Published for SISSA by Springer

Received: *November 24, 2015*
Accepted: *December 24, 2015*
Published: *January 7, 2016*

Renormalization-scale uncertainty in the decay rate of false vacuum

Motoi Endo,[a,b] Takeo Moroi,[a,b] Mihoko M. Nojiri[b,c,d] and Yutaro Shoji[a]

[a] *Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*
[b] *Kavli IPMU (WPI), University of Tokyo, Kashiwa, Chiba 277-8583, Japan*
[c] *KEK Theory Center, IPNS, KEK, Tsukuba, Ibaraki 305-0801, Japan*
[d] *SOKENDAI (The Graduate University for Advanced Studies), Tsukuba, Ibaraki 305-0801, Japan*

E-mail: endo@hep-th.phys.s.u-tokyo.ac.jp, moroi@phys.s.u-tokyo.ac.jp, nojiri@post.kek.jp, yshoji@hep-th.phys.s.u-tokyo.ac.jp

Abstract: We study radiative corrections to the decay rate of false vacua, paying particular attention to the renormalization-scale dependence of the decay rate. The decay rate exponentially depends on the bounce action. The bounce action itself is renormalization-scale dependent. To make the decay rate scale-independent, radiative corrections, which are due to the field fluctuations around the bounce, have to be included. We show quantitatively that the inclusion of the fluctuations suppresses the scale dependence, and hence is important for the precise calculation of the decay rate. We also apply our analysis to a supersymmetric model and show that the radiative corrections are important for the Higgs-stau system with charge breaking minima.

Keywords: Supersymmetry Phenomenology

ArXiv ePrint: 1511.04860

doi:10.1007/JHEP01(2016)031

Contents

1 Introduction

In particle physics and cosmology, decay of false vacua is an important subject. For example, with the observed Higgs and top masses, it has been known that the Higgs quartic coupling constant becomes negative above $\sim 10^{10}$ GeV if the standard model (SM) is a good effective theory up to the scale [1]. Then the electroweak symmetry breaking (EWSB) vacuum is a false vacuum. Even if there exists a true vacuum other than the EWSB vacuum, we may still live in the EWSB vacuum as long as the lifetime of the EWSB vacuum is longer than the present cosmic time. In models beyond the SM, the EWSB vacuum may still be a false vacuum. For example, in supersymmetric (SUSY) models, there may exist a color and/or charge breaking (CCB) vacuum (at which some of the superpartners of quarks and/or leptons acquire non-vanishing expectation values) whose vacuum energy is lower than that of the EWSB vacuum. Existence of such a CCB vacuum imposes important and stringent bounds on SUSY models [2–5].

Precise calculation of the decay rate of the false vacua is important from both theoretical and phenomenological points of view. The procedure to calculate the decay rate was formulated in [6, 7], in which the decay rate is evaluated by performing the path integral around the saddle-point solution (i.e., so-called the "bounce") of the equation of motion in the Euclidean field theory. Given the bounce solution, the decay rate per unit volume is given by

$$\gamma \equiv \mathcal{A} e^{-\mathcal{B}}, \tag{1.1}$$

where $\mathcal{B}$ is the bounce action, which is the Euclidean action of the bounce solution, while the prefactor $\mathcal{A}$ takes account of the effects of fluctuations around the bounce. In many analyses, $\mathcal{B}$ has been evaluated from the tree-level Lagrangian, while an order-of-magnitude estimate has been adopted for $\mathcal{A}$. The main subject of this paper is the calculation of $\mathcal{A}$, which is important to determine the overall scale of the decay rate. Another motivation of the calculation comes from the scale independence of the decay rate. $\mathcal{B}$ inevitably depends on the renormalization scale Q at which the tree-level parameters in the Lagrangian are defined. As we will see, the scale dependence of $\mathcal{B}$ can be sizable. The decay rate of the false vacuum is a physical quantity, and therefore, the scale dependence should be cancelled in the expression of $\gamma = \mathcal{A}e^{-\mathcal{B}}$.

In this paper, we discuss the calculation of the decay rate of false vacua, paying particular attention to the renormalization-scale dependence of the decay rate γ. In section 2, we summarize the formalism to calculate the prefactor. In sections 3 and 4, we perform numerical calculations of the decay rate γ for a simple model of a real scalar field and Higgs-stau system in the minimal SUSY SM (MSSM), respectively. We show that, in those models, $\mathcal{B}$ has sizable dependence on Q, while the scale dependence of $\gamma = \mathcal{A}e^{-\mathcal{B}}$ becomes weak once the effect of the prefactor $\mathcal{A}$ is properly taken into account. Section 5 is devoted to the summary of this paper.

2 Formalism

In order to calculate the decay rate of false vacua, we follow the procedure given in [6–12]. In the calculation, the bounce solution plays an important role. The bounce is the solution of the classical field equations that interpolates between the false and true vacua. It is an $O(4)$ symmetric solution of the four-dimensional Euclidean equation of motion, and it only depends on the radial distance in the Euclidean space $r = \sqrt{x_\mu x_\mu}$. In the following, the bounce solution is denoted as $\sigma(r)$ (or σ_i, when we need to specify the individual fields). We also denote the expectation value of the scalar field at the false vacuum as $\bar{\sigma} \equiv \sigma(r \to \infty)$.

Hereafter, we calculate the prefactor $\mathcal{A}$ at the one-loop level. We consider the prefactor arising from the coupling of the bounce to scalar and spinor fluctuations. Then, the prefactor $\mathcal{A}$ can be decomposed as

$$\mathcal{A} = \frac{\mathcal{B}^2}{4\pi^2} A'_\phi A_\psi, \tag{2.1}$$

where A'_ϕ and A_ψ are scalar- and fermion-loop contributions, respectively. As we see below, A_ψ is dimensionless, while the mass dimension of A'_ϕ is four.

We assume the bosonic contribution arises from the Euclidean Lagrangian of the following form:

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu \phi_i \partial_\mu \phi_i + V(\sigma, \phi_i), \tag{2.2}$$

where ϕ_i denotes scalar fluctuations around the bounce solution $\sigma(r)$, and V is the scalar potential. We take the basis of the scalar fields such that each ϕ_i becomes a mass eigenstate around the false vacuum. Then,

$$A'_\phi = \left| \frac{\mathrm{Det}'[-\partial^2 + V_{ij}(\sigma)]}{\mathrm{Det}[-\partial^2 + \bar{V}_{ij}]} \right|^{-1/2} e^{-S_\phi^{(\mathrm{c.t.})}}, \tag{2.3}$$

where

$$V_{ij}(\sigma) \equiv \left. \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \right|_{\phi=0}, \tag{2.4}$$

$\bar{V}_{ij} \equiv V_{ij}(\bar{\sigma})$, and $S_\phi^{(\text{c.t.})}$ is the counter term to remove the divergences due to ϕ_i. In addition, Det$'$ is the functional determinant with omitting four zero-eigenvalues associated with the translation of the bounce solution. Then, the mass dimension of A'_ϕ is four, that is the mass dimension of γ. A'_ϕ is often estimated to be the fourth power of a typical mass scale in the bounce.

