

# Primary Feynman rules to calculate the $\epsilon$ -dimensional integrand of any 1-loop amplitude

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**ABSTRACT:** When using dimensional regularization/reduction the  $\epsilon$ -dimensional numerator of the 1-loop Feynman diagrams gives rise to rational contributions. I list the set of fundamental rules that allow the extraction of such terms at the integrand level in any theory containing scalars, vectors and fermions, such as the electroweak standard model, QCD and SUSY.

**KEYWORDS:** NLO Computations, QCD, Standard Model, Beyond Standard Model

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**1 Introduction**

New techniques [1–14] for computing 1-loop corrections led to a NLO revolution [15, 16], that, directly or indirectly, has allowed an impressive improvement in our ability to predict physical observables at the NLO accuracy [17–37]. Basically all new methods need a special treatment of the contributions that are not proportional to the scalar 1-loop functions, the so called rational terms [38–41]. That is achieved, in Unitarity and Generalized Unitarity methods, by computing the entire amplitude in different numbers of space-time dimensions [42], or via bootstrapping techniques [43, 44], or through  $d$ -dimensional cuts [45]. The Ossola-Papadopoulos-Pittau (OPP) approach of reference [4] requires, instead, the computation (once for all for the theory at hand) of a special set of tree level Feynman rules [46–50] up to 4-point interactions.<sup>1</sup>

When using dimensional regularization/reduction, the origin of such terms lies in the  $\epsilon$ -dimensional numerator of the 1-loop Feynman diagrams.<sup>2</sup> To be more specific, let us consider the general expression for the integrand of a generic  $m$ -point one-loop (sub-)amplitude

$$\bar{A}(\bar{q}) = \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}, \quad \bar{D}_i = (\bar{q} + p_i)^2 - m_i^2, \quad (1.1)$$

where  $\bar{q}$  is the integration momentum and

$$\bar{q}^2 = q^2 + \tilde{q}^2 \equiv q^2 - \mu^2. \quad (1.2)$$

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<sup>1</sup>The contributions to higher-point functions vanish because of UV finiteness.

<sup>2</sup>Another contribution, which is however directly linkable to the cut-constructible part of the amplitude [51], is generated by the  $\epsilon$ -dimensional 1-loop denominators.

In the previous expressions and in all the following ones, a bar denotes objects living in  $d = 4 + \epsilon$  dimensions, whereas a tilde represents  $\epsilon$ -dimensional quantities.

The numerator function  $\bar{N}(\bar{q})$  can be split into a 4-dimensional plus an  $\epsilon$ -dimensional part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\mu^2, q, \epsilon). \tag{1.3}$$

$N(q)$  lives in 4 dimensions, while  $\tilde{N}(\mu^2, q, \epsilon)$ , which originates from the splitting of  $d$ -dimensional objects

$$\begin{aligned} \bar{q} &= q + \tilde{q}, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_{\mu} + \tilde{\gamma}_{\bar{\mu}}, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\bar{\mu}\bar{\nu}}, \end{aligned} \tag{1.4}$$

gives rise to a rational piece called  $R_2$  in the OPP language:

$$R_2|_{\text{HV}} = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{\tilde{N}(\mu^2, q, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}, \tag{1.5}$$

with

$$\int d^d \bar{q} = \int d^4 q \int d^\epsilon \mu. \tag{1.6}$$

$R_2$  has a pure ultraviolet origin [38, 40], so that eq. (1.5) also holds when infrared/collinear divergences are present in the loop integrals. It can be shown [39, 42] that  $\tilde{N}(\mu^2, q, \epsilon)$  is polynomial in  $\mu^2$  and at most linear in  $\epsilon$ , and that the  $\epsilon$  dependence can be reabsorbed in the regularization scheme [52]. Therefore, beside eq. (1.5), which defines  $R_2$  in the 't Hooft-Veltman (HV) scheme, a Four Dimensional Helicity scheme (FDH) can be used in which the  $\epsilon$  dependence in the numerator function is discarded before integration

$$R_2|_{\text{FDH}} = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{\tilde{N}(\mu^2, q, \epsilon = 0)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}. \tag{1.7}$$

As for the virtual part of the NLO corrections, FDH is equivalent to Dimensional Reduction [53].

It is clear that explicitly using the rules in eq. (1.4) allows an analytic extraction, Feynman diagram by Feynman diagram, of the coefficients of the various powers of  $\mu^2$  and  $\epsilon$ . For example, the GoSam [54, 55] approach achieves this *on the fly*, by linking to algebraical manipulation programs providing the necessary algebra when building the amplitude. The case of QCD is particularly simple, and computations based on a standard Passarino-Veltman [56] decomposition are relatively easy [57, 58]. In addition, for gluonic amplitudes, a super-symmetric decomposition relates the contribution of the rational terms to a scalar massive gluon running in the loop [59], which can also be computed by using massive Cachazo-Svrček-Witten Feynman rules [60–62].

All the approaches mentioned so far have advantages and drawbacks. It is however beyond doubt that it would be desirable to have a way to compute  $R_2$  *independent* of the theory at hand, *four dimensional* and *not requiring the use of analytical manipulations*. In this paper, I provide such a method in the form of primary Feynman rules, which

reproduce the polynomial dependence on  $\mu^2$  of the integrand in eq. (1.7)<sup>3</sup> when working in the renormalizable gauge. As will be explained in the next section, such rules are uniquely determined by reading the original propagators and vertices of the theory, and solely depend on their Lorentz structure. They can therefore be used as any other Feynman rules by programs such as MadLoop [12], HELAC-NLO [13], FeynRules [63] or within a methods such as Open Loops [64] to automatically generate R<sub>2</sub>.

