

Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry

Kiwoon Choi and Sang Hui Im

*Center for Theoretical Physics of the Universe, Institute for Basic Science (IBS),
Daejeon 34051, Korea*

E-mail: kchoi@ibs.re.kr, shim@ibs.re.kr

ABSTRACT: We discuss a scheme to implement the relaxion solution to the hierarchy problem with multiple axions, and present a UV-completed model realizing the scheme. All of the N axions in our model are periodic with a similar decay constant f well below the Planck scale. In the limit $N \gg 1$, the relaxion ϕ corresponds to an exponentially long multi-helical flat direction which is shaped by a series of mass mixing between nearby axions in the compact field space of N axions. With the length of flat direction given by $\Delta\phi = 2\pi f_{\text{eff}} \sim e^{\xi N} f$ for $\xi = \mathcal{O}(1)$, both the scalar potential driving the evolution of ϕ during the inflationary epoch and the ϕ -dependent Higgs boson mass vary with an exponentially large periodicity of $\mathcal{O}(f_{\text{eff}})$, while the back reaction potential stabilizing the relaxion has a periodicity of $\mathcal{O}(f)$. A natural UV completion of our scheme can be found in high scale or (mini) split supersymmetry (SUSY) scenario with the axion scales generated by SUSY breaking as $f \sim \sqrt{m_{\text{SUSY}} M_*}$, where the soft SUSY breaking scalar mass m_{SUSY} can be well above the weak scale, and the fundamental scale M_* can be identified as the Planck scale or the GUT scale.

KEYWORDS: Beyond Standard Model, Supersymmetric Effective Theories, Higgs Physics, Global Symmetries

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1 Introduction

Recently a new approach to address the hierarchy problem has been proposed in [1]. The scheme introduces a scalar degree of freedom, the relaxion ϕ , making the Higgs boson mass a dynamical field depending on ϕ . During the inflationary epoch, the Higgs boson mass-square $\mu_h^2(\phi)$ is scanned by the rolling ϕ from a large positive initial value to zero. Right after the relaxion crosses the point $\mu_h(\phi) = 0$, so that $\mu_h^2(\phi)$ becomes negative, a nonzero Higgs vacuum expectation value (VEV) is developed and a Higgs-dependent back reaction potential begins to operate to stabilize the relaxion.¹ One can then arrange the model parameters in a technically natural way to result in the relaxion stabilized at a point where the corresponding Higgs VEV is much smaller than the initial Higgs boson mass.

An intriguing feature of the relaxion mechanism is that the relaxion potential involves two very different scales. One is the period of the back reaction potential, and the other is the excursion range of the relaxion necessary to scan $\mu_h(\phi)$ from a large initial value to zero. To see this, let us consider the relaxion potential given by

$$V(\phi, h) = V_0(\phi) + \mu_h^2(\phi)|h|^2 + V_{\text{br}}(\phi, h) \tag{1.1}$$

where V_0 is the potential driving the rolling of ϕ during the inflationary epoch and V_{br} is the periodic back reaction potential stabilizing ϕ right after it crosses $\mu_h(\phi) = 0$. In fact, the key feature of the mechanism can be read off from the following form of potential:

$$V_0 = \epsilon_0 f^3 \phi + \dots, \quad \mu_h^2 = M_h^2 + \epsilon_h f \phi + \dots, \quad V_{\text{br}} = \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f}\right), \tag{1.2}$$

where M_h denotes the initial Higgs boson mass, ϵ_0 and ϵ_h are small dimensionless parameters describing the explicit breaking of the relaxion shift symmetry in V_0 and μ_h^2 , respectively, and finally f is the relaxion decay constant in the back reaction potential. In non-supersymmetric theory, the Higgs mass parameter M_h is naturally of the order of the cutoff scale of the model. On the other hand, in supersymmetric theory, it corresponds to

¹A mechanism to cosmologically relax the Higgs boson mass down to a small value through a nucleation of domain wall bubbles has been discussed in [2].

the scale of soft supersymmetry (SUSY) breaking mass which can be well below the cutoff scale of the model. In any case, we are interested in the case that M_h is much larger than the weak scale:

$$M_h \gg v \equiv \langle h \rangle = 174 \text{ GeV}, \tag{1.3}$$

which might be explained by the relaxion mechanism.

Let us now list the conditions for the relaxion mechanism to work. First of all, in order for the rolling relaxion to cross $\mu_h(\phi) = 0$ without a fine tuning of the initial condition, it should experience a field excursion

$$\frac{\Delta\phi}{f} \gtrsim \frac{M_h^2}{\epsilon_h f^2}. \tag{1.4}$$

In order for the scalar potential to be technically natural under radiative corrections, the symmetry breaking parameters ϵ_0 and ϵ_h should obey

$$\epsilon_0 \gtrsim \epsilon_h \frac{M_h^2}{f^2}. \tag{1.5}$$

On the other hand, from the stability condition $\partial_\phi V = 0$, one finds

$$\epsilon_0 \sim \frac{\Lambda_{\text{br}}^4}{f^4}, \tag{1.6}$$

and therefore

$$\frac{\Delta\phi}{f} \gtrsim \frac{M_h^4}{\Lambda_{\text{br}}^4}. \tag{1.7}$$

As for the back reaction potential, generically $\Lambda_{\text{br}}(h=0)$ may not be vanishing, and then one needs

$$\Lambda_{\text{br}}^4(h=v) \gg \Lambda_{\text{br}}^4(h=0). \tag{1.8}$$

Also, in order not to destabilize the weak scale size of the Higgs VEV, its magnitude should be bounded as