The fermionic part of the Euclidean Lagrangian is denoted as

$$\mathcal{L}_\psi = \bar{\psi}\gamma_\mu\partial_\mu\psi + M(\sigma)\bar{\psi}\psi, \tag{2.5}$$

where γ_μ is the γ-matrix, satisfying the anti-commutation relation as $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. Then, fermionic contribution is given by

$$A_\psi = \left[\frac{\text{Det}[-(\partial\!\!\!/ + M)(\partial\!\!\!/ - M)]}{\text{Det}[-\partial^2 + \bar{M}^2]}\right]^{1/2} e^{-S_\psi^{(\text{c.t.})}}, \tag{2.6}$$

where $\bar{M} \equiv M(\bar{\sigma})$, and $S_\psi^{(\text{c.t.})}$ is the counter term.

We first discuss the effect of fluctuations which are not related to the zero-modes. As shown in eqs. (2.3) and (2.6), the prefactor $\mathcal{A}$ is related to the following quantity:

$$A_\varphi = \left(\text{Det}\left[\frac{-\partial^2 + W(r)}{-\partial^2 + \bar{W}}\right]\right)^{(-1)^{F+1}/2} e^{-S_\varphi^{(\text{c.t.})}}, \tag{2.7}$$

with $\varphi = \phi$ or ψ, where $(-1)^F = +1$ and -1 for boson and fermion, respectively, $S_\varphi^{(\text{c.t.})}$ is the counter term, and $\bar{W} \equiv W(r \to \infty)$, which is the value at the false vacuum.

We can obtain a formal expression of A_φ. Expanding W as

$$W(r) = \bar{W} + \delta W(r), \tag{2.8}$$

we obtain

$$\ln A_\varphi = -\sum_{p=1}^{\infty} s_\varphi^{(p)}, \tag{2.9}$$

where

$$s_\varphi^{(p)} \equiv \frac{(-1)^{F+p+1}}{2p}\text{Tr}\left[\delta W \frac{1}{-\partial^2 + \bar{W}}\right]^p + (\text{counter term}), \tag{2.10}$$

with "Tr" denoting the functional trace. In addition, divergences appear only for $p = 1$ and 2, and hence the counter term contributions do not appear for $p \geq 3$. In our analysis, $s_\varphi^{(1)}$ and $s_\varphi^{(2)}$ are evaluated by performing the momentum integration with the $\overline{\text{MS}}$ scheme:

$$s_\varphi^{(1)} = (-1)^{F+1}\sum_i \delta\tilde{W}_{ii}(0)\frac{\bar{W}_{ii}}{32\pi^2}\left[1 - \ln\frac{\bar{W}_{ii}}{Q^2}\right], \tag{2.11}$$

$$\begin{aligned} s_\varphi^{(2)} = {} & (-1)^{F+1}\frac{1}{512\pi^4}\sum_{i,j}\int dk\, k^3 \delta\tilde{W}_{ij}(k)\delta\tilde{W}_{ji}(k) \\ & \times\left[2 - \frac{1}{2}\ln\frac{\bar{W}_{ii}\bar{W}_{jj}}{Q^4} + \frac{\bar{W}_{ii} - \bar{W}_{jj}}{2k^2}\ln\frac{\bar{W}_{ii}}{\bar{W}_{jj}} - \frac{\omega^2}{2k^2}\ln\frac{k^2 + \bar{W}_{ii} + \bar{W}_{jj} + \omega^2}{k^2 + \bar{W}_{ii} + \bar{W}_{jj} - \omega^2}\right], \end{aligned} \tag{2.12}$$

where

$$\omega^2 = \sqrt{(\bar{W}_{ii} + \bar{W}_{jj} + k^2)^2 - 4\bar{W}_{ii}\bar{W}_{jj}}, \tag{2.13}$$

$$\delta\tilde{W}(k) = \frac{4\pi^2}{k}\int dr\, r^2 \delta W(r) J_1(kr), \tag{2.14}$$

with $J_1(x)$ being the modified Bessel function of the first kind. Notice that $\bar{W}$ is a diagonal matrix in our choice of the basis.

Next, let us consider the finite part, i.e., $\sum_{p\geq 3} s_\varphi^{(p)}$. Because the bounce solution has O(4) symmetry, the eigenfunctions of the operator $(-\partial^2 + W)$ can be characterized by the quantum numbers for the rotational group of the four-dimensional Euclidean space, i.e., $\mathrm{SU}(2)_A\times\, \mathrm{SU}(2)_B$. We denote the spin operators for $\mathrm{SU}(2)_A$ and $\mathrm{SU}(2)_B$ as $\hat{A}_i$ and $\hat{B}_i$, respectively, and the eigenvalues of $(\hat{A}^2, \hat{A}_3, \hat{B}^2, \hat{B}_3)$ are denoted as (j_A, m_A, j_B, m_B); $j_A = j_B$ for scalars, and $j_A = j_B \pm \frac{1}{2}$ for fermions. Hereafter, we denote

$$J \equiv \min(j_A, j_B), \tag{2.15}$$

which takes the values of $J = 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ \cdots$. Then, the functional determinant of our interest can be decomposed into the contributions of each J as

$$\mathrm{Det}\left[\frac{-\partial^2 + W}{-\partial^2 + \bar{W}}\right] = \prod_J \mathrm{Det}\left[\frac{-\Delta_J + W}{-\Delta_J + \bar{W}}\right], \tag{2.16}$$

where Δ_J is the four-dimensional Laplace operator acting on the mode with $J = \min(j_A, j_B)$. For scalars,

$$[\Delta_J - W]_\phi = \partial_r^2 + \frac{3}{r}\partial_r - \frac{2J(2J+2)}{r^2} - V_{ij}, \tag{2.17}$$

and for fermions,

$$[\Delta_J - W]_\psi = \partial_r^2 + \frac{3}{r}\partial_r - \begin{pmatrix} 2J(2J+2)r^{-2} + M^2 & \partial_r M \\ \partial_r M & (2J+1)(2J+3)r^{-2} + M^2 \end{pmatrix}. \tag{2.18}$$

