Transverse-momentum-dependent quark splitting functions in k_T-factorization: real contributions

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ABSTRACT: We calculate transverse momentum dependent quark splitting kernels P_{gq} and P_{qq} within k_T-factorization, completing earlier results which concentrated on gluon splitting functions P_{gg} and P_{qg}. The complete set of splitting kernels is an essential requirement for the formulation of a complete set of evolution equations for transverse momentum dependent parton distribution functions and the development of corresponding parton shower algorithms.

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Contents

1 Introduction

The essential theoretical input for experimental findings at the Large Hadron Collider are parton distribution functions (PDFs) which describe momentum distributions of partons in the colliding hadrons in the presence of a hard scale. Together with factorization theorems and hard coefficient functions, PDFs allow to predict new phenomena and to describe existing data. A lot of recent activity in theory and phenomenology of QCD is devoted to so called transverse-momentum-dependent parton distribution functions (TMD PDFs) and TMD factorization (for a review we refer the Reader to [1]). While a rigorous formulation of TMD factorization, valid for all kinematic regions, is still to be achieved (see e.g. [2]), a definition of TMD parton distributions is possible for specific regions of phase space, usually characterized by a hierarchy of scales [3–6]. One of those regions is the high-energy or small-x limit of perturbative QCD, characterized by the hierarchy $\sqrt{s} \gg M \gg \Lambda_{\mathrm{QCD}}$, where $\sqrt{s}$ denotes the center-of-mass energy of the process, M the hard scale of the perturbative event, and Λ_{QCD} the QCD characteristic scale of the order of a few hundred MeV. The underlying theoretical framework for TMD PDFs in this kinematic limit is usually referred to as k_T-factorization or high-energy factorization [7]. During the recent years various hard processes, in particular those associated with the forward region of LHC detectors, characterized by large rapidities, have been studied within the k_T-factorization framework, such as forward jet and forward b-jet production [8–10] and forward Z-production [11–13].

In the following we are in particular interested in the evolution of TMD PDFs, which depends on the parton's longitudinal momentum fraction x, its transverse momentum k_T, and the external hard scale M. An evolution equation which has these elements and is valid in angular ordered phase space for gluon emission is provided by the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) equation [14–17]. The key element of the evolution kernel of

the CCFM equation is the P_{gg} splitting function. At leading order it contains only the most singular pieces at low $z \to 0$ and large $z \to 1$ and appropriate form factors which resum virtual and unresolved real emissions in respectively low and large z regions.

The CCFM equation is restricted to the resummation of purely gluonic emissions in low x and large x domain. In particular this implies that the medium-x behavior of CCFM is not accurate and the formal medium-z limit of the CCFM equation is incomplete. One of the observations based on the Monte-Carlo implementation [18] of the CCFM equation is that the lack of such contributions leads indeed to non-negligible effects. Performing a fit to the proton structure function F_2 at all values of x, it is likely that the gluon contribution is enhanced in regions where quarks in the evolution would contribute. While for inclusive observables, such as the structure function F_2, the overall fit turns out to be satisfactory, see e.g. [19], the predictions based on the gluon density are not satisfactory for exclusive observables, see e.g. [9]. While it is difficult to pinpoint the exact reason for this deficiency, DGLAP resummation definitely suggests that decoupled evolution of quarks and gluons is insufficient. This is further supported by application of the Kutak-Sapeta (KS) gluons densities [20, 21] which account for quark contribution in the evolution [22] and describes production of dijets in p+p collisions at LHC reasonably well [20, 23]. In order to be able to apply CCFM evolution successfully and to provide full parton shower Monte-Carlo description within CCFM, the ultimate goal must be therefore to arrive at a coupled system of equations which in turn requires a full set of k_T-dependent splitting functions [24].

To arrive at a complete and consistent set of evolution equations, it is further necessary to include — apart from the quark splitting functions P_{gq} and P_{qq} — non-singular terms of the P_{gg} splitting function since these corrections are of the same order beyond leading order (LO) CCFM, i.e. beyond large- and small-z enhanced contributions. Note that in [18] it has been observed that inclusion of non-singular pieces of the DGLAP gluon splitting function into CCFM evolution strongly affects the solution of the evolution equation. One may therefore conclude that the effect of quarks in the evolution will be similarly significant.

A first step into this direction has undertaken in [12], where the TMD gluon-to-quark splitting kernel P_{qg} obtained in [25] has been used to define a TMD sea-quark density within k_T-factorization. In the following we extend this result by calculating as a start the unintegrated real emissions kernels for quark-to-quark and quark-to-gluon splitting functions.

From a technical point of view the determination of TMD splitting kernels is based on a generalization of the high energy factorization approach of Catani and Hautmann [25], which itself is based on the formulation of DGLAP evolution in terms of a two-particle-irreducible (2PI) expansion [26] (for overview and recent applications of the method see [27–30])). To guarantee gauge invariance in presence of off-shell particles we follow the proposal made in [12] and make use of the effective action formulation of the high energy factorization in terms of reggeized quarks and gluons [31, 32]. In the case of the gluon channel, consistency of this formalism has been verified up to the 2-loop level through explicit calculations of the higher-order corrections [33–37] and has been recently used to determine the complete next-to-leading order corrections to the jet-gap-jet impact factor [38–40].

