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# Deep inelastic scattering structure functions of holographic spin-1 hadrons with $N_f \geq 1$

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Abstract: Two-point current correlation functions of the large $N$ limit of supersymmetric and non-supersymmetric Yang-Mills theories at strong coupling are investigated in terms of their string theory dual models with quenched flavors. We consider non-Abelian global symmetry currents, which allow one to investigate vector mesons with $N_f > 1$. From the correlation functions we construct the deep inelastic scattering hadronic tensor of spin-one mesons, obtaining the corresponding eight structure functions for polarized vector mesons. We obtain several relations among the structure functions. Relations among some of their moments are also derived. Aspects of the sub-leading contributions in the $1/N$ and $N_f/N$ expansions are discussed. At leading order we find a universal behavior of the hadronic structure functions.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, D-branes

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## Contents





## 1 Introduction

Two-point current correlation functions are relevant for the calculation of important observables of quantum field theories. In particular, for confining gauge theories they allow one to construct the so-called hadronic tensor of deep inelastic scattering (DIS) processes, which is an invaluable tool to extract fundamental information about the structure of hadrons. When considering a DIS process the idea is that a lepton is scattered from a hadron, being the interaction mediated by a virtual photon exchanged from the lepton to the hadron. The process is called *inclusive* since only the scattered lepton is measured, while the hadronic final state is not. The differential cross section of DIS is given by the contraction of a leptonic tensor, which is obtained from Quantum Electrodynamics, and a hadronic tensor, $W_{\mu\nu}$, which carries the information about the strong interaction. By using the optical theorem in quantum field theory the hadronic tensor can be written in terms of the vacuum expectation value of the product of two currents. $W_{\mu\nu}$ has a Lorentz tensor decomposition which depends on the spin of the hadron, and it can be expressed as a sum of several terms. In addition, there are functions multiplying each of these terms. These structure functions, like the tensor $W_{\mu\nu}$, should in principle be derived from QCD. However, the non-perturbative character of QCD makes it extremely difficult to obtain such functions in that way. The structure of the hadronic tensor lies on the two-point current correlation functions, which are affected by the non-perturbative nature of soft processes of QCD.

On the other hand, the gauge/string duality provides holographic dual models which can actually be used to calculate the structure functions of hadrons derived from such models, in terms of two-point current correlation functions. This is so because within this duality the non-perturbative regime of the quantum field theory corresponds to the perturbative regime of the holographic string theory dual model. In this paper we investigate properties of two-point current correlation functions, and therefore the DIS hadronic tensor, using different holographic string theory dual models with flavors in the fundamental representation of the gauge group, and within the quenched approximation. The hadrons we consider are mesons. Notice that these mesons are not exactly those of QCD because at present there is not any holographic dual model which accounts for all the properties of QCD, even in the large $N$ limit. However, it is very interesting to be able to explore their internal structure, since it could manifest a universal character, which obviously is inherent to the two-point current correlation functions. In fact we find such a universal behavior. Particularly, we are interested in the study of two-point correlation functions of non-Abelian symmetry currents, which allows one to describe hadrons with different flavor content.

Polchinski and Strassler proposed a model for the holographic dual description of DIS of confining gauge theories [1] that we briefly describe below. They calculated hadronic structure functions for the Bjorken parameter $x$ of order one within the supergravity approximation. They also considered a small-$x$ calculation by using a dual string theory analysis. Their approach is in the large $N$ limit of confining supersymmetric Yang-Mills theories in four dimensions, such as certain deformations of $\mathcal{N}=4$ SYM, from which they study DIS from glueballs and spin-$\frac{1}{2}$ hadrons. The gauge theories studied in [1] are UV conformal or nearly conformal, which makes the dual string theory defined on a background of the type $\mathrm{AdS}_5\times\mathcal{M}_5$, being $\mathcal{M}_5$ a compact five-dimensional Einstein manifold. Thus, this is a solution of type IIB supergravity whose metric can be written as

$$ds^2=\frac{\rho^2}{R^2}\,\eta_{\mu\nu}\,dy^\mu\,dy^\nu+\frac{R^2}{\rho^2}\,d\rho^2+R^2\,\widehat{ds}^{\,2}_{\mathcal{M}_5}\,, \tag{1.1}$$

where the $\mathrm{AdS}_5$ radius is $R=(4\pi g_s N)^{1/4}\alpha'^{1/2}$ when $\mathcal{M}_5$ is $S^5$. The four-dimensional gauge field theory coordinates are identified with $y^\mu$, while $\rho$ is the holographic radial coordinate related to the dual quantum field theory energy scale. Up to powers of the 't Hooft coupling $\lambda=g_{YM}^2N\equiv 4\pi g_s N$, the ten-dimensional energy scale is given by $R^{-1}$, where the string coupling is denoted by $g_s$. Thus, the four-dimensional energy is given by

$$E^{(4)} \sim \frac{\rho}{R^2} \,. \tag{1.2}$$

In the large $N$ limit of confining gauge theories, the geometry of the holographic dual model whose metric is given by eq. (1.1) must be modified at a radius corresponding to $\rho \sim \rho_0 = \Lambda R^2$. Notice the presence of a confinement scale $\Lambda$. It is worth mentioning that the dynamics of interest for $q \gg \Lambda$ lies on the region where $\rho_{\rm int} \sim q R^2 \gg \rho_0$, where $\rho_{\rm int}$ denotes the bulk region where the relevant interaction occurs. Within this region the conformal metric (1.1) can be used. Thus, it is possible to calculate the dual of the matrix element of the $T^{\mu\nu}$ tensor, which as we shall explain in section 2, is related to the hadronic tensor. As commented before, by using the optical theorem we can write its imaginary part as

$$\begin{aligned}
{\rm Im}\, T^{\mu\nu} &= \pi \sum_{P_X, X} \langle P, \mathcal{Q} | J^\nu(0) | P_X, X\rangle \langle P_X, X | \widetilde{J}^\mu(q) | P, \mathcal{Q}\rangle \\
&= 2\pi^2 \sum_X \delta(M_X^2 + [P+q]^2) \langle P, \mathcal{Q} | J^\nu(0) | P+q, X\rangle \langle P+q, X | J^\mu(0) | P, \mathcal{Q}\rangle \,,
\end{aligned} \tag{1.3}$$

which has been written in terms of the hadron ($P_\mu$) and virtual photon ($q_\mu$) momenta, and the currents $J_\mu$. There is a sum over intermediate states $X$ with mass $M_X$. Notice that $\eta^{\mu\nu}$ raises their Lorentz indices which are four-dimensional ones.

In the large $N$ limit of the gauge theory only single hadron states will contribute. If $-P^2 \ll q^2$, i.e. $|t| \ll 1$, then in the $s$-channel we can approximate

$$s = -(P+q)^2 \simeq q^2 \left( \frac{1}{x} - 1 \right) , \tag{1.4}$$

where we have used the Bjorken variable

$$x \equiv - \frac{q^2}{2P \cdot q} \,, \tag{1.5}$$

and also

$$t \equiv \frac{P^2}{q^2} \,. \tag{1.6}$$

The condition $-P^2 \ll q^2$ is equivalent to $|t| \ll 1$. On the other hand, in ten dimensions the scale $\widetilde{s}$ is set by the relation

$$\widetilde{s} = -g^{MN} P_{X,M} P_{X,N} \leq -g^{\mu\nu} (P+q)_\mu (P+q)_\nu \sim \frac{R^2}{\rho_{\rm int}^2} q^2 \left( \frac{1}{x} - 1 \right) = \frac{\left( x^{-1} - 1 \right)}{\alpha' (4\pi g_s N)^{1/2}} \,. \tag{1.7}$$

The 't Hooft parameter appears in the denominator, so if $(g_s N)^{-1/2} \ll x < 1$ we have $\alpha' \widetilde{s} \ll 1$. Therefore, in this limit only massless string states are produced, and we are dealing with a purely supergravity process [1]. Through this work we assume the Bjorken variable to be within the kinematical regime where the supergravity approximation is reliable.

One can describe the DIS process from the bulk theory perspective. The idea is that within the four-dimensional boundary theory we consider the two-point function of two global symmetry currents inside the hadron. So, let us consider the effect of the insertion of a current operator at the boundary of the $\mathrm{AdS}_5$ space-time. This leads to a perturbation on the boundary condition of a bulk gauge field. This perturbation produces a non-normalizable mode propagating in the bulk [2, 3]. In order to find this mode we should look at the isometry group of the manifold $\mathcal{M}_5$, which corresponds to an $R$-symmetry group on the boundary field theory. If one takes a $\mathrm{U}(1)_R$ subgroup, the associated $R$-symmetry current can be identified with the electromagnetic current inside the hadron. Notice that for the global symmetry group, which corresponds to the isometry of $\mathcal{M}_5$, there is a Killing vector $\upsilon_j$ which produces the non-normalizable mode of a Kaluza-Klein gauge field $A_m(y,r)$. Therefore, the metric perturbation induced by the $R$-symmetry current operator is

$$\delta g_{mj} = A_m(y,r)\,\upsilon_j(\Omega)\,. \tag{1.8}$$

This mode $A_m(y,r)$ propagates in the bulk and couples to a bulk field which is dual to a certain quantum field theory state. For instance when considering glueballs, the holographic dual field in [1] corresponds to the dilaton. Thus, the incoming bulk dilaton field $\Phi_i$ couples to the bulk U(1)-gauge field $A_\mu$ (induced by a current operator inserted at the boundary) and to another dilaton $\Phi_X$, which represents an intermediate hadronic state.[1] The intermediate state propagates in the bulk and couples to an outgoing dilaton $\Phi_f$ (corresponding to the final hadronic state) and a gauge field $A_\nu$ in the bulk which comes from the insertion of a second boundary theory current operator. This is nothing but a holographic dual version of the quantum field theory optical theorem. This can be generalized to other situations, namely mesons including flavors in the fundamental representation of the gauge group. In this case $A_m(y,r)$ in the bulk couples to either scalar or vector fluctuations of flavor probe branes, and the two-point functions which lead to the hadronic tensor correspond to non-Abelian global symmetry currents.

Another interesting issue is related to the role of the sub-leading corrections to the Operator Product Expansion (OPE) of two symmetry currents. From this, the moments of the structure functions can be obtained. These moments have different kind of contributions to the $1/N$ expansion, i.e. while at weak coupling single-trace twist-two operators dominate the expansion, at strong coupling double-trace operators become relevant [1].

We can summarize our main results as follows. We have performed a detailed analysis of the structure of the two-point correlation functions of generic symmetry currents at strong coupling, associated with flavors in the fundamental representation of the gauge group, in the quenched approximation, in terms of the corresponding holographic string theory dual description. This includes the large $N$ limit of supersymmetric and non-supersymmetric Yang-Mills theories in four dimensions. In particular, we have explicitly investigated the cases of the D3D7-brane, the D4D8$\overline{\text{D8}}$-brane, and the D4D6$\overline{\text{D6}}$-brane systems. We would like to emphasize that we have found a *universal* structure of the

[1] The corresponding tree level supergravity Witten's diagram can be seen in figure 1, which describes a forward Compton scattering. Notice that in that figure solid lines indicate mesons.

two-point correlation functions of generic global symmetry currents at strong coupling. For each holographic dual model we have found that the two-point correlation functions of non-Abelian ($N_f > 1$) global symmetry currents can generically be written as the product of a constant, which depends on the particular Dp-brane model, times flavor preserving Kronecker deltas multiplying the corresponding Abelian ($N_f = 1$) result for the same Dp-brane model. We have obtained a universal factorization of the two-point correlation functions for non-Abelian symmetry currents in a model-dependent factor times a model-independent one. More precisely, we should stress that these results strictly hold in the large $N$ limit, i.e. to leading order in the $1/N$ expansion. Sub-leading corrections in this expansion would likely induce some modifications, obviously negligible in the large $N$ limit. The model-dependent and model-independent factorization has already been seen for the two-point functions of Abelian symmetry currents in our previous paper [4]. This factorization comes from the structure of the flavored holographic dual model in the probe approximation, where the probe Dp-brane action is taken to be the non-Abelian version of the Dirac-Born-Infeld action [5]. Thus, in general we can write the $W^{\mu\nu}_{(a)}$ tensor for a holographic dual model corresponding to a certain gauge field theory in the large-$N$ limit as

$$W^{\mu\nu}_{(a)} = A_{(a,b)}(x)\, W^{\mu\nu}_{(b)}\,, \tag{1.9}$$

for models $(a)$ and $(b)$, where $A_{(a,b)}$ is a conversion factor which depends on the pair of Dp-brane models considered. This allows one to write the corresponding structure functions $F_i^{(a)}(x,t)$, where subindex $i$ indicates the $i$-th structure function for every meson in each particular model, as

$$F_i^{(a)}(x,t) = A_{(a,b)}(x)\, F_i^{(b)}(x,t)\,. \tag{1.10}$$

Besides, we have found that a modified version of the Callan-Gross relation is satisfied by the class of flavored holographic dual models we have investigated, when the parameter $t \to 0$. We have obtained new relations between structure functions for the $N_f > 1$ case within each particular model. This confirms our results for $N_f = 1$ given in [4]. This suggests that these relations among structure functions are generic and, therefore it may indicate that they hold for any confining gauge theory in the appropriate kinematical regime. In addition, we have shown that all the moments of certain structure functions satisfy the corresponding inequalities derived from unitarity, as expected [7].

A very interesting aspect of the present work is that we have investigated the $1/N$ and $N_f/N$ contributions to the leading order calculations of the hadronic tensor, from the supergravity dual model point of view. Particularly, we have focused on the structure of the relevant Lagrangians and Witten's diagrams. Indeed, we have derived all relevant Lagrangians. On the other hand, although we have not calculated these Witten's diagrams explicitly, we have discussed how they arise from supergravity. We have pointed out that the $1/N$ and $N_f/N$ expansions of the Witten's diagrams correspond to analogous expansions in the dual quantum field theory. We also have shown how these Witten's diagrams are suppressed by $1/N^2$ and $N_f/N$ powers, respectively, in the supergravity dual models.

This paper essentially contains two parts. The first one, which includes sections 2, 3 and 4, develops a non-trivial generalization of our results of reference [4] when the number of flavors is larger than one, but still within the quenched approximation. In section 3 we begin with a general background metric, which includes the two cases studied in [4], as well as the D4D6$\overline{\text{D6}}$-brane system [6]. We calculate the structure functions for scalar and vector mesons. In section 4, we extend this approach to study flavored vector mesons, which is done in the gravity dual theory by adding $N_f$ flavor probe Dp-branes, with $1 < N_f \ll N$. The second part is introduced in section 5 and it contains very interesting new results about the $1/N$ expansion. We have discussed results corresponding to a DIS process where the lepton is scattered from an entire hadron, which becomes excited but is not fragmented. Beyond it, in section 5 we have considered the $1/N$ and $N_f/N$ expansions. It would be very interesting to investigate the effects of the back-reaction of the probe Dp-branes on the background beyond the probe approximation. Another aspect we have not considered concerns the kinematic regime where the Bjorken parameter is very small, whose holographic dual description goes beyond pure supergravity. In section 6 we carry out a discussion of our results. Two appendices are included to account for details of expressions commented in the main text, and in order to include explicit results of the two-point current correlations functions for the D4D6$\overline{\text{D6}}$-brane model.

## 2 Two-point current correlation functions and DIS

In what follows we adopt the conventions of Manohar [7], except for the Minkowski metric, which we define as being mostly plus. A brief review of the relevant ideas and definitions for the present work can be found in our previous paper [4]. A more detailed derivation of DIS structure functions is available in references [7] and [8].

We consider an incoming lepton beam with four-momentum $k^\mu$ (with $k^0 \equiv E$) which will be scattered from a fixed hadronic target. The four-momentum of the scattered lepton $k'^\mu$ (with $k'^0 \equiv E'$) is measured, but the final hadronic state called $X$ is not. The lepton and the initial hadronic state exchange a virtual photon with four-momentum $q^\mu$. Thus, this virtual photon is able to probe the hadron structure at distances as small as $1/\sqrt{q^2}$.

The DIS differential cross section can be written as

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4}{16\pi^2 q^4}\frac{E'}{ME} l^{\mu\nu} W_{\mu\nu}(P,q)_{h'\,h} \,, \tag{2.1}$$

where we have defined the leptonic tensor as follows

$$l^{\mu\nu} = \sum_{\text{final spin}} \langle k'|J_l^\nu(0)|k, s_l\rangle \langle k, s_l|J_l^\mu(0)|k'\rangle \,, \tag{2.2}$$

which for a spin-$\frac{1}{2}$ lepton becomes

$$l^{\mu\nu} = 2\left[k^\mu k'^\nu + k^\nu k'^\mu - \eta^{\mu\nu}(k\cdot k' - m_l^2) - i\,\epsilon^{\mu\nu\alpha\beta}\, q_\alpha\, s_{l\beta}\right], \tag{2.3}$$

being $m_l$ the lepton mass. In addition, the hadronic tensor is

$$W_{\mu\nu}(P,q)_{h'\,h} = \frac{1}{4\pi}\int d^4x\, e^{iq.x}\,\langle P,h'|[J_\mu(x), J_\nu(0)]|P,h\rangle \,, \tag{2.4}$$

where $P^\mu$ and $P_X^\mu$ denote the hadronic initial and final momenta, $h$ and $h'$ are the polarizations of the initial and final hadronic states, and $M^2 = -P^2$ and $M_X^2 = -P_X^2$ are the initial and final hadronic squared masses, respectively. The hadronic tensor can be recast in terms of its structure functions. In fact, the so-called partonic distribution functions, which can be calculated from the structure functions, give the probability that a hadron contains a given constituent with a given fraction $x$ of its total momentum. Due to the non-perturbative character of QCD, since the partonic distribution functions depend on soft QCD dynamics, they cannot be extracted perturbatively.