Using the technique given in [13–19], it is possible to express the determinant as follows,[1]

$$\mathrm{Det}\left[\frac{-\Delta_J + W}{-\Delta_J + \bar{W}}\right] = \det(\varphi_J/\bar{\varphi}_J)^{N_J}\big|_{r=\infty}, \tag{2.19}$$

where N_J is the degeneracy; $N_J = (2J+1)^2$ for a scalar, and $N_J = 2(2J+1)(2J+2)$ for a fermion. Notice that the factor of 2 in N_J for fermions originates from two choices of $j_A = j_B - \frac{1}{2}$ and $j_A = j_B + \frac{1}{2}$. In addition, φ_J is the function, which is regular in $r = 0$, obeying the following equation:

$$[\Delta_J - W(r)]\varphi_J(r) = 0. \tag{2.20}$$

[1] Here and hereafter, $(\varphi_J/\bar{\varphi}_J)$ should be understood as the product $\varphi_J\bar{\varphi}_J^{-1}$ if φ_J and $\bar{\varphi}_J$ are matrices.

JHEP01(2016)031

The function $\bar{\varphi}_J$, which has the same boundary condition as φ_J at $r=0$, is obtained from eq. (2.20) with W being replaced by $\bar{W}$. We define the function $\varphi_J^{(p)}$ which obeys

$$[\Delta_J - \bar{W}]\varphi_J^{(p)} = \delta W \varphi_J^{(p-1)}, \quad (p \geq 1), \tag{2.21}$$

with $\varphi_J^{(0)} = \bar{\varphi}_J$. Then, $\varphi_J = \sum_{p=0}^{\infty} \varphi_J^{(p)}$, and the following relation holds:

$$\sum_{p\geq 3} s_\varphi^{(p)} = \frac{(-1)^F}{2} \sum_J N_J \left[\mathrm{tr}\ln(\varphi_J/\bar{\varphi}_J) - \tilde{\varphi}_J\right]\Big|_{r=\infty}, \tag{2.22}$$

where

$$\tilde{\varphi}_J \equiv \mathrm{tr}\left[(\varphi_J^{(1)}/\bar{\varphi}_J) - \frac{1}{2}(\varphi_J^{(1)}/\bar{\varphi}_J)^2 + (\varphi_J^{(2)}/\bar{\varphi}_J)\right]. \tag{2.23}$$

Using eqs. (2.11), (2.12), and (2.22), A_φ is given by

$$A_\varphi = e^{-s_\varphi^{(1)} - s_\varphi^{(2)}} \prod_J \left[\det(\varphi_J/\bar{\varphi}_J)e^{-\tilde{\varphi}_J}\right]^{(-1)^{F+1}N_J/2}\Big|_{r=\infty}. \tag{2.24}$$

This expression can be used for numerical calculations. Importantly, the quantities $s_\varphi^{(1)}$ and $s_\varphi^{(2)}$ are finite, while the quantity $\det(\varphi_J/\bar{\varphi}_J)e^{-\tilde{\varphi}_J}$ approaches to 1 as $J \to \infty$, which make it possible to numerically evaluate A_φ with eq. (2.24).

In general, the bounce action cannot be expressed by analytic functions. For our numerical calculations in the following sections, we use `CosmoTransitions` 2.01a [20] to determine the bounce solution as well as $\mathcal{B}$.

In the calculation of γ, the zero-eigenvalues in association with the translation of the bounce should be eliminated from the functional determinant. For this purpose, we add a small constant w to the function $W(r)$ in eq. (2.20) without changing the bounce. With w being small enough, the functional determinant given in eq. (2.19) is proportional to $w^{n_0/2}$, where n_0 is the number of zero-modes. The zero-eigenvalues can be omitted with dividing the functional determinant (for non-vanishing w) by $w^{n_0/2}$ and taking $w \to 0$.

Due to the zero-modes associated with the translation of the bounce, A_ϕ given by eq. (2.24) is proportional to w^{-2} (if there is no other zero-mode). Thus, A_ϕ diverges as $w \to 0$; such a behavior is related to the infinite space-time volume. The dependence of $A_\phi \propto w^{-2}$ originates from the relation of $\det[\phi_{1/2}(r;w)/\bar{\phi}_{1/2}(r)]_{r=\infty, w\to 0} \propto w$, where $\phi_J(r;w)$ obeys

$$\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{2J(2J+2)}{r^2} - W(r) - w\right]\phi_J(r;w) = 0. \tag{2.25}$$

Notice that the zero-modes are involved in the modes with $J = \frac{1}{2}$. After omitting the zero-eigenvalues, we obtain

$$A'_\phi = e^{-s_\phi^{(1)} - s_\phi^{(2)}} \left[\lim_{w\to 0} \det\left(\frac{\partial_w \phi_{1/2}(r;w)}{\bar{\phi}_{1/2}}\right) e^{-\tilde{\phi}_{1/2}}\right]^{-2} \times \prod_{J\neq 1/2} \left[\det(\phi_J/\bar{\phi}_J)e^{-\tilde{\phi}_J}\right]^{-(2J+1)^2/2}\Big|_{r=\infty}. \tag{2.26}$$

Combining eqs. (2.1), (2.24), and (2.26), the decay rate γ is obtained. Defining

$$S_{\rm tot} \equiv \mathcal{B} + \Delta S_\phi + \Delta S_\psi, \tag{2.27}$$

with

$$\Delta S_\phi \equiv - \ln \left[\frac{\mathcal{B}^2}{4\pi^2} \frac{A'_\phi}{\Lambda^4} \right], \tag{2.28}$$

$$\Delta S_\psi \equiv - \ln A_\psi, \tag{2.29}$$

the decay rate is given by

$$\gamma = \Lambda^4 e^{-S_{\rm tot}}, \tag{2.30}$$

where Λ is an arbitrary scale. Notice that the mass dimension in the bracket of ΔS_ϕ is zero, while γ is independent of Λ. Taking $\Lambda = 100\,\mathrm{GeV}$, for example, $S_{\rm tot}$ is required to be larger than 4.0×10^2 to make the quantity $H_0^{-4}\gamma$ smaller than 1, where $H_0 \simeq 67\,\mathrm{km/sec/Mpc}$ [21] is the expansion rate of the present universe.

