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## P-odd and CP-odd four-quark contributions to neutron EDM

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**ABSTRACT:** In a class of beyond-standard-model theories, CP-odd observables, such as the neutron electric dipole moment, receive significant contributions from flavor-neutral P-odd and CP-odd four-quark operators. However, considerable uncertainties exist in the hadronic matrix elements of these operators strongly affecting the experimental constraints on CP-violating parameters in the theories. Here we study their hadronic matrix elements in combined chiral perturbation theory and nucleon models. We first classify the operators in chiral representations and present the leading-order QCD evolutions. We then match the four-quark operators to the corresponding ones in chiral hadronic theory, finding symmetry relations among the matrix elements. Although this makes lattice QCD calculations feasible, we choose to estimate the non-perturbative matching coefficients in simple quark models. We finally compare the results for the neutron electric dipole moment and P-odd and CP-odd pion-nucleon couplings with the previous studies using naive factorization and QCD sum rules. Our study shall provide valuable insights on the present hadronic physics uncertainties in these observables.

**KEYWORDS:** QCD Phenomenology

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**1 Introduction**

Neutron electric dipole moment (nEDM) has attracted considerable attention over more than half a century. For an elementary particle to have non-vanishing intrinsic EDM, simple analysis shows that parity-violating as well as time-reversal-violating interactions must be present. (T-violation is equivalent to CP-violation (combined charge-conjugation and parity) in local quantum field theory.) However, in the standard model (SM) of particle

physics, such interactions arise only from flavor-changing Cabbibo-Kobayashi-Moskawa (CKM) matrix elements, which are strongly suppressed phenomenologically, yielding a very small neutron EDM of order  $10^{-31}$  ecm. Therefore, an experimental observation of a large-size neutron EDM is an unambiguous signal for new physics, widely expected to exit somewhere between the electroweak symmetry breaking and TeV scales.

There has been much speculation in the literature on the nature of new physics. The leading theory is supersymmetry (SUSY) introduced to stabilize the electroweak scale against the fundamental Planck scale (for a good review, see ref. [1]). Various extra dimensional models have been popular in recent years as well (for good reviews see ref. [2, 3]). There are other less dramatic possibilities including technicolor [4], left-right symmetry [5], minimal extensions of the SM [6], little Higgs models [7], etc. In each of these models, there is new CP violation physics giving rise to a significant neutron EDM. To test CP-violation mechanisms, it is important to get accurate predictions of CP-violating observables in these models. In particular, one has to deal with the non-perturbative quantum chromodynamics (QCD) physics present in the structure of the neutron.

An efficient way to calculate the neutron EDM is to use the methodology of effective field theories (EFT). In this approach, one generates P-odd and CP-odd quark and gluon operators after integrating out the heavy particles (including heavy quarks, gauge bosons and new particles) and run these operators to a scale around 1 GeV where non-perturbative QCD physics becomes important. The effective degrees of freedom involves the light quarks (up, down and strange) and gluons. The CP-odd part of the lagrangian is generally written as a sum of CP-odd operators of different mechanical dimensions,

$$\mathcal{L}_{\text{CP-odd}} = \sum_{d=3}^{\infty} \sum_i C_{di}(\mu) \hat{O}_{di}(\mu), \quad (1.1)$$

where  $d = 3, 4, 5$ , etc, is the mechanical dimension of the operators,  $\mu$  is the renormalization scale (taken as  $4\pi F_\pi$  in this paper) and  $i$  sums over operators of the same dimension. The dim-3 operator is the usual CP-odd quark mass term  $\bar{q}i\gamma_5 q$ , which can be rotated away through chiral rotations apart from the  $U_A(1)$  anomaly. The dim-4 operator is the usual  $\theta$  term  $G\tilde{G}$ . Dim-5 operators include quark electric and chromoelectric dipole operators. Dim-6 operators contain various four-quark operators and Weinberg three-gluon operator. The matrix elements of dim-4 and, to less extent, dim-5 operators have been studied extensively in the literature [8–17], and the uncertainty of the estimates are typically at the level of factor 2. The contributions of these operators have also been studied extensively in the context of various new physics models (see refs. [18, 19] for good reviews).

However, the matrix elements of dim-6 operators have been a challenge to estimate. In some beyond-SM theories such as the left-right symmetric model, dim-6 four-quark operators dominate the contributions to nEDM. In the literature, the only serious approach that has been proposed to calculate their matrix elements is the naive factorization method: breaking the four-quark matrix elements into the product of two-quark matrix elements between the nucleon states and between pion and vacuum [20–22]. While the factorization involving mesons can be and has been tested using lattice QCD [23] and the results may be trustable to within a factor of 2, the same is not known for matrix elements involving

the nucleon states. The goal of this paper is to develop a chiral perturbation method combined with simple quark models to estimate the four-quark contribution to the nEDM with hopefully an improved accuracy.

The approach we are going to take is the standard chiral perturbation theory ( $\chi$ PT) (see, for example, ref. [24, 25]) which has been used to calculate the contribution of  $\theta$ -term to nEDM [14]. One of the successes of the chiral approach can be illustrated by the polarizabilities of the nucleon. The electric polarizabilities of the proton and neutron have been extracted from experimental data,  $\alpha_p^{\text{exp}} = (10.4 \pm 0.6) \times 10^{-4} \text{ fm}^3$ ,  $\alpha_n^{\text{exp}} = (12.3 \pm 1.3) \times 10^{-4} \text{ fm}^3$ . The leading contribution in  $\chi$ PT comes from the pion-nucleon intermediate states,

$$\alpha_p = \alpha_n = \frac{5\alpha_{\text{em}}g_A^2}{96\pi F_\pi^2 m_\pi} \approx 11 \times 10^{-4} \text{ fm}^3, \tag{1.2}$$

which diverges linearly as  $m_\pi \rightarrow 0$  and agrees well with the experimental data. One would expect then a similar pion dominance in the neutron EDM because the latter also involves the intermediate electric dipole excitations. Indeed a pioneering calculation by Crewther et al. found that the dominant contribution from the charged-pion chiral-loop diverges logarithmically as  $m_\pi$  goes to zero, and is proportional to the CP-odd pion-nucleon-nucleon coupling  $\bar{g}_{\pi NN}$  [16]. In this paper, we take this contribution as dominating and consider the four-quark operator contribution to  $\bar{g}_{\pi NN}$ . Of course, there are chiral-regular contributions to the nEDM which are of the same order in chiral power counting and numerically competitive or even dominating in the real world [19]. We will consider these contributions as well, although the model-dependence becomes unavoidable.

In the chiral approach, one first writes down the CP-odd and even lagrangian in terms of meson and nucleon fields,

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{Goldstone-boson CP-odd term}} + \mathcal{L}_{\text{nucleon CP-odd mass term}} \\ & + \mathcal{L}_{\text{EDM term}} + \mathcal{L}_{\text{CP-odd } \pi\text{-N coupling}} + (\text{CP - even terms}) \end{aligned} \tag{1.3}$$

where the Goldstone boson CP-odd lagrangian will generate terms annihilating  $\pi^0$  and  $\eta$  in the vacuum, or in other words, will produce meson condensates. The condensates will turn some of the CP-even terms (as we shall see, those proportional to quark masses) in the chiral lagrangian into CP-odd contributions. This will generate an additional CP-odd nucleon-mass term, neutron EDM term and CP-odd pion-nucleon coupling. Once this is done, one can rotate away the CP-odd nucleon mass term, generating further contributions to the neutron EDM terms and the CP-odd pion-nucleon coupling.

The presentation of the paper is organized as follows: In section II, we classify all flavor-neutral P-odd and CP-odd four-quark operators in chiral representations. We also present the leading-order QCD scale evolution of these operators. In section III, we match these operators to the corresponding Goldstone boson operators, baryon operators, and EDM operators in  $\chi$ PT. We also discuss in the case of Peccei-Quinn symmetry the size of the induced  $\theta$  term in the presence of these four-quark operators. In section IV, we calculate their contributions to the P-odd and CP-odd nucleon-pion vertices and the CP-odd nucleon mass using factorization in the case of meson matrix elements and simple

quark models for the nucleon ones. In section V, we study the four-quark contribution to the neutron EDM in the chiral approach supplemented with factorization and quark model estimates of counter terms, and the results are compared with other calculations in the literature. The comparison and analysis show that the hadronic physics uncertainties here can be quantified to within a factor of two for operators generating unsuppressed meson condensate contributions. We conclude the paper in section VI.

To make the convention clear, the EDM operator for spin-1/2 particles is defined as

$$\mathcal{L} = -\frac{1}{2}d^E\psi\sigma_{\mu\nu}i\gamma_5\psi F^{\mu\nu}, \quad (1.4)$$

where  $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$  and  $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$  in the standard Dirac representation.

## 2 P-odd and CP-odd four-quark operators: classification, running and mixing

We consider three light quark flavors: up, down and strange. Flavor-neutral P-odd and CP-odd four-quark operators can be divided into two groups [20]: The first group includes 18 operators made of two different flavors:

$$\begin{aligned} O_{11} &= (\bar{q}i\gamma_5q)(\bar{q}'q'), \\ O_{12} &= (\bar{q}q)(\bar{q}'i\gamma_5q'), \\ O_{21} &= (\bar{q}i\gamma_5t^aq)(\bar{q}'t^aq'), \\ O_{22} &= (\bar{q}t^aq)(\bar{q}'i\gamma_5t^aq'), \\ O_3 &= (\bar{q}i\gamma_5\sigma^{\mu\nu}q)(\bar{q}'\sigma_{\mu\nu}q'), \\ O_4 &= (\bar{q}i\gamma_5\sigma^{\mu\nu}t^aq)(\bar{q}'\sigma_{\mu\nu}t^aq'), \end{aligned} \quad (2.1)$$

where  $q, q' = u, d, s$  and  $q < q'$ . The second group includes 6 operators made of a single quark flavor

$$\begin{aligned} O'_1 &= (\bar{q}i\gamma_5q)(\bar{q}q), \\ O'_2 &= (\bar{q}i\gamma_5t^aq)(\bar{q}t^aq), \end{aligned} \quad (2.2)$$

where  $t^a$  are generators of the SU(3) color group. All other flavor-neutral P-odd and CP-odd four-quark operators can be related to the above through Fierz transformation [20].

To match the above quark operators into the hadronic ones in  $\chi$ Pt, we have to classify the former into irreducible representations of the chiral group  $SU(3)_L \times SU(3)_R$ . Take the operator  $\bar{u}i\gamma_5u\bar{d}d$  as an example, which can be decomposed as

$$\bar{u}i\gamma_5u\bar{d}d = -i\bar{u}_Ru_L\bar{d}_Rd_L + i\bar{u}_Lu_R\bar{d}_Rd_L + \text{h.c.}, \quad (2.3)$$

where  $q_{L,R} = P_{L,R}q$  with  $P_{L,R} = (1 \mp \gamma_5)/2$ . The first term can be further decomposed as

$$\begin{aligned} -i\bar{u}_Ru_L\bar{d}_Rd_L &= -\frac{i}{2}(\bar{u}_Ru_L\bar{d}_Rd_L + \bar{d}_Ru_L\bar{u}_Rd_L) \\ &\quad -\frac{i}{2}(\bar{u}_Ru_L\bar{d}_Rd_L - \bar{d}_Ru_L\bar{u}_Rd_L) \\ &= -\frac{i}{4}S_{kl}^{ij}\bar{q}_Ri^kq_L^k\bar{q}_Rj^lq_L^l - \frac{i}{4}\epsilon_{imn}^jA_j^i\bar{q}_Rkq_L^m\bar{q}_Rlq_L^n, \end{aligned} \quad (2.4)$$

where  $\epsilon_{imn}^{jkl} \equiv \epsilon^{ijkl} \epsilon^{imn}$ , and

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.5)$$

and

$$S_{12}^{12} = S_{12}^{21} = S_{21}^{12} = S_{21}^{21} = 1, \quad (2.6)$$

with other elements vanishing. The second term of eq. (2.3) can be written as

$$i\bar{u}_L u_R \bar{d}_R d_L = iH_{1i}^j H_{2k}^l \bar{q}_{Lj}^k \bar{q}_{Rl}^i q_L^i, \quad (2.7)$$

where

$$H_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.8)$$

In this way the operator  $\bar{u}i\gamma_5 u \bar{d}d$  is decomposed into  $(\bar{3}, 3)$ ,  $(6, \bar{6})$  and  $(8, 8)$  representations of  $SU(3)_L \times SU(3)_R$ , and  $A$ ,  $S$ ,  $H_1$ , and  $H_2$  can be regarded as spurion fields in the sense that if they transform as  $(3, \bar{3})$ ,  $(\bar{6}, 6)$  and  $(8, 8)$  under chiral transformation, the corresponding terms in eqs. (2.4) and (2.7) become invariant. These spurion fields will be used in  $\chi PT$  to construct the effective operators corresponding to the same four-quark operators. All spurion fields for four-quark operators with different Dirac and color structures are shown in table 1. (It is easy to see that there is no  $(1, 1)$  operator because any such operator must be expressible in terms of products of chiral-even quark currents, which cannot yield CP-odd contributions.)

The four-quark operators usually emerge at a high energy scale where some heavy particles have been integrated out. To match them to hadronic operators in effective theories, one must run them down to a low energy scale where non-perturbative physics becomes important. We can choose this to be 1 GeV or the lattice cut-off  $1/a$ , where  $a$  is the lattice spacing. In this work, we take  $\mu = 4\pi F_\pi$ , with  $F_\pi = 93$  MeV. These operators mix with each other when the energy scale changes. Although many of the mixings have been calculated in the literature before (see ref. [26], for example), we recalculate them and present the complete result here for easy reference:

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O_{11} \\ O_{12} \\ O_{21} \\ O_{22} \\ O_3 \\ O_4 \end{pmatrix} = \frac{\alpha_S(\mu)}{4\pi} \begin{pmatrix} 8 & 0 & 0 & 0 & 0 & 1 \\ 0 & 8 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & \frac{2}{9} & \frac{5}{12} \\ 0 & 0 & 0 & -1 & \frac{2}{9} & \frac{5}{12} \\ 0 & 0 & 24 & 24 & -\frac{8}{3} & 0 \\ \frac{16}{3} & \frac{16}{3} & 10 & 10 & 0 & \frac{19}{3} \end{pmatrix} \begin{pmatrix} O_{11} \\ O_{12} \\ O_{21} \\ O_{22} \\ O_3 \\ O_4 \end{pmatrix}, \quad (2.9)$$

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O'_1 \\ O'_2 \end{pmatrix} = \frac{\alpha_S(\mu)}{4\pi} \begin{pmatrix} \frac{40}{9} & -\frac{4}{3} \\ -\frac{80}{27} & -\frac{46}{9} \end{pmatrix} \begin{pmatrix} O'_1 \\ O'_2 \end{pmatrix}. \quad (2.10)$$

Clearly operators with different quark flavor structures do not mix. Since  $SU(3)_L \times SU(3)_R$  symmetry is broken only by quark masses, four-quark operators belonging to different chiral

	$(\bar{3}, 3)$	$(6, \bar{6})$	$(8, 8)$
$q = u, q' = d$	$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$S_{12}^{12} = S_{21}^{12} = S_{12}^{21} = S_{21}^{21} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$q = d, q' = u$	$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$S_{12}^{12} = S_{21}^{12} = S_{12}^{21} = S_{21}^{21} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$q = u, q' = s$	$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$S_{13}^{13} = S_{31}^{13} = S_{13}^{31} = S_{31}^{31} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$q = s, q' = u$	$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$S_{13}^{13} = S_{31}^{13} = S_{13}^{31} = S_{31}^{31} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$q = d, q' = s$	$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$S_{23}^{23} = S_{32}^{23} = S_{23}^{32} = S_{32}^{32} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
$q = s, q' = d$	$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$S_{23}^{23} = S_{32}^{23} = S_{23}^{32} = S_{32}^{32} = 1$ other components are zero	$H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, H_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
$q = u, q' = u$	$A = 0$	$S_{11}^{11} = 4$ other components are zero	$H_1 = 0, H_2 = 0$
$q = d, q' = d$	$A = 0$	$S_{22}^{22} = 4$ other components are zero	$H_1 = 0, H_2 = 0$
$q = s, q' = s$	$A = 0$	$S_{33}^{33} = 4$ other components are zero	$H_1 = 0, H_2 = 0$

**Table 1.** Spurions for CP-odd 4-quark operators. The first six together with three tensor structures yield 18 operators in eq. (4) and the last three with two tensor structures yield six operators in eq. (5).

irreducible representations do not mix either. Therefore, we can further simplify eq. (12),

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O_1^{(3,6)} \\ O_2^{(3,6)} \\ O_3^{(3,6)} \\ O_4^{(3,6)} \end{pmatrix} = \frac{\alpha_S(\mu)}{4\pi} \begin{pmatrix} 8 & 0 & 0 & 1 \\ 0 & -1 & \frac{2}{9} & \frac{5}{12} \\ 0 & 48 & -\frac{8}{3} & 0 \\ \frac{32}{3} & 20 & 0 & \frac{19}{3} \end{pmatrix} \begin{pmatrix} O_1^{(3,6)} \\ O_2^{(3,6)} \\ O_3^{(3,6)} \\ O_4^{(3,6)} \end{pmatrix} \quad (2.11)$$

$$\mu^2 \frac{d}{d\mu^2} \begin{pmatrix} O_1^{(8)} \\ O_2^{(8)} \end{pmatrix} = \frac{\alpha_S(\mu)}{4\pi} \begin{pmatrix} 8 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} O_1^{(8)} \\ O_2^{(8)} \end{pmatrix} \quad (2.12)$$

where  $O_i^{(3,6,8)}$  means the projections of the operator  $O_i$  on the representations  $(\bar{3}, 3)$ ,  $(6, \bar{6})$  and  $(8, 8)$ , respectively. It is easy to see that the  $(\bar{3}, 3)$  and  $(6, \bar{6})$  projections of  $O_{i1}$  and  $O_{i2}$  are the same with  $i = 1, 2$ , whereas their  $(8, 8)$  projections differ only by the sign.

The tensor operators do not have (8, 8) components and therefore do not participate in eq. (2.12).

The four-quark operators may also mix with P-odd and CP-odd operators with dimension less or equal to 6. For mixing with lower-dimensional operators, either quark masses or power divergences will appear. The only other dimension-6 operator is the Weinberg operator [27]

$$O_W = -\frac{1}{6} f^{abc} \epsilon^{\mu\nu\alpha\beta} G_{\mu\rho}^a G_{\nu}^{b\rho} G_{\alpha\beta}^c, \quad (2.13)$$

which is a singlet under chiral transformation. Since the four-quark operators contain no singlet component, the mixing between them and  $O_W$  vanishes. The evolution of the Weinberg operator can be found in ref. [28].