In the case of hadrons composed by massless partons, the probability of finding a parton with a momentum $xP^\mu$ is given by the distribution function $f(x, q^2)$. In the case of free partons this function leads to the Bjorken scaling, which is not actually true for QCD since it is not a free field theory. Notice that the hadronic structure functions are dimensionless functions of $P^2$, $P \cdot q$ and $q^2$. It is usual to write their functional dependence in terms of $t$ and $x$ variables described in the introduction, with $0 < x \leq 1$ and $t \leq 0$. The structure functions are obtained from the most general Lorentz decomposition of the hadronic tensor $W_{\mu\nu}$, satisfying parity invariance, time reversal symmetry, and invariance under translations.

The most general form for spin-zero targets is [1]

$$W_{\mu\nu}^{\text{scalar}} = F_1 \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) - \frac{F_2}{P \cdot q} \left( P_\mu + \frac{q_\mu}{2x} \right) \left( P_\nu + \frac{q_\nu}{2x} \right). \tag{2.5}$$

After contracting with $l^{\mu\nu}$, terms containing $q_\mu$ and $q_\nu$ vanish. Therefore, we can just neglect these terms from the beginning obtaining a simpler expression

$$W_{\mu\nu}^{\text{scalar}} = F_1 \, \eta_{\mu\nu} - \frac{F_2}{P \cdot q} P_\mu P_\nu . \tag{2.6}$$

For spin-one targets, on the other hand, the full general form of the hadronic tensor is [8]

$$\begin{aligned} W_{\mu\nu}^{\text{vector}} = {} & F_1 \, \eta_{\mu\nu} - \frac{F_2}{P \cdot q} P_\mu P_\nu + b_1 r_{\mu\nu} - \frac{b_2}{6} (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) - \frac{b_3}{2} (s_{\mu\nu} - u_{\mu\nu}) \\ & - \frac{b_4}{2} (s_{\mu\nu} - t_{\mu\nu}) - \frac{i \, g_1}{P \cdot q} \, \epsilon_{\mu\nu\lambda\sigma} \, q^\lambda \, s^\sigma - \frac{i \, g_2}{(P \cdot q)^2} \, \epsilon_{\mu\nu\lambda\sigma} \, q^\lambda \, (P \cdot q \, s^\sigma - s \cdot q \, P^\sigma) , \end{aligned} \tag{2.7}$$

where we have already omitted terms proportional to $q_\mu$ and $q_\nu$, as explained before. Functions $r_{\mu\nu}$, $s_{\mu\nu}$, $t_{\mu\nu}$, $u_{\mu\nu}$ and $s^\sigma$, which depend on the hadron polarization, on the hadron and virtual photon momenta, and on the $t$ and $x$ variables, are defined in appendix A.

DIS amplitudes can be obtained from the imaginary part of the forward Compton scattering amplitudes. Thus, it is possible to define the tensor

$$T_{\mu\nu} = i \, \langle P, \mathcal{Q} | \widehat{T} ( \widetilde{J}_\mu(q) \, J_\nu(0) ) | P, \mathcal{Q} \rangle \, , \tag{2.8}$$

where $J_\mu$ and $J_\nu$ are the electromagnetic current operators. In addition, $P$ is the four-momentum of the initial hadronic state, $q$ is the four-momentum of the virtual photon, and $\mathcal{Q}$ is the charge of the hadron. $\widehat{T}(\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2)$ indicates time-ordered product between the operators $\widehat{\mathcal{O}}_1$ and $\widehat{\mathcal{O}}_2$, and the Fourier transform is indicated with a tilde. The tensor



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$T_{\mu\nu} \equiv T_{\mu\nu}(P,q,h)$ has identical symmetry properties as $W_{\mu\nu}(P,q,h)$, thus having similar Lorentz-tensor structure to $W_{\mu\nu}$. By using the optical theorem one obtains

$$\mathrm{Im}\, \widetilde{F_j} = 2\pi\, F_j\,, \tag{2.9}$$

where $\widetilde{F_j}$ is the $j$-th structure function of the $T_{\mu\nu}$ tensor, while $F_j$ is the one corresponding to the $W_{\mu\nu}$ tensor.

## 3 DIS from scalar and vector mesons with $N_f = 1$

### 3.1 General background

In this section we study a general approach to obtain the structure functions for scalar and vector mesons with a single flavor, $N_f = 1$, in terms of two-point correlation functions of global U(1) symmetry currents. This is a holographic dual approach based on [1]. In particular, we show that the structure functions can be written as the product of a model-dependent factor times a model-independent one. We explicitly calculate both factors in terms of the parameters defining a general holographic dual model. This includes the structure functions derived from the D3D7-brane model and from the D4D8$\overline{\text{D8}}$-brane model that we already obtained in our previous paper [4], as well as those obtained from the D4D6$\overline{\text{D6}}$-brane model which we introduce in appendix B of the present work.

Let us consider a general ten-dimensional background metric in the Einstein frame written as

$$ds^2 = \left(\frac{\rho}{R}\right)^{\alpha} \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{\rho}{R}\right)^{\beta} d\vec{Z}\cdot d\vec{Z}\,, \tag{3.1}$$

where $x^\mu = (x^0, \dots, x^3)$, while $\vec{Z} = (Z^1, \dots, Z^6)$, with $\alpha > 1$, $\beta < -1$. We then add a probe Dp-brane with an induced metric of the form

$$ds^2_{Dp} = \left(\frac{\rho}{R}\right)^{\alpha} \eta_{\mu\nu}\, dx^\mu dx^\nu + \left(\frac{\rho}{R}\right)^{\beta} \left[d\rho^2 + \rho^2\, d\Omega^2_{p-4}\right], \tag{3.2}$$

where $\rho$ is the radial direction of the Dp-brane world-volume. The radius $R$ is the length scale of the system, while $\Omega_{p-4}$ indicates coordinates on $S^{p-4}$.

This general induced metric also describes the D3D7, D4D8$\overline{\text{D8}}$, and D4D6$\overline{\text{D6}}$-brane models. In particular, for the D3D7-brane model we must set $p = 7$, $\alpha = 2$, and $\beta = -2$. The asymptotic geometry is AdS$_5 \times S^5$, and $R$ gives the sphere and AdS$_5$ radii. In the case of the D4D8$\overline{\text{D8}}$-brane system we set $p = 8$, $\alpha = \frac{3}{2}$, and $\beta = -\frac{3}{2}$. In addition, for the D4D6$\overline{\text{D6}}$-brane model we have $p = 6$, $\alpha = \frac{3}{2}$, and $\beta = -\frac{3}{2}$. In all these cases, we only recover the asymptotic metric, i.e. for $\rho \gg \rho_0$ ($U \gg U_0$ in the notation of [11]), which is the relevant induced metric to the DIS process.

Scalar and vector mesons correspond to excitations of open strings ending on the probe Dp-brane. The dynamics of the Dp-brane fluctuations is described by the action

$$S_{Dp} = -\mu_p \int d^{p+1}\xi \sqrt{-\det(\hat{P}[g]_{ab} + 2\pi\alpha' F_{ab})} + \frac{(2\pi\alpha')^2}{2}\mu_p \int \hat{P}[C^{(p-3)}] \wedge F \wedge F\,, \tag{3.3}$$

where $g_{ab}$ stands for the metric (3.2), $\mu_p = \left[(2\pi)^p g_s \alpha'^{\frac{p+1}{2}}\right]^{-1}$ is the Dp-brane tension and $\hat{P}$ denotes the pullback of the background fields on the Dp-brane world-volume.



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### 3.2 DIS from scalar mesons

The equations of motion for scalar mesons are obtained from fluctuations of the probe Dp-brane which are orthogonal to the directions of the brane world-volume. Let us take a coordinate $Z^i$ in eq. (3.1), which is perpendicular to the Dp-brane world-volume, and slightly perturb it as follows

$$Z^i = Z_0^i + 2\pi\alpha'\Phi\,, \tag{3.4}$$

where $\Phi$ is a scalar fluctuation whose Lagrangian is straightforwardly derived from the action of eq. (3.3), by setting $F_{ab} = 0$. By expanding to second order in the fluctuation, one obtains

$$S_0^{\rm scalar} = -\mu_p \int d^{p+1}\xi\,\sqrt{-\det g}\left[1+\frac{(2\pi\alpha')^2}{2}\left(\frac{\rho}{R}\right)^{\beta} g^{ab}\,\partial_a\Phi\,\partial_b\Phi\right], \tag{3.5}$$

which corresponds to the Lagrangian

$$\mathcal{L}_0^{\rm scalar} = -\mu_p\,\sqrt{-\det g}\left[1+\frac{(2\pi\alpha')^2}{2}\left(\frac{\rho}{R}\right)^{\beta} g^{ab}\,\partial_a\Phi\,\partial_b\Phi\right], \tag{3.6}$$

where all indices denote directions along the Dp-brane world-volume. The probe brane wraps a $S^{p-4}$. By plugging the metric (3.2) into the quadratic Lagrangian, one obtains the equations of motion (EOM) for scalar fluctuations of the Dp-brane in the probe approximation

$$\partial_a\left[\left(\frac{\rho}{R}\right)^{\theta-\beta}\sqrt{\tilde g}\,g^{ab}\,\partial_b\Phi\right]=0\,, \tag{3.7}$$

where we have defined

$$\theta = 2\,\alpha + \left(\frac{p}{2}-\frac{3}{2}\right)\beta + (p-4)\,. \tag{3.8}$$

Notice that $\tilde g_{ij}$ is the metric on $S^{p-4}$, which together with $\rho$ span coordinates ($Z^1$, $\cdots$, $Z^{p-3}$). The EOM can be more explicitly written as

$$\Box\Phi + \left(\frac{\rho}{R}\right)^{\alpha-\beta-2} R^{-2}\,\nabla_i\nabla^i\Phi + \theta R^{-1}\left(\frac{\rho}{R}\right)^{\alpha-\beta-1}\partial_\rho\Phi + \left(\frac{\rho}{R}\right)^{\alpha-\beta}\partial_\rho^2\Phi = 0\,, \tag{3.9}$$

where $\nabla_i$ is the covariant derivative on $S^{p-4}$.

We propose the following *Ansatz*[2]

$$\Phi^\ell = \phi^\ell(\rho)\,e^{iP\cdot y}\,Y^\ell(S^{p-4})\,, \tag{3.10}$$

where $Y^\ell(S^{p-4})$ are the scalar spherical harmonics on $S^{p-4}$, which satisfy the eigenvalue equation

$$\nabla^i\nabla_i Y^\ell(S^{p-4}) = -\ell\,(\ell+p-5)\,Y^\ell(S^{p-4})\,. \tag{3.11}$$

[2]This solution will be exact for the metric (3.2). Since this metric is only asymptotic for the models that we consider, i.e. D3D7, D4D8$\overline{\rm D8}$, and D4D6$\overline{\rm D6}$-brane models, we must assume the condition $\rho_{\rm int} \sim qR^2 \gg \rho_0 \equiv \Lambda R^2$, where $\rho_{\rm int}$ denotes the interaction region, while $\Lambda$ is an infrared cutoff of the four-dimensional gauge theory.

Now, by replacing the *Ansatz* (3.10) in the EOM (3.9), we obtain

$$\Phi^{\ell}_{\mathrm{IN/OUT}} = c_i \left(\frac{\rho}{R}\right)^{A-\gamma B} e^{iP\cdot y}\, Y^{\ell}(S^{p-4})\,, \tag{3.12}$$

$$\Phi^{\ell}_{X} = c_X\, s^{1/4}\, \Lambda^{-1/2} \left(\frac{\rho}{R}\right)^{A} J_{\gamma}\left[\frac{s^{1/2}R}{B}\left(\frac{\rho}{R}\right)^{-B}\right] e^{iP\cdot y}\, Y^{\ell}(S^{p-4})\,, \tag{3.13}$$

were we have used the full solution for $\Phi$ in the second case, corresponding to the intermediate state $X$, and the leading behavior in the region $\rho \sim \rho_{\mathrm{int}}$ for the initial/final hadronic state (IN/OUT). $J_\gamma$ is the Bessel function of first kind, and $s = -(P+q)^2 = M_X^2$ is the mass-squared of the intermediate state, while $c_X$ and $c_i$ are dimensionless constants. The order of the Bessel function is given by

$$\gamma^2 = \frac{A^2 + \ell(\ell+p-5)}{B^2}\,, \tag{3.14}$$

with the definitions

$$A = \frac{1-\theta}{2}\,, \qquad B = \frac{\alpha-\beta-2}{2}\,. \tag{3.15}$$

These are scalar and pseudoscalar mesons for even and odd values of $\ell$, respectively. This can be seen from the fact that under parity transformation the spherical harmonics satisfy the equation $[Y^{\ell}(S^{p-4})]_{\mathrm{P}} = (-1)^{\ell}\, Y^{\ell}(S^{p-4})$.

By applying the method developed in [4], we couple these holographic scalar mesons to a gauge field in the bulk. This is done by considering a metric fluctuation as given in eq. (1.8). Then, we use the eigenvalue equation $v^j\, \partial_j Y^{\ell}(\Omega) = i\, \mathcal{Q}_{\ell}\, Y^{\ell}(\Omega)$, obtaining the interaction Lagrangian[3]

$$\mathcal{L}^{\mathrm{scalar}}_{\mathrm{interaction}} = i\, \mathcal{Q}\, \mu_p\, (\pi\alpha')^2 \sqrt{-\det g} \left(\frac{\rho}{R}\right)^{\beta} A^m \left(\Phi\, \partial_m \Phi^*_X - \Phi^*_X\, \partial_m \Phi\right). \tag{3.16}$$

The angular dependence on the spherical harmonics corresponds to functions which are charge eigenstates, with charge $\mathcal{Q}$ under the U(1) symmetry group which is induced by transformations on the internal $S^{p-4}$ in the direction of the Killing vector $v^j$.

Alternatively, as we explained in [4], $\mathcal{L}^{\mathrm{scalar}}_{\mathrm{interaction}}$ can be obtained from the coupling of the gauge field $A_m$ to the Noether's current corresponding to the global transformations which leave invariant the Lagrangian (3.6). These are transformations of a $\mathrm{U}(1) \subseteq \mathrm{SO}(p-3)$, being the latter the isometry group of $S^{p-4}$. The referred Noether's current is

$$j^{\mathrm{scalar}}_m = i\, \mu_p\, (\pi\alpha')^2 \left(\frac{\rho}{R}\right)^{\beta} \left(\Phi\, \partial_m \Phi^*_X - \Phi^*_X\, \partial_m \Phi\right), \tag{3.17}$$

and by defining $\mathcal{L}^{\mathrm{scalar}}_{\mathrm{interaction}} = \mathcal{Q}\sqrt{-\det g}\, A^m\, j^{\mathrm{scalar}}_m$, we obtain the same $\mathcal{L}^{\mathrm{scalar}}_{\mathrm{interaction}}$ given by eq. (3.16) from the metric fluctuation. Consequently, $\mathcal{L}^{\mathrm{scalar}}_{\mathrm{interaction}}$ is given by the coupling of the gauge field $A^m$ to the conserved Noether's current $j^{\mathrm{scalar}}_m$. Notice that the scalar fields in eq. (3.10) are charged under the global $\mathrm{U}(1) \subseteq \mathrm{SO}(p-3)$, with $\mathcal{Q}_{\ell} \neq 0$ for $\ell > 0$.

[3] We have dropped the label $\ell$ corresponding to the spherical harmonics from the scalar field $\Phi^{\ell}$, as well as from the charges $\mathcal{Q}_{\ell}$.



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Now let us obtain the relevant matrix element for the hadronic tensor, with the prescription proposed in [1]

$$S_{\text{interaction}} = (2\pi)^4 \, \delta^4(P_X - P - q) \, \tilde{n}_\mu \, \langle P + q, X | J^\mu(0) | P, \mathcal{Q} \rangle \,, \tag{3.18}$$

where $\tilde{n}_\mu$ indicates the polarization unit vector.