$$\Lambda_{\text{br}}(h=v) \lesssim \mathcal{O}(v). \tag{1.9}$$

An immediate consequence of the above conditions is that the relaxion should experience a field excursion much bigger than f in the limit $M_h \gg v$:

$$\frac{\Delta\phi}{f} \gtrsim \frac{M_h^4}{v^4}. \tag{1.10}$$

The required excursion is huge in the case that the back reaction potential is generated by the QCD anomaly, in which $\Lambda_{\text{br}}^4 \sim f_\pi^2 m_\pi^2$ and therefore

$$\frac{\Delta\phi}{f} \gtrsim \mathcal{O}\left(\frac{M_h^4}{f_\pi^2 m_\pi^2}\right) \sim 10^{12} \left(\frac{M_h}{v}\right)^4. \tag{1.11}$$

Even when the scale of the back reaction potential saturates the bound (1.9), the required relaxion excursion is still much larger than f as long as M_h is higher than the weak scale by more than a few orders of magnitudes. Note that the natural size of M_h is the cutoff

scale of the model for non-SUSY case, while it is the soft SUSY breaking scalar mass for SUSY case.

Therefore, in the relaxion scenario, the hierarchy $M_h/v \gg 1$ is replaced with a much bigger hierarchy $\Delta\phi/f \gtrsim M_h^4/v^4$. Although $\Delta\phi \gg f$ might be stable against radiative corrections, it is still crying for an explanation with a sensible UV completion. To incorporate a huge relaxion excursion, one may simply assume that the relaxion is a non-compact field variable. See [3–10] for recent discussions of the related issues. In this paper, we discuss an alternative scenario in which the relaxion corresponds to an exponentially long multi-helical flat direction in the compact field space spanned by N sub-Planckian periodic axions:

$$\phi_i \equiv \phi_i + 2\pi f_i \quad (i = 1, 2, \dots, N)$$

with $f_i \ll M_{\text{Planck}}$. Such a long flat direction is formed by a series of mass mixing between nearby axions, producing a multiplicative sequence of helical windings of flat direction, which results in

$$\frac{\Delta\phi}{f_i} = \mathcal{O}\left(e^{\xi N}\right)$$

for $\xi = \mathcal{O}(1)$. Our scenario is inspired by the recent generalization of the axion alignment mechanism for natural inflation [11] to the case of N axions [12]. Although it requires a rather specific form of axion mass mixings, our scheme does not involve any fine tuning of continuous parameters, nor an unreasonably large discrete parameter.

As we will see, our scheme finds a natural UV completion in high scale or (mini) split supersymmetry (SUSY) scenario with soft SUSY breaking scalar mass $m_{\text{SUSY}} \gg v$. In the UV completed model, the axion scales are generated by SUSY breaking [13–15] as

$$f_i \sim \sqrt{m_{\text{SUSY}} M_*},$$

where M_* can be identified as the Planck scale or the GUT scale. With the $(N - 1)$ hidden Yang-Mills gauge sectors which confine at scales below f_i to generate the desired axion mass mixings, the canonically normalized relaxion has a field range

$$\Delta\phi \equiv 2\pi f_{\text{eff}} \sim 2\pi f_i \left(\prod_{j=1}^{N-1} n_j \right),$$

where $n_j > 1$ corresponds to the number of flavors of the gauge-charged fermions in the j -th hidden sector. One can then arrange the microscopic parameters in a technically natural way to make the resulting relaxion potential $V_0(\phi)$ and the ϕ -dependent Higgs boson mass $\mu_h^2(\phi)$ vary with an exponentially large periodicity of $\mathcal{O}(f_{\text{eff}})$, while the back reaction potential $V_{\text{br}}(h, \phi)$ has a periodicity of $\mathcal{O}(f_i)$. An interesting feature of our model is that the desired $V_0(\phi)$ and $\mu_h^2(\phi)$ arise as a natural consequence of the solution of the MSSM μ -problem advocated in [13–15].

The outline of the paper is as follows. In the next section, we describe the basic idea with a simple toy model and discuss the scheme within the framework of an effective theory of N axions. In section 3, we present a UV model with high scale SUSY, realizing our scheme in the low energy limit. Section 4 is the conclusion.

2 Exponentially long relaxation from multiple axions

To illustrate the basic idea, let us begin with a simple two axion model. The lagrangian density of the model is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 - \left(\tilde{V}_0 + V_0 + \mu_h^2|h|^2 + V_{\text{br}} + \dots\right), \quad (2.1)$$

where h is the SM Higgs doublet and ϕ_i are the periodic axions:

$$\phi_i \equiv \phi_i + 2\pi f_i, \quad (2.2)$$

with a scalar potential

$$\begin{aligned} \tilde{V}_0 &= -\Lambda^4 \cos\left(\frac{\phi_1}{f_1} + n\frac{\phi_2}{f_2}\right), \\ V_0 &= -\epsilon f_2^4 \cos\left(\frac{\phi_2}{f_2} + \delta_2\right), \\ \mu_h^2 &= M_h^2 - \epsilon' f_2^2 \cos\left(\frac{\phi_2}{f_2} + \delta_2'\right), \\ V_{\text{br}} &= -\Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi_1}{f_1} + \delta_1\right), \end{aligned} \quad (2.3)$$

where

$$\Lambda^4 \gg \epsilon f_2^4 \gg \Lambda_{\text{br}}^4. \quad (2.4)$$

Here M_h is an *axion-independent* mass parameter which is comparable to the cutoff scale of the above effective lagrangian, and $n > 1$ is an integer which will be determined by the underlying UV completion. We assume

$$\epsilon f_2^2 \gtrsim \mathcal{O}(\epsilon' M_h^2), \quad \epsilon' f_2^2 \gtrsim \mathcal{O}(M_h^2), \quad (2.5)$$

and therefore the model is stable against the radiative corrections which replace the Higgs operator $|h|^2$ with the cutoff-square of $\mathcal{O}(M_h^2)$, while allowing $\mu_h^2 = 0$ for certain value of ϕ_2 .

