Supersymmetric AdS$_6$ vacua in six-dimensional $N = (1, 1)$ gauged supergravity

Parinya Karndumri[a] **and Jan Louis**[b,c]

[a]*String Theory and Supergravity Group,*
Department of Physics, Faculty of Science, Chulalongkorn University,
254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand

[b]*Fachbereich Physik der Universität Hamburg,*
Luruper Chaussee 149, 22761 Hamburg, Germany

[c]*Zentrum für Mathematische Physik, Universität Hamburg,*
Bundesstrasse 55, D-20146 Hamburg, Germany

E-mail: parinya.ka@hotmail.com, jan.louis@desy.de

ABSTRACT: We study fully supersymmetric AdS$_6$ vacua of half-maximal $N = (1, 1)$ gauged supergravity in six space-time dimensions coupled to n vector multiplets. We show that the existence of AdS$_6$ backgrounds requires that the gauge group is of the form $G' \times G'' \subset$ SO(4, n) where $G' \subset$ SO(3, m) and $G'' \subset$ SO(1, n − m). In the AdS$_6$ vacua this gauge group is broken to its maximal compact subgroup SO(3) × H' × H'' where $H' \subset$ SO(m) and $H'' \subset$ SO(n−m). Furthermore, the SO(3) factor is the R-symmetry gauged by three of the four graviphotons. We further show that the AdS$_6$ vacua have no moduli that preserve all supercharges. This is precisely in agreement with the absence of supersymmetric marginal deformations in holographically dual five-dimensional superconformal field theories.

Contents

1 Introduction

Supersymmetric anti-de Sitter (AdS) vacua and their moduli spaces of gauged supergravities are of particular interest in the AdS/CFT correspondence [1]. The AdS vacua correspond to conformal fixed points of the holographically dual field theories while the moduli spaces describe the conformal manifolds near these fixed points [2, 3]. The latter encode useful information about the exactly marginal deformations of the corresponding superconformal field theories (SCFTs).

AdS backgrounds of gauged supergravities and their moduli spaces have been studied in various space-time dimensions with different numbers of supercharges. In this paper we exclusively focus on the half-maximal gauged $N = (1, 1)$ supergravity in six space-time dimensions $(d = 6)$ and their maximally supersymmetric AdS$_6$ backgrounds.[1] This supergravity is also known as $F(4)$ supergravity and was first constructed in [6]. It is non-chiral and can be coupled to an arbitrary number n of vector multiplets. Each vector multiplet contains four scalars and together with the dilaton in the gravity multiplet, they parametrize the $(4n + 1)$-dimensional coset manifold $\mathbb{R}^+ \times SO(4, n)/SO(4) \times SO(n)$. The corresponding gauged supergravity was constructed in [7, 8] by extending the pure $F(4)$ supergravity using the geometric group manifold approach. [7, 8] also showed that for a gauge group $SU(2)_R \times G$ and $G \subset SO(n)$ a maximally supersymmetric AdS$_6$ vacuum exists where the full $SU(2)_R \times G$ symmetry is realized at the origin of the scalar manifold. This vacuum could be identified with the near horizon geometry of the D4-D8 brane system [9]. For the case of $n = 3$ vector multiplets and $G = SO(3)$, another non-trivial AdS$_6$ vacuum breaking the $SU(2)_R \times SO(3)$ symmetry to $SO(3)_{\text{diag}}$ and preserving the full $N = (1, 1)$ supersymmetry has been identified in [10].

In this paper we do not specify the gauge group upfront but instead follow the strategy of [11–15] in that we first determine the general conditions on the parameters of the gauged

---

[1]In fact, the $N = (1, 1)$ supergravity is the only gauged supergravity in $d = 6$ that admits maximally supersymmetric AdS$_6$ backgrounds [4]. This in turn is consistent with the known classification of the AdS superalgebras given in [5].

supergravity such that AdS_6 backgrounds which preserve all supercharges can exist. In half-maximal supergravities it is then possible to also give all possible gauge groups that can have such vacua. Concretely we find that the gauge group has to be of the form $G' \times G'' \subset SO(4,n)$ where $G' \subset SO(3,m)$ and $G'' \subset SO(1,n-m)$. In the AdS_6 vacua this gauge group is broken to its maximal compact subgroup $SO(3) \times H' \times H''$ where $H' \subset SO(m)$ and $H'' \subset SO(n-m)$. The $SO(3) \sim SU(2)$ factor precisely is the R-symmetry and it is gauged by three of the four graviphotons. Finally, we derive the necessary conditions for the existence of a supersymmetric moduli space near these vacua. For the case at hand we find that no moduli space is possible which is again consistent with the fact that the holographically dual SCFTs have no supersymmetric exactly marginal deformations.

In the AdS/CFT correspondence, AdS_6 vacua are also relevant for studying strongly coupled five-dimensional SCFTs arising from the dynamics of D4-D8 branes [9, 16]. The interpretation in terms of AdS_6 geometry in [17] inspired various studies considering gravity duals of these SCFTs including a recent generalization to quiver gauge theories in [18]. Finding AdS_6 solutions in type II and eleven dimensional supergravities also deserves detailed investigations.[2] In this paper, however, we stay in $d = 6$ throughout the analysis leaving the higher dimensional origins of these vacua for future work.