This paper is organized as follows: in section 2 I give an introductory discussion and fix my conventions. Section 3 introduces vectors, scalars and their mutual interactions. The fermionic case is more delicate, and it is discussed in section 4. In section 5 the generic formulae are specialized to the QCD case and, in section 6, I draw my conclusions. Further details are reported in appendix A.

## 2 Preliminaries and conventions

I am interested in reconstructing the polynomial dependence on  $\mu^2$  of the  $\epsilon$ -dimensional numerator in eq. (1.7). In a renormalizable gauge the relevant expansion is

$$\tilde{N}(\mu^2, q, \epsilon = 0) = \sum_{j=1}^2 (\mu^2)^j c_j(q). \tag{2.1}$$

As already stated in section 1, an explicit dependence on  $\epsilon$ , such as that implied by eq. (1.5), can always be reproduced by a change of regularization scheme. The translation rules relevant for QCD and for the electroweak standard model are given in appendix A.

By inserting eq. (2.1) into eq. (1.7) the relevant integrals are of the kind

$$\frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{(\mu^2)^j q_{\mu_1} \cdots q_{\mu_r}}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \text{with} \quad m \leq 4, \quad 0 < j \leq 2, \quad r \leq 2, \tag{2.2}$$

which give a non-vanishing contribution, in the limit  $\epsilon \rightarrow 0$ , only when  $4 + 2j + r - 2m \geq 0$ . They are computed in [51].

Powers of  $\mu^2$  in eq. (2.1) are generated by the contraction of the integration momentum with itself only in the presence of vectors and fermions, which are indeed the only particles bringing the integration momentum in the numerator. For each vector field  $V_\alpha$ , I therefore introduce a scalar field  $\hat{V}$ , whose propagator corresponds to the propagation of its  $\epsilon$ -dimensional components. In the same way, for any fermion  $F$ , I introduce a fermionic field  $\hat{F}$  and its corresponding propagator. In the following,  $\hat{V}$  and  $\hat{F}$  are called  $\epsilon$ -particles and their propagators are graphically represented by a dashed line and a dashed arrow, respectively.

$\epsilon$ -particles interact among themselves and with the particles of the original theory, bringing an explicit dependence on  $\mu$  and are only allowed to circulate in the loop. I call  $\epsilon$ -vertices the special vertices involving  $\epsilon$ -particles, and  $\epsilon$ -diagrams the Feynman diagrams that contain  $\epsilon$ -vertices. Given the fact that the only possible mechanisms for generating powers of  $\mu^2$  are those illustrated in eq. (1.4), it is easy to determine the  $\epsilon$ -vertices by simply

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<sup>3</sup>The conversion to the HV scheme of eq. (1.5) is presented in appendix A.

looking at the Feynman rules of the original theory. In particular, the Lorentz and color structures are completely dictated by the nature of the particles entering the  $\epsilon$ -vertices, so that, in general, only the relative phases of  $\mu$  and/or  $\sqrt{\mu}$  have to be determined. I fixed them by explicitly writing down all possible classes of 2, 3, and 4-point Feynman diagrams involving scalars, vectors and fermions and by requiring that the  $\epsilon$ -vertices reproduce the results obtained with an explicit calculation of  $R_2$ . It should be possible to determine the  $\epsilon$ -vertices directly from the original Lagrangian, by splitting it into 4 and  $\epsilon$  dimensional parts, but I did not choose such an approach. In the following two sections, I list all possibilities involving vectors, scalars and fermions. I do it in a completely generic way, in the sense that any particular model can be implemented just by giving a specific value to the constants in front of the listed Lorentz structures. For example, the electroweak model is obtained by fixing them according to reference [65] and the minimal supersymmetric standard model by using the rules in [66].

### 3 Vectors, scalars and their interactions

The propagator of a scalar  $\epsilon$ -particle associated with a vector field is given in figure 1. From the original three-vector vertex, two corresponding  $\epsilon$ -vertices are derived, as illustrated in figure 2. To determine the sign of the first  $\epsilon$ -vertex, one should keep track of the flow of the loop momentum  $q$ . The original four-vector vertex gives rise to the two  $\epsilon$ -vertices of figure 3, while two-vector-one-scalar, one-vector-two-scalar and two-vector-two-scalar vertices generate just one  $\epsilon$ -vertex each, as shown in figures 4 to 6.

Although the  $\epsilon$ -vertices can be used in any stage of the calculation to compute  $\epsilon$ -diagrams reproducing the coefficients  $c_j(q)$  of eq. (2.1), it may be useful and illuminating to consider them in strict connection with the original Feynman diagrams. With this kind of reasoning, one may associate to any 1-loop Feynman diagram contributing to the amplitude under study, a set of  $\epsilon$ -diagrams which fully reconstruct its  $\mu^2$  dependence. In particular, when using the same rooting for the loop momentum in each of them, this reconstruction even holds at the integrand level. Furthermore, given the fact that the  $\epsilon$ -vertices bring known powers of  $\mu$  into the  $\epsilon$ -diagrams, one can easily disentangle sub-sets generating specific powers of  $\mu^2$  in eq. (2.1). A concrete example is given in figure 7 for two particular diagrams contributing to a  $VV \rightarrow VV$  scattering, where the sum of the two  $\epsilon$ -diagrams in the second line yields the term  $\mu^4 c_2(q)$  corresponding to the original box diagram.