The outline of the present paper is the following: in section 2 we give a comprehensive review of the results of [12] and explain the strategy of our calculations. In section 3

we determine TMD splitting functions working in the physical light-cone gauge, following closely the setup of [25, 26]. In section 4 we provide an extension of this formalism which makes the gauge invariance of our result explicit, despite of the presence of the off-shell legs in the matrix elements. In section 6 we summarize our results and discuss directions for future research.

2   The method

We start our presentation with a short review of the results of [12, 25] which allowed for the definition of the TMD P_{qg} splitting function and eventually of the sea-quark density. The derivation follows two steps:

a) in [25] a TMD splitting function has been determined to construct a high-energy resummed collinear sea-quark density. Its derivation is based on the two-particle-irreducible (2PI) expansion of [26]. To identify the TMD splitting function, one employs high-energy factorization of the 2PI kernel into a TMD dependent gluon-to-quark splitting, i.e. the TMD splitting function, and the BFKL Green's function, which achieves a resummation of small x logarithms. To obtain the small-x resummed sea-quark distribution, the TMD splitting function is combined with the BFKL Green's function and integrated over the transverse sea-quark momentum, following the conventions of [26].

b) in [12] the limitation to the transverse-momentum-independent sea-quark distributions has been relaxed. To ensure gauge invariance in the presence of off-shell splitting kernels, factorization of the process $qg^* \to qZ$ in the high-energy limit as realized by the reggeized quark formalism [31, 41] has been employed. Generalizing the reggeized quark formalism to finite energies, while taking care of maintaining gauge invariance, it was then possible to factorize the $qg^* \to qZ$ matrix element into a TMD coefficient $qq^* \to Z$ and the TMD gluon-to-quark splitting function of [25]. In particular, combining the TMD gluon-to-quark splitting function with the CCFM resummed TMD gluon distribution, a definition of a TMD sea-quark distribution has been achieved.

In the following we generalize these results to the quark-to-gluon and quark-to-quark splittings, employing the two-step procedure outlined above: we first define the splitting functions within the 2PI expansion of [25, 26] and then generalize our results to the fully off-shell splittings with full dependence on the transverse momentum. Before turning to the derivation we would like to point out a slight extension of the result of [12]. While [12] concentrates on factorization of a particular process, namely $qg^* \to qZ$, one can easily show that the resulting matrix elements and TMD splitting functions are process-independent. To this end we recall the details of the high-energy factorization of the $qg^* \to qZ$ matrix element: within the reggeized quark formalism, the entire process is described using a single diagram, figure 1, with the $qq^* \to Z$ and $g^*q^* \to q$ sub-amplitudes connected by reggeized
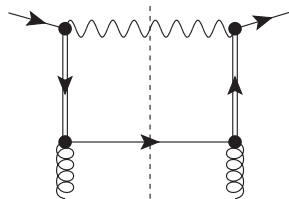
Figure 1. The $g^*q \to Zq$ process within the reggeized quark formalism. Double lines with arrow indicate the effective reggeized quark exchange in the t-channel.

quark propagators,

$$
\left. \Downarrow \right|_q = \frac{(\not{n}\not{p})_{\beta\alpha}}{2p \cdot n} \cdot \frac{i \cdot \not{q}}{q^2 + i\epsilon}
\qquad\qquad
\left. \Uparrow \right|_q = \frac{(\not{p}\not{n})_{\alpha\beta}}{2p \cdot n} \cdot \frac{i \cdot \not{q}}{q^2 + i\epsilon}.
\tag{2.1}
$$

While in the strict high-energy limit the t-channel four-momentum is purely transverse, $q^2 = -\boldsymbol{q}^2$, generalizations to finite energies require to keep the full momentum dependence. The momenta p and n are light-cone momenta $p^2 = n^2 = 0$ associated with the almost light-like momenta of scattering particles normalized to $2p \cdot n = s$, with s the center-of-mass energy of the hadronic process. In e.g. deep-inelastic scattering, n would be associated with the virtual photon and p with the probed hadron. While (generalized) reggeized quark propagators carry at first explicit spin indices and therefore correlate the $qq^* \to Z$ and $g^*q^* \to q$ sub-amplitudes, it is possible to rewrite the high-energy projectors for the cross-section using

$$
\frac{(\not{n}\not{p})_{\beta_1\alpha_1}(\not{p}\not{n})_{\alpha_2\beta_2}}{p \cdot n} = \not{p}_{\alpha_1\alpha_2}\not{n}_{\beta_1\beta_2} + (\gamma_5\not{p})_{\alpha_1\alpha_2}(\gamma_5\not{n})_{\beta_1\beta_2}.
\tag{2.2}
$$

For helicity independent input, the second term can be neglected and one remains with the projector $\not{p}_{\alpha_1\alpha_2}\not{n}_{\beta_1\beta_2}$ which then only contracts the Dirac indices of the $qq^* \to Z$ and $g^*q^* \to q$ sub-amplitudes respectively and therefore leads to a complete factorization of both processes.