The paper is organized as follow. In section 2, we set the stage for our analysis and recall the relevant features of $N = (1,1)$ gauged supergravity. The conditions for the existence of maximally supersymmetric AdS_6 vacua are then derived in section 3, and the analysis of the moduli space is carried out in section 4. We finally end the paper by giving some conclusions and comments on the results in section 5.

2 $N = (1,1)$ gauged supergravity in six dimensions

In this section, we briefly review $N = (1,1)$ gauged supergravity coupled to n vector multiplets in order to set up the notation for the later analysis. More details on this gauged supergravity can be found in [7, 8]. We will follow most of the conventions in these two references.

The possible supermultiplets are the gravitational multiplet and n vector multiplets given respectively by

$$\left(e_\mu^a, \psi_\mu^A, A_\mu^\alpha, B_{\mu\nu}, \chi^A, \sigma\right) \qquad \text{and} \qquad \left(A_\mu, \lambda_A, \phi^\alpha\right)^I . \tag{2.1}$$

The bosonic fields of the supergravity multiplet are given by the graviton e_μ^a, the dilaton σ, four graviphotons A_μ^α, and a two-form field $B_{\mu\nu}$ while each vector multiplet contains a vector, A_μ, and four scalars, ϕ^α. Two sets of indices $\alpha, \beta, \ldots = 0,1,2,3$ and $I, J, \ldots = 1, \ldots, n$ label the $n + 4$ vector fields. Space-time and tangent space indices are denoted respectively by $\mu, \nu = 0, \ldots, 5$ and $a, b = 0, \ldots, 5$. We will also follow the mostly minus space-time signature $(+ - - - - -)$ of [7, 8].

The fermionic fields consist of two gravitini ψ_μ^A, two spin-$\frac{1}{2}$ fields χ^A and $2n$ gauginos λ_A^I. All of these fields and the supersymmetry parameter ϵ^A are eight-component

---

[2]See [19–22] for recent results along this direction and references therein.

pseudo-Majorana spinors and transform in the fundamental representation of the $\mathrm{SU}(2)_R \sim \mathrm{USp}(2)_R$ R-symmetry denoted by indices $A, B = 1, 2$.

The dilaton and the $4n$ scalars $\phi^{\alpha I}$ of the vector multiplets span the coset manifold

$$\mathbb{R}^+ \times \mathrm{SO}(4, n)/\mathrm{SO}(4) \times \mathrm{SO}(n). \tag{2.2}$$

The second factor can in turn be parametrized by the coset representative $L^\Lambda{}_\Sigma$ with $\Lambda, \Sigma, \ldots = 1, 2, \ldots, n + 4$. It is convenient to split the indices transforming under the compact group $\mathrm{SO}(4) \times \mathrm{SO}(n)$ as $\Lambda = (\alpha, I)$ and further under the $\mathrm{SO}(3)_R \times \mathrm{SO}(n)$ as $\Lambda = (0, r, I)$ with $r, s, \ldots = 1, 2, 3$. The $\mathrm{SO}(3)_R$ is identified with the diagonal subgroup of $\mathrm{SO}(3) \times \mathrm{SO}(3) \sim \mathrm{SO}(4)$. The coset representative can be accordingly decomposed as

$$L^\Lambda{}_\Sigma = (L^\Lambda{}_\alpha, L^\Lambda{}_I) = (L^\Lambda{}_0, L^\Lambda{}_r, L^\Lambda{}_I). \tag{2.3}$$

Furthermore, all of the $n+4$ vector fields will be collectively denoted by $A_\mu^\Lambda = (A_\mu^0, A_\mu^r, A_\mu^I)$. Being $\mathrm{SO}(4, n)$ matrices, the $L^\Lambda{}_\Sigma$ satisfy the relation

$$\eta_{\Lambda\Sigma} = L^0{}_\Lambda L^0{}_\Sigma + L^i{}_\Lambda L^i{}_\Sigma - L^I{}_\Lambda L^I{}_\Sigma \tag{2.4}$$

with $\eta_{\Lambda\Sigma} = (1, 1, 1, 1, -1, -1, \ldots, -1)$.

We now turn to the gauged version of this supergravity. The most complete gauged $N = (1, 1)$ supergravity up to now is given in [7, 8]. As in seven dimensions, the full $\mathrm{SO}(4, n)$ covariant formulation in terms of the embedding tensors has not been worked out yet although the corresponding components of the embedding tensor have been identified in [23] using the Kac-Moody approach. In this paper, we will restrict ourselves to the gauged supergravity constructed in [7, 8].

Gauging is implemented by making a particular subgroup G of $\mathrm{SO}(4, n)$ local such that the adjoint representation of G can be embedded in the fundamental representation, $\mathbf{n} + \mathbf{4}$, of $\mathrm{SO}(4, n)$, and $\eta_{\Lambda\Sigma}$ contains the Cartan-Killing form of the gauge group. Consistency with supersymmetry requires that the structure constants are totally anti-symmetric, i.e. $f_{\Lambda\Sigma\Pi} = f_{\Lambda\Sigma}{}^\Gamma \eta_{\Gamma\Pi} = f_{[\Lambda\Sigma\Pi]}$. In the embedding tensor formalism, this condition is called the linear constraint.