The prefactor $\mathcal{A}$ depends on the renormalization scale via $e^{-s_\varphi^{(1)} - s_\varphi^{(2)}}$. Such a scale dependence is necessary to make the decay rate scale-independent. Indeed, in the calculation of the decay rate $\gamma = \mathcal{A}e^{-\mathcal{B}}$, the renormalization-scale dependence of $\mathcal{B}$ is compensated by that of $\mathcal{A}$. The cancellation of the scale dependence at the leading order of $\ln Q$ is shown in the appendix.[2] We also comment that $\sum_{p\geq 3} s_\varphi^{(p)}$ can be as large as $s_\varphi^{(1)}$ and $s_\varphi^{(2)}$. Thus, the calculation of both the divergent and convergent parts of the prefactor $\mathcal{A}$ is needed. Explicit calculations of the prefactor $\mathcal{A}$, including the finite part, are performed in the next sections.

Before closing this section, we comment on the zero-modes in association with the spontaneous breaking of global symmetry. In the following analysis, we also consider the case where a U(1) global symmetry preserved in the false vacuum is broken by the bounce solution. In such a case, another zero-mode appears, which is related to the U(1) transformation of the bounce. The path integral for such a zero-mode can be performed as an integration over the parameter space of the U(1) group. The zero-mode is involved in the $J = 0$ mode, and its effect can be taken care of with the following replacement [23]:

$$\det \left(\phi_0/\bar{\phi}_0\right)^{-1/2}\Big|_{r=\infty} \to 2\pi \sqrt{\int \frac{d^4x}{2\pi} \sum_i q_i^2 \sigma_i^2} \left[\lim_{w\to 0} \det \left(\frac{\partial_w \phi_0(r;w)}{\bar{\phi}_0(r)} \right) \right]^{-1/2} \Bigg|_{r=\infty}, \tag{2.31}$$

where q_i is the charge of the complex scalar field whose real component contains σ_i. The normalization of the U(1) charge is fixed so that the volume of the U(1) group is equal to 2π.

3 Model 1: model with a real scalar field

First let us consider the simplest example with a real scalar field $\Phi = \sigma + \phi$, where σ and ϕ are the bounce and the fluctuation around the bounce, respectively. The scalar potential is

$$V(\Phi) = -\xi_\Phi \Phi + \frac{1}{2} m_\Phi^2 \Phi^2 - \frac{1}{2} T_\Phi \Phi^3 + \frac{1}{8} \lambda_\Phi \Phi^4, \tag{3.1}$$

[2] Discussion on the scale dependence of the decay rate based on the effective potential is given in [22].

with $m_\Phi^2 > 0$ and $\lambda_\Phi > 0$. The bounce σ obeys the following equation:

$$\partial_r^2 \sigma + \frac{3}{r}\partial_r \sigma - \frac{\partial V(\sigma)}{\partial \sigma} = 0. \tag{3.2}$$

We concentrate on the case where the false vacuum is the one around $\Phi = 0$; such a situation is realized for $T_\Phi^2 \gtrsim \lambda_\Phi m_\Phi^2$. The renormalization group equations (RGEs) of the Lagrangian parameters are given by

$$\frac{d\xi_\Phi}{d\ln Q} = \frac{3}{16\pi^2} T_\Phi m_\Phi^2, \tag{3.3}$$

$$\frac{dm_\Phi^2}{d\ln Q} = \frac{3}{16\pi^2}(\lambda_\Phi m_\Phi^2 + 3T_\Phi^2), \tag{3.4}$$

$$\frac{dT_\Phi}{d\ln Q} = \frac{9}{16\pi^2}\lambda_\Phi T_\Phi, \tag{3.5}$$

$$\frac{d\lambda_\Phi}{d\ln Q} = \frac{9}{16\pi^2}\lambda_\Phi^2. \tag{3.6}$$

We calculate the bounce solution with the potential given in eq. (3.1) by varying the renormalization scale. Here, we adopt the following renormalization condition:

$$\xi_\Phi(Q_0) = 0, \tag{3.7}$$

$$m_\Phi^2(Q_0) = m^2, \tag{3.8}$$

$$T_\Phi(Q_0) = T, \tag{3.9}$$

$$\lambda_\Phi(Q_0) = \lambda, \tag{3.10}$$

with $Q_0 = m$. The parameters m and $\lambda(< 1)$ are positive. The Lagrangian parameters for different scale are evaluated by using the RGEs given in eqs. (3.3)–(3.6). We set $T = m$ and $\lambda = 0.6$ in our numerical calculation.

The model involves various mass scales; the scalar mass is m, the true vacuum is at $\sigma \simeq 4.2m$, the potential energy at the true vacuum is $|V|^{1/4} \simeq 1.5m$, the field value at $r = 0$ is $\sigma(0) \simeq 3.7m$, and the barrier hight is $|V|^{1/4} \simeq 0.6m$ at $\Phi \simeq 0.8m$. Since the scales distribute in a wide range, it is difficult to determine which is appropriate for the renormalization scale. We vary the renormalization scale in the range, $Q/m = 0.5$–5.

In figure 1, we plot $\mathcal{B}$ as a function of the renormalization scale, Q. The value of $\mathcal{B}$ has sizable dependence on Q. Hence, it is important to properly calculate the prefactor $\mathcal{A}$ in order to reduce the renormalization-scale uncertainty as well as to determine the overall scale of the decay rate.

Following the procedure explained in the previous section, we calculate $S_{\rm tot}$ in eq. (2.27). The result is also plotted in figure 1. When the prefactor $\mathcal{A}$ is calculated at the one-loop level, the renormalization-scale uncertainty is significantly reduced; $\mathcal{B}$ changes between 404 and 373 for $Q/m = 0.5$–5, while $S_{\rm tot} = \mathcal{B} + \Delta S_\phi$ is stable at $S_{\rm tot} \simeq 400$. Thus, the study of this simple model shows that the proper inclusion of the prefactor $\mathcal{A}$ is necessary for an accurate estimation of the decay rate γ.