3 Splitting functions from the 2 PI expansion in the axial gauge

The decomposition into 2PI diagrams as introduced in [26] is based on the use of axial i.e. light-cone gauge, which allows to analyze collinear singularities on the graph-by-graph basis [42], in contrast to covariant gauges where such a rule is broken. Following [25], we will obtain TMD splitting functions which complete the set of already available evolution kernels. Unlike the case of the gluon-to-quark splitting treated in [25], the resulting splitting kernels have no direct definition as the coefficient of the BFKL Green's function (or it is equivalent in the case of t-channel quark exchange). While the TMD quark-to-quark splitting can be identified as a certain next-to-leading order contributions to the high-energy resummed non-singlet P_{qq} DGLAP splitting function, the TMD quark-to-gluon
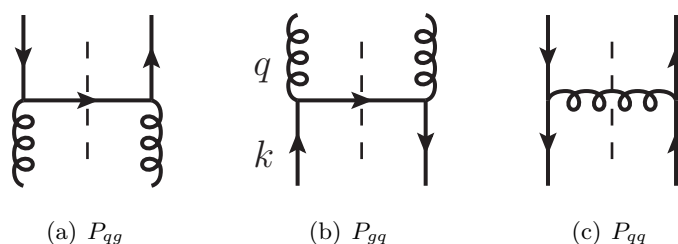
Figure 2. Matrix elements for the determination of splitting functions. Lower (incoming) lines carry always momentum k, upper (outgoing) lines carry momentum q.

splitting is suppressed by a power of x w.r.t. the leading logarithmic small-x resummed P_{gq} DGLAP splitting function. Nevertheless it is possible to attempt a definition of such quantities as matrix elements of reggeized quarks and conventional QCD degrees of freedom in light-cone gauge.

Following the framework set by [25, 26], the starting point for the definition of TMD splitting functions requires determination of the corresponding TMD splitting kernels,

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 - q^2) \mathbb{P}_{j,\,\mathrm{in}} \otimes \hat{K}_{ij}^{(0)}(q,k) \otimes \mathbb{P}_{i,\,\mathrm{out}}. \tag{3.1}$$

Here $\hat{K}_{ij}^{(0)}$, $i,j = q,g$ denotes the actual matrix element, describing the transition of parton j to parton i, see figure 2, which is defined to include the propagators of outgoing lines. In case of gluons, these propagators are taken in $n \cdot A = 0$ light-cone gauge; a similar statement applies to the polarization of real emitted gluons. $\mathbb{P}_{i,\,\mathrm{in/out}}$ are on the other hand semi-projectors on incoming and outgoing lines. The symbol $\otimes$ represents contraction of indices and summation. μ_F denotes the factorization and dimensional regularization in $d = 4 + 2\epsilon$ dimensions is employed with μ^2 the dimensional regularization scale. The Sudakov parametrization for incoming and outgoing momenta, k and q (see figure 2), reads

$$k^\mu = yp^\mu + k_\perp^\mu, \qquad q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \mathbf{q}^2}{2x\,p\cdot n}n^\mu, \qquad \tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}, \tag{3.2}$$

with $z = x/y$. The semi-projectors on outgoing lines, $\mathbb{P}_{j,\,\mathrm{out}}$, are directly taken from [26]:

$$\mathbb{P}_{g,\,\mathrm{out}}^{\mu\nu} = -g^{\mu\nu} \qquad\qquad \mathbb{P}_{q,\,\mathrm{out}} = \frac{\slashed{n}}{2\,q\cdot n} \tag{3.3}$$

While outgoing lines are at first treated in 1-1 correspondence to [26], the on-shell restriction on incoming lines is now relaxed. The corresponding semi-projectors therefore require a slight modification. With the original projectors $\mathbb{P}_{j,\,\mathrm{in}}$ of [26],

$$\mathbb{P}_{g,\,\mathrm{in}}^{[26]\,\mu\nu} = \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n}\right), \qquad\qquad \mathbb{P}_{q,\,\mathrm{in}}^{[26]} = \frac{\slashed{k}}{2}, \tag{3.4}$$

are modified to

$$\mathbb{P}_{g,\,\mathrm{in}}^{\mu\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\mathbf{k}^2}, \qquad\qquad \mathbb{P}_{q,\,\mathrm{in}} = \frac{y\,\slashed{p}}{2}. \tag{3.5}$$

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While the modified gluon projector has been known since long time [25], we emphasize that the modified quark projector follows directly from the decomposition of the high energy projector in eq. (2.2). Its normalization is on the other hand fixed by requiring agreement with the corresponding projector of [26] in the collinear limit. To ensure gauge invariance of the splitting functions in presence of off-shell momenta, it is further necessary to modify standard QCD vertices. The formalism which guarantees that gauge invariance holds is based on the reggeized quark formalism [31, 32, 41] (for more recent re-derivation in spin helicity formalism see [43]). The modification is achieved through adding certain eikonal terms which then in turn arrange gauge invariance of the vertex. Apart from the conventional QCD quark-quark-gluon vertex, $\Gamma^\mu_{qqg} = ig t^a \gamma^\mu$ we have for the off-shell vertex with one reggeized quark q^*

$$\Gamma^\mu_{q^*qg}(p_{q^*}, p_q, p_g) = ig t^a \left(\gamma^\mu + \frac{p^\mu}{p \cdot p_g} \slashed{p}_{q^*} \right) \qquad \text{with} \qquad p_{q^*} \cdot p = 0. \qquad (3.6)$$