As for the back reaction potential, one can consider two different possibilities. One option is to generate it by the coupling of ϕ_1 to the QCD anomaly, yielding

$$\Lambda_{\text{br}}^4(h) \sim y_u \Lambda_{\text{QCD}}^3 h, \quad (2.6)$$

where y_u denotes the up quark Yukawa coupling to the SM Higgs field h , and Λ_{QCD} is the QCD scale. This option corresponds to the minimal model, however generically is in conflict with the axion solution to the strong CP problem. Alternative option is to introduce a new hidden gauge interaction which confines around the weak scale and generates a back reaction potential given by [1, 16]

$$\Lambda_{\text{br}}^4 = m_1^2|h|^2 + m_2^4 \quad (2.7)$$

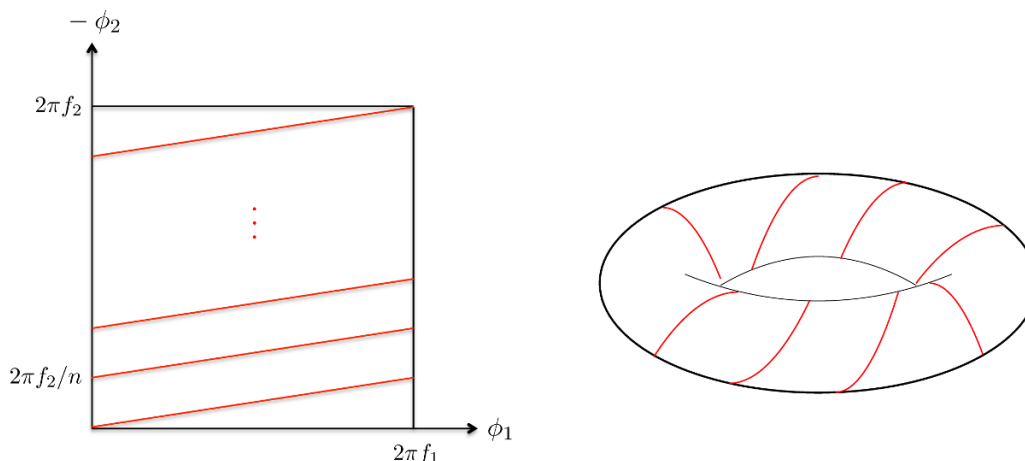


Figure 1. Flat relaxation direction in the two axion model.

with

$$m_2^4 < m_1^2 v^2 \lesssim \mathcal{O}(v^4). \tag{2.8}$$

In order for the model to be technically natural, the underlying dynamics to generate the back reaction potential should be arranged to make sure that the above conditions on m_1 and m_2 are stable against radiative corrections.

The above two axion model involves the shift symmetries

$$U(1)_i : \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + c_i \quad (i = 1, 2), \tag{2.9}$$

which are broken by \tilde{V}_0 down to the relaxation shift symmetry

$$U(1)_\phi : \frac{\phi_1}{f_1} \rightarrow \frac{\phi_1}{f_1} + nc, \quad \frac{\phi_2}{f_2} \rightarrow \frac{\phi_2}{f_2} - c. \tag{2.10}$$

The flat direction associated with $U(1)_\phi$ has a helical winding structure in the compact 2-dim field space of ϕ_i as depicted in figure 1. Then the periodicity of the flat direction is enlarged as

$$\Delta\phi = 2\pi\sqrt{n^2 f_1^2 + f_2^2} \equiv 2\pi f_{\text{eff}}, \tag{2.11}$$

which is larger than the original axion periodicities $2\pi f_1 \sim 2\pi f_2$ by the winding number n .

The relaxation shift symmetry $U(1)_\phi$ is slightly broken by small nonzero values of ϵ, ϵ' and Λ_{br} . Note that this particular form of $U(1)_\phi$ breaking is technically natural as long as the first condition of (2.5) is satisfied. To find the effective potential of the flat relaxation direction, one can rewrite the model in terms of the canonically normalized heavy and light axions [11, 12]:

$$\phi_H = \frac{f_2\phi_1 + nf_1\phi_2}{f_{\text{eff}}}, \quad \phi = \frac{nf_1\phi_1 - f_2\phi_2}{f_{\text{eff}}}, \tag{2.12}$$

for which

$$\begin{aligned}\frac{\phi_1}{f_1} &= n \frac{\phi}{f_{\text{eff}}} + \frac{f_2^2}{n^2 f_1^2 + f_2^2} \frac{\phi_H}{f_H} \\ \frac{\phi_2}{f_2} &= -\frac{\phi}{f_{\text{eff}}} + \frac{n f_1^2}{n^2 f_1^2 + f_2^2} \frac{\phi_H}{f_H},\end{aligned}\tag{2.13}$$

where $f_H = f_1 f_2 / f_{\text{eff}}$. In the limit $\Lambda^4 \gg \epsilon f_2^4 \gg \Lambda_{\text{br}}^4$, it is straightforward to integrate out the heavy axion ϕ_H to derive the low energy effective lagrangian of the light axion ϕ . The resulting effective potential of the canonically normalized ϕ is given by