JHEP01(2016)031

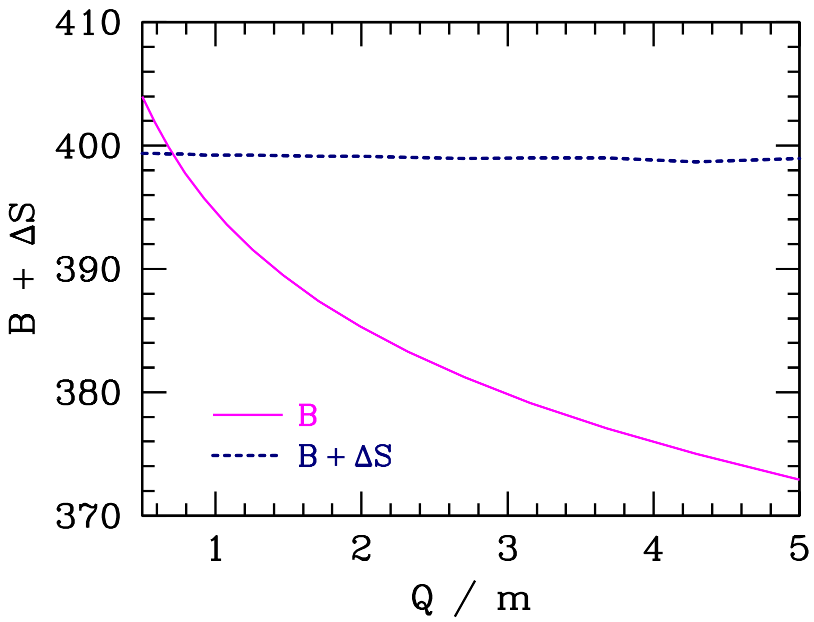

Figure 1. $\mathcal{B}$ and $S_{\rm tot}$ as a function of the renormalization scale Q in the model of a real scalar field with the potential (3.1). We take $T = m$ and $\lambda = 0.6$. Also, $\Lambda = 100\,\mathrm{GeV}$ for ΔS.

4 Model 2: Higgs-Stau system in the MSSM

In SUSY models, the EWSB vacuum becomes a false vacuum if there exists a true vacuum which is CCB or unbounded-from-below directions. The stability of EWSB vacuum often gives significant constraints on the SUSY parameters [2–5, 22, 24–27]. The CCB vacua show up in particular when scalar tri-linear coupling constants are large. Although the decay rate of the EWSB vacuum is important, the prefactor $\mathcal{A}$ is estimated by an order-of-magnitude estimate argument, and is often chosen to be the SUSY scale.

In this section, we consider the case where the tri-linear coupling of the Higgs boson and the scalar taus (staus) is large. Such a setup is attractive because, if we assume the universality of the slepton masses, SUSY contributions to the muon $g-2$ can be large [28–30]. Then, a CCB vacuum may show up in the parameter regions where the muon $g-2$ anomaly is solved [31]. We study the decay rate of the EWSB vacuum in such a case.

For simplicity, we consider the case where masses of all the superparticles and heavy Higgs bosons except for sleptons are much larger than the electroweak scale. We call the mass scale of heavy superparticles as the SUSY scale $M_{\rm SUSY}$. Then, an effective theory is defined between the electroweak scale and the SUSY scale. The effective Lagrangian is described as

$$\begin{aligned}\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm kin} &- y_t(H q_L t_R^c + \mathrm{h.c.}) - m_H^2|H|^2 - \frac{1}{4}\lambda_H|H|^4\\ &- m_{\tilde{\ell}_L}^2|\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2|\tilde{\tau}_R|^2 - T_\tau(H^\dagger\tilde{\ell}_L\tilde{\tau}_R^* + \mathrm{h.c.}) - \frac{1}{4}\kappa^{(1)}|\tilde{\ell}_L|^4 - \frac{1}{4}\kappa^{(2)}|\tilde{\tau}_R|^4\\ &- \frac{1}{4}\lambda^{(1)}|H|^2|\tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(2)}|H^\dagger\tilde{\ell}_L|^2 - \frac{1}{4}\lambda^{(3)}|H|^2|\tilde{\tau}_R|^2 - \frac{1}{4}\kappa^{(3)}|\tilde{\ell}_L|^2|\tilde{\tau}_R|^2,\end{aligned} \tag{4.1}$$

JHEP01(2016)031

where H is the SM-like Higgs doublet, q_L and t^c_R are the third-generation quark doublet and right-handed anti-top, respectively, $\tilde{\ell}_L$ is the third-generation slepton doublet, and $\tilde{\tau}_R$ is the right-handed stau. We denote the kinetic terms as $\mathcal{L}_{\rm kin}$. Terms containing the first- and second-generation sleptons are omitted for simplicity because they are irrelevant for the following discussion.

The scalar potential is significantly affected by the large top-quark Yukawa coupling constant y_t and the tri-linear coupling constant of the stau T_τ. Because the renormalization-scale dependence of $\mathcal{B}$ comes from that of the scalar potential, we concentrate on the RG evolutions of the couplings associated with the bounce fields. The relevant RGEs are given by

$$\frac{dm_H^2}{d\ln Q} = \frac{3y_t^2}{8\pi^2}m_H^2 + \frac{1}{8\pi^2}T_\tau^2, \tag{4.2}$$

$$\frac{dm_{\tilde{\ell}_L}^2}{d\ln Q} = \frac{1}{8\pi^2}T_\tau^2, \tag{4.3}$$

$$\frac{dm_{\tilde{\tau}_R}^2}{d\ln Q} = \frac{1}{4\pi^2}T_\tau^2, \tag{4.4}$$

$$\frac{d\lambda_H}{d\ln Q} = \frac{3y_t^2}{4\pi^2}\lambda_H - \frac{3}{8\pi^2}y_t^4, \tag{4.5}$$

$$\frac{dT_\tau}{d\ln Q} = \frac{3y_t^2}{16\pi^2}T_\tau, \tag{4.6}$$

$$\frac{d\lambda^{(I)}}{d\ln Q} = \frac{3y_t^2}{8\pi^2}\lambda^{(I)}, \tag{4.7}$$

$$\frac{d\kappa^{(I)}}{d\ln Q} = 0, \tag{4.8}$$

with $I = 1, 2, 3$. Because we discuss the renormalization-scale uncertainty at the one-loop level, it is sufficient to consider the leading-logarithmic dependence on the renormalization scale of the parameters which determine the bounce. Hence, we neglect higher loop effects on the vacuum decay rate. In particular, the RG running of y_t is neglected because the top quark does not compose $\mathcal{B}$, and thus, the RG running is two-loop effects.

In the effective Lagrangian, the parameters associated with the SM are determined by the electroweak-scale observables. At the top-quark mass scale, we set them as

$$y_t = \frac{M_t}{v}, \tag{4.9}$$

$$m_H^2(M_t) = -\frac{1}{2}M_h^2, \tag{4.10}$$

$$\lambda_H(M_h) = \frac{M_h^2}{2v^2}, \tag{4.11}$$

where M_t and M_h are the top-quark and Higgs masses, respectively. Numerically, we use $v \simeq 174\,\text{GeV}$, $M_t = 173.5\,\text{GeV}$, and $M_h = 125\,\text{GeV}$ [32]. With the boundary condition, eq. (4.11), $\lambda_H(M_{\rm SUSY})$ may be different from the MSSM prediction at the tree level. We assume that such a deviation is explained by the threshold correction of the scalar-top loops [33].