### 4 Fermions and their interactions with vectors and scalars

The study of the interactions involving fermions is complicated by the presence of  $\gamma_5$ . It is convenient to start from the chiral fermions of the original theory and to split the fermion propagator in chirality flipping and chirality preserving parts, as illustrated in the top part of figure 8. The  $\epsilon$ -propagator corresponding to the  $\epsilon$ -particle associated with a fermion is chirality flipping, being reminiscent of the presence of a  $\not{q}$  in the numerator, as shown in the bottom part of figure 8. Fermions can interact with vectors and scalars with

$$\begin{array}{c}
\boxed{V_\alpha \overset{p}{\rightsquigarrow} V_\beta = -i \frac{g_{\alpha\beta}}{p^2 - M_V^2}} \\
\hat{V} \text{-----} \hat{V} = -i \frac{1}{p^2 - M_V^2}
\end{array}$$

**Figure 1.** Propagator of a vector particle (box on the top) and propagator of its corresponding scalar  $\epsilon$ -particle.

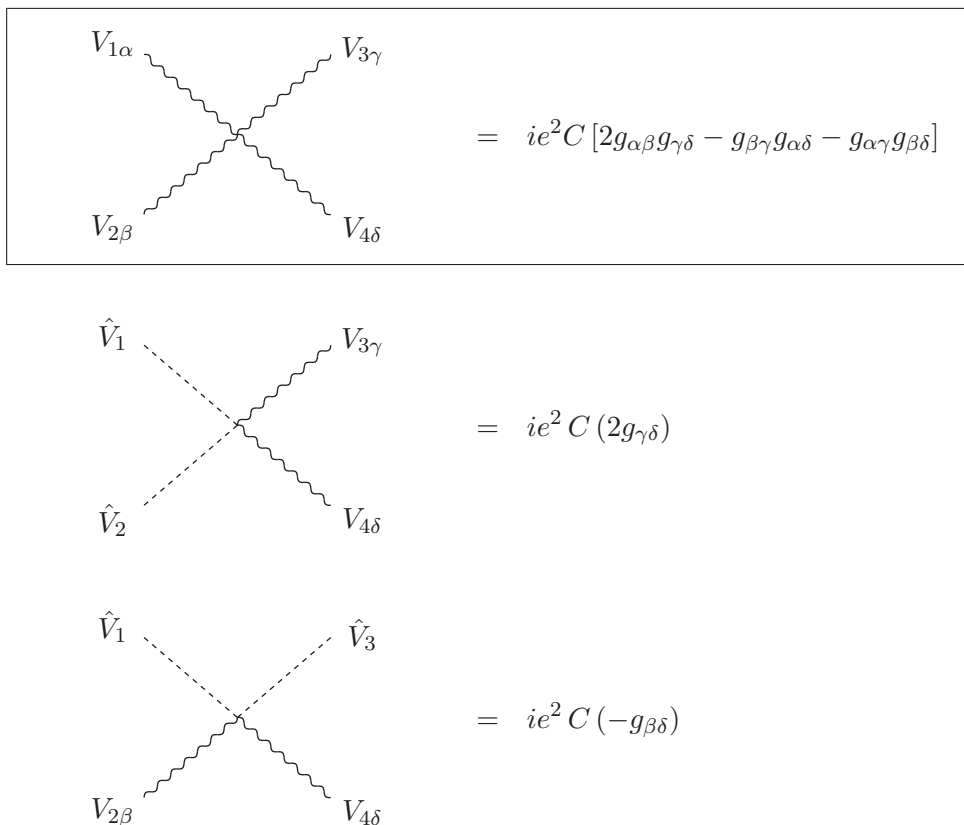
$$\begin{array}{c}
\boxed{V_{1\alpha} \overset{p_1}{\rightsquigarrow} V_{2\beta} \overset{p_2}{\rightsquigarrow} V_{3\gamma} \overset{p_3}{\rightsquigarrow} = -ieC [g_{\alpha\beta}(p_2 - p_1)_\gamma + g_{\beta\gamma}(p_3 - p_2)_\alpha + g_{\gamma\alpha}(p_1 - p_3)_\beta]} \\
\hat{V}_1 \text{-----} V_{2\beta} \overset{p_2}{\rightsquigarrow} V_{3\gamma} \overset{p_3}{\rightsquigarrow} = \hat{V}_1 \text{-----} V_{2\beta} \overset{\pm q}{\rightsquigarrow} V_{3\gamma} \overset{\pm q}{\rightsquigarrow} = -ieC (\pm i\mu) g_{\beta\gamma} \\
V_{1\alpha} \overset{p_1}{\rightsquigarrow} V_{2\beta} \overset{p_2}{\rightsquigarrow} V_{3\gamma} \overset{p_3}{\rightsquigarrow} = -ieC (p_3 - p_2)_\alpha \\
\hat{V}_2 \text{-----} \hat{V}_3 \text{-----}
\end{array}$$

**Figure 2.** Three-vector vertex (box on the top) and its corresponding  $\epsilon$ -vertices.  $q$  represents the flow of the loop momentum.