$$\begin{aligned}V_{\text{eff}} &= -\epsilon f_2^4 \cos\left(\frac{\phi}{f_{\text{eff}}} - \delta_2\right) + \left(M_h^2 - \epsilon' f_2^2 \cos\left(\frac{\phi}{f_{\text{eff}}} - \delta_2'\right)\right) |h|^2 \\ &\quad - \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f} + \delta_1\right),\end{aligned}\tag{2.14}$$

where

$$f_{\text{eff}} = \sqrt{n^2 f_1^2 + f_2^2} \equiv n f.\tag{2.15}$$

We can now generalize the above two axion model to the case of $N > 2$ axions to enlarge the effective axion scale further [12]. The lagrangian density is given by

$$\mathcal{L} = \frac{1}{2} \sum_i (\partial_\mu \phi_i)^2 - \left(\tilde{V}_0 + V_0 + \mu_h^2 |h|^2 + V_{\text{br}} + \dots\right),\tag{2.16}$$

where

$$\begin{aligned}\tilde{V}_0 &= -\sum_{i=1}^{N-1} \Lambda_i^4 \cos\left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}}\right) \\ V_0 &= -\epsilon f_N^4 \cos\left(\frac{\phi_N}{f_N} + \delta_N\right), \\ \mu_h^2 &= M_h^2 - \epsilon' f_N^2 \cos\left(\frac{\phi_N}{f_N} + \delta_N'\right), \\ V_{\text{br}} &= -\Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi_1}{f_1} + \delta_1\right),\end{aligned}\tag{2.17}$$

with $\Lambda_i^4 \gg \epsilon f_N^4 \gg \Lambda_{\text{br}}^4$. The model involves the N axionic shift symmetries:

$$\text{U}(1)_i : \quad \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + c_i\tag{2.18}$$

which are broken by \tilde{V}_0 down to the relaxation shift symmetry:

$$\text{U}(1)_\phi : \quad \frac{\phi_i}{f_i} \rightarrow \frac{\phi_i}{f_i} + \gamma_i c \quad (\gamma_i = -n_i \gamma_{i+1}),\tag{2.19}$$

with the corresponding flat direction given by

$$\phi \propto \sum_{i=1}^N (-1)^{i-1} \left(\prod_{j=i}^{N-1} n_j\right) f_i \phi_i.\tag{2.20}$$

Tuning on small nonzero values of ϵ, ϵ' and Λ_{br} , the relaxion shift symmetry (2.19) is slightly broken and nontrivial potential of ϕ is developed. Although our way to break $U(1)_\phi$ is rather specific, it is technically natural as the model involves many continuous or discrete axionic shift symmetries which are distinguishing our particular way of symmetry breaking from other possibilities. It is again straightforward to integrate out the $(N - 1)$ heavy axions which receive a large mass from \tilde{V}_0 [12]. For the canonically normalized ϕ , the resulting effective potential is given by

$$\begin{aligned}
 V_{\text{eff}} &= V_0(\phi) + \mu_h^2(\phi)|h|^2 + V_{\text{br}}(h, \phi) \\
 &= -\epsilon f_N^4 \cos\left(\frac{\phi}{f_{\text{eff}}} + (-)^{N-1}\delta_N\right) + \left(M_h^2 - \epsilon' f_N^2 \cos\left(\frac{\phi}{f_{\text{eff}}} + (-)^{N-1}\delta'_N\right)\right)|h|^2 \\
 &\quad - \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f} + \delta_1\right), \tag{2.21}
 \end{aligned}$$

where

$$\begin{aligned}
 f_{\text{eff}} &= \sqrt{\sum_{i=1}^N \left(\prod_{j=i}^{N-1} n_j^2\right) f_i^2} \sim \left(\prod_{j=1}^{N-1} n_j\right) f_1 \sim e^{\xi N} f_1 \quad (\xi = \mathcal{O}(1)), \\
 f &= \frac{f_{\text{eff}}}{\left(\prod_{j=1}^{N-1} n_j\right)} \sim f_1. \tag{2.22}
 \end{aligned}$$

For simplicity here we assumed that all f_i are comparable to each other, or f_1 is the biggest among $\{f_i\}$.

Obviously, in the limit $N \gg 1$ the above relaxion potential involves two very different axion scales, an exponentially enhanced effective decay constant f_{eff} and another effective decay constant f which is comparable to the original decay constants f_i . Such a big difference between f_{eff} and f can be understood by noting that in order for the N -th axion ϕ_N to travel one period along the relaxion direction, i.e. $\Delta\phi_N = 2\pi f_N$, the other axions ϕ_i ($i = 1, 2, \dots, N - 1$) should experience a multiple windings as $\Delta\phi_i = 2\pi \left(\prod_{j=i}^{N-1} n_j\right) f_i$. This results in

$$\frac{\phi_i}{f_i} = (-1)^{i-1} \left(\prod_{j=i}^{N-1} n_j\right) \frac{\phi}{f_{\text{eff}}} + \dots, \tag{2.23}$$

where the ellipsis stands for the $(N - 1)$ heavy axions receiving a large mass from \tilde{V}_0 . For an illustration of this feature, we depict in figure 2 the relaxion field direction for the case of $N = 3, n_1 = 2, n_2 = 4$.

