

BV master action for heterotic and type II string field theories

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ABSTRACT: We construct the quantum BV master action for heterotic and type II string field theories.

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1 Introduction

Conventional formulation of superstring theory is based on an on-shell formulation in which the S-matrix of on-shell external states are expressed as correlation functions of conformally invariant vertex operators on a Riemann surface integrated over the moduli space of the Riemann surface. However this approach is not suitable for addressing many issues even within perturbation theory — this includes the problem of mass renormalization and vacuum shift. In the conventional approach, both these problems show up as infrared divergences associated with separating type degenerations of the Riemann surfaces, but there is no suggested cure for this in perturbation theory [1, 2]. In recent papers [3–9] we proposed a possible resolution of these problems based on one particle irreducible (1PI) effective action in which the world-sheet theory is used to first construct a gauge invariant 1PI effective string field theory, and then we use this 1PI action to address the problem of finding the vacuum and the renormalized masses following the usual route of quantum field theory. Since the 1PI action itself does not include contributions from separating type degenerations of the Riemann surface, it does not suffer from any infrared divergences associated with these degenerations. This makes this approach well-suited for addressing the origin and resolution of these divergences in the S-matrix.

However the 1PI effective action does receive contribution from regions of the moduli space associated with non-separating type degenerations. This makes it difficult to address issues related to such divergences using the 1PI action since these divergences are hidden in the building blocks of the theory — the 1PI amplitudes. For this reason it is useful to look for a field theory of strings in which the amplitudes are built from the Feynman diagrams of this string field theory. In this formalism the elementary vertices will be free from infrared divergences associated with both separating and non-separating type degenerations, and

all infrared divergences will appear when we build Feynman diagrams using these vertices. This will make all the infrared divergences manifest in perturbation theory, making it easier to use conventional field theory tools to analyze the effect of these infrared divergences. Needless to say, such a formulation also has the potential of opening the path to studying non-perturbative aspects of string theory.

For bosonic string theory, there has been successful construction of a field theory of open strings as well as closed strings based on the Batalin-Vilkovisky (BV) formalism [10–17]. There have been many attempts in the past to formulate a field theory of closed superstrings / heterotic strings, but for various reasons, none has been completely successful at the quantum level (see [18–34] for a partial list of references). In this paper we generalize the approach of [7, 8] for the construction of closed bosonic string field theory to construct heterotic and type II string field theories.

The main difficulty in constructing a field theory for heterotic and type II strings has been in the Ramond sector since there is no natural way to write down a kinetic term involving Ramond sector fields.¹ In the context of 1PI effective action, this problem was recently addressed in [8] using additional fields in the Ramond sector and then imposing a constraint on the external states that removes the extra states associated with these additional fields. The combination on which we impose the constraint satisfies free field equations of motion, and hence once we set them to zero, they are not produced by interactions. This makes the whole procedure consistent, leading to a set of off-shell amplitudes satisfying the desired Ward identities. This was then used to address the problem of computing renormalized masses, and also computing amplitudes around the shifted vacuum in cases where the perturbative vacuum is destabilized by quantum corrections.

The main observation we make in this paper is that the same trick can be used to construct a BV master action for heterotic and type II string field theory. At the level of the classical theory itself, we introduce an additional set of fields. This doubles the number of degrees of freedom. The resulting gauge invariant theory has the sector that describes correctly the spectrum and interaction of string theory known from the first quantized approach, but there is an additional sector containing free fields. This theory can be quantized using BV formalism following the same procedure as in the case of closed bosonic string field theory, but the quantum theory will also have the additional sector containing free fields. At the end we are free to set the free fields to zero since they are never produced in any interactions (i.e. in the scattering involving external states in the interacting sector, the additional fields will never be produced as intermediate states).

We shall not try to make the paper self-contained. Instead we shall assume that the reader is familiar with the construction of the BV master action in closed string field theory [16]. Some familiarity with the construction of the 1PI action in superstring field theory is also desirable, although we give a brief review of some of the results of [7, 8] in section 2. In section 3 we describe the construction of the action satisfying classical master equation. In section 4 we describe the construction of the full quantum master action and its gauge fixing.

¹A recent proposal for dealing with this problem in classical open superstring field theory can be found in [34].

2 Review

Since our construction will follow closely the conventions used in [7, 8], we shall not give a detailed review of the background material, but only describe a few ingredients that will be used in our analysis. A more detailed review can be found e.g. in [9]. This section will contain three parts. In section 2.1 we review some of the details of the superconformal field theory (SCFT) describing the world-sheet theory of the matter and ghost system [35]. In section 2.2 we review the construction of certain multilinear functions of states of the SCFT and how we use them to construct the 1PI effective action. In section 2.3 we describe the construction of classical (tree level) string field theory from the 1PI action. This classical action will then be used in section 3 for the construction of the classical master action, which will then be generalized to quantum master action in section 4.

2.1 The world-sheet theory

We denote by \mathcal{H} the full Hilbert space of matter ghost SCFT carrying arbitrary picture and ghost numbers, and by \mathcal{H}_T a subspace of \mathcal{H} satisfying the constraints

$$b_0^- |s\rangle = 0, \quad L_0^- |s\rangle = 0, \quad \text{for } |s\rangle \in \mathcal{H}_T, \quad (2.1)$$

where

$$b_0^\pm = b_0 \pm \bar{b}_0, \quad L_0^\pm = L_0 \pm \bar{L}_0, \quad c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0). \quad (2.2)$$

We denote by \mathcal{X} the picture changing operator (PCO) — for type II string theories we also have its anti-holomorphic counterpart $\bar{\mathcal{X}}$. \mathcal{X}_0 and $\bar{\mathcal{X}}_0$ are their zero modes [25, 27, 36]:²

$$\mathcal{X}_0 = \oint \frac{dz}{z} \mathcal{X}(z), \quad \bar{\mathcal{X}}_0 = \oint \frac{d\bar{z}}{\bar{z}} \bar{\mathcal{X}}(\bar{z}) \quad (\text{in type II}). \quad (2.3)$$