Contracting the Lorentz index of this vertex with the gluon momentum yields $p_{g,\mu} \cdot \Gamma^\mu_{q^*qg} = -ig t^a \slashed{p}_q$ which is equivalent to the corresponding expression for the conventional quark-quark-gluon vertex if the quark p^*_q is taken on the mass shell. Moreover, in case the second quark is on the mass shell, we have immediately $p_{g,\mu} \cdot \Gamma^\mu_{q^*qg} \bar{u}(p'_q) = -ig t^a \slashed{p}_q \bar{u}(p'_q) = 0$ with $p^2_{q'} = 0$. We therefore find that using the generalized vertex eq. (3.6), the current conservation holds despite of the quark with momentum p_{q^*} being off-shell.

To determine both angular and transverse momentum dependent splitting functions, we start with eq. (3.1), perform color, Dirac and Lorentz algebra, integrate over q^2 and shift the transverse momenta $\mathbf{q} \to \tilde{\mathbf{q}} = \mathbf{q} - z\mathbf{k}$, following closely the treatment in the seminal work of [25]. We then obtain a set of angular- and transverse momentum dependent splitting functions $\tilde{P}_{ij}$ defined through

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \epsilon, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \int \frac{\mathrm{d}^{2+2\epsilon}\tilde{\mathbf{q}}^2}{\pi^{1+\epsilon}\mu^{2\epsilon}} \frac{e^{-\epsilon\gamma_E}}{\tilde{\mathbf{q}}^2}$$
$$\Theta\left(\mu_F^2 - \frac{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}{1-z}\right) \tilde{P}_{ij}^{(0)}(z, \mathbf{k}, \tilde{\mathbf{q}}, \epsilon) \qquad (3.7)$$

with the $\overline{\text{MS}}$ scheme coupling $\alpha_s = \frac{g^2 \mu^{2\epsilon} e^{\epsilon\gamma_E}}{(4\pi)^{1+\epsilon}}$ and γ_E the Euler-Mascheroni constant. The angular and transverse momentum dependent splitting functions read

$$\tilde{P}_{gq}^{(0)}(z, \mathbf{k}, \tilde{\mathbf{q}}, \epsilon) = C_F \left(\frac{\tilde{\mathbf{q}}^2}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)}\right)^2 \left(\frac{\tilde{\mathbf{q}}^2}{(\tilde{\mathbf{q}} - (1-z)\mathbf{k})^2}\right)$$
$$\times \left\{ \frac{2 - 2z + z^2}{z} + z(1-z)^2(1+z^2)\left(\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right)^2 + 4(1-z)^2\left[\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2} + z\left(\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2}\right)^2\right] \right.$$
$$\left. + 4z^2(1-z)^2 \frac{\mathbf{k}\cdot\tilde{\mathbf{q}}\,\mathbf{k}^2}{\tilde{\mathbf{q}}^4} + 2(1-z)(1+z-z^2)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right\} + C_F \frac{\epsilon\, z\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}} - (1-z)\mathbf{k})^2}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2}, \quad (3.8)$$

$$\tilde{P}_{qg}^{(0)}(z, \mathbf{k}, \tilde{\mathbf{q}}, \epsilon) = T_R \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}\right)^2$$
$$\times \left[1 + 4z^2(1-z)^2\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} + 4z(1-z)(1-2z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2} - 4z(1-z)\left(\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\sqrt{\tilde{\mathbf{q}}^2\mathbf{k}^2}}\right)^2 \right], \qquad (3.9)$$

$$\tilde{P}_{qq}^{(0)}\left(z, \mathbf{k}, \tilde{\mathbf{q}}, \epsilon\right) = C_F \left(\frac{\tilde{\mathbf{q}}^2}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)} \right)^2 \left(\frac{\tilde{\mathbf{q}}^2}{(\tilde{\mathbf{q}} - (1-z)\mathbf{k})^2} \right)$$

$$\times \left\{ \frac{1+z^2}{1-z} + z^2(1-z)(5-4z+z^2)\left(\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right)^2 - 4z(1-z)^2\left(\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2}\right)^2 \right.$$

$$+ 2z(1-2z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2} + 2z(2-7z+7z^2-2z^3)\frac{\mathbf{k}^2\,\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^4} + (1+z+4z^2-2z^3)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}$$

$$+ \epsilon(1-z)\cdot\left[1 - 2(1-2z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2} - 4z(1-z)\left(\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}}{\tilde{\mathbf{q}}^2}\right)^2 + (1-2z+2z^2)\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right.$$

$$\left.\left. + 2z(1-z)(1-2z)\frac{\mathbf{k}\cdot\tilde{\mathbf{q}}\mathbf{k}^2}{\tilde{\mathbf{q}}^4} + z^2(1-z)^2\left(\frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right)^2 \right] \right\}. \quad (3.10)$$

Determination of both angular and transverse momentum dependent splitting functions for the splittings quark-to-gluon and quark-to-quark present, together with the results presented further down in section 5, the central results of this work.