The effective potential (2.21) can easily realize the relaxion mechanism under several consistency conditions. First of all, like (2.5) of the two axion model, we need

$$\epsilon f_N^2 \gtrsim \mathcal{O}(\epsilon' M_h^2), \quad \epsilon' f_N^2 \gtrsim \mathcal{O}(M_h^2), \tag{2.24}$$

in order for the model to be stable against radiative corrections, while allowing $\mu_h = 0$ for certain value of ϕ . Without invoking any fine tuning, there is always a certain range of δ_N and δ'_N for which the relaxion rolls down toward the minimum of $V_0(\phi)$ starting from

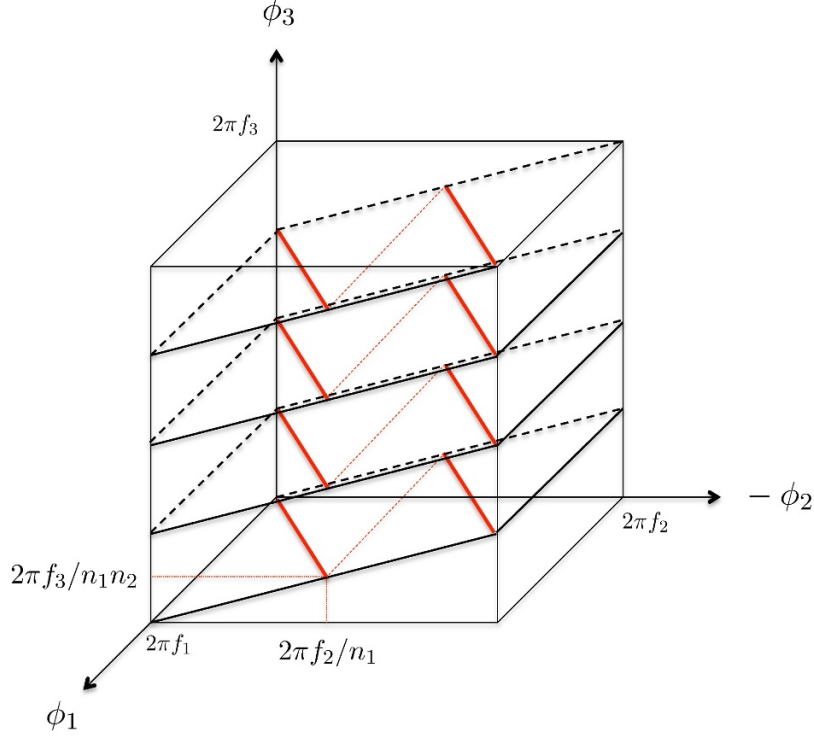


Figure 2. Flat relaxation direction in the three axion case with $n_1 = 2$ and $n_2 = 4$.

a generic initial value ϕ_0 with $\mu_h^2(\phi_0) = \mathcal{O}(M_h^2) > 0$. After a field excursion $\Delta\phi = \mathcal{O}(f_{\text{eff}})$, the relaxation is crossing $\mu_h(\phi) = 0$, and then a nonzero Higgs VEV is developed together with the back reaction potential stabilizing the relaxation at the value giving $\langle h \rangle = v$. The stabilization condition leads to

$$\frac{\epsilon f_N^4}{f_{\text{eff}}} \sim \frac{\Lambda_{\text{br}}^4(h=v)}{f}. \quad (2.25)$$

From (2.24), this then yields a lower bound on f_{eff} :

$$\frac{f_{\text{eff}}}{f} \gtrsim \frac{M_h^4}{\Lambda_{\text{br}}^4(h=v)} = \left(\frac{M_h}{v}\right)^4 \frac{v^4}{\Lambda_{\text{br}}^4(h=v)}, \quad (2.26)$$

where $v^4/\Lambda_{\text{br}}^4(h=v) \sim 10^{12}$ when V_{br} is generated by the QCD anomaly, or $v^4/\Lambda_{\text{br}}^4(h=v)$ has a model-dependent value not exceeding $\mathcal{O}(1)$ when V_{br} is generated by the hidden color dynamics which confines around the weak scale.

To summarize, in our scheme for the relaxation mechanism, $v \ll M_h$ can be technically natural with an exponential hierarchy between the two effective axion scales:

$$\frac{f_{\text{eff}}}{f} = \mathcal{O}(e^{\xi N}) \quad (\xi = \mathcal{O}(1)) \quad (2.27)$$

which is arising as a consequence of a series of mass mixing between nearby axions in the compact field space of N axions. Although it relies on a rather specific form of axion mass mixings, the scheme does not involve any fine tuning of continuous parameters, nor an unreasonably large discrete parameter.

3 A UV model with high scale supersymmetry

In this section, we construct an explicit UV completion of the N axion model discussed in the previous section. We first note that our scheme requires that the axion potential \tilde{V}_0 should dominate over the other part of the potential in (2.17) as it determines the key feature of the model, i.e. an exponentially long flat direction in the compact field space of N axions. Specifically we need

$$f_i^4 \gg |\tilde{V}_0| \gg |V_0| \gtrsim M_h^4. \quad (3.1)$$

On the other hand, we wish to have an explicit UV model providing the full part of the axion potential in (2.17), as well as a mechanism to generate the axion scales f_i . This implies that our UV model should allow the natural size of the Higgs boson mass, i.e. M_h , to be much lower than its cutoff scale. As SUSY provides a natural framework for this purpose, in the following we present a supersymmetric UV completion of the low energy effective potential (2.21).