In heterotic theory we divide \mathcal{H}_T into Neveu-Schwarz (NS) sector \mathcal{H}_{NS} and Ramond (R) sector \mathcal{H}_R . In the type II theory the corresponding division is \mathcal{H}_{NSNS} , \mathcal{H}_{NSR} , \mathcal{H}_{RNS} and \mathcal{H}_{RR} . The operator \mathcal{G} in these theories is defined as

$$\mathcal{G}|s\rangle = \begin{cases} |s\rangle & \text{if } |s\rangle \in \mathcal{H}_{NS} \\ \mathcal{X}_0 |s\rangle & \text{if } |s\rangle \in \mathcal{H}_R \end{cases}, \quad (2.4)$$

in heterotic string theory, and

$$\mathcal{G}|s\rangle = \begin{cases} |s\rangle & \text{if } |s\rangle \in \mathcal{H}_{NSNS} \\ \mathcal{X}_0 |s\rangle & \text{if } |s\rangle \in \mathcal{H}_{NSR} \\ \bar{\mathcal{X}}_0 |s\rangle & \text{if } |s\rangle \in \mathcal{H}_{RNS} \\ \mathcal{X}_0 \bar{\mathcal{X}}_0 |s\rangle & \text{if } |s\rangle \in \mathcal{H}_{RR} \end{cases}, \quad (2.5)$$

in type II theory. It satisfies

$$[Q_B, \mathcal{G}] = 0, \quad [b_0^\pm, \mathcal{G}] = 0. \quad (2.6)$$

²In [34] a different operator with properties similar to \mathcal{X}_0 was used.

The basis states in \mathcal{H}_{NS} are taken to be grassmann even for even ghost number and grassmann odd for odd ghost number. In \mathcal{H}_R the situation is opposite. In type II string theory the basis states are grassmann even for even ghost number and grassmann odd for odd ghost number in \mathcal{H}_{NSNS} and \mathcal{H}_{RR} . In \mathcal{H}_{RNS} and \mathcal{H}_{NSR} the situation is opposite.

In heterotic string theory, we denote by $\widehat{\mathcal{H}}_T$ the subspace of \mathcal{H}_T containing states of picture numbers -1 and $-1/2$ in the NS and R sectors respectively. $\widetilde{\mathcal{H}}_T$ will denote the subspace of states of picture numbers -1 and $-3/2$ in the NS and R sectors. In type II theories $\widehat{\mathcal{H}}_T$ will contain states of picture numbers $(-1, -1)$, $(-1, -1/2)$, $(-1/2, -1)$ and $(-1/2, -1/2)$ in the NSNS, NSR, RNS and RR sectors while $\widetilde{\mathcal{H}}_T$ will contain states of picture numbers $(-1, -1)$, $(-1, -3/2)$, $(-3/2, -1)$ and $(-3/2, -3/2)$ in the NSNS, NSR, RNS and RR sectors.

2.2 The 1PI action

Generalizing the construction of [16] for closed bosonic string theory, in [6–8] we introduced, for the heterotic string, the space $\widetilde{\mathcal{P}}_{g,m,n}$ whose base was the moduli space $\mathcal{M}_{g,m,n}$ of genus g Riemann surface with m NS and n R punctures and whose fiber contains information on local coordinates up to phases and also the locations of $(2g - 2 + m + n/2)$ PCO's. (The generalization to type II strings is straightforward.) Furthermore for m external NS sector states and $n = N - m$ external R-sector states in $\widehat{\mathcal{H}}_T$, collectively called $|A_1\rangle, \dots |A_N\rangle$, we introduced on $\widetilde{\mathcal{P}}_{g,m,n}$ a p -form $\Omega_p^{(g,m,n)}(|A_1\rangle, \dots |A_N\rangle)$ for all integer $p \geq 0$ satisfying certain desired properties. Finally for each g, m, n we introduced a specific subspace of $\mathcal{M}_{g,m,n}$ and (generalized) section³ $\mathcal{R}_{g,m,n}$ of $\widetilde{\mathcal{P}}_{g,m,n}$ on these subspaces satisfying the conditions

$$\begin{aligned} \partial\mathcal{R}_{g,m,n} = & -\frac{1}{2} \sum_{\substack{g_1, g_2 \\ g_1 + g_2 = g}} \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = m + 2}} \sum_{\substack{n_1, n_2 \\ n_1 + n_2 = n}} \mathbf{S}[\{\mathcal{R}_{g_1, m_1, n_1}, \mathcal{R}_{g_2, m_2, n_2}\}] \\ & -\frac{1}{2} \sum_{\substack{g_1, g_2 \\ g_1 + g_2 = g}} \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = m}} \sum_{\substack{n_1, n_2 \\ n_1 + n_2 = n + 2}} \mathbf{S}[\{\mathcal{R}_{g_1, m_1, n_1}; \mathcal{R}_{g_2, m_2, n_2}\}]. \end{aligned} \quad (2.7)$$

Here $\partial\mathcal{R}_{g,m,n}$ denotes the boundary of $\mathcal{R}_{g,m,n}$ and \mathbf{S} denotes the operation of summing over inequivalent permutations of external NS-sector punctures and also external R-sector punctures. $\{\mathcal{R}_{g_1, m_1, n_1}, \mathcal{R}_{g_2, m_2, n_2}\}$ denotes the subspace of $\widetilde{\mathcal{P}}_{g_1+g_2, m_1+m_2-2, n_1+n_2}$ obtained by gluing the Riemann surfaces in $\mathcal{R}_{g_1, m_1, n_1}$ and $\mathcal{R}_{g_2, m_2, n_2}$ at one NS puncture from each via the *special* plumbing fixture relation⁴

$$z w = e^{i\theta}, \quad 0 \leq \theta \leq 2\pi, \quad (2.8)$$

where z and w denote local coordinates around the punctures that are being glued. Similarly $\{\mathcal{R}_{g_1, m_1, n_1}; \mathcal{R}_{g_2, m_2, n_2}\}$ denotes the subspace of $\widetilde{\mathcal{P}}_{g_1+g_2, m_1+m_2, n_1+n_2-2}$ obtained by gluing the Riemann surfaces in $\mathcal{R}_{g_1, m_1, n_1}$ and $\mathcal{R}_{g_2, m_2, n_2}$ at one R puncture from each via the

³Generalized sections include weighted average of sections. Furthermore they may contain ‘vertical segments’ in which the PCO locations may jump discontinuously across codimension 1 subspaces in the interior of $\mathcal{R}_{g,m,n}$ [6, 37].