4   Gauge invariance of TMD splitting functions

The obtained TMD splitting functions will be essential for the definition of set of TMD evolution equations of TMD parton distributions. While the above derivation is based on the 2PI-expansion of [26] the derivation might be at first regarded as not completely satisfactory. While care has been taken to ensure gauge invariance of the off-shell vertex eq. (3.6), the employed formalism heavily relies on the use of the light-cone gauge and gauge invariance of our result is not immediately apparent. This is of particular concern, once we relax the integration over $\tilde{q}$ in eq. (5.1) to allow for TMD factorization in the outgoing momentum $\tilde{q}$ and therefore leave strictly speaking the framework provided by [26]. To ensure gauge invariance also in this more general case, we will provide in the following an explicit gauge invariant extension of the sub-amplitudes figure 2 as well as the projectors. As a consequence we will both obtain explicitly gauge invariant sub-amplitudes and verify that any possible gauge dependence hidden in the propagators of the outgoing parton with momentum q and/or the real produced parton with momentum $p' = k - q$ will cancel. In particular, while calculations are no longer restricted to the light-cone gauge as in section 3, they agree at every stage precisely with the results derived in this gauge. To this end we first generalize the projector of the outgoing gluon in eq. (3.3). Another source of potential violation of gauge invariance is due to the use of the explicit cut-off in eq. (3.1), provided by the constraint $q^2 < \mu_F^2$ which regularizes the integral in the ultra-violet.[1] In physical terms the cut-off separates hard modes $q^2 > \mu_F^2$ (which belong to corresponding coefficient functions) and soft modes $q^2 < \mu_F^2$ (which are absorbed into TMD pdfs). A generalization of our results to a cut-off-independent formulation is left at this stage as a task for future research, restricting ourselves for the time being to the proper definition of gauge-invariant sub-amplitudes.

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[1]Note that this divergence remains otherwise unregularized for positive ϵ.

With the polarization tensor of the gluon propagator in the light-cone gauge given by

$$\Delta_{\mu\mu'}(q) = -g_{\mu\mu'} + \frac{q^{\mu'}n^\mu + n^{\mu'}q^\mu}{q \cdot n}, \tag{4.1}$$

we define the new projector

$$\tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}}(q,n) \equiv \Delta_{\mu\mu'}(q)\mathbb{P}^{\mu'\nu'}_{g,\,\text{out}}\Delta_{\nu'\nu}(q) = -g_{\mu\nu} + \frac{q^\mu n^\nu + n^\mu q^\nu}{q \cdot n} - q^2\frac{n^\mu n^\nu}{(q \cdot n)^2}, \tag{4.2}$$

which fulfills the following properties:

$$0 = \tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}} \cdot q_\mu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}} \cdot q_\nu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}} \cdot n_\mu = \tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}} \cdot n_\nu\,. \tag{4.3}$$

Furthermore

$$\tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}} \cdot q^\mu_\perp q^\nu_\perp = \boldsymbol{q}^2 \tag{4.4}$$

and hence the combination

$$\tilde{\mathbb{P}}^{\mu\nu}_{g,\,\text{out}}(q,n)\tilde{\mathbb{P}}^{\mu'\nu'}_{g,\,\text{in}}(q_\perp), \tag{4.5}$$

is indeed a projector. Due to the properties eq. (4.3), one also has

$$\tilde{\mathbb{P}}^{\mu\nu'}_{g,\,\text{out}}(q,n)\Delta_{\nu'\nu}(q) = \tilde{\mathbb{P}}^{\mu\nu'}_{g,\,\text{out}}(q,n) = \Delta_{\mu\mu'}(q)\tilde{\mathbb{P}}^{\mu'\nu}_{g,\,\text{out}}(q,n)\,. \tag{4.6}$$

Using therefore eq. (4.2) in the analysis of the previous section, will leave our results unchanged. The second modification concerns the sub-amplitudes figure 2. In the high energy limit, corresponding gauge invariant vertices can be easily derived within the reggeized quark formalism. To ensure gauge invariance in presence of both off-shell momenta k and q, with q of the general form eq. (3.2), these vertices require a slight generalization, similar to the one employed already in [12]. The version to be used in the following reads

$$\Gamma^\mu_{q^*g^*q}(q,k,p') = igt^a\left(\gamma^\mu - \frac{n^\mu}{k \cdot n}\slashed{q}\right), \tag{4.7}$$

$$\Gamma^\mu_{g^*q^*q}(q,k,p') = igt^a\left(\gamma^\mu - \frac{p^\mu}{p \cdot q}\slashed{k}\right), \tag{4.8}$$

$$\Gamma^\mu_{q^*q^*g}(q,k,p') = igt^a\left(\gamma^\mu - \frac{p^\mu}{p \cdot p'}\slashed{k} + \frac{n^\mu}{n \cdot p'}\slashed{q}\right). \tag{4.9}$$

where we used $p' = k - q$ for the momentum of the real produced particle and q^*, g^* indicate an off-shell quark and gluon; the momentum k and momentum q refer always to incoming and outgoing particles respectively. In particular, these vertices obey

$$k_\mu \cdot \Gamma^\mu_{q^*g^*q}(k,q)\bar{u}(p') = 0 \qquad q_\mu \cdot \Gamma^\mu_{q^*q^*g}(k,q)\bar{u}(p') = 0 \qquad p'_\mu \cdot \Gamma^\mu_{q^*q^*g}(k,q) = 0\,. \tag{4.10}$$