The P-odd and CP-odd dimension-5 operators are the quark electric dipole moment operators (QEDM) and quark chromo-electric dipole moment operators (QCEDM). In principle, they belong to  $(\bar{3}, 3)$  of the chiral group. However, they can mix logarithmically with four-quark operators multiplied by the quark mass which transforms also like  $(\bar{3}, 3)$  [29, 30].

Finally, the four-quark operators can have mixing with  $m\bar{q}i\gamma_5 q$  with quadratically divergent coefficients. Usually, one defines the four-quark operators with quadratic divergences subtracted, as is natural in dimensional regularization where all quadratically divergent integrals vanish by definition. Equivalently, this can be achieved, for example, by demanding the CP-odd four-quark operators have vanishing contribution between QCD vacuum and CP-odd meson states in perturbation theory. However, as we shall see in the following section, they can have non-perturbative contributions. The exact physical implication of this non-perturbative contribution will be discussed in section V.E.

### 3 Matching to operators in chiral perturbation theory

Generically, any P-odd, CP-odd quark-gluon operator contributes to all P-odd, CP-odd hadronic operators in  $\chi$ PT; the latter are constructed in terms of Goldstone-boson (pion, kaon, eta) fields and baryon fields. Here we consider just the contributions to the Goldstone-boson CP-odd interactions, nucleon CP-odd mass term,  $\pi$ - $N$  CP-odd coupling, as well as the neutron EDM term,

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{Goldstone-boson CP-odd term}} + \mathcal{L}_{\text{nucleon CP-odd mass term}} \\ & + \mathcal{L}_{\text{CP-odd } \pi\text{-}N \text{ coupling}} + \mathcal{L}_{\text{EDM term}} . \end{aligned} \quad (3.1)$$

Following the standard practice in the literature, we imbed the Goldstone-boson fields in the unitary matrix  $U = \exp[2i\Sigma/F_\pi]$  with

$$\Sigma = \begin{pmatrix} \frac{1}{2}\pi^0 + \frac{1}{2\sqrt{3}}\eta & \frac{1}{\sqrt{2}}\pi^+ & \frac{1}{\sqrt{2}}K^+ \\ \frac{1}{\sqrt{2}}\pi^- & -\frac{1}{2}\pi^0 + \frac{1}{2\sqrt{3}}\eta & \frac{1}{\sqrt{2}}K^0 \\ \frac{1}{\sqrt{2}}K^- & \frac{1}{\sqrt{2}}\bar{K}^0 & -\frac{1}{\sqrt{3}}\eta \end{pmatrix}, \quad (3.2)$$

where  $F_\pi$  is the pion decay constant. Under chiral rotations,  $U$  transforms like  $U \rightarrow LUR^\dagger$ , where  $L$  and  $R$  are SU(3) matrices belonging to SU(3) $_L$  and SU(3) $_R$  groups, respectively.



To include the baryon octet, we introduce

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (3.3)$$

Again following the literature, we assume  $B$  transforms nonlinearly under chiral transformation,

$$B \rightarrow K B K^\dagger \quad (3.4)$$

where  $K$  is a unitary matrix defined according to the transformation of  $\xi = U^{1/2}$ .

$$\xi \rightarrow L \xi K^\dagger, \quad \xi \rightarrow K \xi R^\dagger \quad (3.5)$$

It is clear that  $K$  is a nonlinear function of the Goldstone-boson fields.

The quark-mass term breaks chiral symmetry and plays an important role in chiral expansion. To exhibit its physical effect, the usual practice is to introduce the spurion field  $\chi$ , transforming as

$$\chi \rightarrow L \chi R^\dagger. \quad (3.6)$$

However, to combine  $\chi$  with the baryon field  $B$ , we introduce  $\chi_\pm$

$$\chi_\pm = \xi^\dagger \chi \xi^\dagger \pm \xi \chi^\dagger \xi, \quad (3.7)$$

which transform nonlinearly as  $\chi_\pm \rightarrow K \chi_\pm K^\dagger$ .

In the leading order, the chiral lagrangian for meson fields is

$$\mathcal{L} = \frac{1}{4} F_\pi^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{1}{2} F_\pi^2 B \text{Tr}[M^\dagger U + U^\dagger M], \quad (3.8)$$

where  $M = \text{diag}\{m_u, m_d, m_s\}$  is the mass matrix of light quarks. The leading-order chiral lagrangian for the baryon field is [25]

$$\mathcal{L} = \text{Tr} \left\{ \bar{B} i \gamma^\mu D_\mu B - m_0 \bar{B} B + \frac{1}{2} D \bar{B} \gamma^\mu \gamma_5 u_\mu, B + \frac{1}{2} F \bar{B} \gamma_\mu \gamma_5 [u_\mu, B] \right\}, \quad (3.9)$$

where  $u_\mu = i(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$  is an axial vector current,  $D_\mu B = \partial_\mu B + [\Gamma_\mu, B]$  and  $\Gamma_\mu = \{\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\}/2$  is a vector current.

### 3.1 Matching to CP-Odd Goldstone-Boson operators

Once there is a CP-odd term in the QCD lagrangian, it induces CP-odd terms in the effective Goldstone-boson lagrangian. These terms can annihilate odd-number (particularly, one) Goldstone bosons into the vacuum. Because of this CP-odd meson condensate, the original CP-even terms can now contribute to the CP-odd effects. Due to chiral symmetry, a meson condensate can generate physical effects only when the CP-even terms explicitly break the symmetry.

As discussed in the last section, P-odd and CP-odd four-quark operators can be decomposed into chiral  $(\bar{3}, 3)$ ,  $(6, \bar{6})$ ,  $(8, 8)$  and their hermitian conjugate representations. They

Rep.	$(\bar{3}, 3)$	$(6, \bar{6})$	$(8, 8)$
Operator	$O_3^m = i\text{Tr}[AU^\dagger]$	$O_6^m = iS_{kl}^{ij}U_i^kU_j^l$	$O_8^m = i\text{Tr}[H_1UH_2U^\dagger]$

**Table 2.** Leading meson operators in individual irreducible chiral representations where  $A$ ,  $S$ ,  $H_1$  and  $H_2$  are spurion fields in table 1. The appearance of  $i$  in front of each operator indicates that these operators generate P-odd and CP-odd vertices in the meson lagrangian; their Wilson coefficients in the lagrangian are defined to be real.

in turn can be matched to the corresponding chiral operators in the same representations. The leading ones without derivatives are unique and are shown in table 2.

We illustrate the matching process using  $O_{11}^{ud} = \bar{u}i\gamma_5 u\bar{d}d$  as an example. As discussed in the last section, this quark operator can be decomposed into irreducible representations of the chiral group using the spurion fields

$$O_{11}^{ud} = O_{11}^{ud,(\bar{3},3)} + O_{11}^{ud,(6,\bar{6})} + O_{11}^{ud,(8,8)} + \text{h.c.} \quad (3.10)$$

Then, we can match each of the operators to the corresponding one in the meson sector through the non-perturbative Wilson coefficients  $C'$ s

$$O_{11}^{ud,(\bar{3},3)} \sim C^{(\bar{3},3)}O_3, \quad O_{11}^{ud,(6,\bar{6})} \sim C^{(6,\bar{6})}O_6, \quad O_{11}^{ud,(8,8)} \sim C^{(8,8)}O_8. \quad (3.11)$$

The Wilson coefficients can be obtained by matching the simplest matrix elements:  $\langle 0|O|\pi^0\rangle$  and  $\langle 0|O|\eta\rangle$ , which can be calculated using non-perturbative methods such as lattice QCD.

In this paper, we use factorization approximation to estimate these non-perturbative matrix elements. Lattice QCD calculations demonstrate that the matrix elements of four-quark operators can be factorized typically to within a factor of 2. Again take the operator  $O_{11}^{ud}$  as an example, which can annihilate  $\pi^0$  and  $\eta$  to the vacuum. (In principle, it also annihilates  $\eta'$ , but this contribution is suppressed by the mass of  $\eta'$ .) The annihilation amplitude can be estimated using vacuum saturation,

$$\langle 0|\bar{u}i\gamma_5 u\bar{d}d|\pi^0\rangle \approx \langle 0|\bar{d}d|0\rangle\langle 0|\bar{u}i\gamma_5 u|\pi^0\rangle. \quad (3.12)$$

Using chiral symmetry, one can get  $\langle 0|\bar{u}i\gamma_5 u|\pi^0\rangle = \frac{1}{F_\pi}\langle 0|\bar{u}u|0\rangle \equiv -F_\pi B_0$ , and  $\langle 0|\bar{u}i\gamma_5 u|\eta\rangle = -F_\pi B_0/\sqrt{3}$ . (This is consistent with the definition of the chiral rotation of  $U$  defined below eq. (3.2).) Therefore, a term  $C_4 O_{11}^{ud}$  in the QCD lagrangian can be matched to the linear terms in  $\pi^0$  and  $\eta$  in the chiral lagrangian

$$\mathcal{L} = C_4 B_0^2 F_\pi^3 \pi^0 + \frac{1}{\sqrt{3}} C_4 B_0^2 F_\pi^3 \eta + \dots, \quad (3.13)$$

where ... represents higher-power meson fields. Then the leading terms in the potential of  $\pi^0$  and  $\eta$  can be written as

$$V = \frac{1}{2}B_0 \left[ (m_u + m_d)(\pi^0)^2 + \frac{1}{3}(m_u + m_d + 4m_s)\eta^2 \right] + \frac{B_0}{\sqrt{3}}(m_u - m_d)\pi^0\eta - C_4 B_0^2 F_\pi^3 \left( \pi^0 + \frac{1}{\sqrt{3}}\eta \right), \quad (3.14)$$

which can be minimized to yield a condensate  $\langle \pi^0 \rangle$  and  $\langle \eta \rangle$ .

The above discussion can be easily generalized to an arbitrary four-quark operator, for which eq. (3.13) can be written as

$$\mathcal{L} = g_\pi C_4 B_0^2 F_\pi^3 \pi^0 + g_\eta C_4 B_0^2 F_\pi^3 \eta + \dots, \quad (3.15)$$

where  $g_\pi$  and  $g_\eta$  are numerical factors generated through the vacuum saturation approximation. Then, the vevs of meson fields can be written as

$$\begin{aligned} \langle \pi^0 \rangle &= \frac{B_0 F_\pi^3 C_4 [g_\pi(m_u + m_d + 4m_s) - \sqrt{3}g_\eta(m_u - m_d)]}{4(m_u m_d + m_d m_s + m_s m_u)}, \\ \langle \eta \rangle &= \frac{B_0 F_\pi^3 C_4 [-\sqrt{3}g_\pi(m_u - m_d) + 3g_\eta(m_u + m_d)]}{4(m_u m_d + m_d m_s + m_s m_u)}, \end{aligned} \quad (3.16)$$

which is inversely proportional to quark masses! The vev of  $U$  can be written as

$$\langle U \rangle = \exp \left[ i \begin{pmatrix} \langle \pi^0 \rangle + \frac{1}{\sqrt{3}} \langle \eta \rangle & 0 & 0 \\ 0 & -\langle \pi^0 \rangle + \frac{1}{\sqrt{3}} \langle \eta \rangle & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \langle \eta \rangle \end{pmatrix} / F_\pi \right]. \quad (3.17)$$

This defines the vacuum state of Goldstone-boson fields.

Therefore, we can redefine the meson fields in the following way:

$$U = \langle U \rangle U', \quad (3.18)$$

where  $U'$  collects the physical meson excitations. Through this redefinition, the meson lagrangian no longer contains terms annihilating the physical Goldstone bosons. Correspondingly, we redefine the baryon fields,

$$\xi B \xi = \langle U \rangle \xi' B' \xi', \quad (3.19)$$

through a chiral transformation with  $L = \langle U \rangle$  and  $R = 1$ .

The above redefinition can change P-even and CP-even terms with explicit chiral symmetry breaking to P-odd and CP-odd terms. This is particularly true for the CP-even baryon lagrangian with linear dependence on quark masses,

$$\mathcal{L}_c = c_1 \text{Tr}[\bar{B}B] \text{Tr}[MU^\dagger] + c_2 \text{Tr}[M\xi^\dagger \bar{B}B\xi^\dagger] + c_3 \text{Tr}[\bar{B}\xi^\dagger M\xi^\dagger B] + \text{h.c.} \quad (3.20)$$

and

$$\mathcal{L}_d = d_1 \text{Tr}[\bar{B}\gamma_5 B] \text{Tr}[MU^\dagger] + d_2 \text{Tr}[M\xi^\dagger \bar{B}\gamma_5 B\xi^\dagger] + d_3 \text{Tr}[\bar{B}\gamma_5 \xi^\dagger M\xi^\dagger B] + \text{h.c.} \quad (3.21)$$

Substituting  $\langle U \rangle$  to the above equation, we get CP-odd pion-nucleon couplings through

$$c_1 \text{Tr}[\bar{B}'B'] \text{Tr}[\langle U \rangle^\dagger MU'^\dagger] + c_2 \text{Tr}[\langle U \rangle^\dagger M\xi'^\dagger \bar{B}'B'\xi'^\dagger] + c_3 \text{Tr}[\bar{B}'\xi'^\dagger \langle U \rangle^\dagger M\xi'^\dagger B'] + \text{h.c.} \quad (3.22)$$

and the CP-odd masses of baryons

$$d_1 \text{Tr}[\bar{B}'\gamma_5 B'] \text{Tr}[\langle U \rangle^\dagger MU'^\dagger] + d_2 \text{Tr}[\langle U \rangle^\dagger M\xi'^\dagger \bar{B}'\gamma_5 B'\xi'^\dagger] + d_3 \text{Tr}[\bar{B}'\gamma_5 \xi'^\dagger \langle U \rangle^\dagger M\xi'^\dagger B'] + \text{h.c.} \quad (3.23)$$

Rept.	Operators
( $\bar{3}, 3$ )	$O_3^{(1)} = i\text{Tr}[\bar{B}B]\text{Tr}[AU^\dagger], O_3^{(2)} = i\text{Tr}[A\xi^\dagger \bar{B}B\xi^\dagger], O_3^{(3)} = i\text{Tr}[\bar{B}\xi^\dagger A\xi^\dagger B],$ $\tilde{O}_3^{(1)} = i\text{Tr}[\bar{B}\gamma_5 B]\text{Tr}[AU^\dagger], \tilde{O}_3^{(2)} = i\text{Tr}[A\xi^\dagger \bar{B}\gamma_5 B\xi^\dagger], \tilde{O}_3^{(3)} = i\text{Tr}[\bar{B}\gamma_5 \xi^\dagger A\xi^\dagger B],$
( $6, \bar{6}$ )	$O_6^{(1)} = iS_{kl}^{ij}(\xi \bar{B}\xi)_i^k(\xi B\xi)_j^l, O_6^{(2)} = iS_{kl}^{ij}(\xi \bar{B}B\xi)_i^k U_j^l,$ $O_6^{(3)} = iS_{kl}^{ij}(\bar{B}\xi)_i^m(\xi B)_m^k U_j^l, O_6^{(4)} = i\text{Tr}[\bar{B}B]S_{kl}^{ij}U_i^k U_j^l.$ $\tilde{O}_6^{(1)} = iS_{kl}^{ij}(\xi \bar{B}\gamma_5 \xi)_i^k(\xi B\xi)_j^l, \tilde{O}_6^{(2)} = iS_{kl}^{ij}(\xi \bar{B}\gamma_5 B\xi)_i^k U_j^l,$ $\tilde{O}_6^{(3)} = iS_{kl}^{ij}(\bar{B}\gamma_5 \xi)_i^m(\xi B)_m^k U_j^l, \tilde{O}_6^{(4)} = i\text{Tr}[\bar{B}\gamma_5 B]S_{kl}^{ij}U_i^k U_j^l$
( $8, 8$ )	$O_8^{(1)} = i\text{Tr}[\bar{B}\xi^\dagger H_1 U H_2 \xi^\dagger B], O_8^{(2)} = i\text{Tr}[\xi^\dagger \bar{B}B\xi^\dagger H_1 U H_2],$ $O_8^{(3)} = i\text{Tr}[\xi \bar{B}B\xi H_2 U^\dagger H_1], O_8^{(4)} = i\text{Tr}[\bar{B}\xi H_2 U^\dagger H_1 \xi B],$ $O_8^{(5)} = i\text{Tr}[\xi \bar{B}\xi^\dagger H_1]\text{Tr}[\xi^\dagger B\xi H_2], O_8^{(6)} = i\text{Tr}[\xi^\dagger \bar{B}\xi H_2]\text{Tr}[\xi B\xi^\dagger H_1],$ $O_8^{(7)} = i\text{Tr}[\xi \bar{B}\xi H_2 \xi^\dagger B\xi^\dagger H_1], O_8^{(8)} = i\text{Tr}[\xi^\dagger \bar{B}\xi^\dagger H_1 \xi B\xi H_2].$ $\tilde{O}_8^{(1)} = i\text{Tr}[\bar{B}\gamma_5 \xi^\dagger H_1 U H_2 \xi^\dagger B], \tilde{O}_8^{(2)} = i\text{Tr}[\xi^\dagger \bar{B}\gamma_5 B\xi^\dagger H_1 U H_2],$ $\tilde{O}_8^{(3)} = i\text{Tr}[\xi \bar{B}\gamma_5 B\xi H_2 U^\dagger H_1], \tilde{O}_8^{(4)} = i\text{Tr}[\bar{B}\gamma_5 \xi H_2 U^\dagger H_1 \xi B],$ $\tilde{O}_8^{(5)} = i\text{Tr}[\xi \bar{B}\gamma_5 \xi^\dagger H_1]\text{Tr}[\xi^\dagger B\xi H_2], \tilde{O}_8^{(6)} = i\text{Tr}[\xi^\dagger \bar{B}\gamma_5 \xi H_2]\text{Tr}[\xi B\xi^\dagger H_1],$ $\tilde{O}_8^{(7)} = i\text{Tr}[\xi \bar{B}\gamma_5 \xi H_2 \xi^\dagger B\xi^\dagger H_1], \tilde{O}_8^{(8)} = i\text{Tr}[\xi^\dagger \bar{B}\gamma_5 \xi^\dagger H_1 \xi B\xi H_2]$

**Table 3.** Hadronic operators that have the same quantum numbers as four-quark operators in different irreducible representations.

which is part of the CP-odd mass generated by the four-quark operator. Note that since  $\langle U \rangle$  is inversely proportional to the quark mass, the above contribution is not suppressed in the chiral limit.

