$\begin{aligned}
&\theta^u_{2,1}=2\theta^d_2+3\theta^d_3\,,\quad \theta^u_{2,2}=2(3\theta^d_2+2\theta^d_3)\,,\quad \tilde\theta^u_{2,1}=\tilde\theta^u_{2,2}=4\theta^d_2+\theta^d_3\,,\quad \arg[\delta^\eta]=7\theta^d_2\,,\\
&\theta^d_{3,1}=\theta^d_{3,3}=\theta^d_3\,,\quad \tilde\theta^d_{3,1}=\theta^d_2+3\theta^d_3\,,\quad \tilde\theta^d_{3,2_{(4)}}=5\theta^d_2+4\theta^d_3\,,\quad \arg[\tilde\delta^d_{3,3_{(5)}}-\tilde\delta^d_{3,2_{(5)}}]=5\theta^d_2\,,\\
&\theta^d_{2,2}=5\theta^d_2+\theta^d_3\,,\quad \arg[\delta^\nu_{3'}]=\theta^\nu_{2,1}=\theta^\nu_{2,2}=\arg[\delta^\nu_1]=4\theta^d_2-\theta^d_3\,.
\end{aligned}\tag{B.6}$$

C Basis transformations

C.1 Canonical normalisation

In order to find the transformations which map the Kähler potential into its canonical form, we express the hermitian matrix $\mathcal{K}_A$ as in eq. (3.14), i.e. $P_A^\dagger P_A=\mathcal{K}_A$. Note that the matrix P_A is not unique since $P_A\to Q_A P_A$ with unitary Q_A will satisfy eq. (3.14) just as well. Moreover, $\mathcal{K}_A$ can always be decomposed as

$$\mathcal{K}_A\;=\;\left(Q_A^\dagger\sqrt{D_A}Q_A\right)\left(Q_A^\dagger\sqrt{D_A}Q_A\right),\tag{C.1}$$

where D_A is the diagonalised form of $\mathcal{K}_A$. Therefore it is sufficient to find a *hermitian* matrix P_A which satisfies eq. (3.14), i.e. $P_A^\dagger P_A=P_AP_A=\mathcal{K}_A$. Expanding $\mathcal{K}_A$ and P_A in powers of λ,

$$\mathcal{K}_A=\sum_{n=0}^{\infty}k_n\lambda^n\,,\qquad P_A=\sum_{m=0}^{\infty}p_m\lambda^m\,,\tag{C.2}$$

with k_n, p_n being matrices, allows one to calculate P_A iteratively. With $k_0 = \mathbb{1}$, the result reads

$$p_0 = \mathbb{1}\,, \qquad p_1 = \frac{1}{2}k_1\,, \qquad p_n = \frac{1}{2}\left(k_n - \sum_{j=1}^{n-1} p_j p_{n-j}\right)\,. \tag{C.3}$$

C.2 SCKM transformations

The SCKM rotation matrices that diagonalise the Yukawas are found through the singular value decomposition. In particular, if $Y^f = U_L^f \tilde{Y}^f_{\rm diag}(U_R^f)^\dagger$, then U_L^f and U_R^f consist of the eigenvectors of $Y^f(Y^f)^\dagger$ and $(Y^f)^\dagger Y^f$, respectively. These eigenvectors are only defined up to phase transformations

$$U_L^f \to U_L^f \Omega_L^f\,, \qquad \Omega_L^f = \mathrm{diag}\left(e^{i\omega^f_{L1}}, e^{i\omega^f_{L2}}, e^{i\omega^f_{L3}}\right)\,, \tag{C.4}$$

$$U_R^f \to U_R^f \Omega_L^f \Omega_R^f\,, \qquad \Omega_R^f = \mathrm{diag}\left(e^{i\omega^f_{R1}}, e^{i\omega^f_{R2}}, e^{i\omega^f_{R3}}\right)\,. \tag{C.5}$$

We fix the phases of the matrices Ω_L^f by requiring that the CKM and PMNS mixing matrices are given in the standard phase convention, while the phases of Ω_R^f are fixed by demanding real and positive charged fermion masses. To LO, we find the following structure of the SCKM transformation matrices in terms of their λ-suppression.

$$U_L^u \approx \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \lambda^4 & 1 & \lambda^5 \\ \lambda^6 & \lambda^5 & 1 \end{pmatrix}, \qquad U_R^u \approx \begin{pmatrix} 1 & \lambda^4 & \lambda^6 \\ \lambda^4 & 1 & \lambda^5 \\ \lambda^6 & \lambda^5 & 1 \end{pmatrix}, \tag{C.6}$$

$$U_L^d \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \qquad U_R^d \approx \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^4 \\ \lambda^4 & \lambda^4 & 1 \end{pmatrix}, \tag{C.7}$$

$$U_L^e \approx \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^4 \\ \lambda^4 & \lambda^4 & 1 \end{pmatrix}, \qquad U_R^e \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \tag{C.8}$$

With these SCKM transformations, it is straightforward to calculate the CKM mixing to leading order,

$$V_{\mathrm{CKM_{GUT}}} = (U_L^u)^T U_L^{d*} \approx \begin{pmatrix} 1 & \frac{\tilde{x}_2}{y_s}\lambda & \frac{\tilde{x}_2}{y_b}\lambda^3 \\ -\frac{\tilde{x}_2}{y_s}\lambda & 1 & \frac{y_s}{y_b}\lambda^2 \\ -e^{-i\theta_2^d}\frac{\tilde{x}_2^2}{y_s\, y_b}\lambda^4 & -\frac{y_s}{y_b}\lambda^2 & 1 \end{pmatrix}. \tag{C.9}$$

The associated measure of CP violation is given by the Jarlskog invariant $J^q_{\mathrm{CP_{GUT}}}$ and can be calculated from the imaginary part of $V_{\mathrm{CKM_{GUT}}_{21}} V_{\mathrm{CKM_{GUT}}_{32}} V^*_{\mathrm{CKM_{GUT}}_{22}} V^*_{\mathrm{CKM_{GUT}}_{31}}$. The explicit result can be found in eq. (6.5).

D Mass insertion parameters at the GUT scale

In the following we present the explicit expression for the various LO mass insertion parameters at the GUT scale whose λ-suppressions have been stated in eqs. (7.4)–(7.6). Using the definitions of eqs. (7.2), (7.3), we obtain

$$\delta^u_{LL\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & \frac{e^{-i\theta_2^d}\tilde{b}_{12}}{b_{01}}\,\lambda^4 & \frac{e^{-i(4\theta_2^d+\theta_3^d)}\tilde{b}_{13}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\,\lambda^6 \\ \cdot & 1 & \frac{e^{-i(7\theta_2^d+2\theta_3^d)}\tilde{b}_{23}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\,\lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix},$$

$$\delta^u_{RR\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & \frac{e^{-i\theta_2^d}\tilde{b}_{12}}{b_{01}}\,\lambda^4 & \frac{\tilde{b}_{13}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\,\lambda^6 \\ \cdot & 1 & \frac{e^{i(5\theta_2^d+\theta_3^d)}\tilde{b}_{23}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\,\lambda^5 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{D.1}$$

$$\delta^u_{LR\,\mathrm{GUT}} \approx \frac{v_u\,\alpha_0}{m_0}\begin{pmatrix} \frac{\tilde{a}^u_{11}-y_u\frac{\mu}{t_\beta A_0}}{b_{01}}\lambda^8 & 0 & 0 \\ 0 & \frac{\tilde{a}^u_{22}-y_c\frac{\mu}{t_\beta A_0}}{b_{01}}\lambda^4 & \frac{e^{i\theta_2^d}\tilde{a}^u_{23}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\lambda^7 \\ 0 & \frac{e^{i(3\theta_2^d+\theta_3^d)}\tilde{a}^u_{23}}{\sqrt{b_{01}\left(b_{02}+v_u^2\,y_t^2/m_0^2\right)}}\lambda^7 & \frac{\tilde{a}^u_{33}-y_t\frac{\mu}{t_\beta A_0}}{b_{02}+v_u^2\,y_t^2/m_0^2} \end{pmatrix},$$

$$\delta^d_{LL\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & \frac{\tilde{B}_{12}}{b_{01}}\,\lambda^3 & \frac{e^{i\theta_2^d}\tilde{B}_{13}}{\sqrt{b_{01}\,b_{02}}}\,\lambda^4 \\ \cdot & 1 & \frac{\tilde{B}_{23}}{\sqrt{b_{01}\,b_{02}}}\,\lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^d_{RR\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & e^{i\theta_2^d}\tilde{R}_{12}\,\lambda^4 & -\,e^{i\theta_2^d}\tilde{R}_{12}\,\lambda^4 \\ \cdot & 1 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{D.2}$$

$$\delta^d_{LR\,\mathrm{GUT}} \approx \frac{v_d\,\alpha_0}{m_0}\begin{pmatrix} \frac{1}{\sqrt{b_{01}}}\left(\tilde{a}^d_{11}-\frac{\mu\,t_\beta}{A_0}\frac{\tilde{x}_2^2}{y_s}\right)\lambda^6 & \frac{\tilde{a}^d_{12}}{\sqrt{b_{01}}}\,\lambda^5 & \frac{\tilde{a}^d_{12}}{\sqrt{b_{01}}}\lambda^5 \\ -\frac{\tilde{a}^d_{12}}{\sqrt{b_{01}}}\lambda^5 & \frac{1}{\sqrt{b_{01}}}\left(\tilde{a}^d_{22}-\frac{\mu\,t_\beta}{A_0}y_s\right)\lambda^4 & \frac{\tilde{a}^d_{23}}{\sqrt{b_{01}}}\,\lambda^4 \\ e^{-i\theta_2^d}\frac{\tilde{a}^d_{31}}{\sqrt{b_{02}}}\lambda^6 & \frac{\tilde{a}^d_{32}}{\sqrt{b_{02}}}\,\lambda^6 & \frac{1}{\sqrt{b_{02}}}\left(\tilde{a}^d_{33}-\frac{\mu\,t_\beta}{A_0}y_b\right)\lambda^2 \end{pmatrix},$$