Due to these properties, any gauge dependence induced by either the polarization tensor of a t-channel gluon with momentum q, a real produced gluon with momentum $p' = k - q$, or a t-channel gluon with momentum k is canceled and the overall result is gauge-invariant.

In particular it is trivial to check that the results obtained in the previous section using the light-cone gauge, generalize directly to the present formulation. A last comment is in order concerning the universality of our results. As pointed out in the beginning of section 3, unlike the splitting function of [25], our splitting functions cannot be uniquely associated with the e.g. next-to-leading order coefficient of the small-x gluon Green's function etc. Indeed there will be always contributions of similar order of magnitude than elements of our splitting functions, which are not contained in its definition. Our splitting functions comprise however a set of contributions which

- reduces in the collinear limit to collinear splitting functions

- reduces in the high energy limit to corresponding high energy factorized expressions (guaranteed through the use of the reggeized quark and gluon vertices)

- combines both limits in a gauge invariant way.

It is then the combination of these three requirements which provides strong constraints on the terms contained in the definition of our TMD splitting functions.

5 Angular averaged TMD splitting functions and singularity structure

In the following section we further analyze our results of section 3. While the explicit angular-momentum-dependence of our results might be of interest for further Monte-Carlo realizations which aim at description of exclusive final states, the evolution of TMD parton distribution functions generally requires only angular-averaged splitting functions. Furthermore, the splitting functions turn out to be divergent in certain regions of phase space, which will be identified below.

5.1 Angular averaged TMD splitting functions

To arrive at a result similar to the one obtained in [25] for the TMD P_{qg}, it is further necessary to average over the azimuthal angle. With

$$\hat{K}_{ij}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \frac{\alpha_s}{2\pi} z \int_0^{(1-z)(\mu_F^2 - z\mathbf{k}^2)} \frac{\mathrm{d}\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2} \left(\frac{\tilde{\mathbf{q}}^2}{\mu^2}\right)^\epsilon \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P_{ij}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right), \quad (5.1)$$

which then defines the TMD splitting functions P_{ij}, we reproduce for the gluon-to-quark splitting the result of [25], also calculated in [12, 44]

$$P_{qg}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right) = T_R \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2}\right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}\right]. \quad (5.2)$$

For the new TMD splitting functions we obtain

$$P_{gq}^{(0)}\left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon\right) = C_F \left[\frac{2\tilde{\mathbf{q}}^2}{z|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} \right.$$
$$\left. - \frac{\tilde{\mathbf{q}}^2(\tilde{\mathbf{q}}^2(2-z) + \mathbf{k}^2 z(1-z^2)) - \epsilon z(\tilde{\mathbf{q}}^2 + (1-z)^2\mathbf{k}^2)}{(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)^2}\right], \quad (5.3)$$

$$P_{qq}^{(0)} \left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon \right) = C_F \left(\frac{\tilde{\mathbf{q}}^2}{\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2} \right) \left[\frac{\tilde{\mathbf{q}}^2 + (1-z^2)\mathbf{k}^2}{(1-z)|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} \right.$$
$$\left. + \frac{z^2\tilde{\mathbf{q}}^2 - z(1-z)(1 - 3z + z^2)\mathbf{k}^2 + (1-z)^2\epsilon(\tilde{\mathbf{q}}^2 + z^2\mathbf{k}^2)}{(1-z)(\tilde{\mathbf{q}}^2 + z(1-z)\mathbf{k}^2)} \right]. \quad (5.4)$$

As expected from our method to construct TMD splitting functions, we obtain in the collinear limit ($\mathbf{k}^2/\tilde{\mathbf{q}}^2 \to 0$) the well-known real parts of the leading-order Altarelli-Parisi splitting functions in $d = 4 + 2\epsilon$ dimensions:

$$P_{gq}^{(0)}(z, 0, \epsilon) = C_F \frac{1 + (1-z)^2 + \epsilon z^2}{z}, \quad (5.5)$$

$$P_{qg}^{(0)}(z, 0, \epsilon) = T_R \left(z^2 + (1-z)^2 \right), \quad (5.6)$$

$$P_{qq}^{(0)}(z, 0, \epsilon) = C_F \frac{1 + z^2 + \epsilon(1-z)^2}{1 - z}. \quad (5.7)$$