JHEP01(2016)031

The quartic scalar coupling constants, $\lambda^{(I)}$ and $\kappa^{(I)}$, are described by the gauge and tau Yukawa coupling constants at the SUSY scale. At the tree level, they are given by

$$\lambda^{(1)}(M_{\rm SUSY}) = (g^2 + g'^2)\cos 2\beta, \tag{4.12}$$
$$\lambda^{(2)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g^2\cos 2\beta, \tag{4.13}$$
$$\lambda^{(3)}(M_{\rm SUSY}) = 4y_\tau^2 - 2g'^2\cos 2\beta, \tag{4.14}$$
$$\kappa^{(1)}(M_{\rm SUSY}) = \frac{1}{2}(g^2 + g'^2), \tag{4.15}$$
$$\kappa^{(2)}(M_{\rm SUSY}) = -\kappa^{(3)}(M_{\rm SUSY}) = 2g'^2, \tag{4.16}$$

where g and g' are the gauge coupling constants of the $\mathrm{SU(2)_L}$ and $\mathrm{U(1)_Y}$ gauge symmetries, respectively, and $\tan\beta$ is a ratio of the Higgs vacuum expectation values at the EWSB vacuum, $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$. In addition, y_τ is the Yukawa coupling constant of τ lepton, and is given by $y_\tau = M_\tau/v$ (with M_τ being the mass of τ). The SUSY scale, $M_{\rm SUSY}$, is assumed to be 10 TeV, and $\tan\beta = 20$, for our numerical study.

The stau parameters, $m_{\tilde{\ell}_L}$, $m_{\tilde{\tau}_R}$ and T_τ, have not been determined experimentally. As one can expect from the Lagrangian eq. (4.1), CCB vacua show up when the tri-linear scalar coupling T_τ becomes large. As a sample point at which the EWSB vacuum becomes a false vacuum, we choose the following parameters,

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250\,\mathrm{GeV}, \tag{4.17}$$
$$T_\tau = 300\,\mathrm{GeV}. \tag{4.18}$$

at the scale, $Q = m_{\tilde{\tau}}$. Then, the CCB vacuum is at $\langle H^0\rangle \simeq 1.7\,\mathrm{TeV}$, $\langle\tilde{\tau}_L\rangle \simeq 2.5\,\mathrm{TeV}$, and $\langle\tilde{\tau}_R\rangle \simeq 2.5\,\mathrm{TeV}$, where the vacuum energy is smaller than that of the EWSB vacuum.

In order to see the dependence of $\mathcal{B}$ on the renormalization scale, Q is varied from $M_t/2$ to $2m_{\tilde{\tau}}$. Using the Lagrangian parameters at the scale Q, the Euclidean equation of motion is solved to calculate the bounce action. In figure 2, $\mathcal{B}$ is plotted as a function of Q. It changes from 420 to 240 for $Q = M_t/2$ to $2m_{\tilde{\tau}}$, corresponding to 45% scale uncertainty for $\mathcal{B} = 400$.

The prefactor $\mathcal{A}$ is calculated by the procedure explained in section 2. It consists of the fermion and scalar contributions which are denoted as ΔS_t and ΔS_ϕ, respectively; ΔS_t comes from the top quark, while ΔS_ϕ is from H, $\tilde{\ell}_L$, and $\tilde{\tau}_R$. In our analysis, we neglect the $\mathrm{SU(2)_L \times U(1)_Y}$ gauge interactions in the calculation of $\mathcal{A}$ because the gauge coupling constants are numerically small. The inclusion of the gauge boson loops is technically and conceptually complicated, and is beyond the scope of this paper; this issue will be discussed elsewhere [34]. One subtlety is that there exists the $\mathrm{U(1)_{em}}$ symmetry which is preserved in the EWSB vacuum and is broken in the true vacuum. Because we neglect the $\mathrm{U(1)_{em}}$ gauge interaction, the $\mathrm{U(1)_{em}}$ symmetry is treated as a global symmetry, and eq. (2.31) is used to take account of the effect of the associated zero-mode.

In figure 2, the renormalization-scale dependences of $\mathcal{B} + \Delta S_t$, $\mathcal{B} + \Delta S_\phi$, and $S_{\rm tot} = \mathcal{B} + \Delta S_t + \Delta S_\phi$ are displayed. ΔS_t and ΔS_ϕ as well as $\mathcal{B}$ depend on Q, and ΔS_t (ΔS_ϕ) increases (decreases) as Q increases. Importantly, the renormalization-scale dependence of

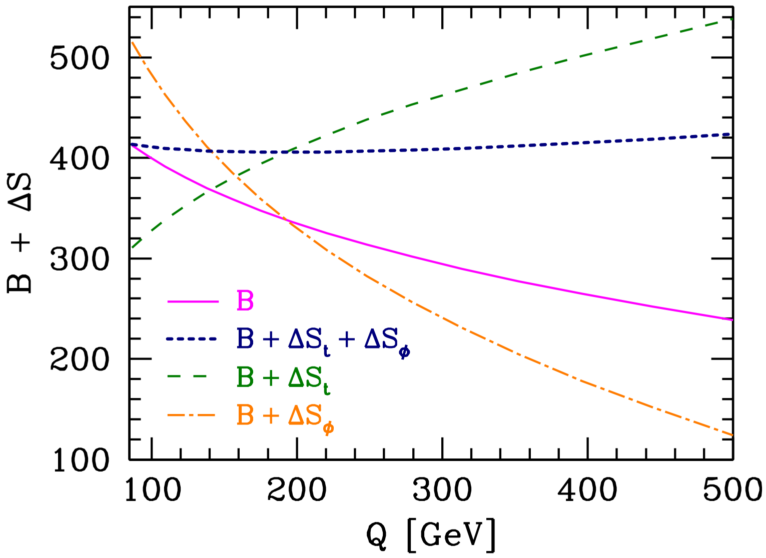

Figure 2. Renormalization-scale dependences of $\mathcal{B}$, $\mathcal{B}+\Delta S_t$, $\mathcal{B}+\Delta S_\phi$, and $S_{\rm tot}=\mathcal{B}+\Delta S_t+\Delta S_\phi$ in the Higgs-stau model. Here, $m_{\tilde{\tau}}\equiv m_{\tilde{\ell}_L}=m_{\tilde{\tau}_R}=250\,\text{GeV}, T_\tau=300\,\text{GeV}$, and $\tan\beta=20$. Also, $\Lambda=100\,\text{GeV}$ is taken for ΔS_ϕ.