$$\delta^e_{LL\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & \tilde{R}_{12}\,\lambda^4 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & 1 & -\,\tilde{R}_{12}\,\lambda^4 \\ \cdot & \cdot & 1 \end{pmatrix}, \quad \delta^e_{RR\,\mathrm{GUT}} \approx \begin{pmatrix} 1 & -\,\frac{e^{i\theta_2^d}\tilde{B}_{12}}{3\,b_{01}}\,\lambda^3 & \frac{\tilde{B}_{13}}{3\sqrt{b_{01}b_{02}}}\,\lambda^4 \\ \cdot & 1 & \frac{3\tilde{B}_{23}}{\sqrt{b_{01}b_{02}}}\,\lambda^2 \\ \cdot & \cdot & 1 \end{pmatrix}, \tag{D.3}$$

$$\delta^e_{LR\,\mathrm{GUT}} \approx \frac{v_d\,\alpha_0}{m_0}\begin{pmatrix} \frac{1}{3\sqrt{b_{01}}}\left(\tilde{a}^d_{11}-\frac{\mu\,t_\beta}{A_0}\frac{\tilde{x}_2^2}{y_s}\right)\lambda^6 & \frac{e^{i\theta_2^d}\tilde{a}^d_{12}}{\sqrt{b_{01}}}\,\lambda^5 & \frac{\tilde{a}^d_{31}}{\sqrt{b_{02}}}\lambda^6 \\ -\frac{e^{-i\theta_2^d}\tilde{a}^d_{12}}{\sqrt{b_{01}}}\lambda^5 & \frac{3}{\sqrt{b_{01}}}\left(\tilde{a}^d_{22}-\frac{\mu\,t_\beta}{A_0}y_s\right)\lambda^4 & \frac{\tilde{a}^e_{23}}{\sqrt{b_{02}}}\,\lambda^6 \\ -\frac{e^{-i\theta_2^d}\tilde{a}^d_{12}}{\sqrt{b_{01}}}\lambda^5 & \frac{3\tilde{a}^d_{23}}{\sqrt{b_{01}}}\,\lambda^4 & \frac{1}{\sqrt{b_{02}}}\left(\tilde{a}^d_{33}-\frac{\mu\,t_\beta}{A_0}y_b\right)\lambda^2 \end{pmatrix}.$$

These δ parameters are expressed in terms of the coefficients of the soft mass matrices in eqs. (6.13)–(6.21), where we have defined

$$\tilde{b}_{12} = (b_2 - b_{01}k_2), \qquad \tilde{b}_{13} = -(b_4 - b_{01}k_4), \qquad \tilde{b}_{23} = -(b_3 - b_{01}k_3), \tag{D.4}$$

$$\tilde{B}_{12} = 2\frac{\tilde{x}_2}{y_s}(b_1 - b_{01}k_1), \quad \tilde{B}_{13} = \frac{\tilde{x}_2^2}{y_b\,y_s}(b_{01} - b_{02}), \quad \tilde{B}_{23} = \frac{y_s}{y_b}(b_{01} - b_{02}), \quad \tilde{R}_{12} = B_3 - K_3,$$

and

$$
\begin{aligned}
\tilde{a}^u_{11} &= a_u e^{i(\theta^a_u-\theta^y_u)}, \quad \tilde{a}^u_{22} = a_c e^{i(\theta^a_c-\theta^y_u)}, \quad \tilde{a}^u_{33} = a_t, \quad \tilde{a}^u_{23} = z^u_2\left(\frac{a_t}{y_t} - e^{i(\theta_2^{z^{ua}}-\theta_2^{z^u})}\frac{z^{u_a}_2}{z^u_2}\right),\\
\tilde{a}^d_{11} &= \frac{\tilde{x}^2_2}{y_s}\left(2\frac{\tilde{x}^a_2}{\tilde{x}_2}e^{i(\theta_2^{\tilde{x}a}-\theta_2^{\tilde{x}})} - \frac{a_s}{y_s}e^{i(\theta^a_s-\theta^y_s)}\right), \quad \tilde{a}^d_{22} = a_s e^{i(\theta^a_s-\theta^y_s)}, \quad \tilde{a}^d_{33} = a_b e^{i(\theta^a_b-\theta^y_b)},\\
\tilde{a}^d_{12} &= \tilde{x}_2\left(\frac{\tilde{x}^a_2}{\tilde{x}_2}e^{i(\theta_2^{\tilde{x}a}-\theta_2^{\tilde{x}})} - \frac{a_s}{y_s}e^{i(\theta^a_s-\theta^y_s)}\right), \quad \tilde{a}^d_{23} = y_s\left(\frac{a_s}{y_s}e^{i(\theta^a_s-\theta^y_s)} - \frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)}\right),\\
\tilde{a}^d_{31} &= z^d_3\left(\frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)} - \frac{z^{d_a}_3}{z^d_3}e^{i(\theta_3^{z^{da}}-\theta_3^{z^d})}\right),\\
\tilde{a}^d_{32} &= \frac{y^2_s}{y_b}\left(\frac{a_s}{y_s}e^{i(\theta^a_s-\theta^y_s)} - \frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)}\right) + z^d_2\left(\frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)} - \frac{z^{d_a}_2}{z^d_2}e^{i(\theta_2^{z^{da}}-\theta_2^{z^d})}\right),\\
\tilde{a}^e_{23} &= 9\frac{y^2_s}{y_b}\left(\frac{a_s}{y_s}e^{i(\theta^a_s-\theta^y_s)} - \frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)}\right) + z^d_2\left(\frac{a_b}{y_b}e^{i(\theta^a_b-\theta^y_b)} - \frac{z^{d_a}_2}{z^d_2}e^{i(\theta_2^{z^{da}}-\theta_2^{z^d})}\right). \qquad \text{(D.5)}
\end{aligned}
$$

The phases $\theta^y_{u,c,s,b}, \theta_i^{z^{u,d}}, \theta_2^{\tilde{x}}$ can be expressed in terms of the flavon phases θ^d_2, θ^d_3 according to eqs. (4.4), (4.10). This has been done in eq. (D.4), but we refrain from doing so in eq. (D.5) in order to highlight the fact that all $\tilde{a}^f_{ij}$ become real in the limit where the contributions of the auxiliary components of the flavon superfields to the A-terms are neglected such that the relation $\theta^a_f = \theta^y_f$ holds.

E Renormalisation group equations in SCKM basis

The renormalisation group equations for the parameters of the superpotential as well as the soft breaking terms are usually given in the gauge flavour basis, see e.g. [17], with the transformation to the SCKM basis being defined only at the electroweak scale. As already discussed in section 6, we find it useful to diagonalise the Yukawa matrices already at the high scale. In such a high scale SCKM basis, the RGEs will explicitly depend on the CKM mixing matrix. Here we define for convenience

$$
V = (U^d_L)^\dagger U^u_L = V^T_{\mathrm{CKM}_{\mathrm{GUT}}}. \qquad \text{(E.1)}
$$

Introducing the parameter $t = \ln(\mu/M_x)$, with μ being the renormalisation scale and M_x the high energy scale, we have for the Yukawas and the trilinear A-parameters,

$$
\begin{aligned}
16\pi^2\frac{d\tilde{Y}^u}{dt} &= \left(3\tilde{Y}^u\tilde{Y}^{u\dagger} + V^\dagger\tilde{Y}^d\tilde{Y}^{d\dagger}V - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{u\dagger}\tilde{Y}^u] + \mathrm{Tr}[\tilde{Y}^{\nu\dagger}\tilde{Y}^\nu]\right)\tilde{Y}^u,\\
16\pi^2\frac{d\tilde{Y}^d}{dt} &= \left(3\tilde{Y}^d\tilde{Y}^{d\dagger} + V\tilde{Y}^u\tilde{Y}^{u\dagger}V^\dagger - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{Y}^d] + \mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{Y}^e]\right)\tilde{Y}^d,\\
16\pi^2\frac{d\tilde{Y}^e}{dt} &= \left(3\tilde{Y}^e\tilde{Y}^{e\dagger} + U^{e\dagger}_L Y^\nu Y^{\nu\dagger}U^e_L - 3g_2^2 - \frac{9}{5}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{Y}^d] + \mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{Y}^e]\right)\tilde{Y}^e, \qquad \text{(E.2)}
\end{aligned}
$$