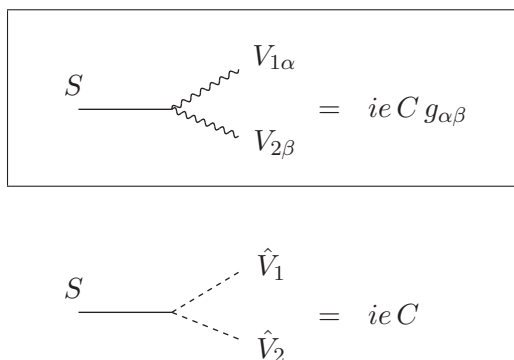
the standard vector-fermion-fermion and scalar-fermion-fermion vertices shown in the top parts of figures 9 and 10, while the corresponding  $\epsilon$ -vertices are drawn in the bottom parts. Notice also that all vertices in figure 9 are chirality flipping, due to the presence of  $\gamma_\alpha$  in the original interaction, while those in figure 10 are chirality preserving, because of their scalar nature. Particularly interesting is the vertex in figure 9 (c), which represents a  $\hat{V}\hat{F}\hat{F}$  interaction. Although no  $\gamma$  matrix is present in that vertex, it should be considered as a chirality flipping one, because of its vectorial origin.

$\epsilon$ -diagrams are built by using the rules of figures 8 to 10 and reading, as usual, the fermionic line backward starting from the arrow. After the last vertex is encountered, a chirality projector  $\omega^\pm = \frac{1}{2}(1 \pm \gamma_5)$  should be inserted, according to the chirality of the fermion entering into it. For example, the diagram in figure 11 generates the fermionic structure

$$\bar{v}(1)\gamma_\beta\gamma_\alpha\gamma^\beta\omega^h u(2). \tag{4.1}$$



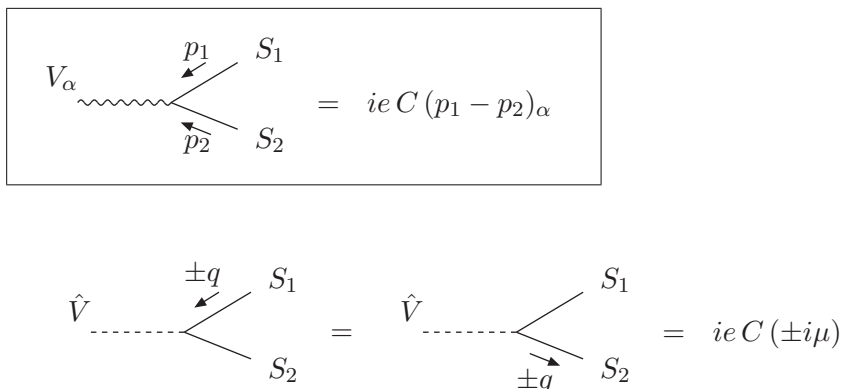
**Figure 3.** Four-vector vertex (box on the top) and its corresponding  $\epsilon$ -vertices.



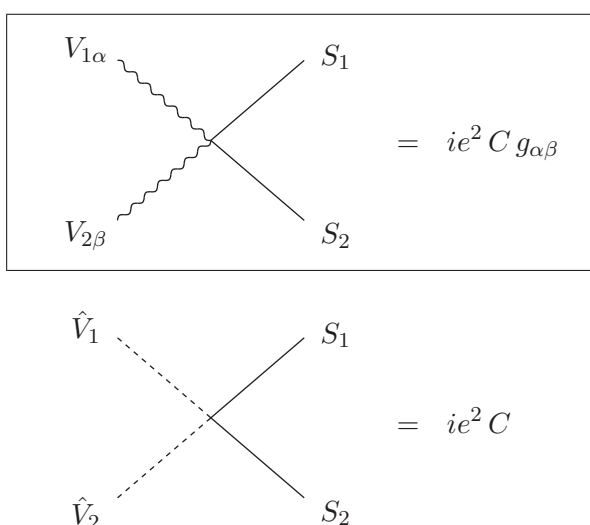
**Figure 4.** Two-vector-one-scalar vertex (box on the top) and its corresponding  $\epsilon$ -vertex.

For fermionic loops, the starting point is arbitrary and should be kept fixed when summing over diagrams and families in order to preserve the right cancellations [47].

When scalars are present, the rules presented in figures 8 to 10 define the diagrams up to a sign, which has to be included by hand. This is due to the anticommutation properties of  $\not{q}$  with the four dimensional  $\gamma$  matrices. The rule for fixing the sign is as follows. For each pair of  $\epsilon$ -fermion lines present in a given diagram the result should be multiplied by  $(-)^{(n_s+n_p)}$ , where  $n_s$  is the number of scalar vertices and  $n_p$  the number of



**Figure 5.** One-vector-two-scalar vertex (box on the top) and its corresponding  $\epsilon$ -vertex.  $q$  represents the flow of the loop momentum.