⁴These correspond to $s = 0$ boundaries of the general plumbing fixture relations given in (2.9).

same special plumbing fixture relation (2.8). There is one additional subtlety in the definition of $\{ ; \}$. The total number of PCO's on the two Riemann surfaces corresponding to a point in $\mathcal{R}_{g_1, m_1, n_1}$ and a point in $\mathcal{R}_{g_2, m_2, n_2}$ is $2(g_1 + g_2) - 4 + (m_1 + m_2) + (n_1 + n_2)/2$. Using the constraints given in the second term in (2.7), this can be written as $(2g - 2) + m + n/2 - 1$, which is one less than the required number of PCO's on a Riemann surface associated with a point in $\tilde{\mathcal{P}}_{g, m, n}$. Therefore in defining $\{ ; \}$ we need to prescribe the location of the additional PCO. A consistent prescription that we shall adopt is to insert a factor of \mathcal{X}_0 around one of the two punctures which are being glued. Which of the two punctures we choose is irrelevant since $\oint dz z^{-1} \mathcal{X}(z) = \oint dw w^{-1} \mathcal{X}(w)$ when z and w are related as in (2.8). In fact in both heterotic and type II string theories, a universal prescription for plumbing fixture rules in all sectors will be to insert the operator \mathcal{G} defined in (2.4), (2.5) at one of the two punctures which are being glued.

$\mathcal{R}_{g, m, n}$'s can be called '1PI subspaces' of $\tilde{\mathcal{P}}_{g, m, n}$ since, as we shall see, they can be used to define 1PI amplitudes. Operationally the regions $\mathcal{R}_{g, m, n}$ are constructed as follows. For $(g = 0, m + n = 3)$ and $(g = 1, m + n = 1)$ we choose $\mathcal{R}_{g, m, n}$ so that its projection to $\mathcal{M}_{g, m, n}$ is the whole moduli space $\mathcal{M}_{g, m, n}$ and the choice of the section encoding choice of local coordinates and PCO locations are arbitrary subject to symmetry restrictions — permutations of punctures for $(g = 0, m + n = 3)$ and modular invariance for $(g = 1, m + n = 1)$. For $(g = 1, m + n = 1)$ the section must also avoid spurious poles [39–41]. Achieving these may involve making use of generalized sections in the sense described in footnote 3. Given these choices we now glue the Riemann surfaces corresponding to points in these $\mathcal{R}_{g, m, n}$'s via the plumbing fixture relations

$$z w = e^{-s+i\theta}, \quad 0 \leq s < \infty, \quad 0 \leq \theta \leq 2\pi. \tag{2.9}$$

While carrying out the plumbing fixture we always choose a pair of punctures on *two different Riemann surfaces* — we never use a pair of punctures on the same Riemann surface. In the first stage these generate subspaces of $\tilde{\mathcal{P}}_{g, m, n}$ for $(g = 0, m + n = 4)$ and $(g = 1, m + n = 2)$ — we ignore the $(g = 2, m + n = 0)$ sector since the associated Riemann surface has no punctures where the vertex operators can be inserted. Typically the projection of these subspaces to $\mathcal{M}_{g, m, n}$ do not cover the whole of $\mathcal{M}_{g, m, n}$ for these values of (g, m, n) . We choose the $\mathcal{R}_{g, m, n}$ for $(g = 0, m + n = 4)$ and $(g = 1, m + n = 2)$ so as to 'fill these gaps'. Only the boundary of $\mathcal{R}_{g, m, n}$ is fixed from this consideration; how we fill the gap is arbitrary, except that we choose them in a manner consistent with the various symmetries e.g. exchange of the NS punctures and exchange of the R punctures and also avoiding spurious poles. The requirement that the boundaries of the new regions $\mathcal{R}_{g, m, n}$ match the $s = 0$ boundaries of the regions of $\tilde{\mathcal{P}}_{g, m, n}$ obtained by plumbing fixture of Riemann surfaces associated with $\mathcal{R}_{g, m, n}$ with $(g = 0, m + n = 3)$ and $(g = 1, m + n = 1)$ leads to the conditions (2.7). We now continue this process, generating new subspaces of $\tilde{\mathcal{P}}_{g, m, n}$ by plumbing fixture of the subspaces $\mathcal{R}_{g', m', n'}$ that have already been determined. We allow the Riemann surfaces associated with these subspaces to be glued multiple number of times, but ensuring that at no stage we glue two punctures situated on the same Riemann surface. We then define new $\mathcal{R}_{g, m, n}$'s by filling the gap left-over from this construction. Continuing this process we construct all the $\mathcal{R}_{g, m, n}$'s.

Once $\mathcal{R}_{g,m,n}$'s have been constructed this way, we define a multilinear function $\{A_1 \cdots A_N\}$ of $|A_1\rangle, \dots |A_N\rangle \in \widehat{\mathcal{H}}_T$ via the relation

$$\{A_1 \cdots A_{m+n}\} = \sum_{g=0}^{\infty} (g_s)^{2g} \int_{\mathcal{R}_{g,m,n}} \Omega_{6g-6+2m+2n}^{(g,m,n)}(|A_1\rangle, \dots |A_{m+n}\rangle). \quad (2.10)$$

Physically these represent 1PI amplitudes with external states $|A_1\rangle, \dots |A_N\rangle$. We also introduced another multilinear function $[A_2 \cdots A_N]$ of $|A_2\rangle, \dots |A_N\rangle \in \widehat{\mathcal{H}}_T$ taking values in $\widetilde{\mathcal{H}}_T$ defined via

$$\langle A_1 | c_0^- | [A_2 \cdots A_N] \rangle = \{A_1 \cdots A_N\} \quad (2.11)$$

for all $|A_1\rangle \in \widehat{\mathcal{H}}_T$. Here $\langle A|B\rangle$ denotes the BPZ inner product between two states $|A\rangle$ and $|B\rangle$ in the full Hilbert space \mathcal{H} . These functions satisfy the identities

$$\{A_1 A_2 \cdots A_{i-1} A_{i+1} A_i A_{i+2} \cdots A_N\} = (-1)^{\gamma_i \gamma_{i+1}} \{A_1 A_2 \cdots A_N\}, \quad (2.12)$$

$$[A_1 \cdots A_{i-1} A_{i+1} A_i A_{i+2} \cdots A_N] = (-1)^{\gamma_i \gamma_{i+1}} [A_1 \cdots A_N], \quad (2.13)$$