JHEP02(2016)118

$$
\begin{aligned}
16\pi^2\frac{d\tilde{A}^u}{dt} &= \left(5\tilde{Y}^u\tilde{Y}^{u\dagger} + V^\dagger\tilde{Y}^d\tilde{Y}^{d\dagger}V - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{u\dagger}\tilde{Y}^u] + \mathrm{Tr}[Y^{\nu\dagger}Y^\nu]\right)\tilde{A}^u + \\
&+ \left(4\tilde{A}^u\tilde{Y}^{u\dagger} + 2V^\dagger\tilde{A}^d\tilde{Y}^{d\dagger}V + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1 + 6\mathrm{Tr}[\tilde{Y}^{u\dagger}\tilde{A}^u] + 2\mathrm{Tr}[Y^{\nu\dagger}A^\nu]\right)\tilde{Y}^u, \\
16\pi^2\frac{d\tilde{A}^d}{dt} &= \left(5\tilde{Y}^d\tilde{Y}^{d\dagger} + V\tilde{Y}^u\tilde{Y}^{u\dagger}V^\dagger - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{Y}^d] + \mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{Y}^e]\right)\tilde{A}^d + \\
&+ \left(4\tilde{A}^d\tilde{Y}^{d\dagger} + 2V\tilde{A}^u\tilde{Y}^{u\dagger}V^\dagger + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{14}{15}g_1^2M_1 + 6\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{A}^d] + 2\mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{A}^e]\right)\tilde{Y}^d, \\
16\pi^2\frac{d\tilde{A}^e}{dt} &= \left(5\tilde{Y}^e\tilde{Y}^{e\dagger} + U_L^{e\dagger}Y^\nu Y^{\nu\dagger}U_L^e - 3g_2^2 - \frac{9}{5}g_1^2 + 3\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{Y}^d] + \mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{Y}^e]\right)\tilde{A}^e + \\
&+ \left(4\tilde{A}^e\tilde{Y}^{e\dagger} + 2U_L^{e\dagger}A^\nu Y^{\nu\dagger}U_L^e + 6g_2^2M_2 + \frac{18}{5}g_1^2M_1 + 6\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{A}^d] + 2\mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{A}^e]\right)\tilde{Y}^e. \qquad \text{(E.3)}
\end{aligned}
$$

The running of the soft scalar masses in the SCKM basis is given by

$$
\begin{aligned}
16\pi^2\frac{d}{dt}(\tilde{m}_u^2)_{LL} &= G_Q\,\mathbb{1} + F_Q^u + V^\dagger F_Q^d V, \\
16\pi^2\frac{d}{dt}(\tilde{m}_d^2)_{LL} &= G_Q\,\mathbb{1} + VF_Q^uV^\dagger + F_Q^d, \\
16\pi^2\frac{d}{dt}(\tilde{m}_e^2)_{LL} &= G_L\,\mathbb{1} + F_L^e + F_L^\nu, \\
16\pi^2\frac{d}{dt}(\tilde{m}_f^2)_{RR} &= G_f\,\mathbb{1} + F_f, \qquad f = u, d, e, \qquad \text{(E.4)}
\end{aligned}
$$

with

$$
\begin{aligned}
F_Q^u &= \tilde{Y}^u\tilde{Y}^{u\dagger}(\tilde{m}_u^2)_{LL} + (\tilde{m}_u^2)_{LL}\tilde{Y}^u\tilde{Y}^{u\dagger} + 2\tilde{Y}^u(\tilde{m}_u^2)_{RR}\tilde{Y}^{u\dagger} + 2(m_{H_u}^2)\tilde{Y}^u\tilde{Y}^{u\dagger} + 2\tilde{A}^u\tilde{A}^{u\dagger}, \\
F_Q^d &= \tilde{Y}^d\tilde{Y}^{d\dagger}(\tilde{m}_d^2)_{LL} + (\tilde{m}_d^2)_{LL}\tilde{Y}^d\tilde{Y}^{d\dagger} + 2\tilde{Y}^d(\tilde{m}_d^2)_{RR}\tilde{Y}^{d\dagger} + 2(m_{H_d}^2)\tilde{Y}^d\tilde{Y}^{d\dagger} + 2\tilde{A}^d\tilde{A}^{d\dagger}, \\
F_L^e &= \tilde{Y}^e\tilde{Y}^{e\dagger}(\tilde{m}_e^2)_{LL} + (\tilde{m}_e^2)_{LL}\tilde{Y}^e\tilde{Y}^{e\dagger} + 2\tilde{Y}^e(\tilde{m}_e^2)_{RR}\tilde{Y}^{e\dagger} + 2(m_{H_d}^2)\tilde{Y}^e\tilde{Y}^{e\dagger} + 2\tilde{A}^e\tilde{A}^{e\dagger}, \\
F_L^\nu &= U_L^{e\dagger}Y^\nu Y^{\nu\dagger}U_L^e(\tilde{m}_e^2)_{LL} + (\tilde{m}_e^2)_{LL}U_L^{e\dagger}Y^\nu Y^{\nu\dagger}U_L^e + 2U_L^{e\dagger}Y^\nu m_N^2 Y^{\nu\dagger}U_L^e + \\
&\quad + 2(m_{H_u}^2)U_L^{e\dagger}Y^\nu Y^{\nu\dagger}U_L^e + 2U_L^{e\dagger}A^\nu A^{\nu\dagger}U_L^e\,,
\end{aligned}
$$

$$
\begin{aligned}
F_u &= 2\left(\tilde{Y}^{u\dagger}\tilde{Y}^u(\tilde{m}_u^2)_{RR} + (\tilde{m}_u^2)_{RR}\tilde{Y}^{u\dagger}\tilde{Y}^u + 2\tilde{Y}^{u\dagger}(\tilde{m}_u^2)_{LL}\tilde{Y}^u + 2(m_{H_u}^2)\tilde{Y}^{u\dagger}\tilde{Y}^u + 2\tilde{A}^{u\dagger}\tilde{A}^u\right), \\
F_d &= 2\left(\tilde{Y}^{d\dagger}\tilde{Y}^d(\tilde{m}_d^2)_{RR} + (\tilde{m}_d^2)_{RR}\tilde{Y}^{d\dagger}\tilde{Y}^d + 2\tilde{Y}^{d\dagger}(\tilde{m}_d^2)_{LL}\tilde{Y}^d + 2(m_{H_d}^2)\tilde{Y}^{d\dagger}\tilde{Y}^d + 2\tilde{A}^{d\dagger}\tilde{A}^d\right), \\
F_e &= 2\left(\tilde{Y}^{e\dagger}\tilde{Y}^e(\tilde{m}_e^2)_{RR} + (\tilde{m}_e^2)_{RR}\tilde{Y}^{e\dagger}\tilde{Y}^e + 2\tilde{Y}^{e\dagger}(\tilde{m}_e^2)_{LL}\tilde{Y}^e + 2(m_{H_d}^2)\tilde{Y}^{e\dagger}\tilde{Y}^e + 2\tilde{A}^{e\dagger}\tilde{A}^e\right),
\end{aligned}
$$

$$
\begin{aligned}
G_Q &= -4\left(\frac{8}{3}g_3^2|M_3|^2 + \frac{3}{2}g_2^2|M_2|^2 + \frac{1}{30}g_1^2|M_1|^2 - \frac{1}{10}g_1^2(m_{H_u}^2 - m_{H_d}^2)\right), \\
G_L &= -4\left(\frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 + \frac{3}{10}g_1^2(m_{H_u}^2 - m_{H_d}^2)\right), \\
G_u &= -4\left(\frac{8}{3}g_3^2|M_3|^2 + \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2(m_{H_u}^2 - m_{H_d}^2)\right), \\
G_d &= -4\left(\frac{8}{3}g_3^2|M_3|^2 + \frac{2}{15}g_1^2|M_1|^2 - \frac{1}{5}g_1^2(m_{H_u}^2 - m_{H_d}^2)\right), \\
G_e &= -4\left(\frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2(m_{H_u}^2 - m_{H_d}^2)\right).
\end{aligned}
$$

JHEP02(2016)118

For completeness, we also show the evolution of the μ parameter, i.e. the coupling of the bilinear superpotential term $H_u H_d$,

$$16\pi^2 \frac{d\mu}{dt} = \left(3\mathrm{Tr}[\tilde{Y}^{u\dagger}\tilde{Y}^u] + 3\mathrm{Tr}[\tilde{Y}^{d\dagger}\tilde{Y}^d] + \mathrm{Tr}[\tilde{Y}^{e\dagger}\tilde{Y}^e] + \mathrm{Tr}[Y^{\nu\dagger}Y^\nu] - 3g_2^2 - \frac{3}{5}g_1^2\right)\mu\,, \quad \text{(E.5)}$$

where $g_i, M_i,\ i = 1, 2, 3$ are the gaugino couplings and masses respectively.

F Renormalisation group running

In this appendix, we provide analytical expression for the RG evolved Yukawa couplings, soft terms and mass insertion parameters. We estimate the effects of RG running using the leading logarithmic approximation. In order to formulate the two-stage running (i) from M_{GUT} to M_R, where the right-handed neutrinos are integrated out, and (ii) from M_R to $M_{\mathrm{SUSY}} \sim M_{\mathrm{W}} \equiv M_{\mathrm{low}}$, we introduce the parameters

$$\eta = \frac{1}{16\pi^2} \ln\left(\frac{M_{\mathrm{GUT}}}{M_{\mathrm{low}}}\right), \quad \eta_N = \frac{1}{16\pi^2} \ln\left(\frac{M_{\mathrm{GUT}}}{M_{\mathrm{R}}}\right). \quad \text{(F.1)}$$

For $M_{\mathrm{GUT}} \approx 2\times 10^{16}\,\mathrm{GeV}$, $M_{\mathrm{R}} \approx 10^{14}\,\mathrm{GeV}$ and $M_{\mathrm{low}} \approx 10^3\,\mathrm{GeV}$, $\eta \approx 0.19$ is of the order of our expansion parameter $\lambda \approx 0.22$ and $\eta_N \approx 0.03$.