In order to calculate the gauge field we have to solve the Maxwell's equation $D_m F^{mn} = 0$, where $m, n = 0, 1, 2, 3, \rho$. We propose the *Ansätze*

$$\begin{aligned} A_\mu &= \widetilde{n}_\mu \, e^{iq\cdot y} \, f(\rho) \,, \\ A_\rho &= e^{iq\cdot y} \, g(\rho) \,, \end{aligned} \tag{3.19}$$

which imply a Lorentz-like gauge. The solution is

$$\begin{aligned} A_\mu &= \widetilde{n}_\mu \, e^{iq\cdot y} \, \frac{1}{\Gamma(n+1)} \left(\frac{qR}{2B}\right)^{n+1} \left(\frac{\rho}{R}\right)^{-(n+1)B} K_{n+1}\left[\frac{qR}{B}\left(\frac{\rho}{R}\right)^{-B}\right] , \\ A_\rho &= -e^{iq\cdot y} \, \frac{i(q\cdot\widetilde{n})}{\Gamma(n+1)} \left(\frac{qR}{2B}\right)^{n+1} \left(\frac{\rho}{R}\right)^{D} K_n\left[\frac{qR}{B}\left(\frac{\rho}{R}\right)^{-B}\right] = -\frac{i}{q^2} \, \eta^{\mu\nu} \, q_\mu \partial_\rho \, A_\nu \,, \end{aligned} \tag{3.20}$$

with

$$D = \frac{-4\alpha + 3\beta + 2}{4} \,, \qquad n = \frac{2+\beta}{4B} \,, \tag{3.21}$$

and $B$ is given in eq. (3.15). The current conservation equation reads[4]

$$\partial \cdot j^{\text{scalar}} = \left(\frac{\rho}{R}\right)^{-\alpha - \frac{(p+3)}{2}\beta - (p-4)} \partial_\rho\left[\left(\frac{\rho}{R}\right)^{2\alpha + \frac{(p-5)}{2}\beta + (p-4)} j_\rho^{\text{scalar}}\right] = 0 \,, \tag{3.22}$$

while the coupling is

$$A_m \, j^m_{\text{scalar}} = \left(\frac{\rho}{R}\right)^{-\alpha} A_\mu \left[j^\mu_{\text{scalar}} - i\,\frac{q^\mu}{q^2} \, (\partial \cdot j_{\text{scalar}})\right] - i\,\frac{q^\nu}{q^2} \left(\frac{\rho}{R}\right)^{3-\theta} \partial_\rho\left[\left(\frac{\rho}{R}\right)^{\theta-1} j_\rho^{\text{scalar}} A_\nu\right] . \tag{3.23}$$

Then, the interaction reads

$$\begin{aligned} S^{\text{scalar}}_{\text{interaction}} &= \mathcal{Q} \int_{\rho_0}^{\infty} d^{p+1}x \, \sqrt{-\det g} \, A_m \, j^m_{\text{scalar}} \\ &= \mathcal{Q} \int_{\rho_0}^{\infty} d^{p+1}x \, \sqrt{-\det g} \left(\frac{\rho}{R}\right)^{-\alpha} A_\mu \left[j^\mu_{\text{scalar}} - i\frac{q^\mu}{q^2} \, (\partial \cdot j_{\text{scalar}})\right] + \\ &\quad \frac{q^\nu}{q^2} \, \mathcal{Q} \int_{\rho_0}^{\infty} d^{p+1}x \, \sqrt{-\det g} \left(\frac{\rho}{R}\right)^{-\theta} \partial_\rho\left[\left(\frac{\rho}{R}\right)^{\theta-1} j_\rho^{\text{scalar}} \, A_\nu\right] \\ &\equiv I_1^{\text{scalar}} + I_2^{\text{scalar}} \,. \end{aligned} \tag{3.24}$$

[4]Note that the subindex $\rho$ only indicates the variable $\rho$, thus there is no sum whenever it appears repeated.

It can be seen that in the limit $\Lambda \ll q$, $I_2^{\rm scalar} \to 0$. On the other hand, by evaluating $I_1^{\rm scalar}$ and using the Ansatz (3.18), we find

$$\begin{aligned}\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= 2^{\gamma+2}\, B^{\gamma+1}\, \pi^2\, \frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}\, c_i\, c_X^*\, \mu_p\, \mathcal{Q}\, \alpha'^2\, (s^{1/4}\Lambda^{-1/2}) \\ &\quad \times \frac{q^{2n+2}\, s^{\frac{\gamma}{2}}\, R^{p-\gamma-5}}{(q^2+s)^{2+\gamma+n}} \left(P^\mu + \frac{q^\mu}{2x}\right). \end{aligned} \tag{3.25}$$

For $|t| \ll 1$ we can approximate $s \simeq q^2(1/x-1)$, thus the above expression becomes

$$\begin{aligned}\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= 2^{\gamma+2}\, B^{\gamma+1}\, \pi^2\, \frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}\, c_i\, c_X^*\, \mu_p\, \mathcal{Q}\, \alpha'^2\, R^{p-3} \\ &\quad \times \left(\frac{\Lambda}{q}\right)^{\gamma+\frac{3}{2}} x^{\frac{\gamma}{2}+\frac{7}{4}+n}(1-x)^{\frac{\gamma}{2}+\frac{1}{4}} \left(P^\mu + \frac{q^\mu}{2x}\right). \end{aligned} \tag{3.26}$$

Following [1] and [4], we can calculate $\mathrm{Im}\, T^{\mu\nu}$ by multiplying eq. (3.26) by its complex conjugate and summing over radial excitations. We estimate the density of states by introducing an IR cutoff at $\rho_0 \equiv \Lambda R^2$. The distance between zeros of the Bessel function of eq. (3.13) is $M_{n'} = n'\pi\Lambda$, which in the large $N$ limit and for large $q$ gives

$$\sum_{n'} \delta(M_{n'}^2 - s) \sim \left(\frac{\partial M_{n'}^2}{\partial n'}\right)^{-1} \sim (2\pi s^{1/2}\Lambda)^{-1}\,. \tag{3.27}$$

Finally, we obtain

$$\begin{aligned}\mathrm{Im}\, T^{\mu\nu} &= 2^{2\gamma+4}\, B^{2\gamma+2}\, \pi^5\, \frac{\Gamma(\gamma+n+2)^2}{\Gamma(n+1)^2}\, |c_i|^2\, |c_X|^2\, \mu_p^2\, \mathcal{Q}^2\, \alpha'^4\, R^{2p-4} \\ &\quad \times \left(\frac{\Lambda^2}{q^2}\right)^{\gamma+2} x^{\gamma+4+2n}\, (1-x)^\gamma \left(P^\mu + \frac{q^\mu}{2x}\right)\left(P^\nu + \frac{q^\nu}{2x}\right). \end{aligned} \tag{3.28}$$

After checking that our $W^{\mu\nu}$ satisfies all the symmetry requirements described above, we obtain the structure functions for the scalar mesons from eq. (2.9):

$$F_1 = 0, \qquad\qquad F_2 = A_0^{\rm scalar} \mu_p^2 \mathcal{Q}^2 \alpha'^4 R^{2p-6} \left(\frac{\Lambda^2}{q^2}\right)^{\gamma+1} x^{\gamma+3+2n}(1-x)^\gamma\,, \tag{3.29}$$

where $A_0^{\rm scalar} = 2^{2\gamma+4}\, B^{2\gamma+2}\, \pi^5\, \frac{\Gamma(\gamma+n+2)^2}{\Gamma(n+1)^2}\, |c_i|^2\, |c_X|^2$ is a dimensionless normalization constant. We can easily check that our previous results for D3D7 and D4D8$\overline{\text{D8}}$-brane systems introduced in [4] are recovered. Also, for the D4D6$\overline{\text{D6}}$-brane system we obtain the results shown in appendix B of the present work.

### 3.3 DIS from vector mesons

In this subsection we calculate the hadronic tensor for vector mesons arising from a single probe brane, i.e. $N_f = 1$. Next, we will decompose this tensor in order to obtain the structure functions. The procedure will be analogous to that developed in last subsection, though the calculations are more tedious.

Vector mesons arise from fluctuations of the vector fields on the Dirac-Born-Infeld (DBI) action of the probe Dp-brane, which are in the directions parallel to the brane world-volume [10]. The starting point is the action (3.3). We calculate the EOM for vector fluctuations, keeping $Z^i$ constant, i.e. $\Phi = 0$, and then by expanding the Lagrangian up to quadratic order in the fluctuation. This new Lagrangian gives the following EOM

$$\partial_a\left(\sqrt{-\det g}F^{ab}\right) = 0\,, \tag{3.30}$$

where $F^{ab} = \partial^a B^b - \partial^b B^a$, and the indices $a, b = 0, \ldots, p$ run over all directions within the Dp-brane world-volume. We have considered only the DBI term of eq. (3.3), since the Wess-Zumino term does not contribute at first order for the set of solutions in which we are interested. By expanding eq. (3.30) we can write

$$\Box B^\mu - \partial^\mu(\partial\cdot B) + \theta R^{-1}\left(\frac{\rho}{R}\right)^{\alpha-\beta-1}\partial_\rho B^\mu + \left(\frac{\rho}{R}\right)^{\alpha-\beta}\partial_\rho^2 B^\mu + \nabla_i\nabla^i B^\mu = 0. \tag{3.31}$$

We propose the same Ansatz used in [10] for the solution of vector mesons $B_\mu$

$$B^\ell_\mu = \zeta_\mu\,\phi^\ell(\rho)\,e^{iP\cdot y}\,Y^\ell(S^{p-4}), \quad P\cdot\zeta = 0, \quad B_\rho = 0, \quad B_i = 0\,, \tag{3.32}$$

where it has been done an expansion in $Y^\ell(S^{p-4})$, which are spherical harmonics on $S^{p-4}$ satisfying eq. (3.11). $\phi^\ell(\rho)$ is a function to be determined, $\zeta_\mu$ is the polarization vector and the relation $\zeta\cdot P = 0$ comes from $\partial^\mu B_\mu = 0$.

By plugging the *Ansatz* (3.32) in eq. (3.31), we obtain

$$B^\ell_{\mu\,\mathrm{IN/OUT}} = \zeta_\mu\Lambda^{-1}c_i\left(\frac{\rho}{R}\right)^{A-\gamma B} e^{iP\cdot y}\,Y^\ell(S^{p-4})\,, \tag{3.33}$$

$$B^\ell_{\mu X} = \zeta_{\mu X}\Lambda^{-1}c_X(s^{-1/4}\Lambda^{1/2})\left(\frac{\rho}{R}\right)^{A} J_\gamma\left[\frac{s^{1/2}R}{B}\left(\frac{\rho}{R}\right)^{-B}\right] e^{iP\cdot y}\,Y^\ell(S^{p-4})\,. \tag{3.34}$$

We have used the full solution for $B^\ell_\mu$ in the second case, corresponding to the intermediate state $X$ and the leading behavior in the region $\rho\sim\rho_{\mathrm{int}}$ for the initial/final hadronic state. As before, $J_\gamma$ is the Bessel function of first kind, $\gamma^2 = \frac{A^2+\ell(\ell+p-5)}{B^2}$ and $s = -(P+q)^2 = M_X^2$ is the mass-squared of the intermediate state, while $c_X$ and $c_i$ are dimensionless constants. We have also used the definitions for $\theta$, $A$, and $B$ in eqs. (3.8) and (3.15). We can classify the solutions as vector mesons for even values of $\ell$, and axial vector mesons for odd values of $\ell$. This comes from the relations $[Y^\ell(\Omega)]_{\mathrm{P}} = (-1)^\ell\,Y^\ell(\Omega)$ and $[\zeta_\mu]_{\mathrm{P}} = (\zeta^0, -\vec{\zeta})$.

From the expansion of $B_\mu$ in spherical harmonics on $S^{p-4}$ it can be seen that the gauge fields on the Dp-brane correspond to charged fields in $\mathcal{M}_5$. Following an analogous procedure as in [4] it can be seen that the modes with $\ell = 0$ correspond to an Abelian gauge field $B^0_\mu$. The rest of the vector fields, $B^\ell_\mu$ with $\ell > 0$, are charged massive fields. Their charges under the $\mathrm{U}(1)\subseteq\mathrm{SO}(p-3)$ of $S^{p-4}$ are $\mathcal{Q}_\ell$, while their masses are $m_\ell^2 = \ell(\ell+p-5)/R^2$. The EOM for the vector mesons in the interaction region, eq. (3.31), can also be derived from the following quadratic Lagrangian,[5]

$$\mathcal{L}_0^{SF} = -\mu_p\,(\pi\alpha')^2\sqrt{-\det g}\;F^{ab}\,F^*_{ab}\,. \tag{3.35}$$

[5] We use $B^\ell_\mu \equiv B_\mu$, with $\ell > 0$, therefore there is a field $B_\mu$ for each $\ell$. The superscript $SF$ stands for single-flavored ($N_f = 1$) vector mesons.

We reproduce the bulk interaction as we have done in the last subsection, by perturbing the metric with the fluctuation (1.8), as explained before. We use again $v^j\,\partial_j Y^\ell(\Omega) = i\,\mathcal{Q}_\ell\,Y^\ell(\Omega)$, obtaining the interaction Lagrangian

$$\mathcal{L}^{SF}_{\text{interaction}} = i\,\mathcal{Q}\,\mu_p\,(\pi\alpha')^2\,\sqrt{-\det g}\,A_m\,[B^*_{Xn}\,F^{nm} - B_n\,(F^{nm}_X)^*]\,, \tag{3.36}$$

where $A_m$ is the five-dimensional gauge field given in eq. (3.20). As in last subsection, the same interaction Lagrangian can be obtained from the coupling of the gauge field $A_m$ to the Noether's current corresponding to the internal global symmetry of the action, in this case eq. (3.35). We can write $\mathcal{L}^{SF}_{\text{interaction}} = \mathcal{Q}\sqrt{-\det g}\,A_m\,j^m_{SF}$ where

$$j^m_{SF} = i\,\mu_p\,(\pi\alpha')^2\,[B^*_{Xn}\,F^{nm} - B_n\,(F^{nm}_X)^*]\,, \tag{3.37}$$

is a conserved current. Following a similar procedure as for scalar mesons, we have the action of the interaction

$$\begin{aligned} S^{SF}_{\text{interaction}} &= \mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\,A_m\,j^m_{SF} \\ &= \mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\left(\frac{\rho}{R}\right)^{-\alpha} A_\mu\left[j^\mu_{SF} - i\,\frac{q^\mu}{q^2}\,(\partial\cdot j_{SF})\right] \\ &\quad + \frac{q^\nu}{q^2}\,\mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\left(\frac{\rho}{R}\right)^{-\theta}\partial_\rho\left[\left(\frac{\rho}{R}\right)^{\theta-1} j^{SF}_\rho\,A_\nu\right] &(3.38)\\ &\equiv I^{SF}_1 + I^{SF}_2\,. &(3.39)\end{aligned}$$

In particular, $I^{SF}_2 \to 0$ when $\Lambda \ll q$. By evaluating $I^{SF}_1$ and using the Ansatz (3.18), we find

$$\begin{aligned}\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= 2^\gamma\,B^{\gamma+1}\,\pi^2\,\frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}\,c_i\,c^*_X\,\mu_p\,\mathcal{Q}\,\alpha'^2\,R^{p-\gamma-5} \\ &\qquad\times(s^{-1/4}\Lambda^{1/2})\,\frac{q^{2\gamma+2}s^{\frac{\gamma}{2}}}{(q^2+s)^{2+\gamma+n}}\,N^\mu\,, \end{aligned}\tag{3.40}$$

with

$$N^\mu = 2\,(\zeta\cdot\zeta_X)\left(P^\mu + \frac{q^\mu}{2x}\right) + (\zeta_X\cdot q)\zeta^\mu - (\zeta\cdot q)\zeta^\mu_X\,. \tag{3.41}$$

We can see that $N^\mu$, which carries all the information about the vector dependence in the matrix element (3.40), and therefore in the structure functions, does not depend on the particular model.

For $|t| \ll 1$ we can approximate $s \simeq q^2(\frac{1}{x} - 1)$, and we obtain

$$\begin{aligned}\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= 2^\gamma\,B^{\gamma+1}\,\pi^2\,\frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}\,c_i\,c^*_X\,\mu_p\,\mathcal{Q}\,\alpha'^2\,R^{p-3} \\ &\qquad\times\left(\frac{\Lambda}{q}\right)^{\gamma+\frac{5}{2}} x^{\frac{\gamma}{2}+\frac{9}{4}+n}(1-x)^{\frac{\gamma}{2}-\frac{1}{4}}N^\mu \\ &\equiv f^{(\gamma)}_\Lambda(x,q)N^\mu. \end{aligned}\tag{3.42}$$

We now multiply eq. (3.42) by its complex conjugate and sum over the radial excitations and over the polarizations of the final hadronic states $\zeta_X^\mu$, since we want to calculate $\mathrm{Im}T^{\mu\nu}$ from eq. (1.3). The density of states is estimated in the same way as we have done for the scalar mesons, obtaining

$$\mathrm{Im}\, T^{\mu\nu} = \frac{\pi f f^*}{\Lambda s^{1/2}} \sum_\lambda N^\mu N^{*\nu} \,. \tag{3.43}$$

By using the solution (3.32), then we normalize the polarizations as $\zeta^\mu(P_X,\lambda)\cdot\zeta^*_\mu(P_X,\lambda') = -M_X^2\delta_{\lambda,\lambda'}$, and by neglecting terms proportional to $q_\mu$ and $q_\nu$, we finally obtain

$$\mathrm{Im}T_{\mu\nu} \equiv 2\frac{\pi x^{\frac{1}{2}} f f^*}{\Lambda q(1-x)^{\frac{1}{2}}} H_{\mu\nu} = 2\frac{\pi x^{\frac{1}{2}} f f^*}{\Lambda q(1-x)^{\frac{1}{2}}} (H^S_{\mu\nu} + H^A_{\mu\nu})\,, \tag{3.44}$$

where $H^S_{\mu\nu}$ and $H^A_{\mu\nu}$ are the symmetric and antisymmetric parts of $H_{\mu\nu}$, respectively,

$$\begin{aligned} H^S_{\mu\nu} &= -\eta_{\mu\nu}(\zeta.q)(\zeta^*.q)(P+q)^2 + P_\mu P_\nu\Big[-4P^2(P+q)^2 + (q.\zeta)(q.\zeta^*)\Big] \\ &\quad + (\zeta_\mu\zeta^*_\nu + \zeta_\nu\zeta^*_\mu)\frac{1}{2}\Big[(P\cdot q)^2 - P^2q^2\Big] \\ &\quad + (P_\mu\zeta^*_\nu + P_\nu\zeta^*_\mu)(\zeta\cdot q)\frac{1}{2}\Big[P\cdot q + q^2\Big] + (P_\mu\zeta_\nu + P_\nu\zeta_\mu)(\zeta^*\cdot q)\frac{1}{2}\Big[P\cdot q + q^2\Big], \end{aligned} \tag{3.45}$$

and

$$\begin{aligned} H^A_{\mu\nu} &= \frac{1}{2}(\zeta_\mu\zeta^*_\nu - \zeta_\nu\zeta^*_\mu)\Big[(P\cdot q)^2 - P^2q^2\Big] \\ &\quad + \frac{1}{2}(P_\nu\zeta^*_\mu - P_\mu\zeta^*_\nu)(\zeta\cdot q)\Big[4P^2 + 7P\cdot q + 3q^2\Big] \\ &\quad + \frac{1}{2}(P_\mu\zeta_\nu - P_\nu\zeta_\mu)(\zeta^*\cdot q)\Big[4P^2 + 7P\cdot q + 3q^2\Big]\,. \end{aligned} \tag{3.46}$$