where γ_i is the grassmannality of $|A_i\rangle$. They also satisfy

$$\begin{aligned} & \sum_{i=1}^N (-1)^{\gamma_1 + \cdots + \gamma_{i-1}} \{A_1 \cdots A_{i-1} (Q_B A_i) A_{i+1} \cdots A_N\} \\ &= -\frac{1}{2} \sum_{\substack{\ell, k \geq 0 \\ \ell + k = N}} \sum_{\substack{\{i_a; a=1, \dots, \ell\}, \{j_b; b=1, \dots, k\} \\ \{i_a\} \cup \{j_b\} = \{1, \dots, N\}}} \sigma(\{i_a\}, \{j_b\}) \{A_{i_1} \cdots A_{i_\ell} \mathcal{G}[A_{j_1} \cdots A_{j_k}]\} \end{aligned} \quad (2.14)$$

and

$$\begin{aligned} & Q_B [A_1 \cdots A_N] + \sum_{i=1}^N (-1)^{\gamma_1 + \cdots + \gamma_{i-1}} [A_1 \cdots A_{i-1} (Q_B A_i) A_{i+1} \cdots A_N] \\ &= - \sum_{\substack{\ell, k \geq 0 \\ \ell + k = N}} \sum_{\substack{\{i_a; a=1, \dots, \ell\}, \{j_b; b=1, \dots, k\} \\ \{i_a\} \cup \{j_b\} = \{1, \dots, N\}}} \sigma(\{i_a\}, \{j_b\}) [A_{i_1} \cdots A_{i_\ell} \mathcal{G}[A_{j_1} \cdots A_{j_k}]] \end{aligned} \quad (2.15)$$

where $\sigma(\{i_a\}, \{j_b\})$ is the sign that one picks up while rearranging b_0^-, A_1, \dots, A_N to $A_{i_1}, \dots, A_{i_\ell}, b_0^-, A_{j_1}, \dots, A_{j_k}$. Finally we also have a relation

$$\{A_1 \cdots A_k \mathcal{G}[\widetilde{A}_1 \cdots \widetilde{A}_\ell]\} = (-1)^{\gamma + \widetilde{\gamma} + \gamma \widetilde{\gamma}} \{\widetilde{A}_1 \cdots \widetilde{A}_\ell \mathcal{G}[A_1 \cdots A_k]\}, \quad (2.16)$$

where γ and $\widetilde{\gamma}$ are the total grassmannalities of A_1, \dots, A_k and $\widetilde{A}_1, \dots, \widetilde{A}_\ell$ respectively.

These ingredients can be used to construct the 1PI action of the theory as follows [7, 8]. We take the string field to consist of two components $|\Psi\rangle$ and $|\widetilde{\Psi}\rangle$. $|\Psi\rangle$ is taken to be an arbitrary element of ghost number 2 in $\widehat{\mathcal{H}}_T$ and $|\widetilde{\Psi}\rangle$ is taken to be an arbitrary element of ghost number 2 in $\widetilde{\mathcal{H}}_T$. Both string fields are taken to be grassmann even. It follows from the paragraph below (2.6) that in the heterotic string theory the expansion coefficients are grassmann even for \mathcal{H}_{NS} and grassmann odd for \mathcal{H}_R , while in type II string theory the

expansion coefficients are grassmann even for \mathcal{H}_{NSNS} and \mathcal{H}_{RR} and grassmann odd for \mathcal{H}_{NSR} and \mathcal{H}_{RNS} . The 1PI action has the form

$$S_{1PI} = g_s^{-2} \left[-\frac{1}{2} \langle \tilde{\Psi} | c_0^- Q_B \mathcal{G} | \tilde{\Psi} \rangle + \langle \tilde{\Psi} | c_0^- Q_B | \Psi \rangle + \sum_{n=1}^{\infty} \frac{1}{n!} \{ \Psi^n \} \right], \quad (2.17)$$

where g_s denotes string coupling and $\{ \Psi^n \}$ means $\{ \Psi \cdots \Psi \}$ with n insertions of $|\Psi\rangle$. It is easy to see that the action (2.17) is invariant under the infinitesimal gauge transformation

$$|\delta\Psi\rangle = Q_B|\Lambda\rangle + \sum_{n=0}^{\infty} \frac{1}{n!} \mathcal{G}[\Psi^n \Lambda], \quad |\delta\tilde{\Psi}\rangle = Q_B|\tilde{\Lambda}\rangle + \sum_{n=0}^{\infty} \frac{1}{n!} [\Psi^n \Lambda], \quad (2.18)$$

where $|\Lambda\rangle \in \hat{\mathcal{H}}_T$, $|\tilde{\Lambda}\rangle \in \tilde{\mathcal{H}}_T$, and both carry ghost number 1.

The 1PI action given in (2.17) is not unique but depends on the choice of $\mathcal{R}_{g,m,n}$, i.e. choice of local coordinates at the punctures and PCO locations. Different choices lead to different definitions of $\{A_1 \cdots A_N\}$. However the corresponding 1PI effective string field theories can be shown to be related by field redefinition, and hence this ambiguity does not affect any of the physical quantities. While we shall not make any specific assumption about the choice of local coordinates and PCO locations, we shall assume that the local coordinates have been scaled by a sufficiently large number so that unit radius circle around the punctures in the local coordinates correspond to physically small disks around the punctures,⁵ and that the PCO's are inserted outside these unit disks. This will ensure that in the 1PR amplitudes obtained by gluing the 1PI amplitudes via (2.9), the PCO's do not collide. This also ensures that as long as the 1PI amplitudes $\{A_1 \cdots A_N\}$ are free from spurious singularities, the 1PR amplitudes built from plumbing fixture of these 1PI amplitudes are also free from spurious singularities.

2.3 Classical action

For the construction of the classical action we can restrict our attention to only the genus zero contribution to the functions $\{A_1 \cdots A_N\}$ and $[A_2 \cdots A_N]$, which we shall denote by $\{A_1 \cdots A_N\}_0$ and $[A_2 \cdots A_N]_0$ respectively. These functions vanish for $N \leq 2$. The classical action of the theory can now be written down from the 1PI effective action (2.17) using the fact that at tree level there is no difference between the classical action and the 1PI action. Therefore it takes the form

$$S_{cl} = g_s^{-2} \left[-\frac{1}{2} \langle \tilde{\Psi} | c_0^- Q_B \mathcal{G} | \tilde{\Psi} \rangle + \langle \tilde{\Psi} | c_0^- Q_B | \Psi \rangle + \sum_{n=3}^{\infty} \frac{1}{n!} \{ \Psi^n \}_0 \right], \quad (2.19)$$

with the gauge transformation taking the form

$$|\delta\Psi\rangle = Q_B|\Lambda\rangle + \sum_{n=1}^{\infty} \frac{1}{n!} \mathcal{G}[\Psi^n \Lambda]_0, \quad |\delta\tilde{\Psi}\rangle = Q_B|\tilde{\Lambda}\rangle + \sum_{n=1}^{\infty} \frac{1}{n!} [\Psi^n \Lambda]_0. \quad (2.20)$$

⁵In string field theory literature this is often described as adding long stubs to the external lines of the vertex.