One can also get a CP-odd dipole moment by considering a photo-pion production term off the nucleon. When the pion is condensed through CP-odd effects, one generates a new contribution to the CP-odd moment, which is beyond the scope of this paper.

### 3.2 Matching to CP-Odd Baryon operators

In this subsection, we construct the leading P-odd and CP-odd baryon operators induced by the CP-odd four-quark operators. These include all the operators with one baryon and one conjugate baryon fields, and without any quark masses or derivatives. All the independent operators are listed in table 3.

There are two types of operators in table 3, those with and without tilde. For the first group without tilde, the expansion of the pion field generates the P-odd and CP-odd nucleon-pion vertices

$$\mathcal{L}_{NN\pi}^{\text{CP-odd}} = (h_c \bar{p} n \pi^+ + \text{h.c.}) + h_n \bar{n} n \pi^0 + h_p \bar{p} p \pi^0. \quad (3.24)$$

For the second group, the leading order expansion is a bilinear-baryon term with a CP-odd mass structure,

$$\mathcal{L}_{\text{mass}}^{\text{CP-odd}} \sim -m_\star \bar{n} i \gamma_5 n. \quad (3.25)$$

This term contributes to the CP-odd baryon wave function.

Traditionally, P-odd and CP-odd pion-nucleon couplings are defined in terms of isospin 0, 1, and 2 of the operators, which can be written as [19]

$$\mathcal{L}_{\pi NN} = \bar{g}_{\pi NN}^{(0)} \bar{N} \tau^a N \pi^a + \bar{g}_{\pi NN}^{(1)} \bar{N} N \phi^0 + \bar{g}_{\pi NN}^{(2)} (\bar{N} \tau^a N \pi^a - 3 \bar{N} \tau^3 N \pi^0), \quad (3.26)$$

where  $\bar{g}_{\pi NN}^{(i)}$  is the coupling of the isospin- $i$  term and  $\tau^i$  are the Pauli matrices. Then, in terms of  $\bar{g}_{\pi NN}^{(i)}$ ,  $h_c$ ,  $h_n$ , and  $h_p$  can be written as

$$h_c = \sqrt{2}(\bar{g}_{\pi NN}^{(0)} + \bar{g}_{\pi NN}^{(2)}), \quad h_n = (-\bar{g}_{\pi NN}^{(0)} + \bar{g}_{\pi NN}^{(1)} + 2\bar{g}_{\pi NN}^{(2)}), \quad h_p = (\bar{g}_{\pi NN}^{(0)} + \bar{g}_{\pi NN}^{(1)} - 2\bar{g}_{\pi NN}^{(2)}), \quad (3.27)$$

where  $h_p$  does not contribute to nEDM.

To match the P-odd and CP-odd four-quark operators to the above baryon operators, one must find ways to calculate the corresponding non-perturbative Wilson coefficients. This can be done by considering the matrix elements of the quark operators in simple states. Take  $O_{11}^{ud} = \bar{u}i\gamma_5 u \bar{d}d$  as an example. As shown in the last section, it can be decomposed into irreducible representations of the chiral group,

$$O_{11}^{ud} = O_{11}^{ud,(\bar{3},3)} + O_{11}^{ud,(6,\bar{6})} + O_{11}^{ud,(8,8)} + \text{h.c.} \quad (3.28)$$

The spurions related to this operator are given in eqs. (2.5), (2.6) and (2.8).  $O_{11}^{ud,(\bar{3},3)}$ ,  $O_{11}^{ud,(6,\bar{6})}$  and  $O_{11}^{ud,(8,8)}$  must be matched to the hadronic operators in the same irreducible representations and with the same spurions. Take the un-tilde hadronic operators as an example:

$$\begin{aligned} O_{11}^{ud,(\bar{3},3)} &= \sum_{i=1}^3 C_3^{(i)} O_3^{(i)} + \dots, \\ O_{11}^{ud,(6,\bar{6})} &= \sum_{i=1}^4 C_6^{(i)} O_6^{(i)} + \dots, \\ O_{11}^{ud,(8,8)} &= \sum_{i=1}^8 C_8^{(i)} O_8^{(i)} + \dots, \end{aligned} \quad (3.29)$$

where “...” represents higher order operators.

Note that, an operator can be separated into hermitian part and anti-hermitian part. Since the QCD Lagrangian is hermitian, the hermitian part and the anti-hermitian part must have the same Wilson coefficient in the effective theory. Take the operator  $\bar{q}_L q_R$  as an example, it is a  $(\bar{3}, 3)$  operator, so it can be matched to  $C U^\dagger$  in the chiral perturbation theory, while its hermitian conjugation  $\bar{q}_R q_L$  is matched to  $C U$  with exactly the same Wilson coefficient since the QCD Lagrangian is invariant under the hermitian conjugate transformation. Therefore, the hermitian part of  $\bar{q}_L q_R$  can be matched to  $C(U^\dagger + U)/2$  whereas the anti-hermitian part can be matched to  $C(U^\dagger - U)/2$ . As a result, one can use either the hermitian part or the anti-hermitian part of the operators to get their Wilson coefficients depending on which way is easier. For the operators without tilde listed in table 3, the anti-hermitian parts contain terms having only one baryon field and one anti-baryon field which is easy to do the matching, while for the operators with tilde, the

hermitian part is easier. Therefore, we choose to match the anti-hermitian part of the operators without a tilde whereas match the hermitian part of the operators with a tilde to get the Wilson coefficients of them. One can show that this matching procedure works when current algebra is valid such as in non-relativistic quark model.

The leading-order expansion of the hadronic operators are, for  $(\bar{3}, 3)$  operators,

$$\begin{aligned}
 O_3^{(1)} &\simeq i\bar{p}p + i\bar{n}n + i\bar{\Lambda}\Lambda + i\bar{\Sigma}^0\Sigma^0 + i\bar{\Sigma}^+\Sigma^+ + i\bar{\Sigma}^-\Sigma^- + i\bar{\Xi}^0\Xi^0 + i\bar{\Xi}^-\Xi^-, \\
 O_3^{(2)} &\simeq i\bar{p}p + i\bar{n}n + \frac{2i}{3}\bar{\Lambda}\Lambda, \\
 O_3^{(3)} &\simeq \frac{2i}{3}\bar{\Lambda}\Lambda + i\bar{\Xi}^0\Xi^0 + i\bar{\Xi}^-\Xi^-.
 \end{aligned} \tag{3.30}$$

Therefore, we can determine the Wilson coefficients with four physical matrix elements,

$$\begin{aligned}
 C_3^{(1)} + C_3^{(2)} &= (-i)\langle p|O_{11}^{ud,(\bar{3},3)}|p\rangle, \\
 C_3^{(1)} &= (-i)\langle \Sigma^0|O_{11}^{ud,(\bar{3},3)}|\Sigma^0\rangle, \\
 C_3^{(1)} + \frac{2}{3}C_3^{(2)} &= (-i)\langle \Lambda|O_{11}^{ud,(\bar{3},3)}|\Lambda\rangle, \\
 C_3^{(1)} + C_3^{(3)} &= (-i)\langle \Xi^0|O_{11}^{ud,(\bar{3},3)}|\Xi^0\rangle,
 \end{aligned} \tag{3.31}$$

where we have chosen the normalization condition

$$\langle \vec{P}|\vec{P}'\rangle = (2\pi)^3\delta^3(\vec{P} - \vec{P}'), \tag{3.32}$$

where  $\vec{P}$  and  $\vec{P}'$  are the momenta of the states.

Since the number of equations is larger than the number of variables, to get a solution the following condition must be satisfied,

$$\det \begin{pmatrix} 1 & 1 & 0 & \langle p|O_{11}^{ud,(\bar{3},3)}|p\rangle \\ 1 & 0 & 0 & \langle \Sigma^0|O_{11}^{ud,(\bar{3},3)}|\Sigma^0\rangle \\ 1 & \frac{2}{3} & 0 & \langle \Lambda|O_{11}^{ud,(\bar{3},3)}|\Lambda\rangle \\ 1 & 0 & 1 & \langle \Xi^0|O_{11}^{ud,(\bar{3},3)}|\Xi^0\rangle \end{pmatrix} = 0, \tag{3.33}$$

which gives a nontrivial relation among these matrix elements;

$$-\frac{2}{3}\langle p|O_{11}^{ud,(\bar{3},3)}|p\rangle - \frac{1}{3}\langle \Sigma^0|O_{11}^{ud,(\bar{3},3)}|\Sigma^0\rangle + \langle \Lambda|O_{11}^{ud,(\bar{3},3)}|\Lambda\rangle = 0. \tag{3.34}$$

This relation must be satisfied in the chiral limit, so it is a test for direct calculations of the matrix elements. Similarly, a simple inspection of eq. (3.30) can give us some more relations among matrix elements

$$\begin{aligned}
 \langle p|O_{11}^{ud,(\bar{3},3)}|p\rangle &= \langle n|O_{11}^{ud,(\bar{3},3)}|n\rangle, \\
 \langle \Sigma^0|O_{11}^{ud,(\bar{3},3)}|\Sigma^0\rangle &= \langle \Sigma^+|O_{11}^{ud,(\bar{3},3)}|\Sigma^+\rangle = \langle \Sigma^-|O_{11}^{ud,(\bar{3},3)}|\Sigma^-\rangle, \\
 \langle \Xi^0|O_{11}^{ud,(\bar{3},3)}|\Xi^0\rangle &= \langle \Xi^-|O_{11}^{ud,(\bar{3},3)}|\Xi^-\rangle.
 \end{aligned} \tag{3.35}$$

Generalizing the above discussion to  $(6, \bar{6})$  and  $(8, 8)$  operators, we write down the leading expansion of the hadronic operators,

$$\begin{aligned}
 O_6^{(1)} &\simeq \frac{i}{3} \bar{\Lambda} \Lambda - i \bar{\Sigma}^0 \Sigma^0 + i \bar{\Sigma}^+ \Sigma^+ + i \bar{\Sigma}^- \Sigma^-, \\
 O_6^{(2)} &\simeq i \bar{p} p + i \bar{n} n + \frac{i}{3} \bar{\Lambda} \Lambda + i \bar{\Sigma}^0 \Sigma^0 + i \bar{\Sigma}^+ \Sigma^+ + i \bar{\Sigma}^- \Sigma^-, \\
 O_6^{(3)} &\simeq \frac{i}{3} \bar{\Lambda} \Lambda + i \bar{\Xi}^0 \Xi^0 + i \bar{\Xi}^- \Xi^- + i \bar{\Sigma}^0 \Sigma^0 + i \bar{\Sigma}^+ \Sigma^+ + i \bar{\Sigma}^- \Sigma^-, \\
 O_6^{(4)} &\simeq 2i (\bar{p} p + \bar{n} n + \bar{\Lambda} \Lambda + i \bar{\Xi}^0 \Xi^0 + \bar{\Xi}^- \Xi^- + \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^+ \Sigma^+ + \bar{\Sigma}^- \Sigma^-) ; \quad (3.36) \\
 O_8^{(1)} &\simeq i \left( \bar{p} p + \frac{1}{6} \bar{\Lambda} \Lambda + \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 + \frac{1}{2\sqrt{3}} \bar{\Sigma}^0 \Lambda + \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^+ \Sigma^+ \right), \\
 O_8^{(2)} &\simeq i \left( \frac{1}{6} \bar{\Lambda} \Lambda + \bar{\Xi}^- \Xi^- + \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 + \frac{1}{2\sqrt{3}} \bar{\Sigma}^0 \Lambda + \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^- \Sigma^- \right), \\
 O_8^{(3)} &\simeq i \left( \frac{1}{6} \bar{\Lambda} \Lambda + \bar{\Xi}^0 \Xi^0 - \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 - \frac{1}{2\sqrt{3}} \bar{\Sigma}^0 \Lambda + \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^+ \Sigma^+ \right), \\
 O_8^{(4)} &\simeq i \left( \bar{n} n + \frac{1}{6} \bar{\Lambda} \Lambda - \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 - \frac{1}{2\sqrt{3}} \bar{\Sigma}^0 \Lambda + \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 + \bar{\Sigma}^- \Sigma^- \right) \\
 O_8^{(5)} &\simeq i \bar{\Sigma}^+ \Sigma^+, \\
 O_8^{(6)} &\simeq i \bar{\Sigma}^- \Sigma^-, \\
 O_8^{(7)} &\simeq i \left( \frac{1}{6} \bar{\Lambda} \Lambda + \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 - \bar{\Sigma}^0 \Lambda - \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 \right), \\
 O_8^{(8)} &\simeq i \left( \frac{1}{6} \bar{\Lambda} \Lambda - \frac{1}{2\sqrt{3}} \bar{\Lambda} \Sigma^0 + \bar{\Sigma}^0 \Lambda - \frac{1}{2} \bar{\Sigma}^0 \Sigma^0 \right), \quad (3.37)
 \end{aligned}$$

from which we can get similar relations among matrix elements just like in the  $(\bar{3}, 3)$  case shown in table 4. The other four-quark operators with the same flavor structures have the same relations among hadronic matrix elements as in this case.

One can either build models or do lattice QCD calculations to get these simplest four-quark matrix elements. Once known, one can get the Wilson coefficients by solving eq. (3.31) and similar equations for  $(6, \bar{6})$  and  $(8, 8)$  operators. Then one can expand these hadronic operators to the first order with one meson field in each term to get the P-odd and CP-odd pion-nucleon vertices. A similar method works for baryon operators with tilde. We will consider these matrix elements in the next section.

### 3.3 Matching to EDM-Type operators

In  $\chi$ PT, any CP-odd quark-gluon operator will generate directly an EDM contribution to the neutron, analytical in the chiral limit. To write down such a contribution, introduce vector and axial vector octet potential  $v_\mu$  and  $a_\mu$ , which transform under local chiral rotations (with space-time dependent chiral transformation) as

$$\begin{aligned}
 r_\mu &\equiv v_\mu + a_\mu \longrightarrow v'_\mu + a'_\mu = R(v_\mu + a_\mu)R^\dagger + iR\partial_\mu R^\dagger, \\
 l_\mu &\equiv v_\mu - a_\mu \longrightarrow v'_\mu - a'_\mu = L(v_\mu - a_\mu)L^\dagger + iL\partial_\mu L^\dagger. \quad (3.38)
 \end{aligned}$$

Rep.	Relations
$(\bar{3}, 3)$	$\langle p O_{11}^{ud,(3,3)} p\rangle = \langle n O_{11}^{ud,(3,3)} n\rangle, \langle \Xi^0 O_{11}^{ud,(3,3)} \Xi^0\rangle = \langle \Xi^- O_{11}^{ud,(3,3)} \Xi^- \rangle,$ $\langle \Sigma^0 O_{11}^{ud,(3,3)} \Sigma^0\rangle = \langle \Sigma^+ O_{11}^{ud,(3,3)} \Sigma^+ \rangle = \langle \Sigma^- O_{11}^{ud,(3,3)} \Sigma^- \rangle,$ $-\frac{2}{3}\langle p O_{11}^{ud,(3,3)} p\rangle - \frac{1}{3}\langle \Sigma^0 O_{11}^{ud,(3,3)} \Sigma^0\rangle + \langle \Lambda O_{11}^{ud,(3,3)} \Lambda\rangle = 0$
$(6, \bar{6})$	$\langle p O_{11}^{ud,(6,\bar{6})} p\rangle = \langle n O_{11}^{ud,(6,\bar{6})} n\rangle, \langle \Sigma^+ O_{11}^{ud,(6,\bar{6})} \Sigma^+ \rangle = \langle \Sigma^- O_{11}^{ud,(6,\bar{6})} \Sigma^- \rangle,$ $\langle \Xi^+ O_{11}^{ud,(6,\bar{6})} \Xi^+ \rangle = \langle \Xi^- O_{11}^{ud,(6,\bar{6})} \Xi^- \rangle$ $2\langle p O_{11}^{ud,6,\bar{6}} p\rangle - \langle \Sigma^0 O_{11}^{ud,(6,\bar{6})} \Sigma^0\rangle - 3\langle \Lambda O_{11}^{ud,(6,\bar{6})} \Lambda\rangle + 2\langle \Xi^0 O_{11}^{ud,(6,\bar{6})} \Xi^0\rangle = 0$
$(8, 8)$	$\langle p O_{11}^{ud,(8,8)} p\rangle + \langle n O_{11}^{ud,(8,8)} n\rangle + \langle \Sigma^0 O_{11}^{ud,(8,8)} \Sigma^0\rangle - 3\langle \Lambda O_{11}^{ud,(8,8)} \Lambda\rangle$ $+ \langle \Xi^0 O_{11}^{ud,(8,8)} \Xi^0\rangle + \langle \Xi^- O_{11}^{ud,(8,8)} \Xi^- \rangle = 0$ $\langle p O_{11}^{ud,(8,8)} p\rangle - \langle n O_{11}^{ud,(8,8)} n\rangle - \langle \Xi^0 O_{11}^{ud,(8,8)} \Xi^0\rangle + \langle \Xi^- O_{11}^{ud,(8,8)} \Xi^- \rangle$ $- \sqrt{3}\langle \Lambda O_{11}^{ud,(8,8)} \Sigma^0\rangle - \sqrt{3}\langle \Sigma^0 O_{11}^{ud,(8,8)} \Lambda\rangle = 0$

**Table 4.** Relations among hadronic matrix elements of the four-quark operators in different chiral representations.

The corresponding gauge fields are defined as

$$\begin{aligned} f_{\mu\nu}^R &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \\ f_{\mu\nu}^L &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]. \end{aligned} \quad (3.39)$$

The gauge fields with definite parity are defined as

$$f_{\mu\nu}^\pm = \xi^\dagger f_{\mu\nu}^R \xi \pm \xi f_{\mu\nu}^L \xi^\dagger, \quad (3.40)$$

which transform under chiral transformation as

$$f_{\mu\nu}^\pm \rightarrow K f_{\mu\nu}^\pm K^\dagger. \quad (3.41)$$

When reducing to the electromagnetic field,  $a_\mu = 0$ ,  $f_{\mu\nu}^\pm = (\xi^\dagger Q \xi \pm \xi Q \xi^\dagger) F_{\mu\nu}$ , where  $Q = \text{diag}(2/3, -1/3, -1/3)$  and  $F_{\mu\nu}$  is the electromagnetic field [25].