$S_{\rm tot}$ is significantly reduced. We can see that $S_{\rm tot}$ is stable around 400; the scale uncertainty becomes about 5%.[3] Thus, the proper inclusion of the prefactor $\mathcal{A}$ stabilizes the decay rate of the EWSB vacuum against the change of the renormalization scale.

As in the case of the previous section, the calculation of the prefactor $\mathcal{A}$ is found to be important to determine the overall scale of the decay rate as well as to reduce the renormalization-scale uncertainty. We also comment that, at the tree level, it is impossible to find an appropriate renormalization scale to estimate the decay rate of the false vacuum, because there is no well-defined procedure to determine $\mathcal{A}$ without performing the loop calculation.

Before closing this section, we comment on other CCB vacua in the MSSM. They also arise in the stop-Higgs potential [5, 22, 24–27]. The calculation of the prefactor $\mathcal{A}$ in this system has not been performed yet despite of its importance. This issue will be discussed elsewhere [34].

5 Summary

We have performed a detailed calculation of the decay rate of the false vacuum $\gamma=\mathcal{A}e^{-\mathcal{B}}$, paying particular attention to its renormalization-scale dependence. The bounce action $\mathcal{B}$ depends on the renormalization scale through the Lagrangian parameters, which makes it difficult to accurately calculate the decay rate at the tree level. Such a scale dependence disappears once we take account of the effects of fluctuations around the bounce, i.e., loop

[3]The renormalization-scale dependence can be improved if we take all the interactions of the effective Lagrangian such as λ_H into account for the beta functions.

JHEP01(2016)031

corrections. In addition, the prefactor $\mathcal{A}$ cannot be determined at the tree level and is often replaced by fourth power of a typical mass scale in the Lagrangian. To resolve this arbitrariness, the calculation of $\mathcal{A}$ is necessary.

We have carefully included one-loop corrections to the decay rate. We have considered a simple model with a scalar field as well as a supersymmetric model in which Higgs-stau system has CCB vacua. With the change of the renormalization scale within the reasonable range, the bounce action can change by $O(10)\,\%$ in these models. We have shown that the renormalization-scale uncertainty is reduced to be $O(1)\,\%$ if the prefactor $\mathcal{A}$ is taken into account properly. Thus, for an accurate calculation of the decay rate, proper inclusion of the loop effects is important.

Acknowledgments

Y.S. is supported by Grant-in-Aid for JSPS Fellows under the program number 26-3171. This work is also supported by Grant-in-Aid for Scientific research Nos. 25105011 (M.E.), 23104008 (T.M.), 26400239 (T.M.), 23104006 (M.M.N.), and 26287039 (M.M.N.), and also by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

A Scale independence at the leading order of $\ln Q$

In this appendix, we show the cancellation of the Q-dependence of $\mathcal{A}e^{-\mathcal{B}}$ at the leading order of $\ln Q$. For simplicity, we assume that there is no kinetic mixing among scalar fields, which is the case in the models we have studied.

Let us denote the tree-level potential of real scalar fields as

$$V = \sum_{n=1}^{4} \sum_{i_1,\cdots,i_n} C^{(n)}_{i_1,\cdots,i_n} \Phi_{i_1} \cdots \Phi_{i_n}, \tag{A.1}$$

where $C^{(n)}_{i_1,\cdots,i_n}$ are Lagrangian parameters. Because of the renormalizability, terms with $n > 4$ do not exist. The bounce action is given by

$$\mathcal{B} = \int d^4x \left[-\frac{1}{2} \sum_i \Phi_i \partial^2 \Phi_i + V \right]_{\Phi\to\sigma}. \tag{A.2}$$

The prefactor $\mathcal{A}$ depends on Q through $s^{(1)}_\varphi$ and $s^{(2)}_\varphi$. The Q-dependent parts of $s^{(1)}_\varphi$ and $s^{(2)}_\varphi$ can be expressed as space-time integrals of local terms; their sum should result in the following form:

$$\sum_\varphi (s^{(1)}_\varphi + s^{(2)}_\varphi) = \int d^4x \left[-\frac{1}{2} \sum_i \zeta_i \sigma_i \partial^2 \sigma_i + \sum_{n=1}^{4} \sum_{i_1,\cdots,i_n} \eta^{(n)}_{i_1,\cdots,i_n} \sigma_{i_1} \cdots \sigma_{i_n} \right] \ln Q + \cdots . \tag{A.3}$$

The renormalization-group equations are expressed by using the wave-function corrections ζ_i and the vertex corrections $\eta^{(n)}_{i_1,\cdots,i_n}$. At the leading order of $\ln Q$, the scale dependence of the Lagrangian parameters is given by

$$C^{(n)}_{i_1,\cdots,i_n}(Q) = C^{(n)}_{i_1,\cdots,i_n}(Q_0) + \left[-\eta^{(n)}_{i_1,\cdots,i_n} + \frac{1}{2} C^{(n)}_{i_1,\cdots,i_n} \sum_{j=i_1}^{i_n} \zeta_j \right] \ln(Q/Q_0) + \cdots . \tag{A.4}$$

Then, the scale dependence of $S_{\rm tot} = \mathcal{B} + \sum_\varphi \sum_p s^{(p)}_\varphi$ is given by

$$S_{\rm tot}(Q) = S_{\rm tot}(Q_0) + \frac{1}{2} \int d^4x \sum_i \zeta_i \left[\Phi_i \left(-\partial^2 \Phi_i + \frac{\partial V}{\partial \Phi_i} \right) \right]_{\Phi \to \sigma} \ln(Q/Q_0) + \cdots . \tag{A.5}$$

The Q-dependent terms in the right-hand side vanish due to the equation of motion for the bounce. Notice that, in order to eliminate the Q-dependence from the decay rate, the wave-function corrections as well as the vertex corrections should be included.

So far, we have neglected the fact that the shape of the bounce solution $\sigma(r)$ depends on the choice of the renormalization scale because of the running of the Lagrangian parameters. Because the bounce is the solution of the equation of motion, the effect of the change of the bounce shape on the bounce action $\mathcal{B}$ is higher order in $\ln Q$.

References

[1] G. Degrassi et al., *Higgs mass and vacuum stability in the Standard Model at NNLO*, *JHEP* **08** (2012) 098 [arXiv:1205.6497] [INSPIRE].

[2] J.M. Frere, D.R.T. Jones and S. Raby, *Fermion masses and induction of the weak scale by supergravity*, *Nucl. Phys.* **B 222** (1983) 11 [INSPIRE].

[3] J.F. Gunion, H.E. Haber and M. Sher, *Charge/color breaking minima and a-parameter bounds in supersymmetric models*, *Nucl. Phys.* **B 306** (1988) 1 [INSPIRE].