F.1 Low energy Yukawas

The SCKM transformations, discussed in section 6, diagonalise the Yukawa matrices at high scales. RG running to low energies re-introduces off-diagonal elements in the low energy Yukawa matrices. These off-diagonal entries in $\tilde{Y}^u_{\mathrm{low}}$ and $\tilde{Y}^d_{\mathrm{low}}$ are proportional to the quark masses and the V_{CKM} elements. As the CKM matrix features only a mild running, the RG corrections can be treated as a perturbation. In $\tilde{Y}^e_{\mathrm{low}}$, the off-diagonal terms are proportional to the charged lepton masses and the elements of Y^ν. The corresponding RG equations are provided explicitly in eq. (E.2) for convenience. To LO in λ, we find,

$$\tilde{Y}^u_{\mathrm{low}} \approx \begin{pmatrix} 1+R^y_u & 0 & 0 \\ 0 & 1+R^y_u & 0 \\ 0 & 0 & 1+R^y_t \end{pmatrix} \tilde{Y}^u_{\mathrm{GUT}} - \eta\, y_b\, y_t \begin{pmatrix} 0 & 0 & \tilde{x}_2\,\lambda^7 \\ 0 & 0 & y_s\,\lambda^6 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{(F.2)}$$

$$\tilde{Y}^d_{\mathrm{low}} \approx \begin{pmatrix} 1+R^y_d & 0 & 0 \\ 0 & 1+R^y_d & 0 \\ 0 & 0 & 1+R^y_b \end{pmatrix} \tilde{Y}^d_{\mathrm{GUT}} + \eta\, y_t^2 \begin{pmatrix} 0 & 0 & e^{i\theta^d_2}\frac{\tilde{x}_2^2}{y_s}\lambda^6 \\ 0 & 0 & y_s\,\lambda^4 \\ 0 & \frac{y_s^2}{y_b}\lambda^6 & 0 \end{pmatrix}, \quad \text{(F.3)}$$

$$\tilde{Y}^e_{\mathrm{low}} \approx \begin{pmatrix} 1+R^y_e & 0 & 0 \\ 0 & 1+R^y_e & 0 \\ 0 & 0 & 1+R^y_e \end{pmatrix} \tilde{Y}^e_{\mathrm{GUT}} + \eta_N\, y_D\, R_\nu \begin{pmatrix} 0 & -3\,y_s\,\lambda^8 & y_b\,\lambda^6 \\ 0 & 0 & y_b\,\lambda^6 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{(F.4)}$$

with

$$R_u^y = \eta \left(\frac{46}{5} g_U^2 - 3y_t^2 \right) - 3\eta_N \, y_D^2, \qquad R_t^y = R_u^y - 3\,\eta\, y_t^2, \tag{F.5}$$

$$R_d^y = \eta \frac{44}{5} g_U^2, \qquad R_b^y = R_d^y - \eta\, y_t^2, \tag{F.6}$$

$$R_e^y = \eta \frac{24}{5} g_U^2 - \eta_N \, y_D^2, \qquad R_\nu = z_1^D - y_D(K_3 + K_3^N). \tag{F.7}$$

where $g_U \approx \sqrt{0.52}$ is the universal gauge coupling constant at the GUT scale.

F.2 Low energy soft terms

Similar to the Yukawa matrices, the parameters of the soft terms have to be run down to low energies. Moreover, it is mandatory to perform further transformations to the "new" SCKM basis which render $\tilde{Y}^f_{\text{low}}$ diagonal again. The running of the trilinear terms is similar to the one of the corresponding Yukawas. To LO in λ, η and η_N, we derive the following expressions in the "new" SCKM basis.

$$\begin{aligned} \frac{\tilde{A}^u_{\text{low}}}{A_0} \approx & \begin{pmatrix} 1+R_u^y & 0 & 0 \\ 0 & 1+R_u^y & 0 \\ 0 & 0 & 1+R_t^y \end{pmatrix} \frac{\tilde{A}^u_{\text{GUT}}}{A_0} - 2 \begin{pmatrix} R_u^a & 0 & 0 \\ 0 & R_u^a & 0 \\ 0 & 0 & R_t^a \end{pmatrix} \tilde{Y}^u_{\text{GUT}} \\ & - 2\eta\, y_t \begin{pmatrix} 0 & 0 & y_b\, \tilde{x}_2^a \, e^{i(\theta_2^{\tilde{x}^a} - \theta_2^{\tilde{x}})}\, \lambda^7 \\ 0 & 0 & y_b\, a_s\, e^{i(\theta_s^a - \theta_s^y)}\, \lambda^6 \\ 0 & y_t e^{i(3\theta_2^d + \theta_3^d)} \tilde{a}^u_{23} \lambda^7 & 0 \end{pmatrix}, \end{aligned} \tag{F.8}$$

$$\begin{aligned} \frac{\tilde{A}^d_{\text{low}}}{A_0} \approx & \begin{pmatrix} 1+R_d^y & 0 & 0 \\ 0 & 1+R_d^y & 0 \\ 0 & 0 & 1+R_b^y \end{pmatrix} \frac{\tilde{A}^d_{\text{GUT}}}{A_0} - 2 \begin{pmatrix} R_d^a & 0 & 0 \\ 0 & R_d^a & 0 \\ 0 & 0 & R_b^a \end{pmatrix} \tilde{Y}^d_{\text{GUT}} \\ & + 2\eta\, y_s\, y_t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & a_t\, \lambda^4 \\ 0 & \frac{1}{y_b} \left(y_s\, a_t - y_t\, \tilde{a}^d_{23} \right) \lambda^6 & 0 \end{pmatrix}, \end{aligned} \tag{F.9}$$

$$\begin{aligned} \frac{\tilde{A}^e_{\text{low}}}{A_0} \approx & \begin{pmatrix} 1+R_e^y & 0 & 0 \\ 0 & 1+R_e^y & 0 \\ 0 & 0 & 1+R_e^y \end{pmatrix} \frac{\tilde{A}^e_{\text{GUT}}}{A_0} - 2R_e^a\, \tilde{Y}^e_{\text{GUT}} \\ & + 2\eta_N\, y_D\, R_\nu\, y_b \begin{pmatrix} 0 & 0 & \frac{\alpha_D}{y_D}\, \lambda^6 \\ 0 & 0 & \frac{R_\nu^a}{R_\nu}\, \lambda^6 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{F.10}$$

with

$$R_u^a = \eta \left(\frac{46}{5} g_U^2 \frac{M_{1/2}}{A_0} + 3a_t\, y_t \right) + 3\eta_N\, y_D\, \alpha_D, \qquad R_t^a = R_u^a + 3\,\eta\, a_t\, y_t, \tag{F.11}$$

$$R_d^a = \eta\, \frac{44}{5} g_U^2 \frac{M_{1/2}}{A_0}, \qquad R_b^a = R_d^a + \eta\, a_t\, y_t, \tag{F.12}$$

$$R_e^a = \eta \frac{24}{5} g_U^2 \frac{M_{1/2}}{A_0} + \eta_N\, y_D \alpha_D, \tag{F.13}$$

$$R_\nu^a = z_1^{D_a} e^{i\theta_1^{z_{D_a}}} - \alpha_D(K_3 + K_3^N). \tag{F.14}$$

The first terms in eqs. (F.8)–(F.10) are analogous to the first terms in eqs. (F.2)–(F.4); they are usually ignored. The second terms contain the universal gaugino mass $M_{1/2}$ contributions, which generate non-zero diagonal trilinear couplings through the running, even for $A_0 \to 0$. The sources of the off-diagonal entries in the Yukawa couplings are also present for the trilinear soft terms. We see that the (13) element in $\tilde{A}^u_{\rm low}$, which was zero in $\tilde{A}^u_{\rm GUT}$, is now filled in, and there is an $\mathcal{O}(\lambda^6)$ contribution (but additionally suppressed by a factor of η) to the (23) element, which was of order λ^7 in $\tilde{A}^u_{\rm GUT}$. The (32) element in $\tilde{A}^u_{\rm low}$, with $\tilde{a}^u_{23}$ given in eq. (D.5), is of the same order in λ as the one that is already present in $\tilde{A}^u_{\rm GUT}$. All the off-diagonal elements generated by the running in $\tilde{A}^d_{\rm low}$ and in $\tilde{A}^e_{\rm low}$ are of the same order in λ as the ones that were already present at the high scale.

Analogously to the trilinear A-terms, we find for the soft scalar mass,

$$\frac{(\tilde{m}^2_u)_{LL_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_u)_{LL_{\rm GUT}}}{m_0^2} + (6.5\,x + T^u_L)\,\mathbb{1} - \eta \begin{pmatrix} 0 & 0 & y_t^2\,\frac{(\tilde{m}^2_u)_{LL_{{\rm GUT}_{13}}}}{m_0^2} \\ \cdot & 0 & y_t^2\,\frac{(\tilde{m}^2_u)_{LL_{{\rm GUT}_{23}}}}{m_0^2} \\ \cdot & \cdot & 2R_q \end{pmatrix}, \tag{F.15}$$

$$\frac{(\tilde{m}^2_u)_{RR_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_u)_{RR_{\rm GUT}}}{m_0^2} + (6.15\,x + T^u_R)\,\mathbb{1} - 2\eta \begin{pmatrix} 0 & 0 & y_t^2\,\frac{(\tilde{m}^2_u)_{RR_{{\rm GUT}_{13}}}}{m_0^2} \\ \cdot & 0 & y_t^2\,\frac{(\tilde{m}^2_u)_{RR_{{\rm GUT}_{23}}}}{m_0^2} \\ \cdot & \cdot & 2R_q \end{pmatrix}, \tag{F.16}$$

$$\frac{(\tilde{m}^2_d)_{LL_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_u)_{LL_{\rm GUT}}}{m_0^2} + (6.5\,x + T^d_L)\,\mathbb{1} + \eta \begin{pmatrix} 0 & 0 & \left(\frac{2R_q}{b_{01}-b_{02}} + y_t^2\right)\frac{(\tilde{m}^2_d)_{LL_{{\rm GUT}_{13}}}}{m_0^2} \\ \cdot & 0 & \left(\frac{2R_q}{b_{01}-b_{02}} + y_t^2\right)\frac{(\tilde{m}^2_d)_{LL_{{\rm GUT}_{23}}}}{m_0^2} \\ \cdot & \cdot & -\,2R_q \end{pmatrix}, \tag{F.17}$$