One can write down a number of EDM type of operators which contain  $\bar{B}$  and  $B$ ,  $f_{\mu\nu}^\pm$ , and the spurion fields  $A$ ,  $H$ , and  $S$ . These contributions are direct matching contributions to the neutron EDM, and cannot be calculated in  $\chi$ PT. These chiral constants can in principle be calculated in lattice QCD. However, we will present quark-model estimates in section V.

### 3.4 Peccei-Quinn symmetry and induced $\theta$ -term

The experimental upper bound on the neutron EDM gives a strong constraint on the P-odd and CP-odd  $\theta$ -term,  $\theta G\tilde{G}$ , in the QCD lagrangian [13, 15, 16]. Using the current experimental limit [31],

$$d_n < 2.9 \times 10^{-26} e \text{ cm}, \quad (3.42)$$

one can get the upper bound,

$$\theta < 10^{-10}. \quad (3.43)$$



On the other hand, it is unnatural for a parameter of the fundamental theory to be so small without fine tuning. There are generally two ways to solve this strong CP problem in the literature. The first is by introducing the spontaneous breaking of parity. Since the  $\theta$ -term also breaks parity, if at some high energy scale parity is conserved, then the  $\theta$ -term at low energy scale can only be generated by loop effects and will be suppressed naturally [32–36].

The other way is to introduce the Peccei-Quinn symmetry,  $U(1)_A$  [37, 38]. After the spontaneous breaking of the symmetry, there emerges a pseudo-goldstone boson,  $a$ , which is called the axion [39–41]. The effective Lagrangian for the axion field can be written as

$$\mathcal{L}_a = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}, \quad (3.44)$$

which includes an effective interaction with  $G\tilde{G}$ . The axion field gets a small mass through the non-vanishing correlation function

$$K = i \left\{ \int d^4x e^{ik \cdot x} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G\tilde{G}(x), \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right) | 0 \rangle \right\}_{k=0}, \quad (3.45)$$

after taking into account the non-perturbative QCD effect [40, 41].

When there is an additional neutral P-odd, CP-odd quark operator,  $O_{\text{CP-odd}}$ , in the lagrangian, the correlation function

$$K_1 = i \left\{ \int d^4x e^{ik \cdot x} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G\tilde{G}(x), O_{\text{CP-odd}}(0) \right) | 0 \rangle \right\}_{k=0} \quad (3.46)$$

will be generally nonzero. Therefore, the vev of  $a$ , which cancels precisely the  $\theta$ -term in the original lagrangian, will now be shifted by a small amount proportional to  $K_1$ . A non-vanishing effective  $\theta$ -term is induced as [42]

$$\theta_{\text{ind}} = -\frac{K_1}{K}, \quad (3.47)$$

which can contribute to the neutron EDM.

Following ref. [42], we take the operator  $\bar{u}i\gamma_5\bar{d}d$  as an example to calculate the contribution to neutron EDM through the induced  $\theta$ -term. Then,  $K_1$  can then be written as

$$K_1 = i \left\{ \int d^4x e^{ik \cdot x} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G\tilde{G}(x), C_4 \bar{u}i\gamma_5\bar{d}d(0) \right) | 0 \rangle \right\}_{k=0}. \quad (3.48)$$

Using the chiral anomaly [43, 44], one can get

$$\frac{\alpha_s}{4\pi} G\tilde{G} = \partial_\mu J_5^\mu - 2m_*(\bar{u}i\gamma_5u + \bar{d}i\gamma_5d + \bar{s}i\gamma_5s), \quad (3.49)$$

where

$$J_5^\mu \equiv \left( \frac{m_*}{m_u} \bar{u}\gamma^\mu\gamma_5u + \frac{m_*}{m_d} \bar{d}\gamma^\mu\gamma_5d + \frac{m_*}{m_s} \bar{s}\gamma^\mu\gamma_5s \right). \quad (3.50)$$

Then one can get

$$K_1 = \frac{i}{2} \int d^4x e^{ik \cdot x} \langle 0 | T (\partial_\mu J_5^\mu(x), C_4 \bar{u}i\gamma_5\bar{d}d(0)) | 0 \rangle_{k=0} \quad (3.51)$$

$$- \frac{i}{2} \int d^4x e^{ik \cdot x} \langle 0 | T (2m_*(\bar{u}i\gamma_5u + \bar{d}i\gamma_5d + \bar{s}i\gamma_5s)(x), C_4 \bar{u}i\gamma_5\bar{d}d(0)) | 0 \rangle_{k=0}.$$

The second term on the right-hand side of the above equation is negligible compared to the first term because it is explicitly proportional to the reduced quark mass  $m_*$  and the operator  $\bar{u}i\gamma_5u + \bar{d}i\gamma_5d + \bar{s}i\gamma_5s$  cannot annihilate light mesons. Therefore  $K_1$  can be calculated as

$$\begin{aligned} K_1 &\approx \frac{i}{2} \int d^4x e^{ik \cdot x} \langle 0 | T(\partial_\mu J_5^\mu(x), C_4 \bar{u}i\gamma_5u\bar{d}d(0)) | 0 \rangle_{k=0} \\ &= -\frac{i}{2} C_4 \langle 0 | [Q_5(0), \bar{u}i\gamma_5u\bar{d}d(0)] | 0 \rangle, \end{aligned} \quad (3.52)$$

where  $Q_5$  is the charge related to the current  $J_5^\mu$  defined in eq. (3.50). In the spirit of large  $N_C$  [45, 46] expansion one can assume that

$$\langle 0 | \bar{u}i\gamma_5u\bar{d}i\gamma_5d | 0 \rangle \ll \langle 0 | \bar{u}u\bar{d}d | 0 \rangle \approx \langle 0 | \bar{u}u | 0 \rangle \langle 0 | \bar{d}d | 0 \rangle. \quad (3.53)$$

Therefore, we can get

$$\begin{aligned} K_1 &\simeq -\frac{i}{2} C_4 \langle 0 | [Q_5(0), \bar{u}i\gamma_5u(0)] | 0 \rangle \langle 0 | \bar{d}d | 0 \rangle = -C_4 \frac{m_*}{m_u} \langle 0 | \bar{u}u | 0 \rangle \langle 0 | \bar{d}d | 0 \rangle \\ &= -\frac{m_*}{m_u} C_4 B_0^2 F_\pi^4. \end{aligned} \quad (3.54)$$

Using the previously known result [41]

$$K = -m_* F_\pi^2 B_0, \quad (3.55)$$

one can get the  $\theta$  angle induced by the operator  $\bar{u}i\gamma_5u\bar{d}d$ ,

$$\theta_{\text{ind}} = -\frac{K_1}{K} = -\frac{C_4 B_0 F_\pi^2}{m_u}. \quad (3.56)$$

A similar result can be obtained for any other CP-odd four-quark operator.

Using the standard chiral result in the literature [14], we write down the effective chiral lagrangian corresponding to this induced  $\theta$  term;

$$\mathcal{L}_\theta = \frac{4\theta m_*}{F_\pi} (c_2 \text{Tr}[\Sigma \bar{B} B] + c_3 \text{Tr}[\bar{B} \Sigma B]) + 2m_* \theta (3d_1 + d_2 + d_3) \text{Tr}[\bar{B} i\gamma_5 B]. \quad (3.57)$$

From the above, we read off the CP-odd pion-nucleon coupling and the CP-odd mass of the neutron;

$$\begin{aligned} h_c &= -\frac{2\sqrt{2}C_4 B_0 F_\pi m_*}{m_u} & h_n &= \frac{2C_4 B_0 F_\pi m_*}{m_u}, \\ M_\star &= \frac{2C_4 B_0 F_\pi^2 m_*}{m_u} (3d_1 + d_2 + d_3). \end{aligned} \quad (3.58)$$

Comparing this with the meson condensates contribution in eq. (4.2), one finds that they are in the same order. If the Peccei-Quinn symmetry exists, one should add this contribution to the neutron EDM. However, since it is not known if the axion mechanism is in operation, we will not include this contribution to the nEDM in the remainder of the paper.

Operator	$g_\pi$	$g_\eta$	Operator	$g_\pi$	$g_\eta$
$\bar{u}i\gamma_5 u \bar{d}d$	1	$1/\sqrt{3}$	$\bar{u}i\gamma_5 t^a u \bar{d}t^a d$	0	0
$\bar{d}i\gamma_5 d \bar{u}u$	-1	$1/\sqrt{3}$	$\bar{d}i\gamma_5 t^a d \bar{u}t^a u$	0	0
$\bar{u}i\gamma_5 u \bar{s}s$	1	$1/\sqrt{3}$	$\bar{u}i\gamma_5 t^a u \bar{s}t^a s$	0	0
$\bar{s}i\gamma_5 s \bar{u}u$	0	$-2/\sqrt{3}$	$\bar{s}i\gamma_5 t^a s \bar{u}t^a u$	0	0
$\bar{d}i\gamma_5 d \bar{s}s$	-1	$1/\sqrt{3}$	$\bar{d}i\gamma_5 t^a d \bar{s}t^a s$	0	0
$\bar{s}i\gamma_5 s \bar{d}d$	0	$-2/\sqrt{3}$	$\bar{s}i\gamma_5 t^a s \bar{d}t^a d$	0	0
$\bar{u}i\gamma_5 u \bar{u}u$	$5/6$	$5/(6\sqrt{3})$	$\bar{u}i\gamma_5 t^a u \bar{u}t^a u$	$-2/9$	$-2/(9\sqrt{3})$
$\bar{d}i\gamma_5 d \bar{d}d$	$-5/6$	$5/(6\sqrt{3})$	$\bar{d}i\gamma_5 t^a d \bar{d}t^a d$	$2/9$	$-2/(9\sqrt{3})$
$\bar{s}i\gamma_5 s \bar{s}s$	0	$-5/(3\sqrt{3})$	$\bar{s}i\gamma_5 t^a s \bar{s}t^a s$	0	$4/(9\sqrt{3})$

**Table 5.**  $g_\pi$  and  $g_\eta$  induced by four-quark operators constructed by scalar currents. Those induced by products of tensor currents are zero.

## 4 P-odd and CP-odd nucleon-pion vertices and CP-odd nucleon mass

In this section, we study the induced physical P-odd and CP-odd nucleon-pion vertices as well as the CP-odd nucleon mass from four-quark operators. There are a number of contributions to consider: First, the CP-odd meson lagrangian will generate meson condensates which can convert a CP-even vertex into a CP-odd one. Second, the baryon wave function contains the CP-odd part due to the CP-odd nucleon mass, which can also rotate a CP-even coupling into a CP-odd one. Finally, there is the contribution from the direct matching operators (without a tilde) in TABLE II. We will consider all of these in this section.

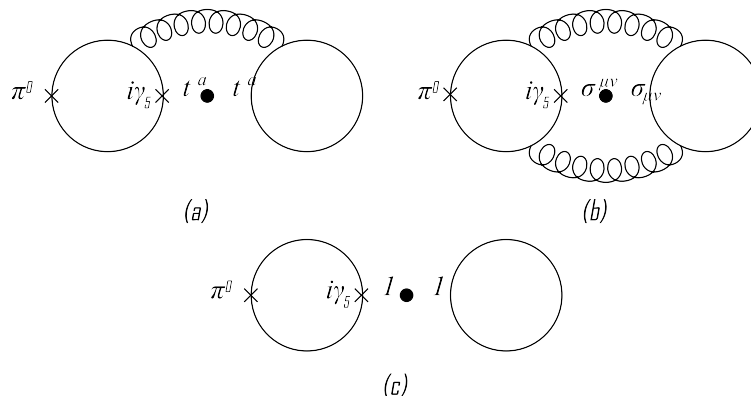
### 4.1 Meson condensates contribution

We use the vacuum saturation approximation to calculate the meson effective lagrangian; the vevs of  $\pi^0$  and  $\eta$  can be obtained from eq. (3.16), where  $g_\pi$  and  $g_\eta$  for all the four-quark operators built with color-singlet and octet scalar currents are listed in table 5. Those induced by tensor operators vanish in this approximation.

In the large  $N_c$  QCD [45] (also see ref. [46] for a good review), the leading contributions for operators constructed from two color-octet currents and two tensor currents are shown as diagrams (a) and (b) in figure 1, respectively. Detailed analysis shows that the diagrams (a) and (b) suffer from  $1/N_c^2$  suppressions compared with (c), which stands for the operator constructed from two scalar color-singlet currents.

Terms contributing to the P-odd, CP-odd nucleon-pion vertices through the condensates of neutral mesons are shown in eq. (3.22). At tree level, one can relate the coefficients  $c_1$ ,  $c_2$ , and  $c_3$  to the mass differences of the baryons and the  $\pi N$   $\sigma$ -term, and their values can be found in the literature [25];

$$c_1 = 2B_0 b_0, \quad c_2 = 2B_0(b_d - b_f), \quad c_3 = 2B_0(b_d + b_f), \quad (4.1)$$



**Figure 1.** Annihilation of pion by four-quark operators: (a) operator constructed from two color-octet current, like  $\bar{u}i\gamma_5 t^a u \bar{d}t^a d$ ; (b) operator from two tensor currents, like  $\bar{u}\sigma^{\mu\nu}i\gamma_5 u \bar{d}\sigma_{\mu\nu}d$ ; (c) operator from two scalar currents, like  $\bar{u}i\gamma_5 u \bar{d}d$ .

where  $b_0 = -0.517 \text{ GeV}^{-1}$ ,  $b_d = 0.066 \text{ GeV}^{-1}$  and  $b_f = -0.213 \text{ GeV}^{-1}$ .

The vertices we are interested in have two nucleons and one pion because of the infrared enhancement in the pion loop [16]. From eq. (3.22) we can read off the relevant terms,

$$\begin{aligned}
 & -\frac{1}{3F_\pi^2} \left\{ c_3 [3\sqrt{2}\langle\pi^0\rangle(m_u - m_d) + \sqrt{6}\langle\eta\rangle(m_u + m_d)] \right\} (\bar{n}p\pi^- + \bar{p}n\pi^+) \\
 & -\frac{2}{3F_\pi^2} \left\{ c_3 m_d (3\langle\pi^0\rangle - \sqrt{3}\langle\eta\rangle) + c_1 [3(m_u + m_d)\langle\pi^0\rangle + \sqrt{3}\langle\eta\rangle(m_u - m_d)] \right\} \bar{n}n\pi^0,
 \end{aligned} \tag{4.2}$$

in which  $\langle\pi^0\rangle$  and  $\langle\eta\rangle$  are given in eq. (3.16). It is customary to define the P-odd, CP-odd nucleon-pion couplings

$$\mathcal{L}_{CPV} = h_c(\bar{p}n\pi^+ + \bar{n}p\pi^-) + h_n\bar{n}n\pi^0, \tag{4.3}$$

where  $h_c$  and  $h_n$  induced by meson condensates are listed in table 6. Typical values of  $h_c$  are one order of magnitude smaller than the value of  $h_n$  because  $\sqrt{2}c_3(m_d - m_u) \ll 4c_1(m_u + m_d)$ . For  $h_c$  or  $h_n$  generated by a certain four-quark operator, if the contribution from  $\langle\pi^0\rangle$  is non-vanishing, the contribution from  $\langle\eta\rangle$  can be neglected since  $\langle\pi^0\rangle/\langle\eta\rangle \simeq m_s/\hat{m} \simeq 30$ . This also explains the contributions from operators with the  $\bar{s}i\gamma_5 s$  factor are much smaller than those without. Finally, the contributions from operators made of color-octet currents are smaller than those from operators made of color-singlet currents because a Fierz transformation is needed for color-octet operators to annihilate the mesons, introducing a suppressing factor of 1/4.

In table 6 one can see that the P-odd and CP-odd pion-nucleon couplings are proportional to  $B_0^2$ , which is related to the quark condensates. The value of  $B_0$  can be extracted from the pion mass

$$m_\pi^2 = B_0(m_u + m_d). \tag{4.4}$$

The natural scale for  $\chi$ PT is  $4\pi F_\pi$  [47], and for simplicity we use the same scale to define the quark masses to get  $B_0$ . The quark masses we use are  $m_u = 2.4 \text{ MeV}$  and  $m_d = 4.75 \text{ MeV}$

Operator	Meson condensates contribution		Factorization	
	$h_c (C_4 B_0^2)$	$h_n (C_4 B_0^2)$	$h_c (C_4 B_0^2)$	$h_n (C_4 B_0^2)$
$\bar{u}i\gamma_5 u \bar{d}d$	-0.0117	0.225	0.0063	-0.24
$\bar{d}i\gamma_5 d \bar{u}u$	0.0130	-0.227	0.0063	0.19
$\bar{u}i\gamma_5 u \bar{s}s$	-0.0117	0.225	0	-0.088
$\bar{s}i\gamma_5 s \bar{u}u$	-0.00122	0.000864	0	0
$\bar{d}i\gamma_5 d \bar{s}s$	0.0130	-0.227	0	0.087
$\bar{s}i\gamma_5 s \bar{d}d$	-0.00122	0.000864	0	0
$\bar{u}i\gamma_5 u \bar{u}u$	-0.00976	0.188	0	-0.16
$\bar{d}i\gamma_5 d \bar{d}d$	0.0108	-0.189	0	0.20
$\bar{s}i\gamma_5 s \bar{s}s$	-0.00102	0.000722	0	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \sigma_{\mu\nu} d$	0	0	0.076	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{s} \sigma_{\mu\nu} s$	0	0	0	0
$\bar{d}i\gamma_5 \sigma^{\mu\nu} d \bar{s} \sigma_{\mu\nu} s$	0	0	0	0
$\bar{u}i\gamma_5 t^a u \bar{d} t^a d$	0	0	0.0085	0
$\bar{d}i\gamma_5 t^a d \bar{u} t^a u$	0	0	0.0085	0
$\bar{u}i\gamma_5 t^a u \bar{s} t^a s$	0	0	0	0
$\bar{s}i\gamma_5 t^a s \bar{u} t^a u$	0	0	0	0
$\bar{d}i\gamma_5 t^a d \bar{s} t^a s$	0	0	0	0
$\bar{s}i\gamma_5 t^a s \bar{d} t^a d$	0	0	0	0
$\bar{u}i\gamma_5 t^a u \bar{u} t^a u$	0.00261	-0.0501	0	0.042
$\bar{d}i\gamma_5 t^a d \bar{d} t^a d$	-0.00288	0.0503	0	-0.054
$\bar{s}i\gamma_5 t^a s \bar{s} t^a s$	0.000272	-0.000192	0	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{d} \sigma_{\mu\nu} t^a d$	0	0	0.101	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{s} \sigma_{\mu\nu} t^a s$	0	0	0	0
$\bar{d}i\gamma_5 \sigma^{\mu\nu} t^a d \bar{s} \sigma_{\mu\nu} t^a s$	0	0	0	0

**Table 6.** CP-odd pion-nucleon couplings induced by meson condensates.  $C_4$  is the Wilson coefficient of the corresponding four-quark operator. The two columns on the right side shows the P-odd and CP-odd pion-nucleon vertices calculated using factorization which will be discussed in section V.

in  $\overline{\text{MS}}$  at 2 GeV. Using the one-loop renormalization group to run them down to  $\mu = 4\pi F_\pi$ , we have

$$B_0 = 2.2 \text{ GeV} . \quad (4.5)$$

Here we have used one-loop  $\Lambda_{\text{QCD}} = 250 \text{ MeV}$ .