It is straightforward to calculate the tensor $W_{\mu\nu}$ from $\mathrm{Im}T_{\mu\nu}$. By comparing the $W_{\mu\nu}$ tensor obtained in this way with the general form of eq. (2.7) we can extract the eight structure functions (recall that we have derived these equations for $|t| \ll 1$)

$$\begin{aligned} F_1 &= A^{SF}(x)\frac{1}{12x^3}(1 - x - 2xt - 4x^2t + 4x^3t + 8x^3t^2)\,, \\ F_2 &= A^{SF}(x)\frac{1}{6x^3}(1 - x + 12xt - 14x^2t - 12x^2t^2)\,, \\ b_1 &= A^{SF}(x)\frac{1}{4x^3}(1 - x - xt)\,, \\ b_2 &= A^{SF}(x)\frac{1}{2x^3}(1 - x - x^2t)\,, \\ b_3 &= A^{SF}(x)\frac{1}{24x^3}(1 - 4x + 8x^2t)\,, \\ b_4 &= A^{SF}(x)\frac{1}{12x^3}(-1 + 4x - 2x^2t)\,, \\ g_1 &= A^{SF}(x)\frac{t}{8x^2}(-7 + 6x + 8xt)\,, \\ g_2 &= A^{SF}(x)\frac{1}{16x^4}(3 - 3x - 4xt + 2x^2t)\,, \end{aligned} \tag{3.47}$$

where
$$A^{SF}(x) = A_0^{SF}\mu_p^2 \mathcal{Q}^2\alpha'^4 R^{2p-6}\left(\frac{\Lambda^2}{q^2}\right)^{\gamma} x^{\gamma+2n+5}(1-x)^{\gamma-1}\,, \tag{3.48}$$

and $A_0^{SF} = (2B)^{2\gamma+2}\,\pi^5\,\frac{\Gamma(\gamma+n+2)^2}{\Gamma(n+1)^2}\,|c_i|^2\,|c_X|^2$ is a dimensionless normalization constant. Recall that $\gamma$ is given in eq. (3.14) and $n$ in eq. (3.21). Constants $\alpha$ and $\beta$ come from the definition of the general background metric (3.1), $A$ and $B$ are defined in eqs. (3.15) and $\theta$ is given by eq. (3.8).

We can see that, as it happened with the scalar mesons, the results for D3D7 and D4D8$\overline{\text{D8}}$-brane systems from [4] are recovered,[6] as well as that for D4D6$\overline{\text{D6}}$-brane system given in appendix B.

## 4 DIS from vector mesons with $N_f > 1$

### 4.1 General background calculations

In this section, we study a general approach to obtain the structure functions for polarized vector mesons with $N_f > 1$ flavors. From the string theory dual model these mesons arise by considering $N_f > 1$ probe Dp-branes. In particular, we show that the structure functions can be decomposed in model-dependent and model-independent factors, as it occurs when $N_f = 1$. We calculate both factors for a general model with an induced metric given by eq. (3.2). All the calculations in this section are within the tree-level approximation. One-loop corrections are discussed in section 5.

We consider the same background as in section 3, given by the induced metric (3.2) on the $N_f$ probe Dp-branes

$$ds^2 = \left(\frac{\rho}{R}\right)^{\alpha}\eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{\rho}{R}\right)^{\beta}\left[d\rho^2 + \rho^2\, d\Omega_{p-4}^2\right],$$

at least in the asymptotic region, $\rho_{\text{int}} \gg \rho_0 = \Lambda R^2$.

We start from the non-Abelian Dirac-Born-Infeld action [5]

$$S_0^{MF} = -\mu_p \int d^{p+1}\xi\,\sqrt{-\det g}\,(\pi\alpha')^2\,\mathrm{Tr}(F^2)\,, \tag{4.1}$$

where $F_{ab} = \partial_a B_b - \partial_b B_a + i\,[B_a, B_b]$ and $\mu_p = [(2\pi)^p g_s \alpha'^{\frac{p+1}{2}}]^{-1}$. This is the generalization of eq. (3.35) for the case of mesons with $N_f > 1$.

In order to calculate the hadronic tensor using the holographic dual prescription we consider that the holographic meson couples to a gauge field (3.20) in the bulk of the string theory dual model as in section 3.

We can expand the action eq. (4.1) in terms of $B_\mu$ obtaining[7]

$$\mathcal{L}_0^{MF} = -\mu_p\,(\pi\alpha')^2\,\sqrt{-\det g}\,Tr\left\{\widehat{F}^*_{ab}\widehat{F}^{ab} + \left(i\,\widehat{F}^*_{ab}[B^a, B^b] + c.c.\right) - [B_a^*, B_b^*][B^a, B^b]\right\}, \tag{4.2}$$

[6] We have found a mistake in the equation for $g_1$ in our previous work [4]. The present solution is the correct one.

[7] We write $\mathrm{Tr}(F^*_{ab}F^{ab})$ instead of $\mathrm{Tr}(F_{ab}F^{ab})$ since their EOM's are the same.



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where we have defined $\widehat{F}_{ab} = \partial_a B_b - \partial_b B_a = F_{ab} - i\,[B_a, B_b]$. As we shall see in section 5.4, the last two terms are sub-leading with respect to the first one in the $1/N$ expansion. Therefore, at leading order we only keep the first term. Thus, we obtain the same interaction Lagrangian as for vector mesons with $N_f = 1$.

The EOM can be expanded and we obtain

$$\Box B^\mu - \partial^\mu(\partial \cdot B) + \theta\, R^{-1}\left(\frac{\rho}{R}\right)^{\alpha-\beta-1}\partial_\rho B^\mu + \left(\frac{\rho}{R}\right)^{\alpha-\beta}\partial_\rho^2 B^\mu + \nabla_i\nabla^i B^\mu = 0, \tag{4.3}$$

which is the same as eq. (3.31). The *Ansatz* for the solution of the vector mesons $B_\mu$ is

$$\begin{aligned} B_\mu^\ell &= \sum_{\mathcal{A}=1}^{N_f} B_\mu^{(\mathcal{A})\ell}\,\tau_{\mathcal{A}}\,, \\ B_\mu^{(\mathcal{A})\ell} &= \zeta_\mu\, c^{(\mathcal{A})}\phi^\ell(\rho)\, e^{iP\cdot y}\, Y^\ell(S^{(p-4)}), \quad P\cdot\zeta = 0, \quad B_\rho^{(\mathcal{A})\ell} = 0, \quad B_i^{(\mathcal{A})\ell} = 0\,, \end{aligned} \tag{4.4}$$

where $\tau_{\mathcal{A}}$ are the generators of the flavor group $\mathrm{SU}(N_f)$, which satisfy the Lie algebra

$$[\tau_{\mathcal{A}}, \tau_{\mathcal{B}}] = i\, f_{\mathcal{ABC}}\, \tau_{\mathcal{C}}\,. \tag{4.5}$$

We have also expanded $B_\mu^{(\mathcal{A})}$ in spherical harmonics $Y^\ell(S^{(p-4)})$, satisfying eq. (3.11). The radial dependence $\phi(\rho)$ is to be determined, $\zeta_\mu$ is the polarization vector and the relation $\zeta\cdot P = 0$ comes from $\partial^\mu B_\mu = 0$. By using the *Ansatz* (4.4) in eq. (4.3), we obtain the solution for each component $B_\mu^{(\mathcal{A})}$ which coincides with the vector mesons with $N_f = 1$ studied in previous section, namely

$$B_{\mu\,\mathrm{IN/OUT}}^{(\mathcal{A})\ell} = \zeta_\mu \Lambda^{-1} c_i^{(\mathcal{A})}\left(\frac{\rho}{R}\right)^{A-\gamma B} e^{iP\cdot y}\, Y^\ell(S^{(p-4)})\,, \tag{4.6}$$

$$B_{\mu X}^{(\mathcal{A})\ell} = \zeta_{\mu X}\Lambda^{-1} c_X^{(\mathcal{A})}(s^{-1/4}\Lambda^{1/2})\left(\frac{\rho}{R}\right)^{A} J_\gamma\left[\frac{s^{1/2}R}{B}\left(\frac{\rho}{R}\right)^{-B}\right] e^{iP\cdot y}\, Y^\ell(S^{(p-4)})\,. \tag{4.7}$$

We have used the full solution for $B_\mu^{(\mathcal{A})\ell}$ in the second case, corresponding to the intermediate state $X$ and the leading behaviour in the region $\rho \sim \rho_{\mathrm{int}}$ for the initial/final hadronic state (IN/OUT). As before, $J_\gamma$ is the Bessel function of first kind, $\gamma^2 = \frac{A^2+\ell(\ell+p-5)}{B^2}$ and $s = -(P+q)^2 = M_X^2$ is the mass-squared of the intermediate state, while $c_X^{(\mathcal{A})}$ and $c_i^{(\mathcal{A})}$ are dimensionless constants. We have also used the definitions for $\theta$, $A$, and $B$ in eqs. (3.8) and (3.15). From the expansion of $B_\mu^{(\mathcal{A})}$ in spherical harmonics on $S^{(p-4)}$, it can be seen that the gauge fields on the branes correspond to charged massive fields in the five-dimensional space spanned by coordinates $0, 1, 2, 3$, and $\rho$, for $\ell > 1$, and a gauge field $B_\mu^0$.

By considering the metric fluctuation from eq. (1.8), and equation $v^j\,\partial_j Y^\ell(\Omega) = i\,\mathcal{Q}_\ell\, Y^\ell(\Omega)$, we obtain the interaction Lagrangian[8]

$$\begin{aligned} \mathcal{L}_{\mathrm{interaction}}^{MF} &= i\mathcal{Q}\mu_p(\pi\alpha')^2\sqrt{-\det g}\,\Big\{A_m Tr\big(B_{Xn}^*\widehat{F}^{nm} - B_n(\widehat{F}_X^{nm})^*\big) \\ &\qquad + iA_m Tr\big(B_n[B_X^{m*}, B_X^{n*}]\big) + iA_m Tr\big(B_{Xn}^*[B^m, B^n]\big)\Big\} \\ &\equiv \mathcal{L}_{\mathrm{interaction}_1}^{MF} + \mathcal{L}_{\mathrm{interaction}_2}^{MF} + \mathcal{L}_{\mathrm{interaction}_3}^{MF}. \end{aligned} \tag{4.8}$$

[8]In what follows we denote $B_\mu \equiv B_\mu^\ell$, i.e. we omit the $\ell$ index to make our notation simpler. The label $MF$ stands for multi-flavored vector mesons, i.e. those with $N_f > 1$.

It is easy to see that the term $\mathcal{L}^{MF}_{\text{interaction}_3}$ does not contribute to the process of interest, since it involves two initial states for the hadron $B_\mu$. On the other hand, we will show in the next section that the term $\mathcal{L}^{MF}_{\text{interaction}_2}$ contributes only to diagrams which are sub-leading in the $1/N$ expansion. Therefore, the only diagram which contributes to leading order is that of figure 1, which only involves the first term $\mathcal{L}^{MF}_{\text{interaction}_1}$. This is the same diagram present in the $N_f = 1$ vector mesons studied in last section. We can see it as the coupling of the gauge field $A_m$ to a certain current $j^m_{MF}$. Therefore, $\mathcal{L}^{MF}_{\text{interaction}_1} = \mathcal{Q}\sqrt{-\det g}\, A_m\, j^m_{MF}$ where

$$j^m_{MF} = i\,\mu_p\,(\pi\alpha')^2\, Tr\big(B^*_{Xn}F^{nm} - B_n(F^{nm}_X)^*\big)\,. \tag{4.9}$$

The action of interaction is then

$$\begin{aligned} S^{MF}_{\text{interaction}} &= \mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\,A_m\,j^m_{MF} \\ &= \mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\left(\frac{\rho}{R}\right)^{-\alpha} A_\mu\left[j^\mu_{MF} - i\,\frac{q^\mu}{q^2}\,(\partial\cdot j_{MF})\right] \\ &\quad + \frac{q^\nu}{q^2}\mathcal{Q}\int_{\rho_0}^{\infty} d^{p+1}x\,\sqrt{-\det g}\left(\frac{\rho}{R}\right)^{-\theta}\partial_\rho\left[\left(\frac{\rho}{R}\right)^{\theta-1} j^{MF}_\rho A_\nu\right] \\ &\equiv I^{MF}_1 + I^{MF}_2\,. \end{aligned} \tag{4.10}$$

As we have seen, $I^{MF}_2 = I^{SF}_2 \to 0$ in the limit of interest, $\Lambda \ll q$. On the other hand, by evaluating $I^{MF}_1$ we can see that

$$\begin{aligned} \langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= C_f\,\delta_{\mathcal{AB}}\,I^{SF}_1 \\ &= C_f\,\delta_{\mathcal{AB}}\,(2B)^{\gamma+2}\,\pi^2\,\frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}\,c_i\,c^*_X\,\mu_p\,\mathcal{Q}\,\alpha'^2\,R^{p-\gamma-5} \\ &\quad (s^{-1/4}\Lambda^{1/2})\,\frac{q^{2\gamma+2}s^{\frac{\gamma}{2}-n}}{(q^2+s)^{2+\gamma+n}}\,N^\mu \\ &= C_f\,\delta_{\mathcal{AB}}\,f^{(\gamma)}_\Lambda(x,q)\,N^\mu\,, \end{aligned} \tag{4.11}$$

where we define $f^{(\gamma)}_\Lambda(x,q)$ as in last section and have used

$$\begin{aligned} B_{\mu\,\text{IN/OUT}} &= B^{(\mathcal{A})}_{\mu\,\text{IN/OUT}}\,\tau_{\mathcal{A}}, &&\text{(no sum)} \\ B_{\mu X} &= B^{(\mathcal{B})}_{\mu X}\,\tau_{\mathcal{B}}, &&\text{(no sum)} \end{aligned} \tag{4.12}$$

and

$$N^\mu = 2(\zeta\cdot\zeta_X)\left(P^\mu + \frac{q^\mu}{2x}\right) + (\zeta_X\cdot q)\zeta^\mu - (\zeta\cdot q)\zeta^\mu_X\,. \tag{4.13}$$

We also use

$$[\tau_{\mathcal{A}},\tau_{\mathcal{B}}] = i\,f_{\mathcal{ABC}}\,\tau_{\mathcal{C}}\,, \qquad Tr(\tau_{\mathcal{A}}\tau_{\mathcal{B}}) = C_f\,\delta_{\mathcal{AB}}\,, \tag{4.14}$$

being $C_f$ the Casimir of SU($N_f$).

For $|t| \ll 1$ we can approximate $s \simeq q^2(\frac{1}{x}-1)$, as we have done in the previous section

$$\begin{aligned} \langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle &= C_f\delta_{AB}\mathcal{Q}\mu_p(2)^\gamma B^{\gamma+1}\frac{\Gamma(\gamma+n+2)}{\Gamma(n+1)}(\pi\alpha')^2 c_i c^*_X N^\mu R^{p-3} \\ &\quad \times\left(\frac{\Lambda}{q}\right)^{\gamma+\frac{5}{2}} x^{\frac{\gamma}{2}+n+\frac{9}{4}}(1-x)^{\frac{\gamma}{2}-\frac{1}{4}}\,. \end{aligned} \tag{4.15}$$

### 4.2 Results for the structure functions

In order to obtain $\mathrm{Im}T^{\mu\nu}$, we multiply eq. (4.15) by its complex conjugate and sum over the radial excitations and over the polarizations of the final hadronic states $\zeta_X^\mu$. The density of states can be estimated as for the scalar and vector mesons. We then sum over polarizations and neglect terms proportional to $q_\mu$ and $q_\nu$ as in the previous section, obtaining

$$\mathrm{Im}\, T^{\mu\nu} = C_f^2\, \delta_{\mathcal{AB}_X}\, \frac{\pi f f^*}{\Lambda s^{1/2}} \sum_\lambda N^\mu N^{*\nu} = C_f^2\, \delta_{\mathcal{AB}_X}\, \frac{\pi\, x^{\frac{1}{2}} f f^*}{\Lambda q(1-x)^{\frac{1}{2}}} (H^S_{\mu\nu} + H^A_{\mu\nu})\,, \tag{4.16}$$

where we have defined $\langle P+q, X|J^\mu(0)|P,\mathcal{Q}\rangle = C_f\, \delta_{\mathcal{AB}_X}\, f_\Lambda^{(\gamma)}(x,q)\, N^\mu$, while $H^S_{\mu\nu}$ and $H^A_{\mu\nu}$ are exactly the same as eqs. (3.45) and (3.46).