$$\frac{(\tilde{m}^2_d)_{RR_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_d)_{RR_{\rm GUT}}}{m_0^2} + (6.1\,x + T^d_R)\,\mathbb{1}\,, \tag{F.18}$$

$$\frac{(\tilde{m}^2_e)_{LL_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_e)_{LL_{\rm GUT}}}{m_0^2} + (0.5\,x + T^e_L - 2\eta_N\,R_l)\,\mathbb{1} - 2\eta_N \begin{pmatrix} 0 & \tilde{E}_{12} & -\,\tilde{E}^*_{12} \\ \cdot & 0 & -\,\tilde{E}_{12} \\ \cdot & \cdot & 0 \end{pmatrix}\lambda^4, \tag{F.19}$$

$$\frac{(\tilde{m}^2_e)_{RR_{\rm low}}}{m_0^2} \approx \frac{(\tilde{m}^2_e)_{RR_{\rm GUT}}}{m_0^2} + (0.15\,x + T^e_R)\,\mathbb{1}, \tag{F.20}$$

where we have introduced the ratio $x = M^2_{1/2}/m_0^2$ and

$$R_q = (2b_{02} + c_{H_u})\,y_t^2 + \alpha_0^2\,a_t^2\,, \tag{F.21}$$

$$\tilde{E}_{12} = y_D^2\left(\tilde{R}_{12} + B^N_3 - K^N_3 B^N_0\right) + R'_l - (K_3 + K^N_3)R_l\,, \tag{F.22}$$

$$R_l = (1 + B^N_0 + c_{H_u})y_D^2 + \alpha_0^2\alpha_D^2\,, \tag{F.23}$$

$$R'_l = (1 + B^N_0 + c_{H_u})y_D\,z_1^D + \alpha_0^2\alpha_D\,z_1^{D_a}\,e^{i\theta_1^{z_{D_a}}}\,, \tag{F.24}$$

with $c_{H_u} = m^2_{H_u\mathrm{GUT}}/m_0^2$. Furthermore, the small quantities $T^f_{L,R}$ are defined as

$$T_L^u = \frac{1}{m_0^2}\left(\frac{1}{20}T + \Delta_L^u\right), \qquad T_R^u = \frac{1}{m_0^2}\left(-\frac{1}{5}T + \Delta_R^u\right), \tag{F.25}$$

$$T_L^d = \frac{1}{m_0^2}\left(\frac{1}{20}T + \Delta_L^d\right), \qquad T_R^d = \frac{1}{m_0^2}\left(\frac{1}{5}T + \Delta_R^d\right), \tag{F.26}$$

$$T_L^e = \frac{1}{m_0^2}\left(-\frac{3}{20}T + \Delta_L^e\right), \qquad T_R^e = \frac{1}{m_0^2}\left(\frac{3}{10}T + \Delta_R^e\right), \tag{F.27}$$

with $T = \frac{1}{4\pi^2}\int\limits_{\ln(M_{\mathrm{GUT}})}^{\ln(M_{\mathrm{low}})} g_U^2\,(m^2_{H_u} - m^2_{H_d})$, as well as

$$\Delta_L^u = \left(\frac{1}{2} - \frac{2}{3}\sin^2(\theta_W)\right)\cos(2\beta)M_Z^2\,, \qquad \Delta_R^u = \frac{2}{3}\sin^2(\theta_W)\cos(2\beta)M_Z^2\,, \tag{F.28}$$

$$\Delta_L^d = \left(-\frac{1}{2} + \frac{1}{3}\sin^2(\theta_W)\right)\cos(2\beta)M_Z^2\,, \qquad \Delta_R^d = -\frac{1}{3}\sin^2(\theta_W)\cos(2\beta)M_Z^2\,, \tag{F.29}$$

$$\Delta_L^e = \left(-\frac{1}{2} + \frac{1}{2}\sin^2(\theta_W)\right)\cos(2\beta)M_Z^2\,, \qquad \Delta_R^e = -\sin^2(\theta_W)\cos(2\beta)M_Z^2\,. \tag{F.30}$$

The contributions $T^f_{L,R}$ to the running soft masses are usually ignored, and it is common practice to set them to zero in a numerical scan. In our study, we will therefore not consider them any further.

The off-diagonal entries in the soft scalar masses which are induced by the running are of the same order in λ as the high scale ones, with an additional suppression by η. Only for the LL masses of the down-squarks and charged sleptons, the contributions due to R_q and $R_l^{(\prime)}$ can be relatively large as those factors take values up to ~ 35 in a numerical scan. Generally, however, the main effect of the RG evolution on the scalar masses is the change of the diagonal elements. The masses of the first two generations of $(\tilde{m}^2_u)_{LL_{\mathrm{low}}}$, $(\tilde{m}^2_u)_{RR_{\mathrm{low}}}$, $(\tilde{m}^2_d)_{LL_{\mathrm{low}}}$ and all three generations of $(\tilde{m}^2_d)_{RR_{\mathrm{low}}}$, $(\tilde{m}^2_e)_{RR_{\mathrm{low}}}$ are increased at low energy scales due to the second terms in eqs. (F.15)–(F.20). The (33) elements of $(\tilde{m}^2_u)_{LL_{\mathrm{low}}}$, $(\tilde{m}^2_u)_{RR_{Low}}$ and $(\tilde{m}^2_d)_{LL_{\mathrm{low}}}$ can still remain relatively light, as they also feel the effect of R_q, defined in eq. (F.21), entering with a negative sign. Similarly, the enhancement of all three diagonal entries of $(\tilde{m}^2_e)_{LL_{\mathrm{low}}}$ is reduced due to the term $-2\eta_N R_l$ which encodes seesaw effects.

F.3 Low energy mass insertion parameters

With these preparations, we can now formulate the mass insertion parameters at the low energy scale.

Up-type quark sector.

$$(\delta^u_{LL})_{12} = \frac{1}{(p^u_{L1G})^2} e^{-i\theta^d_2}\, \tilde{b}_{12}\, \lambda^4, \tag{F.31}$$
$$(\delta^u_{LL})_{13} = \frac{1}{p^u_{L1G} p^u_{L3G}} e^{-i(4\theta^d_2+\theta^d_3)} (1-\eta\, y_t^2)\, \tilde{b}_{13}\, \lambda^6, \tag{F.32}$$
$$(\delta^u_{LL})_{23} = \frac{1}{p^u_{L1G} p^u_{L3G}} e^{-i(7\theta^d_2+2\theta^d_3)} (1-\eta\, y_t^2)\, \tilde{b}_{23}\, \lambda^5, \tag{F.33}$$

$$(\delta^u_{RR})_{12} = \frac{1}{(p^u_{R1G})^2} e^{-i\theta^d_2}\, \tilde{b}_{12}\, \lambda^4, \tag{F.34}$$
$$(\delta^u_{RR})_{13} = \frac{1}{p^u_{R1G} p^u_{R3G}} (1-2\eta\, y_t^2)\, \tilde{b}_{13}\, \lambda^6, \tag{F.35}$$
$$(\delta^u_{RR})_{23} = \frac{1}{p^u_{R1G} p^u_{R3G}} e^{i(5\theta^d_2+\theta^d_3)} (1-2\eta\, y_t^2)\, \tilde{b}_{23}\, \lambda^5, \tag{F.36}$$

$$(\delta^u_{LR})_{11} = \frac{\alpha_0\, v_u}{m_0\, p^u_{L1G}\, p^u_{R1G}} y_u (1+R^y_u) \left(\frac{\tilde{a}^u_{11}}{y_u} - \frac{\mu(1+R_\mu)}{A_0\, t_\beta} - 2\frac{R^a_u}{1+R^y_u} \right) \lambda^8, \tag{F.37}$$
$$(\delta^u_{LR})_{22} = \frac{\alpha_0\, v_u}{m_0\, p^u_{L1G}\, p^u_{R1G}} y_c (1+R^y_u) \left(\frac{\tilde{a}^u_{22}}{y_c} - \frac{\mu(1+R_\mu)}{A_0\, t_\beta} - 2\frac{R^a_u}{1+R^y_u} \right) \lambda^4, \tag{F.38}$$
$$(\delta^u_{LR})_{33} = \frac{\alpha_0\, v_u}{m_0\, p^u_{L3G}\, p^u_{R3G}} y_t (1+R^y_t) \left(\frac{\tilde{a}^u_{33}}{y_t} - \frac{\mu(1+R_\mu)}{A_0\, t_\beta} - 2\frac{R^a_t}{1+R^y_t} \right), \tag{F.39}$$

$$(\delta^u_{LR})_{12} = (\delta^u_{LR})_{21} = (\delta^u_{LR})_{31} = 0, \tag{F.40}$$
$$(\delta^u_{LR})_{13} = -\frac{\alpha_0\, v_u}{m_0\, p^u_{L1G}\, p^u_{R3G}} \tilde{x}_2\, y_b\, y_t \left(\frac{\tilde{x}^a_2}{\tilde{x}_2} e^{i(\theta^{\tilde{x}_a}_2 - \theta^{\tilde{x}}_2)} + \frac{R^a_t}{1+R^y_t} \right) 2\eta\lambda^7, \tag{F.41}$$