## 4.2 Direct contribution from matching

To get the P-odd and CP-odd meson-nucleon coupling through direct matching, one needs to calculate the matrix elements listed in table 4. Lattice QCD is perhaps the ultimate choice for calculating hadronic matrix elements. However, it is still quite difficult to directly calculate the matrix elements of four-quark operators between baryons. Therefore, we

resort to quark models to get an estimate. In the remainder of this subsection we will use two different quark models to calculate these hadronic matrix elements: the simple non-relativistic quark model [48–52] and the MIT bag model [52–56]. We also discuss the significance of the model calculations from the viewpoint of naive factorization.

#### 4.2.1 Non-relativistic quark model

Here we consider the simplest version of the non-relativistic quark model with harmonic oscillator interacting potentials,

$$H = - \sum_{i=1}^3 \frac{1}{2m} \nabla_i^2 + \frac{1}{2} \frac{m_c}{3} \omega^2 [(\vec{r}_1 - \vec{r}_2)^2 + (\vec{r}_2 - \vec{r}_3)^2 + (\vec{r}_3 - \vec{r}_1)^2] , \quad (4.6)$$

where  $\vec{r}_1$ ,  $\vec{r}_2$ , and  $\vec{r}_3$  are positions of the three quarks inside the baryon,  $m_c$  is the mass of the constituent quarks and  $\omega$  is the angular frequency. One can isolate the center of mass by introducing the Jacobi coordinates,

$$\begin{aligned} \vec{R} &= \frac{1}{\sqrt{3}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) , \\ \vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) , \\ \vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) . \end{aligned} \quad (4.7)$$

Then the spatial wave function of the nucleon can be written as

$$f(\vec{r}_1, \vec{r}_2, \vec{r}_3; \vec{k}) = (3\sqrt{3})^{-1/2} \exp(i\vec{P} \cdot \vec{R}/\sqrt{3}) \psi(\vec{\rho}, \vec{\lambda}) , \quad (4.8)$$

where  $\psi(\vec{\rho}, \vec{\lambda}) = (\alpha^3/\pi^{3/2}) \exp[-\alpha^2(\rho^2 + \lambda^2)/2]$  in which  $\alpha = (m\omega)^{1/2} \approx 0.41$  GeV [51] is the oscillator parameter, and  $\vec{P}$  is the nucleon momentum. It is easy to check that the wave function is normalized to  $(2\pi)^3 \delta^3(\vec{P} - \vec{P}')$ . The internal part of the wave function is assumed to have SU(6) spin-flavor symmetry. For example, the spin-up proton state has the following wave function,

$$|p_\uparrow\rangle = \frac{1}{\sqrt{18}} \int d^3r_1 d^3r_2 d^3r_3 f(\vec{r}_1, \vec{r}_2, \vec{r}_3) \epsilon^{abc} \left[ u_\downarrow^{a\dagger}(\vec{r}_1) d_\uparrow^{b\dagger}(\vec{r}_2) - u_\uparrow^{a\dagger}(\vec{r}_1) d_\downarrow^{b\dagger}(\vec{r}_2) \right] u_\uparrow^{c\dagger}(\vec{r}_3) |0\rangle , \quad (4.9)$$

where  $a$ ,  $b$ , and  $c$  are color indices and the anti-commutation relation of the non-relativistic quark creation and annihilation operators is defined as  $\{u_\alpha^{a\dagger}(\vec{x}), u_\beta^b(\vec{y})\} = \delta_{ab} \delta_{\alpha\beta} \delta^3(\vec{x} - \vec{y})$  with  $\alpha$  and  $\beta$  as spin indices. The spatial part of the wave functions is common for all members of the baryon octet. The SU(6) internal wave functions are listed in table 7 for easy reference.

Using eqs. (2.3)–(2.8), one can project operator  $\bar{u}i\gamma_5 u \bar{d}d$  into different irreducible representations of the chiral group,  $O_{11}^{ud,(3,3)}$ ,  $O_{11}^{ud,(6,6)}$ , and  $O_{11}^{ud,(8,8)}$  as in eq. (3.28). Restricting

$ p_\uparrow\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[u_\downarrow^{a\dagger}d_\uparrow^{b\dagger} - u_\uparrow^{a\dagger}d_\downarrow^{b\dagger}]u_\uparrow^{c\dagger} 0\rangle;$
$ n_\uparrow\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[d_\uparrow^{a\dagger}u_\downarrow^{b\dagger} - d_\downarrow^{a\dagger}u_\uparrow^{b\dagger}]d_\uparrow^{c\dagger} 0\rangle;$
$ \Lambda_\uparrow\rangle \sim \frac{1}{\sqrt{12}}\epsilon^{abc}[u_\uparrow^{a\dagger}d_\downarrow^{b\dagger} - u_\downarrow^{a\dagger}d_\uparrow^{b\dagger}]s_\uparrow^{c\dagger} 0\rangle;$
$ \Sigma_\uparrow^+\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[s_\downarrow^{a\dagger}u_\uparrow^{b\dagger} - s_\uparrow^{a\dagger}u_\downarrow^{b\dagger}]u_\uparrow^{c\dagger} 0\rangle;$
$ \Sigma_\uparrow^0\rangle \sim \frac{1}{6}\epsilon^{abc}[s_\uparrow^{a\dagger}d_\downarrow^{b\dagger}u_\uparrow^{c\dagger} + s_\uparrow^{a\dagger}d_\uparrow^{b\dagger}u_\downarrow^{c\dagger} - 2s_\downarrow^{a\dagger}d_\uparrow^{b\dagger}u_\uparrow^{c\dagger}] 0\rangle;$
$ \Sigma_\uparrow^-\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[s_\uparrow^{a\dagger}d_\downarrow^{b\dagger} - s_\downarrow^{a\dagger}d_\uparrow^{b\dagger}]d_\uparrow^{c\dagger} 0\rangle;$
$ \Xi_\uparrow^0\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[s_\downarrow^{a\dagger}u_\uparrow^{b\dagger} - s_\uparrow^{a\dagger}u_\downarrow^{b\dagger}]s_\uparrow^{c\dagger} 0\rangle;$
$ \Xi_\uparrow^-\rangle \sim \frac{1}{\sqrt{18}}\epsilon^{abc}[s_\uparrow^{a\dagger}d_\downarrow^{b\dagger} - s_\downarrow^{a\dagger}d_\uparrow^{b\dagger}]s_\uparrow^{c\dagger} 0\rangle.$

**Table 7.** SU(6) wave functions of baryon spin-1/2 octet.

to the non-relativistic case, these operators become

$$\begin{aligned}
 O_{11}^{ud,(3,3)}(x) &\simeq -\frac{i}{8} : \left( u_\alpha^{a\dagger}(x)u_\alpha^a(x)d_\beta^{b\dagger}(x)d_\beta^b(x) - d_\alpha^{a\dagger}(x)u_\alpha^a(x)u_\beta^{b\dagger}(x)d_\beta^b(x) \right) : \\
 O_{11}^{ud,(6,\bar{6})}(x) &\simeq -\frac{i}{8} : \left( u_\alpha^{a\dagger}(x)u_\alpha^a(x)d_\beta^{b\dagger}(x)d_\beta^b(x) + d_\alpha^{a\dagger}(x)u_\alpha^a(x)u_\beta^{b\dagger}(x)d_\beta^b(x) \right) : \\
 O_{11}^{ud,(8,8)}(x) &\simeq \frac{i}{4} : u_\alpha^{a\dagger}(x)u_\alpha^a(x)d_\beta^{b\dagger}(x)d_\beta^b(x) : .
 \end{aligned} \tag{4.10}$$

where  $u$  and  $d$  are non-relativistic two-component quark annihilation operators,  $a$  and  $b$  label the color,  $\alpha$  and  $\beta$  label the spin, and the “: :” means that the products of the constituent quark fields are normal-ordered.

Considering the  $(6, \bar{6})$  component as an example, the simple quark model gives the following matrix elements:

$$\begin{aligned}
 \langle p_\uparrow(P) | O_{11}^{ud,(6,\bar{6})} | p_\uparrow(P) \rangle &= \langle n_\uparrow(P) | O_{11}^{ud,(6,\bar{6})} | n_\uparrow(P) \rangle = -\frac{i}{8}a, \\
 \langle \Sigma_\uparrow^+(P) | O_{11}^{ud,(6,\bar{6})} | \Sigma_\uparrow^+(P) \rangle &= \langle \Sigma_\uparrow^-(P) | O_{11}^{ud,(6,\bar{6})} | \Sigma_\uparrow^-(P) \rangle = 0, \\
 \langle \Sigma_\uparrow^0(P) | O_{11}^{ud,(6,\bar{6})} | \Sigma_\uparrow^0(P) \rangle &= -\frac{i}{4}a, \\
 \langle \Lambda_\uparrow(P) | O_{11}^{ud,(6,\bar{6})} | \Lambda_\uparrow(P) \rangle &= 0, \\
 \langle \Xi_\uparrow^0(P) | O_{11}^{ud,(6,\bar{6})} | \Xi_\uparrow^0(P) \rangle &= \langle \Xi_\uparrow^-(P) | O_{11}^{ud,(6,\bar{6})} | \Xi_\uparrow^-(P) \rangle = 0,
 \end{aligned} \tag{4.11}$$

where  $a = \int d^3r f^*(\vec{P}; \vec{x}, \vec{x}, \vec{r}) f(\vec{P}; \vec{x}, \vec{x}, \vec{r})$  is independent of  $\vec{x}$ . It is easy to check that these matrix elements satisfy the symmetry conditions listed in table 4. Using eq. (3.36), one can get the Wilson coefficients for  $(6, \bar{6})$  hadronic operators defined in eq. (3.29);

$$\begin{aligned}
 C_6^{(1)} &= -C_6^{(2)} = \frac{1}{8} \frac{\alpha^3}{(2\pi)^{3/2}}; \\
 C_6^{(3)} &= C_4^{(4)} = 0.
 \end{aligned} \tag{4.12}$$

Expanding the hadronic operators to the first order, one can get the P-odd, CP-odd three-point nucleon-pion couplings,  $h_c$  and  $h_n$ . The result induced by  $O_{11}^{ud,(6,\bar{6})}$  is

$$h_c = C_{11}^{ud} \alpha^3 / (8\pi^{3/2} F_\pi) \simeq 0.022 C_{11}^{ud} \alpha^3 / F_\pi, \quad h_n = 0. \tag{4.13}$$

In the same way one can calculate  $h_c$  and  $h_n$  induced by the  $(\bar{3}, 3)$  and  $(8, 8)$  components of  $\bar{u}i\gamma_5 u\bar{d}d$ . Taking into account the hermitian conjugate part of each component, the contribution for  $h_c$  and  $h_n$  is doubled.

### 4.2.2 MIT bag model

The basic idea of the bag model is that valence quarks are confined in a bag where the vacuum is in a phase different from the true QCD vacuum. The inside has a constant energy-momentum density generating a negative pressure,  $B$ , which is balanced by the positive pressure of the quarks. The bag is usually taken as a sphere of radius  $R_0$ . The quarks inside the bag move freely with the following wave functions,

$$\psi_{n,-1,1/2,m}(\vec{r}, t) = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} ij_0(\omega_{n,-1}r/R_0)\chi_m \\ -j_1(\omega_{n,-1}r/R_0)\vec{\sigma} \cdot \hat{r}\chi_m \end{pmatrix}. \quad (4.14)$$

The normalization factor of the above is

$$N(\omega_{n\kappa}) = \left( \frac{\omega_{n\kappa}^3}{2R_0^3(\omega_{n\kappa} + \kappa)\sin^2\omega_{n\kappa}} \right)^{1/2}. \quad (4.15)$$

The boundary condition gives the energy eigenvalue equation,

$$\tan\omega_{n\kappa} = \frac{\omega_{n\kappa}}{\omega_{n\kappa} + \kappa}, \quad (4.16)$$

and numerical calculation gives  $\omega_0 = 2.043$ . The ground state of quarks is  $\kappa = -1$ ,  $n = 0$  state. For the baryon octet, all the quarks are in this state. Keeping only this, the quark operator can be written as

$$q(x) = \psi_{0,-1,1/2,m}(\vec{x})e^{-i\omega_{0,-1}t/R_0}b_{0,-1,1/2,m} + (\text{anti-quark creation}). \quad (4.17)$$

The physical meaning of the operator  $b_m(0)$  is that it annihilates a quark with quantum number described by the wave function  $\psi_{0,-1,1/2,m}$ . Due to the assumption that inside the bag the interaction between quarks and gluons is negligible, flavor and spin automatically become good quantum numbers.

We again take the  $(6, \bar{6})$  component of  $\bar{u}i\gamma_5 u\bar{d}d$  as an example, which can be written as

$$O_{11}^{ud(6,\bar{6})} \sim -\frac{i}{2}\bar{\psi}_\lambda(\vec{x})\mathbb{P}_L\psi_\sigma(\vec{x})\bar{\psi}_\rho(\vec{x})\mathbb{P}_L\psi_\tau(\vec{x}) \times \left[ u_\lambda^{a\dagger}u_\sigma^a d_\rho^{b\dagger}d_\tau^b + d_\lambda^{a\dagger}u_\sigma^a u_\rho^{b\dagger}d_\tau^b \right], \quad (4.18)$$

where  $a$  and  $b$  are indices of color,  $\lambda, \sigma, \rho, \tau$  labeling the spin. The creation and annihilation operators here are just like  $b_{0,-1,1/2,m}$  in eq. (4.17). Using

$$\begin{aligned} &\bar{\psi}_\lambda(x)\mathbb{P}_L\psi_\sigma(x) \\ &= \frac{N^2}{8\pi} \left\{ [j_0^2(\omega_0 r/R_0) - j_1^2(\omega_0 r/R_0)] \delta_{\lambda\sigma} - 2ij_0(\omega r/R_0)j_0(\omega r/R_0)\chi_\lambda^\dagger\vec{\sigma} \cdot \hat{r}\chi_\sigma \right\}, \quad (4.19) \end{aligned}$$



Operators	NR quark model		MIT bag model	
	$h_c/(\alpha^3/F_\pi)$	$h_n/(\alpha^3/F_\pi)$	$h_c/(1/(R_0^3 F_\pi))$	$h_n/(1/(R_0^3 F_\pi))$
$\bar{u}i\gamma_5 u \bar{d}d$	0.045	0.13	0.029	-0.024
$\bar{d}i\gamma_5 d \bar{u}u$	0.045	-0.13	0.029	0.024
$\bar{u}i\gamma_5 u \bar{s}s$	0	0	0	0
$\bar{s}i\gamma_5 s \bar{u}u$	0	0	0	0
$\bar{d}i\gamma_5 d \bar{s}s$	0	0	0	0
$\bar{s}i\gamma_5 s \bar{d}d$	0	0	0	0
$\bar{u}i\gamma_5 u \bar{u}u$	0.045	0	0.029	0
$\bar{d}i\gamma_5 d \bar{d}d$	0.045	-0.13	0.029	-0.083
$\bar{s}i\gamma_5 s \bar{s}s$	0	0	0	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \sigma_{\mu\nu} d$	0.18	0	0.12	0
$\bar{d}i\gamma_5 \sigma^{\mu\nu} d \bar{u} \sigma_{\mu\nu} u$	0.18	0	0.12	0
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{s} \sigma_{\mu\nu} s$	0	0	0	0
$\bar{s}i\gamma_5 \sigma^{\mu\nu} s \bar{u} \sigma_{\mu\nu} u$	0	0	0	0
$\bar{d}i\gamma_5 \sigma^{\mu\nu} d \bar{s} \sigma_{\mu\nu} s$	0	0	0	0
$\bar{s}i\gamma_5 \sigma^{\mu\nu} s \bar{d} \sigma_{\mu\nu} d$	0	0	0	0

**Table 8.** P-odd, CP-odd three-point pion-nucleon vertices generated by P-odd, CP-odd four-quark operators. The couplings induced by operators constructed by two color-octet currents are equal to the couplings induced by corresponding color-singlet operators multiplying by  $-2/3$ .

and only keeping the terms which give non-vanishing contributions after integrating over a spherical region, we have

$$\begin{aligned}
 & \bar{\psi}_\lambda(x) \mathbb{P}_L \psi_\sigma(x) \bar{\psi}_\rho(x) \mathbb{P}_L \psi_\tau(x) \\
 & \simeq \frac{N^4}{64\pi^2} \left\{ [j_0^2(\omega_0 r/R_0) - j_1^2(\omega_0 r/R_0)]^2 \delta_{\lambda\sigma} \delta_{\rho\tau} \right. \\
 & \quad \left. - 4j_0^2(\omega_0 r/R_0) j_1^2(\omega_0 r/R_0) (\chi_\lambda^\dagger \vec{\sigma} \cdot \hat{r} \chi_\sigma) (\chi_\rho^\dagger \vec{\sigma} \cdot \hat{r} \chi_\tau) \right\},
 \end{aligned}$$

where we neglect the term proportional to  $\vec{\sigma} \cdot \hat{r}$ . In a proton state normalized to our convention before, the expectation value of the operator can be written as

$$\begin{aligned}
 \langle p_\uparrow | O_{11}^{ud(6,\bar{6})} | p_\uparrow \rangle &= -\frac{i}{2} N(w_0)^4 \frac{1}{64\pi^2} \int d^3x \\
 & \times \left\{ [j_0^2(\omega_0 r/R_0) - j_1^2(\omega_0 r/R_0)]^2 \langle p_\uparrow | A | p_\uparrow \rangle - 4j_0^2(\omega_0 r/R_0) j_1^2(\omega_0 r/R_0) \langle p_\uparrow | B | p_\uparrow \rangle \right\},
 \end{aligned} \tag{4.20}$$

where

$$\begin{aligned}
 A &= : u_\lambda^{a\dagger} u_\lambda^a d_\rho^{b\dagger} d_\rho^b : + : d_\lambda^{a\dagger} u_\lambda^a u_\rho^{b\dagger} d_\rho^b : \\
 B &= : u_\lambda^{a\dagger} (\chi_\lambda^\dagger \vec{\sigma} \cdot \hat{r} \chi_\sigma) u_\sigma^a d_\rho^{b\dagger} (\chi_\rho^\dagger \vec{\sigma} \cdot \hat{r} \chi_\tau) d_\tau^b : \\
 & \quad + : d^{a\dagger} (\chi_\lambda^\dagger \vec{\sigma} \cdot \hat{r} \chi_\sigma) u_\sigma^a u_\rho^{b\dagger} (\chi_\rho^\dagger \vec{\sigma} \cdot \hat{r} \chi_\tau) d_\tau^b : .
 \end{aligned} \tag{4.21}$$