The equations of motion derived from (2.19) can be written as

$$Q_B(|\Psi\rangle - \mathcal{G}|\tilde{\Psi}\rangle) = 0, \tag{2.21}$$

$$Q_B|\tilde{\Psi}\rangle + \sum_{n=3}^{\infty} \frac{1}{(n-1)!} [\Psi^{n-1}]_0 = 0. \tag{2.22}$$

A priori this theory has too many degrees of freedom. For example at the linearized level, the gauge inequivalent solutions to (2.21) and (2.22) are given by the elements of BRST cohomology in the ghost number 2 sectors of $\hat{\mathcal{H}}_T$ and $\tilde{\mathcal{H}}_T$. This will double the number of physical states.⁶ To circumvent this difficulty we observe that given any solution to the equations of motion (2.21), (2.22), we can generate new solutions by adding to $|\tilde{\Psi}\rangle$ arbitrary BRST invariant states keeping $|\Psi\rangle$ fixed. This suggests the following two step process for solving the equations of motion. First by adding \mathcal{G} operated on the second equation to the first equation we write the independent equations as

$$Q_B|\Psi\rangle + \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \mathcal{G}[\Psi^{n-1}]_0 = 0, \tag{2.23}$$

and

$$Q_B|\tilde{\Psi}\rangle + \sum_{n=3}^{\infty} \frac{1}{(n-1)!} [\Psi^{n-1}]_0 = 0. \tag{2.24}$$

In the first step we find general solutions of (2.23) without any reference to (2.24), and then, for each of these solutions, pick a particular $|\tilde{\Psi}\rangle$ that solves (2.24).⁷ We could implement this by imposing some specific condition like $|\Psi\rangle - \mathcal{G}|\tilde{\Psi}\rangle = 0$, but this will not be necessary. In the second step, for each of the solutions obtained at the first step, we add to $|\tilde{\Psi}\rangle$ an arbitrary element of the BRST cohomology in the ghost number 2 sector of $\tilde{\mathcal{H}}_T$. This generates the most general solution to the full set of equations of motion. Since the deformation of the solution generated in the second step do not get modified by interactions, and do not affect the solution generated in the first step, upon quantization they will represent free particles which do not scatter with each other or with the particles associated with the solutions to (2.23). Thus this sector decouples from the theory at tree

⁶The doubling trick for dealing with Ramond sector in Berkovits version of open string field theory has been explored previously in [38]. The relationship between our approach and the approach of [38] is not completely clear. In particular one of the key features of our approach is that the field $\tilde{\Psi}$ enters the action only in quadratic terms. This features seems to be absent in [38].

⁷One might wonder whether given a solution to (2.23), one can always find a solution to (2.24). One class of solutions to these equations may be obtained by starting with a seed solution to the linearized equations of motion carrying some generic momentum, and then correcting it iteratively using the general procedure described e.g. in [7–9]. In this case one can relate possible obstruction to finding iterative solutions to these equations to the question of whether or not the non-linear terms in the equations of motion are BRST trivial. Using the isomorphism between BRST cohomologies in different picture number sector for generic momenta given in [36], one can then show that if the non-linear terms in (2.23) are BRST trivial, then the non-linear terms in (2.24) are also BRST trivial. Therefore given a solution to (2.23) one can find a solution to (2.24). This leaves open the possibility that there may be ‘large’ classical solutions to (2.23) for which there is no solution to (2.24). In such cases we can simply discard these solutions without violating anything that we know in perturbative string theory.

level. This can also be seen from the analysis of Feynman diagrams [7–9]. It follows from the analysis of [7–9] — restricted to tree level string theory — that the interacting part of the theory describes correctly the spectrum and S-matrix of string theory at tree level.

The gauge inequivalent solutions to the *linearized* equations of motion at the first step are characterized by the elements of the BRST cohomology in the ghost number two sector of $\widehat{\mathcal{H}}_T$, whereas the gauge inequivalent solutions to the *linearized* equations of motion at the second step are characterized by the elements of the BRST cohomology in the ghost number two sector of $\widetilde{\mathcal{H}}_T$. This shows that the physical states in the interacting part of the theory are in the BRST cohomology in $\widehat{\mathcal{H}}_T$ while the physical states which decouple are in the BRST cohomology in $\widetilde{\mathcal{H}}_T$. The two are isomorphic at non-zero momentum, but not at zero momentum [36].

We shall see in eq. (4.4) that the interaction terms in the action in the full quantum theory continue to be independent of $|\widetilde{\Psi}\rangle$. Hence the particles associated with the modes where we deform $|\widetilde{\Psi}\rangle$ by adding a BRST invariant state keeping $|\Psi\rangle$ fixed will never appear as intermediate states in an amplitude even in the full quantum theory. This will be demonstrated explicitly in section 4.2 where we shall derive the Feynman rules in the full quantum theory. In what follows we shall work with the full classical action (2.19) and its quantum generalization (4.4) at intermediate stages, and discuss the decoupling of the modes of $|\widetilde{\Psi}\rangle$ only at the very end.

3 Classical master action

We shall now construct the classical master action corresponding to the BV quantization of the action (2.19). We follow the procedure described in [16] for closed bosonic string field theory. This is done in several steps.