The $f_{\Lambda\Sigma}{}^\Gamma$ appear as structure constants in the gauge algebra

$$[T_\Lambda, T_\Sigma] = f_{\Lambda\Sigma}{}^\Gamma T_\Gamma \tag{2.5}$$

in which T_Λ are gauge generators. These structure constants must satisfy the Jacobi identity

$$f_{[\Sigma\Gamma}{}^\Delta f_{\Lambda]\Delta}{}^\Pi = 0 \tag{2.6}$$

which in the embedding tensor formalism is the so-called quadratic constraint. In general, this constraint comes from the requirement that the gauge generators, obtained from appropriate projections of the global symmetry generators by the embedding tensor, form a closed Lie algebra of the corresponding gauge group.

The bosonic Lagrangian with only the metric and scalars non-vanishing reads

$$e^{-1}\mathcal{L} = -\frac{1}{4}R + \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4}(P_\mu^{I0} P_{I0}^\mu + P_\mu^{Ir} P_{Ir}^\mu) - V, \tag{2.7}$$

where the scalar kinetic term is written in terms of the Maurer-Cartan one-forms

$$P^I_{\ 0} = (L^{-1})^I_{\ \Lambda}(dL^\Lambda_{\ 0} + f^\Lambda_{\ \Gamma\Pi}A^\Gamma L^\Pi_{\ 0}), \qquad P^I_{\ r} = (L^{-1})^I_{\ \Lambda}(dL^\Lambda_{\ r} + f^\Lambda_{\ \Gamma\Pi}A^\Gamma L^\Pi_{\ r}). \qquad (2.8)$$

The scalar potential V is given by

$$V = -5\left[\frac{1}{144}(Ae^\sigma + 6me^{-3\sigma}L_{00})^2 + \frac{1}{16}(B_ie^\sigma - 2me^{-3\sigma}L_{0i})^2\right]$$
$$+ \frac{1}{144}(Ae^\sigma - 18me^{-3\sigma}L_{00})^2 + \frac{1}{16}(B_ie^\sigma + 6me^{-3\sigma}L_{0i})^2 \qquad (2.9)$$
$$+ \frac{1}{4}(C^I_{\ t}C_{It} + 4D^I_{\ t}D_{It})e^{2\sigma} - m^2e^{-6\sigma}L_{0I}L^{0I}$$

where m is the mass of the two-form in the gravitational multiplet and we abbreviated

$$A = \epsilon^{rst}K_{rst}, \qquad B^i = \epsilon^{ijk}K_{jk0},$$
$$C_I^{\ t} = \epsilon^{trs}K_{rIs}, \qquad D_{It} = K_{0It}, \qquad (2.10)$$

with the "dressed" structure constants given by

$$K_{rst} = f_{\Lambda\Sigma\Pi}L^\Lambda_{\ r}(L^{-1})_s^{\ \Sigma}L^\Pi_{\ t}, \qquad K_{rs0} = f_{\Lambda\Sigma\Pi}L^\Lambda_{\ r}(L^{-1})_s^{\ \Sigma}L^\Pi_{\ 0},$$
$$K_{rIt} = f_{\Lambda\Sigma\Pi}L^\Lambda_{\ r}(L^{-1})_I^{\ \Sigma}L^\Pi_{\ t}, \qquad K_{0It} = f_{\Lambda\Sigma\Pi}L^\Lambda_{\ 0}(L^{-1})_I^{\ \Sigma}L^\Pi_{\ t}. \qquad (2.11)$$

The supersymmetry transformations of the fermions which will play an important role in the following analysis are given by

$$\delta\psi_{A\mu} = D_\mu\epsilon_A + S_{AB}\gamma_\mu\epsilon^B,$$
$$\delta\chi_A = \frac{i}{2}\gamma^\mu\partial_\mu\sigma\epsilon_A + N_{AB}\epsilon^B, \qquad (2.12)$$
$$\delta\lambda^I_A = -iP^{\ AB}_{ri}\sigma^r\partial_\mu\phi^i\gamma^\mu\epsilon_B + iP^I_{0i}\epsilon^{AB}\partial_\mu\phi^i\gamma^7\gamma^\mu\epsilon_B + M^I_{AB}\epsilon^B,$$

where the fermion-shift matrices are defined as

$$S_{AB} = \frac{i}{24}\left[Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB} - \frac{i}{8}\left[B_te^\sigma - 2me^{-3\sigma}(L^{-1})_{t0}\right]\gamma^7\sigma^t_{AB},$$

$$N_{AB} = \frac{1}{24}\left[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB} + \frac{1}{8}\left[B_te^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}\right]\gamma^7\sigma^t_{AB}, \qquad (2.13)$$

$$M^I_{AB} = (-C^I_{\ t} + 2i\gamma^7D^I_{\ t})e^\sigma\sigma^t_{AB} - 2me^{-3\sigma}(L^{-1})^I_{\ 0}\gamma^7\epsilon_{AB}.$$

In the present convention, the anti-symmetric matrix $\epsilon_{AB} = -\epsilon_{BA}$ is taken to be $\epsilon_{12} = \epsilon^{12} = 1$. The σ^t_{AB} matrices are related to the usual Pauli matrices $\sigma^{tA}_{\ \ B}$ by the relation[3]

$$\sigma^t_{AB} = \sigma^{tC}_{\ \ B}\epsilon_{CA}. \qquad (2.14)$$

Finally, the chirality matrix γ_7 is defined by

$$\gamma_7 = i\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5 \qquad (2.15)$$

with $\gamma_7^2 = -\mathbb{I}$ and $\gamma_7^T = -\gamma_7$.