A straightforward calculation gives

$$\begin{aligned}
 \langle p_\uparrow | A | p_\uparrow \rangle &= \langle n_\uparrow | A | n_\uparrow \rangle = 1 \\
 \langle \Sigma_\uparrow^+ | A | \Sigma_\uparrow^+ \rangle &= \langle \Sigma_\uparrow^- | A | \Sigma_\uparrow^- \rangle = 0 \\
 \langle \Sigma_\uparrow^0 | A | \Sigma_\uparrow^0 \rangle &= 2 \\
 \langle \Lambda_\uparrow | A | \Lambda_\uparrow \rangle &= 0 \\
 \langle \Xi_\uparrow^0 | A | \Xi_\uparrow^0 \rangle &= \langle \Xi_\uparrow^- | A | \Xi_\uparrow^- \rangle = 0,
 \end{aligned} \tag{4.22}$$

and

$$\begin{aligned}
 \langle p_\uparrow | B | p_\uparrow \rangle &= \langle n_\uparrow | B | n_\uparrow \rangle = 1/3 \\
 \langle \Sigma_\uparrow^+ | B | \Sigma_\uparrow^+ \rangle &= \langle \Sigma_\uparrow^- | B | \Sigma_\uparrow^- \rangle = 0 \\
 \langle \Sigma_\uparrow^0 | B | \Sigma_\uparrow^0 \rangle &= 2/3 \\
 \langle \Lambda_\uparrow | B | \Lambda_\uparrow \rangle &= 0 \\
 \langle \Xi_\uparrow^0 | B | \Xi_\uparrow^0 \rangle &= \langle \Xi_\uparrow^- | B | \Xi_\uparrow^- \rangle = 0.
 \end{aligned} \tag{4.23}$$

Therefore we can get in the MIT bag model

$$\langle p_\uparrow | O_{11}^{ud(6,\bar{6})}(x) | p_\uparrow \rangle = i\mathbb{A} + \frac{i}{3}\mathbb{B}, \tag{4.24}$$

where

$$\begin{aligned}
 \mathbb{A} &= -\frac{1}{2}N(\omega_0)^4 \frac{1}{16\pi} R_0^3 \int_0^1 \left(\frac{r}{R_0}\right)^2 d\left(\frac{r}{R_0}\right) [j_0^2(\omega_0 r/R_0) - j_1^2(\omega_0 r/R_0)]^2, \\
 \mathbb{B} &= \frac{1}{2}N(\omega_0)^4 \frac{1}{4\pi} R_0^3 \int_0^1 \left(\frac{r}{R_0}\right)^2 d\left(\frac{r}{R_0}\right) j_0^2(\omega_0 r/R_0) j_1^2(\omega_0 r/R_0),
 \end{aligned} \tag{4.25}$$

and similarly for other matrix elements.

Then, using the method we used in the non-relativistic quark model, we can get  $h_c$  and  $h_n$  induced by  $O_{11}^{ud(6,\bar{6})}$ ,

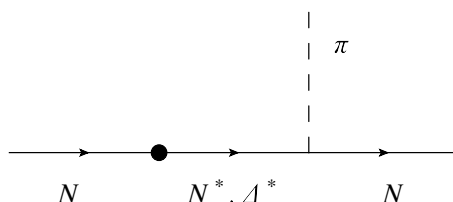
$$h_c = \frac{0.015 C_{11}^{ud}}{R_0^3 F_\pi}, \quad h_n = 0. \tag{4.26}$$

One can compare this with the result from the non-relativistic quark model in eq. (4.13), where  $h_c$  is proportional to  $\alpha^3$ . From the definition of  $\psi$  below eq. (4.8),  $1/\alpha$  can also be seen as the radius of the baryon. It is well known that  $1/\alpha = 0.5$  fm gives a too small value for the proton's charge radius and the pion cloud is usually invoked to gap it. On the other hand, the bag radius is usually taken to be 1.0 fm, which will give a considerably smaller  $h_c$ . In any case, it is reasonable to consider  $R_0 \sim 1/\alpha$  and take the non-relativistic quark model result as the representative.

The couplings  $h_c$  and  $h_n$  induced by color-singlet four-quark operators are listed in table 8 and those by color-octet operators are equal to the above multiplying by  $-2/3$ . In table 8, many four-quark operators yield zero  $h_c$  and  $h_n$  because we neglect the ‘‘sea quark’’ contribution. By making the four-quark operators normal ordered in eq. (4.10)

Operators	NR quark model		MIT bag model	
	$h_c/(C_4\alpha^3/F_\pi)$	$h_n/(C_4\alpha^3/F_\pi)$	$h_c/(C_4/(R_0^3F_\pi))$	$h_n/(C_4/(R_0^3F_\pi))$
$\bar{u}i\gamma_5u\bar{d}d$	0.0883	0.374	0.0560	-0.0690
$\bar{d}i\gamma_5d\bar{u}u$	0.0883	-0.374	0.0560	0.0690
$\bar{u}i\gamma_5t^au\bar{d}t^ad$	-0.0343	-0.0759	-0.0222	0.0140
$\bar{d}i\gamma_5t^ad\bar{u}t^au$	-0.0343	0.0759	-0.0222	-0.0140
$\bar{u}i\gamma_5u\bar{u}u$	0.0883	0	0.0569	0
$\bar{d}i\gamma_5d\bar{d}d$	0.0883	-0.255	0.0569	-0.163
$\bar{u}i\gamma_5t^au\bar{u}t^au$	-0.0343	0	-0.0221	0
$\bar{d}i\gamma_5t^ad\bar{d}t^ad$	-0.0343	0.0991	-0.0221	0.0633
$\bar{u}i\gamma_5\sigma^{\mu\nu}u\bar{d}\sigma_{\mu\nu}d$	-0.0397	0	-0.0230	0
$\bar{u}i\gamma_5\sigma^{\mu\nu}t^au\bar{d}\sigma_{\mu\nu}t^ad$	-0.268	0	-0.180	0

**Table 9.** Same as table 8, except the matrix elements are quoted here at the scale  $\mu = 4\pi F_\pi$  assuming the quark model scale of 400 MeV.



**Figure 2.** P-odd and CP-odd pion-nucleon coupling generated by the four-quark operators through parity-odd resonances, where the black dot is the CP-odd, four-quark operator,  $N^*$  and  $\Delta^*$  are the CP-odd excited states.

and (4.21), one cannot get any contribution to  $h_c$  and  $h_n$  from four-quark operators containing strange quarks.

Model calculations do not have explicit QCD scale dependence. To match the results with QCD matrix elements, we have to assume a model scale and using perturbative QCD (pQCD) evolution to run them to appropriate perturbative scale, for which we choose to be  $\mu = 4\pi F_\pi$ . In this work, we assume the model scale to be at 400 MeV and  $\Lambda_{\text{QCD}} = 250$  MeV and take into account the pQCD effect using one-loop renormalization group equation to run the operators down to the energy scale of the model. At this low energy regime the strong coupling is large and the one-loop pQCD evolution is by no means accurate, but it may still serve as an estimate of the pQCD effect. The matrix elements at scale  $\mu$  are shown in table 9.

### 4.2.3 Contribution from odd-parity resonances

The P-odd and CP-odd quark operators can also generate a CP-odd pion-nucleon interaction through the parity-odd excited resonances which is shown in figure 2. The P-odd and CP-odd quark operators can generate mixings between nucleons and parity-odd excited resonances which can be calculated using quark models [57]. Take the operator

$O_{11}^{(ud)} = \bar{u}i\gamma_5 u \bar{d}d$  and the intermediate state  $N(1535)$  as an example, using the harmonic oscillator non-relativistic quark model the mass mixing between neutron and  $N(1535)$  resonance can be estimated as  $\delta = m_c \omega^2 / (8\sqrt{3}\pi^{3/2})$ , where  $m_c \approx \omega \approx 300$  MeV are the constituent quark mass and the frequency of the harmonic oscillator, respectively. The resonance can decay into a nucleon plus a pion, the partial decay width is about 50 MeV [58]. The effective Lagrangian for this process can be written as

$$\mathcal{L}_{N^*} = g_{N^*} \bar{N} N^* \pi + h.c., \tag{4.27}$$

where as an order-of-magnitude estimate we discard the isospin quantum number. Then, from the partial decay width one can get  $g_{N^*} \sim \mathcal{O}(1)$ . The P-odd and CP-odd pion-nucleon coupling induced by this mixing can be written as

$$h_{mix} = \frac{C_4 g_{N^*} \delta}{M_{N^*} - m_n} \approx 6 \times 10^{-4} C_4 \text{GeV}^2, \tag{4.28}$$

where  $C_4$  is the Wilson coefficient of the four-quark operator. Compared with the direct matching contribution listed in table 8, one can see that  $h_{mix}$  is about two orders of magnitude smaller and therefore its contribution to nEDM is negligible.

The contribution from figure 2 can be seen as a one-loop contribution since the intermediate resonances may also be described as scattering states of pion and nucleon. Therefore, this contribution is suppressed by a loop factor.

### 4.3 Tree-level CP-Odd mass of neutron

The nucleon CP-odd observables receive contributions from its CP-odd mass term  $m' \bar{\psi} i \gamma_5 \psi$ . In  $\chi$ PT, there are also two sources of CP-odd mass: that induced by the condensates of meson fields, namely  $\langle \pi^0 \rangle$  and  $\langle \eta \rangle$ , and that from the direct matching contribution of the four-quark operators.

#### 4.3.1 Meson condensates

The relevant terms contributing to the CP-odd mass of neutron can be read from expanding eq. (3.23), which gives

$$\bar{n} i \gamma_5 n \frac{1}{F_\pi} \left\{ -d_1 \left[ (m_u - m_d) \langle \pi^0 \rangle + \frac{1}{\sqrt{3}} (m_u + m_d) \langle \eta \rangle - \frac{2}{\sqrt{3}} m_s \langle \eta \rangle \right] + d_2 \frac{2}{\sqrt{3}} m_s \langle \eta \rangle + d_3 m_d (\langle \pi^0 \rangle - \frac{1}{\sqrt{3}} \langle \eta \rangle) \right\}, \tag{4.29}$$

where  $d_1$ ,  $d_2$  and  $d_3$  can be related to the discrepancy of the Goldberger-Treiman relation, and the values  $d_2$  and  $d_3$  have been determined in the literature [59].

$$d_2 = -2B_0 m_0 (D_{19} - F_{19}), \quad d_3 = -2B_0 m_0 (D_{19} + F_{19}), \tag{4.30}$$

where  $m_0$  is the common octet mass in the chiral limit, and

$$m_0 F_{19} \approx -0.2, \quad m_0 D_{19} \approx -0.4. \tag{4.31}$$

Operators	$m'_n/(10^{-3}C_4B_0^2 \text{ GeV})$	Operators	$m'_n/(10^{-3}C_4B_0^2 \text{ GeV})$
$\bar{u}i\gamma_5u\bar{d}d$	-8.8	$\bar{u}t^a i\gamma_5u\bar{d}t^a d$	0
$\bar{d}i\gamma_5d\bar{u}u$	5.7	$\bar{d}t^a i\gamma_5d\bar{u}t^a u$	0
$\bar{u}i\gamma_5d\bar{s}s$	-8.8	$\bar{u}t^a i\gamma_5u\bar{s}t^a s$	0
$\bar{s}i\gamma_5s\bar{u}u$	3.2	$\bar{s}t^a i\gamma_5s\bar{u}t^a u$	0
$\bar{d}i\gamma_5d\bar{s}s$	5.7	$\bar{d}t^a i\gamma_5d\bar{s}t^a s$	0
$\bar{s}i\gamma_5s\bar{d}d$	3.2	$\bar{s}t^a i\gamma_5s\bar{d}t^a d$	0
$\bar{u}i\gamma_5u\bar{u}u$	-7.4	$\bar{u}t^a i\gamma_5u\bar{u}t^a u$	2.0
$\bar{d}i\gamma_5d\bar{d}d$	4.7	$\bar{d}t^a i\gamma_5d\bar{d}t^a d$	-1.3
$\bar{s}i\gamma_5s\bar{s}s$	2.6	$\bar{s}t^a i\gamma_5s\bar{s}t^a s$	0.7

**Table 10.** CP-odd mass of the neutron induced by meson condensates. Contributions from operators made of tensor currents are neglected due to the large- $N_C$  suppression.

Note that the signs of the  $F_{19}$  and  $D_{19}$  here are different from those in ref. [59]. Since  $d_1$  has not been determined from isospin-violation effect, we will set it to be zero in the following calculation. One should note that disregarding  $d_1$  leads to some errors because  $m_s\langle\eta\rangle$  might be the same order as  $m_d\langle\pi^0\rangle$ .

### 4.3.2 Direct contribution

The leading-order expansion of the tilded hadronic operators listed in table 3 are hermitian. Take  $\tilde{O}_6^{(2)}$  as an example. It can be written as

$$\tilde{O}_6^{(2)} \simeq \bar{p}i\gamma_5p + \bar{n}i\gamma_5n + \frac{1}{3}\bar{\Lambda}i\gamma_5\Lambda + \bar{\Sigma}^0i\gamma_5\Sigma^0 + \bar{\Sigma}^+i\gamma_5\Sigma^+ + \bar{\Sigma}^-i\gamma_5\Sigma^-, \quad (4.32)$$

which gives a CP-odd mass of neutron. To calculate the matching coefficients, we can see from above that the leading-order expansion is parity-odd, and we need to calculate a parity-odd quantity. The simplest is  $\Delta\vec{s} \cdot \Delta\vec{p}$ , where  $\Delta\vec{s}$  is the spin difference between the initial and final states and  $\Delta\vec{p}$  is the momentum difference between the initial and final states.

In the non-relativistic quark model, take the  $(6, \bar{6})$  components of  $\bar{u}i\gamma_5u\bar{d}d$ , as an example, to calculate the matrix elements proportional to  $\Delta\vec{s} \cdot \Delta\vec{p}$ ; the relevant part of the four-quark operator can be written as

$$O_{11}^{ud,(6,\bar{6})} \sim -\frac{i}{8} \left\{ \frac{i}{2m_C} : [\nabla \cdot (u^\dagger \vec{\sigma} u)] (d^\dagger d) : + \frac{i}{2m_C} : (u^\dagger u) [\nabla \cdot (d^\dagger \vec{\sigma} d)] : \right. \\ \left. - \frac{i}{2m_C} : [\nabla \cdot (d^\dagger \vec{\sigma} u)] (u^\dagger d) : - \frac{i}{2m_C} : (d^\dagger u) [\nabla \cdot (u^\dagger \vec{\sigma} d)] : \right\}, \quad (4.33)$$

where  $u$  and  $d$  are two-component quark operators,  $m_C$  is the mass of the constituent quark which is set to be one-third of the nucleon mass. The wave functions of baryons in the non-relativistic quark model are listed in eq. (4.9) and table 7. Then using the same method as described in the last section one can get the CP-odd mass of the neutron directly induced by the tilded operators, and the results are listed in table 11. After the leading-order QCD evolution to the scale where  $\mu = 4\pi F_\pi$ , the result is shown in table 12.

Operators	CP-odd mass/ $(\alpha^3 C_4)$	Operators	CP-odd mass / $(\alpha^3 C_4)$
$\bar{u}i\gamma_5 u \bar{d}d$	0.0635	$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \sigma_{\mu\nu} d$	-0.127
$\bar{d}i\gamma_5 d \bar{u}u$	-0.127	—	—
$\bar{u}i\gamma_5 u \bar{s}s$	0	$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{s} \sigma_{\mu\nu} s$	0
$\bar{s}i\gamma_5 s \bar{u}u$	0	—	—
$\bar{d}i\gamma_5 d \bar{s}s$	0	$\bar{d}i\gamma_5 \sigma^{\mu\nu} d \bar{s} \sigma_{\mu\nu} s$	0
$\bar{s}i\gamma_5 s \bar{d}d$	0	—	—
$\bar{u}i\gamma_5 u \bar{u}u$	0	—	—
$\bar{d}i\gamma_5 d \bar{d}d$	-0.127	—	—
$\bar{s}i\gamma_5 s \bar{s}s$	0	—	—

**Table 11.** CP-odd mass of neutron induced directly by color-singlet four-quark operators. The CP-odd mass induced by color-octet four-quark operators are equal to the one induced by corresponding color-singlet operators multiplied by  $-2/3$ .

Operators	CP-odd mass/ $(\alpha^3 C_4)$	Operators	CP-odd mass / $(\alpha^3 C_4)$
$\bar{u}i\gamma_5 u \bar{d}d$	0.212	$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \sigma_{\mu\nu} d$	0.0280
$\bar{d}i\gamma_5 d \bar{u}u$	-0.336	$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{d} \sigma_{\mu\nu} t^a d$	0.189
$\bar{u}i\gamma_5 t^a u \bar{d} t^a d$	-0.0314	—	—
$\bar{d}i\gamma_5 t^a d \bar{u} t^a u$	0.0799	—	—
$\bar{u}i\gamma_5 u \bar{u}u$	0	$\bar{u}i\gamma_5 t^a u \bar{u} t^a u$	0
$\bar{d}i\gamma_5 d \bar{d}d$	-0.249	$\bar{d}i\gamma_5 t^a d \bar{d} t^a d$	0.0968

**Table 12.** Same as figure 11. The matrix elements are now evolved to the scale where  $\mu = 4\pi F_\pi$ .

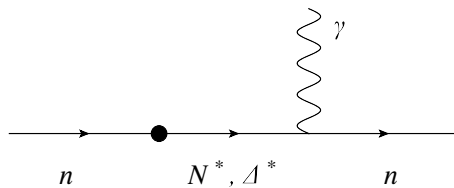
### 4.3.3 Contribution to CP-Odd Meson-Nucleon coupling

If rotating away the CP-odd nucleon mass through  $U_A(1)$  transformation, one can generate new contributions to the CP-odd meson-nucleon coupling from CP-even chiral operators. However, this contribution is of higher order in chiral power counting because all the CP-even meson-nucleon interactions are suppressed in the chiral limit, whereas the CP-odd coupling we considered in the previous subsections are not.