1. First we relax the constraint on the ghost number and let $|\Psi\rangle$ and $|\widetilde{\Psi}\rangle$ be arbitrary states in $\widehat{\mathcal{H}}_T$ and $\widetilde{\mathcal{H}}_T$. The grassmannality of the coefficients are chosen such that the string field is always even.
2. We divide $\widehat{\mathcal{H}}_T$ and $\widetilde{\mathcal{H}}_T$ into two subsectors: $\widehat{\mathcal{H}}_+$ and $\widetilde{\mathcal{H}}_+$ will contain states in $\widehat{\mathcal{H}}_T$ and $\widetilde{\mathcal{H}}_T$ of ghost numbers ≥ 3 , while $\widehat{\mathcal{H}}_-$ and $\widetilde{\mathcal{H}}_-$ will contain states in $\widehat{\mathcal{H}}_T$ and $\widetilde{\mathcal{H}}_T$ of ghost numbers ≤ 2 . We introduce basis states $|\widehat{\varphi}_r^-\rangle$, $|\widetilde{\varphi}_r^-\rangle$, $|\widehat{\varphi}_r^+\rangle$ and $|\widetilde{\varphi}_r^+\rangle$ of $\widehat{\mathcal{H}}_-$, $\widetilde{\mathcal{H}}_-$, $\widehat{\mathcal{H}}_+$ and $\widetilde{\mathcal{H}}_+$ satisfying orthonormality conditions

$$\langle \widehat{\varphi}_r^- | c_0^- | \widetilde{\varphi}_+^s \rangle = \delta_r^s = \langle \widetilde{\varphi}_+^s | c_0^- | \widehat{\varphi}_r^- \rangle, \quad \langle \widetilde{\varphi}_r^- | c_0^- | \widehat{\varphi}_+^s \rangle = \delta_r^s = \langle \widehat{\varphi}_+^s | c_0^- | \widetilde{\varphi}_r^- \rangle, \quad (3.1)$$

and expand the string fields $|\Psi\rangle$, $|\widetilde{\Psi}\rangle$ as

$$\begin{aligned} |\widetilde{\Psi}\rangle &= \sum_r |\widetilde{\varphi}_r^-\rangle \widetilde{\psi}^r + \sum_r (-1)^{g_r^*+1} |\widetilde{\varphi}_r^+\rangle \widetilde{\psi}_r^*, \\ |\Psi\rangle - \frac{1}{2} \mathcal{G} |\widetilde{\Psi}\rangle &= \sum_r |\widehat{\varphi}_r^-\rangle \psi^r + \sum_r (-1)^{\widetilde{g}_r^*+1} |\widehat{\varphi}_r^+\rangle \psi_r^*. \end{aligned} \quad (3.2)$$

Here g_r^* , g_r , \tilde{g}_r^* and \tilde{g}_r label the grassmann parities of ψ_r^* , ψ^r , $\tilde{\psi}_r^*$ and $\tilde{\psi}^r$ respectively. They in turn can be determined from the assignment of grassmann parities to the basis states as described below (2.6) and the fact that $|\Psi\rangle$ and $|\tilde{\Psi}\rangle$ are both even.

3. We shall identify the variables $\{\psi^r, \tilde{\psi}^r\}$ as ‘fields’ and the variables $\{\psi_r^*, \tilde{\psi}_r^*\}$ as the conjugate ‘anti-fields’ in the BV quantization of the theory. It can be easily seen that ψ^r and ψ_r^* carry opposite grassmann parities and $\tilde{\psi}^r$ and $\tilde{\psi}_r^*$ carry opposite grassmann parities. This is consistent with their identifications as fields and conjugate anti-fields.
4. Given two functions F and G of all the fields and anti-fields, we now define their anti-bracket in the standard way:

$$\{F, G\} = \frac{\partial_R F}{\partial \psi^r} \frac{\partial_L G}{\partial \psi_r^*} + \frac{\partial_R F}{\partial \tilde{\psi}^r} \frac{\partial_L G}{\partial \tilde{\psi}_r^*} - \frac{\partial_R F}{\partial \psi_r^*} \frac{\partial_L G}{\partial \psi^r} - \frac{\partial_R F}{\partial \tilde{\psi}_r^*} \frac{\partial_L G}{\partial \tilde{\psi}^r}, \quad (3.3)$$

where the subscripts R and L of ∂ denote left and right derivatives respectively.

5. The anti-bracket can be given the following interpretation in the world-sheet SCFT. Given a function $F(|\Psi\rangle, |\tilde{\Psi}\rangle)$ let us define $\langle F_R|$, $\langle \tilde{F}_R|$, $|F_L\rangle$, $|\tilde{F}_L\rangle$ such that under an infinitesimal variation of $|\Psi\rangle$, $|\tilde{\Psi}\rangle$ we have

$$\delta F = \langle F_R|c_0^-|\delta\tilde{\Psi}\rangle + \langle \tilde{F}_R|c_0^-|\delta\Psi\rangle = \langle \delta\tilde{\Psi}|c_0^-|F_L\rangle + \langle \delta\Psi|c_0^-|\tilde{F}_L\rangle. \quad (3.4)$$

Then using completeness of the basis states and using (3.1)–(3.3) one can show that the anti-bracket between two functions F and G is given by

$$\{F, G\} = - \left(\langle F_R|c_0^-|\tilde{G}_L\rangle + \langle \tilde{F}_R|c_0^-|G_L\rangle + \langle \tilde{F}_R|c_0^-|\mathcal{G}|\tilde{G}_L\rangle \right). \quad (3.5)$$

6. The classical BV master action of string field theory is now taken to be of the same form as (2.19) but with $|\Psi\rangle$, $|\tilde{\Psi}\rangle$ containing states of all ghost numbers:

$$S = g_s^{-2} \left[-\frac{1}{2} \langle \tilde{\Psi}|c_0^-Q_B\mathcal{G}|\tilde{\Psi}\rangle + \langle \tilde{\Psi}|c_0^-Q_B|\Psi\rangle + \sum_{n=3}^{\infty} \frac{1}{n!} \{\Psi^n\}_0 \right]. \quad (3.6)$$

We shall now check that this action satisfies the classical master equation. Using (3.6) and (3.4) we get

$$\begin{aligned} \langle S_R| &= -\langle \Psi|Q_B + \langle \tilde{\Psi}|Q_B\mathcal{G}, & \langle \tilde{S}_R| &= -\langle \tilde{\Psi}|Q_B + \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \langle [\Psi^{n-1}]_0|, \\ |S_L\rangle &= Q_B|\Psi\rangle - Q_B\mathcal{G}|\tilde{\Psi}\rangle, & |\tilde{S}_L\rangle &= Q_B|\tilde{\Psi}\rangle + \sum_{n=3}^{\infty} \frac{1}{(n-1)!} |[\Psi^{n-1}]_0\rangle. \end{aligned} \quad (3.7)$$