[4] J.A. Casas, A. Lleyda and C. Muñoz, *Strong constraints on the parameter space of the MSSM from charge and color breaking minima*, *Nucl. Phys.* **B 471** (1996) 3 [hep-ph/9507294] [INSPIRE].

[5] A. Kusenko, P. Langacker and G. Segre, *Phase transitions and vacuum tunneling into charge and color breaking minima in the MSSM*, *Phys. Rev.* **D 54** (1996) 5824 [hep-ph/9602414] [INSPIRE].

[6] S.R. Coleman, *The fate of the false vacuum. 1. Semiclassical theory*, *Phys. Rev.* **D 15** (1977) 2929 [*Erratum ibid.* **D 16** (1977) 1248] [INSPIRE].

[7] C.G. Callan Jr. and S.R. Coleman, *The fate of the false vacuum. 2. First quantum corrections*, *Phys. Rev.* **D 16** (1977) 1762 [INSPIRE].

[8] J. Avan and H.J. De Vega, *Inverse scattering transform and instantons of four-dimensional Yukawa and ϕ^4 theories*, *Nucl. Phys.* **B 269** (1986) 621 [INSPIRE].

[9] G. Isidori, G. Ridolfi and A. Strumia, *On the metastability of the standard model vacuum*, *Nucl. Phys.* **B 609** (2001) 387 [hep-ph/0104016] [INSPIRE].

[10] J. Baacke and G. Lavrelashvili, *One loop corrections to the metastable vacuum decay*, *Phys. Rev.* **D 69** (2004) 025009 [hep-th/0307202] [INSPIRE].

[11] G.V. Dunne, *Functional determinants in quantum field theory*, *J. Phys.* **A 41** (2008) 304006 [arXiv:0711.1178] [INSPIRE].

[12] G.V. Dunne and H. Min, *Beyond the thin-wall approximation: Precise numerical computation of prefactors in false vacuum decay*, *Phys. Rev.* **D 72** (2005) 125004 [hep-th/0511156] [INSPIRE].

[13] J.H. Van Vleck, *The correspondence principle in the statistical interpretation of quantum mechanics*, *Proc. Nat. Acad. Sci.* **14** (1928) 178 [INSPIRE].

[14] R.H. Cameron and W.T. Martin, *Evaluation of various Wiener integrals by use of certain Sturm-Liouville differential equations*, *Bull. Am. Math. Soc.* **51** (1945) 73.

[15] I.M. Gelfand and A.M. Yaglom, *Integration in functional spaces and it applications in quantum physics*, *J. Math. Phys.* **1** (1960) 48 [INSPIRE].

[16] R.F. Dashen, B. Hasslacher and A. Neveu, *Nonperturbative methods and extended hadron models in field theory. 1. Semiclassical functional methods*, *Phys. Rev.* **D 10** (1974) 4114 [INSPIRE].

[17] S.R. Coleman, *The uses of instantons*, *Subnucl. Ser.* **15** (1979) 805 [INSPIRE].

[18] K. Kirsten and A.J. McKane, *Functional determinants by contour integration methods*, *Annals Phys.* **308** (2003) 502 [math-ph/0305010] [INSPIRE].

[19] K. Kirsten and A.J. McKane, *Functional determinants for general Sturm-Liouville problems*, *J. Phys.* **A 37** (2004) 4649 [math-ph/0403050] [INSPIRE].

[20] C.L. Wainwright, *CosmoTransitions: computing cosmological phase transition temperatures and bubble profiles with multiple fields*, *Comput. Phys. Commun.* **183** (2012) 2006 [arXiv:1109.4189] [INSPIRE].

[21] Planck collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, arXiv:1502.01589 [INSPIRE].

[22] J.E. Camargo-Molina, B. O'Leary, W. Porod and F. Staub, *Stability of the CMSSM against sfermion VEVs*, *JHEP* **12** (2013) 103 [arXiv:1309.7212] [INSPIRE].

[23] A. Kusenko, K.-M. Lee and E.J. Weinberg, *Vacuum decay and internal symmetries*, *Phys. Rev.* **D 55** (1997) 4903 [hep-th/9609100] [INSPIRE].

[24] D. Chowdhury, R.M. Godbole, K.A. Mohan and S.K. Vempati, *Charge and color breaking constraints in MSSM after the Higgs discovery at LHC*, *JHEP* **02** (2014) 110 [arXiv:1310.1932] [INSPIRE].

[25] N. Blinov and D.E. Morrissey, *Vacuum stability and the MSSM Higgs mass*, *JHEP* **03** (2014) 106 [arXiv:1310.4174] [INSPIRE].

[26] J.E. Camargo-Molina, B. Garbrecht, B. O'Leary, W. Porod and F. Staub, *Constraining the natural MSSM through tunneling to color-breaking vacua at zero and non-zero temperature*, *Phys. Lett.* **B 737** (2014) 156 [arXiv:1405.7376] [INSPIRE].

[27] M. Endo, T. Moroi and M.M. Nojiri, *Footprints of supersymmetry on Higgs decay*, *JHEP* **04** (2015) 176 [arXiv:1502.03959] [INSPIRE].

[28] J.L. Lopez, D.V. Nanopoulos and X. Wang, *Large* $(g-2)_\mu$ *in* SU(5) × U(1) *supergravity models*, *Phys. Rev.* **D 49** (1994) 366 [hep-ph/9308336] [INSPIRE].

JHEP01(2016)031

[29] U. Chattopadhyay and P. Nath, *Probing supergravity grand unification in the Brookhaven $g-2$ experiment*, Phys. Rev. **D 53** (1996) 1648 [hep-ph/9507386] [INSPIRE].

[30] T. Moroi, *The Muon anomalous magnetic dipole moment in the minimal supersymmetric standard model*, Phys. Rev. **D 53** (1996) 6565 [*Erratum ibid.* **D 56** (1997) 4424] [hep-ph/9512396] [INSPIRE].

[31] M. Endo, K. Hamaguchi, T. Kitahara and T. Yoshinaga, *Probing bino contribution to muon $g-2$*, JHEP **11** (2013) 013 [arXiv:1309.3065] [INSPIRE].

[32] Particle Data Group collaboration, K.A. Olive et al., *Review of particle physics*, Chin. Phys. **C 38** (2014) 090001 [INSPIRE].

[33] Y. Okada, M. Yamaguchi and T. Yanagida, *Renormalization group analysis on the Higgs mass in the softly broken supersymmetric standard model*, Phys. Lett. **B 262** (1991) 54 [INSPIRE].

[34] M. Endo, T. Moroi, M. M. Nojiri and T. Shoji, work in progress.