By rewriting the hadronic tensor for spin-1 hadrons $W_{\mu\nu}$ from eq. (2.7), we obtain the following structure functions

$$\begin{aligned}
F_1 &= A^{MF}(x)\frac{1}{12x^3}(1-x-2xt-4x^2t+4x^3t+8x^3t^2)\,,\\
F_2 &= A^{MF}(x)\frac{1}{6x^3}(1-x+12xt-14x^2t-12x^2t^2)\,,\\
b_1 &= A^{MF}(x)\frac{1}{4x^3}(1-x-xt)\,,\\
b_2 &= A^{MF}(x)\frac{1}{2x^3}(1-x-x^2t)\,,\\
b_3 &= A^{MF}(x)\frac{1}{24x^3}(1-4x+8x^2t)\,,\\
b_4 &= A^{MF}(x)\frac{1}{12x^3}(-1+4x-2x^2t)\,,\\
g_1 &= A^{MF}(x)\frac{t}{8x^2}(-7+6x+8xt)\,,\\
g_2 &= A^{MF}(x)\frac{1}{16x^4}(3-3x-4xt+2x^2t)\,,
\end{aligned} \tag{4.17}$$

where

$$A^{MF}(x) = A_0^{MF}\mu_p^2\mathcal{Q}^2\alpha'^4R^{2p-6}\left(\frac{\Lambda^2}{q^2}\right)^\gamma x^{\gamma+2n+5}(1-x)^{\gamma-1}\,, \tag{4.18}$$

and $A_0^{MF} = 2C_f^2\,\delta_{\mathcal{AB}_X}\,(2B)^{2\gamma+2}\,\pi^5\,\frac{\Gamma(\gamma+n+2)^2}{\Gamma(n+1)^2}\,|c_i^{(\mathcal{A})}|^2|\,c_X^{(\mathcal{B}_X)}|^2$ is a dimensionless normalization constant. Subindex $\mathcal{A}$ labels the flavor of the incoming meson state, and $\mathcal{B}_X$ that of the intermediate state. These equations have been obtained in the limit $|t|\ll 1$.

Notice that if we take the Abelian (single-flavored) limit we obtain the same full set of structure functions calculated in section 3. Some particular cases have been calculated in [4] and in appendix B.[9] Thus, we can summarize our results in a compact form

$$F_i^{(a)MF}(x,t) = C_f^2\,\delta_{\mathcal{AB}_X}\,F_i^{(a)SF}(x,t)\,, \tag{4.19}$$

for each holographic dual model $(a)$, where $i$ indicates the each particular structure function, $i=1,\cdots,8$.

[9] We can redefine the normalization constants as $c_i^{(A)} = c_i/\sqrt{C_f}$ and $c_X^{(\mathcal{B}_X)} = c_X/\sqrt{C_f}$ for the Abelian case.

On the other hand, for each pair of holographic dual models $(a)$ and $(b)$ we find the relation $F_i^{(a)}(x,t) = A_{(a,b)}(x)\, F_i^{(b)}(x,t)$, which leads to the following relation for the hadronic tensor

$$W^{\mu\nu}_{(a)} = A_{(a,b)}(x)\, W^{\mu\nu}_{(b)} \,. \tag{4.20}$$

Very interestingly, the set of eqs. (4.17) leads to the following inequality[10]

$$F_1 \geq |g_1| \,, \tag{4.21}$$

which holds for $|t| \ll 1$ for each Dp-brane model. This relation implies the following inequality among moments of the structure functions

$$M_n(F_1) \geq |M_n(g_1)| \quad n = 1, 2, \ldots \tag{4.22}$$

which must be satisfied from unitarity [7]. On the other hand, since $F_1 \geq 0$ and $0 \leq x \leq 1$, the chain of inequalities

$$M_n(F_1) \geq M_{n+1}(F_1) \quad n = 1, 2, \ldots \tag{4.23}$$

is satisfied. The moments of the structure functions $F_1$ and $g_1$ are defined as follows

$$M_n(F_1) = \int_0^1 dx\, x^{n-1}\, F_1(x, q^2) \,, \tag{4.24}$$

$$M_n(g_1) = \int_0^1 dx\, x^{n-1}\, g_1(x, q^2) \,. \tag{4.25}$$

In addition, we have found relations between different structure functions that we shall discuss in the conclusions.

## 5 Sub-leading contributions to the $1/N$ expansion

In this section we investigate the sub-leading contributions to the $1/N$ and $N_f/N$ expansions of the two-point correlation functions of global symmetry currents. We explain why in the large $N$ limit we only have to consider the tree-level Witten's diagram displayed in figure 1, which is the holographic dual version of the Feynman's diagram of the forward Compton scattering of a charged lepton by a hadron. We consider the full relevant Lagrangians of the three cases studied in sections 3 and 4, corresponding to scalar mesons, $N_f = 1$ vector mesons, and $N_f > 1$ vector mesons, respectively. We study the sub-leading contributions given by one-loop diagrams.

### 5.1 Five-dimensional reduction of type IIB supergravity

We very briefly review the five-dimensional reduction of type IIB supergravity on $S^5$ as presented in [12] (other relevant references for this section are [13–15]), in order to give an example of the $N$-power counting in supergravity Feynman's diagrams.

[10] These comments also hold for $N_f = 1$, see eqs. (3.47).

Let us begin with the ten-dimensional type IIB supergravity action written in the Einstein frame, which contains the graviton, dilaton $\phi$, the Ramond-Ramond axion field $\mathcal{C}$ and the five-form field strength $F_5$

$$S_{IIB}^{\rm SUGRA} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{|\det g|}\left[\mathcal{R}_{10} - \frac{1}{2}\,(\partial\phi)^2 - \frac{1}{2}\,e^{2\phi}\,(\partial\mathcal{C})^2 - \frac{1}{4\cdot 5!}\,(F_5)^2\right]. \qquad (5.1)$$

We consider the $\mathrm{AdS}_5\times S^5$ metric with the radius $R^4 = 4\pi g_s N\alpha'^2$.

Now, the five-dimensionally reduced action, in terms of the five-dimensional dilaton $\phi_5(x)$ takes the form

$$S_{5d}^{\rm SUGRA} = -\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{|\det g_5|}\left[\mathcal{R}_5 - \frac{1}{2}\,(\partial\phi_5)^2 + \cdots\right], \qquad (5.2)$$

where dots indicate other terms which are not relevant for the present discussion, since we are only interested in the $N$-power counting. We consider the constant $\kappa_5$, which is defined as

$$\frac{1}{2\kappa_5^2} = \frac{N^2}{8\pi^2}\,. \qquad (5.3)$$

Hence, we can see how the factor $N^2$ appears in the five-dimensional action. When we consider the D3D7-brane system, for instance, the power-counting structure for the pure five-dimensional supergravity action plus the DBI-action of the $N_f$ probe D7-branes schematically reads

$$S = N^2\left[\tilde{S}_{IIB}^{\rm SUGRA} + \frac{N_f}{N}\,\tilde{S}_{DBI}\right], \qquad (5.4)$$

where $\tilde{S}$ indicates the corresponding actions with kinetic terms which do not depend on $N$. Thus, in order to obtain canonically normalized fields we redefine the five-dimensional dilaton as $\tilde{\phi}_5 \equiv N\phi_5$, and similarly for the graviton. By plugging the normalized fields into the action $S$ one obtains the correct power of $N$ in each interaction vertex. Therefore, one can construct the Witten's diagrams for holographic dual processes, displaying the corresponding $N$-power counting in each case.

### 5.2 Scalar mesons

The relevant part of the free Lagrangian for scalar mesons eq. (3.6) can be rewritten as[11]

$$\mathcal{L}_0^{\rm scalar} = -\mu_p(2\pi\alpha')^2\,\sqrt{-\det g}\left(\frac{\rho}{R}\right)^{\beta}\frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi^*\,. \qquad (5.5)$$

By factorizing the scalar field as

$$\Phi(\rho, y^\mu, \Omega) = \sum_\ell \varphi^\ell(\rho, y^\mu)\,Y^\ell(\Omega)\,, \qquad (5.6)$$

and by defining the squared root of the determinant of the five-dimensional piece of the metric as

$$\sqrt{-\det g^5} = \left(\frac{\rho}{R}\right)^{\alpha+\frac{\beta}{2}}, \qquad (5.7)$$

[11] We exclude the first term in eq. (3.6) since it does not contribute to the EOM.

we can write down the reduced five-dimensional free action for each $\varphi^\ell$ as follows

$$S_0^{\rm scalar} = -\mu_p\,(\pi\alpha')^2\,R^{p-4}\int d^5x\,\sqrt{-{\rm det}g^5}\,\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}\left[\partial^m\varphi^\ell\,\partial_m\varphi^{\ell *}+M_\ell^2\,\varphi^\ell\,\varphi^{\ell *}\right], \tag{5.8}$$

where we have used

$$\int d^{p-4}\Omega\,\sqrt{{\rm det}\widetilde{g}}\,Y^\ell\,Y^{\ell' *} = \delta_{\ell\ell'}\,, \tag{5.9}$$

and we have defined

$$\int d^{p-4}\Omega\,\sqrt{{\rm det}\widetilde{g}}\,\eta^{ij}\,\partial_i Y^\ell\,\partial_j Y^{\ell' *} \equiv \delta_{\ell\ell'} M_\ell^2\,. \tag{5.10}$$

Notice that eq. (5.8) is the action for a scalar complex field.[12] Recall the interaction Lagrangian given in eq. (3.16)

$$\mathcal{L}_{\rm interaction}^{\rm scalar} = i\,\mathcal{Q}\,\mu_p\,(\pi\alpha')^2\,\sqrt{-{\rm det}g}\,\left(\frac{\rho}{R}\right)^{\beta}A^m\left(\Phi\partial_m\,\Phi^* - \Phi^*\,\partial_m\Phi\right),$$

which under the same five-dimensional reduction becomes[13]

$$S_{\rm interaction}^{\rm scalar} = i\mathcal{Q}\mu_p(\pi\alpha')^2\,R^{p-4}\int d^5x\,\sqrt{-{\rm det}g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}A^m\left[\varphi\,\partial_m\varphi^* - \varphi^*\,\partial_m\varphi\right]. \tag{5.11}$$

At the order at which we are interested in, graviton-like perturbations are relevant. By considering the $h_{mn}$ fluctuation on the metric

$$g_{mn} \to g_{mn} + h_{mn}\,, \tag{5.12}$$

this induces the interaction terms[14]

$$h^{mn}\,\partial_m\Phi\,\partial_n\Phi^*; \qquad h^{mn}\,A_m\,\Phi\,\partial_n\Phi^*, \tag{5.13}$$

which, upon five-dimensional reduction of the action given in eq. (5.8), become

$$h^{mn}\,\partial_m\varphi\,\partial_n\varphi^*\,. \qquad h^{mn}\,A_m\,\varphi\,\partial_n\varphi^*. \tag{5.14}$$

By assembling all factors, we obtain the full five-dimensional action

$$\begin{aligned} S_{\rm total}^{\rm scalar} &= S_0^{\rm scalar} + S_{\rm interaction}^{\rm scalar} \\ &= -\mu_p(\pi\alpha')^2 R^{p-4}\int d^5x\sqrt{-{\rm det}g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}\Big[\partial^m\varphi\partial_m\varphi^* + M_\ell^2\varphi\varphi^* \\ &\quad + h^{mn}\partial_m\varphi\partial_n\varphi^* - i\mathcal{Q}\,A^m(\varphi\partial_m\varphi^* - \varphi^*\partial_m\varphi) - i\mathcal{Q}h^{mn}A_m(\varphi\partial_n\varphi^* - \varphi^*\partial_n\varphi)\Big], \end{aligned} \tag{5.15}$$

[12] Recall that we are using the signature (-,+,+,+,+). The full action is $S_0^{\rm scalar} = \sum_\ell S_0^{{\rm scalar}\,\ell}$, and we are writing only one $S_0^{{\rm scalar}\,\ell}$ in eq. (5.8).

[13] Notice that on the last equation we have dropped the subindex $X$ from the interaction Lagrangians. We keep this convention in the rest of this section. Besides, we shall not write the superscript $\ell$.

[14] The kinetic term for the graviton as well as that for the gauge field $A^m$ come from the ten-dimensional supergravity action discussed in the last subsection. Here we only consider the $S_{DBI}$ discussed in sections 3 and 4, defined in the probe-brane worldvolume.

which includes the kinetic term for the scalar mesons, as well as the interaction terms with the graviton and the gauge field $A_m$. The last two terms can be seen as part of a covariant derivative in the kinetic term for the scalar meson. We can redefine the fields in order to be canonically normalized

$$\begin{aligned}\widetilde{\varphi} &\equiv \sqrt{N}\,\varphi\,,\\ \widetilde{A}^m &\equiv N\,A^m\,,\\ \widetilde{h}^{mn} &\equiv N\,h^{mn}\,.\end{aligned} \tag{5.16}$$

By considering that

$$\mu_p(\pi\alpha')^2 = \frac{(\pi\alpha')^2}{(2\pi)^p g_s \alpha'^{\frac{p+1}{2}}} = \frac{1}{2^p\pi^{p-2}}\frac{N}{\lambda\alpha'^{\frac{p-3}{2}}}\,, \tag{5.17}$$

we can write $S^{\text{scalar}}_{\text{total}}$ explicitly in terms having different powers of $N$ as

$$\begin{aligned}S^{\text{scalar}}_{\text{total}} = \frac{R^{p-4}}{2^p\pi^{p-2}\lambda\alpha'^{\frac{p-3}{2}}}\int d^5x\sqrt{-\det g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}\\ \Big[\partial^m\widetilde{\varphi}\partial_m\widetilde{\varphi}^* + M_\ell^2\widetilde{\varphi}\widetilde{\varphi}^* + N^{-1}\widetilde{h}^{mn}\partial_m\widetilde{\varphi}\partial_n\widetilde{\varphi}^*\\ - N^{-1}i\mathcal{Q}\widetilde{A}^m(\widetilde{\varphi}\partial_m\widetilde{\varphi}^* - \widetilde{\varphi}^*\partial_m\widetilde{\varphi}) - N^{-2}i\mathcal{Q}\widetilde{h}^{mn}\widetilde{A}_m(\widetilde{\varphi}\partial_n\widetilde{\varphi}^* - \widetilde{\varphi}^*\partial_n\widetilde{\varphi})\Big]\,.\end{aligned} \tag{5.18}$$

Now, we can construct the relevant diagrams to our process as shown in figures 1 and 2. We can see that only the tree-level diagram of figure 1 contributes to leading order in $N$, namely, $N^{-2}$, while diagrams of figure 2 are sub-leading, i.e. of order $N^{-4}$.

### 5.3 Vector mesons with $N_f = 1$

Let us consider the relevant part of the free Lagrangian for $N_f = 1$ vector mesons given in eq. (3.35)

$$\mathcal{L}_0^{SF} = -\mu_p\,(\pi\alpha')^2\,\sqrt{-\det g}\,F^{ab}F^*_{ab}\,,$$

and define

$$\begin{aligned}B^a(\rho, y^\mu, \Omega) &= \sum_\ell b^{a\,\ell}(\rho, y^\mu)\,Y^\ell(\Omega)\,,\\ f^\ell_{mn} &= \partial_m b^\ell_n - \partial_n b^\ell_m\,.\end{aligned} \tag{5.19}$$

Then, we can write the reduced five-dimensional free action for each $b^\ell_n$ as[15]

$$S_0^{SF} = -\mu_p\,(2\pi\alpha')^2 R^{p-4}\int d^5x\,\sqrt{-\det g^5}\,\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}\left[\frac{1}{4}f^{mn}f^*_{mn} + \frac{1}{2}M_\ell^2\,b^m b^*_m\right], \tag{5.20}$$

which is a Proca-like Lagrangian.

[15] From now on we drop the superscript $\ell$ in the rest of this subsection. The following equation is the part of the action corresponding to only one $b^\ell_n$.

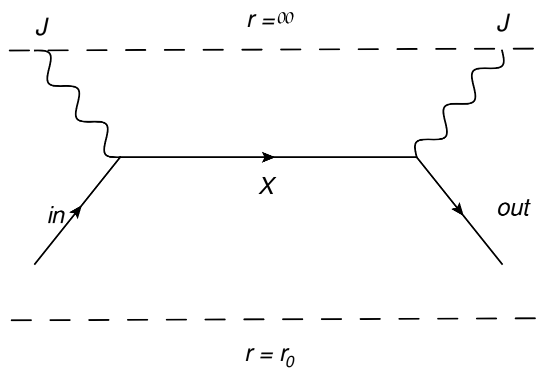


**Figure 1**. Five-dimensional tree-level Witten's diagram which is the holographic dual version of the forward Compton scattering of a charged lepton by a hadron in four-dimensions. This diagram is made of five-dimensional fields obtained from dimensional reduction of ten-dimensional supergravity on a compact Einstein manifold. The dual field of the hadron is indicated with a solid line. The hadron can be a scalar, a $N_f = 1$ vector meson, or a $N_f > 1$ vector meson. Wavy lines indicate the dual field corresponding to a virtual photon exchanged from the lepton (not drawn in this diagram) and the hadron. This field (wavy line) is a fluctuation induced by the insertion of the global symmetry current operator at the boundary theory. This Witten's diagram gives the leading contribution to the $1/N$ expansion. Notice that $r \to \infty$ corresponds to the boundary where the four-dimensional gauge theory is defined and where current operators $J$ are inserted.