First of all, to have N axions with the decay constants $f_i \ll M_{\text{Planck}}$, we introduce N global U(1) symmetries under which

$$\text{U}(1)_i : \quad X_i \rightarrow e^{i\beta_i} X_i, \quad Y_i \rightarrow e^{-3i\beta_i} Y_i \quad (i = 1, 2, \dots, N), \quad (3.2)$$

where X_i and Y_i are gauge-singlet chiral superfields with the $\text{U}(1)_i$ -invariant superpotential

$$W_1 = \sum_i \frac{X_i^3 Y_i}{M_*}, \quad (3.3)$$

where M_* corresponds to the cutoff scale of the model, which might be identified as the GUT scale or the Planck scale. Here and in the following, we ignore the dimensionless coefficients of order unity in the lagrangian. We assume that SUSY is softly broken with SUSY breaking soft masses

$$m_{\text{SUSY}} \sim M_h \ll M_*. \quad (3.4)$$

In particular, the model involves the soft SUSY breaking terms of X_i and Y_i , given by

$$\mathcal{L}_{\text{soft}} = -m_{X_i}^2 |X_i|^2 - m_{Y_i}^2 |Y_i|^2 + \left(A_i \frac{X_i^3 Y_i}{M_*} + \text{h.c.} \right), \quad (3.5)$$

where

$$m_{X_i} \sim m_{Y_i} \sim A_i \sim m_{\text{SUSY}}.$$

To achieve the N axions in the low energy limit, we need all $m_{X_i}^2$ are tachyonic, which results in

$$\langle X_i \rangle \equiv x_i \sim \sqrt{m_{\text{SUSY}} M_*}, \quad \langle Y_i \rangle \equiv y_i \sim \sqrt{m_{\text{SUSY}} M_*}. \quad (3.6)$$

Then the canonically normalized axion components ϕ_i can be identified as

$$X_i \propto e^{i\phi_i/f_i}, \quad Y_i \propto e^{-3i\phi_i/f_i} \quad (3.7)$$

with

$$f_i = \sqrt{2(x_i^2 + 9y_i^2)} \sim \sqrt{m_{\text{SUSY}} M_*}. \quad (3.8)$$

Now we need a dynamics to generate the axion potential \tilde{V}_0 in (2.17), developing an exponentially long flat direction as described in the previous section. For this purpose, we introduce $(N - 1)$ hidden Yang-Mills sectors associated with the gauge group $G = \prod_{i=1}^{N-1} \text{SU}(k_i)$, including also the charged matter fields

$$\Psi_i + \Psi_i^c, \quad \Phi_{ia} + \Phi_{ia}^c \quad (i = 1, 2, \dots, N - 1; a = 1, 2, \dots, n_i), \quad (3.9)$$

where Ψ_i and Φ_{ia} are the fundamental representation of $\text{SU}(k_i)$, while Ψ_i^c and Φ_{ia}^c are anti-fundamentals. These gauged charged matter fields couple to the $\text{U}(1)_i$ -breaking fields X_i through the superpotential

$$W_2 = \sum_{i=1}^{N-1} (X_i \Psi_i \Psi_i^c + X_{i+1} \Phi_{ia} \Phi_{ia}^c). \quad (3.10)$$

Note that X_i couples to a single flavor of the $\text{SU}(k_i)$ -charged hidden quark $\Psi_i + \Psi_i^c$, while X_{i+1} couples to n_i flavors of the $\text{SU}(k_i)$ -charged hidden quarks $\Phi_{ia} + \Phi_{ia}^c$. With this form of hidden Yang-Mills sectors, the N global $\text{U}(1)$ symmetries are explicitly broken down to a single $\text{U}(1)$ by the $\text{U}(1)_i \times \text{SU}(k_j) \times \text{SU}(k_j)$ anomalies. The charged matter fields $\Psi_i + \Psi_i^c$ and $\Phi_{ia} + \Phi_{ia}^c$ get a heavy mass of $\mathcal{O}(f_i)$, so can be integrated out at scales below f_i . This yields an axion-dependent threshold correction to the holomorphic gauge kinetic function τ_i of $\text{SU}(k_i)$ at scales below f_i :

$$\tau_i = \frac{1}{g_i^2} + \frac{i}{8\pi^2} \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) + \theta^2 M_{\lambda_i}, \quad (3.11)$$

where we ignored the dependence on $|X_i|$, while including the soft SUSY breaking by the gaugino masses $M_{\lambda_i} \sim m_{\text{SUSY}}$. As a consequence, at scales below f_i , the global symmetry breaking by the $\text{U}(1)_i \times \text{SU}(k_j) \times \text{SU}(k_j)$ anomalies is described by the following axion effective interactions:

$$\sum_{i=1}^{N-1} \frac{1}{32\pi^2} \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) (F \tilde{F})_{\text{SU}(k_i)}, \quad (3.12)$$

where $(F)_{\text{SU}(k_i)}$ denotes the gauge field strength of $\text{SU}(k_i)$ and $(\tilde{F})_{\text{SU}(k_i)}$ is its dual. As we wish to generate the axion potential $|\tilde{V}_0| \gg M_h^4 \sim m_{\text{SUSY}}^4$ from the above axion couplings, we assume

$$\tilde{\Lambda}_i \gg m_{\text{SUSY}}, \quad (3.13)$$

where $\tilde{\Lambda}_i$ denotes the confining scale of the hidden gauge group $\text{SU}(k_i)$. In such case, the resulting axion potential is determined by the non-perturbative effective superpotential describing the formation of the $\text{SU}(k_i)$ gaugino condensation [17]:

$$W_{\text{np}} \sim \langle \lambda_i \lambda_i \rangle \propto \left(e^{-8\pi^2 \tau_i} \right)^{1/k_i}, \quad (3.14)$$

yielding

$$\tilde{V}_0 = - \sum_{i=1}^{N-1} \Lambda_i^4 \cos \left(\frac{1}{k_i} \left(\frac{\phi_i}{f_i} + n_i \frac{\phi_{i+1}}{f_{i+1}} \right) \right) \quad (3.15)$$

with

$$\Lambda_i^4 \sim \frac{8\pi^2}{k_i} M_{\lambda_i} \tilde{\Lambda}_i^3. \quad (3.16)$$