$$\begin{aligned}(\delta^u_{LR})_{23} = \frac{\alpha_0\, v_u}{m_0\, p^u_{L1G}\, p^u_{R3G}} \Bigg\{ &- y_s\, y_b\, y_t \left(\frac{a_s}{y_s} e^{i(\theta^a_s - \theta^y_s)} + \frac{R^a_t}{1+R^y_t} \right) 2\eta\lambda^6 + \\ &+\lambda^7 \Bigg[e^{i\theta^d_2} \tilde{a}^u_{23} (1+R^y_t - \eta\, y_t^2) + 2\eta\, y_b\, y_t \Bigg(e^{i\theta^d_2} \tilde{a}^d_{12} + \left(\frac{a_s}{y_s} e^{i(\theta^a_s-\theta^y_s)} + \frac{R^a_t}{1+R^y_t} \right) \times \\ &\times (\tilde{x}_2 \cos(\theta^d_2) - z^d_4 \cos(4\theta^d_2+\theta^d_3)) + z^d_4 e^{i(4\theta^d_2+\theta^d_3)} \left(e^{i(\theta^a_s-\theta^y_s)} - \frac{z^{d_a}_4}{z^d_4} e^{i(\theta^{z_{da}}_4 - \theta^{z_d}_4)} \right) \Bigg) \Bigg] \Bigg\},\end{aligned} \tag{F.42}$$

$$(\delta^u_{LR})_{32} = \frac{\alpha_0\, v_u}{m_0\, p^u_{L3G}\, p^u_{R1G}} (1+R^y_t - 2\eta\, y_t^2) e^{i(3\theta^d_2+\theta^d_3)} \tilde{a}^u_{23}\, \lambda^7, \tag{F.43}$$

where, in eq. (F.42), z^d_4 and $z^{d_a}_4$ parameterise the $\mathcal{O}(\lambda^5)$ NLO corrections of the (22) and (23) elements of the down-type Yukawa and soft trilinear structures, respectively. Originating from the second term of eq. (4.7), $z^d_4 e^{i\theta^{z_d}_4} = y^d_2\, \tilde{\delta}^d_{3,2_{(4)}}\, \phi^d_2$, so that $\theta^{z_d}_4 = 6\theta^d_2 + 4\theta^d_3$. We see that the term proportional to $\eta\,\lambda^6$, which was generated in $\tilde{A}^u_{\mathrm{low}_{23}}$ via th RG evolution, is the source of the associated term in $(\delta^u_{LR})_{23}$, which was of order λ^7 at the GUT scale. In eqs. (F.31)–(F.43) we have defined the factors

$$
\begin{aligned}
p^u_{L1G} &= \sqrt{b_{01} + 6.5\, x}, \qquad p^u_{L3G} = \sqrt{b_{02} + 6.5\, x - 2\eta R_q + \frac{v_u^2}{m_0^2} y_t^2 (1 + R_t^y)^2}\,, \\
p^u_{R1G} &= \sqrt{b_{01} + 6.15\, x}, \qquad p^u_{R3G} = \sqrt{b_{02} + 6.15\, x - 4\eta R_q + \frac{v_u^2}{m_0^2} y_t^2 (1 + R_t^y)^2}\,,
\end{aligned}
\tag{F.44}
$$

which are related to the full sfermion mass matrices by

$$
\begin{aligned}
m_{\tilde{u}_{LL}} &\approx m_{\tilde{c}_{LL}} \approx m_0\, p^u_{L1G}\,, \qquad m_{\tilde{t}_{LL}} \approx m_0\, p^u_{L3G}\,, \\
m_{\tilde{u}_{RR}} &\approx m_{\tilde{c}_{RR}} \approx m_0\, p^u_{R1G}\,, \qquad m_{\tilde{t}_{RR}} \approx m_0\, p^u_{R3G}\,,
\end{aligned}
\tag{F.45}
$$

whose GUT scale definitions are given in eq. (7.1). The μ parameter at the low energy scale can be estimated by

$$
\mu_{\text{low}} \approx \mu\,(1 + R_\mu)\,, \qquad R_\mu = 4\eta \left(0.9\, g_U^2 - \frac{3}{4} y_t^2\right) - 3\eta_N\, y_D^2\,. \tag{F.46}
$$

Down-type quark sector.

$$
\begin{aligned}
(\delta^d_{LL})_{12} &= \frac{1}{(p^d_{L1G})^2} \tilde{B}_{12}\, \lambda^3, && \text{(F.47)} \\
(\delta^d_{LL})_{13} &= \frac{1}{p^d_{L1G} p^d_{L13}} e^{i\theta^d_2} \frac{\tilde{x}_2^2}{y_b\, y_s} \left(b_{01} - b_{02} + 2\eta\, R_q\right) \left(1 + \frac{\eta\, y_t^2}{1 + R_b^y}\right) \lambda^4, && \text{(F.48)} \\
(\delta^d_{LL})_{23} &= \frac{1}{p^d_{L1G} p^d_{L13}} \frac{y_s}{y_b} \left(b_{01} - b_{02} + 2\eta\, R_q\right) \left(1 + \frac{\eta\, y_t^2}{1 + R_b^y}\right) \lambda^2, && \text{(F.49)}
\end{aligned}
$$

$$
\begin{aligned}
(\delta^d_{RR})_{12} &= -(\delta^d_{RR})_{13} = \frac{1}{(p^d_R)^2} e^{i\theta^d_2}\, \tilde{R}_{12}\, \lambda^4, && \text{(F.50)} \\
(\delta^d_{RR})_{23} &= -\frac{1}{(p^d_R)^2} \tilde{R}_{12}\, \lambda^4, && \text{(F.51)}
\end{aligned}
$$

$$
\begin{aligned}
(\delta^d_{LR})_{11} &= \frac{\alpha_0\, v_d}{m_0\, p^d_{L1G}\, p^d_R} \frac{\tilde{x}_2^2}{y_s} (1 + R_d^y) \left(\frac{\tilde{a}^d_{11}}{\tilde{x}_2^2 / y_s} - \frac{\mu\, t_\beta (1 + R_\mu)}{A_0} - 2\frac{R_d^a}{1 + R_d^y}\right) \lambda^6, && \text{(F.52)} \\
(\delta^d_{LR})_{22} &= \frac{\alpha_0\, v_d}{m_0\, p^d_{L1G}\, p^d_R} y_s (1 + R_d^y) \left(\frac{\tilde{a}^d_{22}}{y_s} - \frac{\mu\, t_\beta (1 + R_\mu)}{A_0} - 2\frac{R_d^a}{1 + R_d^y}\right) \lambda^4, && \text{(F.53)} \\
(\delta^d_{LR})_{33} &= \frac{\alpha_0\, v_d}{m_0\, p^d_{L3G}\, p^d_R} y_b (1 + R_b^y) \left(\frac{\tilde{a}^d_{33}}{y_b} - \frac{\mu\, t_\beta (1 + R_\mu)}{A_0} - 2\frac{R_b^a}{1 + R_b^y}\right) \lambda^2, && \text{(F.54)}
\end{aligned}
$$

$$
\begin{aligned}
(\delta^d_{LR})_{12} &= -(\delta^d_{LR})_{21} = (\delta^d_{LR})_{13} = \frac{\alpha_0\, v_d}{m_0\, p^d_{L1G}\, p^d_R} (1 + R_d^y) \tilde{a}^d_{12}\, \lambda^5, && \text{(F.55)} \\
(\delta^d_{LR})_{23} &= \frac{\alpha_0\, v_d}{m_0\, p^d_{L1G}\, p^d_R} y_s (1 + R_d^y) \left(\frac{\tilde{a}^d_{23}}{y_s} + 2\frac{\eta\, y_t^2}{1 + R_b^y} \left(\frac{a_t}{y_t} + \frac{R_d^a}{1 + R_d^y}\right)\right) \lambda^4, && \text{(F.56)} \\
(\delta^d_{LR})_{31} &= \frac{\alpha_0\, v_d}{m_0\, p^d_{L3G}\, p^d_R} e^{-i\theta^d_2} (1 + R_b^y) \tilde{a}^d_{31}\, \lambda^6, && \text{(F.57)}
\end{aligned}
$$

JHEP02(2016)118

$$
\begin{aligned}
(\delta^d_{LR})_{32} \;=\; & \frac{\alpha_0\,\upsilon_d}{m_0\,p^d_{L^3G}\,p^d_R}(1+R^y_b)y_b\left(\frac{\tilde{a}^d_{32}}{y_b}+2\eta y_t^2\,\frac{y_s^2}{y_b^2}\left[\frac{2(1+R^y_b)+\eta y_t^2}{2(1+R^y_b)^2}\,\frac{\tilde{a}^d_{23}}{y_s}\right.\right. \\
& \left.\left.+\left(\frac{a_t}{y_t}+\frac{R^a_d}{1+R^y_d}\right)\frac{(1+R^y_d)^2}{(1+R^y_b)^3}\right]\right)\lambda^6,
\end{aligned} \tag{F.58}
$$

where

$$
p^d_{L^1G} \;=\; \sqrt{b_{01}+6.5\,x}, \qquad p^d_{L^3G} = \sqrt{b_{02}+6.5\,x-4\eta R_q}, \qquad p^d_R = \sqrt{1+6.1x}, \tag{F.59}
$$

such that

$$
\begin{aligned}
& m_{\tilde{d}_{LL}} \approx m_{\tilde{s}_{LL}} \approx m_0\,p^d_{L^1G}\,, \qquad m_{\tilde{b}_{LL}} \approx m_0\,p^d_{L^3G}\,, \\
& m_{\tilde{d}_{RR}} \approx m_{\tilde{s}_{RR}} \approx m_{\tilde{b}_{RR}} \approx m_0\,p^d_R\,.
\end{aligned} \tag{F.60}
$$

Charged lepton sector.