After five-dimensional reduction the interaction Lagrangian (3.36)

$$\mathcal{L}^{SF}_{\text{interaction}} = i\, \mathcal{Q}\, \mu_p\, (\pi\alpha')^2 \sqrt{-\det g}\, A_m \left[B^*_n F^{nm} - B_n (F^{nm})^*\right],$$

can be written as

$$S^{SF}_{\text{interaction}} = i\mathcal{Q}\, \mu_p\, (\pi\alpha')^2 R^{p-4} \int d^5x \sqrt{-\det g^5} \left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} A^m \left[b^*_n f^{mn} - b_n f^{mn*}\right]. \tag{5.21}$$

By considering a metric fluctuation, it introduces the interaction terms

$$h^{mq}\, F^n_q\, F^*_{mn}, \qquad h^{mq}\, A_m\, (B^*_n\, F^n_q - B^*_n\, F^{n*}_q)\,, \tag{5.22}$$

which, after dimensional reduction, become

$$h^{mq}\, f^n_q f^*_{mn}; \qquad h^{mq}\, A_m\, (b^*_n f^n_q - b^*_n\, f^{n*}_q)\,, \tag{5.23}$$

If we gather all these terms we obtain the full action

$$\begin{aligned} S^{SF}_{\text{total}} &= S^{SF}_0 + S^{SF}_{\text{interaction}} \\ &= -4\mu_p (\pi\alpha')^2 R^{p-4} \int d^5x \sqrt{-\det g^5} \left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} \left[\frac{1}{4} f^{mn} f^*_{mn} + \frac{1}{2} M^2_\ell b^m b^*_m \right. \\ &\quad \left. + \frac{1}{4} h^{mq} f^n_q f^*_{mn} - \frac{i}{4} \mathcal{Q} A^m \left(b^*_n f^{mn} - b_n f^{mn*}\right) - \frac{i}{4} \mathcal{Q} h^{mq} A_m \left(b^*_n f^n_q - b^*_n f^{n*}_q\right)\right]. \end{aligned} \tag{5.24}$$

We can redefine the fields in order to make the kinetic terms canonically normalized in terms of powers of $N$, thus

$$\begin{aligned} \tilde{b}^m &\equiv \sqrt{N}\, b^m\,, \qquad (\tilde{f}^{mn} \equiv \sqrt{N}\, f^{mn}\,) \\ \tilde{A}^m &\equiv N\, A^m\,, \\ \tilde{h}^{mn} &\equiv N\, h^{mn}\,. \end{aligned} \tag{5.25}$$

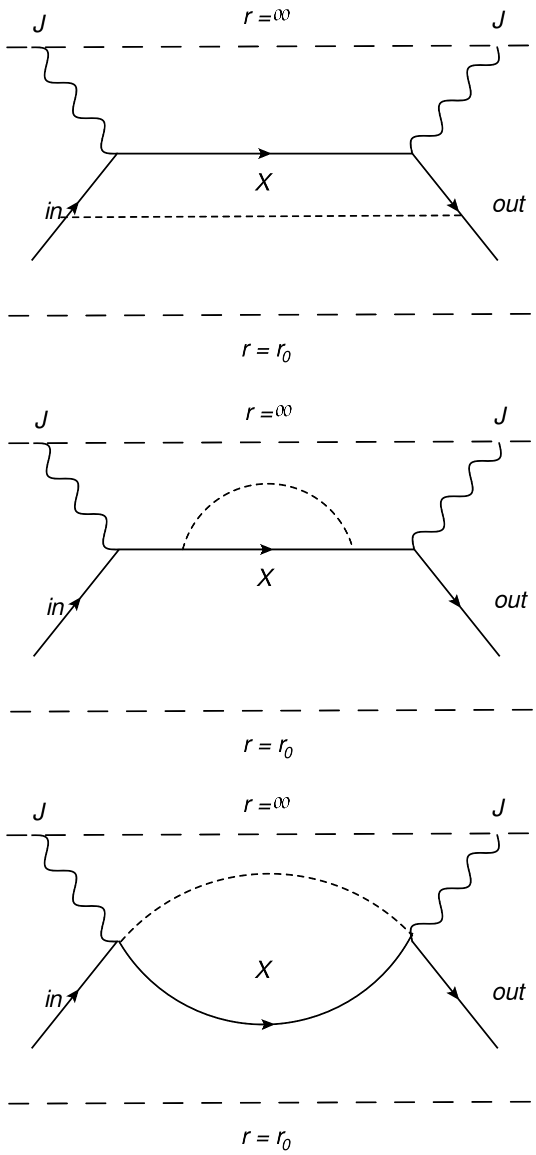


**Figure 2**. Illustration of some of one-loop Witten's diagrams. Five-dimensional one-loop ladder graviton (top), rainbow graviton (middle), fish graviton (bottom) Witten's diagrams corresponding to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions. These diagrams are made of five-dimensional fields obtained from dimensional reduction on a five-dimensional Einstein manifold. The dashed line indicates a graviton $h_{mn}$. These Witten's diagrams contribute to order $N^{-2}$ relative to the leading-order contribution in the $1/N$ expansion displayed in figure 1.

By using eq. (5.17) we can write $S^{SF}_{total}$ in terms of the powers of $N$ as

$$S^{SF}_{total} = \frac{R^{p-4}}{2^{p-2}\pi^{p-2}\lambda\alpha'^{\frac{p-3}{2}}} \int d^5x\sqrt{-\det g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})}$$
$$\left[\frac{1}{4}\widetilde{f}^{mn}\widetilde{f}^{*}_{mn} + \frac{1}{2}M_\ell^2\widetilde{b}^m\widetilde{b}^*_m + N^{-1}\frac{1}{4}\widetilde{h}^{mq}\widetilde{f}^n_q\widetilde{f}^*_{mn}\right.$$
$$\left. - N^{-1}\frac{i}{4}\mathcal{Q}\widetilde{A}^m\left(\widetilde{b}^*_n\widetilde{f}^{mn} - \widetilde{b}_n f^{mn*}\right) - N^{-2}\frac{i}{4}\mathcal{Q}\widetilde{h}^{mq}\widetilde{A}_m\left(\widetilde{b}^*_n\widetilde{f}^n_q - \widetilde{b}^*_n\widetilde{f}^{n*}_q\right)\right]. \tag{5.26}$$

The relevant diagrams to our process are very similar to the ones in the case of scalar mesons. They are those in figures 1 and 2, just noting that the meson line now corresponds to the vector meson $b^m$ instead of the scalar meson $\varphi$ in last subsection. We can see again that only the tree-level diagram in figure 1 contributes to leading order in $N$, namely, $N^{-2}$, while diagrams in figure 2 are sub-leading, i.e. order $N^{-4}$.

### 5.4 Vector mesons with $N_f > 1$

We begin with the relevant part of the Lagrangian for vector mesons with $N_f > 1$ from eq. (4.2)

$$\mathcal{L}_0^{MF} = -\mu_p(\pi\alpha')^2\sqrt{-\det g}\,Tr\Big\{\widehat{F}_{ab}\widehat{F}^{ab\,*} + \Big(i\widehat{F}^*_{ab}[B^a,B^b] + c.c.\Big) - [B^*_a,B^*_b][B^a,B^b]\Big\},$$

with $\widehat{F}_{ab} = \partial_a B_b - \partial_b B_a = F_{ab} - i[B_a,B_b]$, and we define

$$\begin{aligned} B^a(\rho,y^\mu,\Omega) &= \sum_\ell b^{a\,\ell}(\rho,y^\mu)Y^\ell(\Omega), \\ \widehat{f}^\ell_{mn} &= \partial_m b^\ell_n - \partial_n b^\ell_m = f^\ell_{ab} - i[b^\ell_a, b^\ell_b]. \end{aligned} \tag{5.27}$$

Then, we can write the reduced 5-dimensional free action as[16]

$$\begin{aligned} S_0^{MF} = -2\mu_p(\pi\alpha')^2R^{p-4}\int d^5x\sqrt{-\det g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} Tr\Big\{\frac{1}{2}\widehat{f}^{mn\,\ell}\widehat{f}^{\ell\,*}_{mn} + M^2_\ell b^{m\,\ell}b^{\ell\,*}_m \\ + \sum_{\ell'\,\ell''}\Big[\frac{i}{2}a_{\ell\,\ell'\,\ell''}[b^{\ell'}_m, b^{\ell''}_n]\widehat{f}^{mn\,\ell*} + c.c.\Big] - \sum_{\ell'\,\ell''\,\ell'''}\frac{c_{\ell\,\ell'\,\ell''\,\ell'''}}{2}[b^\ell_m, b^{\ell'}_n][b^{m\,\ell''*}, b^{n\,\ell'''*}]\Big\}, \end{aligned} \tag{5.28}$$

where we have used eqs. (5.9) and (5.10), and defined

$$a_{\ell\,\ell'\,\ell''} \equiv \int d^{p-4}\Omega\,\sqrt{\tilde{g}}\,Y^{\ell*}\,Y^{\ell'}\,Y^{\ell''}, \tag{5.29}$$

$$c_{\ell\,\ell'\,\ell''\,\ell'''} \equiv \int d^{p-4}\Omega\,\sqrt{\tilde{g}}\,Y^{\ell}\,Y^{\ell'}\,Y^{\ell''*}\,Y^{\ell'''*}. \tag{5.30}$$

After five-dimensional reduction the interaction Lagrangian (4.8)

$$\begin{aligned} \mathcal{L}^{MF}_{\text{interaction}} &= i\mathcal{Q}\,\mu_p\,(\pi\alpha')^2\sqrt{-\det g}\,\Big\{A_m\,Tr\big(B^*_n\,\widehat{F}^{nm} - B_n\,(\widehat{F}^{nm})^*\big) \\ &\quad + i\,A_m\,Tr\big(B^*_n\,[B^m\,,\,B^n]\big) + i\,A_m\,Tr\big(B_n\,[B^{m*}\,,\,B^{n*}]\big)\Big\}, \qquad (5.31) \\ &\equiv \mathcal{L}^{MF}_{\text{interation}_1} + \mathcal{L}^{MF}_{\text{interation}_2} + \mathcal{L}^{MF}_{\text{interation}_3}\,, \end{aligned}$$

can be written as

$$\begin{aligned} S^{MF}_{\text{interaction}} = \mathcal{Q}\mu_p(\pi\alpha')^2\int d^5x\sqrt{-\det g^5}\left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} \\ \Big\{iA^m Tr\Big(b^{\ell*}_n\widehat{f}^{mn\,\ell} - b^\ell_n\widehat{f}^{mn\,\ell*}\Big) - \sum_{\ell'\,\ell''}\Big(a_{\ell\,\ell'\,\ell''}A_m Tr\Big(b^{\ell*}_n[b^{m\ell'}, b^{n\ell''}]\Big) + c.c.\Big)\Big\}. \end{aligned} \tag{5.32}$$

[16] We write the piece of the action corresponding to a single $b^\ell$.



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Graviton-like perturbations are relevant, thus it introduces the interaction terms

$$h^{mq} Tr(\widehat{F}^n_q \widehat{F}^*_{mn})\,; \qquad h^{mq} Tr[A_m(B^*_n \widehat{F}^n_q - B^*_n \widehat{F}^{n*}_q)], \tag{5.33}$$

which, after dimensional reduction, become

$$h^{mq} Tr(\widehat{f}^n_q \widehat{f}^*_{mn})\,; \qquad h^{mq} Tr[A_m(b^*_n \widehat{f}^n_q - b^*_n \widehat{f}^{n*}_q)]. \tag{5.34}$$

By assembling all factors, we obtain the full action

$$\begin{aligned}
S^{MF}_{\text{total}} &= S^{MF}_0 + S^{MF}_{\text{interaction}} \\
&= -2\mu_p(\pi\alpha')^2 R^{p-4} \int d^5x \sqrt{-\det g^5} \left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} \\
&\quad Tr\Bigg\{\frac{1}{2}\widehat{f}^{mn\,\ell}\widehat{f}^{\ell\,*}_{mn} + M^2_\ell b^{m\,\ell} b^{\ell\,*}_m + \sum_{\ell'\,\ell''}\left(\frac{i}{2}a_{\ell\,\ell'\,\ell''}[b^{\ell'}_m, b^{\ell''}_n]\widehat{f}^{mn\,\ell*} + c.c.\right) \\
&\quad - \sum_{\ell'\,\ell''\,\ell'''} \frac{c_{\ell\,\ell'\,\ell''\,\ell'''}}{2}[b^\ell_m, b^{\ell'}_n][b^{m\,\ell''*}, b^{n\,\ell'''*}]\Bigg\} + \frac{1}{2}h^{mq} Tr(\widehat{f}^{n\,\ell}_q \widehat{f}^{\ell\,*}_{mn}) \\
&\quad -\frac{i}{2}\mathcal{Q}A^m Tr\left(b^*_{n\,\ell}\widehat{f}^{mn\,\ell} - b^\ell_n \widehat{f}^{mn\,\ell*}\right) - \frac{i}{2}\mathcal{Q}h^{mq}A_m Tr\left(b^{\ell\,*}_n \widehat{f}^{n\,\ell}_q - b^{\ell\,*}_n \widehat{f}^{n\,\ell\,*}_q\right) \\
&\quad +\left[\frac{i}{2}\sum_{\ell'\,\ell''} a_{\ell\,\ell'\,\ell''} A_m Tr\left(b^*_{n\,\ell}[b^{m\,\ell'}, b^{n\,\ell''}]\right) + c.c.\right]\Bigg\}.
\end{aligned} \tag{5.35}$$

As before, we redefine the fields in order to be canonically normalized:

$$\begin{aligned}
\widetilde{b}^m &\equiv \sqrt{N}\, b^m\,, \quad (\widetilde{\widehat{f}}^{mn} \equiv \sqrt{N}\,\widehat{f}^{mn}\,) \\
\widetilde{A}^m &\equiv N A^m, \\
\widetilde{h}^{mn} &\equiv N h^{mn}\,.
\end{aligned} \tag{5.36}$$

By using eq. (5.17), we can write $S^{MF}_{\text{total}}$ in terms of the powers $1/N$ as

$$\begin{aligned}
S^{MF}_{\text{total}} &= \frac{R^{p-4}}{2^{p-1}\pi^{p-2}\lambda\alpha'^{\frac{p-3}{2}}} \int d^5x \sqrt{-\det g^5} \left(\frac{\rho}{R}\right)^{(\alpha+\beta)+(p-4)(1+\frac{\beta}{2})} \\
&\quad Tr\Bigg\{\frac{1}{2}\widetilde{\widehat{f}}^{mn\,\ell}\widetilde{\widehat{f}}^{\ell\,*}_{mn} + M^2_\ell \widetilde{b}^{m\,\ell}\widetilde{b}^{\ell\,*}_m + N^{-\frac{1}{2}}\sum_{\ell'\,\ell''}\left(\frac{i}{2}a_{\ell\,\ell'\,\ell''}[\widetilde{b}^{\ell'}_m, \widetilde{b}^{\ell''}_n]\widetilde{\widehat{f}}^{mn\,\ell*} + c.c.\right) \\
&\quad -N^{-1}\sum_{\ell'\,\ell''\,\ell'''} \frac{c_{\ell\,\ell'\,\ell''\,\ell'''}}{2}[\widetilde{b}^\ell_m, \widetilde{b}^{\ell'}_n][\widetilde{b}^{m\,\ell''*}, \widetilde{b}^{n\,\ell'''*}]\Bigg\} + N^{-1}\frac{1}{2}\widetilde{h}^{mq} Tr(\widetilde{\widehat{f}}^{n\,\ell}_q \widetilde{\widehat{f}}^{\ell\,*}_{mn}) \\
&\quad -N^{-1}\frac{i}{2}\mathcal{Q}A^m Tr\left(\widetilde{b}^*_{n\,\ell}\widetilde{\widehat{f}}^{mn\,\ell} - \widetilde{b}^\ell_n \widetilde{\widehat{f}}^{mn\,\ell*}\right) - N^{-2}\frac{i}{2}\mathcal{Q}\widetilde{h}^{mq}A_m Tr\left(\widetilde{b}^{\ell\,*}_n \widetilde{\widehat{f}}^{n\,\ell}_q - \widetilde{b}^{\ell\,*}_n \widetilde{\widehat{f}}^{n\,\ell\,*}_q\right) \\
&\quad +N^{-\frac{3}{2}}\left[\frac{i}{2}\sum_{\ell'\,\ell''} a_{\ell\,\ell'\,\ell''} A_m Tr\left(\widetilde{b}^*_{n\,\ell}[\widetilde{b}^{m\,\ell'}, \widetilde{b}^{n\,\ell''}]\right) + c.c.\right]\Bigg\}.
\end{aligned} \tag{5.37}$$

Since we have more vertices, we can construct more diagrams relevant to our process. These are, in addition to the ones in figures 1 and 2, those in figure 3. These new diagrams

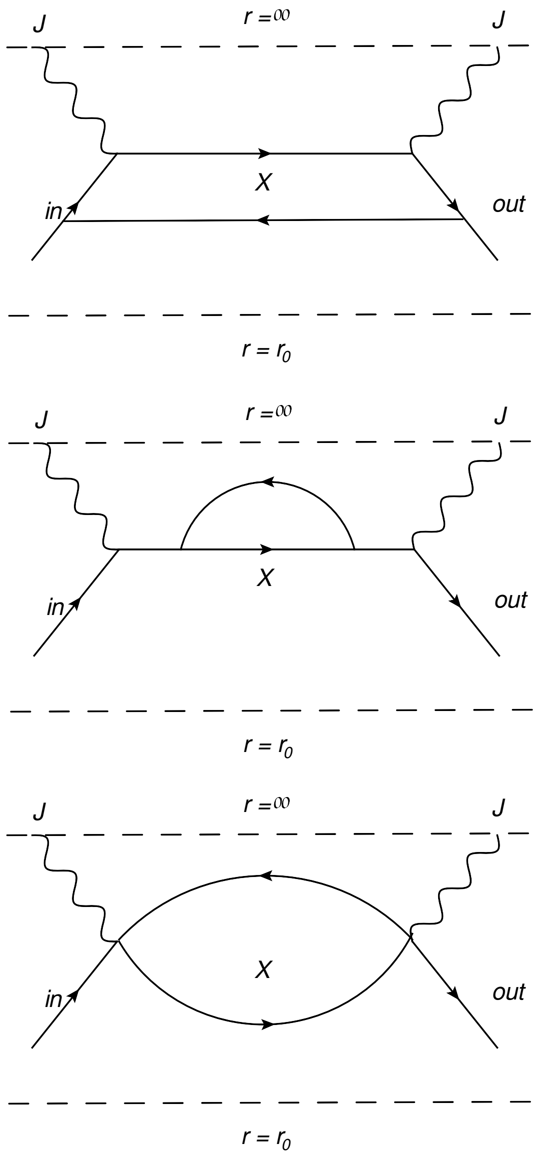


**Figure 3**. Illustration of some one-loop Witten's diagrams. Five-dimensional one-loop ladder meson (top), rainbow meson (middle), fish-like meson (bottom) Witten's diagrams corresponding to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions. These diagrams are made of five-dimensional fields obtained from dimensional reduction on a five-dimensional Einstein manifold. The solid line indicates a vector meson with $N_f > 1$. These Witten's diagrams contribute to order $N_f/N$ relative to the leading-order contribution in the $1/N$ expansion displayed in figure 1.

are sub-leading in $N$ as well but, since they have a meson loop, we have to sum over all the different flavors, obtaining then a factor $N_f$. In addition, notice that these three last diagrams, while sub-leading with respect to the tree-level diagram of figure 1, are dominant with respect to those of figures 2, 3 and 4. The suppression of the diagrams of figure 2 with respect of that of figure 1 is of order $1/N^2$.