Therefore from (3.5) we have

$$\begin{aligned} \{S, S\} &= - \left(\langle S_R|c_0^-|\tilde{S}_L\rangle + \langle \tilde{S}_R|c_0^-|S_L\rangle + \langle \tilde{S}_R|c_0^-|\mathcal{G}|\tilde{S}_L\rangle \right) \\ &= -2 \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \langle \Psi|c_0^-Q_B[\Psi^{n-1}]_0\rangle - \sum_{m=3}^{\infty} \sum_{n=3}^{\infty} \frac{1}{(m-1)!(n-1)!} \langle \mathcal{G}[\Psi^{m-1}]_0|c_0^-|[\Psi^{n-1}]_0\rangle \\ &= -2 \sum_{n=3}^{\infty} \frac{1}{(n-1)!} \{\Psi^{n-1}Q_B\Psi\}_0 - \sum_{m=3}^{\infty} \sum_{n=3}^{\infty} \frac{1}{(m-1)!(n-1)!} \{\mathcal{G}[\Psi^{m-1}]_0\Psi^{n-1}\}_0 \\ &= 0, \end{aligned} \quad (3.8)$$

where in the last step we have used (2.14). Note that the $|\tilde{\Psi}\rangle$ dependent terms cancel in going from the first to the second line itself, and this cancelation does not require any details of the interaction terms except that they depend only on $|\Psi\rangle$. The manipulations leading from second to the fourth line are identical to what is done in closed bosonic string field theory [16], except for insertion of factor of \mathcal{G} on $[\dots]_0$. Eq. (3.8) shows that the action S satisfies the classical master equation $\{S, S\} = 0$.

4 Quantum master action

Given the construction of the classical master action and the definitions of fields and anti-fields given in section 3, the construction of the quantum master action can be given using the same steps as in [16], with the necessary modifications for superstrings read out from the results of [7, 8]. For this reason we shall only sketch the steps, omitting the details of the proofs. In section 4.1 we give the construction of the master action and in section 4.2 we discuss gauge fixing and Feynman rules.

4.1 Action

The first step in the analysis will be to introduce new subspaces $\overline{\mathcal{R}}_{g,m,n}$ of $\tilde{\mathcal{P}}_{g,m,n}$ satisfying relations similar to — but not quite the same — as (2.7):

$$\begin{aligned} \partial \overline{\mathcal{R}}_{g,m,n} = & -\frac{1}{2} \sum_{\substack{g_1, g_2 \\ g_1 + g_2 = g}} \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = m+2}} \sum_{\substack{n_1, n_2 \\ n_1 + n_2 = n}} \mathbf{S}[\{\overline{\mathcal{R}}_{g_1, m_1, n_1}, \overline{\mathcal{R}}_{g_2, m_2, n_2}\}] \\ & -\frac{1}{2} \sum_{\substack{g_1, g_2 \\ g_1 + g_2 = g}} \sum_{\substack{m_1, m_2 \\ m_1 + m_2 = m}} \sum_{\substack{n_1, n_2 \\ n_1 + n_2 = n+2}} \mathbf{S}[\{\overline{\mathcal{R}}_{g_1, m_1, n_1}, \overline{\mathcal{R}}_{g_2, m_2, n_2}\}] \\ & -\Delta_{NS} \overline{\mathcal{R}}_{g-1, m+2, n} - \Delta_R \overline{\mathcal{R}}_{g-1, m, n+2}, \end{aligned} \tag{4.1}$$

where Δ_{NS} and Δ_R are two new operations defined as follows. Δ_{NS} takes a pair of NS punctures on a Riemann surface corresponding to a point in $\overline{\mathcal{R}}_{g-1, m+2, n}$ and glues them via the special plumbing fixture relation (2.8). Δ_R represents a similar operation on a pair of R punctures, but we must also insert a factor of \mathcal{X}_0 around one of the punctures. The generalization to type II string theory is straightforward, with the general principle that we always insert the operator \mathcal{G} introduced in (2.4), (2.5) at one of the punctures which is being glued.

Operationally the construction of $\overline{\mathcal{R}}_{g,m,n}$ follows a procedure similar to the one for $\mathcal{R}_{g,m,n}$, except that now while generating higher genus Riemann surfaces from gluing of lower genus surfaces via the relation (2.9), we also allow gluing of a pair of punctures on the same Riemann surface. Therefore we begin with a three punctured sphere with arbitrary choice of local coordinates and PCO locations consistent with exchange symmetries, and in the first step either glue two puncture on a three punctured sphere via (2.9) to generate a family of one punctured tori, or two punctures on two three punctured spheres to generate a family of four punctured spheres. These generate certain subspaces of $\tilde{\mathcal{P}}_{g,m,n}$ with $(g = 1, m + n = 1)$ and $(g = 0, m + n = 4)$ whose projection to $\mathcal{M}_{g,m,n}$ generically does not cover

the whole of $\mathcal{M}_{g,m,n}$. We then fill the gap with the subspaces $\overline{\mathcal{R}}_{g,m,n}$ of $\widetilde{\mathcal{P}}_{g,m,n}$. Again the choice of this subspace is arbitrary except that its boundaries are fixed and it must obey exchange and other symmetries and avoid spurious poles. Continuing this process we can generate all the $\overline{\mathcal{R}}_{g,m,n}$'s.

Once $\overline{\mathcal{R}}_{g,m,n}$'s are constructed we define new multilinear functions $\{\{A_1 \cdots A_N\}\}$ of $|A_1\rangle, \dots, |A_N\rangle \in \widehat{\mathcal{H}}_T$ via the relation

$$\{\{A_1 \cdots A_{m+n}\}\} = \sum_{g=0}^{\infty} (g_s)^{2g} \int_{\overline{\mathcal{R}}_{g,m,n}} \Omega_{6g-6+2m+2n}^{(g,m,n)}(|A_1\rangle, \dots, |A_{m+n}\rangle). \quad (4.2)$$

We also introduce another multilinear function $[[A_2 \cdots A_N]]$ of $|A_2\rangle, \dots, |A_N\rangle \in \widehat{\mathcal{H}}_T$ taking values in $\widetilde{\mathcal{H}}_T$, defined via

$$\langle A_1 | c_0^- [[A_2 \cdots A_N]] \rangle = \{\{A_1 \cdots A_N\}\} \quad (4.3)$$

for all $|A_1\rangle \in \widehat{\mathcal{H}}_T$. These new functions satisfy relations similar to those given in (2.12)–(2.16), except that the right hand sides of (2.14) and (2.15) contain new terms involving contraction of a pair of states inside the same bracket. Since these relations have form identical to those given in [16], except for the insertion of a \mathcal{X}_0 operator when we contract a pair of R-sector states, we shall not write down these relations.