---

[3]Note that $\sigma^t_{AB} = \sigma^t_{(AB)}$.

3 Maximally supersymmetric AdS$_6$ vacua

We now determine the maximally supersymmetric AdS$_6$ vacua preserving all sixteen supercharges. In order to do so, we impose that the following conditions vanish for all supercharges in the background

$$\langle\delta\psi_{\mu A}\rangle = 0\,, \qquad \langle\delta\chi_A\rangle = 0\,, \qquad \langle\delta\lambda_A^I\rangle = 0\,. \tag{3.1}$$

Due to the symmetries of $\sigma_{AB}^t = \sigma_{(AB)}^t$ and $\epsilon_{AB} = \epsilon_{[AB]}$, the linear independence of γ_7 and $\mathbb{I}$ and by using (2.12) and (2.13) we infer that the second and third equations imply

$$\langle Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}\rangle = 0, \tag{3.2}$$
$$\langle e^{-3\sigma}(L^{-1})^I{}_0\rangle = 0, \tag{3.3}$$
$$\langle B_t e^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}\rangle = 0, \tag{3.4}$$
$$\langle C^I{}_t\rangle = \langle D^I{}_t\rangle = 0\,. \tag{3.5}$$

From (2.10) we learn that the first condition in (3.5) is equivalent to

$$\epsilon^{trs}K_{rIs} = 0\,. \tag{3.6}$$

The second condition in (3.5) yields $K_{0It} = 0$ so that together we have

$$K_{rIs} = K_{0It} = 0\,. \tag{3.7}$$

Using (2.10) we can rewrite condition (3.2) as

$$\epsilon^{rst}K_{rst} = 18me^{-4\langle\sigma\rangle}\langle(L^{-1})_{00}\rangle \tag{3.8}$$

which is solved by

$$K_{rst} = g\epsilon_{rst} \tag{3.9}$$

for an arbitrary SU(2)$_R$ gauge coupling g. We can accordingly determine the background value of the dilaton by inserting (3.9) into (3.8)

$$e^{-4\langle\sigma\rangle}\langle(L^{-1})_{00}\rangle = \frac{g}{3m}\,. \tag{3.10}$$

The remaining conditions (3.3) and (3.4) give

$$\langle(L^{-1})^I{}_0\rangle = 0\,, \qquad \langle B_t\rangle = -6me^{-4\langle\sigma\rangle}\langle(L^{-1})_{t0}\rangle\,. \tag{3.11}$$

Using the component-$(0I)$ and -$(0i)$ of the relation (2.4) and the identity $L^{-1} = \eta L^T\eta$, we find that $L_{0I} = 0$ implies $L_{0i} = 0$ and thus

$$\langle B_t\rangle = 0\,. \tag{3.12}$$

Using the definition of B_t given in (2.10) we thus arrive at

$$K_{jk0} = 0\,. \tag{3.13}$$

By taking the (00)-component of the relation (2.4), we find that $L_{0I} = L_{0i} = 0$ also implies $L_{00} = 1$. Inserting the results obtained so far into (2.9) we conclude that the background value of the scalar potential (related to the cosmological constant) in an AdS$_6$ vacuum is given by

$$\langle V \rangle = -20m^2 \left(\frac{g}{3m} \right)^{\frac{3}{2}} . \tag{3.14}$$

We see that AdS vacua do not exist for $m = 0$ as already pointed out in [7, 8].[4] This is very similar to AdS backgrounds of half-maximal supergravities in seven dimensions [13, 24, 25]. Note also that by shifting the value of $\langle \sigma \rangle$ we can choose $g = 3m$ as in [7, 8].

In order to continue let us recall that we are left with the unconstrained structure constants

$$K_{rst} , \qquad K_{rIJ} , \qquad K_{0IJ} , \qquad K_{IJK} , \tag{3.15}$$

whose choice specify the particular supergravity at hand. We can now use the quadratic constraint to determine the corresponding gauge groups. These are the gauge groups which can occur in the supergravities that admit maximally supersymmetric AdS$_6$ vacua. For the case at hand the quadratic constraint reduces to the usual Jacobi identity given in (2.6).

As a warm up let us first consider the simple situation where $K_{rIJ} = K_{0IJ} = K_{IJK} = 0$ and we only have K_{rst} non-zero. In this case, equation (2.6) reduces to the Jacobi identity of an SO(3) algebra with the structure constants $K_{rst} = g\epsilon_{rst}$. We then simply recover the pure $F(4)$ gauged supergravity with an SU(2) $\sim$ SO(3) gauge group constructed in [6].

For $K_{rIJ} = K_{0IJ} = 0$ but $K_{IJK} \neq 0$, the condition (2.6) gives rise to two separate Jacobi identities for K_{rst} and K_{IJK} which correspond to two commuting compact groups. The gauge group is accordingly SO(3) $\times H$ with $H \subset$ SO(n) and compact. This gauge group and the resulting AdS$_6$ vacuum together with the dual five-dimensional SCFT have already been studied in [7, 8].