**Figure 6.** Two-vector-two-scalar vertex (box on the top) and its corresponding  $\epsilon$ -vertex.

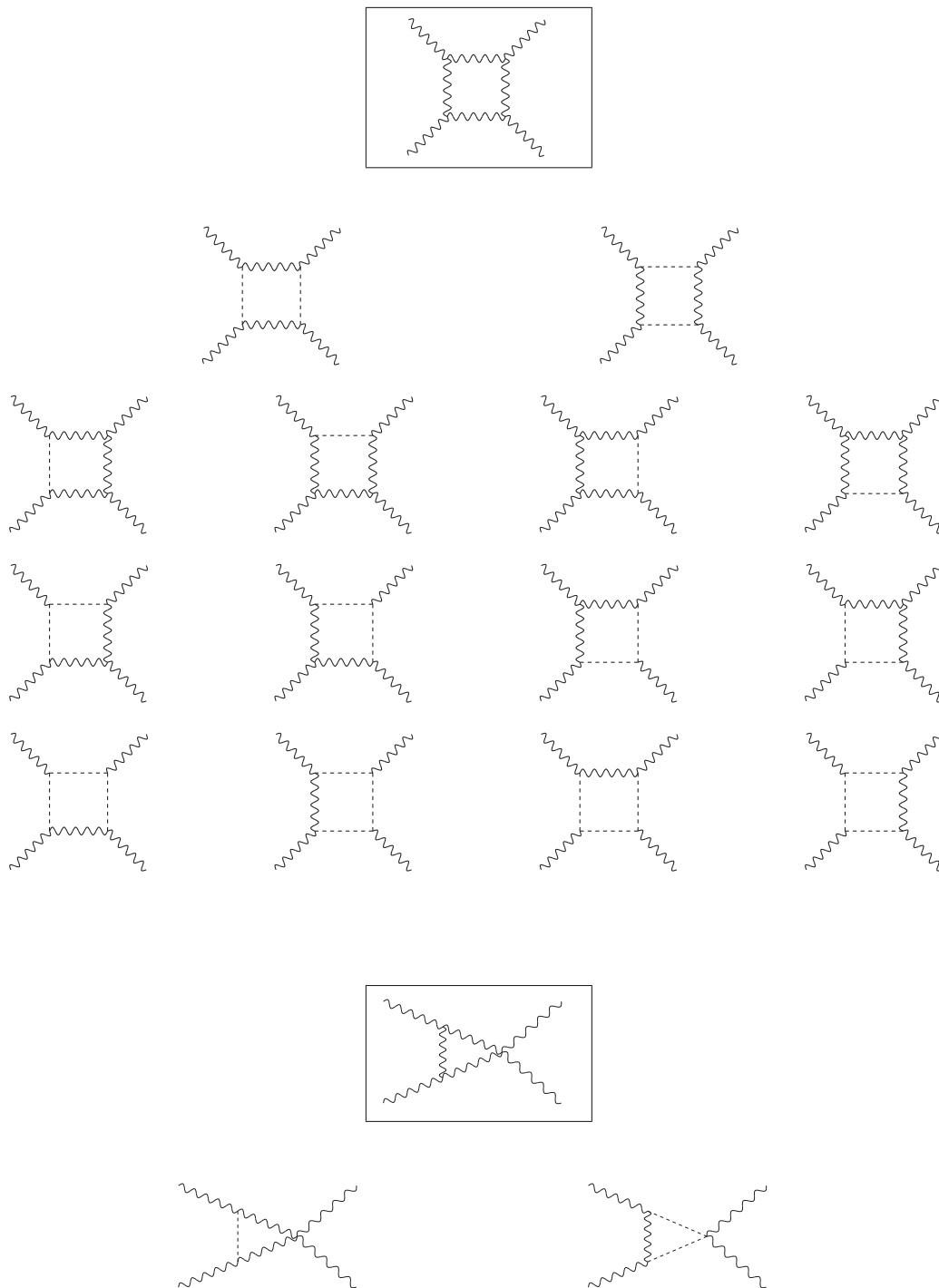
chirality preserving propagators of the kind  $i \frac{m_F}{p^2 - m_F^2}$  between them. For example, a minus sign should be assigned to the diagrams in figure 12 (a), (b) and (d) whereas a plus sign should be given to that one in figure 12 (c).

A last subtlety concerns again the vertex in figure 9 (c). The scalar  $\epsilon$ -dimensional degrees of freedom brought by the field  $\hat{V}$  can only originate from one of the vertices introduced in section 3. In fact,  $\hat{V}$  fields are only needed for non abelian theories because of the momentum dependent three-vector vertex of figure 2.<sup>4</sup> Therefore, diagrams where both ends of a  $\hat{V}$   $\epsilon$ -particle connect to a fermion line should be discarded, such as that one given in figure 13.

Finally, in the case of Majorana fermions, such as neutralinos and gluinos in SUSY theories, the relative sign of interfering Feynman diagrams can be determined as described in [67, 68].

<sup>4</sup>In pure QED, one just needs to introduce one  $\hat{F}$  field for each fermion family.





**Figure 7.** Examples of  $\epsilon$ -diagrams. In the two boxes I draw the original diagrams. The  $\epsilon$ -diagrams below each box reconstruct the complete  $\mu^2$  dependence of the integrand of the corresponding diagram, provided the same rooting for the loop momentum is chosen in each of them.

$$\begin{aligned}
 \boxed{
 \begin{aligned}
 \frac{F \xrightarrow{p} \bar{F}}{F \xrightarrow{p} \bar{F}} &= \frac{i}{\not{p} - m_F} = \frac{-h \xrightarrow{p} h}{F \xrightarrow{p} \bar{F}} + \frac{h \xrightarrow{p} h}{F \xrightarrow{p} \bar{F}} \\
 \frac{-h \xrightarrow{p} h}{F \xrightarrow{p} \bar{F}} &= i \frac{\not{p}}{p^2 - m_F^2}, \quad \frac{h \xrightarrow{p} h}{F \xrightarrow{p} \bar{F}} = i \frac{m_F}{p^2 - m_F^2}.
 \end{aligned}
 } \\
 \frac{\hat{F} \xrightarrow{p} \hat{F}}{\hat{F} \xrightarrow{p} \hat{F}} &= \frac{i}{p^2 - M_F^2} = \frac{-h \xrightarrow{p} h}{\hat{F} \xrightarrow{p} \hat{F}}
 \end{aligned}$$