## 5 Four-quark contribution to neutron EDM in $\chi$ PT

In this section, we study the CP-odd four-quark contributions to the neutron EDM in  $\chi$ PT. The approach here is completely general and is applicable to any CP-odd quark-gluon operators. Some results presented can be found in the literature; however, to our knowledge, this is the most systematic and thorough discussion in the context of the CP-odd four-quark operators. In the last subsection, we make a comparison of the four-quark contributions in different approximations of non-perturbative QCD physics.

In  $\chi$ PT, the leading contributions come from many different sources. Since the CP-violating pion-nucleon couplings are  $\mathcal{O}(1)$ , the pion loop contribution to the neutron EDM is  $\mathcal{O}(1)$ , apart from possible enhancement by chiral logarithms. On the other hand, the direct matching contribution is also  $\mathcal{O}(1)$ , along with the pion condensate contribution



**Figure 3.** Direct calculation of the neutron EDM in quark models. The neutron makes a transition to a CP-odd excited state and goes back via electromagnetic interaction, where the black dot is the CP-odd, four-quark operator,  $N^*$  and  $\Delta^*$  are the CP-odd excited states.

through photo-production amplitudes. Finally, the CP-odd mass terms contribute through the nucleon magnetic moment after chiral rotation. This contribution is again  $\mathcal{O}(1)$  in chiral power counting. We will consider all these leading contributions in the following subsections. We ignore the subleading contribution in this work.

### 5.1 Direct matching from quark model

We have first considered the direct matching contribution from the four-quark operators to the neutron EDM in section IV. When any CP-odd quark-gluon operator is matched in  $\chi$ PT, there appear many tree-level neutron EDM-like operators in the chiral Lagrangian [14]. We do not have much to say about the size of the Wilson coefficients other than they are  $\mathcal{O}(1)$  in chiral power counting. Since they also serve as the counter terms for ultraviolet-divergent chiral-loop calculations, they depend on the regularization scheme and subtraction scale. In this work, we choose to estimate this contribution using nucleon models with dipole excitations into odd-parity resonances, such as  $S_{11}$ , following the work in [57].

We use the non-relativistic quark model with harmonic oscillator potentials to estimate the contribution from the first CP-odd excited states, which is shown in figure 3. The wave functions of the lowest CP-odd excited states can be written as

$$\begin{aligned}
 |N_{\uparrow}^*\rangle &= N_1 \epsilon^{abc} \int d^3r_1 d^3r_2 d^3r_3 \exp\left(\frac{i\vec{P}\cdot\vec{R}}{\sqrt{3}} - \frac{\alpha^2}{2}(\rho^2 + \lambda^2)\right) \\
 &\quad \left\{ (\lambda_x + i\lambda_y) [u_{\downarrow}^{a\dagger}(r_1) d_{\uparrow}^{b\dagger}(r_2) d_{\downarrow}^{c\dagger}(r_3) - u_{\uparrow}^{a\dagger}(r_1) d_{\downarrow}^{b\dagger}(r_2) d_{\downarrow}^{c\dagger}(r_3)] |0\rangle \right. \\
 &\quad \left. - \lambda_z [u_{\uparrow}^{a\dagger}(r_1) d_{\downarrow}^{b\dagger}(r_2) d_{\uparrow}^{c\dagger}(r_3) - u_{\downarrow}^{a\dagger}(r_1) d_{\uparrow}^{b\dagger}(r_2) d_{\uparrow}^{c\dagger}(r_3)] |0\rangle \right\}; \\
 |\Delta_{\uparrow}^*\rangle &= N_2 \epsilon^{abc} \int d^3r_1 d^3r_2 d^3r_3 \exp\left(\frac{i\vec{P}\cdot\vec{R}}{\sqrt{3}} - \frac{\alpha^2}{2}(\rho^2 + \lambda^2)\right) \\
 &\quad \left\{ (\lambda_x + i\lambda_y) [2u_{\downarrow}^{a\dagger}(r_1) d_{\downarrow}^{b\dagger}(r_2) d_{\uparrow}^{c\dagger}(r_3) + d_{\downarrow}^{a\dagger}(r_1) d_{\downarrow}^{b\dagger}(r_2) u_{\uparrow}^{c\dagger}(r_3)] |0\rangle \right. \\
 &\quad \left. - \lambda_z [2u_{\uparrow}^{a\dagger}(r_1) d_{\uparrow}^{b\dagger}(r_2) d_{\downarrow}^{c\dagger}(r_3) + d_{\uparrow}^{a\dagger}(r_1) d_{\uparrow}^{b\dagger}(r_2) u_{\downarrow}^{c\dagger}(r_3)] |0\rangle \right\}. \quad (5.1)
 \end{aligned}$$

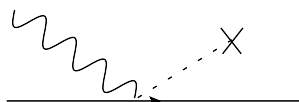
In the above formulas  $\lambda_x$ ,  $\lambda_y$  and  $\lambda_z$  are the  $x$ ,  $y$  and  $z$  components of  $\lambda$ , respectively.  $N_1$  and  $N_2$  are normalization factors of the states with  $N_1 = 2^{1/2} \alpha^4 / (3^{9/4} \pi^{3/2})$ ,  $N_2 = \alpha^4 / (2^{1/2} 3^{9/4} \pi^{3/2})$ .

Operators	nEDM/( $e\alpha C_4$ )	Operators	nEDM/( $e\alpha C_4$ )
$\bar{u}i\gamma_5 u \bar{d}d$	$-\frac{1}{6\sqrt{2}\pi^{3/2}}$	$\bar{u}i\gamma_5 t^a u \bar{d}t^a d$	$\frac{1}{9\sqrt{2}\pi^{3/2}}$
$\bar{d}i\gamma_5 d \bar{u}u$	$-\frac{1}{3\sqrt{2}\pi^{3/2}}$	$\bar{d}i\gamma_5 t^a d \bar{u}t^a u$	$\frac{\sqrt{2}}{9\pi^{3/2}}$
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d}\sigma_{\mu\nu} d$	$\frac{1}{\sqrt{2}\pi^{3/2}}$	$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{d}\sigma_{\mu\nu} t^a d$	$-\frac{\sqrt{2}}{3\pi^{3/2}}$
$\bar{u}i\gamma_5 u \bar{u}u$	0	$\bar{u}i\gamma_5 t^a u \bar{u}t^a u$	0
$\bar{d}i\gamma_5 d \bar{d}d$	0	$\bar{d}i\gamma_5 t^a d \bar{d}t^a d$	0

**Table 13.** nEDM contributed from first excited CP-odd states in the non-relativistic quark model, where  $C_4$  is the Wilson coefficients of the quark models,  $\alpha$  is defined below eq. (4.8). The unit of nEDM used here is  $e \cdot \text{GeV}^{-1}$ , which is different from the traditional one  $e \cdot \text{cm}$  due to that the Wilson coefficients of the four-quark operators are unknown which are always in the unit of  $\text{GeV}^{-2}$ . The translation between the two units is  $e \cdot \text{GeV}^{-1} \simeq 2 \times 10^{-14} e \cdot \text{cm}$ .

Operators	nEDM/( $10^{-3}eC_4\text{GeV}$ )	Operators	nEDM/( $10^{-3}eC_4\text{GeV}$ )
$\bar{u}i\gamma_5 u \bar{d}d$	-37.6	$\bar{u}i\gamma_5 t^a u \bar{d}t^a d$	3.80
$\bar{d}i\gamma_5 d \bar{u}u$	-62.6	$\bar{d}i\gamma_5 t^a d \bar{u}t^a u$	8.87
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d}\sigma_{\mu\nu} d$	77.5	$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{d}\sigma_{\mu\nu} t^a d$	-103
$\bar{u}i\gamma_5 u \bar{u}u$	0	$\bar{u}i\gamma_5 t^a u \bar{u}t^a u$	0
$\bar{d}i\gamma_5 d \bar{d}d$	0	$\bar{d}i\gamma_5 t^a d \bar{d}t^a d$	0

**Table 14.** Same as table XIII, except the renormalization scale is now at  $4\pi F_\pi$ .



**Figure 4.** Pion-photoproduction diagram with the pion field annihilated by the four-quark operator into the vacuum, where the cross is a four-quark operator.

The results are shown in table 13, which agree with the results extracted from ref. [57]. We also need to take into account the evolution of the operators between  $4\pi F_\pi$  and the energy scale of the quark model. The results are shown in table 14 with  $\alpha = 0.41 \text{ GeV}$ .

## 5.2 Meson condensate contribution through Photo-Pion production

In photon-pion production, there are CP-even electric-dipole couplings between the baryon-octet and electromagnetic fields through using  $f_\pm^{\mu\nu}$  [14]. Some of these couplings can generate the neutron EDM if they violate the chiral symmetry through the quark masses and at the same time the meson fields acquire vacuum condensates through the CP-odd four-quark operators. In more physical language, the contact terms for the pion-photoproduction processes give rise to the neutron EDM through the diagram in figure 4. Although the electromagnetic field also violates chiral symmetry, it cannot generate an EDM through meson condensates by itself — a quark mass factor is essential.

The terms of interest are made of linear products of baryon fields  $\bar{B}$  and  $B$ ,  $\chi_-$



Operators	$d_{\pi\gamma}/(10^{-3}eC_4B_0^2 \text{ GeV}^{-1})$	Operators	$d_{\pi\gamma}/(10^{-3}eC_4B_0^2 \text{ GeV}^{-1})$
$\bar{u}i\gamma_5 u \bar{d}d$	10.6	$\bar{u}t^a i\gamma_5 u \bar{d}t^a d$	0
$\bar{d}i\gamma_5 d \bar{u}u$	-10.5	$\bar{d}t^a i\gamma_5 d \bar{u}t^a u$	0
$\bar{u}i\gamma_5 d \bar{s}s$	10.6	$\bar{u}t^a i\gamma_5 u \bar{s}t^a s$	0
$\bar{s}i\gamma_5 s \bar{u}u$	-0.12	$\bar{s}t^a i\gamma_5 s \bar{u}t^a u$	0
$\bar{d}i\gamma_5 d \bar{s}s$	-10.5	$\bar{d}t^a i\gamma_5 d \bar{s}t^a s$	0
$\bar{s}i\gamma_5 s \bar{d}d$	-0.12	$\bar{s}t^a i\gamma_5 s \bar{d}t^a d$	0
$\bar{u}i\gamma_5 u \bar{u}u$	8.87	$\bar{u}t^a i\gamma_5 u \bar{u}t^a u$	-2.37
$\bar{d}i\gamma_5 d \bar{d}d$	-8.77	$\bar{d}t^a i\gamma_5 d \bar{d}t^a d$	2.34
$\bar{s}i\gamma_5 s \bar{s}s$	-0.10	$\bar{s}t^a i\gamma_5 s \bar{s}t^a s$	0.03

**Table 15.** nEDM induced by meson condensates through pion-photoproduction. Contribution from operators constructed by tensor operators are neglected due to the large- $N_C$  suppression.

and  $f_+$  [14],

$$\mathcal{L}_{\pi\gamma}^C = \frac{1}{16\pi^2 F_\pi^2} [\delta_1 \text{Tr}[\bar{B}\sigma_{\mu\nu}\gamma_5\{\chi_-, f_+^{\mu\nu}\}B] + \delta_2 \text{Tr}[\bar{B}\sigma_{\mu\nu}\gamma_5 f_+^{\mu\nu}B] \text{Tr}[\chi_-] + \dots] , \quad (5.2)$$

where we have shown two of the ten possible terms. It is difficult, however, to extract the Wilson coefficients  $\delta_i$  directly from experimental data. Some of the coefficients have been estimated by calculating the contribution from the excited baryon states in the context of the two-flavor scenario [60]. In the two-flavor scenario, neglecting the isospin violation generated by the difference between the up and down quark masses, the terms relevant to nEDM can be written as

$$\mathcal{L}_{\pi\gamma}^{2-flavor} = \bar{N}\gamma_5\sigma_{\mu\nu}[(a_1^p - a_1^n)f_+^{\mu\nu} + a_1^n \text{Tr}(f_+^{\mu\nu})]\chi_- N , \quad (5.3)$$

where  $N = \begin{pmatrix} p \\ n \end{pmatrix}$ , and in the two-flavor case,  $f_+^{\mu\nu} \equiv e(\xi^\dagger Q \xi + \xi Q \xi^\dagger)F^{\mu\nu}$ , in which  $Q = (1 + \tau^3)/2$ . Expanding  $f_+^{\mu\nu}$  and  $\chi_-$ , we can get the nEDM induced by the condensate of  $\pi^0$ ;

$$d_{\pi\gamma} = -\frac{8ea_1^n B_0(m_u + m_d)\langle\pi^0\rangle}{F_\pi} . \quad (5.4)$$

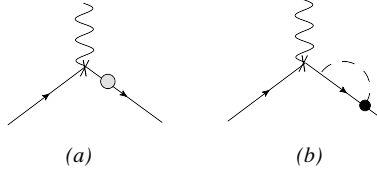
From ref. [60], one can get the contribution to  $a_1$  from  $\Delta$  and  $\rho$  internal states, which is

$$a_1 = -0.156 \text{ GeV}^{-3} . \quad (5.5)$$

Using this, one can estimate the nEDM induced by the pion condensate, as shown in table 15.

### 5.3 CP-Odd Baryon mass contribution

The CP-odd baryon-mass terms considered in the previous section generate a CP-odd part of the baryon wave function. This part can transform a magnetic moment term into an EDM contribution. The physics of this is shown in figure 5.



**Figure 5.** The CP-odd mass of neutron turns the tree level magnetic moment into an EDM. The cross is the tree level magnetic moment, the gray dot is the CP-odd mass of the neutron and the black dot is the CP-odd pion-nucleon coupling.

The mass terms of the neutron can be written as

$$\mathcal{L}_{\text{mass}} = -m_n \bar{n}n - m'_n \bar{n}i\gamma_5 n . \quad (5.6)$$

Note that the neutron field  $n$  here is already redefined using the transformation in eq. (3.19) after taking into account the meson condensate effect as discussed in the previous section. Redefining the neutron field again through a chiral rotation,

$$n = \exp\left(-i\frac{m'_n}{2m_n}\right)\gamma_5 n' , \quad (5.7)$$

the mass term becomes the standard one,

$$\mathcal{L}_{\text{mass}} = -m_n \bar{n}'n' . \quad (5.8)$$

On the other hand, the tree level anomalous magnetic moment of the neutron can be written as

$$\mathcal{L}_{\text{mag.mom.}} = -\frac{1}{4}\frac{\kappa_n}{m_n}\bar{n}\sigma^{\mu\nu}nF_{\mu\nu} . \quad (5.9)$$

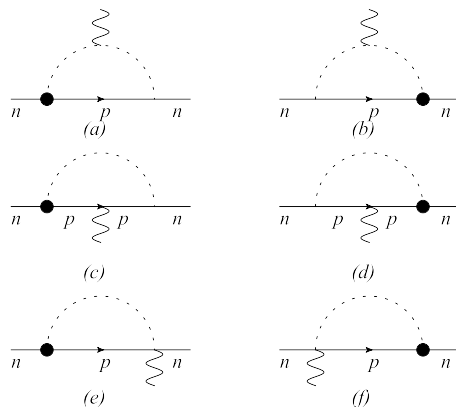
The redefinition in eq. (5.7) generates a neutron EDM,

$$d_{\text{CP-oddmass}}^{\text{EDM}} = -\frac{\kappa_n m'_n}{2m_n^2} . \quad (5.10)$$

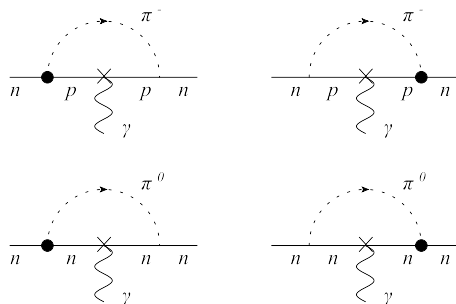
The experimental values of the anomalous magnetic dipole moments of the nucleons are  $\kappa_p = 1.7928$ ,  $\kappa_n = -1.9131$ . The numerical values of this contribution have been shown in tables 16 and 17. For the tensor operator  $\bar{u}i\gamma_5\sigma^{\mu\nu}u\bar{d}\sigma_{\mu\nu}d$ , the contribution from the CP-odd mass of the nucleon is particularly large. The CP-odd mass also gets a quantum correction shown in diagram (b) of figure 5. It is easy to see that this term does not have any chiral enhancement and is of a higher-order effect.

#### 5.4 Leading chiral loop contribution

The contribution we have considered so far has a smooth chiral limit, i.e., regular as the quark masses go to zero. The leading contribution in the chiral limit, however, involves the pion loop with an infrared divergence. This contribution was first calculated by Crewther et al [16], and has been studied thoroughly in the literature (see figure 6). Diagrams (a)



**Figure 6.** Charged-pion loop contribution to neutron EDM (without the anomalous magnetic moment), where the black dots represent the CP-odd vertices.