### 5.5 Higher-order contributions to the supergravity calculation

In the previous subsections we have discussed the next-to-leading order terms corresponding to the $1/N$ expansion from the scalar and vector mesons. We have obtained those terms after re-scaling the meson fields. The result is that one obtains more vertices in comparison with the leading order Lagrangian. Therefore, we can construct more diagrams which are relevant to the holographic dual description of the forward Compton scattering process. The additional diagrams are of the type presented in figures 2, 3 and 4. In figure 2 we display three types of five-dimensional one-loop Witten's diagrams: a ladder-graviton diagram (top), a rainbow-graviton diagram (middle), and a fish-graviton diagram (bottom). They correspond to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions. Notice that these diagrams are made of five-dimensional fields obtained from dimensional reduction of the type IIA or IIB supergravity (depending on the model we consider) on a five-dimensional Einstein manifold. The dashed line indicates a graviton $h_{mn}$. These Witten's diagrams contribute to order $N^{-2}$ relative to the leading-order contribution in the $1/N$ expansion displayed in figure 1. Notice that it can be additional one-loop and multi-loop diagrams to the ones indicated in the figures, however the present analysis of their contributions to the $1/N$ and $N_f/N$ expansions will be valid.

In figure 3 we display five-dimensional one-loop ladder meson (top), rainbow meson (middle), fish-like meson (bottom) Witten's diagrams corresponding to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions. As in figure 2 these diagrams are made of five-dimensional fields obtained from dimensional reduction of ten-dimensional supergravity on a five-dimensional Einstein manifold. The solid line in the top figure indicates a vector meson with $N_f > 1$. Thus, these diagrams are sub-leading in the $1/N$ expansion, but since they have a meson loop, we have to sum over all different flavors, obtaining a factor $N_f$.

Therefore, the suppression of the diagrams in figure 2 is of order $1/N^2$ while the suppression of diagrams in figure 3 is of the order $N_f/N$.

In addition, we can also consider multi-loop contributions as in figure 4. Five-dimensional two-loop meson (top), $n$-loop ladder graviton (middle), $n$-loop rainbow graviton (bottom) Witten's diagrams corresponding to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions. These diagrams are made of five-dimensional fields obtained from dimensional reduction on a five-dimensional Einstein manifold. The solid line in the top diagram indicates a vector meson with $N_f > 1$. These Witten's diagrams contribute to order $(N_f/N)^2$ (top) and $(1/N^2)^n$ (middle and bottom) relative to the leading-order contribution in the $1/N$ expansion displayed in figure 1. Therefore, we can summarize the contributions as:

- Figure 1: leading contribution.
- Figure 2: sub-leading contributions, of order $N^{-2}$ relative to the one in figure 1.
- Figure 3: sub-leading contributions, of order $N_f/N$ relative to the one in figure 1.



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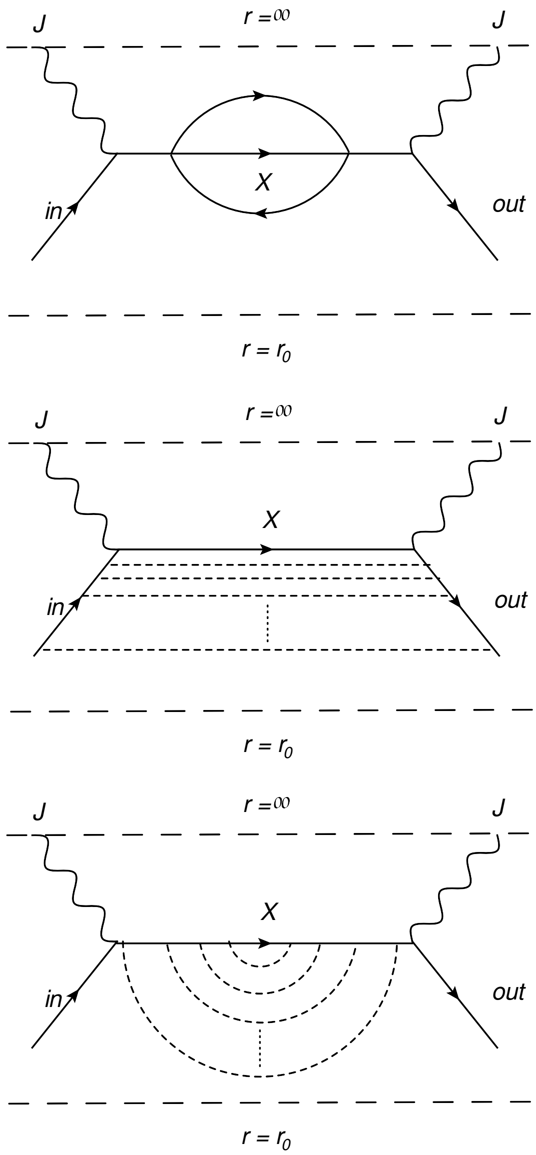


**Figure 4**. Illustration of some multi-loop Witten's diagrams. Multi-loop contributions from Witten's diagrams corresponding to sub-leading corrections to the forward Compton scattering Feynman's diagrams contributing to DIS in four-dimensions.

- Figure 4: sub-leading contributions, of order $(N_f/N)^2$ and $(1/N^2)^n$ relative to the one in figure 1.

We have not explicitly obtained these sub-leading contributions from the diagrams illustrated in figures 2–4. Notice that the UV completion of these diagrams should be done in terms of string theory calculations.

### 5.6 Comments on the quantum field theory OPE

In this section we aim at relating the $1/N$ and $N_f/N$ expansions discussed in the previous subsection from the supergravity point of view with the corresponding expansions from the operator product expansion of two-currents in the four-dimensional dual gauge theories. First notice that the $T_{\mu\nu}$ tensor, whose expectation value enters the definition of the hadronic tensor $W_{\mu\nu}$, is given by the product of two currents

$$\hat{T}_{\mu\nu} \equiv i \int d^4x \, e^{iq\cdot x} \hat{T}(\hat{J}_\mu(x)\, \hat{J}_\nu(0)) \,.$$

For deep inelastic scattering the leading operators in the OPE of two currents are twist two when the gauge theory is weakly coupled. So, to zeroth order in QCD one can write[17]

$$\begin{aligned}
\hat{T}_{\mu\nu} = & \sum_{n=2,4,\cdots}^{\infty} C_n^{(1)} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{2^n q_{\mu_1} \cdots q_{\mu_n}}{(-q^2)^n} \, \hat{O}_V^{\mu_1\cdots\mu_n} \\
& + \sum_{n=2,4,\cdots}^{\infty} C_n^{(2)} \left( g_{\mu\mu_1} - \frac{q_\mu q_{\mu_1}}{q^2} \right) \left( g_{\nu\mu_2} - \frac{q_\nu q_{\mu_2}}{q^2} \right) \frac{2^n q_{\mu_3} \cdots q_{\mu_n}}{(-q^2)^{n-1}} \, \hat{O}_V^{\mu_1\cdots\mu_n} \\
& + \sum_{n=1,3,\cdots}^{\infty} C_n^{(3)} \, i \, \epsilon_{\mu\nu\lambda\mu_1} \, q^\lambda \, \frac{2^n q_{\mu_2} \cdots q_{\mu_n}}{(-q^2)^n} \, \hat{O}_A^{\mu_1\cdots\mu_n} \,,
\end{aligned} \tag{5.38}$$

where $C_n^{(1)} = C_n^{(2)} = C_n^{(3)} = 1 + \mathcal{O}(\alpha_s)$, where $\alpha_s$ is the QCD coupling. The operators are defined as follows

$$\hat{O}_V^{\mu_1\cdots\mu_n} = \frac{1}{2} \left( \frac{i}{2} \right)^{n-1} \hat{S} \Big( \bar{\psi} \, \gamma^\mu \, \hat{D}^{\mu_1} \cdots \hat{D}^{\mu_n} \, \hat{\mathcal{Q}}_{qcm}^2 \, \psi \Big) \,, \tag{5.39}$$

$$\hat{O}_A^{\mu_1\cdots\mu_n} = \frac{1}{2} \left( \frac{i}{2} \right)^{n-1} \hat{S} \Big( \bar{\psi} \, \gamma^\mu \, \hat{D}^{\mu_1} \cdots \hat{D}^{\mu_n} \, \gamma_5 \, \hat{\mathcal{Q}}_{qcm}^2 \, \psi \Big) \,, \tag{5.40}$$

where the derivative operator $\hat{D}^\mu$ acts on left and right, while $\hat{\mathcal{Q}}_{qcm}$ is quark charge matrix, $\hat{S}$ indicates symmetrization and it removes all traces over $\mu_1 \cdots \mu_n$.

Now, in order to calculate the structure functions in this regime one has to calculate the matrix element of $T_{\mu\nu}$ between two hadronic states, in the present case of spin-1, which leads to

$$< P, E | \hat{O}_V^{\mu_1\cdots\mu_n} | P, E > = \hat{S} [ a_n \, P^{\mu_1} \cdots P^{\mu_n} + d_n \left( E^{*\mu_1} E^{\mu_2} - \frac{1}{3} P^{\mu_1} \, P^{\mu_2} \right) P^{\mu_3} \cdots P^{\mu_n} ] \,, \tag{5.41}$$

$$< P, E | \hat{O}_A^{\mu_1\cdots\mu_n} | P, E > = \hat{S} [ r_n \, \epsilon^{\lambda\sigma\tau\mu_1} \, E^*_\lambda \, E_\sigma \, P_\tau \, P^{\mu_2} \cdots P^{\mu_n} ] \,, \tag{5.42}$$

which define the coefficients $a_n$, $d_n$ and $r_n$.

The leading diagram in the parton model is displayed in figure 5 where the virtual photon strikes a parton. This is a tree-level perturbative QFT calculation in the weakly coupled theory. Thus, the operators which appear in the $JJ$ OPE at weak coupling have twist $\tau = 2, 4, \cdots$, even, and therefore twist-two single-trace operators dominates the OPE. Notice that at finite coupling these operators develop anomalous dimensions $\gamma_n$, where

[17] This expression of the OPE follows the notation and metric convention of [8], which has an overall minus sign in the metric.

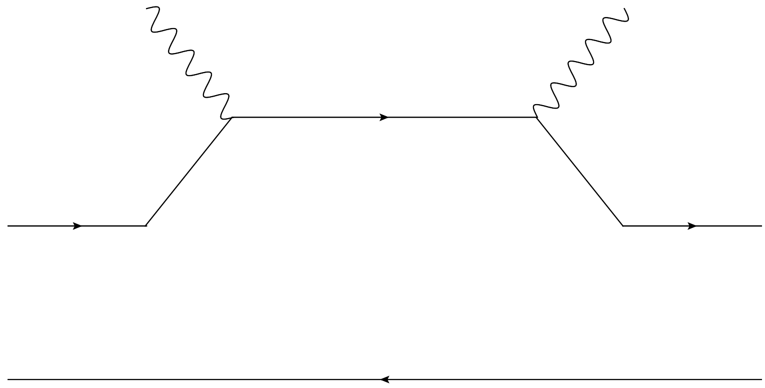

**Figure 5**. Forward Compton scattering from a meson in the parton model. A parton is struck by the virtual photon indicated with a wavy line.

the subindex stands for the quantum numbers of the corresponding operator. In leading perturbation theory $\gamma_n \sim \alpha_s(q^2)\,N$, and in this regime the parton model for spin-1/2 partons leads to the Callan-Gross relation $F_2 = 2xF_1$, where the Bjorken variable $x$ is the fraction of the total momentum $(P^\mu)$ of the hadron carried by the specified parton. The idea is that a parton evolves, which means that it splits into more partons which leads to reduce the momentum carried by each individual parton.

On the other hand, at large coupling the situation changes dramatically because the above operators have large anomalous dimensions and then they no longer dominate the OPE. The point is that on general grounds there are double-trace operators which do not receive large anomalous dimensions for any value of the 't Hooft coupling. It turns out that these operators dominate the OPE at strong coupling. They are protected operators. Basically, the discussion is similar to that presented for the case of a theory with adjoint fields, where leptons are scattered by glueballs [1], but now there are contributions from fields in the fundamental representation of the gauge group, which leads us to replace the factor $N$ by $\sqrt{N}$ wherever it corresponds when considering fundamental fields instead of adjoint ones. Also, there will be a $N_f$ factor coming from summing over flavor loops. The first difference with respect to the weak coupling situation is that the lepton cannot strike individual partons any more, and instead it strikes the hole hadron. This can be represented by a quark-gluon diagram as in figure 6, which represents a multi-gluon exchange in a planar diagram which can be calculated in terms of its dual tree-level Witten's diagram of figure 1, and they are the calculations that we have presented in sections 2 and 3. In principle, one can go beyond the planar limit and include non-planar diagrams for gluon exchange as depicted in figure 7. This is the quark-model diagram which corresponds to sub-leading supergravity dual calculations of the type given by the one-loop Witten's diagram of figure 2. Moreover, it is also possible to consider multi-flavor loops as shown in figure 8.

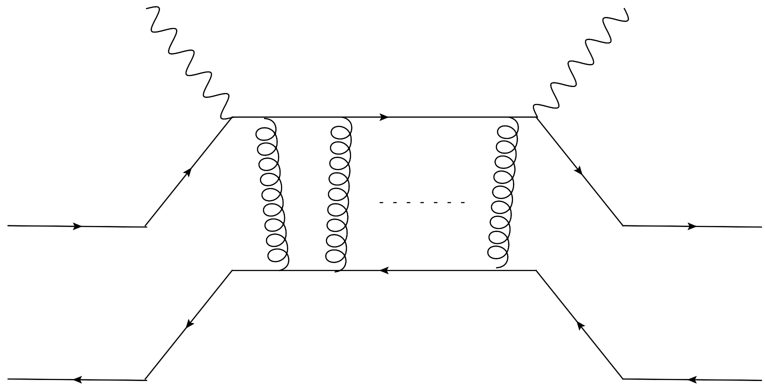

**Figure 6**. Forward Compton scattering at strong coupling. The meson is struck by the virtual photon indicated with a wavy line. This is the quark-model diagram which corresponds to the leading supergravity dual calculation given by the tree-level Witten's diagram of figure 1. Multi-gluon exchange between quark-anti quark pair is shown. This is a planar diagram.

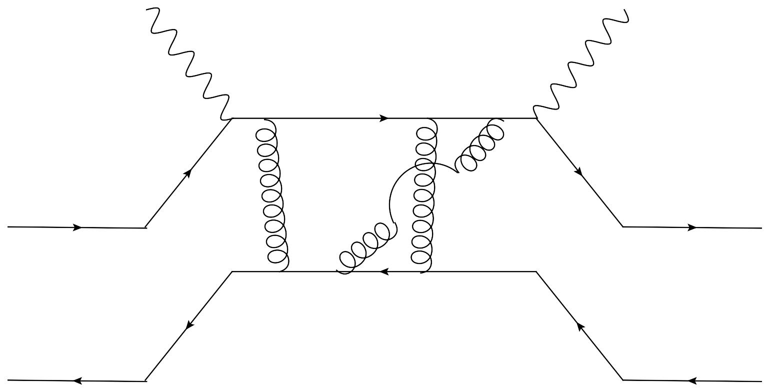

**Figure 7**. Forward Compton scattering at strong coupling. The meson is struck by the virtual photon indicated with a wavy line. Non-planar multi-gluon exchange between quark-anti quark pair is shown.

## 6 Discussion

As we mentioned in the introduction we have performed a detailed analysis of the structure of the two-point correlation functions of generic global symmetry currents at strong coupling, associated with flavors in the fundamental representation of the gauge group, in the quenched approximation, in terms of the corresponding holographic string theory dual description. This includes the large $N$ limit of supersymmetric and non-supersymmetric Yang-Mills theories in four dimensions. In particular, we have explicitly investigated the cases of the D3D7-brane, the D4D8$\overline{\text{D8}}$-brane, and the D4D6$\overline{\text{D6}}$-brane systems.