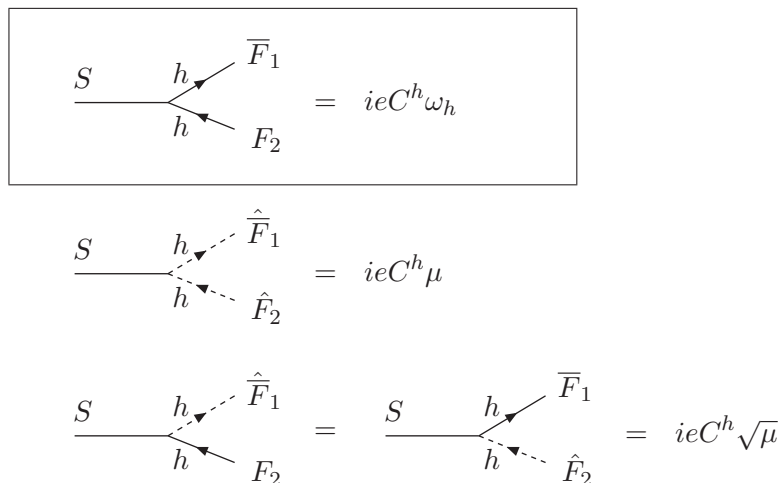
**Figure 8.** In the box on the top a fermion propagator is split in chirality flipping and chirality preserving parts.  $h = \pm$  denotes right-handed or left-handed components and the dashed line on the bottom represents the propagator of the  $\epsilon$ -particle associated with a fermion.

$$\begin{aligned}
 &\boxed{
 \begin{aligned}
 V_\alpha \begin{array}{l} \nearrow -h \bar{F}_1 \\ \searrow h F_2 \end{array} &= ie\gamma_\alpha C^h \omega_h
 \end{aligned}
 } \\
 \text{(a)} \quad &V_\alpha \begin{array}{l} \nearrow -h \hat{F}_1 \\ \searrow h \hat{F}_2 \end{array} = ie\gamma_\alpha C^h \mu \\
 \text{(b)} \quad &V_\alpha \begin{array}{l} \nearrow -h \hat{F}_1 \\ \searrow h F_2 \end{array} = V_\alpha \begin{array}{l} \nearrow -h \bar{F}_1 \\ \searrow h \hat{F}_2 \end{array} = ie\gamma_\alpha C^h \sqrt{\mu} \\
 \text{(c)} \quad &\hat{V} \begin{array}{l} \nearrow -h \bar{F}_1 \\ \searrow h \hat{F}_2 \\ \pm q \end{array} = \hat{V} \begin{array}{l} \nearrow \pm q \hat{F}_1 \\ \searrow -h F_2 \\ \pm q \end{array} = ieC^h (\pm i\sqrt{\mu})
 \end{aligned}$$

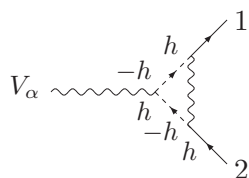
**Figure 9.** Vector-fermion-fermion vertex (box on the top) and its corresponding  $\epsilon$ -vertices.  $h = \pm$  denotes right-handed or left-handed fermion components,  $\omega_h$  is a chirality projector and  $q$  represents the flow of the loop momentum.

## 5 QCD

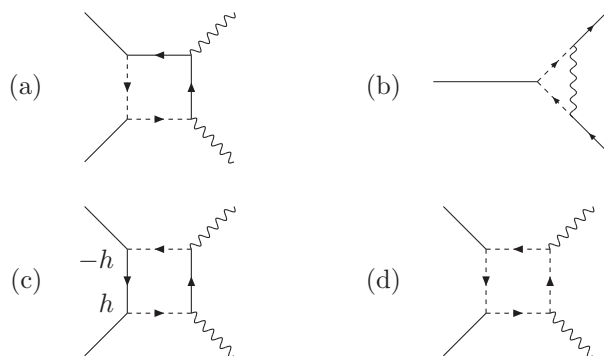
In the case of QCD no  $\gamma_5$  is present and splitting fermions into right-handed and left-handed components is no longer necessary. No additional sign needs to be inserted, since no scalar particles are involved in the  $\epsilon$ -fermionic vertices, although diagrams in which a scalar  $\epsilon$ -gluon connects 2 fermionic lines should be discarded, as explained in the previous section. I list the relevant QCD  $\epsilon$ -propagators and  $\epsilon$ -vertices in figure 14. As can be



**Figure 10.** Scalar-fermion-fermion vertex (box on the top) and its corresponding  $\epsilon$ -vertices.  $h = \pm$  denotes right-handed or left-handed fermion components and  $\omega_h$  is a chirality projector.

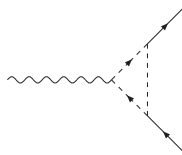


**Figure 11.**  $\epsilon$ -diagram generating the fermionic structure  $\bar{v}(1)\gamma_\beta\gamma_\alpha\gamma^\beta\omega^hu(2)$ .



**Figure 12.** Fermionic  $\epsilon$ -diagrams. A minus sign should be assigned to the diagrams (a), (b) and (d), while no additional sign is required for (c).

seen, they have exactly the same general Lorentz structure described in sections 3 and 4, while terms of the original color structures split among the various contributions in a well defined way.



**Figure 13.** Example of fermionic  $\epsilon$ -diagram that should be discarded because both ends of the  $\epsilon$ -gluon connect to a fermionic line.

## 6 Conclusions

I presented the set of special Feynman rules allowing the reconstruction of the  $\epsilon$ -dimensional part of 1-loop amplitudes in theories with vectors, scalars and fermions. The rules are quite simple, when assuming a renormalizable gauge, and easily derivable from the vertices of the original theory. They can be used to extract the  $\mu^2$  dependence from the integrand of any contributing 1-loop Feynman diagram, namely the  $\epsilon$ -dimensional part generated by self contractions of the loop momentum.