The quantum master action is given by

$$S_q = g_s^{-2} \left[-\frac{1}{2} \langle \widetilde{\Psi} | c_0^- Q_B \mathcal{G} | \widetilde{\Psi} \rangle + \langle \widetilde{\Psi} | c_0^- Q_B | \Psi \rangle + \sum_{n=1}^{\infty} \frac{1}{n!} \{\{\Psi^n\}\} \right]. \quad (4.4)$$

Following the analysis of [16], this can be shown to satisfy the quantum master equation

$$\frac{1}{2} \{S_q, S_q\} + \Delta S_q = 0, \quad (4.5)$$

where, for any function F of the fields and anti-fields,

$$\Delta F \equiv \frac{\partial_R}{\partial \psi^s} \frac{\partial_L F}{\partial \psi_s^*}. \quad (4.6)$$

The main point to note in this analysis is that on the left hand side of (4.5) the $\widetilde{\Psi}$ dependent terms cancel at the first step as in (3.8). After this the $|\Psi\rangle$ dependent terms have structure identical to what appears in the closed bosonic string field theory of [16] except for insertion of \mathcal{X}_0 factors on the R sector propagators. The resulting expression can be manipulated in the same way as in [16].⁸

⁸There is a slightly different sign convention between [16] and [6–8]. In [16] the vacuum was normalized to satisfy $\langle 0 | \bar{c}_{-1} c_{-1} \bar{c}_0 c_0 \bar{c}_1 c_1 | 0 \rangle = 1$ while in [6–8] the normalization was $\langle 0 | c_{-1} \bar{c}_{-1} c_0 \bar{c}_0 c_1 \bar{c}_1 e^{-2\phi} | 0 \rangle = 1$ where ϕ is the bosonized superconformal ghost. This leads to a non-standard sign convention for the moduli space integration measure described in [9]. Alternatively one can continue to use the standard integration measure and include an additional factor of $(-1)^{3g-3+N}$ in the definition of $\{A_1 \cdots A_N\}$. However this difference is irrelevant for the present analysis since the identities (2.11)–(2.16) and their quantum generalizations take the same form in [16] and [6–8].

4.2 Gauge fixing and Feynman rules

In the BV formalism, given the master action we compute the quantum amplitudes by carrying out the usual path integral over a Lagrangian submanifold of the full space spanned by ψ^r and ψ_r^* . It is most convenient to work in the Siegel gauge

$$b_0^+|\Psi\rangle = 0, \quad b_0^+|\tilde{\Psi}\rangle = 0 \quad \Rightarrow \quad b_0^+ \left(|\Psi\rangle - \frac{1}{2}\mathcal{G}|\tilde{\Psi}\rangle \right) = 0. \quad (4.7)$$

To see that this describes a Lagrangian submanifold, we divide the basis states used in the expansion (3.2) into two classes: those annihilated by b_0^+ and those annihilated by c_0^+ . These two sets are conjugates of each other under the inner product (3.1). Now in the expansion given in (3.2), Siegel gauge condition sets the coefficients of the basis states annihilated by c_0^+ to zero. Since in this expansion the fields and their anti-fields multiply conjugate pairs of basis states, it follows that if the Siegel gauge condition sets a field to zero then its conjugate anti-field remains unconstrained, and if it sets an anti-field to zero then its conjugate field remains unconstrained. Therefore this defines a Lagrangian submanifold.

In the Siegel gauge the propagator in $|\tilde{\Psi}\rangle, |\Psi\rangle$ space takes the form (see [9] for the sign conventions)

$$-g_s^2 b_0^+ b_0^- (L_0^+)^{-1} \delta_{L_0, \bar{L}_0} \begin{pmatrix} 0 & 1 \\ 1 & \mathcal{G} \end{pmatrix}. \quad (4.8)$$

Only the lower right corner of the matrix is important for computing amplitudes since the interaction vertices only involve $|\Psi\rangle$ and not $|\tilde{\Psi}\rangle$. We can now use standard procedure to express the different contributions to the amplitude as integrals over subspaces of the moduli space of punctured Riemann surfaces, and the relation (4.1) ensures that the sum over all Feynman diagrams cover the whole moduli space [16, 42]. Note that only states in $\hat{\mathcal{H}}_T$ propagate along internal lines but they can carry arbitrary ghost number.⁹

Since the field $|\tilde{\Psi}\rangle$ continues to appear only in the kinetic term even in the full quantum BV action, the additional modes we have introduced via $|\tilde{\Psi}\rangle$ decouple from the interacting part of the theory. Indeed 1PI amplitudes computed using the master action would reproduce the 1PI action given in (2.17), with the only difference that the $\mathcal{R}_{g,m,n}$'s involved

⁹Some qualification is warranted here. The states which enter the vertex are states in $\hat{\mathcal{H}}_T$ annihilated by b_0^+ . But the propagator itself consists of the operator $-g_s^2 b_0^+ b_0^- (L_0^+)^{-1} \mathcal{G}$ sandwiched between a pair of basis states in the conjugate sector, which are states in \mathcal{H} carrying picture numbers $(-1, -3/2)$ and annihilated by c_0^+ and c_0^- . Since there is no restriction on the ghost number, there are apparently infinite number of states at each mass level obtained by repeated application of the zero mode β_0 of β in the R sector. However the operator \mathcal{X}_0 in the propagator annihilates all but a finite number of these states. This can be seen using the fact that the application of β_0 reduces the ghost number of the state. On the other hand the application of \mathcal{X}_0 produces a state of picture number $-1/2$ for which β_0 annihilates the vacuum and hence at a given level, we can no longer have states of arbitrarily small (i.e. large negative) ghost number. Therefore a state of picture number $-3/2$ must be annihilated by \mathcal{X}_0 for sufficiently small ghost number since there will be no candidate state with the right quantum numbers. This in turn shows that only a finite number of states propagate at each mass level. This property is manifest if instead of \mathcal{X}_0 we use the kinetic operator used in [34], but at this stage it is not clear how to write down a fully gauge invariant closed string field theory action based on this kinetic operator.

in the definitions of $\{A_1 \cdots A_N\}$ are not defined independently, but constructed from the $\overline{\mathcal{R}}_{g,m,n}$'s used for defining $\{\{A_1 \cdots A_N\}\}$ by plumbing fixture of $\overline{\mathcal{R}}_{g,m,n}$'s in all possible ways via the relation (2.9), but keeping only the '1PI contributions'. In (2.17) $|\tilde{\Psi}\rangle$ appears only in the kinetic term, showing that its equation of motion leads to free fields even in the full quantum theory.

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