In the large $N$ limit we have found a universal structure of the two-point correlation functions of generic global symmetry currents at strong coupling. For each holographic dual model we have found that the two-point correlation functions of non-Abelian ($N_f > 1$) global symmetry currents can generically be written as the product of a constant, which depends on the particular Dp-brane model, times flavor preserving Kronecker deltas multi-

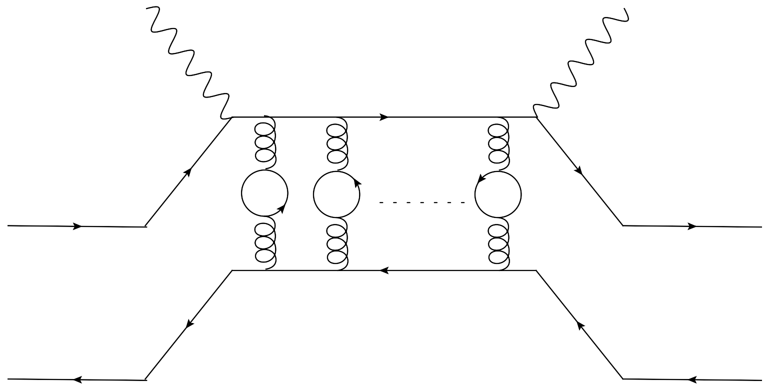

**Figure 8**. Forward Compton scattering at strong coupling. The meson is struck by the virtual photon indicated with a wavy line. This is the quark-model diagram which corresponds to subleading supergravity dual calculations of the type given by the $n$-flavor loop Witten's diagram. These are planar diagrams.

plying the corresponding Abelian ($N_f = 1$) result for the same Dp-brane model. We have obtained a universal factorization of the two-point correlation functions for non-Abelian symmetry currents in a model-dependent factor times a model-independent one. This has already been seen for the two-point functions of Abelian symmetry currents in our previous paper [4]. This factorization comes from the structure of the flavored holographic dual model in the probe approximation, where the probe Dp-brane action is taken to be the non-Abelian version of the Dirac-Born-Infeld action [5]. Thus, in general we can write the hadronic tensor $W^{\mu\nu}_{(a)}$ for a holographic dual model corresponding to a certain gauge field theory in the large-$N$ limit as

$$W^{\mu\nu}_{(a)} = A_{(a,b)}\, W^{\mu\nu}_{(b)}\,, \tag{6.1}$$

for models $(a)$ and $(b)$, where $A_{(a,b)}(x)$ is a conversion factor which depends on the pair of Dp-brane models considered. This allows one to write the corresponding structure functions $F_i^{(a)}(x,t)$, where subindex $i$ indicates the $i$-th structure function for every meson in each particular model, as $F_i^{(a)}(x,t) = A_{(a,b)}\, F_i^{(b)}(x,t)$ as we explained in the Introduction. Besides, we have found that a modified version of the Callan-Gross relation is satisfied for a large class of flavored holographic dual models, $F_2 = 2F_1$ without multiplying by the Bjorken parameter, when the parameter $t \to 0$, which is an indication that the coupling is strong and therefore there are no partons. In fact, we have found a number of additional relations among the structure functions which hold in every Dp-brane model studied in the present context. They are

$$b_2 = 2\, b_1\,, \tag{6.2}$$

$$b_1 = 3\, F_1\,, \tag{6.3}$$

$$g_2 = \frac{9}{4x}\, F_1 \tag{6.4}$$

$$b_4 = -2\, b_3\,. \tag{6.5}$$

The last three relations are predictions as in our previous work [4]. In addition, we have shown that all the moments of the structure functions satisfy the corresponding inequalities derived from unitarity, as expected [7].

These results, which hold for a number Dp-brane models, seem to suggest that there is a universal structure of the two-point current correlation functions, and therefore for the hadronic tensor. Moreover, this might be an indication that the structure of actual QCD polarized vector mesons at strong coupling should have the above relations among their structure functions. QCD lattice calculations could confirm these predictions, it would be very interesting to know it. In the affirmative case, it would imply that any candidate for a holographic QCD model in the large $N$ limit should lead to two-point current correlation functions with the properties indicated above.

On the other hand, it would be very interesting to know how the above relations become modified at strong coupling for the kinematical region where the Bjorken parameter is very small. Additionally, it would also be extremely interesting to investigate the fate of these new structure function relations at weak coupling.

A very interesting aspect of the present work is that we have investigated the $1/N$ and $N_f/N$ contributions to the leading order calculations of the hadronic tensor, from the supergravity dual model point of view. Particularly, we have focused on the structure of the relevant Lagrangians and Witten's diagrams. Indeed, we have derived all relevant Lagrangians. On the other hand, although we have not calculated these Witten's diagrams explicitly, we have discussed how they arise from supergravity, how their $1/N$ and $N_f/N$ powers match those in the corresponding expansions in quantum field theory, and how these Witten's diagrams are suppressed by $1/N^2$ and $N_f/N$ powers, respectively, in the supergravity dual models.

Other papers where holographic description of DIS has been investigated include [16–19]. However, we follow a different approach to construct the interactions derived formally from the DBI action of the probe branes in a way that explicitly manifests the global symmetry. Also, other regimes of the Bjorken parameter have been considered through their holographic dual description as for instance in references [20–23]. In addition, DIS and current correlators in SYM plasmas have also been investigated [24, 25]. Particularly, $\alpha'^3$ type IIB string theory corrections to current correlators in SYM plasmas have been investigated in [26–30].

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## A Hadronic tensor of spin-one mesons

In the definition of the hadronic tensor of spin-1 mesons in section 2 we have used the following functions

$$r_{\mu\nu} \equiv \frac{1}{(P\cdot q)^2}\left(q\cdot\zeta^*\, q\cdot\zeta - \frac{1}{3}(P\cdot q)^2\kappa\right)\eta_{\mu\nu}\,, \tag{A.1}$$

$$s_{\mu\nu} \equiv \frac{2}{(P\cdot q)^3}\left(q\cdot\zeta^*\, q\cdot\zeta - \frac{1}{3}(P\cdot q)^2\kappa\right)P_\mu P_\nu\,, \tag{A.2}$$

$$t_{\mu\nu} \equiv \frac{1}{2(P\cdot q)^2}\left(q\cdot\zeta^*\, P_\mu\zeta_\nu + q\cdot\zeta^*\, P_\nu\zeta_\mu + q\cdot\zeta\, P_\mu\zeta^*_\nu + q\cdot\zeta\, P_\nu\zeta^*_\mu - \frac{4}{3}(P\cdot q)P_\mu P_\nu\right), \tag{A.3}$$

$$u_{\mu\nu} \equiv \frac{1}{P\cdot q}\left(\zeta^*_\mu\zeta_\nu + \zeta^*_\nu\zeta_\mu + \frac{2}{3}M^2\eta_{\mu\nu} - \frac{2}{3}P_\mu P_\nu\right), \tag{A.4}$$

$$s^\sigma \equiv \frac{-i}{M^2}\epsilon^{\sigma\alpha\beta\rho}\zeta^*_\alpha\zeta_\beta P_\rho\,, \tag{A.5}$$

being $\kappa = 1 - 4x^2 t$ and $s^\sigma$ a four-vector analogous to the spin four-vector in the case of spin-$\frac{1}{2}$ particles. Besides, $\zeta_\mu$ and $\zeta^*_\mu$ denote the initial and final hadronic polarization vectors, respectively. The condition $P\cdot\zeta = 0$ is satisfied, and the normalization is given by $\zeta^2 = -M^2$.

## B Meson structure functions from the D4D6$\overline{\text{D6}}$-brane model

In this appendix we extend the calculations developed in [4] to the model consisting of $N$ D4 and $N_f$ D6 branes described in [6]. All the results obtained in this section can be derived as particular cases of the calculations in section 3.[18] This model is similar to that of [11], whose DIS calculations where done in our previous paper [4]. Therefore, the results will be similar.

The model in reference [6] consists of branes in the following configuration:

$$\begin{aligned} N\ D4 &: 0\ 1\ 2\ 3\ 4\ -\ -\ -\ -\ - \\ N_f\ D6 &: 0\ 1\ 2\ 3 -\ 5\ \ 6\ \ 7\ - -. \end{aligned} \tag{B.1}$$

Note that the D4 and the D6 branes may be separated from each other along the directions $x_8$ and $x_9$. In the decoupling limit for the D4-branes this system provides a non-conformal version of the AdS/CFT correspondence. This means that on the gauge theory side there is a supersymmetric five-dimensional SU($N$) gauge theory coupled to a four-dimensional defect. The system is dual to $\mathcal{N} = 2$ supersymmetric Yang-Mills theory in $d = 4$. The degrees of freedom localized on the defect are $N_f$ hypermultiplets in the fundamental representation of SU($N$), which arise from the open strings connecting the D4 and the D6-branes. Each hypermultiplet consists on two Weyl fermions of opposite chiralities, $\psi_L$ and $\psi_R$, and two complex scalars.

By identifying the direction 4 as $x_4 \sim x_4 + \frac{2\pi}{M_{KK}}$, where $M_{KK}$ is the mass scale for the Kaluza-Klein modes, and by imposing anti-periodic conditions for the D4-brane fermions,

[18] We have studied the backgrounds D3D7-brane and D4D8$\overline{\text{D8}}$-brane systems in [4] the case $N_f = 1$.

all of the supersymmetries are broken and the theory becomes a four-dimensional one for energies $E \ll M_{KK}$, while the adjoint fermions and scalars become massive. Generation of mass for the fundamental fermions is forbidden by a chiral $\mathrm{U}(1)_A$ symmetry that rotates $\psi_L$ and $\psi_R$ with opposite phases.

In the limit $N_f \ll N$, the back-reaction of the D6-branes on the supergravity background is negligible, therefore, they can be treated as probe branes. In the string description, the $\mathrm{U}(1)_A$ symmetry corresponds to the rotation symmetry in the 89−plane.

We adopt, as in [6], the solution in which there are $N_f$ D6-branes and $N_f$ anti-D6-branes.

**Background of D4-branes.** The background metric of $N$ D4-branes in this configuration is

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (\eta_{\mu\nu} dy^\mu dy^\nu + f(U) d\tau^2) + \frac{(R^3 U)^{\frac{1}{2}}}{\rho(U)^2} \overrightarrow{dz} \cdot \overrightarrow{dz}, \tag{B.2}$$

with $\mathrm{U}(\rho) = \left(\rho^{3/2} + \frac{U_{KK}^3}{4\rho^{3/2}}\right)$, $f(U) = 1 - \frac{U_{KK}^3}{U^3}$, and $\vec{z} = (z^5, \ldots, z^9)$.

The dynamics of interest for DIS corresponds to the limit $q \gg \Lambda$, where $\Lambda$ is the confinement energy scale of the gauge theory. Thus, we shall consider the interaction in the UV limit, being the interaction region given by $U_{\mathrm{int}} \sim q^2 R^3 \gg U_0 = \Lambda^2 R^3 \equiv U_{KK}$. In this limit, the induced metric on the D6-branes takes the form

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \eta_{\mu\nu} dy^\mu dy^\nu + \left(\frac{R}{U}\right)^{\frac{3}{2}} dU^2 + R^{\frac{3}{2}} U^{\frac{1}{2}} d\Omega_2^2\,, \tag{B.3}$$

which is the same as eq. (96) in [4], coming from the same limit ($U \gg U_{KK}$) taken in the context of the D4D8$\overline{\text{D8}}$-brane system model [11]. The difference is that the coordinates $z^8$ and $z^9$ do not belong to the probe brane in this case. We can see that this metric is a particular case of (3.2) with $p = 6$ ; $\alpha = -\beta = \frac{3}{2}$. Therefore, all the analysis done in section 3 applies. We then write the main results, avoiding further details.

**The gauge field.** By proposing the *Ansatz* (3.19) we obtain the solution (3.20) which reads ($\alpha = -\beta = \frac{3}{2}$ ; $p = 6$)

$$A_\mu = \frac{2}{\Gamma(5/4)} n_\mu\, e^{iq\cdot y} \left(\frac{q^2 R^3}{U}\right)^{5/8} K_{5/4}\left(\left[\frac{4q^2R^3}{U}\right]^{\frac{1}{2}}\right), \tag{B.4}$$

$$A_U = -\frac{2i(q\cdot n)}{\Gamma(5/4) q} \frac{1}{(qR)^3}\, e^{iq\cdot y} \left(\frac{q^2R^3}{U}\right)^{17/8} K_{1/4}\left(\left[\frac{4q^2R^3}{U}\right]^{\frac{1}{2}}\right) = -\frac{i}{q^2}\eta^{\mu\nu} q_\mu \partial_U A_\nu\,, \tag{B.5}$$

where $K_{5/4}$ and $K_{1/4}$ are modified Bessel functions, and $q \equiv \sqrt{q^2}$.

**DIS from scalar mesons.** The EOM for scalar mesons arises from the transversal fluctuation

$$z^8 = 0 + 2\pi\alpha'\chi\,, \qquad z^9 = 0 + 2\pi\alpha'\varphi\,, \tag{B.6}$$

where the coordinates $z^8$ and $z^9$ lie on the (8, 9) plane, transversal to the D6-brane. Note that we are perturbing around the solution $z^8 = z^9 = 0$, i.e. , $r = 0$ which corresponds to the D4D6$\overline{D6}$-brane system solution. The scalar fluctuations around this background are $\chi$ and $\varphi$.

The Lagrangian for the scalar fluctuations at leading order is

$$\mathcal{L} = -\mu_6\sqrt{|\det g|}\left[1 + \left(\frac{R}{U}\right)^{3/2} g^{ab}(\pi\alpha')^2\partial_a\Phi\partial_b\Phi^*\right], \tag{B.7}$$

where $\Phi \equiv \chi + i\varphi$. The solutions are, after imposing an *Ansatz* like eq. (3.10),

$$\Phi_{\mathrm{IN/OUT}} = c_i(\Lambda U)^{-\frac{7}{8}-\frac{\gamma}{2}} e^{iP\cdot y} Y(S^2), \tag{B.8}$$

$$\Phi_X = c_X(s^{\frac{1}{4}}\Lambda^{-\frac{1}{2}})(\Lambda U)^{-\frac{7}{8}} J_\gamma\left(\left[\frac{4sR^3}{U}\right]^{\frac{1}{2}}\right) e^{iP_X\cdot y} Y(S^2), \tag{B.9}$$

with $\gamma = (1/4)\sqrt{49 + 64\ell(\ell+1)}$. After perturbing the metric as in (1.8) we obtain

$$\mathcal{L}^{\mathrm{scalar\ D4D6}}_{\mathrm{interaction}} = i\mathcal{Q}\mu_6(\pi\alpha')^2\sqrt{|\det g|}\left(\frac{R}{U}\right)^{\frac{3}{2}} A_m(\Phi\partial^m\Phi^*_X - \Phi^*_X\partial^m\Phi) = \mathcal{Q}\sqrt{|\det g|}A_m j^m, \tag{B.10}$$

with

$$j^m = i\mu_6(\pi\alpha')^2\left(\frac{R}{U}\right)^{\frac{3}{2}}(\Phi\partial^m\Phi^*_X - \Phi^*_X\partial^m\Phi). \tag{B.11}$$

By using the current conservation of eq. (3.22) and the *Ansatz* (3.18), we obtain the structure functions

$$F_1 = 0, \qquad F_2 = A_{0\ D4D6}^{scalar}\mathcal{Q}^2\left(\frac{\mu_6^2\alpha'^4}{\Lambda^6}\right)\left(\frac{\Lambda^2}{q^2}\right)^{\gamma+1} x^{\gamma+7/2}(1-x)^\gamma, \tag{B.12}$$

where $A_{0\ D4D6}^{\mathrm{scalar}} = 4\pi^5|c_i|^2|c_X|^2[\Gamma(9/4+\gamma)]^2[\Gamma(5/4)]^{-2}$ is a normalization dimensionless constant. We can see that this solution is a particular case of eq. (3.29).

**DIS from vector mesons.** From the DBI action we derive the EOM (3.31), then we propose a quadratic Lagrangian from which we obtain exactly the same EOM. The *Ansatz* for the solution is (3.32) and the solutions (3.33) and (3.34) become

$$B_{\mu\,\mathrm{IN/OUT}} = \zeta_\mu c_i\Lambda^{-1}(\Lambda U)^{-\gamma/2-7/8} e^{iP\cdot y}\, Y^\ell(S^4), \tag{B.13}$$

$$B_{X\mu} = \zeta_{X\mu} c_X\Lambda^{-1}(s^{-1/4}\Lambda^{-1/2})\left(\frac{U}{\Lambda^2R^3}\right)^{-7/8} J_\gamma\left[\left(\frac{4sR^3}{U}\right)^{\frac{1}{2}}\right] e^{iP_X\cdot y}\, Y^\ell(S^2), \tag{B.14}$$

with $\gamma = (1/4)\sqrt{49 + 64\ell(\ell+3)}$. The quadratic Lagrangian is exactly that in eq. (3.35), the interaction Lagrangian that in (3.36), and by repeating the calculations as in section 3, we finally obtain the solutions (3.47) with

$$A^{\mathrm{vect}}_{D4D6}(x) = A^{\mathrm{vect}}_{0\ D4D6}\mathcal{Q}^2\left(\frac{\mu_6^2(\alpha')^4}{\Lambda^6}\right)\left(\frac{\Lambda^2}{q^2}\right)^\gamma x^{\gamma+11/2}(1-x)^{\gamma-1}, \tag{B.15}$$

$$A^{\mathrm{vect}}_{0\ D4D6} = \pi^5|c_i|^2|c_X|^2[\Gamma(\gamma+9/4)]^2[\Gamma(5/4)]^{-2}. \tag{B.16}$$

We can observe that they have the same form as the ones obtained with the D3D7 and D4D8$\overline{\mathrm{D8}}$-brane model studied in our previous work [4], and summarized in section 3.



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