As a next step let us also take $K_{rIJ} \neq 0$ but still have $K_{0IJ} = 0$. In this case the SO(3)-singlet graviphoton A^0 decouples from all other gauge bosons. This is very similar to the seven-dimensional case studied in [13] where the gauge groups are embedded in SO(3, n) $\subset$ SO(4, n). If one additionally assumes that the gauge group is semi-simple one can in fact list all possibilities. The Cartan-Killing form of these gauge groups must be embeddable in the SO(3, n) invariant tensor $\eta = (\delta_{rs}, -\delta_{IJ})$ which imposes a strong constraint. Furthermore, the existence of supersymmetric AdS$_6$ vacua requires that the gauge groups must contain SO(3) as a subgroup. The only possible semisimple gauge groups are then given by

$$\tilde{G} \times H \tag{3.16}$$

where $\tilde{G} = $ SO(3), SO(3, 1) or SL(3, $\mathbb{R}$) and $H \subset$ SO($n + 3 - \dim \tilde{G}$) is a compact group.

We finally consider the most general case with all structure constants in (3.15) non-zero. Follow a similar analysis in [14] we introduce the gauge generators embedded in SO(4, n) as

$$(T_\Lambda)_\Gamma{}^\Pi = f_\Lambda{}^{\Sigma\Delta} (t_{\Sigma\Delta})_\Gamma{}^\Pi = f_{\Lambda\Gamma}{}^\Pi \tag{3.17}$$

---

[4]Recall that m is the mass parameter of the two-form in the gravitational multiplet.

where $(t_{\Sigma\Delta})_\Gamma{}^\Pi = \delta_{[\Sigma}^\Pi \eta_{\Delta]\Gamma}$ are SO$(4,n)$ generators in the vector representation. Splitting the indies $\Lambda = (0, i, I)$ decomposes the gauge generators as

$$(T_0)_\Gamma{}^\Pi = f_{0\Gamma}{}^\Pi, \qquad (T_i)_\Gamma{}^\Pi = f_{i\Gamma}{}^\Pi, \qquad (T_I)_\Gamma{}^\Pi = f_{I\Gamma}{}^\Pi, \tag{3.18}$$

which couple to the vector fields A^0, A^i and A^I, respectively.

It is more convenient to write down the various independent components of the Jacobi identity. They read

$$K_{[ij}{}^l K_{k]l}{}^m = 0\,, \tag{3.19}$$

$$K_{iJ}{}^I K_{IK}{}^j + K_{Ki}{}^I K_{IJ}{}^j + K_{JK}{}^r K_{ri}{}^j = 0\,, \tag{3.20}$$

$$K_{iJ}{}^I K_{IK}{}^L + K_{Ki}{}^I K_{IJ}{}^L + K_{JK}{}^I K_{Ii}{}^L = 0\,, \tag{3.21}$$

$$K_{0I}{}^J K_{Jj}{}^K + K_{Ij}{}^J K_{J0}{}^K = 0\,, \tag{3.22}$$

$$K_{IJ}{}^K K_{K0}{}^L + K_{0I}{}^K K_{KJ}{}^L + K_{J0}{}^K K_{KI}{}^L = 0\,, \tag{3.23}$$

$$K_{[IJ}{}^0 K_{K]0}{}^M + K_{[IJ}{}^r K_{K]r}{}^M + K_{[IJ}{}^L K_{K]L}{}^M = 0\,. \tag{3.24}$$

The first two relations (3.19), (3.20) imply that the SO(3) generators T_i have non-vanishing elements in both SO(3) and SO(n) blocks. We therefore split the indices $I, J, K, \dots$ into two sets $\hat{I}, \hat{J}, \hat{K} = 1, \dots, m$ and $\tilde{I}, \tilde{J}, \tilde{K} = 1, \dots n - m$ such that only the $\hat{I}, \hat{J}, \hat{K}$ indices mix with r, s, t indices. Or, in other word, we have $K_{r\hat{I}\hat{j}} \neq 0$ and $K_{r\tilde{I}\tilde{j}} = 0$. With this convention the SO(3) generators take the form

$$T_i = \begin{pmatrix} 0 & & & \\ & K_{ij}{}^k & & \\ & & K_{i\hat{j}}{}^{\hat{K}} & \\ & & & 0_{n-m} \end{pmatrix}, \tag{3.25}$$

where 0_n indicates an $n \times n$ zero matrix.

The relation (3.22) corresponds to $[T_i, T_0] = 0$ and thus T_0 and T_i cannot have common I, J, K indices or equivalently $K_{0\hat{I}\hat{j}} = K_{0\tilde{I}\hat{j}} = 0$. This determines the T_0 generator to be

$$T_0 = \begin{pmatrix} 0 & & & \\ & 0_3 & & \\ & & 0_m & \\ & & & K_{0\tilde{J}}{}^{\tilde{K}} \end{pmatrix}. \tag{3.26}$$

Equation (3.21) and the $(\hat{I}, \hat{J}, \hat{K}, \hat{M})$ components of relation (3.24) imply that the $T_{\hat{I}}$ generators are given by

$$T_{\hat{I}} = \begin{pmatrix} 0 & & & \\ & 0_3 & K_{\hat{I}r}{}^{\hat{K}} & \\ & K_{\hat{I}\hat{j}}{}^r & K_{\hat{I}\hat{j}}{}^{\hat{K}} & \\ & & & 0_{n-m} \end{pmatrix}. \tag{3.27}$$

Therefore, the $(T_i, T_{\hat{I}})$ generators together form a non-compact group $G' \subset$ SO$(3, m)$, $m \leq n$.