**Figure 7.** Contribution from the tree level anomalous magnetic moments of proton and neutron, where the crosses are anomalous magnetic moments of nucleons and the dots are CP-odd vertices.

and (b) in figure 6 contain an infrared divergence which is regularized by the mass of pion and an analytical part. The constant part is canceled by diagrams (c) and (d). Diagrams (e) and (f) cancel with each other [14]. Therefore, up to terms of order  $(m_\pi/m_n)$ , the neutron EDM generated by the charged pion loop can be written as [14]

$$d_{\pi^+}^n = -\frac{e\sqrt{2}}{16\pi^2 F_\pi} h_c (D + F) \ln(m_\pi^2/m_n^2), \tag{5.11}$$

where  $D + F = -g_A = -1.26$  is the CP-even pion-nucleon coupling (the signs of  $D$  and  $F$  is different from that in ref. [25] because we are using a different definition of the chiral transformation of  $U$ ), and  $h_c$  is the CP-odd pion-nucleon coupling defined in eq. (4.3). Note that, in figure 6, the contribution from the proton’s anomalous magnetic moment has not been included. To include this contribution, we consider all these diagrams in figure 7 where the neutral pion loop is also present, and the result is ref. [22].

$$d_{\pi_0+\kappa}^n = \frac{e}{16\pi^2} \frac{D + F}{F_\pi} \left( -\sqrt{2} h_c \kappa_p + h_n \kappa_n \right) F_n \left( \frac{m_\pi^2}{m_n^2} \right) \tag{5.12}$$

operators	nEDM from different contributions / ( $10^{-3}eC_4$ GeV)								
	contact term	meson photo-production	$\pi$ - $N$ coupling direct		$\pi$ - $N$ coupling, $\langle\pi^0\rangle, \langle\eta\rangle$		CP-odd mass direct	CP-odd mass $\langle\pi^0\rangle, \langle\eta\rangle$	total
			figure 6	figure 7	figure 6	figure 7			
$\bar{u}i\gamma_5 u\bar{d}d$	-37.6	52.9	-30.9	71.3	27.2	202.1	15.8	-47.6	253.2
$\bar{u}u\bar{d}i\gamma_5 d$	-62.6	-52.3	-30.9	-37.3	-30.2	-202.3	-25.1	30.6	-410.2
$\bar{u}i\gamma_5 u\bar{s}s$	0	52.9	0	0	27.2	202.1	0	-47.6	234.6
$\bar{u}u\bar{s}i\gamma_5 s$	0	-0.6	0	0	2.8	-0.7	0	17.1	18.5
$\bar{d}i\gamma_5 d\bar{s}s$	0	-52.3	0	0	-30.2	-202.3	0	30.6	-254.3
$\bar{d}d\bar{s}i\gamma_5 s$	0	-0.6	0	0	2.8	-0.7	0	17.1	18.5
$\bar{u}i\gamma_5 u\bar{u}u$	0	44.1	-30.9	17.0	22.7	168.9	0	-39.7	182.1
$\bar{d}i\gamma_5 d\bar{d}d$	0	-43.6	-30.9	-20.0	-25.1	-168.5	-18.6	25.5	-281.2
$\bar{s}i\gamma_5 s\bar{s}s$	0	-0.5	0	0	2.4	-0.6	0	14.2	15.5
$\bar{u}i\gamma_5\sigma^{\mu\nu}u\bar{d}\sigma_{\mu\nu}d$	77.5	0	13.9	-7.7	0	0	2.1	0	85.8
$\bar{u}i\gamma_5\sigma^{\mu\nu}u\bar{s}\sigma_{\mu\nu}s$	0	0	0	0	0	0	0	0	0
$\bar{d}i\gamma_5\sigma^{\mu\nu}d\bar{s}\sigma_{\mu\nu}s$	0	0	0	0	0	0	0	0	0

**Table 16.** nEDM from the P-odd and CP-odd four-quark operators composed of color-singlet currents. Different contributions are shown.

operators	nEDM from different contributions / ( $10^{-3}eC_4$ GeV)								
	contact term	meson photo-production	$\pi$ - $N$ coupling direct		$\pi$ - $N$ coupling, $\langle\pi^0\rangle, \langle\eta\rangle$		CP-odd mass direct	CP-odd mass $\langle\pi^0\rangle, \langle\eta\rangle$	total
			figure 6	figure 7	figure 6	figure 7			
$\bar{u}i\gamma_5 t^a u\bar{d}t^a d$	3.8	0	12.0	-17.6	0	0	-2.34	0	-4.2
$\bar{u}t^a u\bar{d}i\gamma_5 t^a d$	8.9	0	12.0	4.42	0	0	6.0	0	31.2
$\bar{u}i\gamma_5 t^a u\bar{s}t^a s$	0	0	0	0	0	0	0	0	0
$\bar{u}t^a u\bar{s}i\gamma_5 t^a s$	0	0	0	0	0	0	0	0	0
$\bar{d}i\gamma_5 t^a d\bar{s}t^a s$	0	0	0	0	0	0	0	0	0
$\bar{d}t^a d\bar{s}i\gamma_5 t^a s$	0	0	0	0	0	0	0	0	0
$\bar{u}i\gamma_5 t^a u\bar{u}t^a u$	0	-11.8	12.0	-6.6	-6.1	-45.0	0	10.6	-46.8
$\bar{d}i\gamma_5 t^a d\bar{d}t^a d$	0	11.6	12.0	7.8	6.7	44.8	7.2	-6.8	83.4
$\bar{s}i\gamma_5 t^a s\bar{s}t^a s$	0	0.1	0	0	-0.6	0.2	0	-3.8	-4.1
$\bar{u}i\gamma_5\sigma^{\mu\nu}t^a u\bar{d}\sigma_{\mu\nu}t^a d$	-103.1	0	93.8	-51.6	0	0	14.1	0	-46.9
$\bar{u}i\gamma_5\sigma^{\mu\nu}t^a u\bar{s}\sigma_{\mu\nu}t^a s$	0	0	0	0	0	0	0	0	0
$\bar{d}i\gamma_5\sigma^{\mu\nu}t^a d\bar{s}\sigma_{\mu\nu}t^a s$	0	0	0	0	0	0	0	0	0

**Table 17.** Neutron EDM generated by P-odd and CP-odd four-quark operators composed of color-octet currents. The labels have the same meaning as in table 16.

where  $\kappa_n$  and  $\kappa_n$  are tree-level anomalous magnetic moments of protons and neutrons, respectively, and

$$F_n(s) = \frac{3}{2} - s - \frac{3s - s^2}{2} \ln s + \frac{s(5s - s^2) - 4s}{2\sqrt{s - s^2/4}} \arctan \frac{\sqrt{s - s^2/4}}{s/2}. \quad (5.13)$$

We can see that there is no chiral enhancement in  $F_n(s)$ .

Using the above, we estimate the pion-loop and the CP-odd mass contributions to neutron EDM due to the P-odd and CP-odd four-quark operators. The results are listed in tables 16 and 17. Although the charged pion-loop (figure 6) dominates in the chiral limit, its numerical value is actually about an order of magnitude smaller than the analytical chiral-loop contribution (figure 7). This is due to the enhancement of  $h_n$  relative to  $h_c$  in the large  $N_c$  limit.

The P-odd and CP-odd four-quark operators can also lead to nonvanishing P-odd and CP-odd interaction like  $n \rightarrow K\Sigma$  and  $n \rightarrow \Lambda\eta$ . These interactions can generate nEDM through kaon- or eta-loop diagrams. However, there is no reason to believe that the kaon- or eta-loop contribution should be more important than the pion-loop contribution so that it would not change the order-of-magnitude estimate of nEDM generated by those four-quark operators without the strange quark. For those operators containing strange quark the estimation may not be reliable and the kaon- or eta-loop contributions should be included.

### 5.5 Comparison with other calculations and the error-bars of this calculation

The P-odd and CP-odd four-quark contributions to neutron EDM have been studied using different approximation methods in the literature [20–22]. The problem is that it is difficult to get an estimate on the errors in any of these methods. This is the strong motivation for the alternative study presented here. By using a completely different approach, we hope to get a better idea how well one actually estimates these hadronic matrix elements.

In ref. [20], the authors used the external field method, factorization and QCD sum rules to make a direct calculation of the neutron EDM. Their result is supposed to be the total contribution, although it is unclear how the chiral physics would be included in this approach. Their numbers are listed in table 18 as “factorization and QCD sum rule.” The result is, in general, comparable to the charged pion-loop contribution, although the contribution to the tensor operator is particularly large.

In ref. [21], the authors also calculated the contributions of the pion-loop as we do in this paper. They used entirely the factorization method to calculate the CP-odd pion-nucleon couplings, including the effects that the CP-odd operators can annihilate the neutral pion in the vacuum. Taking the operator  $\bar{u}i\gamma_5 u \bar{d}d$  as an example, their factorization works like this:

$$\begin{aligned} \langle n\pi^0 | \bar{u}i\gamma_5 u \bar{d}d | n \rangle &= \langle n | \bar{d}d | n \rangle \langle \pi^0 | \bar{u}i\gamma_5 u | 0 \rangle \\ &+ \langle 0 | \bar{d}d | 0 \rangle \left( \langle n\pi^0 | \bar{u}i\gamma_5 u | n \rangle - \frac{1}{m_\pi^2} \langle n\pi^0 | \mathcal{L}_{QCD}^m | n\pi^0 \rangle \langle \pi^0 | \bar{u}i\gamma_5 u | 0 \rangle \right), \end{aligned} \tag{5.14}$$

where  $\mathcal{L}_{QCD}$  is the usual QCD Lagrangian. The terms inside the bracket on the second line of the above formula cancel each other. The reason is that  $\bar{u}i\gamma_5 u$  is just a CP-odd mass of the up-quark which can be rotated away through chiral transformation, except for a possible  $U_A(1)$  contribution. Thus these two contributions should cancel with each other exactly. This is first noticed in ref. [61] in the spirit of the Feinberg-Weinberg-Kabir theorem [62]. Using this method, one can get the CP-odd vertices,  $h_c$  and  $h_n$  as shown in table 6. For the charged coupling  $h_c$ , one needs to do a Fierz transformation, from which one can get a suppression factor of 1/12, where 1/3 is from the color factor and the other 1/4 is from the spin. Therefore,  $h_c$  is one order of magnitude smaller than  $h_n$ . The corresponding nEDM calculated using this method is included in table 18 as well.

From table 18, taking the operator  $\bar{u}i\gamma_5 u \bar{d}d$  as an example, one can see that the magnitude of our result is comparable with what obtained using naïve factorization method but with a different sign; also the our result is about one order of magnitude larger than

Operators	Our results	Naive factorization	Factorization and QCD sum rules
$\bar{u}i\gamma_5 u \bar{d}d$	253	-248	17.5
$\bar{d}i\gamma_5 d \bar{u}u$	-410	177	-17.5
$\bar{u}i\gamma_5 d \bar{s}s$	235	-85.8	-
$\bar{s}i\gamma_5 s \bar{u}u$	18.5	0	-
$\bar{d}i\gamma_5 d \bar{s}s$	-254	85.8	-
$\bar{s}i\gamma_5 s \bar{d}d$	18.5	0	-
$\bar{u}i\gamma_5 u \bar{u}u$	182	-154	-17.7
$\bar{d}i\gamma_5 d \bar{d}d$	-281	203	15.2
$\bar{s}i\gamma_5 s \bar{s}s$	15.5	0	-
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \sigma_{\mu\nu} d$	85.8	-79.4	-127.5
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{s} \sigma_{\mu\nu} s$	0	0	-
$\bar{d}i\gamma_5 \sigma^{\mu\nu} d \bar{s} \sigma_{\mu\nu} s$	0	0	-
$\bar{u}t^a i\gamma_5 u \bar{d}t^a d$	-4.2	-8.88	-3.18
$\bar{d}t^a i\gamma_5 d \bar{u}t^a u$	31.3	-8.88	3.18
$\bar{u}t^a i\gamma_5 u \bar{s}t^a s$	0	0	-
$\bar{s}t^a i\gamma_5 s \bar{u}t^a u$	0	0	-
$\bar{d}t^a i\gamma_5 d \bar{s}t^a s$	0	0	-
$\bar{s}t^a i\gamma_5 s \bar{d}t^a d$	0	0	-
$\bar{u}t^a i\gamma_5 u \bar{u}t^a u$	-46.8	39.5	-23.5
$\bar{d}t^a i\gamma_5 d \bar{d}t^a d$	83.4	-51.1	9.3
$\bar{s}t^a i\gamma_5 s \bar{s}t^a s$	-4.12	0	-
$\bar{u}t^a i\gamma_5 \sigma^{\mu\nu} u \bar{d}t^a \sigma_{\mu\nu} d$	-46.9	-106	14.3
$\bar{u}t^a i\gamma_5 \sigma^{\mu\nu} u \bar{s}t^a \sigma_{\mu\nu} s$	0	0	-
$\bar{d}t^a i\gamma_5 \sigma^{\mu\nu} d \bar{s}t^a \sigma_{\mu\nu} s$	0	0	-

**Table 18.** Comparison of different methods, nEDM calculated by factorization in ref. [21, 22] are shown as “naive factorization”. The column on the right side shows nEDM calculated using factorization and QCD sum rules [20]. The unit of the numbers is  $10^{-3}eC_4\text{GeV}$ .

the result estimated using QCD sum rules. In our calculation, we separate the contribution into the meson condensate contribution and the direct matching contribution. The vacuum saturation method is used to calculate the meson condensate contribution to  $h_c$  and  $h_n$ . This vacuum saturation method using to calculate the meson matrix elements is accurate in the large- $N_C$  limit, which means the calculation for this contribution is accurate up to  $1/N_C$  [46]. From table 16, one can see that the meson condensate contributions dominate over the direct matching contributions. Therefore, for operators generating unsuppressed meson condensates (see Sec IV for detailed discussions), a conservative uncertainty can be set to be a factor of two.

In ref. [21], the authors also used the vacuum saturation approach to get the factorization result as shown in eq. (5.14). However, in the case of baryon matrix element, the non-factorized contribution is not suppressed in the large- $N_C$  limit [46], therefore the

missed non-factorized contribution should be of the same order as the factorized contribution shown in eq. (5.14). The calculation using QCD sum rules in ref. [20] did not include the meson condensate contribution, therefore, their calculation might miss an important contribution.

The factor of two uncertainty can also be seen from the Feinberg-Weinberg-Kabir theorem [62]. Applying to this context, the theorem dictates that CP-odd  $(3, \bar{3})$  two-quark operators give no contribution to CP-odd processes. However, since we are using a hybrid method, this theorem may not be satisfied. Therefore, the amount of violation of this theorem can be seen as an estimate of the error of this calculation. Take the operator  $\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d$  as an example, following the prescription in sections III and IV, one can get meson-condensate contribution to the neutral CP-odd pion-nucleon coupling which can be written as

$$h_n^{mc} = \frac{2C_3(2c_1 + c_3)}{F_\pi} \approx -\frac{10C_3}{F_\pi}, \quad (5.15)$$

where  $m_u = m_d = \bar{m}$  is assumed for the sake of simplicity,  $C_3$  is the Wilson coefficient of the two-quark operator and the definitions of  $c_1$  and  $c_3$  can be found in eq. (4.1). If the  $\sigma$ -term is also employed to do the direct matching, one can easily show that the direct matching contribution cancels the meson condensate contribution exactly. Instead, in order to get the uncertainty of our calculation we need to do the direct matching using the quark model. Since the operator includes only products of two quark fields, the calculation using the quark model is straightforward, which gives

$$h_n^{dir} = R \frac{3C_3}{F_\pi} \approx \frac{5C_3}{F_\pi}, \quad (5.16)$$

where the factor of 3 is due to that in the quark model the nucleon contains three constituent quarks.  $R \approx 1.7$  comes from the perturbative QCD effect as discussed in section V. The anomalous dimensions of the operator discussing here is as the same as the anomalous dimensions of the quark mass. The relative sign between the direct contribution and the meson condensate contribution is as desired. However, the magnitude of the direct contribution is about two times smaller than the meson condensate contribution. The mismatch between the two contributions is due to that quark model does not differentiate  $\langle N|\bar{q}q|N\rangle$  and  $\langle N|q^\dagger q|N\rangle$ . From this mismatch one can see that the inaccuracy of the direct contribution calculated using quark model might be a factor of two. Therefore, conservatively, the total inaccuracy for those operators having unsuppressed vacuum condensate contributions can be seen as a factor of two.

## 6 Conclusion

In this paper, we studied the four-quark contributions to the neutron EDM, which dominate over other QCD operators in some new physics models. Our approach was based on chiral expansion and simple quark models. It is well known in the literature that the leading chiral contribution comes from one-pion loop which dominates in the chiral limit  $m_\pi \rightarrow 0$ , just like in the case of the nucleon electric polarizability. Therefore, one needs to calculate the four-quark contribution to the CP-odd pion nucleon couplings. We studied these couplings

Operators	Upper bound of $ C_4 /(\text{GeV}^{-2})$
$\bar{u}i\gamma_5 u \bar{d}d$	$5 \times 10^{-12}$
$\bar{u}u \bar{d}i\gamma_5 d$	$4 \times 10^{-12}$
$\bar{u}i\gamma_5 u \bar{s}s$	$6 \times 10^{-12}$
$\bar{d}i\gamma_5 d \bar{s}s$	$6 \times 10^{-12}$
$\bar{u}i\gamma_5 u \bar{u}u$	$8 \times 10^{-12}$
$\bar{d}i\gamma_5 d \bar{d}d$	$5 \times 10^{-12}$
$\bar{u}i\gamma_5 \sigma^{\mu\nu} u \bar{d} \bar{\sigma}_{\mu\nu} d$	$2 \times 10^{-11}$
$\bar{u}i\gamma_5 t^a u \bar{d} t^a d$	$4 \times 10^{-10}$
$\bar{u} t^a u \bar{d} i\gamma_5 t^a d$	$4 \times 10^{-11}$
$\bar{u}i\gamma_5 t^a u \bar{u} t^a u$	$3 \times 10^{-11}$
$\bar{d}i\gamma_5 t^a d \bar{d} t^a d$	$2 \times 10^{-11}$
$\bar{u}i\gamma_5 \sigma^{\mu\nu} t^a u \bar{d} \bar{\sigma}_{\mu\nu} t^a d$	$3 \times 10^{-11}$

**Table 19.** Upper bound on the Wilson coefficients of P-odd, CP-odd four-quark operators, calculated using the experimental data and hadronic matrix elements in this work.

in simple quark models, as an alternate to large- $N_c$  factorization. We also considered  $\mathcal{O}(1)$  contribution from direct matching and pion-condensation to the dipole moment, as well as the CP-odd nucleon mass contribution through the magnetic moment. The resulting nEDM can be compared with those from the naive factorization and QCD sum rules. The comparison provides us some idea on the hadronic physics uncertainty in the neutron EDM calculation. Our approach also provides a formalism for lattice QCD calculations of the nucleon matrix elements of the four-quark operators.

Using the matrix elements thus obtained, we obtain new-physics-independent upper bounds on the Wilson coefficients of four-quark operators from the experimental data. The current experimental upper bound on neutron EDM is  $2.9 \times 10^{-26} e \text{ cm}$  [31]. If we assume that there is no significant cancellations among the contributions from these operators, we can use the experiment limit to give upper bounds to the Wilson coefficients of individual operators. In our calculation, the strange quark effects were ignored, and we considered only operators composed of up and down quarks. The final results are shown in table 19.

It is interesting to note that the chiral-enhanced contribution is actually large- $N_c$  suppressed. In fact, the non-singular part of the chiral-loop contribution numerically dominates over the singular one. This suggests a large- $N_c$  analysis of the neutron EDM, including the delta resonance contribution. However, this is beyond the scope of this paper.

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