$$
(\delta^e_{LL})_{12} \;=\; -(\delta^e_{LL})_{23} = \frac{1}{(p^e_L)^2}\left(\tilde{R}_{12}-2\eta_N\tilde{E}_{12}\right)\lambda^4, \tag{F.61}
$$

$$
(\delta^e_{LL})_{13} \;=\; -\frac{1}{(p^e_L)^2}\left(\tilde{R}_{12}-2\eta_N\tilde{E}^*_{12}\right)\lambda^4, \tag{F.62}
$$

$$
(\delta^e_{RR})_{12} \;=\; -\frac{1}{(p^e_{R^1G})^2}e^{i\theta^d_2}\frac{\tilde{B}_{12}}{3}\,\lambda^3, \tag{F.63}
$$

$$
(\delta^e_{RR})_{13} \;=\; \frac{1}{p^e_{R^1G}\,p^e_{R^3G}}\,\frac{\tilde{B}_{13}}{3}\,\lambda^4, \tag{F.64}
$$

$$
(\delta^e_{RR})_{23} \;=\; \frac{1}{p^e_{R^1G}\,p^e_{R^3G}}3\tilde{B}_{23}\,\lambda^2, \tag{F.65}
$$

$$
(\delta^e_{LR})_{11} \;=\; \frac{1}{p^e_L\,p^e_{R^1G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}\,\frac{\tilde{x}_2^2}{3\,y_s}(1+R^y_e)\left(\frac{y_s}{\tilde{x}_2^2}\tilde{a}^d_{11}-\frac{\mu\,t_\beta}{A_0}(1+R_\mu)-2\frac{R^a_e}{1+R^y_e}\right)\lambda^6, \tag{F.66}
$$

$$
(\delta^e_{LR})_{22} \;=\; \frac{1}{p^e_L\,p^e_{R^1G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}3\,y_s(1+R^y_e)\left(\frac{\tilde{a}^d_{22}}{y_s}-\frac{\mu\,t_\beta}{A_0}(1+R_\mu)-2\frac{R^a_e}{1+R^y_e}\right)\lambda^4, \tag{F.67}
$$

$$
(\delta^e_{LR})_{33} \;=\; \frac{1}{p^e_L\,p^e_{R^3G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}y_b(1+R^y_e)\left(\frac{\tilde{a}^d_{33}}{y_b}-\frac{\mu\,t_\beta}{A_0}(1+R_\mu)-2\frac{R^a_e}{1+R^y_e}\right)\lambda^2, \tag{F.68}
$$

$$
(\delta^e_{LR})_{12} \;=\; \frac{1}{p^e_L\,p^e_{R^1G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}(1+R^y_e)e^{i\theta^d_2}\tilde{a}^d_{12}\,\lambda^5, \tag{F.69}
$$

$$
(\delta^e_{LR})_{13} \;=\; \frac{1}{p^e_L\,p^e_{R^3G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}\left((1+R^y_e)\tilde{a}^d_{31}+2\eta_N\,y_D\,R_\nu\,y_b\left(\frac{\alpha_D}{y_D}+\frac{R^a_e}{1+R^y_e}\right)\right)\lambda^6, \tag{F.70}
$$

$$
(\delta^e_{LR})_{21} \;=\; (\delta^e_{LR})_{31} = -\frac{1}{p^e_L\,p^e_{R^1G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}(1+R^y_e)e^{-i\theta^d_2}\tilde{a}^d_{12}\,\lambda^5, \tag{F.71}
$$

$$
(\delta^e_{LR})_{23} \;=\; \frac{1}{p^e_L\,p^e_{R^3G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}\left((1+R^y_e)\tilde{a}^e_{23}+2\eta_N\,y_D\,R_\nu\,y_b\left(\frac{R^a_\nu}{R_\nu}+\frac{R^a_e}{1+R^y_e}\right)\right)\lambda^6, \tag{F.72}
$$

$$
(\delta^e_{LR})_{32} \;=\; \frac{1}{p^e_L\,p^e_{R^1G}}\,\frac{\upsilon_d\,\alpha_0}{m_0}(1+R^y_e)3\,\tilde{a}^d_{23}\,\lambda^4, \tag{F.73}
$$

where

$$p_L^e = \sqrt{1 + 0.5\,x - 2\eta_N\,R_l}, \qquad p_{R1G}^e = \sqrt{b_{01} + 0.15\,x}, \qquad p_{R3G}^e = \sqrt{b_{02} + 0.15\,x}, \tag{F.74}$$

such that

$$\begin{aligned} &m_{\tilde{e}_{LL}} \approx m_{\tilde{\mu}_{LL}} \approx m_{\tilde{\tau}_{LL}} \approx m_0\,p_L^e\,, \\ &m_{\tilde{e}_{RR}} \approx m_{\tilde{\mu}_{RR}} \approx m_0\,p_{R1G}^e\,, \qquad m_{\tilde{\tau}_{RR}} \approx m_0\,p_{R3G}^e\,. \end{aligned} \tag{F.75}$$

References

[1] S.F. King, *Neutrino mass models*, *Rept. Prog. Phys.* **67** (2004) 107 [hep-ph/0310204] [INSPIRE].

[2] G. Altarelli and F. Feruglio, *Discrete Flavor Symmetries and Models of Neutrino Mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [arXiv:1002.0211] [INSPIRE].

[3] S.F. King and C. Luhn, *Neutrino Mass and Mixing with Discrete Symmetry*, *Rept. Prog. Phys.* **76** (2013) 056201 [arXiv:1301.1340] [INSPIRE].

[4] S.F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino Mass and Mixing: from Theory to Experiment*, *New J. Phys.* **16** (2014) 045018 [arXiv:1402.4271] [INSPIRE].

[5] S.F. King, *Models of Neutrino Mass, Mixing and CP-violation*, *J. Phys.* **G 42** (2015) 123001 [arXiv:1510.02091] [INSPIRE].

[6] R.S. Chivukula and H. Georgi, *Composite Technicolor Standard Model*, *Phys. Lett.* **B 188** (1987) 99 [INSPIRE].

[7] L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939 [INSPIRE].

[8] A.J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, *Phys. Lett.* **B 500** (2001) 161 [hep-ph/0007085] [INSPIRE].

[9] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: An Effective field theory approach*, *Nucl. Phys.* **B 645** (2002) 155 [hep-ph/0207036] [INSPIRE].

[10] V. Cirigliano, B. Grinstein, G. Isidori and M.B. Wise, *Minimal flavor violation in the lepton sector*, *Nucl. Phys.* **B 728** (2005) 121 [hep-ph/0507001] [INSPIRE].

[11] C. Bobeth, T. Ewerth, F. Krüger and J. Urban, *Enhancement of $B(\bar{B}_d \to \mu^+\mu^-)/B(\bar{B}_s \to \mu^+\mu^-)$ in the MSSM with minimal flavor violation and large tan beta*, *Phys. Rev.* **D 66** (2002) 074021 [hep-ph/0204225] [INSPIRE].

[12] B. Grinstein, M. Redi and G. Villadoro, *Low Scale Flavor Gauge Symmetries*, *JHEP* **11** (2010) 067 [arXiv:1009.2049] [INSPIRE].

[13] D. Guadagnoli, R.N. Mohapatra and I. Sung, *Gauged Flavor Group with Left-Right Symmetry*, *JHEP* **04** (2011) 093 [arXiv:1103.4170] [INSPIRE].

[14] R.T. D'Agnolo and D.M. Straub, *Gauged flavour symmetry for the light generations*, *JHEP* **05** (2012) 034 [arXiv:1202.4759] [INSPIRE].

[15] G. Krnjaic and D. Stolarski, *Gauging the Way to MFV*, *JHEP* **04** (2013) 064 [arXiv:1212.4860] [INSPIRE].

[16] R. Franceschini and R.N. Mohapatra, *New Patterns of Natural R-Parity Violation with Supersymmetric Gauged Flavor*, *JHEP* **04** (2013) 098 [arXiv:1301.3637] [INSPIRE].

[17] D.J.H. Chung, L.L. Everett, G.L. Kane, S.F. King, J.D. Lykken and L.-T. Wang, *The soft supersymmetry breaking Lagrangian: theory and applications*, *Phys. Rept.* **407** (2005) 1 [hep-ph/0312378] [INSPIRE].

[18] H. Georgi and S.L. Glashow, *Unity of All Elementary Particle Forces*, *Phys. Rev. Lett.* **32** (1974) 438 [INSPIRE].

[19] J.C. Pati and A. Salam, *Unified Lepton-Hadron Symmetry and a Gauge Theory of the Basic Interactions*, *Phys. Rev.* **D 8** (1973) 1240 [INSPIRE].

[20] J.C. Pati and A. Salam, *Lepton Number as the Fourth Color*, *Phys. Rev.* **D 10** (1974) 275 [*Erratum ibid.* **D 11** (1975) 703] [INSPIRE].

[21] T. Feldmann, *See-Saw Masses for Quarks and Leptons in* SU(5), *JHEP* **04** (2011) 043 [arXiv:1010.2116] [INSPIRE].

[22] T. Feldmann, F. Hartmann, W. Kilian and C. Luhn, *Combining Pati-Salam and Flavour Symmetries*, *JHEP* **10** (2015) 160 [arXiv:1506.00782] [INSPIRE].

[23] R. Barbieri and F. Senia, *Minimal flavour violation and* SU(5)*-unification*, *Eur. Phys. J.* **C 75** (2015) 602 [arXiv:1506.09201] [INSPIRE].

[24] G.G. Ross and O. Vives, *Yukawa structure, flavor and CP-violation in supergravity*, *Phys. Rev.* **D 67** (2003) 095013 [hep-ph/0211279] [INSPIRE].

[25] G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and nonAbelian family symmetry in SUSY*, *Nucl. Phys.* **B 692** (2004) 50 [hep-ph/0401064] [INSPIRE].