Finally, the relation (3.23) and the $(\tilde{I}, \tilde{J}, \tilde{K}, \tilde{M})$ components of relation (3.24) determine the structure of $T_{\tilde{I}}$ to be

$$T_{\tilde{I}} = \begin{pmatrix} 0 & & & K_{\tilde{I}0}{}^{\tilde{K}} \\ & 0_3 & & \\ & & 0_m & \\ K_{\tilde{I}\tilde{J}}{}^{0} & & & K_{\tilde{I}\tilde{J}}{}^{\tilde{K}} \end{pmatrix}. \tag{3.28}$$

These generators together with T_0 form another non-compact group $G'' \subset \mathrm{SO}(1, n-m)$. We then conclude that the general gauge group admitting maximally supersymmetric AdS$_6$ vacua take the form

$$G' \times G'' \tag{3.29}$$

where $G' \subset \mathrm{SO}(3, m)$ and $G'' \subset \mathrm{SO}(1, n-m)$. In an AdS$_6$ background, the gauge group is broken to its maximal compact subgroup $\mathrm{SO}(3) \times H' \times H''$ in which $H' \subset \mathrm{SO}(m)$ and $H'' \subset \mathrm{SO}(n-m)$.

To confirm this, we inspect the massive vector fields arising from the above symmetry breaking. Defining $A^{\hat{I}} = (L^{-1})^{\hat{I}}{}_\Lambda A^\Lambda$ and $A^{\tilde{I}} = (L^{-1})^{\tilde{I}}{}_\Lambda A^\Lambda$, we find that various components of the Maurer-Cartan one-form $P^I{}_\alpha$ are given by

$$\begin{aligned}
P^{\hat{I}}{}_0 &= (L^{-1})^{\hat{I}}{}_\Lambda dL^\Lambda{}_0, & P^{\hat{I}}{}_r &= (L^{-1})^{\hat{I}}{}_\Lambda dL^\Lambda{}_r + K^{\hat{I}}{}_{\hat{j}r} A^{\hat{J}}, \\
P^{\tilde{I}}{}_0 &= (L^{-1})^{\tilde{I}}{}_\Lambda dL^\Lambda{}_0 + K^{\tilde{I}}{}_{\tilde{j}0} A^{\tilde{J}}, & P^{\tilde{I}}{}_r &= (L^{-1})^{\tilde{I}}{}_\Lambda dL^\Lambda{}_r.
\end{aligned} \tag{3.30}$$

By computing the scalar kinetic terms, we can indeed see that there is precisely one massive vector field corresponding to each non-compact generators $K_{\hat{I}\hat{j}}{}^r$ and $K_{\tilde{I}\tilde{j}}{}^0$. These massive vectors correspond to the broken non-compact generators of the full gauge group.

4 Moduli space of supersymmetric AdS$_6$ vacua

In this section, we determine the flat directions of the scalar potential V which preserve all 16 supercharges. These are the moduli of the AdS$_6$ backgrounds corresponding to supersymmetric marginal deformations of the five-dimensional superconformal field theories dual to the AdS$_6$ vacua identified in the previous section.

Similar to the analysis of [12–14], a necessary condition for the existence of these moduli can be determined by considering possible deformations of the supersymmetry conditions (3.1) near the AdS$_6$ vacua. By varying the conditions in (3.5), we find

$$\delta(e^{4\sigma} A) = 4\langle A \rangle \delta\sigma + e^{4\langle\sigma\rangle} \delta A = 0, \tag{4.1}$$

$$\delta C^I{}_t = \delta D^I{}_t = \delta B_t = 0. \tag{4.2}$$

We now introduce a parametization of the variation of the coset representative with respect to the $4n$ scalars $\phi^{\alpha I}$

$$\delta L^\Lambda{}_\alpha = \langle L^\Lambda{}_I \rangle \delta\phi^{\alpha I}, \qquad \delta L^\Lambda{}_I = \langle L^\Lambda{}_\alpha \rangle \delta\phi^{\alpha I} \tag{4.3}$$

and their inverse

$$\delta(L^{-1})^\Lambda{}_\alpha = -\langle (L^{-1})^\Lambda{}_I \rangle \delta\phi^{\alpha I}, \qquad \delta(L^{-1})^\Lambda{}_I = -\langle (L^{-1})^\Lambda{}_\alpha \rangle \delta\phi^{\alpha I}. \tag{4.4}$$

Using these relations and the decomposition of indices $\alpha = (0, r)$, we find

$$\delta L^\Lambda{}_0 = \langle L^\Lambda{}_I \rangle \delta\phi^{0I}, \qquad \delta L^\Lambda{}_i = \langle L^\Lambda{}_I \rangle \delta\phi^{iI}, \qquad \delta L^\Lambda{}_I = \langle L^\Lambda{}_0 \rangle \delta\phi^{0I} + \langle L^\Lambda{}_r \rangle \delta\phi^{rI} \tag{4.5}$$

and

$$\delta(L^{-1})^\Lambda{}_0 = -\langle (L^{-1})^\Lambda{}_I \rangle \delta\phi^{0I}, \qquad\qquad \delta(L^{-1})^\Lambda{}_i = -\langle (L^{-1})^\Lambda{}_I \rangle \delta\phi^{iI},$$
$$\delta(L^{-1})^\Lambda{}_I = -\langle (L^{-1})^\Lambda{}_0 \rangle \delta\phi^{0I} - \langle (L^{-1})^\Lambda{}_r \rangle \delta\phi^{rI}. \tag{4.6}$$