Our next mission is to generate the axion potential V_0 and the axion-dependent Higgs mass-square μ_h^2 in (2.17), driving the evolution of the relaxion during the inflationary epoch, while scanning the Higgs mass-square from an initial value of $\mathcal{O}(m_{\text{SUSY}}^2)$ to zero. This can be done by introducing a superpotential term given by

$$W_3 = \left(\frac{X_{N-1}^2}{M_*} + \frac{X_N^2}{M_*} \right) H_u H_d, \quad (3.17)$$

together with the associated Kähler potential term:

$$\Delta K = \frac{X_{N-1}^2 X_N^{*2}}{M_*^2} + \text{h.c.} \quad (3.18)$$

Here we ignore the irrelevant terms such as $|X_N|^4$ or $|X_{N-1}|^4$ in the Kähler potential. Note that the couplings in W_3 leads to a logarithmically divergent radiative correction to ΔK [18], and our model is stable against such radiative correction as long as the coefficient of ΔK is of order unity. Note also that W_3 provides a solution to the MSSM μ problem as it yields naturally the Higgsino mass $\mu_{\text{eff}} \sim m_{\text{SUSY}}$ [13–15].

After integrating out the $(N-1)$ axions which receive a heavy mass from \tilde{V}_0 , while leaving the light relaxion ϕ as described in the previous section, the Kähler potential term ΔK gives rise to

$$V_0 = -m_0^4 \cos \left(2(n_{N-1} + 1) \frac{\phi}{f_{\text{eff}}} + \delta \right), \quad (3.19)$$

where

$$m_0^4 \sim \frac{f_{N-1}^2 f_N^2}{M_*^2} m_{\text{SUSY}}^2 \sim m_{\text{SUSY}}^4, \quad (3.20)$$

$$f_{\text{eff}} = \sqrt{\sum_{i=1}^N \left(\prod_{j=i}^{N-1} n_j^2 \right) f_i^2} \sim \left(\prod_{j=1}^{N-1} n_j \right) f_1,$$

and δ is a phase angle which is generically of order unity. In our scheme, the MSSM Higgsino mass μ_{eff} originates from W_3 , and therefore is naturally of the order of m_{SUSY} [13–15]. Again, after integrating out the $(N-1)$ heavy axions, we find the MSSM Higgs parameters μ_{eff} and $B\mu_{\text{eff}}$ depend on the relaxion ϕ as

$$\begin{aligned} \mu_{\text{eff}} &= \mu_{N-1} \exp(-i2n_{N-1}\phi/f_{\text{eff}}) + \mu_N \exp(i2\phi/f_{\text{eff}}), \\ B\mu_{\text{eff}} &= b_{N-1} \exp(-i2n_{N-1}\phi/f_{\text{eff}}) + b_N \exp(i2\phi/f_{\text{eff}}), \end{aligned} \quad (3.21)$$

where

$$|\mu_N| \sim |\mu_{N-1}| \sim \frac{f^2}{M_*} \sim m_{\text{SUSY}}, \quad |b_N| \sim |b_{N-1}| \sim m_{\text{SUSY}}^2. \quad (3.22)$$

Then the determinant of the MSSM Higgs mass matrix

$$\mathcal{D} = (m_{H_u}^2 + |\mu_{\text{eff}}|^2)(m_{H_d}^2 + |\mu_{\text{eff}}|^2) - |B\mu_{\text{eff}}|^2 \quad (3.23)$$

also depends on ϕ via

$$\begin{aligned} |\mu_{\text{eff}}|^2 &= |\mu_N|^2 + |\mu_{N-1}|^2 + 2|\mu_N\mu_{N-1}| \cos\left(2(n_{N-1} + 1)\frac{\phi}{f_{\text{eff}}} + \delta_{\mu_N} - \delta_{\mu_{N-1}}\right), \\ |B\mu_{\text{eff}}|^2 &= |b_N|^2 + |b_{N-1}|^2 + 2|b_N b_{N-1}| \cos\left(2(n_{N-1} + 1)\frac{\phi}{f_{\text{eff}}} + \delta_{b_N} - \delta_{b_{N-1}}\right), \end{aligned} \quad (3.24)$$

where δ_μ and δ_b are the phases of μ and b , respectively. Obviously, for an appropriate range of δ_μ and δ_b , the determinant \mathcal{D} can flip its sign from positive to negative as the relaxion experiences an excursion of $\mathcal{O}(f_{\text{eff}})$. Once the relaxion is stabilized near the point of $\mathcal{D} = 0$, the MSSM Higgs doublets H_u and H_d can be decomposed into the light SM Higgs h with a mass of $\mathcal{O}(v)$ and the other heavy Higgs bosons having a mass of the order of $m_{\text{SUSY}} \gg v$.