The complete electroweak model can be studied by simply fixing the constants appearing in the vertices of figures 1 to 6 and figures 8 to 10 to their standard model values, while the interactions relevant for QCD are explicitly listed in figure 14.

A four dimensional helicity scheme is used in this work, but simple translation rules to the 't Hooft Veltman scheme are collected in an appendix.

SUSY and BSM theories sharing the same Lorentz structures studied in this paper can be treated in the same way.

The special vertices presented here may also be considered as a practical tool to determine the counter-terms needed to restore gauge invariance in calculations where the numerator function of the 1-loop Feynman diagrams is computed in four dimensions. This possibility is particularly appealing in conjunction with schemes such as dimensional reduction, where the use of particular classes of four-dimensional identities involving  $\gamma_5$  is forbidden.

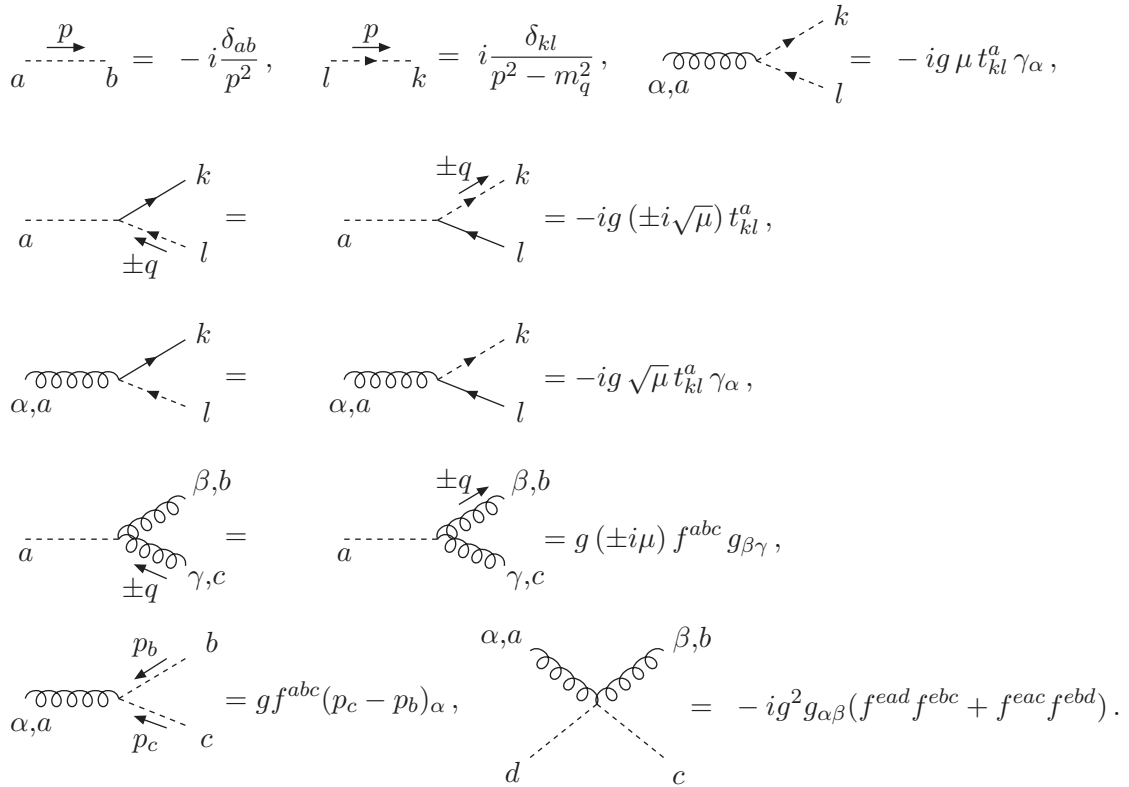
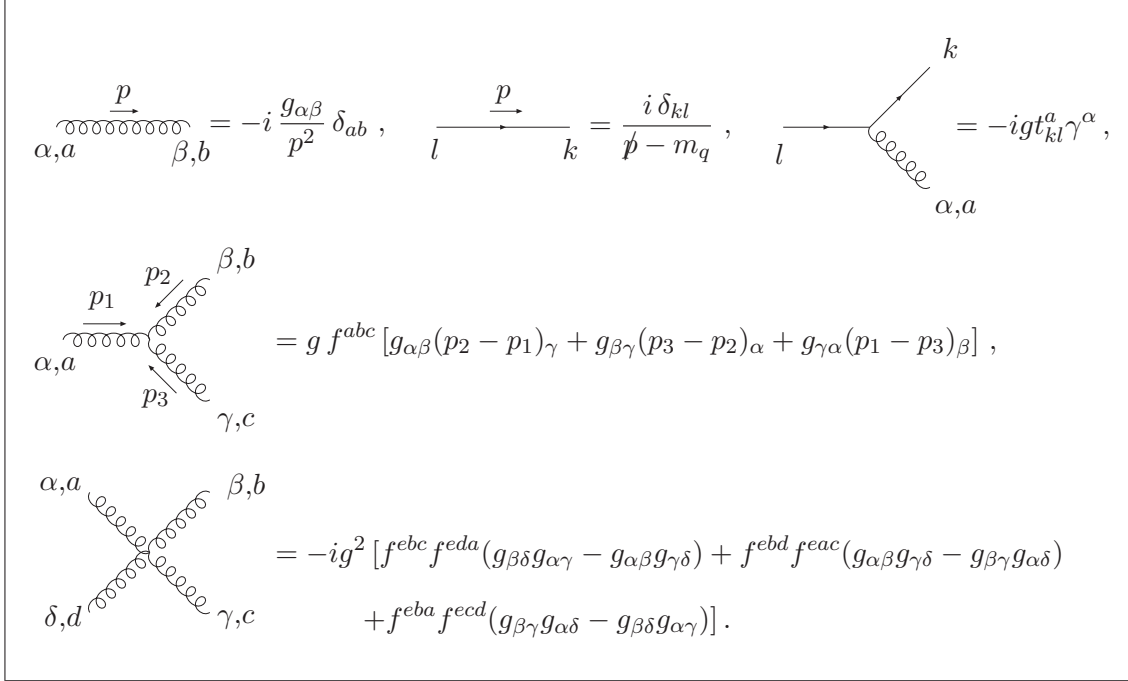
It would also be interesting to generalize this approach to the Unitary gauge and beyond 1-loop. I leave these two issues to future investigations.