With the help of these relations, we can rewrite the conditions (4.1) and (4.2) as

$$0 = \delta(e^{4\sigma} A) = 4e^{4\langle\sigma\rangle} \langle A \rangle \delta\sigma + 3e^{4\langle\sigma\rangle} \langle C_{Ir} \rangle \delta\phi^{rI}, \tag{4.7}$$

$$0 = \delta B_t = \langle C_{It} \rangle \delta\phi^{0I} + 2\epsilon_{rtk} \langle D_{Ik} \rangle \delta\phi^{rI}, \tag{4.8}$$

$$0 = \delta C^I{}_t = 2\epsilon^{trs} K_{[rIJ} \delta\phi_{s]J} - \epsilon^{trs} K_{r0s} \delta\phi^{0I} - \epsilon^{trs} K_{ris} \delta\phi^{iI}, \tag{4.9}$$

$$0 = \delta D^I{}_t = K_{0It} \delta\phi^{0I} + K_{0IJ} \delta\phi^{tJ} - K_{0rt} \delta\phi^{rI} \tag{4.10}$$

where

$$K_{0IJ} = f_{\Lambda\Sigma\Pi} L^\Lambda{}_0 (L^{-1})_I{}^\Sigma L^\Pi{}_J, \qquad K_{rIJ} = f_{\Lambda\Sigma\Pi} L^\Lambda{}_r (L^{-1})_I{}^\Sigma L^\Pi{}_J. \tag{4.11}$$

Using the AdS$_6$ conditions (3.5), (3.7) and (3.13) obtained in the previous section, we find

$$\delta\sigma = 0, \quad K_{0\tilde{I}\tilde{J}} \delta\phi_{t\tilde{J}} = 0, \quad K_{rst} \delta\phi_{t\tilde{I}} = 0, \quad 2\epsilon^{rst} K_{[r\hat{I}\hat{j}} \delta\phi_{s]\hat{j}} + K_{rst} \delta\phi_{t\hat{I}} = 0. \tag{4.12}$$

From these conditions, we immediately obtain $\delta\phi_{t\tilde{I}} = 0$ for $K_{rst} \neq 0$.

The last equation in (4.12) is similar to the one considered in [12, 13], and it has been shown in [12] that this equation has general solutions of the form

$$\delta\phi_{s\hat{I}} = K_{s\hat{I}\hat{j}} \lambda^{\hat{J}}. \tag{4.13}$$

The remaining scalars that are not fixed by the above conditions are $\delta\phi_{0\tilde{I}}$. We can readily recognize that $\delta\phi_{s\hat{I}}$ and $\delta\phi_{0\tilde{I}}$ correspond to Goldstone bosons of the symmetry breaking $G' \times G'' \to \mathrm{SO}(3) \times H' \times H''$, with $H' \subset \mathrm{SO}(m)$ and $H'' \subset \mathrm{SO}(n-m)$ in the AdS$_6$ vacuum. These massless scalars are eaten by the massive gauge fields mentioned in the previous section. Thus, all of the flat directions correspond to Goldstone bosons and no moduli exist. This in turn is consistent with the fact that there are no marginal deformations preserving all supersymmetry in the dual five-dimensional SCFTs.

5 Conclusions

In this paper, we have analyzed the general conditions for the existence of maximally supersymmetric AdS$_6$ vacua in the $N = (1, 1)$ half-maximal gauged supergravities in six dimensions. We have found that three of the graviphotons have to gauge an $\mathrm{SU}(2)_R$ R-symmetry while the forth one can be used to gauge a commuting non-compact group.

The fact that the $SU(2)_R$ R-symmetry must be gauged is similar to the results in $d = 4, 6, 7$. This is in general a necessary condition for the existence of AdS vacua as shown in [4]. It is also consistent with the important role played by the corresponding R-symmetry in the dual field theories [16]. Furthermore, all vacua we have identified have no flat directions which preserve all supercharges corresponding to the absence of supersymmetric exactly marginal deformations in the dual five-dimensional SCFTs.

We end the paper by briefly giving some comments on the $\mathbb{R}^+ \times SO(4, n)$ covariant formulation in term of the embedding tensor. As shown in [23], there are two components of the embedding tensor given by ξ^Λ and $f_{\Lambda\Sigma\Gamma}$ as well as a massive deformation of the two-form field. The ξ^Λ is involved in gauging of the $\mathbb{R}^+$ factor. Due to many similarities between the six-dimensional $N = (1, 1)$ gauged supergravity considered here and the $N = 2$ gauged supergravity in seven dimensions, we expect that the $\mathbb{R}^+$ gauging and the massive deformation could not be turned on simultaneously. Therefore, the existence of maximally supersymmetric AdS$_6$ vacua would require $\xi^\Lambda = 0$ as shown in [13] for the seven-dimensional case. It would be of particular interest to explore this issue in particular to construct the complete gauging of $N = (1, 1)$ supergravity in the embedding tensor formulation.