5.2 Singularity structure of the TMD splitting functions

Unlike the P_{qg} TMD splitting function, the splitting functions in eq. (5.2) and eq. (5.4) develop singularities in certain regions of phase space. These singularities can be organized into two groups: those associated with the limit $z \to 1$, only present for the splitting P_{qq}, and those associated with the limit $|\tilde{\mathbf{q}}| \to (1-z)|\mathbf{k}|$, present for both P_{gq} and P_{qq}. The coefficient of the $z \to 1$ singularity reads

$$\lim_{z \to 1} P_{qq}^{(0)} \left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2} \right) = C_F \frac{2}{1 - z} \quad (5.8)$$

and coincides with the $z \to 1$ singularity of the conventional collinear splitting functions where it is known to be regularized by corresponding virtual corrections to the splitting kernel. We expect a similar mechanism to be realized in the case of the P_{qq} splitting kernel with full transverse momentum dependence. The nature of the second singularity is more intriguing, since it is present for both diagonal (P_{qq}) and off-diagonal (P_{gq}) splitting kernels. The coefficients of this singularity is provided by

$$\lim_{|\tilde{\mathbf{q}}| \to (1-z)|\mathbf{k}|} P_{gq}^{(0)} \left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon \right) = \frac{2C_F}{z} \frac{\tilde{\mathbf{q}}^2}{|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|}$$
$$\lim_{|\tilde{\mathbf{q}}| \to (1-z)|\mathbf{k}|} P_{qq}^{(0)} \left(z, \frac{\mathbf{k}^2}{\tilde{\mathbf{q}}^2}, \epsilon \right) = \frac{2C_F}{1-z} \frac{\tilde{\mathbf{q}}^2}{|\tilde{\mathbf{q}}^2 - (1-z)^2\mathbf{k}^2|} \quad (5.9)$$

For the P_{qq} splitting function, this singularity always overlaps with the $z \to 1$ singularity. At the level of the angular-dependent splitting kernels eq. (3.8) and eq. (3.10), this singularity is easily identified with the vanishing of the transverse momentum of the real, emitted parton i.e. of the real gluon (P_{qq}) and of the real quark (P_{gq}) respectively. To analyze the precise structure of the singularities within dimensional regularization it is convenient to

switch to the re-scaled momentum $\tilde{\mathbf{p}} = \frac{\mathbf{k}-\mathbf{q}}{1-z}$ instead of $\tilde{\mathbf{q}}$. We then obtain

$$
\hat{K}_{qq}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}\mu^{2\epsilon}} e^{-\epsilon\gamma_E} \Theta\left(\mu_F^2 - (1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 - z\mathbf{k}^2\right)
$$
$$
\times \left\{ \frac{1}{(1-z)^{1-2\epsilon}} \left(\frac{1}{\tilde{\mathbf{p}}^2} + \frac{\mathbf{k}^2 + z(1-z)\tilde{\mathbf{p}}^2}{\tilde{\mathbf{p}}^2[(1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 + z\mathbf{k}^2]} \right) \right.
$$
$$
\left. - \frac{\tilde{\mathbf{p}}^2 z(1-z)^{2+2\epsilon}(1+\epsilon)}{[(1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 + z\mathbf{k}^2]^2} + \frac{\epsilon(1-z)^{1+2\epsilon}}{(1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 + z\mathbf{k}^2} \right\} \quad (5.10)
$$

$$
\hat{K}_{gq}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s\right) = \frac{\alpha_s}{2\pi} z \int \frac{d^{2+2\epsilon}\tilde{\mathbf{p}}}{\pi^{1+\epsilon}\mu^{2\epsilon}} e^{-\epsilon\gamma_E} \Theta\left(\mu_F^2 - (1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 - z\mathbf{k}^2\right)
$$
$$
\times \left\{ \frac{(1-z)^{2\epsilon}}{z} \frac{2}{\tilde{\mathbf{p}}^2} + \frac{\tilde{\mathbf{p}}^2 z(1-z)^{2+2\epsilon}(1+\epsilon)}{[(1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 + z\mathbf{k}^2]^2} \right.
$$
$$
\left. - \frac{2(1-z)^{1+2\epsilon}}{(1-z)(\tilde{\mathbf{p}}-\mathbf{k})^2 + z\mathbf{k}^2} \right\} \quad (5.11)
$$

It is now possible to isolate the singularities of interest using a phase space slicing parameter $\lambda \to 0$ which splits the integration over $\tilde{\mathbf{p}}$ into regions $|\tilde{\mathbf{p}}| < \lambda$, $|\tilde{\mathbf{p}}| > \lambda$. Defining[2] $K_{qq}^{(0)\text{fin.}}$ and $K_{gq}^{(0)\text{fin.}}$ as the kernels given in eq. (5.10) and eq. (5.11), but with the integration measure $d^{2+2\epsilon}\tilde{\mathbf{p}}$ replaced appropriately by $d^{2+2\epsilon}\tilde{\mathbf{p}} \cdot \Theta(\tilde{\mathbf{p}}^2 - \lambda^2)$ and $d^{2+2\epsilon}\tilde{\mathbf{p}} \cdot \Theta(\lambda^2 - \tilde{\mathbf{p}}^2)$ we have