[26] S. Antusch, S.F. King and M. Malinsky, *Solving the SUSY Flavour and CP Problems with* SU(3) *Family Symmetry*, *JHEP* **06** (2008) 068 [arXiv:0708.1282] [INSPIRE].

[27] S. Antusch, S.F. King, M. Malinsky and G.G. Ross, *Solving the SUSY Flavour and CP Problems with Non-Abelian Family Symmetry and Supergravity*, *Phys. Lett.* **B 670** (2009) 383 [arXiv:0807.5047] [INSPIRE].

[28] W. Altmannshofer, A.J. Buras, S. Gori, P. Paradisi and D.M. Straub, *Anatomy and Phenomenology of FCNC and CPV Effects in SUSY Theories*, *Nucl. Phys.* **B 830** (2010) 17 [arXiv:0909.1333] [INSPIRE].

[29] A.J. Buras and J. Girrbach, *BSM models facing the recent LHCb data: A First look*, *Acta Phys. Polon.* **B 43** (2012) 1427 [arXiv:1204.5064] [INSPIRE].

[30] D.M. Straub, *Overview of Constraints on New Physics in Rare B Decays*, arXiv:1205.6094 [INSPIRE].

[31] S.F. King and I.N.R. Peddie, *Canonical normalization and Yukawa matrices*, *Phys. Lett.* **B 586** (2004) 83 [hep-ph/0312237] [INSPIRE].

[32] S.F. King, I.N.R. Peddie, G.G. Ross, L. Velasco-Sevilla and O. Vives, *Kähler corrections and softly broken family symmetries*, *JHEP* **07** (2005) 049 [hep-ph/0407012] [INSPIRE].

[33] P. Minkowski, *$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?*, *Phys. Lett.* **B 67** (1977) 421 [INSPIRE].

[34] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories*, *Conf. Proc.* **C 790927** (1979) 315 [arXiv:1306.4669] [INSPIRE].

[35] T. Yanagida, *Horizontal Symmetry And Masses Of Neutrinos*, *Conf. Proc.* **C 7902131** (1979) 95.

[36] R.N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Violation*, *Phys. Rev. Lett.* **44** (1980) 912 [INSPIRE].

[37] Y. Yamada, *SUSY and GUT threshold effects in SUSY* SU(5) *models*, *Z. Phys.* **C 60** (1993) 83 [INSPIRE].

[38] S. Antusch and M. Spinrath, *Quark and lepton masses at the GUT scale including SUSY threshold corrections*, *Phys. Rev.* **D 78** (2008) 075020 [arXiv:0804.0717] [INSPIRE].

[39] C. Hagedorn, S.F. King and C. Luhn, *A SUSY GUT of Flavour with $S_4 \times$ SU(5) to NLO*, *JHEP* **06** (2010) 048 [arXiv:1003.4249] [INSPIRE].

[40] C. Hagedorn, S.F. King and C. Luhn, *SUSY $S_4 \times$ SU(5) revisited*, *Phys. Lett.* **B 717** (2012) 207 [arXiv:1205.3114] [INSPIRE].

[41] S.P. Martin, *A Supersymmetry primer*, hep-ph/9709356 [INSPIRE].

[42] Y. Shirman, *Introduction to Supersymmetry and Supersymmetry Breaking*, arXiv:0907.0039 [INSPIRE].

[43] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, *Nucl. Phys.* **B 720** (2005) 64 [hep-ph/0504165] [INSPIRE].

[44] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing, A_4 and the modular symmetry*, *Nucl. Phys.* **B 741** (2006) 215 [hep-ph/0512103] [INSPIRE].

[45] L. Wolfenstein, *Parametrization of the Kobayashi-Maskawa Matrix*, *Phys. Rev. Lett.* **51** (1983) 1945 [INSPIRE].

[46] G.-J. Ding, S.F. King, C. Luhn and A.J. Stuart, *Spontaneous CP-violation from vacuum alignment in S_4 models of leptons*, *JHEP* **05** (2013) 084 [arXiv:1303.6180] [INSPIRE].

[47] F. Feruglio, C. Hagedorn and R. Ziegler, *A realistic pattern of lepton mixing and masses from S_4 and CP*, *Eur. Phys. J.* **C 74** (2014) 2753 [arXiv:1303.7178] [INSPIRE].

[48] C. Luhn, *Trimaximal TM_1 neutrino mixing in S_4 with spontaneous CP-violation*, *Nucl. Phys.* **B 875** (2013) 80 [arXiv:1306.2358] [INSPIRE].

[49] R. Gatto, G. Sartori and M. Tonin, *Weak Selfmasses, Cabibbo Angle and Broken* SU(2) $\times$ SU(2), *Phys. Lett.* **B 28** (1968) 128 [INSPIRE].

[50] H. Georgi and C. Jarlskog, *A New Lepton-Quark Mass Relation in a Unified Theory*, *Phys. Lett.* **B 86** (1979) 297 [INSPIRE].

[51] X.-G. He and A. Zee, *Minimal modification to the tri-bimaximal neutrino mixing*, *Phys. Lett.* **B 645** (2007) 427 [hep-ph/0607163] [INSPIRE].

[52] W. Grimus and L. Lavoura, *A model for trimaximal lepton mixing*, *JHEP* **09** (2008) 106 [arXiv:0809.0226] [INSPIRE].

[53] C.H. Albright, A. Dueck and W. Rodejohann, *Possible Alternatives to Tri-bimaximal Mixing*, *Eur. Phys. J.* **C 70** (2010) 1099 [arXiv:1004.2798] [INSPIRE].

[54] H. Ishimori, Y. Shimizu, M. Tanimoto and A. Watanabe, *Neutrino masses and mixing from S_4 flavor twisting*, *Phys. Rev.* **D 83** (2011) 033004 [arXiv:1010.3805] [INSPIRE].

[55] Y. Shimizu, M. Tanimoto and A. Watanabe, *Breaking Tri-bimaximal Mixing and Large θ_{13}*, *Prog. Theor. Phys.* **126** (2011) 81 [arXiv:1105.2929] [INSPIRE].

[56] E. Ma and D. Wegman, *Nonzero θ_{13} for neutrino mixing in the context of A_4 symmetry*, *Phys. Rev. Lett.* **107** (2011) 061803 [arXiv:1106.4269] [INSPIRE].

[57] X.-G. He and A. Zee, *Minimal Modification to Tri-bimaximal Mixing*, *Phys. Rev.* **D 84** (2011) 053004 [arXiv:1106.4359] [INSPIRE].

[58] S.F. King and C. Luhn, *Trimaximal neutrino mixing from vacuum alignment in A4 and S4 models*, *JHEP* **09** (2011) 042 [arXiv:1107.5332] [INSPIRE].

[59] I.K. Cooper, S.F. King and C. Luhn, *$A4 \times \mathrm{SU}(5)$ SUSY GUT of Flavour with Trimaximal Neutrino Mixing*, *JHEP* **06** (2012) 130 [arXiv:1203.1324] [INSPIRE].

[60] S.F. King, C. Luhn and A.J. Stuart, *A Grand $\Delta(96) \times \mathrm{SU}(5)$ Flavour Model*, *Nucl. Phys.* **B 867** (2013) 203 [arXiv:1207.5741] [INSPIRE].

[61] C. Luhn, K.M. Parattu and A. Wingerter, *A Minimal Model of Neutrino Flavor*, *JHEP* **12** (2012) 096 [arXiv:1210.1197] [INSPIRE].

[62] F. Feruglio, C. Hagedorn and L. Merlo, *Vacuum Alignment in SUSY A4 Models*, *JHEP* **03** (2010) 084 [arXiv:0910.4058] [INSPIRE].

[63] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Running neutrino masses, mixings and CP phases: Analytical results and phenomenological consequences*, *Nucl. Phys.* **B 674** (2003) 401 [hep-ph/0305273] [INSPIRE].

[64] S. Antusch, J. Kersten, M. Lindner, M. Ratz and M.A. Schmidt, *Running neutrino mass parameters in see-saw scenarios*, *JHEP* **03** (2005) 024 [hep-ph/0501272] [INSPIRE].

[65] C. Jarlskog, *Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP-violation*, *Phys. Rev. Lett.* **55** (1985) 1039 [INSPIRE].

[66] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A Complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387] [INSPIRE].

[67] M. Misiak, S. Pokorski and J. Rosiek, *Supersymmetry and FCNC effects*, *Adv. Ser. Direct. High Energy Phys.* **15** (1998) 795 [hep-ph/9703442] [INSPIRE].

[68] M. Dimou, S.F. King and C. Luhn, *Phenomenological Implications of an $S_4 \times \mathrm{SU}(5)$ SUSY GUT of Flavour*, arXiv:1512.09063 [INSPIRE].

[69] F. Feruglio, C. Hagedorn and R. Ziegler, *Lepton Mixing Parameters from Discrete and CP Symmetries*, *JHEP* **07** (2013) 027 [arXiv:1211.5560] [INSPIRE].

[70] M. Holthausen, M. Lindner and M.A. Schmidt, *CP and Discrete Flavour Symmetries*, *JHEP* **04** (2013) 122 [arXiv:1211.6953] [INSPIRE].

[71] W. Grimus and M.N. Rebelo, *Automorphisms in gauge theories and the definition of CP and P*, *Phys. Rept.* **281** (1997) 239 [hep-ph/9506272] [INSPIRE].

[72] M.-C. Chen, M. Fallbacher, K.T. Mahanthappa, M. Ratz and A. Trautner, *CP Violation from Finite Groups*, *Nucl. Phys.* **B 883** (2014) 267 [arXiv:1402.0507] [INSPIRE].

JHEP02(2016)118