Acknowledgments

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[2] O. Aharony, B. Kol and S. Yankielowicz, *On exactly marginal deformations of N = 4 SYM and type IIB supergravity on $AdS_5 \times S^5$*, *JHEP* **06** (2002) 039 [hep-th/0205090] [INSPIRE].

[3] B. Kol, *On conformal deformations*, *JHEP* **09** (2002) 046 [hep-th/0205141] [INSPIRE].

[4] J. Louis and S. Lüst, *Classification of maximally supersymmetric backgrounds in supergravity theories*, arXiv:1607.08249 [INSPIRE].

[5] W. Nahm, *Supersymmetries and their Representations*, *Nucl. Phys.* **B 135** (1978) 149 [INSPIRE].

[6] L.J. Romans, *The F(4) Gauged Supergravity in Six-dimensions*, *Nucl. Phys.* **B 269** (1986) 691 [INSPIRE].

[7] R. D'Auria, S. Ferrara and S. Vaula, *Matter coupled F(4) supergravity and the AdS_6/CFT_5 correspondence*, *JHEP* **10** (2000) 013 [hep-th/0006107] [INSPIRE].

[8] L. Andrianopoli, R. D'Auria and S. Vaula, *Matter coupled F(4) gauged supergravity Lagrangian*, *JHEP* **05** (2001) 065 [hep-th/0104155] [INSPIRE].

[9] A. Brandhuber and Y. Oz, *The D4-D8 brane system and five-dimensional fixed points*, *Phys. Lett.* **B 460** (1999) 307 [hep-th/9905148] [INSPIRE].

[10] P. Karndumri, *Holographic RG flows in six dimensional F(4) gauged supergravity*, *JHEP* **01** (2013) 134 [*Erratum ibid.* **06** (2015) 165] [arXiv:1210.8064] [INSPIRE].

[11] S. de Alwis, J. Louis, L. McAllister, H. Triendl and A. Westphal, *Moduli spaces in AdS_4 supergravity*, *JHEP* **05** (2014) 102 [arXiv:1312.5659] [INSPIRE].

[12] J. Louis and H. Triendl, *Maximally supersymmetric AdS_4 vacua in N = 4 supergravity*, *JHEP* **10** (2014) 007 [arXiv:1406.3363] [INSPIRE].

[13] J. Louis and S. Lüst, *Supersymmetric AdS_7 backgrounds in half-maximal supergravity and marginal operators of (1,0) SCFTs*, *JHEP* **10** (2015) 120 [arXiv:1506.08040] [INSPIRE].

[14] J. Louis, H. Triendl and M. Zagermann, $\mathcal{N} = 4$ *supersymmetric AdS_5 vacua and their moduli spaces*, *JHEP* **10** (2015) 083 [arXiv:1507.01623] [INSPIRE].

[15] J. Louis and C. Muranaka, *Moduli spaces of AdS_5 vacua in $\mathcal{N} = 2$ supergravity*, *JHEP* **04** (2016) 178 [arXiv:1601.00482] [INSPIRE].

[16] N. Seiberg, *Five-dimensional SUSY field theories, nontrivial fixed points and string dynamics*, *Phys. Lett.* **B 388** (1996) 753 [hep-th/9608111] [INSPIRE].

[17] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, *AdS_6 interpretation of 5D superconformal field theories*, *Phys. Lett.* **B 431** (1998) 57 [hep-th/9804006] [INSPIRE].

[18] O. Bergman and D. Rodriguez-Gomez, *5d quivers and their AdS_6 duals*, *JHEP* **07** (2012) 171 [arXiv:1206.3503] [INSPIRE].

[19] A. Passias, *A note on supersymmetric AdS_6 solutions of massive type IIA supergravity*, *JHEP* **01** (2013) 113 [arXiv:1209.3267] [INSPIRE].

[20] F. Apruzzi, M. Fazzi, A. Passias, D. Rosa and A. Tomasiello, *AdS_6 solutions of type-II supergravity*, *JHEP* **11** (2014) 099 [*Erratum ibid.* **05** (2015) 012] [arXiv:1406.0852] [INSPIRE].

[21] H. Kim, N. Kim and M. Suh, *Supersymmetric AdS_6 Solutions of Type IIB Supergravity*, *Eur. Phys. J.* **C 75** (2015) 484 [arXiv:1506.05480] [INSPIRE].

[22] E. D'Hoker, M. Gutperle and C.F. Uhlemann, *Holographic duals for five-dimensional superconformal quantum field theories*, arXiv:1611.09411 [INSPIRE].

[23] E.A. Bergshoeff, J. Gomis, T.A. Nutma and D. Roest, *Kac-Moody Spectrum of (Half-)Maximal Supergravities*, *JHEP* **02** (2008) 069 [arXiv:0711.2035] [INSPIRE].

[24] P. Karndumri, *RG flows in 6D N = (1, 0) SCFT from* SO(4) *half-maximal 7D gauged supergravity*, *JHEP* **06** (2014) 101 [arXiv:1404.0183] [INSPIRE].

[25] P. Karndumri, *Noncompact gauging of N = 2 7D supergravity and AdS/CFT holography*, *JHEP* **02** (2015) 034 [arXiv:1411.4542] [INSPIRE].