$$
\hat{K}_{qq}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \hat{K}_{qq}^{\text{fin.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) + \hat{K}_{qq}^{\text{div.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right)
$$
$$
\hat{K}_{gq}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \hat{K}_{gq}^{\text{fin.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) + \hat{K}_{gq}^{\text{div.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) \quad (5.12)
$$

with

$$
\hat{K}_{qq}^{\text{div.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s, \epsilon\right) = \frac{\alpha_s}{\pi} \Theta(\mu_F^2 - \mathbf{k}^2) \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} \frac{\lambda^{2\epsilon}}{\epsilon(1-z)^{1-2\epsilon}} + \mathcal{O}(\lambda)
$$
$$
= \frac{\alpha_s}{\pi} \Theta(\mu_F^2 - \mathbf{k}^2) \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} \left(\frac{\lambda^2}{\mu^2}\right)^\epsilon \frac{1}{\epsilon} \left(\frac{1}{2\epsilon}\delta(1-z) + \frac{1}{(1-z)_+^{1-2\epsilon}} \right) + \mathcal{O}(\lambda)
$$
$$
\hat{K}_{gq}^{\text{div.}}\left(z, \frac{\mathbf{k}^2}{\mu_F^2}, \alpha_s\right) = \frac{\alpha_s}{\pi} \Theta(\mu_F^2 - \mathbf{k}^2) \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} \left(\frac{\lambda^2}{\mu^2}\right)^\epsilon \frac{((1-z))^{2\epsilon}}{\epsilon z} + \mathcal{O}(\lambda) \quad (5.13)
$$

where we made use of the limit $\lambda \to 0$. We furthermore introduced the usual plus-prescription

$$
\int_0^1 dz \frac{f(z)}{(1-z)_+} \equiv \int_0^1 dz \frac{f(z) - f(1)}{(1-z)}, \quad (5.14)
$$

and made use of the identity

$$
\frac{1}{(1-z)^{1-2\epsilon}} = \frac{1}{2\epsilon}\delta(1-z) + \frac{1}{(1-z)_+^{1-2\epsilon}}
$$
$$
= \frac{1}{2\epsilon}\delta(1-z) + \frac{1}{(1-z)_+} + 2\epsilon \left(\frac{\ln(1-z)}{(1-z)} \right)_+ + \mathcal{O}(\epsilon^2). \quad (5.15)
$$

---

[2]Note that the kernels $K_{qq}^{(0)\text{fin.}}$ and $K_{gq}^{(0)\text{fin.}}$ still contain divergences due to the $z \to 1$ singularity.

Note that since the real emitted particle is on-shell, the vanishing of its transverse momentum $\tilde{\mathbf{p}}$ implies also vanishing of the component parallel to n. As a consequence the momentum of the emitted particle is in this case collinear to the initial proton momentum p. For a hands-on approach, it appears therefore to be natural to avoid this singularity by introducing a cut-off, similar to λ in the $K_{qq}^{(0)\text{fin.}}$ and $K_{gq}^{(0)\text{fin.}}$ terms or by imposing an angular ordering inspired constrained on the t-channel momenta such as $|\tilde{\mathbf{q}}|/(1-z) > |\mathbf{k}|$ which avoids the singular region. Such a treatment would then allow for first numerical tests of the proposed TMD splitting functions and for their application to phenomenological studies. A complete theoretical treatment of this singularity would on the other hand require the determination of virtual corrections (in the case of the P_{qq} splitting) and most likely the realization of a systematic subtraction mechanism which removes parton emission collinear to the initial proton momentum from the TMD splitting kernels. Both tasks are beyond the scope of this work and are left as a task for future research.

6   Summary and outlook

In this paper we extended the method developed by Catani and Hautmann for the determination of transverse-momentum-dependent parton splitting functions to splittings of initial k_T-dependent quarks, based on factorization of cross-sections in the high energy limit. Gauge invariance of underlying amplitudes in presence of off-shell partons is achieved due to the reggeized quark calculus, which supplements conventional QCD vertices by certain eikonal contributions. While our approach is heavily based on the 2PI expansion in the light-cone gauge by Curci et al., we have been able to verify that it is possible to generalize the employed projectors in a way, such that the choice of gauge for the sub-amplitudes, which underlie the derivation of our splitting kernels, becomes irrelevant i.e. our TMD splitting kernels are independent of the employed gauge. While our splitting kernels are in this way well defined objects, there are not necessarily universal, since they cannot be directly defined as the coefficients of e.g. the high energy resummation of a certain TMD parton distribution function, such as the TMD gluon-to-quark splitting functions. They are merely constrained by the requirement to reduce in the collinear and high energy limit to the well-known exact expressions.

The current study determines only the real contribution to the TMD quark-to-quark and quark-to-gluon splitting kernels. Future studies will have to focus on the determination of the corresponding virtual corrections for the TMD quark-to-quark splitting function, the development of a coherent framework which allows for a systematic subtraction of singularities not canceled by virtual corrections and finally the formulation of appropriate coupled evolution equations for TMD parton distribution functions. As a long term goal, a matching of TMD evolution based on factorization in the soft-collinear limit, see e.g. [45–47] is a task which needs to be addressed.

Acknowledgments

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