To complete the model, we need to generate the back reaction potential V_{br} . In regard to this, we simply adopt the schemes suggested in [1]. One option is to generate V_{br} through the QCD anomaly. For this, one can introduce

$$W_{\text{br}} = X_1 Q Q^c, \quad (3.25)$$

where $Q + Q^c$ is an exotic quark which receive a heavy mass by $\langle X_1 \rangle \sim f_1$. Once this heavy quark is integrated out, the axion ϕ_1 couples to the gluons as

$$\frac{1}{32\pi^2} \frac{\phi_1}{f_1} (F\tilde{F})_{\text{QCD}}. \quad (3.26)$$

After the $(N - 1)$ heavy axions are integrated out, this leads to the relaxion-gluon coupling

$$\frac{1}{32\pi^2} \frac{\phi}{f} (F\tilde{F})_{\text{QCD}}, \quad (3.27)$$

where

$$f = \frac{f_{\text{eff}}}{\left(\prod_{j=1}^{N-1} n_j\right)} \sim f_1. \quad (3.28)$$

Then the resulting back reaction potential is obtained to be

$$V_{\text{br}}(h, \phi) \approx -y_u \Lambda_{\text{QCD}}^3 h \cos\left(\frac{\phi}{f} + \delta_{\text{br}}\right), \quad (3.29)$$

where y_u is the up quark Yukawa coupling to the SM Higgs field h , and δ_{br} is a phase angle of order unity.

Alternatively, we can consider a back reaction potential generated by an $\text{SU}(n_{HC})$ hidden color gauge interaction which confines at scales around the weak scale [1, 16]. For this, one can introduce the hidden colored matter superfields

$$L + L^c, \quad N + N^c \quad (3.30)$$

with the superpotential couplings

$$W_{\text{br}} = \kappa_1 \frac{X_1^2}{M_*} LL^c + \kappa_u H_u LN^c + \kappa_d H_d L^c N, \quad (3.31)$$

where L is an $SU(n_{HC})$ -fundamental and $SU(2)_L$ -doublet with the $U(1)_Y$ charge $1/2$, L^c is its conjugate representation, N is an $SU(n_{HC})$ -fundamental but $SU(2)_L \times U(1)_Y$ singlet, and N^c is its conjugate representation. At scales below m_{SUSY} , all superpartners can be integrated out, leaving the following Yukawa interactions between the relevant light degrees of freedom:

$$\mathcal{L}_{\text{br}} = m_L e^{2i\phi_1/f_1} LL^c + \kappa_u \sin \beta h LN^c + \kappa_d \cos \beta h^\dagger L^c N + m_N NN^c, \quad (3.32)$$

where $L + L^c$ and $N + N^c$ denote the fermion components of the original superfields, $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$, and

$$m_L \sim \kappa_1 \frac{f_1^2}{M_*} \sim \kappa_1 m_{\text{SUSY}} \quad (3.33)$$

is presumed to be lighter than m_{SUSY} . Note that a nonzero Dirac mass of $N + N^c$ is induced by radiative corrections below m_{SUSY} , giving

$$m_N \sim \frac{1}{16\pi^2} \sin(2\beta) \kappa_u \kappa_d m_L^* e^{-2i\phi_1/f_1} \ln \left(\frac{m_{\text{SUSY}}}{m_L} \right). \quad (3.34)$$

Now this effective theory at scales below m_{SUSY} corresponds to the non-QCD model proposed in [1, 16], yielding a back reaction potential of the form

$$V_{\text{br}} = m_1^2 h h^\dagger \cos \left(2\frac{\phi}{f} + \delta_1 \right) + m_2^4 \cos \left(2\frac{\phi}{f} + \delta_2 \right), \quad (3.35)$$

where we have expressed the axion component ϕ_1 in terms of the light relaxion field ϕ , and

$$m_1^2 \sim \frac{\kappa_u \kappa_d \sin(2\beta)}{m_L} \Lambda_{\text{HC}}^3, \quad m_2^4 \sim m_N \Lambda_{\text{HC}}^3 \quad (3.36)$$

for the $SU(n_{HC})$ confinement scale Λ_{HC} . If $m_2^4 < m_1^2 v^2$ with $m_1^2 \lesssim \mathcal{O}(v^2)$, which can be achieved for $m_L < 4\pi v$ [1], this back reaction potential can successfully stabilize the relaxion at a value giving $v = \langle h \rangle \ll m_{\text{SUSY}}$.

4 Conclusion

In this paper, we have addressed the problem of huge scale hierarchy in the relaxion mechanism, i.e. a relaxion excursion $\Delta\phi \sim 2\pi f_{\text{eff}}$ which is bigger than the period $2\pi f$ of the back reaction potential by many orders of magnitudes. We proposed a scheme to yield an exponentially long relaxion direction within the compact field space of N periodic axions with decay constants well below the Planck scale, giving $f_{\text{eff}}/f \sim e^{\xi N}$ with $\xi = \mathcal{O}(1)$. Although it relies on a specific form of the mass mixing between nearby axions, our scheme does not involve any fine tuning of continuous parameters, nor an unreasonably large discrete parameter. Furthermore, our scheme finds a natural UV completion in high scale or (mini)

split SUSY scenario, in which all decay constants of the N periodic axions are generated by SUSY breaking as $f_i \sim \sqrt{m_{\text{SUSY}} M_*}$, where m_{SUSY} denotes the soft SUSY breaking scalar masses and M_* is the fundamental scale such as the Planck scale or the GUT scale. In our model, the required relaxion potential and the relaxion-dependent Higgs boson mass are generated through a superpotential term providing a natural solution to the MSSM μ -problem.

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