## Acknowledgments

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## A From the FDH scheme to the HV scheme

In this appendix, I give the translation rules from the FDH scheme (or dimensional reduction) reproduced by the Feynman rules given in sections 3 to 5 and the HV scheme of eq. (1.5). For QCD, once a 1-loop amplitude  $A^{(1)}$  has been computed in FDH, the



**Figure 14.** QCD Feynman rules (box on the top) and corresponding  $\epsilon$ -propagators and  $\epsilon$ -vertices.  $q$  represents the flow of the loop momentum.

corresponding HV result can be obtained with the help of the formula [69]

$$A_{\text{HV}}^{(1)} = A_{\text{FDH}}^{(1)} + A^{(0)} \frac{g^2}{16\pi^2} \left[ \frac{N_c}{6} (n_q + n_Q - 2) - \frac{n_q}{2} \frac{N_c^2 - 1}{2N_c} \right], \quad (\text{A.1})$$

where  $A^{(0)}$  is the tree level result,  $N_c$  the number of colors,  $n_q$  the number of massless quarks and  $n_Q$  the number of massive quarks. For the electroweak model, once the renormalized 1-loop amplitude has been determined in FDH, since the terms proportional to  $\epsilon$  in eq. (1.5) are separately gauge invariant, their total contribution can be completely reabsorbed by shifts of the renormalization constants. The HV result can then be obtained through the replacements<sup>5</sup>

$$\begin{aligned} \delta t &\rightarrow \delta t - \frac{e}{8\pi^2 s} M_W^3 \left( 1 + \frac{1}{2c^4} \right) \\ \delta M_H^2 &\rightarrow \delta M_H^2 + 3 \frac{e^2}{16\pi^2 s^2} M_W^2 \left( 1 + \frac{1}{2c^4} \right) \\ \delta Z_H &\rightarrow \delta Z_H \\ \delta M_W^2 &\rightarrow \delta M_W^2 + \frac{e^2}{24\pi^2 s^2} M_W^2 \\ \delta Z_W &\rightarrow \delta Z_W - \frac{e^2}{24\pi^2 s^2} \\ \delta M_Z^2 &\rightarrow \delta M_Z^2 + \frac{e^2 c^2}{24\pi^2 s^2} M_Z^2 \\ \delta Z_{ZZ} &\rightarrow \delta Z_{ZZ} - \frac{e^2 c^2}{24\pi^2 s^2} \\ \delta Z_{AZ} &\rightarrow \delta Z_{AZ} + \frac{e^2 c}{12\pi^2 s} \\ \delta Z_{ZA} &\rightarrow \delta Z_{ZA} \\ \delta Z_{AA} &\rightarrow \delta Z_{AA} - \frac{e^2}{24\pi^2} \\ \delta m_{f,i} &\rightarrow \delta m_{f,i} - \frac{m_{f,i}}{2} \frac{e^2}{16\pi^2} \left( \frac{1}{4s^2 c^2} - 6 \frac{Q_f I_{W,f}^3}{c^2} + 6 \frac{Q_f^2}{c^2} + \frac{1}{2s^2} \right) \\ \delta Z_{ii}^{f,L} &\rightarrow \delta Z_{ii}^{f,L} + \frac{e^2}{16\pi^2} \left( \frac{1}{4s^2 c^2} - 2 \frac{Q_f I_{W,f}^3}{c^2} + \frac{Q_f^2}{c^2} + \frac{1}{2s^2} \right) \\ \delta Z_{ii}^{f,R} &\rightarrow \delta Z_{ii}^{f,R} + \frac{e^2}{16\pi^2} \frac{Q_f^2}{c^2}. \end{aligned} \quad (\text{A.2})$$

From eq. (A.2), the necessary shifts in the charge renormalization constant and in the sine and cosine of the weak mixing angle (relevant when using the on-shell scheme) can be

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<sup>5</sup>I use the same notations and conventions of [65] and assume a unit CKM matrix.

determined from the equations

$$\begin{aligned}
 \delta Z_e &= -\frac{1}{2} \left( \delta Z_{AA} + \frac{s}{c} \delta Z_{ZA} \right) \\
 \frac{\delta c}{c} &= \frac{1}{2} \left( \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right) \\
 \frac{\delta s}{s} &= -\frac{c^2}{s^2} \frac{\delta c}{c}.
 \end{aligned}
 \tag{A.4}$$

The rules in eq. (A.2) are easily derived from the explicit knowledge of the part proportional to  $\lambda_{\text{HV}}$  in the 2-point functions listed in [47].

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